ALGEBRAIC GELFAND COMPACTNESS

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ABSTRACT.

INTRODUCTION

In this paper the rings are associative but we do not ask them to have unit.

1. PRELIMINARIES

[2]

If A and B are commutative C^*-algebras, and $f\colon A \longrightarrow B$ is an $*$-algebra morphism such that the linear span of $\{f(a)b \mid a \in A, b \in B\}$ is dense in B, then f is called non-degenerate We denote by $\mathcal{C}^*$ the category of commutative C^*-algebras and non-degenerate morphisms.

If X and Y are topological spaces and $\alpha\colon X \longrightarrow Y$ is a continuous map such that the inverse image of compact subsets of Y are compact subsets of X, then we call α a proper map. We denote by $\mathcal{T}$ the category of Hausdorff locally compact topological spaces and proper continuous maps. If $\alpha\colon X \longrightarrow \mathbb{C}$ is a continuous map such that for every $\epsilon > 0$ there is K_ϵ compact subset of X with $|\alpha(x)| < \epsilon$ for all $x \in X \setminus K_\epsilon$,then we say that α vanishes at infinity.

Proposition 1.1 (Non-unital Gelfand Duality)**.** *There is an equivalence of categories: $\mathcal{T}^{op} \cong \mathcal{C}^*$.*

[1]

2. RINGS WITH LOCAL UNITS

Definition 2.1. *Let R be a ring. We say that R is a ring with local units, if for any finitely many $x_1, \ldots, x_n \in R$ there is an idempotent $e \in R$ with $a_1, \ldots, a_n \in eRe$.*

3. RINGS WITH LOCAL UNITS

Definition 3.1. *Let R be a ring. We say that R has enough idempotents, if there exists a family $\{e_\lambda\}_{\lambda\in\Lambda}$ of pairwise orthogonal idempotents of elements in R with $R = \bigoplus_{\lambda\in\Lambda} Re_\lambda = \bigoplus_{\lambda\in\Lambda} e_\lambda R$. In this case $\{e_\lambda\}_{\lambda\in\Lambda}$ is called a complete family of idempotents in R.*

4. RINGS WITH LOCAL UNITS

Definition 4.1. *Let R be a ring. We say that R is a s-unital ring, if for any finitely many $x_1, \ldots, x_n \in R$ there is $y \in R$ with $x_i y = x_i = y x_i$ for $i = 1, \ldots, n$.*

Date: October 15, 2019.

2000 *Mathematics Subject Classification.* Primary ****, ****; Secondary ****, ****.

Key words and phrases. ,,

5. Rings with local units

Definition 5.1. *Let R be a ring. We say that R is a firm ring, if the canonical morphism $\mu\colon R \otimes_R R \longrightarrow R$ given by $\mu(r \otimes s) = rs$ is an isomorphism.*

6. Rings with local units

Definition 6.1. *Let R be a ring. We say that R is idempotent, if $R^2 = R$, that is, that for any $x \in R$ there are $x_1, \ldots, x_n, y_1, \ldots, y_n \in R$ such that $x = \sum_{i=1}^{n} x_i y_i$.*

7. Compact Dimension

Definition 7.1. *Let A be a commutative C^*-algebra, and $\{e_\lambda\}_{\lambda \in \Lambda}$ a directed family of self adjoint elements of A. We say that $\{e_\lambda\}_{\lambda \in \Lambda}$ is an approximate identity, if $xe_\lambda \to x$, for all $x \in A$.*

It is well know that any commutative C^*-algebra has an approximate identity. In fact there is a canonical approximate identity given by the family of all positive self adjoint elements with norm less or equal than one with its natural order.

Definition 7.2. *Let A be a commutative C^*-algebra. We define the compact dimension of A as the least cardinality of an approximate unit of A. We denote this cardinal by $dim_C(A)$*

As any commutative C^*-algebra has an approximate identity, the compact dimension is well defined. Also we have that a commutative C^*-algebra has unit if and only if $dim_C(A) = 1$.

Proposition 7.1. *Let A be non-unital commutative C^*-algebra. Then $dim_C(A)$ is a limit cardinal.*

Proof. $\square$

References

[1] Gerald J Murphy. *C*-algebras and operator theory*. Academic press, 2014.
[2] Robert Wisbauer. *Foundations of module and ring theory*. Routledge, 2018.

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