# Filtros 2 - Complementar Kalman

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w/ noise

$$\begin{cases} \dot{\varphi} = r + b \\ \dot{b} = 0 \end{cases} \qquad y = \varphi \qquad x = \begin{bmatrix} \varphi \\ b \end{bmatrix} \quad \dot{x} = \begin{bmatrix} \dot{\varphi} \\ \dot{b} \end{bmatrix} \quad u = r$$

$$\begin{cases} \dot{\varphi} = r + b + \xi_1 \\ \dot{b} = \xi_2 \end{cases} \qquad y = \varphi + \eta \to \text{noise} \qquad w = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} \quad v = [\eta]$$

$$\dot{x} = Ax + Bu + Gw \qquad A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad G = I$$

$$y = Cx + v \qquad C = [1 \quad 0]$$

System

continuos kalman equation

$$\dot{P} = AP + PA^T + GwG^T - PC^T v^{-1} C P \qquad P = \begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix}$$

$$K = PC^T v^{-1} \qquad \dot{P} = 0 \text{ (steady state)}$$

P - covariance matrix

K = kalman gain

continuos kalman equation

$$AP = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} p_{21} & p_{22} \\ 0 & 0 \end{bmatrix}$$

$$PA^T = \begin{bmatrix} p_{11} & p_{21} \\ p_{21} & p_{22} \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} p_{21} & 0 \\ p_{22} & 0 \end{bmatrix}$$

$$GwG^T = \begin{bmatrix} \xi_1 & 0 \\ 0 & \xi_2 \end{bmatrix} = Q$$

$$PC^T = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} p_{11} \\ p_{21} \end{bmatrix}$$

$$CP = [1 \quad 0]\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = [p_{11} \quad p_{12}]$$

$$v^{-1} = \frac{1}{\eta}$$

$$0 = \begin{bmatrix} p_{21} & p_{22} \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} p_{21} & 0 \\ p_{22} & 0 \end{bmatrix} + \begin{bmatrix} \xi_1 & 0 \\ 0 & \xi_2 \end{bmatrix} - \frac{1}{\eta}\begin{bmatrix} p_{11} \\ p_{21} \end{bmatrix}[p_{11} \quad p_{12}]$$

$$\begin{cases} 0 = p_{21} + p_{21} + \xi_1 - \frac{1}{\eta} p_{11}^2 & (1) \\ 0 = p_{22} + 0 + 0 - \frac{1}{\eta} p_{11} p_{12} & (2) \\ 0 = 0 + p_{22} + 0 - \frac{1}{\eta} p_{11} p_{21} & (3) \\ 0 = 0 + \xi_2 - \frac{1}{\eta} p_{21} p_{12} & (4) \end{cases}$$

$p_{21} = p_{12} \to (2) = (3)$

$$\begin{cases} p_{11} = \sqrt{\eta\left(2 p_{12} + \xi_1\right)} \\ p_{22} = \frac{1}{\eta} p_{11} p_{12} \end{cases} \qquad \begin{cases} p_{11} = \sqrt{\eta\left(2\sqrt{\xi_2 \eta} + \xi_1\right)} \\ p_{22} = \dots \end{cases}$$

$$\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} p_{21} & p_{22} \end{bmatrix}$$

$$r^{-1} = \frac{1}{\eta}$$

$$\begin{cases} p_{22} = \frac{1}{\eta} p_{11} \, p_{12} \\ p_{12} = \sqrt{\xi_2 \eta} \end{cases} \qquad \begin{cases} p_{22} = \dots \\ p_{12} = \sqrt{\xi_2 m} \end{cases}$$

$$k = \frac{1}{\eta}\begin{bmatrix} p_{11} \\ p_{21} \end{bmatrix} \Rightarrow k = \begin{bmatrix} \frac{\sqrt{m}\left(2\sqrt{\xi_2 m} + \xi_1\right)}{\eta} \\ \frac{\sqrt{\xi_2 m}}{\eta} \end{bmatrix}$$

CA

$$\frac{2\sqrt{\xi_2 m}}{\sqrt{m}} + \frac{\xi_1}{\sqrt{m}} = 2\sqrt{\xi_2} + \frac{\xi_1}{\sqrt{m}}$$

$$k = \begin{bmatrix} 2\sqrt{\xi_2} + \frac{\xi_1}{\sqrt{m}} \\ \frac{\xi_2}{\sqrt{\eta}} \end{bmatrix}$$