SIMPLICIAL COVERINGS

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Abstract.

Introduction

1. Preliminaries

We denote by $\mathcal{S}_f$ the category of finite sets.

We denote by Δ the simplex category and by $s\mathcal{S}$ the category of simplicial sets. [?] [?]

An abstract simplicial complex is a pair $(S, \mathcal{K})$ where S is a set and $\mathcal{K}$ is a family of non-empty finite subsets of S such that, if $\sigma \subseteq \tau$, $\sigma \neq \emptyset$ and $\tau \in \mathcal{K}$ then $\sigma \in \mathcal{K}$. A morphism between abstract simplicial complexes $f\colon (S_1, \mathcal{K}_1) \to (S_2, \mathcal{K}_2)$ it's a funtion $f\colon S_1 \to S_2$ such that $f(\sigma) \in \mathcal{K}_2$ for any simplex $\sigma \in \mathcal{K}_1$. We will denote $\mathcal{C}$ to the category of abstract simplicial complexes.

The geometric realization of an abstract simplical complex $(S, \mathcal{K})$ is given by the following formula: first we give a total order to S. Then, for any simplex $\sigma = \{s_0 < s_1 < \ldots < s_q\}$ we define $|\sigma| = \Delta^q$, the standar topological q-simplex and we asociate to the vertex s_q the q-th vertex of Δ^q. If $\tau = \{s_0 < \ldots < s_q\}$ is a simplex and $\sigma = \{s_{q_1} < \ldots < s_{q_k}\} \subseteq \tau$, we define $i^\tau_\sigma\colon |\sigma| \to |\tau|$ to be the affine function such that maps the j-th vertex of $|\sigma|$ to the q_j-th vertex of $|\tau|$. Thus we take $|S|$ as the colimit over this system. If $f\colon (S_1, \mathcal{K}_1) \to (S_2, \mathcal{K}_2)$ it's a morphism of abstract simplicial complexes, then we can define $|f|\colon |S_1| \to |S_2|$ as the colimit of the affine functions $|f|_\sigma|\colon |\sigma| \to |f(\sigma)|$. $|-|\colon \mathcal{C} \to \mathcal{T}$ it's a functor, were $\mathcal{T}$ is the category of topological spaces and continuous maps.

We shall recall that, given a topological space X, a covering space on X it's a continuous map $p\colon E \to X$, such that for every $x \in X$, there is an open neighborhood U such that $p^{-1}(U)$ it's a disjoint union of open sets U_λ, $\lambda \in \Lambda$, and $p|_{U_\lambda}\colon U_\lambda \to U$ it's a homeomorphism.

2. Abstract Simplicial Coverings

Definition 2.1. *Let $(S, \mathcal{K})$ an abstract simplicial complex. An abstract simplicial covering on $(S, \mathcal{K})$ it's a simplicial map $p\colon (E, \mathcal{L}) \to (S, \mathcal{K})$ such that for each simplex $\sigma \in \mathcal{K}$, $p^{-1}(\sigma) = \amalg_{\lambda \in \Lambda} \sigma_\lambda$ were $\sigma_\lambda \in \mathcal{L}$ and $p|_{\sigma_\lambda}\colon \sigma_\lambda \to \sigma$ it is onto and one to one, for each $\lambda \in \Lambda$.*

Proposition 2.1. *Let $p\colon (E, \mathcal{L}) \to (S, \mathcal{K})$ be an abstract simplicial covering. Then $|p| : |E| \to |S|$ it's a covering space.*

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3. Simplicial Coverings

Definition 3.1. *Let X be a simplicial set. A simplicial covering on X is a pair (Y, p) where Y is a simplicial set and $p \colon Y \longrightarrow X$ is a simplicial map such that*

Proposition 3.1. *Let X be a simplicial set and (Y, p) a simplicial covering of X. Then $(|Y|, |p|)$ is covering of $|X|$.*

Proof. □

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