SIMPLICIAL COVERINGS

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Abstract.

Introduction

1. Preliminaries

We recall that, given a topological space X, a covering space on X it's a continuous map $p\colon E \to X$, such that for every $x \in X$, there is an open neighborhood U such that $p^{-1}(U)$ it's a disjoint union of open sets U_λ, $\lambda \in \Lambda$, and $p|_{U_\lambda} : U_\lambda \to U$ it's a homeomorphism. We recommend the book of P. May [1] as reference of covering spaces.

An abstract simplicial complex is a pair $(S, \mathcal{K})$ where S is a set and $\mathcal{K}$ is a family of non-empty finite subsets of S such that, if $\sigma \subseteq \tau$ and $\tau \in \mathcal{K}$ then $\sigma \in \mathcal{K}$. If $\sigma \in \mathcal{K}$, then the dimension of σ is $|\sigma| - 1$, and we denote it by $dim(\sigma)$. The elements of $\mathcal{K}$ of dimension n are called n-simplices, and we denote the set of n-simplices by $\mathcal{K}_n$. The 0-simplices are called vertices.

The geometric realization of an abstract simplicial complex $(S, \mathcal{K})$ is the set of all function $\phi\colon S \longrightarrow [0,1]$ such that:

- $supp(\phi) \in \mathcal{K}$
- $\sum_{s \in S} \phi(s) = 1$

We denote this set by $|(S, \mathcal{K})|$. We may think $[0,1]^S$ as the direct limit of $[0,1]^A$ where Aranges over all finite subsets of S. So we give the $|(S, \mathcal{K})|$ the subspace topology.

A morphism between abstract simplicial complexes $(S_1, \mathcal{K}_1)$ and $(S_2, \mathcal{K}_2)$ is a map $f\colon S_1 \longrightarrow S_2$ such that $f(\sigma) \in \mathcal{K}_2$ for any $\sigma \in \mathcal{K}_2$.

2. Abstract Simplicial Coverings

Definition 2.1. *Let $(S, \mathcal{K})$ be an abstract simplicial complex. An abstract simplicial covering of $(S, \mathcal{K})$ is a pair $((T, \mathcal{L}), p)$ where $(T, \mathcal{L})$ is a abstract simplicial complex and $p\colon T \longrightarrow S$ is a morphism of abstract simplicial complexes such that...*

Proposition 2.1. *Let $(S, \mathcal{K})$ be an abstract simplicial complex and $((T, \mathcal{L}), p)$ an abstract simplicial covering of $(S, \mathcal{K})$. Then $(|(T, \mathcal{L})|, |p|)$ is covering of $|(S, \mathcal{K})|$.*

Proof. □

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References

[1] J Peter May. *A concise course in algebraic topology*. University of Chicago press, 1999.

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