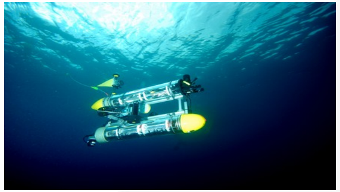
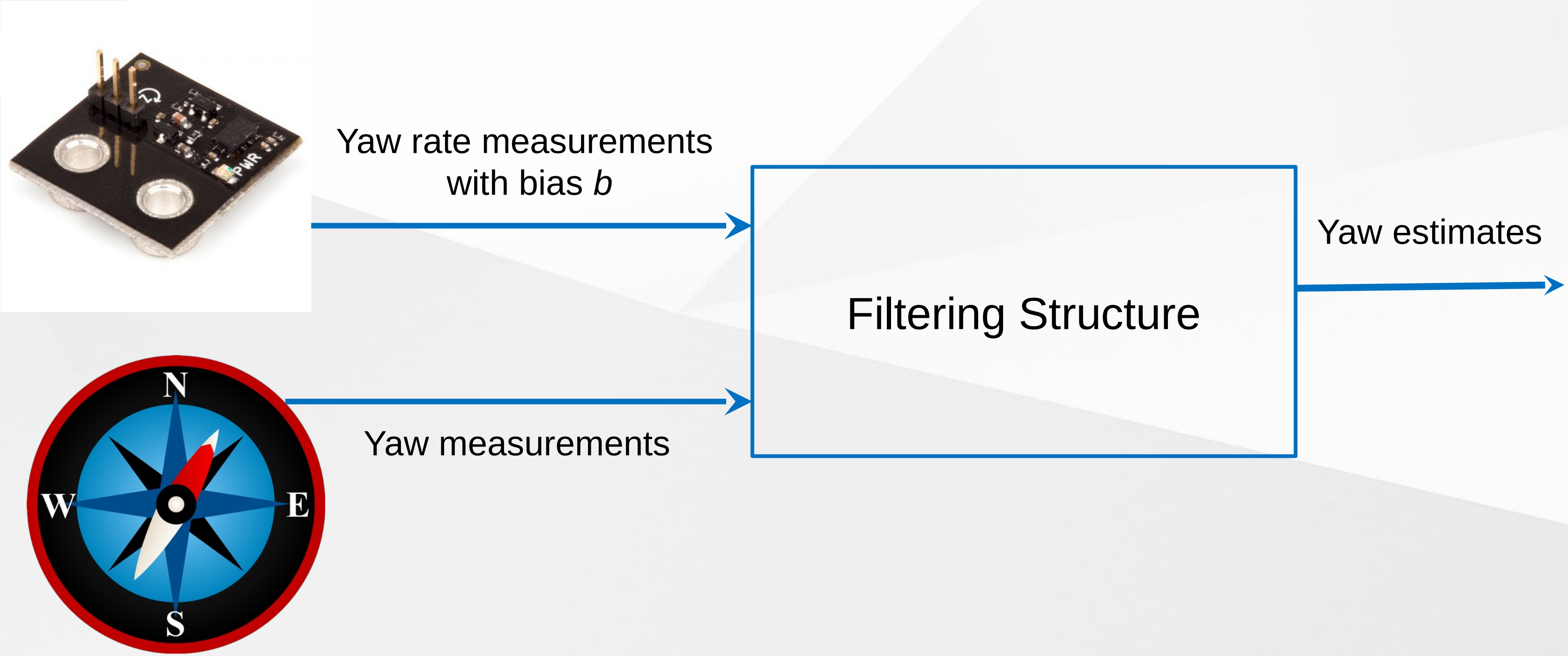
DSOR Group Meeting
27 November and
5 December 2020

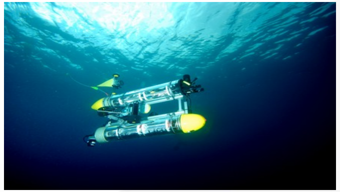
Navigation: from concept to filter implementation (use-cases)



Example 1

Yaw estimation from Rate Gyro and Compass Measurements



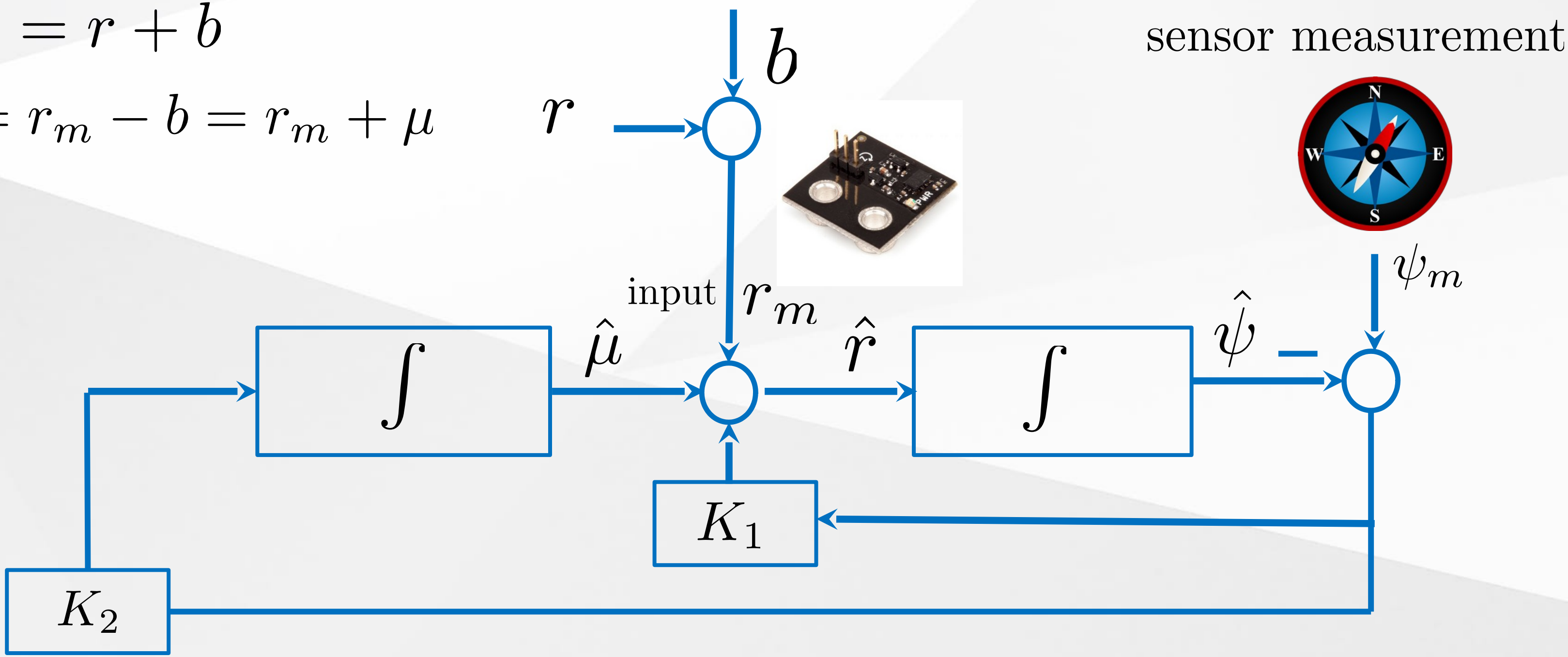


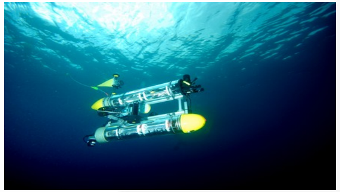
Example 1

Complementary Filter Structure

$$r_m = r + b$$

$$r = r_m - b = r_m + \mu$$





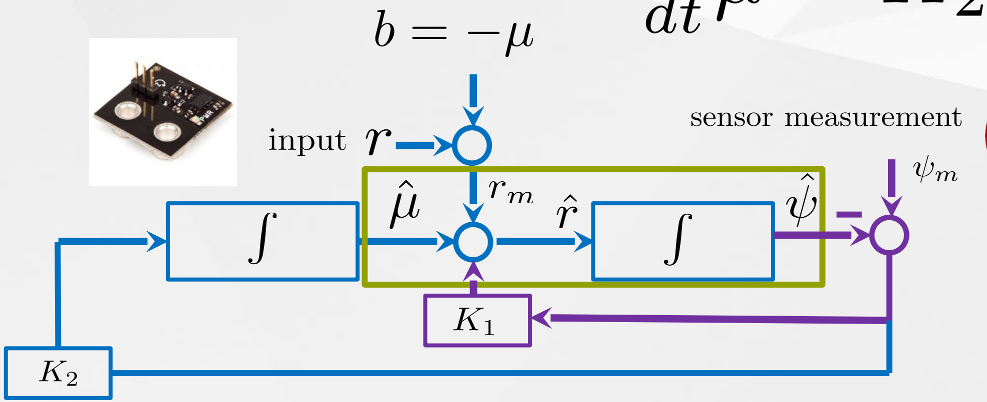
Example 1

Complementary Filter Structure

Prediction | *Correction*

$$\frac{d}{dt}\hat{\psi} = \underbrace{\hat{\mu} + r_m}_{\text{Prediction}} + \underbrace{K_1(\psi_m - \hat{\psi})}_{\text{Correction}}$$

$$\frac{d}{dt}\hat{\mu} = K_2(\psi_m - \hat{\psi})$$



Underlying Design Model

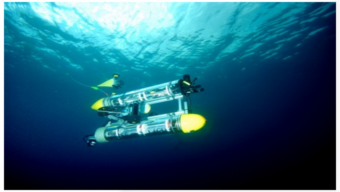
$$\frac{d}{dt}\psi = r_m + \mu + \xi_1$$

$$\frac{d}{dt}\mu = 0 + \xi_2$$

state noise

$$\psi_m = \psi + \eta$$

measurement noise



Example 1- a Sensor Fusion perspective

Underlying Design Model

$$\frac{d}{dt}\psi = r + \xi_1$$

$$\frac{d}{dt}r = 0 + \xi_2$$

state noise

$b - rate\ gyro\ bias$

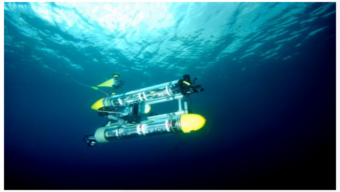
$$\frac{d}{dt}b = 0 + \xi_2$$

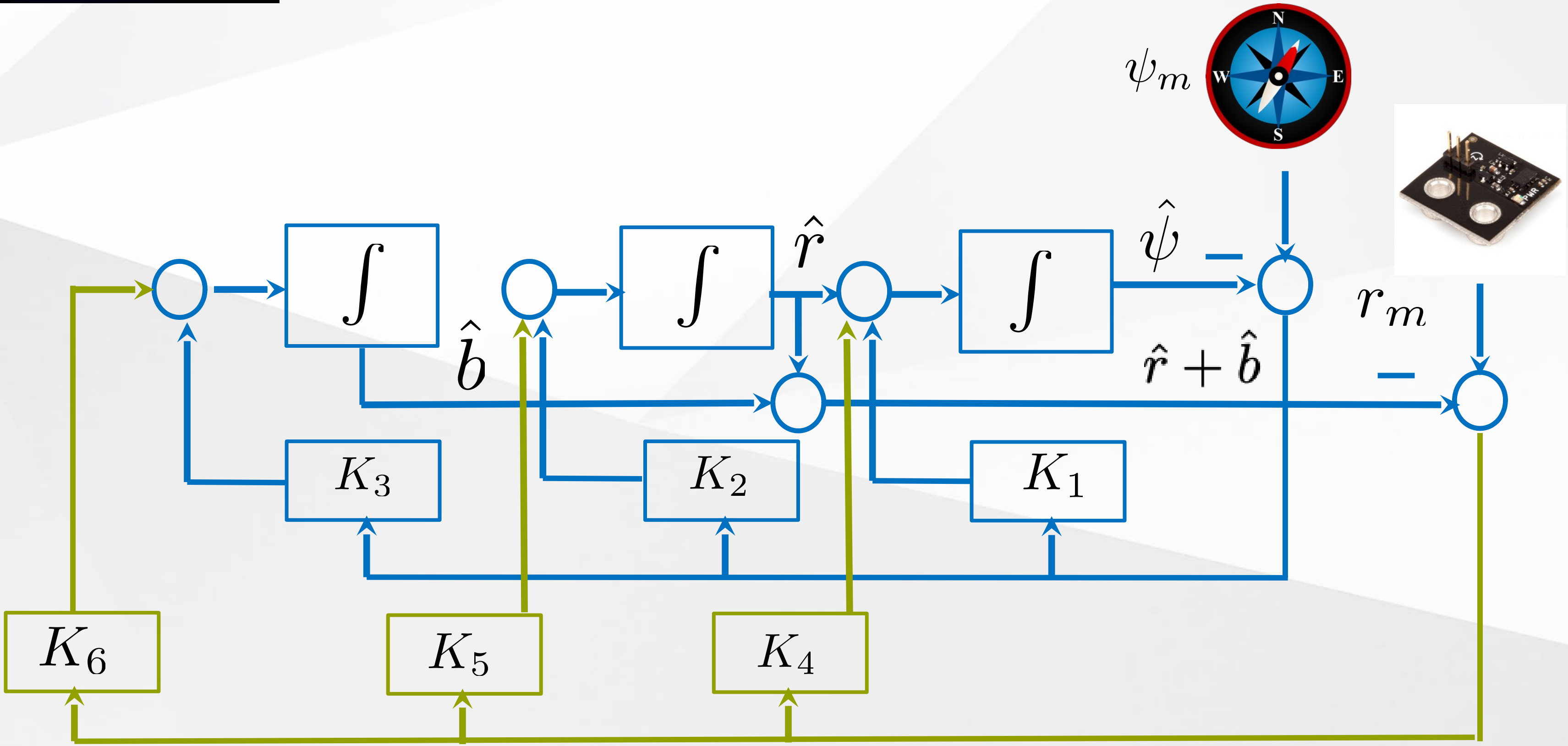
$$\psi_m = \psi + \eta_1$$

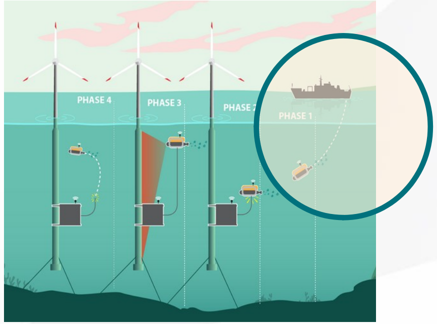
$$r_m = r + b + \eta_2$$

measurement noise

Example 1- a Sensor Fusion perspective







Example 2

AUV-borne position estimation from Doppler, AHRS, inverted USBL, and GPS measurements (2D)

Doppler – velocity measurements

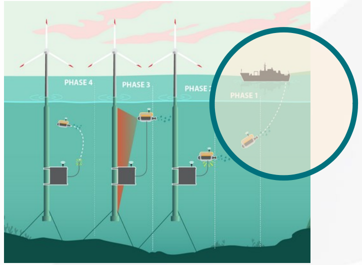
Yaw measurements (AHRS)

USBL – position wrt ship measurements + GPS data

(comms and relative positioning)

Filtering Structure

Position and velocity estimates



Example 2

Complementary Filter Structure $v_m = v_w = v - v_c$

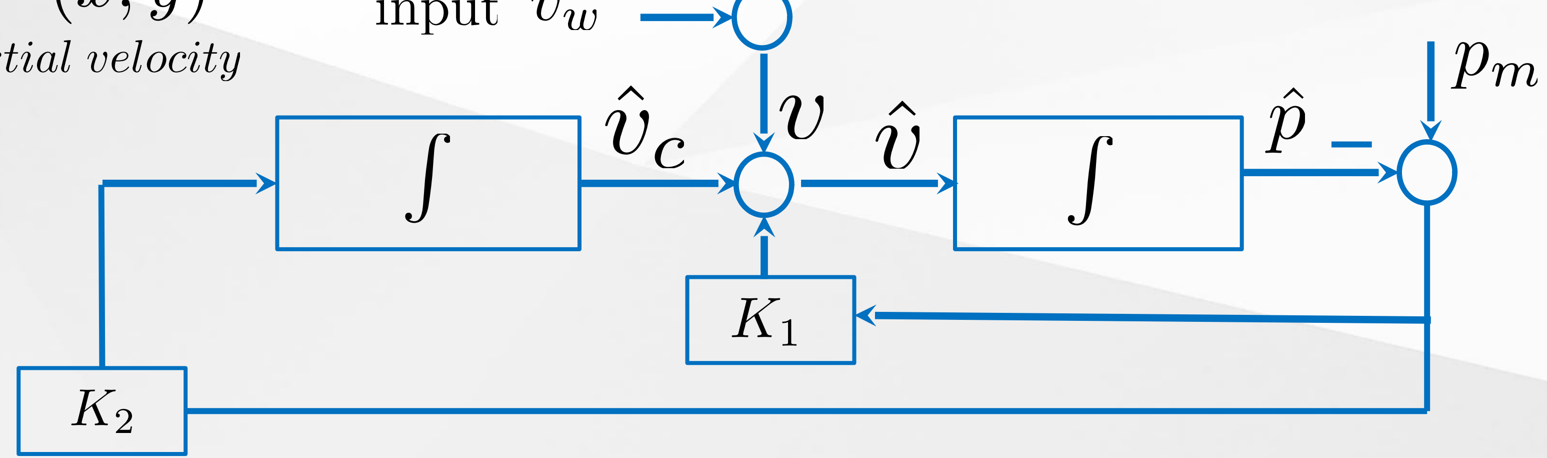
$$v = v_w + v_c$$

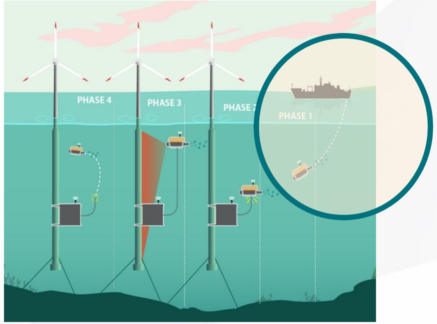
$$p = (x, y)^T$$

inertial position

$$\dot{v} = (\dot{x}, \dot{y})^T$$

inertial velocity





Example 2

Complementary Filter Structure

$$\frac{d}{dt}\hat{p} = \hat{v}_c + v_m + K_1(p_m - \hat{p})$$

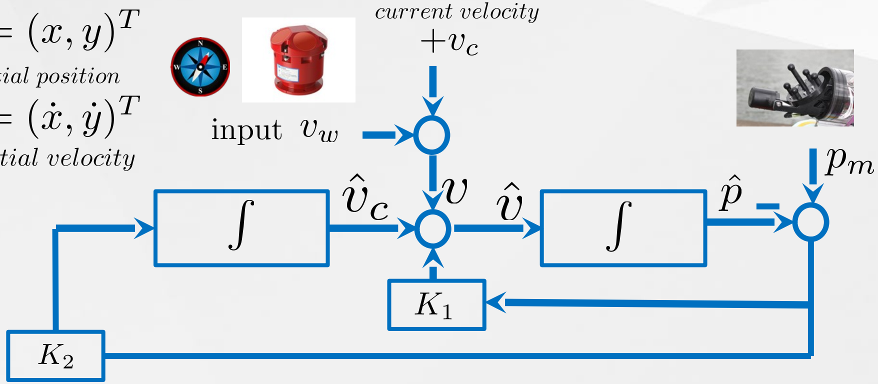
$$\frac{d}{dt}\hat{v}_c = K_2(p_m - \hat{p})$$

$p = (x, y)^T$

inertial position

$v = (\dot{x}, \dot{y})^T$

inertial velocity



Underlying Design Model

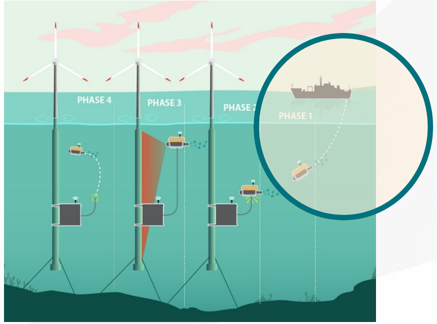
$$\frac{d}{dt}p = v_c + v_w + \xi_1$$

$$\frac{d}{dt}v_c = 0 + \xi_2$$

state noise

$$p_m = p + \eta$$

measurement noise



Example 3

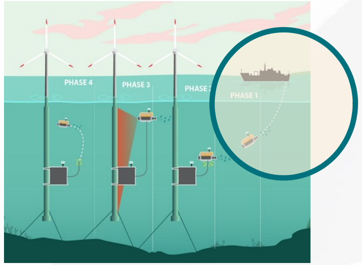
Ship borne tracking of AUV position
from USBL and GPS measurements (2D)

USBL – AUV position wrt
ship measurements

GPS-based
Ship position
measurements

Filtering Structure

Position and velocity
estimates



Example 3

p_m obtained from USBL and GPS data

$$p = (x, y)^T$$

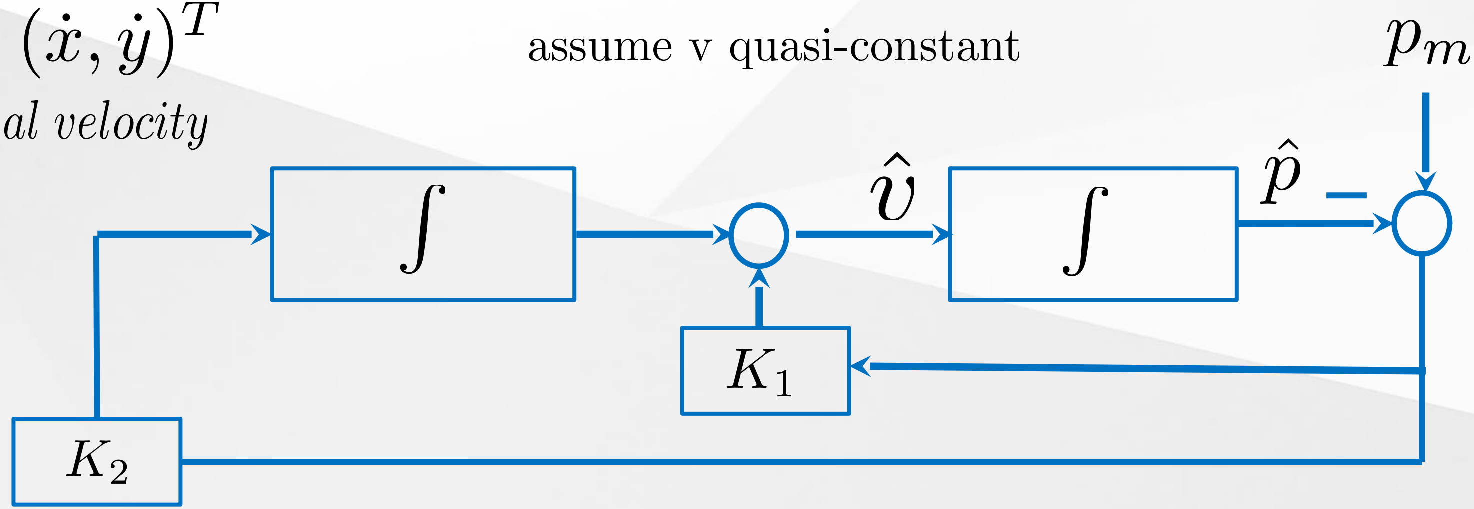
inertial position

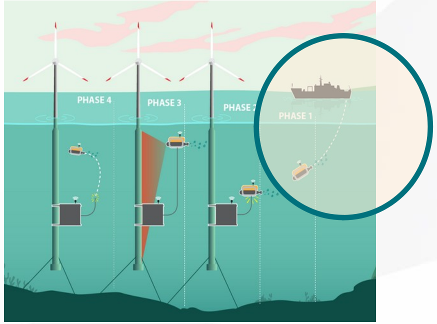
$$v = (\dot{x}, \dot{y})^T$$

inertial velocity

there is no direct or indirect measurement of v !

assume v quasi-constant





Example 3

$$p = (x, y)^T$$

inertial position

$$v = (\dot{x}, \dot{y})^T$$

inertial velocity

there is no direct or
indirect measurement of v !

Underlying Design Model

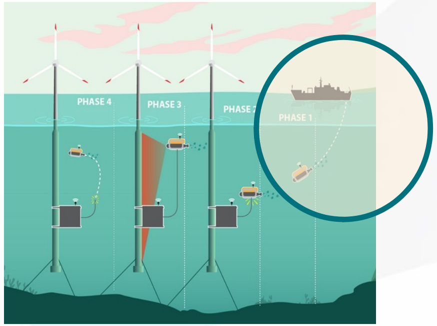
$$\frac{d}{dt} p = v + \xi_1 \longleftarrow$$

state noise

$$\frac{d}{dt} v = 0 + \xi_2 \longleftarrow$$

$$p_m = p + \eta \longleftarrow$$ **Measurement noise**

p_m obtained from
USBL and GPS data



Example 4

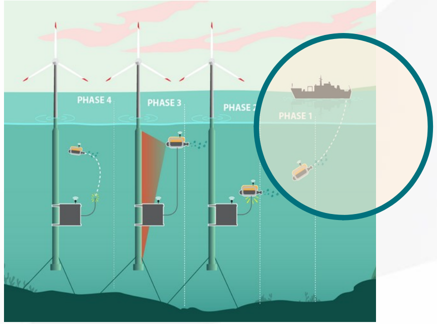
Ship borne tracking of AUV position
from USBL and GPS measurements (2D)

USBL – AUV position wrt
ship measurements

GPS-based
Ship position
measurements

Filtering Structure

Position / velocity +
Yaw / Yaw rate estimates



Example 4

Underlying Design Model (small sideslip angle) *

$$\boxed{\frac{d}{dt}p = \mathcal{R}(\psi)(u,0)^T + \xi_p}$$

$$\boxed{\begin{aligned}\frac{d}{dt}x &= u\cos(\psi) + \xi_x \\ \frac{d}{dt}y &= u\sin(\psi) + \xi_y\end{aligned}}$$

$$p = (x,y)^T$$

inertial position

$$\psi - yaw\ angle$$

$$r - yaw\ rate$$

$$u - surge\ speed$$

$$\frac{d}{dt}u = 0 + \xi_u$$

$$\frac{d}{dt}\psi = r + \xi_\psi$$

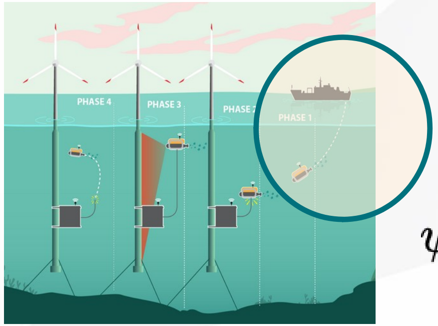
state noise

$$\frac{d}{dt}r = 0 + \xi_r$$

$$p_m = p + \eta$$

Measurement noise

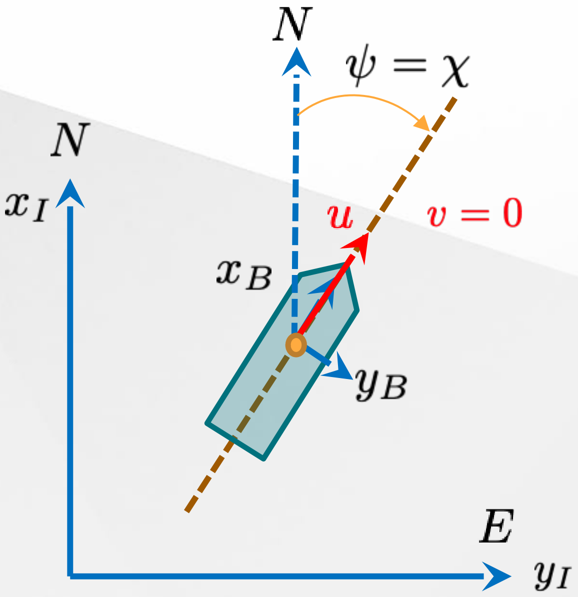
* A rigorous interpretation is to view *u* as the absolute value of the total velocity vector and ψ as the course angle



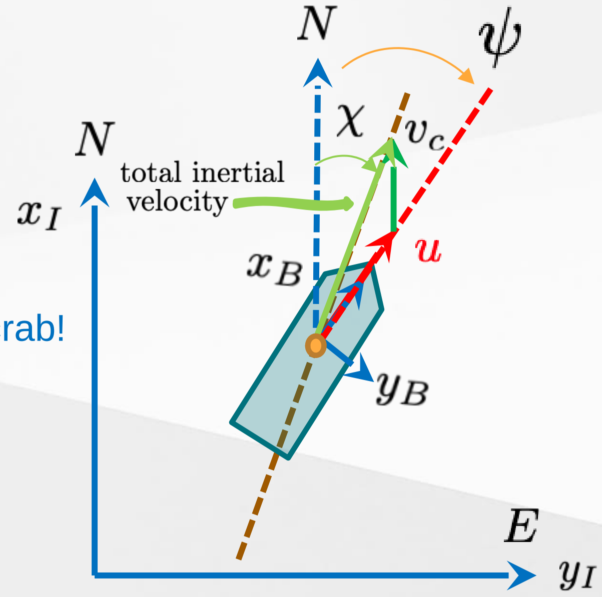
A word about notation

$\psi - yaw\ angle \quad \chi - course\ angle \quad v_c - velocity\ of\ current$

No current, no sideslip

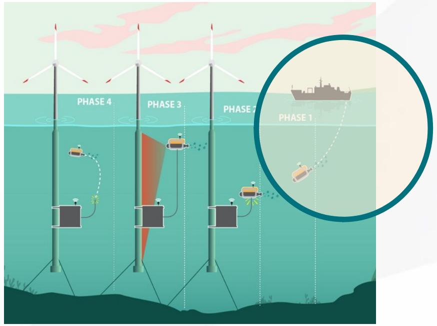


Current, no sideslip



Vehicle “flying” crab!

Vehicle along straight line



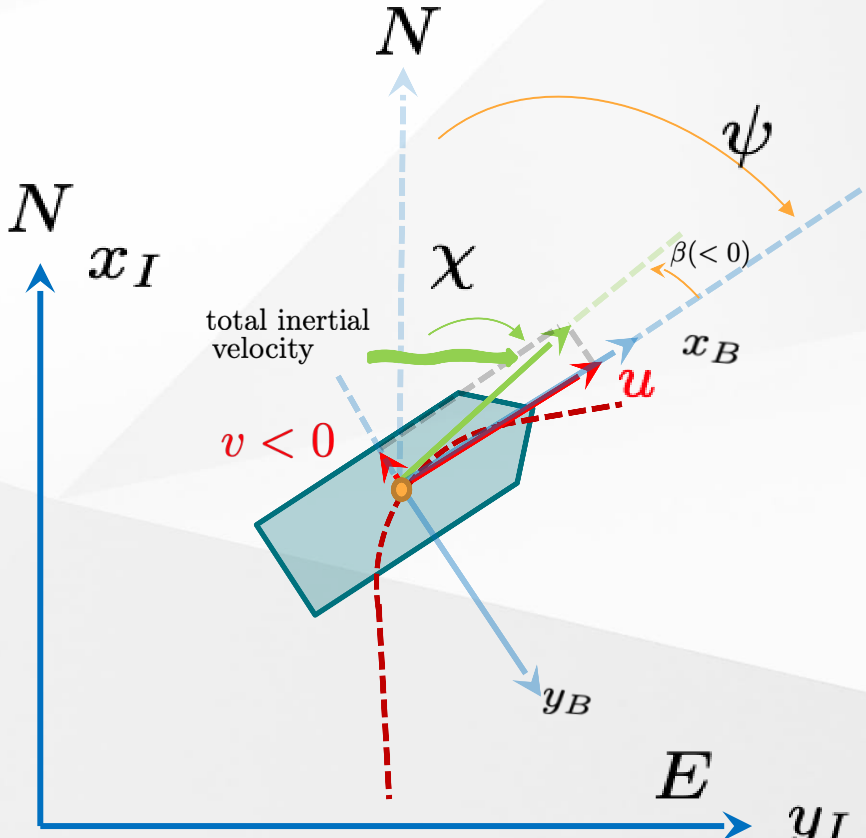
A word about notation

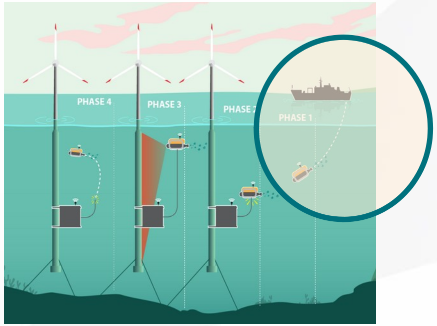
No current, with sideslip - vehicle moving along a curved line

ψ – *yaw angle*

β – *sideslip angle*

χ – *course angle*





A word about notation

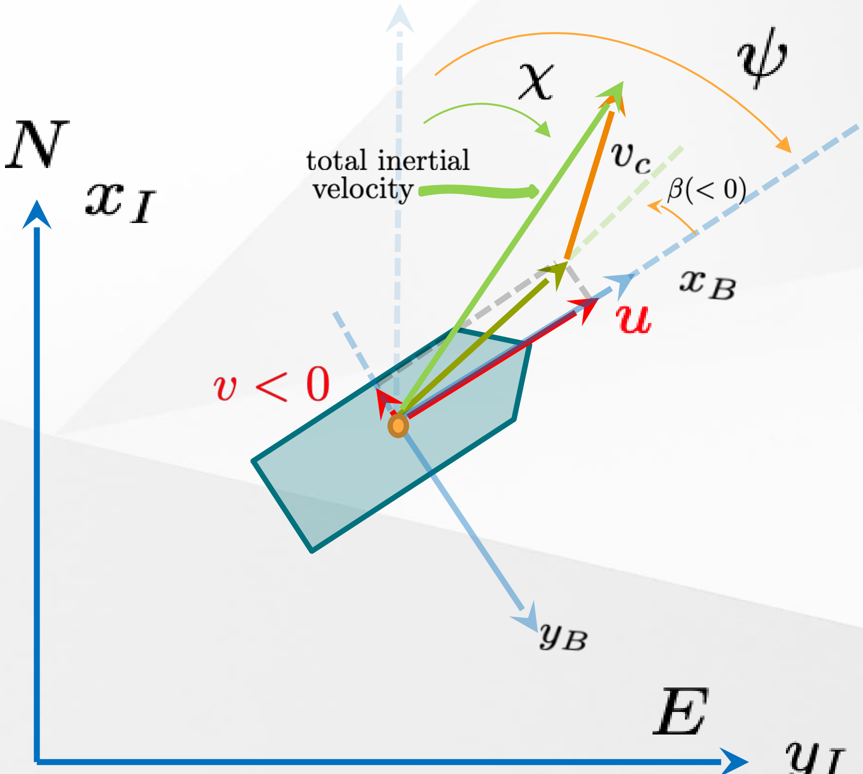
Current, with sideslip

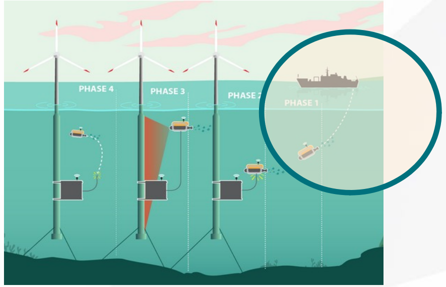
ψ – *yaw angle*

β – *sideslip angle*

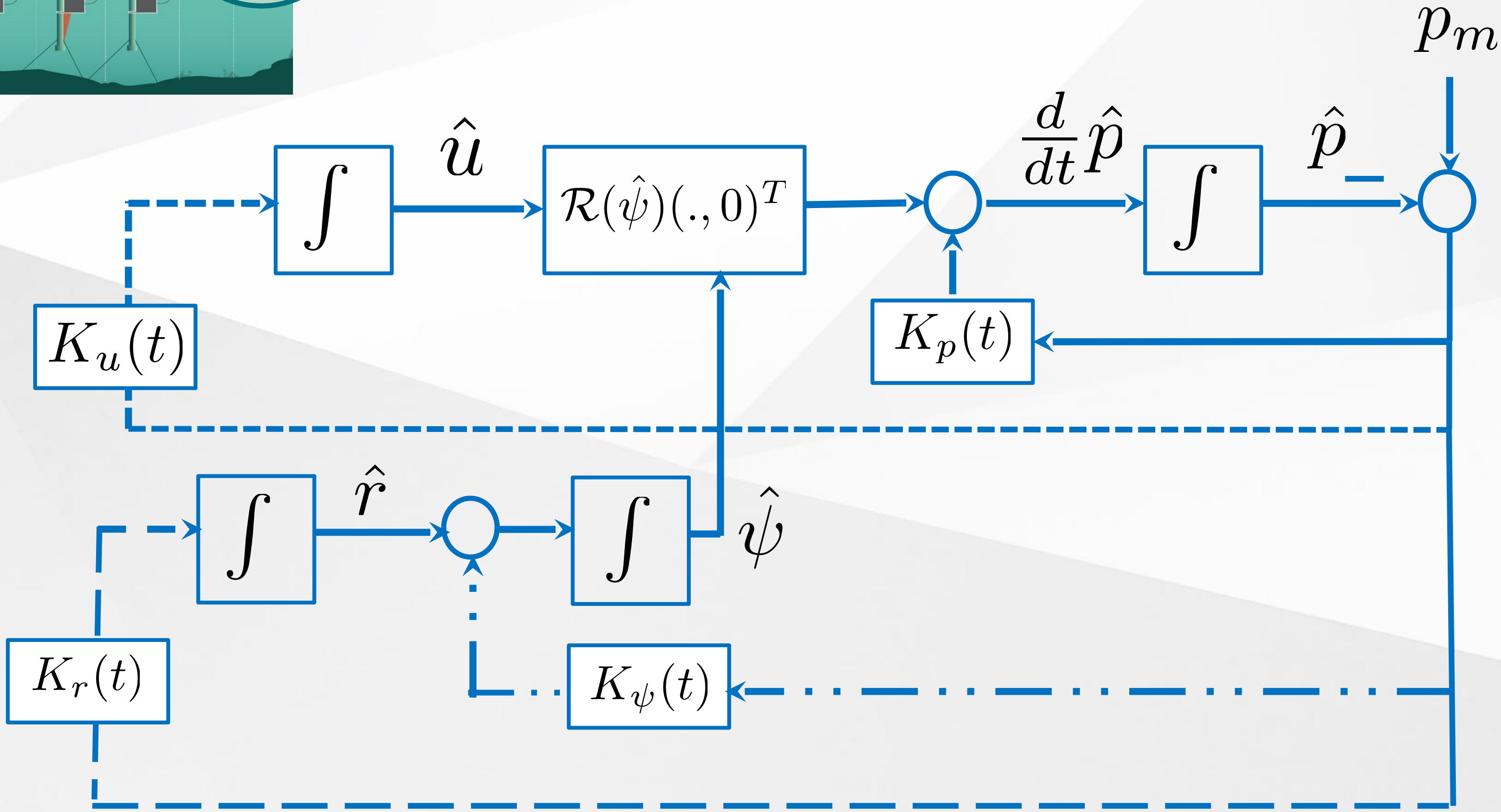
χ – *course angle*

v_c – *velocity of current*

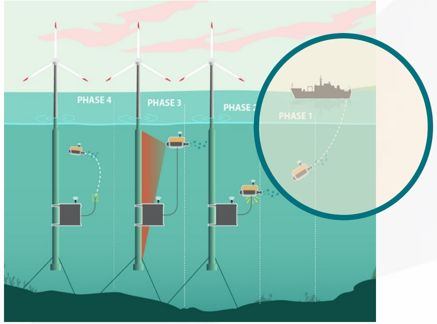



PHASE 4
PHASE 3
PHASE 1

Example 4


p_m
$\int$
$\hat{u}$
$\mathcal{R}(\hat{\psi})(.,0)^T$
$\frac{d}{dt}\hat{p}$
$\int$
$\hat{p}$
$-$
$K_u(t)$
$K_p(t)$
$\int$
$\hat{r}$
$\int$
$\hat{\psi}$
$K_r(t)$
$K_\psi(t)$

Example 5



AUV-borne position estimation from Doppler, AHRS, and measurements of ranges-only to known beacon (e.g. on docking station)

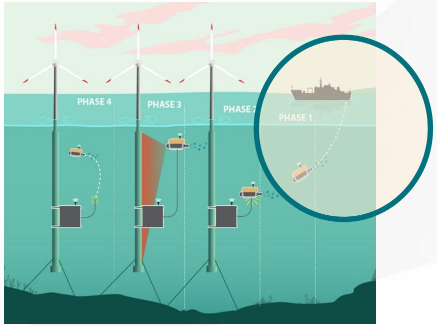
Doppler – velocity measurements

Yaw measurements (AHRS)

Range measuring device – measurements of range wrt fixed beacon

Filtering Structure

Position and velocity estimates



Example 5

Complementary Filter Structure

Underlying Design Model

current velocity

Doppler measurement

$$v_m = v_w = v - v_c$$

$$v = v_w + v_c$$

$$p = (x, y)^T$$

inertial position

$$v = (\dot{x}, \dot{y})^T$$

inertial velocity

p_b – *beacon position*

r_m - range measurement

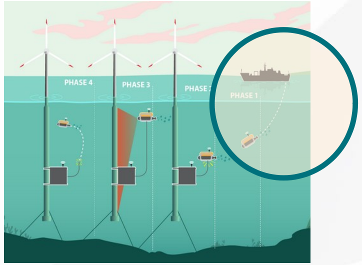
$$\frac{d}{dt}p = v_c + v_w + \xi_1$$

$$\frac{d}{dt}v_c = 0 + \xi_2$$

state noise

$$r_m = ||p - p_b|| + \eta$$

measurement noise



Example 5

Complementary Filter Structure

$$v_m = v_w = v - v_c$$

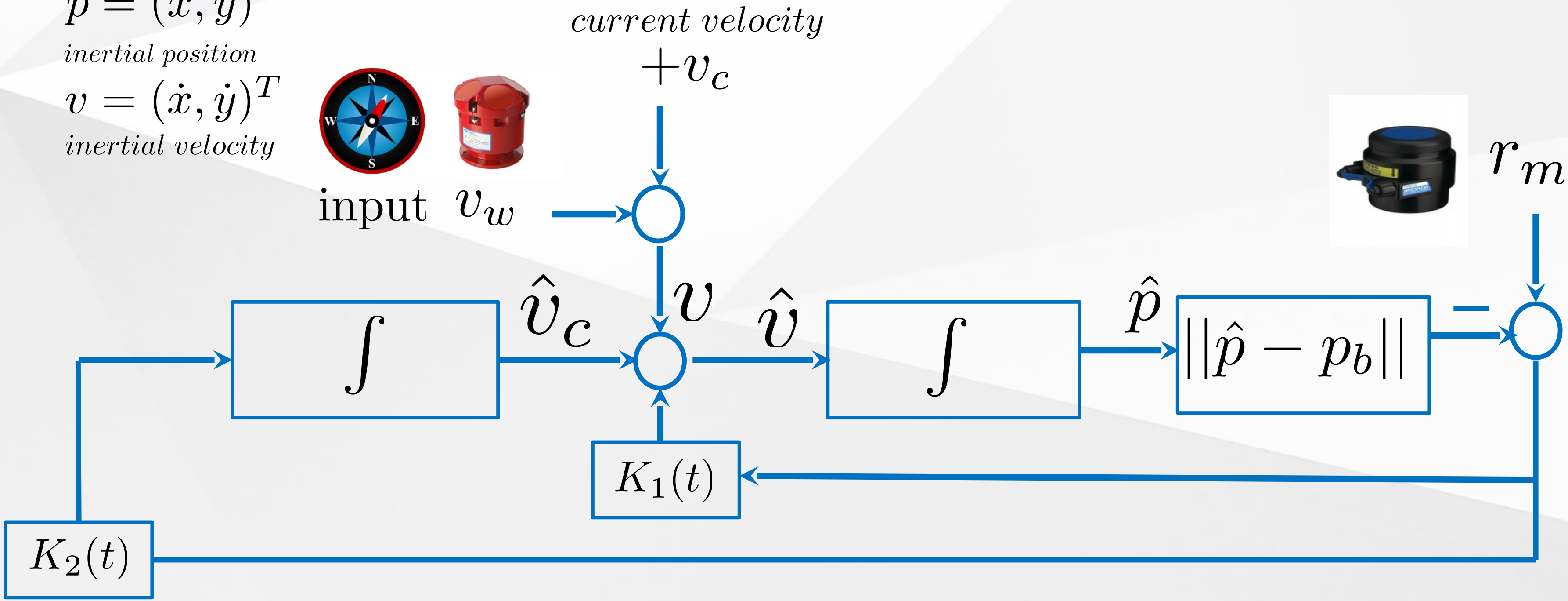
$$v = v_w + v_c$$

$p = (x, y)^T$

inertial position

$v = (\dot{x}, \dot{y})^T$

inertial velocity



medusa-vx

System - Implementation

$$\tilde{y} = z_k - C_k \hat{x}_{k|k-1}$$

$$S_k = C_k P_{k|k-1} C_k^T + R_k$$

$$K_k = P_{k|k-1} C_k^T S_k^{-1}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \tilde{y}_k$$

$$P_{k|k-1} = (1 - k_k C_k) P_{k|k-1}$$

where

- $\tilde{y}$ = Innovation Factor Vector
- z_k = Measurement Vector
- C_k = Observation Matrix
- R_k = Covariance of the Measurement Noise
- S_k = Innovation Matrix
- K_k = Kalman Gain Matrix

$$\hat{x}_{k|k-1} = A_k \hat{x}_{k-1|k-1} + B_k u_k + w_k$$

$$P_{k|k-1} = A_k P_{k-1|k-1} A_k^T + Q_k$$

where

- A_k = State Transition Matrix
- B_k = Control-Input Model
- u_k = Input Vector
- w_k = Zero-mean Gaussian Process Noise
- Q_k = Covariance of the Process Noise

$$x = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \\ \dot{x_c} \\ \dot{x_y} \end{bmatrix} \quad A_k = \begin{bmatrix} 1 & 0 & \delta t & 0 & 0.5\delta t^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & \delta t & 0 & 0.5\delta t^2 & 0 & 0 \\ 0 & 0 & 1 & 0 & \delta t & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \delta t & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad B_k = [0]_{8\times 8}$$

where

- δt = predict_period

Example 1: Yaw estimation from Rate Gyro and Compass Measurements

Kalman Filter with Yaw and Yaw Rate as measurement vector

$$\tilde{y} = z_k - C_k \hat{x}_{k|k-1}$$

$$S_k = C_k P_{k|k-1} C_k^T + R_k$$

$$K_k = P_{k|k-1} C_k^T S_k^{-1}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \tilde{y}_k$$

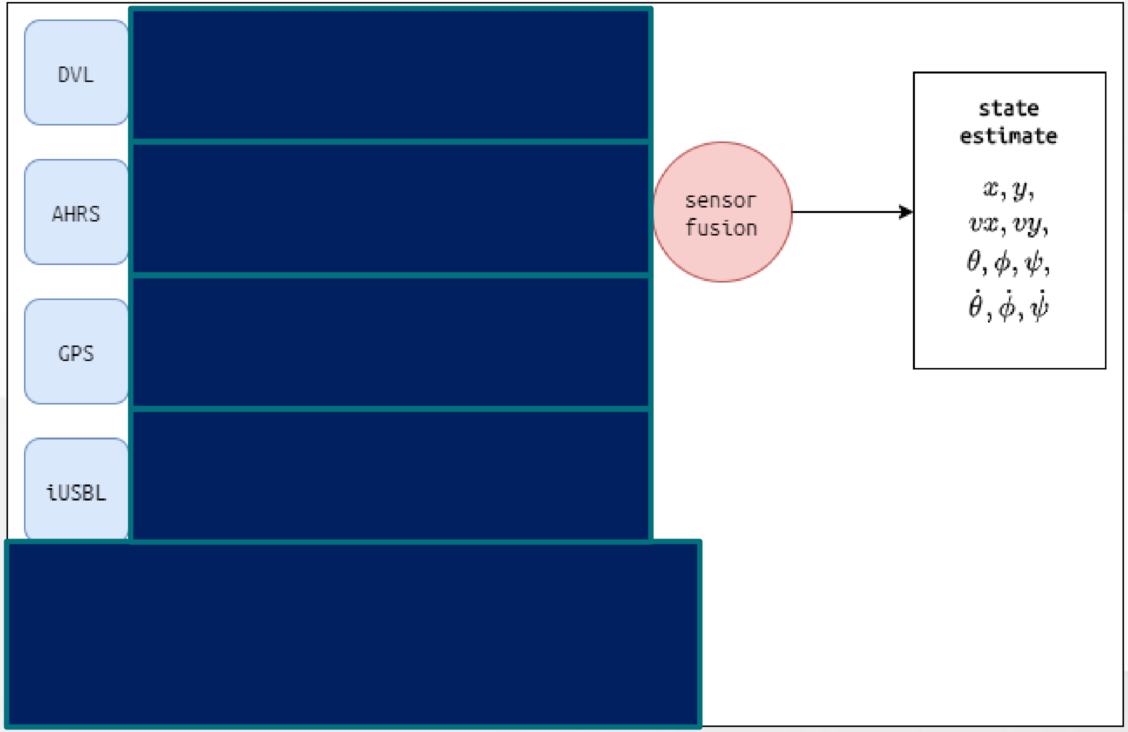
$$P_{k|k-1} = (1 - k_k C_k) P_{k|k-1}$$

where

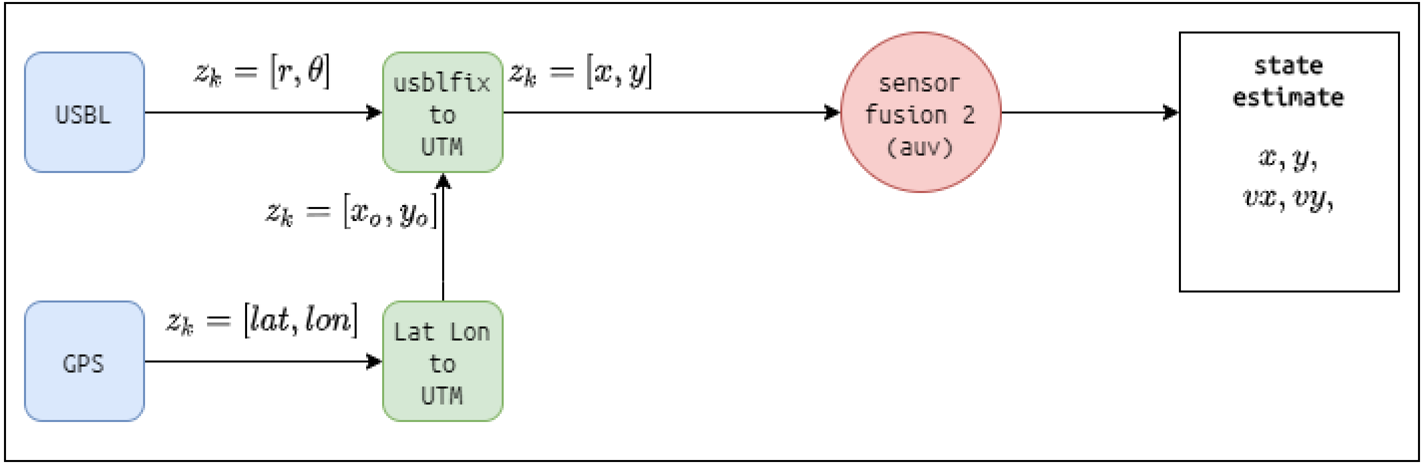
- $\tilde{y}$ = Innovation Factor Vector
- z_k = Measurement Vector
- C_k = Observation Matrix
- R_k = Covariance of the Measurement Noise
- S_k = Innovation Matrix
- K_k = Kalman Gain Matrix

Example 2: AUV-borne position estimation from Doppler, AHRS, inverted USBL, and GPS measurements (2D)

- Doppler updates velocity
- AHRS updates roll, pitch, yaw, roll rate, pitch rate, yaw rate
- Inverted USBL updates position
- GPS updates position (at surface)

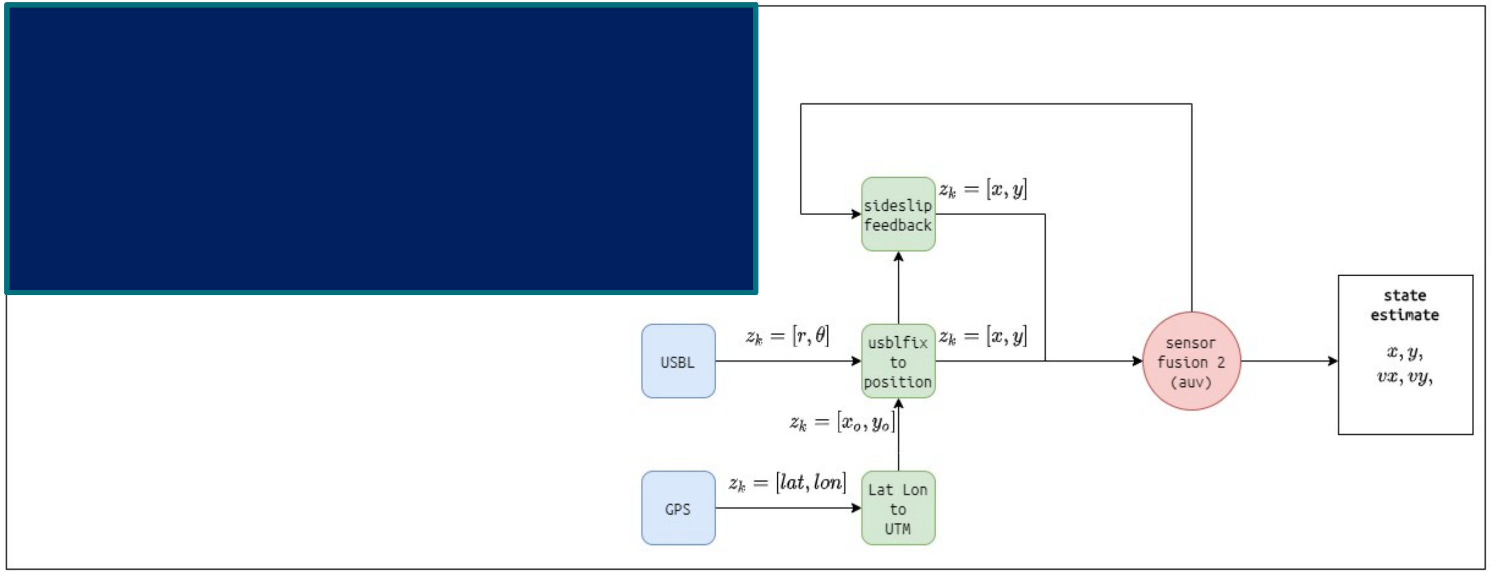


Example 3: Ship borne tracking of AUV position from USBL and GPS measurements (2D)



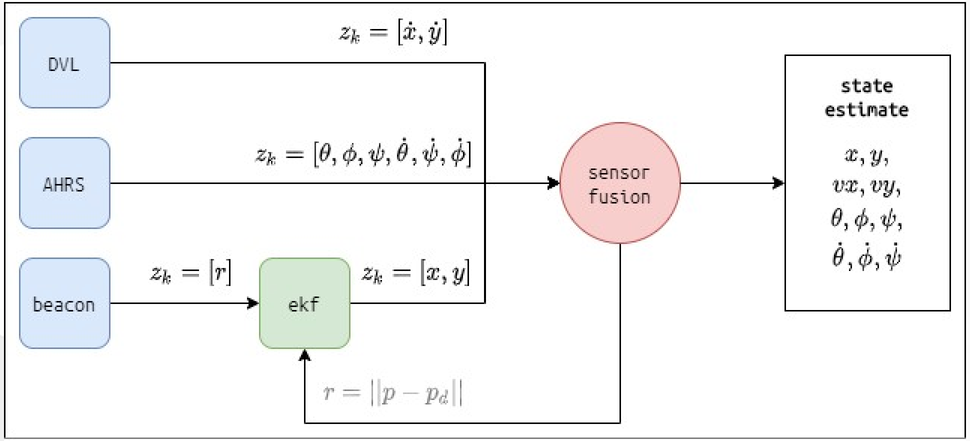
- Absolute Position of the Ship (GPS) + Relative Position of AUV (USBL)= Absolute position of the AUV

Example 4: Ship borne tracking of AUV position from USBL and GPS measurements (2D)



- Relative Position of AUV (USBL) + Absolute Position of the Ship (GPS) = Absolute position of the AUV
- Sideslip angle feedback provides position updates using “predict” stage

Example 5: AUV-borne position estimation from Doppler, AHRS, and measurements of ranges-only to known beacon (e.g. on docking station)



- Ekf to incorporate range as position updates

DSOR LAB

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5 December 2020