

# GELFAND COMPACTNESS

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ABSTRACT.

## INTRODUCTION

In this paper the rings are associative but we do not ask them to have unit.

## 1. PRELIMINARIES

[2]

We denote by  $\mathcal{C}^*$  the category of commutative  $C^*$ -algebras and morphisms of  $C^*$  algebras. We denote by  $\mathcal{T}$  the category of Hausdorff locally compact topological spaces and proper continuous maps.

## 2. RINGS WITH LOCAL UNITS

**Definition 2.1.** Let  $R$  be a ring. We say that  $R$  is a ring with local units, if for any finitely many  $x_1, \dots, x_n \in R$  there is an idempotent  $e \in R$  with  $a_1, \dots, a_n \in eRe$ .

**Definition 2.2.** Let  $R$  be a ring. We say that  $R$  has enough idempotents, if there exists a family  $\{e_\lambda\}_{\lambda \in \Lambda}$  of pairwise orthogonal idempotents of elements in  $R$  with  $R = \bigoplus_{\lambda \in \Lambda} Re_\lambda = \bigoplus_{\lambda \in \Lambda} e_\lambda R$ . In this case  $\{e_\lambda\}_{\lambda \in \Lambda}$  is called a complete family of idempotents in  $R$ .

**Definition 2.3.** Let  $R$  be a ring. We say that  $R$  is a  $s$ -unital ring, if for any finitely many  $x_1, \dots, x_n \in R$  there is  $y \in R$  with  $x_i y = x_i = y x_i$  for  $i = 1, \dots, n$ .

**Definition 2.4.** Let  $R$  be a ring. We say that  $R$  is a firm ring, if the canonical morphism  $\mu: R \otimes_R R \rightarrow R$  given by  $\mu(r \otimes s) = rs$  is an isomorphism.

**Definition 2.5.** Let  $R$  be a ring. We say that  $R$  is idempotent, if  $R^2 = R$ , that is, that for any  $x \in R$  there are  $x_1, \dots, x_n, y_1, \dots, y_n \in R$  such that  $x = \sum_{i=1}^n x_i y_i$ .

## 3. PARACOMPACT SPACE

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## 4. METACOMPACT SPACE

## 5. ORTHOCOMPACT SPACE

## REFERENCES

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- [2] Robert Wisbauer. *Foundations of module and ring theory*. Routledge, 2018.

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