

Braids in Crisis: A Topological Exploration of Market Dynamics

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Abstract

This project applies braid theory to analyze stock market behavior during major financial crises. By converting daily stock price rankings into braids, the study captures the underlying patterns of market volatility as stocks cross over and under one another. Building on Racorean's crossing stocks method, we refine this approach by quantitatively distinguishing overcrossings from undercrossings based on the magnitude of price changes. We apply the method to historical data from Black Monday (1987), the 2008 Global Financial Crisis, and the COVID-19 Crash (2020), as well as to recent market trends (2024–2025). Furthermore, by closing the braids into knots and computing the Jones polynomial, this work quantifies knot complexity as a potential indicator of market stress. The findings indicate that periods of high market volatility correspond to more complex braid structures. This topological approach offers a complementary perspective to traditional volatility measures such as the VIX, providing additional insight into the interconnectedness of market dynamics.

1 Introduction

Financial markets are complex and often show unexpected fluctuations in prices, which we call volatility, during major economic events; however, we will strictly be exploring financial crises, such as Black Monday (1987), the global financial crisis (2008), and the COVID-19 crash (2020). Many traditional methods of quantifying this volatility, like the CBOE Volatility Index (VIX), are based on quantifying the magnitude of volatility - the size of this fluctuation in price. VIX specifically focuses on implied volatility, reflecting expectations of future market volatility as opposed to the realized volatility currently being felt. Therefore, it can be affected by sentiment, speculation, and other factors. This study adopts O. Racorean's *crossing stocks method* to present a more classical topological approach, compared to O. Racorean's more quantum approach, which uses braid and knot theory to give a more complete picture of market dynamics—creating a braid of our stock prices and their "overcrossings" and "undercrossings"—ultimately, recording the intricate patterns/complexities of volatility as opposed to solely the magnitude. In analyzing these braids, we are offered a way to quantify the *structural* complexity and interconnectedness of price movements/volatility; that, if used in conjunction with classic metrics, provides us with a more complete and richer picture of the market than traditional metrics.

This study is organized as follows: firstly, we review O. Racorean's findings and explain how we convert daily stock price changes into braids and later knots/links. Before explaining our methodology, we describe how we implement O. Racorean's crossing stocks method and detail the modifications made to improve the method's potential application and results. Ultimately, we present our findings and cross-reference them with standard volatility metrics (i.e., the VIX index) to validate our data before further analyzing and interpreting the topological information. Before discussing the potential of our topological approach, we examine its strengths, weaknesses, and possible directions for future investigation.

2 Background: Racorean's Topological Approach

Ovidiu Sorin Racorean (2014) pioneered the application of topology, specifically braids and knots, to model stock market interactions. He proposed that daily stock price fluctuations can be represented as braids, capturing the intricate interdependencies between these fluctuations. This section introduces and explains the ideas behind Racorean's work and the steps behind his *crossing stocks method*.

2.1 Crossing Diagrams and Permutations

Racorean's method begins with constructing a "crossing diagram," which visualizes when one stock's price overtakes another in a given period. Each day, stock prices are arranged in order. This order is similar to a permutation diagram, where the position of each stock reflects its relative price. A crossing is recorded when the order changes from one day to the next. Consider the following diagram in Figure 1, which illustrates the initial arrangement of stocks and visually graphs them to map how stock prices change over time.

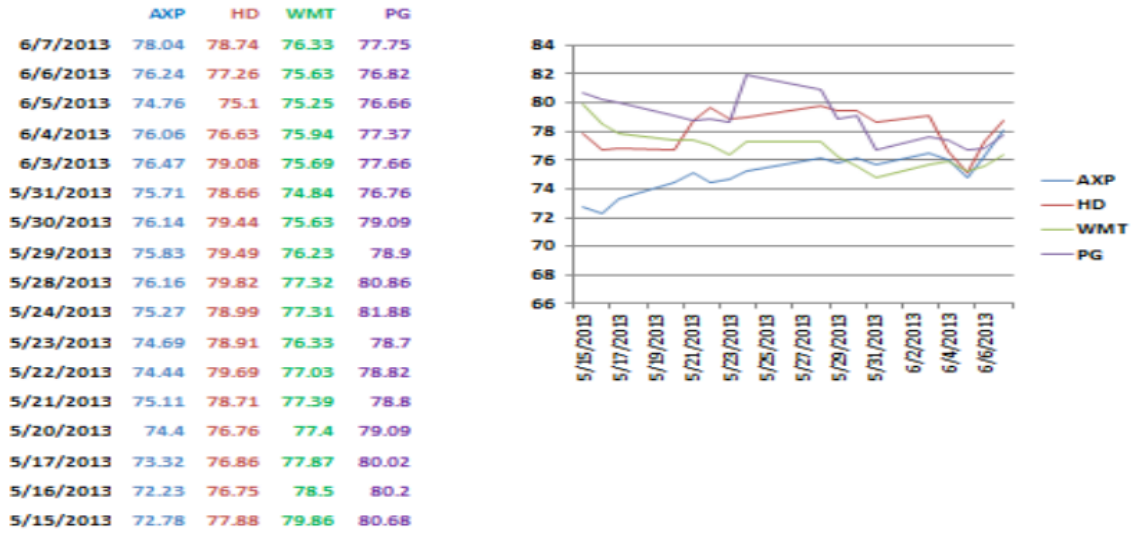


Figure 1: Ovidiu Racorean's representation of all stock prices time series in one chart

It is evident that as stock prices change, they overlap and cross each other as one stock's price rises above that of another, or inversely, when one stock's price falls below that of another. By focusing specifically on these moments of stocks crossing each other, we end up with a more explicit *crossing of stocks diagram* -or crossing diagram, as shown in Figure 2, that will go on to form the basis for translating our time-series stock price data into a topological braid.

2.2 Defining Over- and Undercrossings

While the crossing diagram identifies all the crossings, we have yet to distinguish whether one is an overcrossing or an undercrossing, and therefore have yet to translate our stock price data into a braid fully. Thus, our next step is to distinguish our crossings and classify them as either an overcrossing or an undercrossing. To categorize these crossings, Racorean introduced a convention based on the magnitude of price changes:

- **Overcrossing** (σ_i): If the overtaking stock experiences a larger price increase than the stock it overtakes, the crossing is an overcrossing.
- **Undercrossing** (σ_i^{-1}): If the overtaking stock has a smaller price increase, the crossing is an undercrossing.



Figure 2: Ovidiu Racorean's Crossing of Stocks diagram of the data in Figure 1

This method provides a quantitative way to define crossings, considering not just positional changes but also the relative strength of price movements. Considering the relative strength or magnitude of price changes provides a way to gauge market momentum, as already known by existing volatility metrics. By using this difference in price movements, this method avoids arbitrary choices that detract from an accurate reflection of underlying market dynamics, or more specifically, filter out "noise volatility" while highlighting volatility that stems from more significant structural shifts in market behavior.

2.3 From Crossings to Braid Words

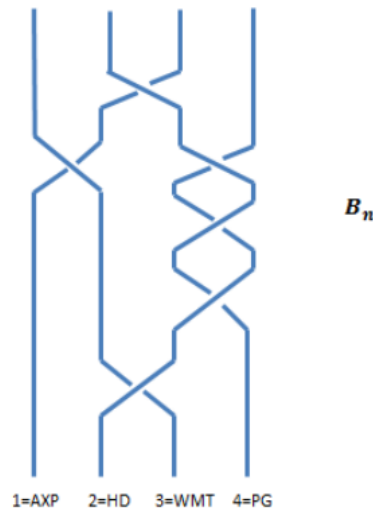


Figure 3: Ovidiu Racorean's Figure 2 crossing diagram with defined under/overcrossings

Once crossings are identified and classified we practically have our braid. And, like any braid, we can express it as a braid word of the sequence of adjacent strands crossing over and under each other, the

one below being that of our Figure 3 braid:

$$\sigma_2 \sigma_2 \sigma_3 \sigma_3^{-1} \sigma_2^{-1} \sigma_3^{-1} \sigma_2 \sigma_1 \sigma_3 \sigma_2 \sigma_1.$$

2.3.1 Braid Word Breakdown

Recall our overcrossings (σ_i) and undercrossings (σ_i^{-1}), in our braid word are distinguished by the superscript (negative for undercrossing and positive for overcrossing), with the strand being distinguished by the subscript. Breaking down the braid in Figure 3, examining the crossings sequentially we see that:

1. Strand 2 over strand 3 $\rightarrow \sigma_2$
2. Strand 2 over strand 4: *To jump two strands, HD must cross over WMT and then PG $\rightarrow \sigma_2 \sigma_3$ [Since we already did one σ_2 in step 1, the new adjacent generator here is σ_3 .]*
3. Strand 2 under strand 4: *Undoing the above jump under PG $\rightarrow \sigma_3^{-1} \sigma_2^{-1}$*
4. Strand 4 under strand 3 $\rightarrow \sigma_3^{-1}$
5. Strand 3 over strand 1: *WMT must cross HD and then AXP $\rightarrow \sigma_2 \sigma_1$*
6. Strand 4 over strand 1: *PG must cross WMT, HD, then AXP in succession $\rightarrow \sigma_3 \sigma_2 \sigma_1$*

Thus, when we put this all together we see our 6 crossings form our braid word - a compact representation of the market's changes

$$\underbrace{\sigma_2}_{(1)} \underbrace{\sigma_2 \sigma_3}_{(2)} \underbrace{\sigma_3^{-1} \sigma_2^{-1}}_{(3)} \underbrace{\sigma_3^{-1}}_{(4)} \underbrace{\sigma_2 \sigma_1}_{(5)} \underbrace{\sigma_3 \sigma_2 \sigma_1}_{(6)}.$$

Ultimately, setting up the possibility of applying further topological analysis.

2.4 Knots, Links and Market Dynamics

Racorean further extends this model by closing the braid to form a knot. As by the Alexander Theorem, every closed braid results in a knot. There are two ways for closure of braids into knots explored by Racorean: trace closure, and plat closure. However, we are strictly focused on trace closure, which connects each top-strand directly to the corresponding bottom-strand with an arc that lies entirely behind the braid; resulting in either a single closed component(knot) or a multi-component link. The resulting knot/link is analyzed using topological invariants that provide insights into the link/knot's complexity. While Racorean focuses on both the Alexander polynomial and the Jones polynomial, but due to limitations in computational power and the limitations of the Alexander polynomial - with its inability to detect chirality or distinguish many distinct knots, we'll be strictly focusing on the Jones polynomial as its better suited for alternating knot, pretzel knots, and multi-component links. With this knot/complexity being used as an indicator of market stress. The underlying idea is that a more complicated knot (one with higher complexity) indicates more turbulent market conditions. By studying how the complexity of these knots changes over time, it is possible to track and analyze shifts in market behavior.

2.5 Connection to Topological Quantum Computing

Although the main goal is to study market dynamics, Racorean also presents a valuable connection to topological quantum computing. In topological quantum computing, information is encoded in the braiding of anyons. Ultimately, he suggests that the tools developed for quantum computing could help refine analyses of market dynamics as both stock prices and non-Abelian anyons (that make up quantum computers) follow braid-like trajectories that mimic each other. While presenting great potential for further exploration, our approach will solely be an exploration of the classical topological ideas presented in Racorean's work, solely translating the time-series stock price data into a braid that will be closed to compute invariants to quantify complexity. Instead, we focus on creating a more deterministic indicator of market structure instead of a more speculative means of forecasting market dynamics.

3 Methodology

Raw Code Can be Accessed on the Following Repository: <https://github.com/FMK2K/Knot-Theory-Financial-Metric>

3.1 Data Collection Pipeline

The following pseudocode script is of the sophisticated pipeline implemented in this project. Let's break down how it works and why its approach is advantageous:

Algorithm 1 Topological Financial Crisis Detection Pipeline

```
Input: EVENTS (dictionary of financial events with date ranges and tickers),  
COMMON_TICKERS (list of reliable tickers)  
Output: optimized_data (dictionary containing processed data for each event)  
Initialize optimized_data  $\leftarrow \{\}$   
for all event_name, event in EVENTS do  
    is_sub_event  $\leftarrow$  check if event has 'sub_phase_start'  
    event_start  $\leftarrow$  event['sub_phase_start'] if is_sub_event else event['era_start']  
    event_end  $\leftarrow$  event['sub_phase_end'] if is_sub_event else event['era_end']  
    all_tickers  $\leftarrow$  COMMON_TICKERS  $\cup$  event['tickers']  
    valid_tickers  $\leftarrow$  filter all_tickers where validate_ticker(t, event_start, event_end) is True  
    if valid_tickers is empty then  
        raise ValueError("No valid tickers for " + event_name)  
    end if  
    date_range  $\leftarrow$  (pd.to_datetime(event['sub_phase_start']) -  $\Delta t$ , pd.to_datetime(event['sub_phase_end']) +  $\Delta t$ ) if is_sub_event else (event['era_start'], event['era_end'])  
    combined_data  $\leftarrow$  fetch_market_data(valid_tickers, date_range.start, date_range.end)  
    periods  $\leftarrow$  generate_adaptive_windows(event, event_name)  
    event_processed_data  $\leftarrow$  process_windows(combined_data, periods)  
    if is_sub_event then  
        event_processed_data  $\leftarrow$  {'crisis' : event_processed_data['crisis']}  
    end if  
    optimized_data[event_name]  $\leftarrow$  event_processed_data Exception as e  
    optimized_data[event_name]  $\leftarrow$  {'error' : str(e)}  
    Log error: e  
end for  
verify_tickers(optimized_data)  
Output optimized_data
```

Helper Functions

- **validate_ticker**(*ticker, start, end*): Checks if ticker has valid historical data within the given range.
- **fetch_market_data**(*tickers, start, end*): Fetches historical market data for given tickers and date range, with caching and error handling.
- **generate_adaptive_windows**(*event, event_name*): Generates pre-crisis, crisis, and non-crisis time windows based on the event definition, with adaptive buffer periods.
 - **Pre-crisis window**: A period leading up to the crisis.
 - **Crisis window**: The defined crisis period itself.
 - **Non-crisis window**: Control periods outside the crisis to establish a baseline. The function uses adaptive buffer periods (e.g., 10-day pre-crisis, 3-day post-crisis) and for sub-events, it uses shorter buffers. It also attempts to select non-crisis periods that don't overlap with other defined events.
- **process_windows**(*combined_data, periods*): Processes market data for each time window, calculates returns and normalized returns.
- **verify_tickers**(*optimized_data*): Generates a report on the processed data, checking for valid tickers and data availability.

3.1.1 Configuration

Prior to running our pipeline, we set up the necessary libraries (pandas for data manipulation, numpy for numerical operations, etc.) and configure essential parameters for asynchronous operations. Before initializing a `diskcache` to store fetched financial data, the cache has size and compression limits to manage resources effectively, improving efficiency by avoiding redundant API calls.

3.1.2 Data Definitions: Events and Tickers

It defines a dictionary where each key represents a historical financial event. With the events focused on all being significant moments of financial turmoil, each exemplifying a relatively distinct kind of volatility that cannot be captured in real time by traditional metrics, as not all volatility is the same, as shown in Figure 4.

Volatility Type	Normal Market	Credit Crisis (CC)	Financial Crisis (FC)	Recession
Episodic	Low, short-lived spikes	Moderate, potentially larger & more frequent spikes	High, frequent, intertwined with crisis	Moderate, increased spikes
Economic Cycle-Driven	Low, gradual changes	Moderate increase	High, sustained	High, sustained
Existential	Extremely Rare	Low to Potentially High (if severe CC escalates)	Extremely High, defining characteristic	Low to Potentially Moderate (if severe recession threatens financial stability)

Figure 4: (*Not all market volatility is created equal*, 2025)

With these events including sub-phases to better examine the moment of crisis itself, and how this crisis was instigated, where it peaked in volatility, and the first point of intervention:

- **Black Monday (1987):** A sharp, largely episodic crash, amplified by market mechanisms and investor panic, with underlying economic concerns but a relatively quick recovery. Whose volatility could be characterized as being more episodic, as the crash was sudden, severe, and marked by a rapid increase in trades, which was only amplified by existing market mechanisms, all in all creating a feedback loop that led to a shortage of liquid assets.
- **2008 Financial Crisis:** A prolonged period of existential volatility stemming from systemic financial vulnerabilities and leading to a deep economic recession. Whose volatility could be characterized as being primarily existential, with episodic spikes, given the systemic collapse of key parts of the US (and larger global) financial system, given the bursting of the mortgage bubble that spread through financial instruments/institutions and worsened this financial crisis to a prolonged recession.
- **COVID-19 Crash (2020):** A rapid onset of existential volatility triggered by an external shock, with significant economic implications but a faster recovery due to aggressive policy responses. Whose volatility can be characterized similar to that of the 2008 Financial Crisis albeit with a far swifter and more substantial recovery due to intervention.

Our last event being an analysis of current financial conditions to see what our data reveals about the current market and any parallels it may have to the examined past events.

- **Recent Market Data (2025):** A test case reflecting current market trends centered around the tariffs of President Trump's 2nd term.

For each event, we specify:

- **era_start** and **era_end**: The broader time period surrounding the event.
- **crisis_start** and **crisis_end**: The specific dates defining the crisis period within the era.
- **Sub-phases**: Sub-phases with their own start and end dates, allowing for more granular analysis of different stages (the trigger, the peak of volatility, and the initial point of intervention) within a crisis.

- **tickers:** A list of relevant stock tickers to represent diverse market sectors and allow for accurate analysis of the event.
 - **Common Tickers:** SPY, AGG, AAPL, JPM, XOM (Reliable tickers that are relatively consistent across all events and represent the whole market)
 - **Black Monday:** IBM, KO (Additional tickers to better reflect 1980s market leaders) - *however, we were met with issues drawing data on the SP 500 index and the AGG index on the performance of the total U.S. investment-grade bond market*
 - **2008 Crisis:** BAC, C (Additional tickers to better reflect 2008 banking leaders)
 - **COVID-19 Crash:** AMZN, PFE (Additional tickers to better encompass pandemic-relevant stocks.)
 - **Recent Data:** NVDA, WMT (Additional tickers to better reflect current tech and blue-chip dominance)

3.1.3 Anti-Blocking Mechanisms

After being being blocked by financial data providers due to excessive requests, the pipeline implements the following to prevent this:

- **Rotating User Agents:** It cycles through a list of different browser identifiers in the HTTP headers, making requests appear to come from various users.
- **Security Headers:** It includes standard HTTP security headers that mimic browser behavior, which can help in avoiding detection as a bot.

3.1.4 Rate Limit Controls:

This section defines parameters to manage the rate at which API data requests are made.

- **max_concurrent:** Limits the number of simultaneous API calls.
- **delay_range:** Introduces random delays between requests.
- **batch_size** and **batch_delay:** Fetches data for tickers in batches with a pause between batches.
- **max_retries** and **retry_backoff:** Implements a retry mechanism with exponential backoff for failed requests.

3.1.5 Normalization

Rather than using min-max normalization to compare stocks with different price scales, we use log normalization with the function implemented using a rolling window for calculating the mean and standard deviation data, with enhancements for short data windows and protection against division by zero. Ultimately, returning an **optimized_data** that better preserves the shape of price trajectories for relative comparisons than a min-max normalization and effectively highlights outlier and extreme movements better than min-max normalization.

3.2 Braid Word Construction

This algorithm takes our formatted stock price data as input and transforms this financial data into a representation of a braid - we run all our event data through this in a later `process_braid_pipeline`. Here's how it works:

Algorithm 2 Construct Braid from Price Matrix

```

Input: price_matrix ( $T \times N$ ), tickers (list of length  $N$ )
Output: braid_representation (dictionary with 'braid_word', 'n_strands', 'tickers')
 $T, N \leftarrow \text{shape of } price\_matrix$ 
 $ranks \leftarrow \text{argsort}(price\_matrix, \text{axis}=1)$ 
 $braid\_gens \leftarrow []$ 
for  $t$  from 1 to  $T - 1$  do
     $prev\_order \leftarrow ranks[t - 1]$ 
     $curr\_order \leftarrow ranks[t]$ 
    for  $i$  from 0 to  $N - 2$  do
         $a \leftarrow curr\_order[i]$ 
         $b \leftarrow curr\_order[i + 1]$ 
         $pos\_a \leftarrow \text{index of } a \text{ in } prev\_order$ 
         $pos\_b \leftarrow \text{index of } b \text{ in } prev\_order$ 
        if  $pos\_a > pos\_b$  then
             $da \leftarrow |price\_matrix[t, a] - price\_matrix[t - 1, a]|$ 
             $db \leftarrow |price\_matrix[t, b] - price\_matrix[t - 1, b]|$ 
            if  $da$  is not NaN and  $db$  is not NaN then
                if  $da > db$  then
                    Append " $\sigma_{i+1}$ " to  $braid\_gens$ 
                else
                    Append " $\sigma_{i+1}^{-1}$ " to  $braid\_gens$ 
                end if
            end if
        end if
    end for
end for
 $braid\_word \leftarrow \text{join elements of } braid\_gens \text{ with " "}$ 
 $braid\_representation \leftarrow \{ 'braid\_word' : braid\_word, 'n\_strands' : N, 'tickers' : tickers \}$ 
return  $braid\_representation$ 

```

3.2.1 Detecting Crossings

Given we've already captured the relative ordering of the assets over time via our `ranks[t]` function; we then iterate through the relative price order of two stocks, represented by adjacent "strands" at positions i and $i+1$ on the current day, and compare their positions in the price ranking on the previous day to identify instances of price ranking change(our crossings).

3.2.2 Racorean's Magnitude Rule for Overcrossings and Undercrossings

If the relative order of these two strands has changed (i.e., if stock a was ranked lower than stock b on the previous day but is now ranked higher), a "crossing" is detected. The direction of this crossing (which strand goes "over" the other in the braid analogy) is determined by comparing the absolute change in price (price—) for the two stocks between the previous and current day. The stock with the larger absolute price change is considered to have "passed over" the other.

3.2.3 Braid Generator Assignment

Based on which stock passed over, a braid generator is appended to the `braid_gens` list:

- If the stock at the lower index (i) in the current ranking had a larger price change, it's assigned σ_{i+1} .
- If the stock at the higher index ($i+1$) in the current ranking had a larger price change, it's assigned σ_{i+1}^{-1} . The subscript $i+1$ refers to the crossing occurring between the $(i+1)$ th and $(i+2)$ th strands in the braid.

Ultimately returning a data structure containing:

braid_word: A string representing the constructed braid as a sequence of σ_i and σ_i^{-1} generators, separated by spaces.

n_strands: The number of tickers (which corresponds to the number of strands in the braid).

tickers: The original list of tickers.

3.3 Braid To Knot

This bash script processes a series of JSON files, each containing our braid data from the former `braid_construction` algorithm. Using SageMath, a powerful open-source mathematics software system, to close our braids, turning them into knots/links and returning invariants(our final results) that we'll later analyze. Here's a breakdown of how it works:

Algorithm 3 Analyze Braids from JSON Files

Input: *json_files* (list of paths to JSON braid data)
Output: *output_file* (JSON file containing analysis results)
output_file \leftarrow "braid_analysis_results.json"
temp_file \leftarrow create temporary file
Initialize empty list *results*
for all *json_path* in *json_files* **do**
 Print "Processing: *json_path*"
 Run SageMath script (with timeout) to:
 Load braid data from *json_path*
 Construct braid group and braid object
 Construct link from the braid
 if link has 1 component (is a knot) **then**
 Calculate Alexander polynomial
 Calculate Jones polynomial
 Calculate determinant
 Calculate signature
 Calculate determinant square
 Calculate genus
 Store results in *result* dictionary
 else
 Calculate Jones polynomial
 Store results (is_knot=False) in *result* dictionary
 end if Exception as *e*
 Store error message in *result* dictionary
 Append *result* (as JSON) to *temp_file*
end for
Format *temp_file* content into a JSON array in *output_file*
Remove *temp_file*
Print "Full results saved to: *output_file*"

3.3.1 Knot/Link Analysis

If the link has only one component ($L.number_of_components() == 1$), it means that all strands are connected to form a single Alexander polynomial. Jones polynomial: A knot polynomial that can distinguish many knots.

Determinant Signature: A numerical invariant of a knot that tell us more about its structure.

Determinant square: A boolean indicating if the determinant is a perfect square.

Genus: The minimum genus of a surface whose boundary is the knot. These values tell us the complexity of the simplest surface you could stretch the knot over.

If the link has more than one component, it's a link (multiple separate, possibly intertwined loops), the Jones polynomial becomes particularly useful. As we want an invariant that is very good at telling us how our strands (the stock tickers) are interacting or not interacting. Given the Jones polynomial is like a more sensitive detector for these inter-group relationships than the other invariants, which tend to focus more on the properties of a single closed loop (a knot). It helps us understand if the market is becoming more fragmented (more linked components with specific Jones polynomial characteristics) or more unified in its relative price actions.

4 Results

4.1 Knot/Link Invariant Data

Idx	File	Jones polynomial
0	black_monday_pre-crisis_period1_1987-10-02_1987-10-11.json	$-t^{21/2} + 2t^{19/2} - 4t^{17/2} + 5t^{15/2} - 7t^{13/2} + 5t^{11/2} - 7t^{9/2} + 3t^{7/2} - 4t^{5/2} + t^{3/2} - t^{1/2}$
1	black_monday_non-crisis_period1_1987-08-17_1987-08-20.json	$-t^{1/2} - t^{-1/2}$
2	black_monday_non-crisis_period2_1987-11-02_1987-11-08.json	$-t^{5/2} - t^{1/2}$
3	black_monday_crisis_period1_1987-10-10_1987-10-29.json	$-t^4 + t + t^{-1} + t^{-2} + 2$
4	black_monday:initial_collapse_crisis_period1_1987-10-12_1987-10-16.json	$-t^{5/2} - t^{1/2}$
5	black_monday:peak_volatility_crisis_period1_1987-10-19_1987-10-23.json	$-t^{5/2} - t^{-5/2}$
6	2008_crisis_non-crisis_period1_2007-12-03_2007-12-06.json	$t + t^{-1} + 2$
7	2008_crisis_non-crisis_period2_2008-10-06_2008-10-12.json	$t^{-1} + t^{-2} + t^{-3} + 1$
8	2008_crisis_crisis_period1_2008-09-13_2008-10-02.json	$t^{3/2} - 2t^{1/2} + 4t^{-1/2} - 8t^{-3/2} + 9t^{-5/2} - 13t^{-7/2} + 12t^{-9/2} - 11t^{-11/2} + 9t^{-13/2} - 5t^{-15/2} + 3t^{-17/2} - t^{-19/2}$
9	2008_crisis:lehman_collapse_crisis_period1_2008-09-15_2008-09-19.json	$t^{-1} + t^{-2} + t^{-3} + 1$
10	2008_crisis:credit_freeze_crisis_period1_2008-09-22_2008-09-26.json	$t^2 + t + t^{-1} - t^{-4} + 2$
11	covid_pre-crisis_period1_2020-02-20_2020-02-29.json	$t^{11} - t^{10} - 2t^8 - t^7 + t^6 + t^5 + 3t^4 + t^3 + t^2$
12	covid_non-crisis_period1_2019-12-02_2019-12-08.json	$-t^{-1/2} - 2t^{-3/2} - 2t^{-5/2} - t^{-7/2} - t^{-11/2} - t^{-13/2}$
13	covid_non-crisis_period2_2020-03-22_2020-03-28.json	$t^{-1} + t^{-2} + t^{-3} + 1$
14	covid_crisis_period1_2020-02-28_2020-03-18.json	$-2t^{11/2} - 3t^{7/2} - 2t^{3/2} - t^{1/2}$
15	covid:global_spread_crisis_period1_2020-03-02_2020-03-06.json	$-t^{15/2} - 3t^{11/2} - 3t^{7/2} - t^{3/2}$
16	covid:circuit_breakers_crisis_period1_2020-03-02_2020-03-13.json	$t^2 + 2t + 3 + 3t^{-1} + 2t^{-2} + 2t^{-3} + t^{-4} + t^{-5} + t^{-6}$
17	covid:initial_stabilization_relief_crisis_period1_2020-03-16_2020-04-03.json	$-t^8 + t^7 - 4t^6 + 5t^5 - 6t^4 + 10t^3 - 4t^2 + 10t - 1 + 5t^{-1} - t^{-3}$
18	current:tariff_announcement_crisis_period1_2025-02-01_2025-02-14.json	$t + 3t^{-1} - t^{-2} + 3t^{-3} - 2t^{-4} + t^{-5} - t^{-6}$
19	current:market_reaction_crisis_period1_2025-02-15_2025-03-15.json	$t^{-1} - 2t^{-2} + 4t^{-3} - 4t^{-4} + 6t^{-5} - 5t^{-6} + 6t^{-7} - 4t^{-8} + 4t^{-9} - 2t^{-10} + t^{-11} - t^{-12}$
20	current:stabilization_after_reaction_crisis_period1_2025-03-16_2025-04-01.json	$t^{7/2} - 4t^{5/2} + 8t^{3/2} - 18t^{1/2} + 25t^{-1/2} - 38t^{-3/2} + 42t^{-5/2} - 48t^{-7/2} + 43t^{-9/2} - 37t^{-11/2} + 27t^{-13/2} - 16t^{-15/2} + 9t^{-17/2} - 9t^{-19/2} + t^{-21/2}$

Table 1: Knot Data: Index, Filename, and Jones Polynomial

Idx	Jones polynomial	VIX_Close	CN \geq Span	Non-Alternating?	Non-Palindromic?	Extremal ± 1 ?	Det	IZ	Composite?	GB \leq Span/2
0	$-t^{21/2} + 2t^{19/2} - 4t^{17/2} + 5t^{15/2} - 7t^{13/2} + 5t^{11/2} - 7t^{9/2} + 3t^{7/2} - 4t^{5/2} + t^{3/2} - t^{1/2}$	30.06	20	False	True	False	40	NaN	No clear evidence	10
1	$-t^{1/2} - t^{-1/2}$	30.06	2	True	False	True	0	NaN	No clear evidence	1
2	$-t^{5/2} - t^{1/2}$	37.27	4	True	False	True	2	NaN	No clear evidence	2
3	$-t^4 + t + t^{-1} + t^{-2} + 2$	57.19	5	False	True	False	0	NaN	No clear evidence	2.5
4	$-t^{5/2} - t^{1/2}$	45.32	4	True	False	True	2	NaN	No clear evidence	2
5	$-t^{5/2} - t^{-5/2}$	53.12	5	True	False	True	0	NaN	No clear evidence	2.5
6	$t + t^{-1} + 2$	15.20	2	False	True	True	4	NaN	No clear evidence	1
7	$t^{-1} + t^{-2} + t^{-3} + 1$	25.30	3	True	False	True	0	NaN	No clear evidence	1.5
8	$t^{3/2} - 2t^{1/2} + 4t^{-1/2} - 8t^{-3/2} + 9t^{-5/2} - 13t^{-7/2} + 12t^{-9/2} - 11t^{-11/2} + 9t^{-13/2} - 5t^{-15/2} + 3t^{-17/2} - t^{-19/2}$	23.10	22	False	True	True	9	NaN	No clear evidence	11
9	$t^{-1} + t^{-2} + t^{-3} + 1$	28.50	3	True	False	True	0	NaN	No clear evidence	1.5
10	$t^2 + t + t^{-1} - t^{-4} + 2$	29.70	6	False	True	True	2	NaN	No clear evidence	3
11	$t^{11} - t^{10} - 2t^8 - t^7 + t^6 + t^5 + 3t^4 + t^3 + t^2$	12.40	9	False	True	True	0	NaN	No clear evidence	4.5
12	$-t^{-1/2} - 2t^{-3/2} - 2t^{-5/2} - t^{-7/2} - t^{-11/2} - t^{-13/2}$	11.80	12	True	False	True	0	NaN	No clear evidence	6
13	$t^{-1} + t^{-2} + t^{-3} + 1$	14.60	3	True	False	True	0	NaN	No clear evidence	1.5
14	$-2t^{11/2} - 3t^{7/2} - 2t^{3/2} - t^{1/2}$	37.30	10	True	False	False	0	NaN	No clear evidence	5
15	$-t^{15/2} - 3t^{11/2} - 3t^{7/2} - t^{3/2}$	82.69	12	True	False	True	0	NaN	No clear evidence	6
16	$t^2 + 2t + 3 + 3t^{-1} + 2t^{-2} + 2t^{-3} + t^{-4} + t^{-5} + t^{-6}$	66.13	8	False	True	True	4	NaN	No clear evidence	4
17	$-t^8 + t^7 - 4t^6 + 5t^5 - 6t^4 + 10t^3 - 4t^2 + 10t - 1 + 5t^{-1} - t^{-3}$	55.46	11	False	True	True	39	NaN	No clear evidence	5.5
18	$t + 3t^{-1} - t^{-2} + 3t^{-3} - 2t^{-4} + t^{-5} - t^{-6}$	19.80	7	False	True	True	12	NaN	No clear evidence	3.5
19	$t^{-1} - 2t^{-2} + 4t^{-3} - 4t^{-4} + 6t^{-5} - 5t^{-6} + 6t^{-7} - 4t^{-8} + 4t^{-9} - 2t^{-10} + t^{-11} - t^{-12}$	22.10	11	False	True	True	29	NaN	No clear evidence	5.5
20	$t^{7/2} - 4t^{5/2} + 8t^{3/2} - 18t^{1/2} + 25t^{-1/2} - 38t^{-3/2} + 42t^{-5/2} - 48t^{-7/2} + 43t^{-9/2} - 37t^{-11/2} + 27t^{-13/2} - 16t^{-15/2} + 9t^{-17/2} - 9t^{-19/2} + t^{-21/2}$	21.75	28	False	True	True	95	NaN	No clear evidence	14

Table 2: Periods annotated with original Jones polynomials, VIX closes, and knot-theory-derived volatility metrics (Adjusted for Landscape A4).

4.2 Theorems Referenced:

Given that our algorithms only returned the Jones polynomial, in order to better see the complexities and interwoven-ness of our braid, we will need to compute other invariants/indicators using our Jones polynomial before using them in contrast and conjunction with standard metrics (VIX in this case) using the following theorems/axioms

4.2.1 Crossing Number and Span

Theorem 4.19 (*Chang, On the Jones polynomial and its applications, 2011*)

The span of the Jones polynomial gives a lower bound on the crossing number. Furthermore, if a knot is an alternating knot, then equality holds

4.2.2 Alternating

Theorem (*Thistlethwaite 1987, A spanning tree expansion of the Jones polynomial, Topology*):

The coefficients of the Jones polynomial of a non-split alternating link alternate in sign (up to an overall sign).

4.2.3 Palindromic

Corollary 4.4.6 (*Ranicki Hoste, Knotnotes, UCSD*)

Any knot K whose Jones polynomial $V_K(t)$ is not palindromic (i.e. symmetrical under exchanging t and t^{-1}) is chiral, i.e distinct from its mirror-image

4.2.4 Extremal Coefficients

Theorem 9.3.1 (Lickorish, *An Introduction to Knot Theory*, Springer, 1997)

For any non-split alternating link L , the coefficients of the extremal terms in $V_L(t)$ (i.e. the terms of highest and lowest degree) are 1. Equivalently, if either extremal coefficient vanishes or has absolute value 1, then this signals a failure of the usual “alternating-like” spanning-tree structure.

4.2.5 Determinant

By standard definition is obtained by evaluating the absolute value of the Jones polynomial $V_K(t)$ at $t = -1$: $det(K) = |V_K(-1)|$.

4.2.6 Interior Zeros

Thistlethwaite (1987), *A spanning tree expansion of the Jones polynomial*:

“Any vanishing coefficient in an interior exponent corresponds to the absence of spanning trees of a certain writhe-parity.”

4.2.7 Composite Links/Factorization

Theorem 9.3.1 (*Lickorish, An Introduction to Knot Theory, 1997*):

“If either extremal coefficient vanishes or has absolute value 1, then no reduced alternating diagram of L exists; in particular, L is non-adequate.

4.2.8 Genus Bound(Span)

Morton’s Inequality (*Morton, The Jones Polynomial and the Seifert Genus, 1986*):

“For any knot K with Seifert genus $g(K)$, $2g(K) \leq \text{span } V_K(t)$.”

5 Analysis of Results

5.1 Topological Data's Alignment with Standard Volatility Metrics

If we perform an examination on our table data to see how well it aligns with standard metrics like VIX. Focusing on our identified period of high VIX (indexed at rows 3, 5, 15, 16, 17) to see whether braid complexity increased accordingly, and contrast those with our periods of low VIX (indexed at rows 8, 11, 12, 20).

In doing so we see that generally, the lower bounds on the crossing number seem to be higher during some of the high VIX periods (e.g., Idx 15, 17) and also during some lower VIX periods (e.g., Idx 8, 20). However, there isn't a perfectly consistent trend of higher CN during high VIX.

Similar to the crossing number, some high VIX periods have higher genus bounds (Idx 15, 17), but so do some lower VIX periods (Idx 8, 20). With nearly everything else varying in high and low VIX periods. Now when we focus on non-crisis, and crisis periods as opposed to strictly VIX we do see a higher determinant, genus bound, and crossing number on average during periods of crisis (albeit given our small sample this may need to be further explored)

Table 3: Comparison of Crisis vs. Non-Crisis Metrics

Metric	Average Crisis	Average Non-Crisis
CN \geq Span	10.36	5.17
Det	16.07	0.83
GB \leq Span/2	5.5	2.83

Thus, there might be a weak positive correlation between the VIX and the lower bound on the crossing number and the genus bound (metrics related to the complexity of our knot/link). As periods with higher VIX tend to have, on average, higher values for these metrics. Additionally, we have a potential positive correlation between the VIX and the determinant (an invariant metric for the interconnectedness of our knot/link), as the average determinant is notably higher during crisis periods, suggesting a possible positive correlation with volatility. This aligns with our initial idea that increased market instability could manifest as more complex topological structures in our market braid.

5.2 What Else Does Our Data Tell Us About the Type of Volatility

Recall the Volatility Types in our Crises:

- Black Monday (1987): Sharp, episodic, driven by market mechanisms and panic, relatively quick recovery.
- 2008 Global Financial Crisis: Prolonged, existential, driven by systemic financial collapse and a deep economic downturn.
- COVID-19 Pandemic Crash (2020): Rapid onset of existential volatility due to an external shock and economic shutdowns, but with a faster recovery due to policy responses.

When we analyze our topological metrics alongside VIX during these crisis periods, we see that:

5.2.1 Black Monday

- VIX: Shows a clear spike during the crisis periods (57.19, 45.32, 53.12) compared to the pre-crisis period (30.06).
- CN Span: The lower bound on crossing number is relatively high in the pre-crisis period (20) and then fluctuates during the immediate crisis (5, 4, 5). It doesn't show a consistent spike with the peak VIX.
- Non-Alternating?: Shows both True and False, no clear pattern.
- Det: Values are 40, 0, 2, 0. The pre-crisis determinant is high.
- GB Span/2: Relatively high pre-crisis (10) and then lower during the immediate crisis (2.5, 2, 2.5).

With our decrease in the genus bound from pre-crisis to crisis potentially reflecting the sharp, sudden nature of Black Monday - suggesting a quick change in the market's "topological" state. With the high pre-crisis crossing number and determinant potentially indicating a complex market structure that was susceptible to a rapid unraveling. Both of which align with the episodic volatility of that crisis.

5.2.2 2008 Global Financial Crisis

- VIX: Rises from non-crisis levels (15.20, 25.30) to a crisis level (23.10, 28.50, 29.70). The peak VIX during the most intense phase of the crisis (not captured in these specific snapshots but known to be much higher) might correlate with more extreme knot metrics if we had that data.
- CN Span: Shows a jump to the highest value in the entire dataset during the crisis period (Idx 8: 22).
- Det: Values are 4, 0, 9, 0, 2. A moderate increase during the crisis.
- GB Span/2: Also shows the highest value in the dataset during the crisis (Idx 8: 11).

With the sustained higher levels of crossing number lower bound and genus bound during the 2008 crisis periods in our snapshots, potentially reflecting the prolonged complexity and interconnectedness of the financial system that contributed to and was exacerbated by the crisis - the higher determinant also aligns with this idea of increased interconnectedness. Once more, this aligns with the economic cycle-driven volatility of the Great Recession.

5.2.3 COVID-19 Pandemic

- VIX: Shows a dramatic increase from pre-crisis/non-crisis levels (12.40, 11.80, 14.60) to very high crisis levels (37.30, 82.69, 66.13, 55.46).
- CN Span: Shows a mix, with some higher values during the crisis (12, 8, 11) but also high in a non-crisis period (12).
- Det: Values are 0, 0, 0, 0, 0, 4, 39. A notable increase in the later crisis stages in our snapshots.
- GB Span/2: Shows higher values during the crisis (6, 6, 4, 5.5) compared to the immediate pre-crisis period.

The rapid spike in VIX aligns with the sudden onset of the crisis. This aligns with the existential volatility and rapid recovery; however, this alignment, although present, is weaker in our knot metrics, while the genus bound later the determinant also shows increases during the crisis periods in our snapshots, potentially reflecting the market's grappling with the sudden systemic shock. It's not enough to conclude alignment with our volatility type.

While not a perfect one-to-one mapping, there are indications that the topological metrics, particularly the lower bound on crossing number, the determinant, and the genus bound, tend to reflect the nature of the crises' volatility.

With our genus bound and crossing number (which can be interpreted as proxies for complexity) tend to be higher during the crisis periods for both the 2008 and COVID-19 events compared to the immediate pre-crisis snapshots. This aligns with the idea that periods of high systemic risk involve greater market complexity.

When summarizing the behavior of these topological metrics we see sustained or increasing levels of complexity-related knot metrics in our snapshots during prolonged, systemic crises (2008, COVID); and rapid shifts from a potentially complex pre-crisis states in sharper, episodic crises (Black Monday).

This lines up with our initial hypothesis that the patterns in the topological metrics can offer additional insights into the underlying complexity and interconnectedness of the market during different types of volatility events. However, this idea warrants further exploration to be solidified.

5.3 Implications About the Present

When looking at the increasing trends in the crossing number lower bound, determinant, and genus bound across these periods could suggest a market that is:

- Initially reacting with moderate complexity and interconnectedness
- Experiencing increased complexity and significantly higher interconnectedness during the market reaction.
- Potentially moving towards a more intricate and highly interconnected state even as the VIX shows signs of stabilization (Stabilization After Reaction).

This doesn't neatly fit into the extreme "existential" volatility of the 2008 or COVID crashes in terms of VIX magnitude in these specific snapshots. However, the increasing complexity and interconnectedness metrics might suggest a period of elevated volatility characterized by a trend towards increasing topological complexity, which itself may be indicative of an underlying market adjustment to the news that involves a more intricate web of relationships and market interconnectedness as the market reacts and attempts to stabilize.

6 Conclusion

In this study, we applied braid theory to analyze stock market behavior during major financial crises such as Black Monday (1987), the Global Financial Crisis (2008), and the COVID-19 Crash (2020), along with recent market dynamics (2024-2025). By translating daily stock price rankings into braids and then closing those braids into knots or links, we developed a new representation of market volatility that captures the structure and interconnections of stock price movements. Our approach builds on Racorean's crossing stocks method by distinguishing between overcrossings and undercrossings based on the relative magnitude of price changes. This method allowed us to create a symbolic braid word that represents the dynamic behavior of the market over time. We then computed topological invariants, with a special focus on the Jones polynomial, to quantify the complexity of these knots. The analysis indicates that periods of high market stress tend to correspond to a higher level of knot complexity. For example, crisis periods often display higher values in the lower bound of the crossing number, determinant, and genus bound when compared to non-crisis periods. The results of our study support the hypothesis that topological measures can provide additional insight into market dynamics. In particular, our findings suggest that increased market instability may be reflected in more complex braids and knots. When compared with standard metrics like the VIX, the topological approach appears to capture aspects of market interconnectedness and structural complexity that traditional measures might miss.

Several limitations remain. Data quality issues, especially in historical datasets such as Black Monday and the 2008 crisis, have introduced some challenges in the braid construction process. The sensitivity of our method to the normalization technique and the presence of noise in the data suggest the need for further refinement. In addition, while the Jones polynomial is a useful tool, interpreting its invariants in direct relation to market stress requires additional work. Future research should focus on improving data cleaning methods and exploring alternative normalization techniques to minimize noise. Expanding the analysis to a wider set of financial instruments and incorporating additional statistical tools may further solidify the relationship between topological invariants and market behavior. A deeper investigation into the mapping between knot complexity and market measures may also yield a more robust indicator of systemic risk. Our work demonstrates that applying braid and knot theory to financial data offers a promising new avenue for analyzing market volatility. This topological approach has the potential to complement existing methods, providing a richer picture of market dynamics during periods of crisis. Continued refinement of the methodology and further validation with larger datasets will be key to developing a practical tool for forecasting and understanding financial instability.

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