This article has been accepted for publication in IEEE Access. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2022.3221476

Digital Object Identifier

Robust MPC for Systems With Model Uncertainties and Measurement Outliers

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ABSTRACT This paper considers the observer-based output feedback robust model predictive control (RMPC) problem for systems with model uncertainties and possible measurement outliers. For the sake of alleviating the effects from possible abnormal measurements, we design a set of observer-based output feedback RMPC controllers with the saturation constraint where the saturation level is adaptive according to the estimation errors. The purpose of the addressed problem is to design a set of desired RMPC controllers so as to guarantee the robustness and the asymptotical stability of the closed-loop system. Sufficient stability conditions are obtained by solving a time-varying terminal constraint set of an auxiliary optimization problem, and the corresponding control law and the upper bound of the quadratic cost function are derived. In addition, an algorithm including both off-line and on-line parts is provided to find a sub-optimal solution. Finally, two simulation examples are employed to illustrate the effectiveness of the proposed RMPC approach.

INDEX TERMS Robust model predictive control, measurement outliers, observer-based output feedback, time-varying terminal constraint set.

I. INTRODUCTION

OVER the past several decades, model predictive control (MPC), namely receding horizon control has been developed rapidly because of its great advantage of solving optimization problems that are limited by hard constraints and multi-variables, see e.g. [1]- [13]. At each time instant, by handling the online optimization issue based on the present measured values, a series of control moves within the range of future forecast are calculated, and only the first component is executed on the plant. At the next moment, according to the new measurement results, the optimization issue is required to be formulated again, the control input fixed by this issue is implemented on the system. In view of those achievements for nominal systems, the unavoidable parameter uncertainties have been addressed in the process of modeling due to many extensions, which brings about the robust MPC (RMPC) strategy, see e.g. [14]- [21]. RMPC strategy has been getting more and more attention and many novel RMPC strategies have been put forward in recent years, see e.g. [22]- [29].

Assuming states of the system are completely available, a majority of the existing outcomes involving the aforementioned ones are got. However, the systems to be handled have been becoming more and more complicated nowadays, so that assumption could not be always correct in actual engineering, and the states of the system cannot be always obtained in actual time, which means that the above RMPC strategy may not be valid. Accordingly, for the unmeasurable system states, it is of great practical significance to propose a novel RMPC strategy with output feedback control. For instance, to guarantee the stability of the system along with parameter uncertainties, the static output feedback controller based on the RMPC method has been presented [30]. The remarkable thing is that the conditions got by using RMPC strategy based on static output feedback are somewhat conservative due to the fact that output matrices usually need to be sorted in all rows, or the input matrices need to be sorted in all columns. In order to overcome this obstacle, the dynamic output feedback RMPC issue has turned into a charming research subject, then many outcomes have been achieved [31]- [33]. In addition, the output feedback RMPC strategy based on the observer is also an effective method to solve the problem [34]- [36]. However, to the author's knowledge, the observer-based dynamic output feedback RMPC problem has not been well studied. Therefore, filling this gap is our motivation.

It is well acknowledged that measurement outliers have become an inevitable phenomenon, which may give rise to the worsening of estimation performance. In [37], the state estimation issues have been handled with the Kalman filtering framework, and the algorithm is robust to the outliers.

In order to estimate the states of LTI systems in case of measurement outliers in [38], the moving horizon technology has been employed. In [39], an observer for the LTI system has been devised, and the effects of measurement outliers are mitigated due to introducing a saturated output injection. It is very important to consider the effect of measurement outliers fully and reduce their influence effectively.

Based on the previous discussion, the design of observer-based output feedback controllers for polytopic uncertain systems is studied by using the saturation function, which ensures the robustness and asymptotic stability of the system. In this paper, the main contributions are presented as follows: (1) Because of the difficulties in achieving the state measurements in practice, the output feedback control based on the observer is applied in the framework of RMPC. The technique of output matrix singular decomposition is used to handle the equality limit, which originates from the observer-based RMPC strategy. (2) There is a technology of saturating innovations, which is employed to reduce the negative influences of possible measurement outliers. At each time step, the adaptive saturation level is defined recursively by previous errors. Compare with the saturation mechanism that along with fixed levels, the dynamical level that we propose can be adjusted adaptively with the error accuracy and can present better performance. (3) In the matter of dynamic output feedback RMPC strategy, some effects of the polytopic uncertainties as well as the saturation function are all reflected in the controller establishment. (4) Sufficient conditions are obtained by utilizing the Lyapunov-like method along with the positive robust control invariant set technology.

The remaining in this paper is arranged as below. In Section II, the dynamic output feedback RMPC (OFRMPC) issue is designed for systems, which along with polytopic uncertainties being subject to measurement outliers, and lots of necessary definitions are presented. In Section III, with regard to the polytopic uncertain systems without or with hard constraints, sufficient conditions are obtained to guarantee stability, and the related dynamic OFRMPC algorithm is designed. In Section IV, two illustrative simulation examples are given to verify the effectiveness as well as the validity of the dynamic OFRMPC strategy that we propose. We give the conclusion in Section V at the end.

$Notations.$ About this paper, the notations are standard unless otherwise specified. $\mathbb{R}^n$ means the n-dimensional Euclidean space, $\mathbb{R}^{n_1 \times n_2}$ means the set of all $n_1 \times n_2$ real matrices. In regard to a matrix P, if $P > 0$, then P is positive definite and symmetric. $\|x\|$ means the Euclidean norm and $\|x\|^2 = x^T x$. $\|x\|_M^2 = x^T M x$ where $M > 0$ is a positive definite and symmetric weighting matrix. A scalar $|a|$ means the absolute value of a. 0 and I mean the zero matrix and the identity matrix with appropriate dimensions separately. The symbol "*" means the symmetric part in a symmetric matrix. The shorthand diag$\{\cdots\}$ stands for a block-diagonal matrix. The superscript "T" means the transpose and the superscript "-1" means the inverse (if invertible) for a matrix.

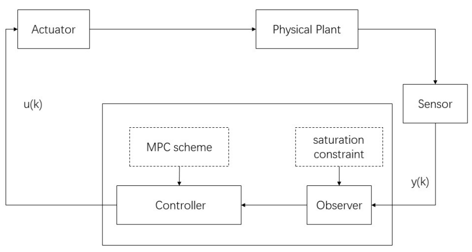

FIGURE 1. Structure of RMPC-based system subject to measurement outliers .

II. PROBLEM STATEMENT AND PRELIMINARIES

A. SYSTEM MODELS

The polytopic uncertain discrete-time linear system that we consider as follows:

$$\begin{cases} x(k+1) = A(k)x(k) + B(k)u(k) \\ y(k) = C(k)x(k) \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$, $u(k) \in \mathbb{R}^{n_u}$ and $y(k) \in \mathbb{R}^{n_y}$ are the state, the control input and the measurement output. And $A(k)$, $B(k)$, as well as $C(k)$, are unknown matrices with appropriate dimensions that belong to a polytope which is given by

$$\Delta := \left\{\Theta | \Theta = \sum_{l=1}^{L} \kappa_l \Theta^{(l)}, \sum_{l=1}^{L} \kappa_l = 1, 0 \le \kappa_l \le 1\right\}, \tag{2}$$

where $\Theta := \big(A(k),\ B(k),\ C(k)\big) \in \Delta$, matrices $\Theta^{(l)}$ are known that are defined as $\Theta^{(l)} := \big(A^{(l)}, B^{(l)}, C^{(l)}\big)(l = 1, 2, \ldots, L)$, which are the vertices of the convex hull Δ.

As shown in **FIGURE 1**, the measurement outliers may exist in the data transmission where from the sensor to the controller, thus the stability of the system will be threatened, so the observer-based controller with saturation function is discussed.

Denote $\hat{y}(k) \triangleq C(k)\hat{x}(k)$ and $r(k) \triangleq y(k) - \hat{y}(k)$, the observer-based system with the controller is to be defined in the following form:

$$\begin{cases} \hat{x}(k+1) = A^{(0)}\hat{x}(k) + B^{(0)}u(k) + H(k)\mathbf{S}at_{\sigma(k)}(y(k) - \hat{y}(k)) \\ \hat{y}(k) = C^{(0)}\hat{x}(k) \end{cases} \tag{3}$$

where $\hat{x}(k) \in \mathbb{R}^{n_{\hat{x}}}$ and $\hat{y}(k) \in \mathbb{R}^{n_{\hat{y}}}$ are the state estimate and output estimate of the observer of the system at time interval k. $H(k)$ is the parameter matrix that we will design. $A^{(0)}, B^{(0)}, C^{(0)}$ are the nominal matrices, and the nonlinear mapping $\mathbf{S}at_{\sigma(k)}(\cdot) : \mathbb{R}^{n_y} \mapsto \mathbb{R}^{n_y}$ in observer-based system (3) is a saturation function described below:

$$\mathbf{S}at_{\sigma(k)}(r(k)) \triangleq \begin{bmatrix} \mathbf{S}at^{(1)}_{\sigma(k)}(r(k)) \\ \mathbf{S}at^{(2)}_{\sigma(k)}(r(k)) \\ \vdots \\ \mathbf{S}at^{(n_y)}_{\sigma(k)}(r(k)) \end{bmatrix},$$

where $\mathbf{S}at^{(l)}_{\sigma(k)}(r(k)) \triangleq sign(r^{(l)}(k)) \cdot min\{|r^{(l)}(k)|, \sigma(k)\}$, and $r^{(l)}(k)$ denotes the l-th entry of the vector $r(k)$. At each step, the saturation level $\sigma(k)$ varies along with time, and it is defined according to the difference equation as follows:

$$\sigma(k+1) = \lambda\sigma(k) + (y(k) - \hat{y}(k))^T \Re(k)(y(k) - \hat{y}(k)), \tag{4}$$

where the matrix $\lambda \in [0, 1)$, $\Re(k)$ is positive definite and determined before.

Remark 1. In an observer-based system (3), a saturation function is used to alleviate some influences from possible measurement outliers with the help of limiting the innovations which are fed to the observer, the innovations mean differences between the estimated and measurement outputs. Unlike in most existing literature, the saturation level is always fixed [40], on the contrary, we adopt the iterative function (4) to fix the saturation level $\sigma(k)$ in this paper, which takes advantage of the innovation at the related time step. Specifically, we can see from (4) that when the innovation gets smaller at time step k, namely the estimation error also gets smaller, then the saturation level $\sigma(k+1)$ will get lower, and the related limit that is imposed on innovation will be more strict at time step $k+1$. With the help of introducing this mechanism, we can adjust the saturation level adaptively, and we will present the related superiority in two simulation examples later.

For conciseness of presentation, $\phi_k(\cdot) \triangleq \mathbf{S}at_{\sigma(k)}(\cdot)$ is denoted. By [41], there is a diagonal matrix $\Lambda(k)$ which satisfies $0 \leq \Lambda(k) \leq I$ such that

$$(\phi_k(r(k)) - \Lambda(k)r(k))^T(\phi_k(r(k)) - r(k)) \leq 0, \tag{5}$$

where $\Lambda(k) \triangleq diag\{\nu(1k), \nu(2k), ..., \nu(n_y k)\}$ with $0 \leqslant \nu(ik) \leqslant 1$.

Remark 2. In most existing documents, it should be noted that similar techniques are used to handle the saturation phenomenon, and it is assumed that the matrix $\Lambda(k)$ has been determined because the saturation level is determined. But about this paper, we can know from (4) and (5) that $\Lambda(k)$ ought to be a time-varying matrix because the saturation level $\sigma(k)$ varies along with the time. In fact, based on the values of $\sigma(k)$ as well as $r(k)$ at the related time step, the value of the time-varying $\Lambda(k)$ ought to be fixed.

B. PROBLEM OF INTERESTS

Consider the following controllers in the framework of MPC for system (3):

$$u(k+n|k) = F(k+n|k)\hat{x}(k+n|k), n = 0, 1, 2, ... \tag{6}$$

where $u(k+n|k)$ and $\hat{x}(k+n|k)$ are the nth step prediction of control input and state estimate at time k. We will determine the feedback gain $F(k+n|k)$ by optimisation. In the light of the actual requirement of engineering, the constraints on control input and state are given by

$$\| u(k+n|k) \|_2 \leq \bar{u}(k), n \geq 0 \tag{7}$$

$$\| x(k+n|k) \|_2 \leq \bar{x}(k), n \geq 0 \tag{8}$$

where $\bar{u} > 0, \bar{x} > 0$ are known scalars.

Denoting the estimate error $e(k) = x(k) - \hat{x}(k)$, the closed-loop system within the predicted horizon can be written as below:

$$\begin{aligned} \eta(k) &= \begin{bmatrix} \zeta(k) & x^T(k) & \hat{x}^T(k) & \phi^T(k) \end{bmatrix}^T \\ &= \begin{bmatrix} 1 & e^T(k) & x^T(k) & \hat{x}^T(k) & \phi^T(k) \end{bmatrix}^T, \end{aligned} \tag{9}$$

$$\zeta(k) = \begin{bmatrix} 1 & e^T(k) \end{bmatrix}^T,$$

we can obtain the following augmented system:

$$\zeta(k+1) = \tilde{f}(k)\eta(k), \tag{10}$$

where

$$\tilde{f}(k) \triangleq \begin{bmatrix} \hat{f}(k) & \hat{A}(k) & \hat{B}(k) & \hat{H}(k) \end{bmatrix}, \tag{11}$$

$$\hat{f}(k) \triangleq \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \hat{A}(k) \triangleq \begin{bmatrix} 0 \\ A(k) \end{bmatrix}, \phi(k) \triangleq \phi_k(r(k)),$$

$$\hat{B}(k) \triangleq \begin{bmatrix} 0 \\ B(k)F(k) - A^{(0)} - B^{(0)}F(k) \end{bmatrix}, \hat{H}(k) \triangleq \begin{bmatrix} 0 \\ -H(k) \end{bmatrix}.$$

We could find that system (10) is polytopic uncertain. In addition, concerning the above augmented closed-loop system (10), we consider a min-max cost function of the following form to design the controllers:

$$\min_{F(k), H(k)} \max_{(A(k), B(k), C(k)) \in \Delta} J_\infty(k), \tag{12}$$

where the objective function $J_\infty(k)$ is determined by

$$J_\infty(k) \triangleq \sum_{n=0}^{\infty} [\zeta^T(k)Q\zeta(k) + u^T(k)Ru(k)] \tag{13}$$

with Q_1, Q_2, Q_3, Q_4, Q_5 and R expressing the positive definite and symmetric weighting matrices, $Q = diag\{Q_1, Q_2, Q_3, Q_4, Q_5\}$.

In view of the min-max problem Eq.(12), the online optimization issue is shown to determine the controllers based on out-feedback MPC:

$$\mathbf{Op1}: \min_{F(k),H(k)} \max_{(A(k),B(k),C(k))\in\Delta} J_\infty(k),$$

$$s.t. \max_c |[u(k+n|k)]_c| \le \bar{u},$$

$$s.t. \max_d |[x(k+n|k)]_d| \le \bar{x},$$

$$\zeta(k+n|k) \in \Upsilon(P(k+n|k),5\rho), \quad n=0,1,2,...$$

where Υ is the terminal constraint set, which is fixed by

$$\Upsilon \triangleq \{\zeta(k+n|k)|\zeta(k+n|k)^T P(k+n|k)\zeta(k+n|k) \leqslant 5\rho\}, \tag{14}$$

and $P(k+n|k)$ expressing a positive definite matrix of a quadratic function. We will cover further particulars in the next part. Note that the first component $u(k)$ of a group of predicted inputs $\{u(k), u(k+1), u(k+2), ...\}$ will be employed on the plant at every moment. Significantly, according to an online optimization problem, the terminal constraint set Υ concerning the varying time is usually obtained, and the initial system state is required to be inside. In addition, infinite horizon control laws have been shown to ensure nominal stability as described by [22]. Therefore, we have adopted the prediction horizon of this paper as ∞ to guarantee at least nominal stability.

In this paper, the dynamic output feedback controllers Eq.(6) based on RMPC are designed to make the system (1) asymptotically stable under introducing the observer with a saturation function. To be more specific, an auxiliary optimization issue **Op1** is given to find the required parameter matrices $F(k)$ and $H(k)$, so that we can ensure stability in the closed-loop system. In order to reach this aim, about any admissible parameter k, two requirements want to be satisfied at the same time as follows:

R1. an auxiliary optimization issue is presented to denote the issue **Op1**, so that we can get the sub-optimization solution;

R2. according to the parameter matrices $F(k)$ and $H(k)$ that we obtain, the observer-based closed-loop system with a saturation function Eq.(3) is asymptotically stable.

III. MAIN RESULTS

In establishing our main outcomes, the following lemmas are useful.

Lemma 1. (Schur Complement Equivalence) Given constant matrices $M_k(k=1,2,3)$ where $M_1 = M_1^T$ and $0 < M_2 = M_2^T$, then $M_1 + M_3^T M_2^{-1} M_3 < 0$ if and only if

$$\begin{bmatrix} M_1 & M_3^T \\ M_3 & -M_2 \end{bmatrix} < 0,$$

or

$$\begin{bmatrix} -M_2 & M_3 \\ M_3^T & M_1 \end{bmatrix} < 0.$$

A. OBSERVER-BASED CONTROLLER DESIGN OF MPC WITHOUT HARD CONSTRAINTS

In this part, we will provide a number of sufficient conditions in constrained systems, which assure the performance that we desire by means of the quadratic function method. Therefore, we can get the dynamic output feedback controllers which are based on RMPC. To be exact, firstly, sufficient conditions are given to meet the terminal constraint set conditions in **Op1**, i.e., $\zeta(k+n|k) \in \Upsilon(P(k+n|k),5\rho)$. After that, an auxiliary optimization issue is proposed to seek out the suboptimal solution for the unconstrained system. In addition, the inequation analysis technique is used to deal with the unavailable state $x(k)$ problem of the obtained auxiliary issue, another auxiliary issue is presented for solvability. In the end, by solving this kind of on-line auxiliary optimization problem, we get sufficient conditions to ensure stability in the closed-loop system.

1) Terminal constraint set

The following significant definition is presented before developing the mean results.

Definition 1. Under the control law (6), the set Υ is a robust positive invariant (RPI) set for system (1) if $\zeta(k) \in \Upsilon$ implies $\zeta(k+1) \in \Upsilon$.

On the basis of the on-line optimization issue **Op1**, we need to satisfy the following conditions so that the set $\Upsilon(P(k+n|k),5\rho)$ is the terminal constraint set for **Op1**:

C1: there is a quadratic function determined by

$$V(\zeta(k+n|k)) \triangleq \zeta^T(k+n|k)P(k+n|k)\zeta(k+n|k) \tag{15}$$

such that

$$V(\zeta(k+n+1|k)) - V(\zeta(k+n|k))$$

$$\leqslant -\zeta^T(k+n|k)Q\zeta(k+n|k) - u^T(k+n|k)Ru(k+n|k) \tag{16}$$

C2: the set $\Upsilon(P(k+n|k),5\rho)$ is an RPI set.

Next, we will discuss two conditions that we mention above one after another.

Firstly, according to the RLMI approach, the theorem provides a sufficient condition for the system (10) as follows.

Theorem 1 : We give $\gamma > 0, \Gamma > 0$ and $\{F(k),H(k)\}_{0\le k\le N}$. If there are a series of positive definite matrices $\{P(k)\}_{0\le k\le N+1}$ with $P(0) \le \gamma^2\bar{\Gamma}(\bar{\Gamma} \triangleq$

$diag\{0,\Gamma\})$, many positive scalars $\{\tau(k)\}_{0\leq k\leq N}$, a series of real value scalars $\{\varepsilon(k)\}_{0\leq k\leq N}$ such that

$$\Omega(k) \triangleq \bar{\Omega}(k) + \check{\Phi} + \bar{R} - \tau(k)\Omega(ik) - \varepsilon(k)\Omega(\sigma k) \leq 0, \tag{17}$$

where

$$\bar{\Omega}(k) \triangleq \tilde{f}^T(k)P(k+1)\tilde{f}(k) - diag\{P(k),0,0,0\}, \tag{18}$$

$$\check{\Phi} \triangleq diag\{Q,0,0,0\},$$

$$\bar{R} \triangleq diag\{0,0,0,\sqrt{R}F(k),0\},$$

$$\begin{aligned}\Omega(ik) \triangleq\ & diag\{0,0,0,0,I\} - \pi^T\Xi(k)\tilde{g}(k)\\ &-\tilde{g}^T(k)\Xi^T(k)\pi + \tilde{g}^T(k)\Lambda(k)\tilde{g}(k),\end{aligned} \tag{19}$$

$$\Omega(\sigma k) \triangleq diag\{-\sigma(k+1)+\lambda\sigma(k),0,0,0,0\} + \tilde{g}^T(k)\Re(k)\tilde{g}(k), \tag{20}$$

$$\tilde{g}(k) \triangleq \begin{bmatrix}0 & 0 & C(k) & -C(k) & 0\end{bmatrix}.$$

The following theorem proves the condition $\mathbf{C}1$.

Lemma 2. Let positive definite and symmetric matrices Q_1,Q_2,Q_3,Q_4,Q_5 as well as R be provided. For system (10) which is controlled by Eq.(6), if there exists a positive scalar $\rho > 0$, positive definite and symmetric matrices $\tilde{Q}(k),\tilde{Q}(k+1)$ and matrices ω, $l = 1,2,3,...,L$ so that the conditions hold as follows:

$$\begin{bmatrix}\vec{Q}^l(k) & * & * & *\\ \tilde{\Phi}^l & \bar{\rho}I & * & *\\ \check{Y} & 0 & \rho I & *\\ \vec{\Psi}^l & 0 & 0 & \tilde{Q}(k+1)\end{bmatrix} \geq 0, \tag{21}$$

where $\tilde{Q}(k) = diag\{\tilde{Q}_i\}(i=1,2,3,4,5)$, $\tilde{Q}(k+1) = diag\{\tilde{Q}_j\}(j=1,2,3,4,5)$,

$$\vec{Q}^l(k) = \begin{bmatrix}\bar{Q}_1 & 0 & 0 & 0 & 0\\ 0 & \bar{Q}_2 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\tilde{\Phi}^l = \begin{bmatrix}\sqrt{Q_1}S_{11} & 0 & 0 & 0 & 0\\ 0 & \sqrt{Q_2}S_{11} & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\check{Y} = \begin{bmatrix}0 & 0 & 0 & \sqrt{R}\omega & 0\end{bmatrix},$$

$$\vec{\Psi}^l = \begin{bmatrix}S_{11} & 0 & 0 & 0 & 0\\ 0 & 0 & A^l & B^l\omega - A^l - B^{(0)}\omega & -H(k)\end{bmatrix},$$

$$\bar{\rho}I = \begin{bmatrix}\rho I & 0 & 0 & 0 & 0\\ 0 & \rho I & 0 & 0 & 0\\ 0 & 0 & \rho I & 0 & 0\\ 0 & 0 & 0 & \rho I & 0\\ 0 & 0 & 0 & 0 & \rho I\end{bmatrix},$$

$$\bar{Q}_1 = (S_{11})^T + S_{11} - \tilde{Q}_1,$$

$$\bar{Q}_2 = (S_{11})^T + S_{11} - \tilde{Q}_2,$$

$$\omega = F(k)\tilde{Q}_4,$$

we have Eq.(16) with $V(k+n)$ determined by Eq.(15). In addition, the related output feedback gains of control law (6) is provided by

$$F(k) = \omega\tilde{Q}_4^{-1}. \tag{22}$$

Proof : Determine a quadratic function according to (15), i.e.,

$$V(\zeta(k+n|k)) \triangleq \zeta^T(k+n|k)P(k+n|k)\zeta(k+n|k), \tag{23}$$

where $P(k+n) = diag\{P_i(k+n)\}(i=1,2,3,4,5)$ is the positive definite and symmetric matric to be fixed.

Calculate the difference of Eq.(15) according to system (10) yields.

$$\begin{aligned}\triangle V(\zeta(k+n|k)) &= V(\zeta(k+n+1|k)) - V(\zeta(k+n|k))\\ &= \zeta^T(k+n+1|k)P(k+n+1)\zeta(k+n+1|k)\\ &- \zeta^T(k+n|k)P(k+n)\zeta(k+n|k)\\ &= \eta^T(k+n|k)\digamma\eta(k+n|k)\end{aligned} \tag{24}$$

where

$$P1 = \tilde{f}^T(k+n|k)P(k+n+1|k)\tilde{f}(k+n|k)$$

$$P2 = P(k+n|k)diag\{1,0,0,0\}$$

$$\digamma = (P1 - P2).$$

Next, we provide a free matrix:

$$S = \begin{bmatrix}S_{11} & 0\\ 0 & S_{22}\end{bmatrix}$$

where the matrix S_{11} is arbitrary diagonal, and the matrix S_{22} is arbitrary with the appropriate dimension.

Substitute the conditions$(i=1,2)$

$$\begin{aligned}&S_{11} + (S_{11})^T - \tilde{Q}_i - (S_{11})^T\tilde{Q}_i^{-1}S_{11}\\ &= -(\tilde{Q}_i - S_{11})\tilde{Q}_i^{-1}(\tilde{Q}_i - S_{11})^T \leq 0,\end{aligned}$$

into Eq.(21), we will get

$$\begin{bmatrix}\overleftarrow{Q}^l(k) & * & * & *\\ \tilde{\Phi}^l & \bar{\rho}I & * & *\\ \check{Y} & 0 & \rho I & *\\ \vec{\Psi} & 0 & 0 & \tilde{Q}(k+1)\end{bmatrix} \geq 0, \tag{25}$$

where

$$\overleftarrow{Q}^l(k)=\begin{bmatrix}(S_{11})^T\tilde{Q}_1^{-1}(S_{11}) & 0 & 0 & 0 & 0\\ 0 & (S_{11})^T\tilde{Q}_2^{-1}(S_{11}) & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}.$$

Afterward, it can be known that we will get Eq.(25) if pre- and post-multiplying the following inequality with $\mathrm{diag}\{S_{11}, S_{11}, I, \tilde{Q}_4, \underbrace{I, ..., I}_{9}\}$ and its transpose:

$$\begin{bmatrix}\overrightarrow{Q}^l(k) & * & * & *\\ \sqrt{\breve{Q}} & \bar{\rho}I & * & *\\ \bar{Y} & 0 & \rho I & *\\ \hat{f}^l & 0 & 0 & \tilde{Q}(k+1)\end{bmatrix}\geq 0, \tag{26}$$

where

$$\overrightarrow{Q}^l(k)=\begin{bmatrix}\tilde{Q}_1^{-1} & 0 & 0 & 0 & 0\\ 0 & \tilde{Q}_2^{-1} & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\sqrt{\breve{Q}}=\begin{bmatrix}\sqrt{Q_1} & 0 & 0 & 0 & 0\\ 0 & \sqrt{Q_2} & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\bar{Y}=\begin{bmatrix}0 & 0 & 0 & \sqrt{R}F & 0\end{bmatrix},$$

$$\hat{f}^l=\begin{bmatrix}1 & 0 & 0 & 0 & 0\\ 0 & 0 & A^l & B^lF-A^l-B^{(0)}F & -H(k)\end{bmatrix},$$

Because system (10) is polytopic uncertain, i.e., Eq.(26) is affine in Ω and means that

$$\begin{bmatrix}\overrightarrow{Q}^l(k) & * & * & *\\ \sqrt{\breve{Q}} & \bar{\rho}I & * & *\\ \bar{Y} & 0 & \rho I & *\\ \hat{f}^l(k) & 0 & 0 & \tilde{Q}(k+1)\end{bmatrix}\geq 0. \tag{27}$$

According to the Schur Complement, deduce from Eq.(27) that

$$\tilde{f}^T(k)\tilde{Q}^{-1}(k+1)\tilde{f}(k)+\bar{Y}^T\rho^{-1}\bar{Y}+\rho^{-1}\breve{Q}-\tilde{Q}^{-1}(k)\leq 0. \tag{28}$$

Multiply both sides of Eq.(28) with $\rho > 0$ and define $P(k)=\rho\tilde{Q}^{-1}(k), P(k+1)=\rho\tilde{Q}^{-1}(k+1)$, we can get

$$\tilde{f}^T(k)P(k+1)\tilde{f}(k)+\bar{Y}^T\bar{Y}+\breve{Q}-P(k)\leq 0. \tag{29}$$

Pre- and post-multiplying Eq.(29) with $\eta^T(k+n|k)$ and its transpose means

$$\begin{aligned}\Gamma(k)-\eta^T(k+n|k)P(k)\eta(k+n|k)+\\ \eta^T(k+n|k)\breve{Q}\eta(k+n|k)+\Omega\leq 0.\end{aligned} \tag{30}$$

where

$$\Gamma(k)=(\tilde{f}(k)\eta(k+n|k))^TP(k+1)(k+n+1)(\tilde{f}(k)\eta(k+n|k)),$$

$$\Omega=(\bar{Y}\eta(k+n|k))^T(\bar{Y}\eta(k+n|k)).$$

Notice Eqs.(6), (10), and (24), condition (16) can be assured by Eq.(30). So we can complete the proof.

Let us handle the saturation function $\phi(k)$. We can know from (5) that

$$(\phi(k)-\Lambda(k)r(k))^T(\phi(k)-r(k))\leq 0, \tag{31}$$

which, by noticing $\Xi(k)\triangleq 1/2(\Lambda(k)+I)$, can be represented by

$$\begin{aligned}\phi^T(k)\phi(k)-\phi^T(k)\Xi(k)r(k)-r^T(k)\Xi^T(k)\phi(k)\\ +r^T(k)\Lambda(k)r(k)\leq 0,\end{aligned} \tag{32}$$

We know that

$$\begin{aligned}r(k)&=y(k)-\hat{y}(k)\\ &=C(k)x(k)-C(k)\hat{x}(k)\\ &=\tilde{g}(k)\eta(k).\end{aligned} \tag{33}$$

In the end, (32) can be descried by

$$\begin{aligned}&\eta^T(k)diag\{0,0,0,0,I\}\eta(k)-\eta^T(k)\pi^T\Xi(k)\tilde{g}(k)\eta(k)\\ &-\eta^T(k)\tilde{g}^T(k)\Xi^T(k)\pi\eta(k)+\eta^T(k)\tilde{g}^T(k)\Lambda(k)\tilde{g}(k)\eta(k)\leq 0,\end{aligned} \tag{34}$$

equivalently,

$$\eta^T\Omega(ik)\eta(k)\leq 0, \tag{35}$$

where $\Omega(ik)$ is given in (19) and

$$\pi=\begin{bmatrix}0 & 0 & 0 & 0 & I\end{bmatrix},$$

According to the limit (4) which is imposed on the saturation level, we have

$$\sigma(k+1)=\lambda\sigma(k)+(y(k)-\hat{y}(k))^T\Re(k)(y(k)-\hat{y}(k)),$$

which can be described by

$$\begin{aligned}\eta^T(k)(diag\{-\sigma(k+1)+\lambda\sigma(k),0,0,0,0\}\\ +\tilde{g}^T(k)\Re(k)\tilde{g}(k))\eta(k)=0\end{aligned}$$

equivalently,

$$\eta^T(k)\Omega(\sigma k)\eta(k)=0.$$

where $\Omega(\sigma k)$ is given in (20).

Remark 3. Specifically, for the polytopic uncertainties of system matrices in (1) and hard constraints in (7) and (8), some theoretical analysis techniques which may generate sufficient and necessary conditions like Riccati equations are usually invalid. By comparison, the Lyapunov-like function approach makes the solution adequate.

Next, we are going to study the condition **C**2 in the terminal constraint set. Namely, we need to seek out sufficient

This article has been accepted for publication in IEEE Access. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2022.3221476

conditions, which can satisfy that the set $\Upsilon(P(k), 5\rho)$ is an RPI set.

From Definition 1, we know the requirements should be met to ensure the RPI set $\Upsilon(P(k), 5\rho)$ as follows.

$R1$. at the time instant $n = 0$, the initial state is part of the set Υ, i.e,

$$\eta^T(k)\tilde{Q}^{-1}(k)\eta(k) \leqslant 5;$$

$R2$. future states $\eta(k+n|k),\ n > 0$ are part of the set Υ.

In the following content, we handle the above requirements one by another.

According to the Schur Complement, $\mathbf{R}1$ holds if and only if

$$\begin{bmatrix} 5 & * \\ \eta(k) & \tilde{Q}(k) \end{bmatrix} \geq 0. \tag{36}$$

In addition, on account of Lemma 2 and $\zeta(k) = [I\ 0\ 0\ 0\ 0]\eta(k)$, it is easy to see from Eq.(36) that

$$V(\zeta(k+n+1)) \leq V(\zeta(k+n)) \leq ... \leq V(\zeta(k)) \leq 5\rho. \tag{37}$$

which means that predicted states $\zeta(k+n|k)$ are part of the set Υ so long as Υ contains the initial state $\zeta(k)$. And this guarantees that the set Υ is an RPI set.

Up to now, by conditions (21) and (36), we can ensure the terminal constraint set. That is to say, the condition $\zeta(k+n|k) \in \Upsilon(P(k), 5\rho)$ of **Op1** is met.

2) Auxiliary optimization problems

About this part, for the unconstrained system in this paper, we will discuss how to deal with the **Op1**.

Op1 is an optimization issue that includes parameter uncertainties in an infinite time horizon, handle it directly is not easy. On the contrary, to seek out a sub-optimal solution, we will propose a certain auxiliary optimization issue. We try to present this auxiliary problem next.

Evidently, if condition (21) holds, we have Eq.(16). This means that $\zeta(\infty|k) = 0$ and $V(\infty) = 0$. Sum up both sides of Eq.(16) from $n = 0$ to $n = \infty$ and use Eq.(12) yields

$$J_\infty(k) \leq V(k) = \zeta^T(k)P(k)\zeta(k) \leqslant 5\rho, \tag{38}$$

which means

$$\max_{(A(k),B(k),C(k))\in\Delta} J_\infty(k) \leqslant 5\rho. \tag{39}$$

This provides a superior limit of the objective function of **Op1**.

On the basis of the above analysis, we are ready to present an auxiliary optimization issue for the system without constraints:

$$\mathbf{Op2}: \min_{\tilde{Q}_i(i=1,2,3,4,5),F(k),H(k)} 5\rho, \quad s.t\ Eqs.(21)\ and\ (36).$$

The condition (36) can't be checked online because of the immeasurable state $x(k)$. In what follows, we will handle the problem of unavailable states in Eq.(36). The following significant assumption is shown before proceeding with the endeavor.

Assumption 1. On the basis of the initial state of the system (1), we show a known set:

$$x(0) \in \{x(k)|x^T(k)S^{-1}x(k) \leq 1\}, \tag{40}$$

where matrix $S > 0$ can be specified in advance from actual experience.

Lemma 3. In view of the system (1) which is controlled by Eq.(7), if there are positive definite and symmetric matrices $\hat{Q}_i(i = 1, 2, 3, 4, 5)$, for Assumption 1, such that

$$\begin{bmatrix} 2 & * & * \\ \hat{x}(k) & \tilde{Q}_4 & * \\ \phi(k) & 0 & \tilde{Q}_5 \end{bmatrix} \geq 0, \tag{41}$$

$$\begin{bmatrix} \tilde{Q}(k) & * \\ \hat{f}^l\tilde{Q}(k) & \hat{Q} \end{bmatrix} \geq 0, \tag{42}$$

$$\tilde{Q}_1 \geq \hat{Q}_1, \tilde{Q}_2 \geq \hat{Q}_2, \tilde{Q}_3 \geq \hat{Q}_3, \tilde{Q}_4 \geq \hat{Q}_4, \tilde{Q}_5 \geq \hat{Q}_5, \tilde{Q}_1 \geq S, \tag{43}$$

$$\begin{bmatrix} \tilde{Q}(k) & * & * \\ [0,0,I,0]\hat{f}^l\tilde{Q}(k) & 2/5\hat{Q}_4 & * \\ [0,0,0,I]\hat{f}^l\tilde{Q}(k) & 0 & 2/5\hat{Q}_5 \end{bmatrix} \geq 0. \tag{44}$$

hold, where $\hat{Q} \triangleq diag\{\hat{Q}_1, \hat{Q}_2, \hat{Q}_3, \hat{Q}_4, \hat{Q}_5\}$ and $\hat{f}^l$ mean the vertices of $\tilde{f}(k),\ l = 1, 2, 3, ..., L$, so we can always ensure the condition (36).

Proof : By taking the similar to [41], the above lemma can be gotten and is omitted. So the proof is completed.

According to Assumption 1 and Lemma 3, we are able to transform the problem **Op2** into the approximate optimization for the solvability as follows:

$$\mathbf{Op3}: \min_{\hat{Q}_i>0,\tilde{Q}_i>0(i=1,2,3,4,5),F(k),H(k)} 5\rho,$$

$$s.t\ Eqs.(22),\ (41),\ (42)\ and\ (44).$$

3) Feasibility and stability

In what follows, we will make the feasibility of the presented problems clear. And to move forward a single step, we will represent the stability of the system (1), which is controlled by Eq.(6).

Theorem 2. The positive definite and symmetric matrices $Q_i(i = 1, ..., 5)$ and R are given. We take the system (1) controlled by Eq.(7) into account. If a feasible solution to the optimization issue **Op3** at the initial time instant k exists, after that, the corresponding feasible solution also exists at any future time instant $t > k$. In addition, the closed-loop system is asymptotically stable and Eq.(7) determines feedback gains.

Proof :
(1)*Feasibility*. At the initial moment k, let us assume that the optimization issue **Op3** is feasible. For all the future time instant $k+n,\ n \geq 1$, we have to prove that the issue **Op3** is feasible too. It is not hard to see that only the condition (21) depends on the states, and other conditions are feasible at any future time instant $t > k$ so long as they are feasible at the time instant k. To this extent, we need to prove that condition (21) is feasible for the future time instant. Namely, the feasibility of condition (36) in **Op2** needs to be proved. From Eqs.(1) and (10), we can get the following relations:

$$\zeta(k+n|k+n) = \zeta(k+n), n \geq 1 \tag{45}$$

$$\zeta(k+1|k) = \tilde{f}(k)\eta(k|k), \tag{46}$$

$$\zeta(k+1) = \tilde{f}(k)\eta(k), \tag{47}$$

for some $\tilde{f}(k) \in \triangle$. On the basis of RPI set and $\zeta(k) = [I\ 0\ 0\ 0\ 0]\eta(k)$, we have

$$\eta^T(k+1|k)Q^{-1}(k+1)\eta(k+1|k) < \eta^T(k)Q^{-1}(k)\eta(k) < 5.$$

and we get

$$\zeta^T(k+1|k)Q^{-1}(k+1)\zeta(k+1|k) < \zeta^T(k)Q^{-1}(k)\zeta(k) < 5. \tag{48}$$

Using Eqs.(46) and (47), we can get the following condition from Eq.(48)

$$\zeta^T(k+1)Q^{-1}(k+1)\zeta(k+1) < 5. \tag{49}$$

This indicates that condition (36) is feasible at the time instant $k+1$. In addition, this process is able to go on for any time $k+2, k+3, ...$ in the future.

(2)*Stability*. We need to build a strictly decreasing quadratic function $\bar{V}(\zeta(k)) = \zeta^T(k)P^*(k)\zeta(k)$, which is used to prove that the system (1) controlled by Eq.(6) is asymptotically stable, the subscript "*" means the optimal solution of issue **Op3** at the time instant k. On the basis of the above Feasibility, we have

$$\begin{aligned}\zeta^T(k+1)P^*(k+1)\zeta(k+1) &\leq \zeta^T(k+1)P^*(k)\zeta(k+1)\\ &< \zeta^T(k)P^*(k)\zeta(k)\end{aligned} \tag{50}$$

where $P(k), P(k+1)$ without the subscript "*" means the feasible solution. So the quadratic function $\bar{V}(\zeta(k))$ is strictly decreasing, we can end the proof of the theorem.

Remark 4. As a note, it is very difficult to provide sufficient and necessary standards to deal with the RMPC problem, which is subject to hard constraints. Inequality transformation techniques may lead to some conservatism, see e.g. [42]. However, due to the property of the MPC strategy, if it is feasible at the initial time instant, the solvable optimization problem is feasible for any future time instant.

B. OBSERVER-BASED CONTROLLER DESIGN OF MPC WITH HARD CONSTRAINTS

About this part, the MPC issue for the polytopic systems that along with hard limits is going to be handled, which is based on the establishments made before. After that, a number of sufficient conditions are gotten. In the end, subject to some conditions, an algorithm is proposed to solve an online optimization issue.

1) Controller design with a saturation function

In the first place, to ensure the hard constraints on the inputs and states Eqs.(7), (8), and a few inequalities are proposed. Afterwards, based on MPC for the constrained system, the controllers are designed and the related algorithm is proposed according to the optimization problem.

Lemma 4. If there are positive definite and symmetric matrices $\tilde{Q}_3, \tilde{Q}_4$, ω, hard constraints on the inputs and the states Eqs. (7) and (8) are met so that the following conditions

$$\begin{bmatrix} I & * \\ \omega^T & \bar{u}^2\tilde{Q}_4 \end{bmatrix} \geq 0, \tag{51}$$

$$\begin{bmatrix} I & * \\ \tilde{Q}_3 & \bar{x}^2\tilde{Q}_3 \end{bmatrix} \geq 0. \tag{52}$$

hold.

Proof : Taking into account the limitations on the input predictions at first, for any $n \geq 0$, we get from Eq.(6) that

$$\begin{aligned}|[u(k+n|k)]_c|^2 &= |r_c(F\hat{x}(k+n|k))|^2\\ &= |r_c\omega\tilde{Q}_4^{-1}\hat{x}(k+n|k)|^2\\ &= |r_c\omega\tilde{Q}_4^{-1/2}{S_{11}}^{-1/2}\hat{x}(k+n|k)|^2\\ &\leq \parallel r_c\omega\tilde{Q}_4^{-1/2}\parallel^2 \parallel \tilde{Q}_3^{-1/2}\hat{x}(k+n|k)\parallel^2\\ &(from\ Cauchy\ Schwarz\ inequality)\\ &\leq \parallel r_c\omega\tilde{Q}_4^{-1/2}\parallel^2\\ &= r_c(\omega\tilde{Q}_4^{-1}\omega^T)r_c^T\\ &\leq \bar{u}^2\end{aligned} \tag{53}$$

where r_c is the cth row of an n_u-ordered identity matrix. Eq.(53) holds if and only if Eq.(51) holds by the Schur Complement.

Afterwards, in terms of the constraint on the state predictions, we can get the following by the similar technique presented above

$$\begin{aligned}|[x(k+n|k)]_d|^2 &= |r_d\tilde{Q}_3^{1/2}\tilde{Q}_3^{-1/2}[x(k+n|k)]|^2\\ &\leq \| r_d\tilde{Q}_3^{1/2} \|^2 \| \tilde{Q}_3^{-1/2}[x(k+n|k)] \|^2\\ &(from\ Cauchy\ Schwarz\ inequality)\\ &\leq \| r_d\tilde{Q}_3^{1/2} \|^2\\ &= r_d\tilde{Q}_3 r_d^T\\ &\leq \bar{x}^2\end{aligned} \tag{54}$$

where r_d is the dth row of an n_x-ordered identity matrix. Eq.(54) holds if and only if Eq.(52) holds by the Schur Complement.

For a constrained system, according to Lemma 4, a further auxiliary optimization issue can be achieved by

$$\mathbf{Op4}: \min_{\hat{Q}_i>0,\tilde{Q}_i>0(i=1,2,3,4,5),F(k),H(k)} 5\rho,$$

$$s.t\ Eqs.(21),\ (40),\ (41),\ (43)\ and\ (51),\ (52).$$

On the basis of the above discussion, we get ready to show the following theorem so that the system (1) with hard constraints controlled by Eq.(6) is asymptotically stable.

Remark 5. Significantly, it is difficult to directly solve the online optimization problem Op1 which includes parameter uncertainties over an infinite horizon. To deal with such a difficulty, the auxiliary optimization problem Op2 has been formulated by giving a certain upper bound of the objective function of Op1. However, Op2 is still unsolvable due to the immeasurable state $x(k)$ of the condition (28). So the optimization problem Op2 has been transformed into the problem Op3 by using Assumption 1 and Lemma 3. Based on the established results for unconstrained systems and Lemma 3 for hard constraints, a further auxiliary optimization problem Op4 has been presented for the constrained system.

Theorem 3. Eq.(1) and (7), (8) are controlled by Eq.(6) and a system with hard constraints is considered. At the initial time instant k, if the optimization problem **Op4** is feasible, then for all the future time instants $t > k$, the optimization problem **Op4** is feasible too. In addition, with the feedback gains $F(k) = \omega(k)\tilde{Q}_4^{-1}$, the closed-loop system is stable.

Proof : The proof procedure is ignored because it is similar to that in Theorem 2.

2) Algorithm of RMPC for constrained system

About this part, consider the dynamic OFRMPC strategy, the algorithm for the constrained observer-based systems with a saturation function is to be shown.

Algorithm:

$Off-linepart:$

Define an initial state $\eta(0) = [1\ e^T(0)\ x^T(0)\ \hat{x}^T(0)\ \phi^T(0)]^T$, proper matrix S so that $x(0) \in \{x(k)|x^T(k)S^{-1}x(k) \leq 1\}$ is feasible at $k = 0$.

$On-linepart:$

Step 1. Firstly, at the moment k, address the optimization issue **Op4** to get the controller gain $F(k)$ and $H(k)$ by the observer and parameters in the Off-line part.

Step 2. Secondly, calculate $F(k) = \omega(k)\tilde{Q}_4^{-1}$, act $u(k) = F(k)\hat{x}(k)$ on the plant and return to *Step* 1.

By the way, with the help of the "Algorithm", the optimization problems can be solved online to obtain the controller gain $F(k)$. It should be noted that only the first component $u(k)$ of a set of predicted inputs $u(k), u(k+1|k), u(k+2|k), ...$ will be acted on the plant at every time instant.

Note that the controllers we design are determined by the "min-max" problem over an infinite time horizon. Although the MPC strategy we proposed in this paper is based on an infinite horizon, the optimization problem needs to be reconstructed by the new measurements. In other words, at the next time step, a new optimization problem will be reformulated to solve a new controller in the framework of the RMPC approach.

Remark 6. The dynamic OFRMPC issue is handled for observer-based linear systems which along with polytopic uncertainties. The main unique properties of our outcomes are outlined: (1)because of the existence of measurement outliers, for systems with polytopic uncertainties, the observer with a saturation function is provided to handle the dynamic OFRMPC problem; (2)a number of methodologies are made to obtain desired results on the saturation function; (3)the optimization issue **Op4** is formed to seek out a certain superior limit of the quadratic cost function with the derivation; (4)an online dynamic OFRMPC algorithm is put forward to get some controllers, which makes the closed-loop constrained system mentioned before asymptotically stable.

IV. ILLUSTRATIVE EXAMPLE

About this part, we demonstrate the effectiveness of the proposed OFRMPC strategy via two illustrative examples. The system in a distillation process has two manipulated variables, boil-up radio and reflux; two controlled variables, bottom composition and top.

Example 1: We get the discrete-time system according to selecting the same sampling period. From the practical viewpoint, the parameter uncertainties in system matrices need to be considered. We consider this system model:

$$\begin{aligned}x(k+1) &= \begin{bmatrix}0.948 & 0\\ 0 & 0.948\end{bmatrix}x(k) + \begin{bmatrix}0.512 & 0.015\\ 0.086 & 0.469\end{bmatrix}u(k)\\ &\triangleq Ax(k) + Bu(k),\\ y(k) &= \begin{bmatrix}1.5 & 0\\ 0 & 1.5\end{bmatrix}x(k) \triangleq Cx(k),\end{aligned}$$

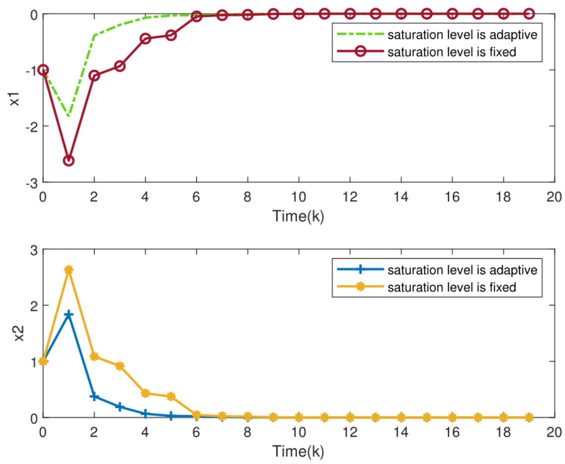

FIGURE 2. The estimation error subject to the different saturation level.

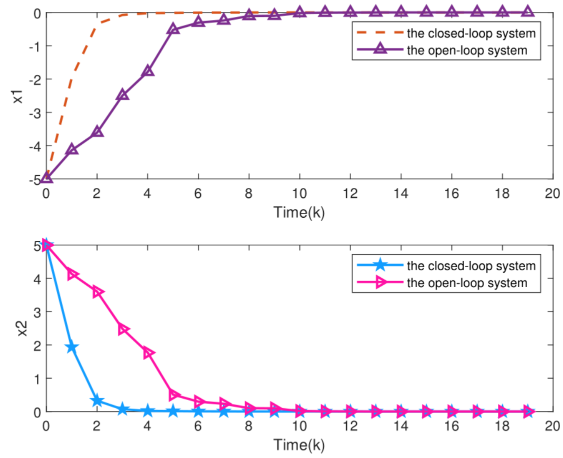

FIGURE 3. State responses for the open-loop and closed-loop systems.

with the initial value

$$x(0) = \begin{bmatrix} -5 \\ 5 \end{bmatrix}, \ \hat{x}(0) = \begin{bmatrix} -4 \\ 4 \end{bmatrix}.$$

The initial condition $x(0)$ corresponds to the feasible system states of the distillation column, where two components denote the initial values of top composition and bottom composition, respectively. After that, to better satisfy the polytopic uncertainties requirements of the practical system, we choose system parameters:

$$A^{(1)} = \begin{bmatrix} 0.948 & 0 \\ 0 & 0.948 \end{bmatrix}, \quad A^{(2)} = \begin{bmatrix} 0.096 & 0 \\ 0 & 0.089 \end{bmatrix},$$

$$A^{(0)} = 1/2(A^{(1)} + A^{(2)}).$$

$$B^{(1)} = B^{(2)} = B = \begin{bmatrix} 0.512 & 0.015 \\ 0.086 & 0.469 \end{bmatrix},$$

$$B^{(0)} = 1/2(B^{(1)} + B^{(2)}).$$

$$C^{(1)} = C^{(2)} = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix},$$

$$Q_1 = Q_2 = Q_3 = Q_4 = Q_5 = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix},$$

$$R = 0.01 * \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}.$$

The superior limits of input, as well as state, are given by $\bar{u} = 55$ and $\bar{x} = 950$ separately. We define the weighting matrix as

$$S = \begin{bmatrix} 0 & 0.9 \\ 0.9 & 0 \end{bmatrix}.$$

The simulation results are shown in FIGURE 2 and FIGURE 3. To be exact, FIGURE 2 plots the trend of estimation errors $e(k)$ for a system with the fixed saturation level and a system with the adaptive saturation level. It is not difficult to know that the proposed algorithm, which along with adaptive changed saturation level is able to alleviate the impact of measurement outliers effectively, so the estimation performance is improved. FIGURE 3 depicts the state trends of systems without and with controllers. And we can find that the closed-loop system addressed is more stable than the open-loop system with the proposed dynamic OFRMPC algorithm.

Example 2: We consider the second example as an unstable system without control. The parameters are given as follows:

$$A^{(1)} = A = \begin{bmatrix} 1.501 & 0.1 \\ -0.13 & -0.1 \end{bmatrix}, \ A^{(2)} = \begin{bmatrix} 1 & 1.5 \\ -0.1 & -0.2 \end{bmatrix},$$

$$B^{(1)} = B^{(2)} = \begin{bmatrix} 1.24 & 0.51 \\ 0.18 & 0.69 \end{bmatrix}, C^{(1)} = C^{(2)} = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}.$$

As can be seen from FIGURE 4, the trend of estimation errors $e(k)$ for a system with the adaptive saturation level shows that the proposed algorithm can effectively alleviate the impact of measurement outliers. FIGURE 5 depicts the state trends of systems without and with controllers. From FIGURE 5, it can be seen that the system is stable with the designed RMPC controller, which means that the presented RMPC scheme is necessarily effective.

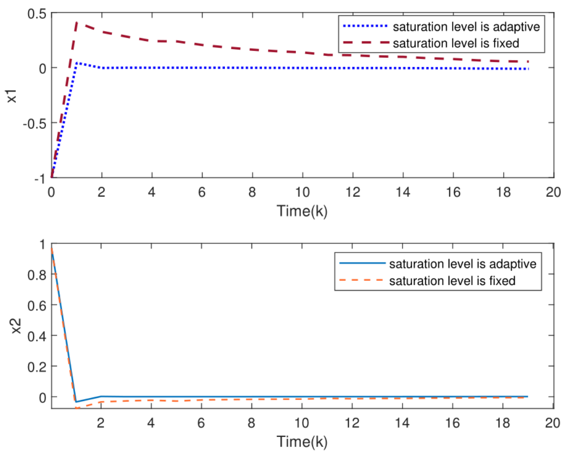

FIGURE 4. The estimation error subject to the different saturation level.

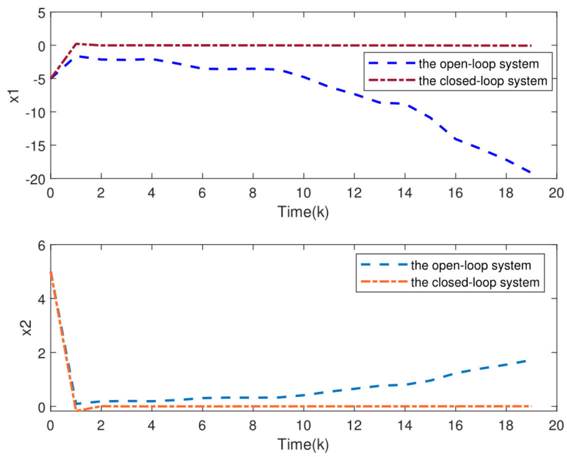

FIGURE 5. State responses for the open-loop and closed-loop systems.

V. CONCLUSION

In this paper, we have investigated the dynamic OFRMPC problem for the discrete-time polytopic uncertain system subject to possible measurement outliers. For the sake of alleviating the effects from possible measurement outliers, an observer has been designed with a saturated output injection where the saturation level is defined dynamically according to the errors. Considering the states are often unmeasurable in the practical system, the observer-based output feedback controller has been designed. In addition, the control law has been obtained by solving an optimization problem with convex constraints. And an iterative algorithm has been developed to find the sub-optimal solution. In the end, two simulation examples have been applied to demonstrate the effectiveness of the proposed RMPC algorithm.

ACKNOWLEDGEMENTS

This work was supported in part by the Natural Science Foundation of Zhejiang LQ22F030011, Public Welfare Technology Application and Research Projects of Zhejiang LGG22F030016 and Natural Science Foundation of Huzhou 2021YZ05.

REFERENCES

[1] H. Li, Y. Shi, "Robust distributed model predictive control of constrained continuous-time nonlinear systems: a robustness constraint approach," IEEE Trans. Autom. Control, vol. 59, no. 6, pp. 1673–1678, Jul. 2014, 10.1109/ACC.2014.6858906.

[2] H. Li, Y. Shi, "Event-triggered robust model predictive control of continuous-time nonlinear systems," Automatica, vol. 50, no. 5, pp. 1507–1513, Mar. 2014, 10.1016/j.automatica.2014.03.015.

[3] L. Dai, Y. Xia, Y. Gao, M. Cannon, "Distributed stochastic MPC for systems with parameter uncertainty and disturbances," Int. J. Robust Nonlin. Control, vol. 28, no. 6, pp. 2424–2441, Apr. 2018, 10.1002/rnc.4024.

[4] X. Mi, Y. Zou, S. Li, "Event-triggered MPC design for distributed systems toward global performance," Int. J. Robust Nonlin. Control, vol. 28, no. 4, pp. 1474–1495, Oct. 2018, 10.1002/rnc.3969.

[5] Y. Song, Z. Wang, D. Ding, G. Wei, "Robust H_2/H_∞ model predictive control for linear systems with polytopic uncertainties under weighted MEF-TOD protocol," IEEE Trans. Syst. Man Cyber. Syst, vol. 49, no. 7, pp. 1470–1481, Jul. 2019, 10.1109/TSMC.2017.2757760.

[6] H. Li, W. Yan, Y. Shi, "Triggering and control codesign in self-triggered model predictive control of constrained systems: with guaranteed performance," IEEE Trans. Autom. Control, vol. 63, no. 11, pp. 4008–4015, Nov. 2018, 10.1109/TAC.2018.2810514.

[7] Y. Zou, J. Lam, Y. Niu, D. Li, "Constrained predictive control synthesis for quantized systems with Markovian data loss," Automatica, vol. 55, no. C, pp. 217–225, May. 2015, 10.1016/j.automatica.2015.03.016.

[8] Y. Xia, H. Yang, P. Shi, M. Fu, "Constrained infinite-horizon model predictive control for fuzzy-discrete-time systems," IEEE Trans. Fuzzy Syst, vol. 18, no. 2, pp. 429–436, Apr. 2010, 10.1109/TFUZZ.2010.2043441.

[9] Z. Li, J. Deng, R. Lu, Y. Xu, J. Bai, C. Y. Su, "Trajectory-tracking control of mobile robot systems incorporating neural-dynamic optimized model predictive approach," IEEE Trans. Syst. Man Cyber. Syst, vol. 46, no. 6, pp. 740–749, Jun. 2016, 10.1109/TSMC.2015.2465352.

[10] J. Zhang, H. Yang, X. Jia, "Model predictive control with mixed performances for uncertain positive systems," IEEE Access, vol. 6, pp. 10221–10230, 2018, 10.1109/ACCESS.2018.2799159.

[11] D.Q. Mayne, H. Michalska, "Receding horizon control of nonlinear systems," IEEE Trans. Autom. Control, vol. 35, no. 7, pp. 814–824, Jul. 1990, 10.1109/CDC.1989.70083.

[12] H. Chen, F. Allgower, "A quasi-infinite horizon nonlinear model predictive control scheme with guaranteed stability," Automatica, vol. 34, no. 10, pp. 1205–1217, Oct. 1998, 10.23919/ECC.1997.7082300.

[13] Z. Li, H. Xiao, C. Yang, Y. Zhao, "Model predictive control of nonholonomic chained systems using general projection neural networks optimization," IEEE Trans. Syst. Man Cyber. Syst, vol. 45, no. 10, pp. 1313–1321, Oct. 2015, 10.1109/TSMC.2015.2398833.

[14] K. Zhu, Y. Song, D. Ding, "Resilient RMPC for polytopic uncertain systems under TOD protocol: a switched system approach," Int. J. Robust Nonlin. Control, vol. 28, no. 16, pp. 5103–5117, Nov. 2018, 10.1002/rnc.4307.

[15] L. Zhang, B. Wang, Y. Li, Y. Tang, "Distributed stochastic model predictive control for cyber physical systems with multiple state delays and probabilistic saturation constraints," Automatica, vol. 129, pp. 109574, Jul. 2021.

[16] J. Wang, Y. Song, "Resilient RMPC for cyber-physical systems with polytopic uncertainties and state saturation under TOD scheduling: An ADT approach," IEEE Trans. Indust. Inform, vol. 16, no. 7, pp. 4900–4908, Jul. 2020, 10.1109/TII.2019.2938889.

[17] H. Huang, D. Li, Y. Xi, "Mixed h_2/h_∞ robust model predictive control with saturated inputs," Int. J. Syst. Sci, vol. 45, no. 12, pp. 2565–2575, 2014.

[18] J. Li, D. Li, Y. Xi, Y. Xu, Z. Gan, "Output-feedback model predictive control for stochastic systems with multiplicative and additive uncertainty," Int. J. Robust Nonlin. Control, vol. 28, no. 1, pp. 86–102, Jan. 2018, 10.1002/rnc.3856.

[19] Y. Song, Z. Wang, L. Zou, S. Liu, "Endec-decoder-based N-Step model predictive control: Detectability, stability and optimization," Automatica, vol. 135, Jan. 2022, 10.1016/j.automatica.2021.109961.

[20] J. Wang, Y. Song, G. Wei, "Security-based resilient robust model predictive control for polytopic uncertain systems subject to decep-

tion attacks and RR protocol," IEEE Trans. Syst. Man Cyber. Syst, 10.1109/TSMC.2021.3103538, 2021.
[21] C. Liu, H. Li, J. Gao, D. Xu, "Robust self-triggered mincmax model predictive control for discrete-time nonlinear systems," Automatica, vol. 89, pp. 333–339, 2018, 10.1016/j.automatica.2017.12.034.
[22] M. Kothare, V. Balakrishnan, M. Morari, "Robust constrained model predictive control using linear matrix inequalities," Automatica, vol. 32, no. 10, pp. 1361–1379, Oct. 1996, 10.1016/0005-1098(96)00063-5.
[23] L. Zhang, B. Wang, Y. Zheng, A. Zemouche, X. Zhao, C. Shen, "Robust Packetized MPC for Networked Systems Subject to Packet Dropouts and Input Saturation With Quantized Feedback," IEEE Trans. Cyber, Mar. 2022, 10.1109/TCYB.2022.3166855.
[24] Y. Song, Z. Wang, D. Ding, G. Wei, "Robust model predictive control under redundant channel transmission with applications in networked DC motor systems," Int. J. Robust Nonlin. Control, vol. 26, no. 18, pp. 3937–3957, Dec. 2016, 10.1002/rnc.3542.
[25] T. Zhang, G. Feng, J. Lu, W. Xiang, "Robust constrained fuzzy affine model predictive control with application to a fluidized bed combustion plant," IEEE Trans. Control Syst. Technol, vol. 16, no. 5, pp. 1047–1056, Sept. 2008, 10.1109/TCST.2007.916320.
[26] W. Yang, G. Feng, T. Zhang, "Robust model predictive control for discrete-time Takagi-Sugeno fuzzy systems with structured uncertainties and persistent disturbances," IEEE Trans. Fuzzy Syst, vol. 22, no. 5, pp. 1213–1228, 2014.
[27] J. Wang, Y. Song, S. Zhang, S. Liu, "Robust model predictive control for linear discrete-time system with saturated inputs and randomly occurring uncertainties," Asian J. Control, vol. 20, no. 1, pp. 425–436, 2018.
[28] L. Zhang, S. Zhuang, R. D. Braatz, "Switched model predictive control of switched linear systems: feasibility, stability and robustness," Automatica, vol. 67, pp. 8–21, May. 2016, 10.1016/j.automatica.2016.01.010.
[29] H. Michalska, D. Q. Mayne, "Robust receding horizon control of constrained nonlinear systems," IEEE Trans. Autom. Control, vol. 38, no. 11, pp. 1623–1633, Nov. 1993, 10.1109/9.262032.
[30] T. Shi, R. Lu, L. Qiang, "Robust static output feedback infinite horizon RMPC for linear uncertain systems," J. Frankl. Inst, vol. 353, no. 4, pp. 891–902, Mar. 2016, 10.1016/j.jfranklin.2016.01.012.
[31] B. Ding, "Constrained robust model predictive control via parameter-dependent dynamic output feedback," Automatica, vol. 46, no. 9, pp. 1517–1523, Sept. 2010, 10.1016/j.automatica.2010.06.014.
[32] C. Zhu, P. Zhang, L. Li, B. Yang, Z. Lu, "Dynamic output feedback control of networked systems with medium access constraints," IEEE Access, vol. 9, pp. 123075–123087, 2021, 10.1109/ACCESS.2021.3107324.
[33] Y. Song, G. Wei, S. Liu, "Distributed output feedback MPC with randomly occurring actuator saturation and packet loss," Int. J. Robust Nonlin. Control, vol. 26, no. 14, pp. 3036–3057, Nov. 2016, 10.1002/rnc.3489.
[34] D. Li, Y. Xi, F. Gao, "Synthesis of dynamic output feedback RMPC with saturated inputs," Automatica, vol. 49, no. 4, pp. 949–954, 2013, 10.1016/j.automatica.2013.01.010.
[35] J. Qiu, S. Ding, H. Gao, S. Yin, "Fuzzy-model-based reliable static output feedback H_∞ control of nonlinear hyperbolic PDE systems," IEEE Trans. Fuzzy Syst, vol. 24, no. 2, Apr. 2016, 10.1109/TFUZZ.2015.2457934.
[36] K. Zhu, Y. Song, S. Zhang, et al., "Non-fragile observer-based output feedback control for polytopic uncertain system under distributed model predictive control approach," Int. J.Syst. Sci, vol. 48, no. 9-12, pp. 1891–1901, Jul. 2017, 10.1080/00207721.2017.1295329.
[37] R. Gibbs, "New Kalman filter and smoother consistency tests," Automatica, vol. 49, no. 10, pp. 3141–3144, Oct. 2013, 10.1016/j.automatica.2013.07.013.
[38] A. Alessandri, M. Awawdeh, "Moving-horizon estimation with guaranteed robustness for discrete-time linear systems and measurements subject to outliers," Automatica, vol. 67, no. C, May. 2016, pp. 85–93, 10.1016/j.automatica.2016.01.015.
[39] A. Alessandri, L. Zaccarian, "Stubborn state observers for linear time-invariant systems," Automatica, vol. 88, pp. 1–9, Feb. 2018, 10.1016/j.automatica.2017.10.022.
[40] Y. Chen, Z. Wang, S. Fei, Q.-L. Han, "Regional stabilization for discrete time-delay systems with actuator saturations via a delay-dependent polytopic approach," IEEE Trans. on Autom. Control, vol. 64, no. 3, pp. 1257–1264, Mar. 2019, 10.1109/TAC.2018.2847903.
[41] Z. Wang, B. Shen, X. Liu, "H_∞ filtering with randomly occurring sensor saturations and missing measurements," Automatica, vol. 48, no. 3, pp. 556–562, Mar. 2012, 10.1016/j.automatica.2012.01.008.
[42] J. Hu, H. Zhang, X. Yu, H. Liu, D Chen, "Design of sliding-mode-based control for nonlinear systems with mixed-delays and packet losses under uncertain missing probability," IEEE Trans. Syst. Man Cyber. Syst, vol. 51, no. 5, pp. 3217-3228, May. 2021.

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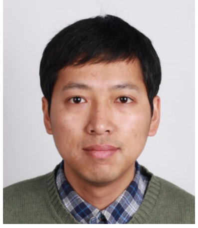

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