Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

Model Predictive Dynamic Control Allocation with Actuator Dynamics†

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Abstract— **A model predictive, dynamic control allocation algorithm is developed in this paper for the inner loop of a re-entry vehicle guidance and control system. The purpose of the control allocation portion of the guidance and control architecture is to distribute control power among redundant control effectors to meet the desired control objectives under a set of constraints. Most existing algorithms neglect the actuator dynamics or deal with the actuator dynamics separately, thereby assuming a static relationship between actuator outputs (in our case, control surface deflections) and plant inputs (i.e., moments about the three body axis). In this paper, we propose a dynamic control allocation scheme based on Model-based Predictive Control (MPC) that directly takes into account actuators with nonnegligible dynamics and hard constraints. Model-based Predictive Control schemes compute the control inputs by optimizing an open-loop control objective over a future time interval at each control step. In our setup, the model-predictive control allocation problem is posed as a sequential quadratic programming problem with dynamic constraints, which can be cast into a linear complementary problem (LCP) and therefore solved by linear programming approaches in a finite number of iterations. The time-varying affine internal model used in the MPC design enhances the ability of the control loop to deal with unmodeled system nonlinearities. The approach can be easily extended to encompass a variety of linear actuator dynamics without the need to redesign the overall scheme. Results are based on the model of an experimental reusable launch vehicle, and compared with that of existing static control allocation schemes.**

I. INTRODUCTION

Control allocation approaches for advanced aircraft, in particular re-entry vehicles (RVs), have received increased attention as modern aircraft employ more complicated control architectures. Generally speaking, systems of this kind are characterized by the presence of more control effectors than controlled variables, meaning that the control system possesses a certain degree of redundancy, and can in principle achieve multiple control objectives. However, this increased capability requires intelligent schemes, usually satisfying some criteria for optimality, to select in real time the control configuration for the available actuators. Typically, hard performance constraints exist, so that the control allocation scheme must distribute the available control authority among redundant control effectors to meet the control objectives, and satisfy the constraints at the same time (see, for instance, [2]). Control allocation modules and closed-loop control laws are generally designed separately. The basic control allocation objective, then, is to generate appropriate commands to the actuators in order to produce the desired control at the plant input. To date, most existing algorithms for control allocation neglect actuator dynamics, or deal with the actuator dynamics separately. In that case, the interaction among control law, control allocation module and dynamics of actuators and aircraft may cause the control commands to lose their effectiveness in providing adequate tracking performance or even stability of the closed loop. One reason for this is that the presence of actuator dynamics can decrease the overall effective bandwidth of the control system, and can even accentuate the effect of unmodeled nonlinearities. While some existing approaches have considered the problem to a certain extent, extension to general cases remains elusive.

In this paper, a model predictive dynamic control allocation scheme is proposed to account for actuator dynamics and constraints. A time-varying affine internal model, based on a high-fidelity simulation of an experimental RV, is used in the MPC design. One advantage of this approach is that its basic structure can be easily extended to a variety of linear actuator models without the need to redesign the control scheme. The MPC problem herein is posed as a sequential quadratic programming problem with constraints. Subsequently, the problem is cast into a linear complementary problem (LCP), which is solvable by linear programming in a finite number of iterations. Results for the proposed technique are based on the RV model simulation, and are compared with that of existing static control allocation schemes. The paper is organized as follows. The next section discusses traditional control allocation schemes, followed by an overview of the aircraft model with actuator dynamics, and an explanation of the baseline control allocation scheme developed by Doman et al. [5]. The MPC based control scheme is developed in detail in Section III, while simulation results and comparison with the static control allocation scheme of Doman et al. are presented in Section IV.

II. STATIC CONTROL ALLOCATION

Control allocation schemes are used to distribute control authority among a set of control effectors such that a desired set of moments or accelerations are produced by the controls. A traditional control allocation framework is shown in Figure 1, where u_{des} represents the desired

† This work is supported by the AFRL/AFOSR Collaborative Center of Control Science at the Ohio State University.

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Fig. 1. Control Allocation

moments/accelerations generated by the outer loop control law, δ_{cmd} is the command signal to the actuators computed by the control allocation scheme, δ_{act} is the output of the actuator subsystem embodying the actuator dynamics, and u_{act} is the actual moment/angular acceleration generated by the control effectors in response to u_{des}. In general, u_{act} is related to the actuator output δ_{act} by means of a nonlinear mapping $u_{act} = g(t, \delta_{act})$, with amplitude and rate constraints

$$\delta_L \leq \delta \leq \delta_U \,, \qquad |\dot{\delta}| \leq \bar{\dot{\delta}} \,,$$

where δ stands either for δ_{cmd} or δ_{act}, δ_L and δ_U are the lower and upper limits of the amplitude, and $\bar{\dot{\delta}}$ is the rate limit. As the flight control system is a sampled-data system, using the simple first-order difference approximation

$$\dot{\delta} \approx \frac{\delta(t) - \delta(t-T)}{T} \,,$$

where T is the sampling period, the rate limit can be absorbed into the amplitude limit as follows:

$$-\bar{\dot{\delta}} \leq \frac{\delta(t) - \delta(t-T)}{T} \leq \bar{\dot{\delta}}$$

$$\delta(t-T) - \bar{\dot{\delta}}T \leq \delta(t) \leq \delta(t-T) + \bar{\dot{\delta}}T \,.$$

New lower and upper amplitude limits can therefore be defined as

$$\underline{\delta} = \max\{\delta_L,\, \delta(t-T) - \bar{\dot{\delta}}T\} \,, \quad \bar{\delta} = \min\{\delta_U,\, \delta(t-T) + \bar{\dot{\delta}}T\}$$

allowing us to consider only amplitude limits in the sequel. In most traditional existing static control allocation schemes, the control effectiveness mapping in Figure 1 is assumed to be linear and time-invariant, i.e.,

$$g(t, \delta_{act}) = B\delta_{act} \,,$$

leading to a drastic simplification in the design[1]. In such a representation, the actuator dynamics are neglected (that is, $\delta_{cmd} = \delta_{act}$), since it is commonly assumed that the dynamics of the actuators are relatively fast compared with that of the aircraft. In this simplified scenario the *static control allocation problem* takes the following form: find δ_{cmd} such that

$$u_{des} = B\delta_{cmd} \,, \quad \underline{\delta} \leq \delta_{cmd} \leq \bar{\delta} \,,$$

where $u_{act} \in \mathbb{R}^m$, $B \in \mathbb{R}^{m \times n}$ and $\delta_{cmd} \in \mathbb{R}^n$. The control effector redundancy is expressed by the fact that $\mathrm{rank}(B) = m$, and $n > m$. If a feasible solution exists, it is readily obtained by means of pseudo-inversion. In this case, the available redundancy may be employed to satisfy a sub-objective of the form

$$J_{sub} = \min_{\delta_{cmd}} \|\delta_{cmd} - \delta_p\|_{W_p} \,,$$

where W_p is a weighting matrix, and δ_p is a preferred control input chosen to meet additional requirements on control deflections, drag minimization and so forth. For example, if δ_p is chosen to be δ_{act} at the previous sample, then the sub-objective is to minimize the control energy. If this is the case, the solution becomes

$$\delta_{cmd} = \delta_p + W_p^{-1}B^T(BW_p^{-1}B^T)^{-1}(u_{des} - B\delta_p) \,.$$

In solving the static (linear) control allocation problem, linear programming (LP) methods have many advantages. For example, the LP problem can be solved within a finite number of iterations, which makes it amenable for a real-time implementation on flight control systems operating with a sampling rate in the 50-100 Hz range. To be cast into the realm of linear programming, the control allocation problem is formulated as a 1-norm optimization problem of the form

$$\begin{aligned} &\min_{\delta} J_d = \|u_{des} - B\delta\|_1 \\ &\text{subject to:} \quad \underline{\delta} \leq \delta \leq \bar{\delta} \,. \end{aligned} \tag{1}$$

Note that the 1-norm allows the optimization problem (1 to be cast into the following LP problem:

$$\min_{\delta} J_d = \left[\; 0 \cdots 0 \; 1 \cdots 1 \;\right] \begin{bmatrix} \delta \\ \delta_s \end{bmatrix}$$

subject to:

$$\left[\; -\delta_s \,,\;\; \delta \,,\;\; -\delta \,,\;\; B\delta - \delta_s \,,\;\; -B\delta - \delta_s \;\right]^T \leq \left[\; 0 \,,\;\; \bar{\delta} \,,\;\; -\underline{\delta} \,,\;\; u_{des} \,,\;\; -u_{des} \;\right]^T ,$$

where $\delta_s \in \mathbb{R}^n$ is a slack variable vector with the same dimension as δ. If the optimal solution is such that $J_d = 0$, the solution is not unique, and "excess" control power can be utilized to optimize additional objectives. For example, the following 1-norm optimization sub-objective can be used to drive the control inputs to some preferred values

$$\begin{aligned} &\min_{\delta} J_S = \min_{\delta} \|W_p^T(\delta - \delta_p)\|_1 \,, \\ &\text{subject to:} \quad u_{des} = B\delta \,, \quad \underline{\delta} \leq \delta \leq \bar{\delta} \,, \end{aligned}$$

where $W_p \in \mathbb{R}^n$ is a vector of weights to make the optimization problem more flexible. The additional sub-optimal problem can also be cast into an LP problem of the form

$$\min_{\delta} J_S = W_p^T\delta_s \,,$$

subject to:

$$B\delta = u_{des}$$

$$\left[\; -\delta_s \,,\;\; \delta \,,\;\; -\delta \,,\;\; \delta - \delta_s \,,\;\; -\delta - \delta_s \;\right]^T \leq \left[\; 0 \,,\;\; \bar{\delta} \,,\;\; -\underline{\delta} \,,\;\; \delta_p \,,\;\; -\delta_p \;\right]^T .$$

The LP problem can be solved by the familiar simplex method, for which numerous well-developed implementations are available.

[1]Nonlinear control allocation methods are not considered in this paper, but are the subject of the companion work [8]

III. MODEL PREDICTIVE DYNAMIC CONTROL ALLOCATION

For systems with actuator dynamics and constraints, the interactions between the advanced control law, the control allocation algorithm, and the actuator dynamics with the aircraft body becomes increasingly complicated. Hence, the control allocation becomes more difficult. Existing control allocation schemes are not amenable to mixing control effects of actuators with varying time constants. For example, the dynamics of control surfaces can be classified as *fast* modes, while the dynamics of the engine are relatively *slow*, perhaps even with inherent delay; traditionally, engine dynamics are treated separately for these reasons. All of these factors make the overall control design (inner- and outer-loop) complicated, particularly because of the interactions among the varying dynamical systems. Model predictive control (MPC) or receding horizon control (RHC) are optimization based control techniques which can effectively be used for such stringent requirements. MPC utilizes a model of the plant to predict the output during a future time interval (horizon), and computes the control commands by minimizing an objective function. Thus, the control calculated can pre-act to the system dynamics and achieve a better performance. Such a methodology is most effective when the dynamics of the plant can be determined with sufficient accuracy. For the problem of dynamic control allocation at issue, the dynamics of the actuator have been well studied and a relatively accurate model of the plant is available.

A. RV model with dynamic inversion control law

In what follows, we consider explicitly the control architecture for reentry vehicles proposed in Doman and Ngo [4] as a baseline for the design of the MPC-based control allocation algorithm. Simulation results for the proposed technique will then be compared with those obtained employing the static mixed optimization control allocation with intercept correction (MOIC) developed in [5].

Following [4], a dynamic inversion-based control law is employed in conjunction with a control allocation module having the structure depicted in Figure 1. As shown in Figure 2, the dynamic inversion module generates $\dot{\omega}_{BAE}$, a variable containing nonlinear terms independent of the control effective mapping, interpreted as the angular acceleration due to the base-aircraft aerodynamics. The prediction module generates a prediction of the desired angular acceleration reference over the receding horizon. The control commands

$$CV^{cmd} = [p \quad q \quad r]^{cmd}$$

are body-axis angular velocity commands to be tracked by the vehicle rotational dynamics. The actuator dynamics is modeled as a second-order system for each control channel (control surface) expressed by equations of the form

$$\dot{\delta}(t) = A_\delta \delta(t) + B_\delta \delta_{cmd}(t)\ ,\ \underline{\delta} \le \delta \le \overline{\delta}\ .$$

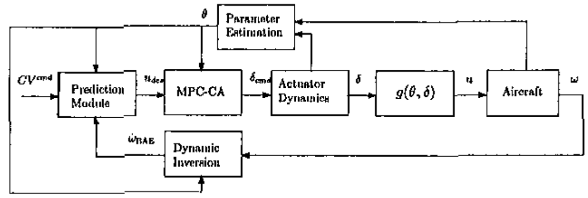

Fig. 2. RLV with MPC control allocation

The estimation module in Figure 2 is required to provide the necessary information on the model parameters to the MPC control allocator (represented by the MPC&CA block in the figure). The aircraft rotational dynamics can be stated as

$$\dot{\omega} = f(\omega, \theta) + g(\theta, \delta)\,, \tag{2}$$

where $\omega = [p \ q \ r]^T$, and the vector θ contains the plant parameters associated with the relevant operating point. In (2), $g(\theta, \delta)$ represents the control dependent accelerations, while $f(\omega, \theta)$ includes accelerations due to the aircraft body and engine. Most modern control allocation schemes assume that the control effective mapping is linear, in order to reduce the complexities of the control allocation design algorithm as well as the advanced control law design. That is, the control dependent part of the rotation equation is simplified as

$$g(\theta, \delta) = B(t)\delta\,.$$

Typically, this assumption is valid when the deflections of the control surfaces are within their linear region. If one or some of the control deflections are close to the position limits, nonlinear behavior is likely observed, the individual control effectiveness coefficients could be inaccurate, and instabilities may result.

In the proposed MPC control allocation technique, the control effective mapping is modeled as a time-varying affine mapping of the form

$$g(\theta, \delta) = B(t)\delta + \epsilon(\delta)\,,$$

yielding the resulting model

$$\begin{aligned} \dot{\omega}(t) &= f(\omega(t), \theta) + B(t)\delta(t) + \epsilon(\delta(t)) \\ \dot{\delta}(t) &= A_\delta \delta(t) + B_\delta \delta_{cmd}(t)\ ,\ \underline{\delta} \le \delta \le \overline{\delta}\,. \end{aligned}$$

The problem is then that of finding the input command $\delta_{cmd}(t)$ such that $\omega(t)$ tracks $\omega_{des}(t)$ as closely as possible. If we let

$$\begin{aligned} u(t) &= \dot{\omega}(t) - f(\omega(t), \theta) - \epsilon(\delta(t)) \\ &= B(t)\delta(t)\,, \end{aligned} \tag{3}$$

the MPC control allocation problem is posed as follows: for the constrained system

$$\begin{aligned} \dot{\delta}(t) &= A_\delta \delta(t) + B_\delta \delta_{cmd}(t) \\ y(t) &= B(t)\delta(t)\ ,\ \underline{\delta} \le \delta \le \overline{\delta} \end{aligned} \tag{4}$$

find $\delta_{cmd}(t)$ such that $y(t)$ tracks $u_{des}(t)$ as closely as possible, being u_{des} the predicted value for $u(t)$ obtained from (3). The MPC approach utilizes (4) as an internal model of the actuator dynamics to predict $y(t)$ at a future discrete time instants $[\hat{y}(k+N_1|k), \cdots, \hat{y}(k+N_2|k)]$. In this representation, $\hat{y}(k+j|k)$ denotes the optimal j-step ahead prediction of the system output, based on data up to time k, while N_1 and N_2 are respectively the lower and upper limits of the receding horizon. The pseudo-control commands $[u_{des}(k+N_1), \cdots, u_{des}(k+N_2|k)]$ are computed from (3) on the basis of predicted outputs of the dynamic inversion module. The MPC algorithm applies a control sequence that minimizes a multistage cost function of the form

$$J(N_1, N_2, N_u) = \sum_{j=N_1}^{N_2} W_y(j)[\hat{y}(k+j|k) - u_{des}(k+j)]^2 + \sum_{j=1}^{N_u} \lambda(j)[\Delta\delta_{cmd}(k+j-1)]^2 \tag{5}$$

subject to

$$\underline{\delta} \leq \delta_{cmd} \leq \overline{\delta} \tag{6}$$

where N_u is the horizon of the control command, $W_y(j)$ and $\lambda(j)$ are weighting functions, and $\Delta\delta_{cmd}$ is the incremental control command representing the energy needed to actuate the control effector. The objective of predictive control is to compute the future control sequence $\delta_{cmd}(k), \delta_{cmd}(k+1), \ldots$ in such a way that the future plant output $y(k+j)$ is driven close to $u_{des}(k+j|k)$, through minimization of the objective function.

B. From MPC to LCP

According to [3], the MPC problem is then cast into a linear complimentary problem (LCP) of the following form: given a vector $q \in \mathbb{R}^n$ and matrix $M \in \mathbb{R}^{n\times n}$, find two vectors s and z satisfying

$$s - Mz = q, \quad s, z \geq 0, \quad < s, z >= 0. \tag{7}$$

The pivotal algorithm developed by Lemke [6], can then be used to solve the LCP. To transform our MPC problem to the LCP (7), define

$$\delta = e_n\underline{\delta} + x \tag{8}$$

where e_n is the n-dimensional vector $e_n = [1, 1, \ldots, 1]^T$. The constraints can be expressed in condensed form as

$$x \geq 0, \quad Rx \leq c, \tag{9}$$

where R and c are, respectively, an appropriate matrix and a column vector related to the specific constraints. The MPC objective function, (5), can be transformed into the form

$$J(\delta) = \frac{1}{2}\delta^T H\delta + b\delta + f_0,$$

which, using (8), becomes

$$J = \frac{1}{2}x^T Hx + ax + f_1,$$

where

$$a = b + \underline{\delta}e_n^T H, \quad f_1 = f_0 + \underline{\delta}^2 e_n^T He_n + b\underline{\delta}.$$

Let v and v_1 be the vectors of Lagrange multipliers associated rispectively with the first and the second set of constraints in (9), and let v_2 be the vector of slack variables. The Karush-Kuhn-Tucker (KKT) conditions [1] read as

$$Rx = v_2 = c, \quad -Hx - R^T v = v_1 = a,$$
$$x^T v_1 = 0, \quad v^T v_2 = 0, \; x, v, v_1, v_2 \geq 0$$

or, in matrix form,

$$\begin{bmatrix} I_{m\times m} & 0_{m\times N} & 0_{m\times m} & R \\ 0_{N\times m} & I_{N\times N} & -R^T & -H \end{bmatrix} \begin{bmatrix} v_2 \\ v_1 \\ v \\ x \end{bmatrix} = \begin{bmatrix} c \\ a \end{bmatrix}.$$

The KKT conditions can be further expressed as the linear complementary problem

$$s - Mz = q, \quad s^T z = 0, \quad s, z \geq 0$$

where

$$M = \begin{bmatrix} 0 & -R \\ R^T & H \end{bmatrix}, q = \begin{bmatrix} c \\ a \end{bmatrix}, s = \begin{bmatrix} v_2 \\ v_1 \end{bmatrix}, z = \begin{bmatrix} v \\ x \end{bmatrix}$$

The above LPC can be solved using Lemke's algorithm; as a matter of fact, if the matrix H is positive definite, convergence to the optimal solution in a finite number of iterations is guaranteed [1]. Moreover, a carefully chosen initial solution can greatly reduce the computational requirements. A standard choice is to let the solution of the unconstrained MPC problem, expressed in a quadratic programming form, be the starting point. In this way the initial point is close to the solution of an LCP problem derived from a constrained MPC problem.

IV. SIMULATION RESULTS

Because very few results have appeared for control allocation approaches which account for actuator dynamics, we will compare our proposed MPC algorithm presented above to existing methods designed by neglecting actuator dynamics. A leading algorithm along these lines is a mixed optimization control allocation scheme of Doman et al. [5]. In this work, the control effective mapping is assumed to be affine, and the algorithm provides intercept corrections to the standard linear formulation (we will refer to this baseline scheme as MOIC). The specific model of a six-degree of freedom reentry vehicle endowed with a dynamic inversion based control architecture described in [4] has been adopted as a testbed for the comparative study between the MPC and the mixed-optimization approach, as discussed in Section III. Specifically, the model has four control surfaces: right flap, left flap, right tail and left tail, with upper deflection limit 30°, lower deflection limit −30°, and rate limit 60°/s for all control surfaces under nominal conditions. The dynamics of the four actuators are second-order system with amplitude and rate limits, and damping ratio equal to 0.7.

Different natural frequencies, ranging from 20Hz and 5Hz, have been independently assigned and tested for each pair of tail and flap effectors. According to the scheme in figure 2, the desired rolling, pitching and yawing rates serving as the command trajectories to the system are provided by a higher level planning module. In our test, we have employed a feasible reference trajectory corresponding to an approach and landing maneuver. The segment of command trajectory employed in the test has duration equal to 53 seconds. For the chosen reference trajectory, the only non-zero command is the pitch rate, while the roll and yaw rate commands are kept to a zero setpoint. In the prediction module of Fig. 2, a cubic curve fitting is used to smooth the control command and to generate smooth pseudo-control commands u_{des}. Results from several tests performed changing the natural frequencies are summarized in Table I. The performance criteria are the mean square error (MSE) and the maximum error with respect to the reference trajectory over the entire test interval. Results indicate that the errors for MPC are approximately one order to two orders of magnitude smaller than those achived by the MOIC, depending on the test conditions. For example, a significant improvement in the pitch rate tracking performance is shown in Fig. 3 for the specific case of the bandwidth of the tail actuator dynamics equal to 6 Hz. It is worth noting that, when ω_n decreases, the performance of MPC does not vary much, while that of the MOIC scheme degenerates sharply. At this regard, figures 4 and 5 show the time history of the pitch rate and the pitch rate error obtained by the MPC scheme during the first 25 seconds of the simulation for decreasing values of the actuator bandwidth, while the results of the same test for the MOIC algorithm are shown in figures 6 and 7 respectively. It can be noticed that the MOIC-based control loop is driven to the verge of instability by the reduced actuator bandwidth, while the MPC-based control is capable of maintaining adequate tracking performance.

TABLE I

MPC AND MOIC UNDER DIFFERENT CONDITIONS

ω_n Hz	Maximum Error		MSE	
	MPC	MOIC	MPC	MOIC
20	3.56e-03	1.19e-02	7.49e-04	2.11e-02
15	3.74e-03	1.23e-02	8.81e-04	2.23e-02
12	3.74e-03	1.25e-02	1.03e-03	2.37e-02
10	3.69e-03	1.34e-02	1.19e-03	2.51e-02
8	3.68e-03	1.34e-02	1.47e-03	2.77e-02
7	3.72e-03	1.61e-02	1.68e-03	3.52e-02
6	3.88e-03	3.97e-02	1.91e-03	7.81e-02
5	4.25e-03	4.32e-02	2.16e-03	1.41e-01

V. CONCLUSIONS AND FUTURE RESEARCH

In this paper, a model predictive dynamic control allocation scheme has been proposed to account for actuator dynamics and constraints. A time-varying affine internal model, based on a high-fidelity simulation of an experimental RV, has been used in the MPC design. The proposed scheme provides a generic approach to distribute control authority among different types of actuators. Results indicate that the proposed approach performs better than traditional static control allocation algorithms in presence of significant time lags due to actuator dynamics. The MPC control allocation method proposed in this paper has however other advantages than an improvement in tracking accuracy. For example, the proposed technique can deal, in principle, with different types of actuators exhibiting a time-scale separation into fast and slow dynamics. Moreover, preliminary studies have shown that a large class of failure conditions can also be dealt with by the proposed algorithm, without the need to redesign the control allocation scheme. This renders the proposed methodology extremely appealing for reconfigurable control. As for the stability of the proposed MPC algorithm, several recent works [7] have addressed the essential principles for ensuring closed-loop stability, and results following these lines will appear in future versions of this study. Finally, it is worth noting that the computational burden of the MPC technique has not been addressed and compared with that of MOIC, primarily because at the present stage the MPC algorithm has been developed in MATLAB and Simulink only. This important issue will be addressed in the future, when the MPC algorithm will be implemented as a stand-alone module.

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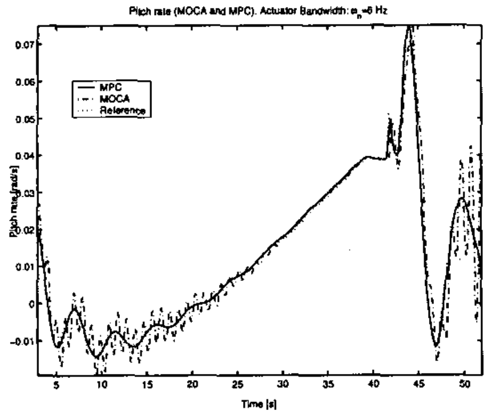

Fig. 3. Performance of MPC and MOIC with $\omega_n = 6Hz$: pitch rate.

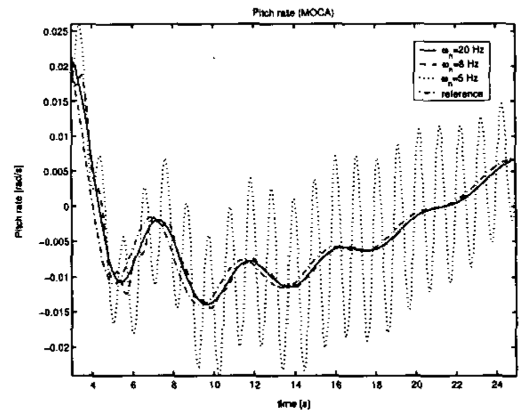

Fig. 6. Performance of MOIC with $\omega_n = 20, 8, 5$ Hz: pitch rate.

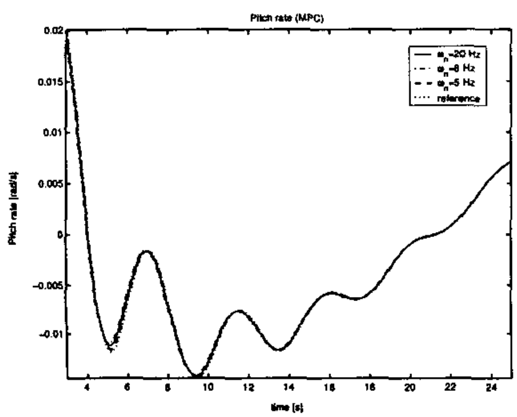

Fig. 4. Performance of MPC with $\omega_n = 20, 8, 5Hz$: pitch rate.

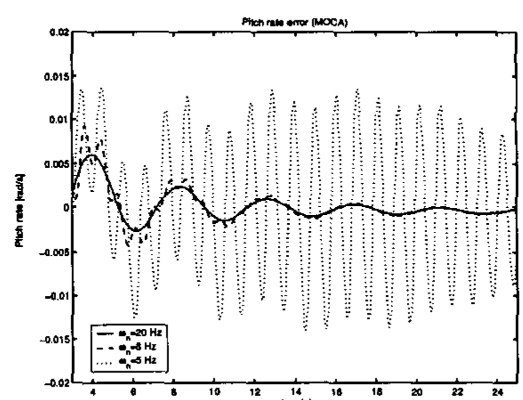

Fig. 7. Performance of MOIC with $\omega_n = 20, 8, 5$ Hz: pitch rate error.

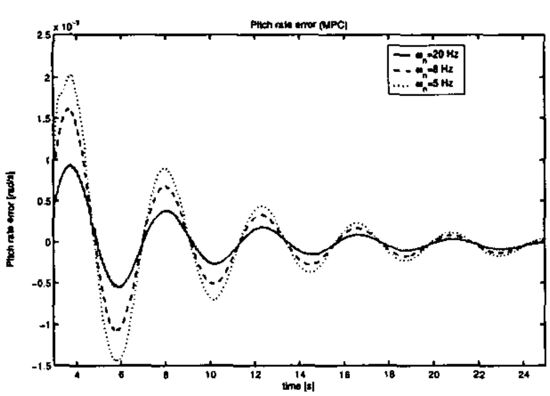

Fig. 5. Performance of MPC with $\omega_n = 20, 8, 5Hz$: pitch rate error.