Filtro - Complementar frequency

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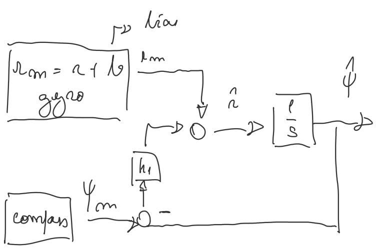
Complementary Filter

structure

$$\psi(s) = \underbrace{\frac{k}{s+k}}_{LPF}\psi(s) + \underbrace{\frac{s}{s+k}}_{HPF}\psi(s)$$



With integrator removes bias of the filters

1st order

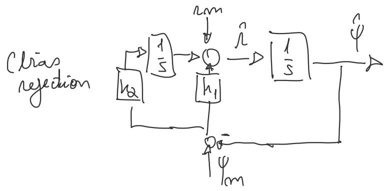
$$\begin{cases}\dot{\hat{\psi}} = \frac{1}{s}(\hat{r}) \\ \hat{r} = r_m - K(\hat{\psi} - \psi_m)\end{cases}$$

$$\Leftrightarrow \hat{\psi} = \frac{1}{s}r_m - \frac{k\hat{\psi}}{s} + \frac{k\psi_m}{s}$$

$$\hat{\psi}\left(1 + \frac{k}{s}\right) = \frac{1}{s}r_m + \frac{k\psi_m}{s}$$

$$\hat{\psi} = \underbrace{\frac{1}{s+k}}_{LPF}r_m + \underbrace{\frac{k}{s+k}}_{HPF}\psi_m$$

bias rejection



2nd order

$$\begin{cases}\hat{\psi} = \frac{1}{s}\hat{r} \\ \hat{r} = r_m + \frac{1}{s}k_2(\psi_m - \hat{\psi}) + k_1(\psi_m - \hat{\psi})\end{cases}$$

$$\Downarrow$$

$$\boxed{s\hat{\psi}} = \boxed{r_m} + \boxed{\frac{k_2\psi_m}{s}} - \boxed{\frac{k_2\hat{\psi}}{s}} + \boxed{k_1\psi_m} - \boxed{k_1\hat{\psi}}$$

$$(s)\ \boxed{\hat{\psi}}\left(s + k_1 + \frac{k_2}{s}\right) = \boxed{r_m} + \boxed{\psi_m}\left(\frac{k_2}{s} + k_1\right)(s)$$

$$\hat{\psi} = F_r r_m + F_\psi \psi$$

$$F_r = \frac{s}{s^2 + k_1 s + k_2} \qquad F_\psi = \underbrace{\frac{k_1 s + k_2}{s^2 + k_1 s + k_2}}_{LPF}$$

$$s F_r = \frac{1}{s}\underbrace{\frac{s^2}{s^2 + k_1 s + k_2}}_{HPF}$$

$$\frac{w_n^2}{s^2 + \xi w_n + w_n^2}$$

$$\begin{cases}w_n^2 = k_2 \\ \xi w_n = k_1\end{cases} \Leftrightarrow \begin{cases}w_n = \sqrt{k_2} \\ \xi = \frac{k_1}{\sqrt{k_2}}\end{cases}$$

Kalman Filter

$$x_k \xrightarrow{\text{predict}} x_{k+1} \xrightarrow{\text{update}}$$

u_k $\qquad$ y_{k+1}

extended Kalman Filter (For non-linear dynamics)

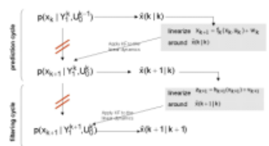


Figure 5.1: Extented Kalman filter dynamic concept

steady state KF

for LTI systems

(particularization of Kalman filter)

Only one gain

Uses Kalman logic to decide k_1, k_2

bias $= b$

$\mu = -b \rightarrow$ bias estimation

Design model

$$\begin{cases}\frac{d}{dt}\psi = r + \xi_1 \\ \frac{d}{dt}r = 0 + \xi_2\end{cases}$$

$$\psi_m = \psi + \eta_1$$

$$r_m = r + b + \eta_2$$

$$\psi = x_1 \qquad r = x_2 \quad \rightarrow \quad \begin{aligned}\dot{x}_1 &= x_2 + \xi_1 \\ \dot{x}_2 &= \xi_2 \\ \psi &= \psi_m + \eta_1 \\ r_m &= r + b + \eta_2\end{aligned} \quad \rightarrow$$

Continuous

$$\dot{x} = Ax + bw$$

$$y = cx + v$$

$$A = \begin{bmatrix}0 & 1 \\ 0 & 0\end{bmatrix} \quad b = \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix} \quad w = \begin{bmatrix}\xi_1 \\ \xi_2\end{bmatrix}$$

$$c = \begin{bmatrix}1 & 0\end{bmatrix} \quad V = \begin{bmatrix}\eta_1\end{bmatrix}$$

Discrete

$$x(k+1) = A\,x(k) + G\,w$$

$$y(k) = C\,x(k) + V$$

$$A = \begin{bmatrix}1 & T \\ 0 & 1\end{bmatrix} \quad G = \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix} \quad w = \begin{bmatrix}w_1 \\ w_2\end{bmatrix}$$

$$C = \begin{bmatrix}1 & 0\end{bmatrix} \quad V = [v_1]$$

Kalman logic

$$\bar{P} = \begin{pmatrix}P_{11} & P_{12} \\ P_{21} & P_{22}\end{pmatrix}$$

Steady state Riccati equation

$$\bar{P} = A\bar{P}A^T + NWN^T - (A\bar{P}C^T)(C\bar{P}C^T + V)^{-1}(A\bar{P}C^T)^T$$

$$\hat{y}(k|k) = A\hat{x}(k-1|k-1) + h[y(k) - C\hat{x}(k-1|k-1)]$$

$$\hat{\psi} = r_m + k_1[\psi_m -$$

$$A\bar{P}A^T = \begin{bmatrix}1 & T \\ 0 & 1\end{bmatrix}\begin{bmatrix}P_{11} & P_{12} \\ P_{21} & P_{22}\end{bmatrix}\begin{bmatrix}1 & 0 \\ T & 1\end{bmatrix}$$

$$= \begin{bmatrix}P_{11} + TP_{21} & P_{12} + TP_{22} \\ P_{21} & P_{22}\end{bmatrix}\begin{bmatrix}1 & 0 \\ T & 1\end{bmatrix}$$

Assuming $W = \begin{bmatrix}w_1 & 0 \\ 0 & w_2\end{bmatrix}$

$V = \eta_1$

$$= \begin{bmatrix} p_{11}+Tp_{21} & p_{12}+Tp_{22} \\ p_{21} & p_{22} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ T & 1 \end{bmatrix}$$

$$= \begin{bmatrix} p_{11}+(p_{21}+p_{12})T+p_{22}T^2, & p_{12}+Tp_{22} \\ p_{21}+p_{22}T & p_{22} \end{bmatrix}$$

$$A\bar{P}C^T = \begin{bmatrix} p_{11}+Tp_{21} & p_{12}+Tp_{22} \\ p_{21} & p_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} p_{11}+Tp_{21} \\ p_{21} \end{bmatrix}$$

$$C\bar{P}C^T = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} p_{12} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = p_{12}$$

$$C\bar{P}C^T + V = p_{12} + \eta_1$$

$$(C\bar{P}C^T + V)^{-1} = \frac{1}{p_{12}+\eta_1}$$

$$V = \eta_1$$

$$\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} p_{11}+(p_{21}+p_{12})T+p_{22}T^2, & p_{12}+Tp_{22} \\ p_{21}+p_{22}T & p_{22} \end{bmatrix} + \begin{bmatrix} w_1 & 0 \\ 0 & w_2 \end{bmatrix} - \frac{1}{p_{12}+\eta_1} \begin{bmatrix} p_{11}+Tp_{21} \\ p_{21} \end{bmatrix} \begin{bmatrix} p_{11}+Tp_{21} & p_{21} \end{bmatrix}$$

$$\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} p_{11}+(p_{21}+p_{12})T+p_{22}T^2, & p_{12}+Tp_{22} \\ p_{21}+p_{22}T & p_{22} \end{bmatrix} + \begin{bmatrix} w_1 & 0 \\ 0 & w_2 \end{bmatrix} - \frac{1}{p_{12}+\eta_1} \begin{bmatrix} p_{11}+2(Tp_{21})+T^2p_{21}^2 & p_{21}(p_{11}+Tp_{21}) \\ p_{21}(p_{11}+Tp_{21}) & p_{21}^2 \end{bmatrix}$$

$$\begin{cases} p_{11} = p_{11} + (p_{21}+p_{12})T + p_{22}T^2 + w_1 - \dfrac{p_{11}+2Tp_{21}+T^2p_{21}}{p_{12}+\eta_1} \\ p_{12} = p_{12}+Tp_{22} - \dfrac{p_{21}(p_{11}+Tp_{21})}{p_{12}+\eta_1} \\ p_{21} = p_{21}+Tp_{22} - \dfrac{p_{21}(p_{11}+Tp_{21})}{p_{12}+\eta_1} \\ p_{22} = p_{22}+w_2 - \dfrac{p_{21}^2}{p_{12}+\eta_1} \end{cases}$$

$$\begin{cases} \cancel{p_{11}} = \cancel{p_{11}} + 2\sqrt{w_2}\,T + p_{22}T^2 + w_1 - \dfrac{p_{11}+2T\sqrt{w_2}+T^2\sqrt{w_2}}{\sqrt{w_2}+\eta_1} \\ \cancel{p_{12}} = \cancel{p_{12}} + Tp_{22} - \dfrac{p_{21}(p_{11}+Tp_{21})}{p_{12}+\eta_1} \iff p_{22} = \dfrac{1}{T}\,\dfrac{\sqrt{w_2}(p_{11}+T\sqrt{w_2})}{\sqrt{w_2}+\eta_1} \\ \cancel{p_{22}} = \cancel{p_{22}} + w_2 \iff p_{21} = \sqrt{w_2} = p_{12} \end{cases}$$

$$\begin{cases} p_{11} + 2T\sqrt{w_2} + T^2\sqrt{w_2} = (\sqrt{w_2}+\eta_1)(2\sqrt{w_2}T + w_1) + T\sqrt{w_2}\,p_{11} + T^2\sqrt{w_2} \\ p_{21} = \dfrac{1}{T}\,\dfrac{\sqrt{w_2}(p_{11}+T\sqrt{w_2})}{\sqrt{w_2}+\eta_1} \\ p_{22} = \sqrt{w_2} \end{cases}$$

$$\begin{cases} p_{11}(1-T\sqrt{w_2}) = \boxed{2w_2T} \boxed{+w_1\sqrt{w_2}} \boxed{+\eta_1 2\sqrt{w_2}T} \boxed{+\eta_1 w_1} \boxed{-2T\sqrt{w_2}} \boxed{+T^2\sqrt{w_2}} \\ p_{22} = \dfrac{1}{T}\,\dfrac{\sqrt{w_2}(p_{11}+T\sqrt{w_2})}{\sqrt{w_2}+\eta_1} \\ p_{12} = \sqrt{w_2} \end{cases}$$

$$\begin{cases} p_{11} = \left[w_1(\eta_1+\sqrt{w_2}) + (2w_2+2\sqrt{w_2}(\eta_1-1))T + \sqrt{w_2}\,T^2\right] / (1-T\sqrt{w_2}) \\ p_{22} = \ldots \\ p_{12} = \sqrt{w_2} \end{cases}$$

$$\boxed{h = \bar{P}C^T(CPC^T+V)^{-1}}$$

$$h = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \times \frac{1}{p_{12}+\eta_1}$$

$$h = \begin{bmatrix} p_{11} \\ p_{21} \end{bmatrix} \times \frac{1}{p_{12}+\eta_1}$$

$$h = \begin{bmatrix} \dfrac{p_{11}}{\sqrt{w_2}+\eta_1} \\ \dfrac{\sqrt{w_2}}{\sqrt{w_2}+\eta_1} \end{bmatrix}$$