Aufgabe3

January 17, 2017

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from scipy.optimize import curve_fit
plt.style.use('ggplot')
plt.rcParams['text.usetex'] = True
plt.rcParams['text.latex.unicode'] = True
plt.rcParams['font.family'] = 'lmodern'
%matplotlib inline
```

In [2]:
```
df = pd.read_csv('aufg_a.csv')
print(df)
print(df.columns)
```

```
     x     "y_0"
0  0.5  0.132939
1  1.5  0.204351
2  2.5  0.197394
3  3.5  0.157457
4  4.5  0.139232
5  5.5  0.060324
6  6.5  0.056360
7  7.5  0.051944
Index(['x', ' "y_0"'], dtype='object')
```

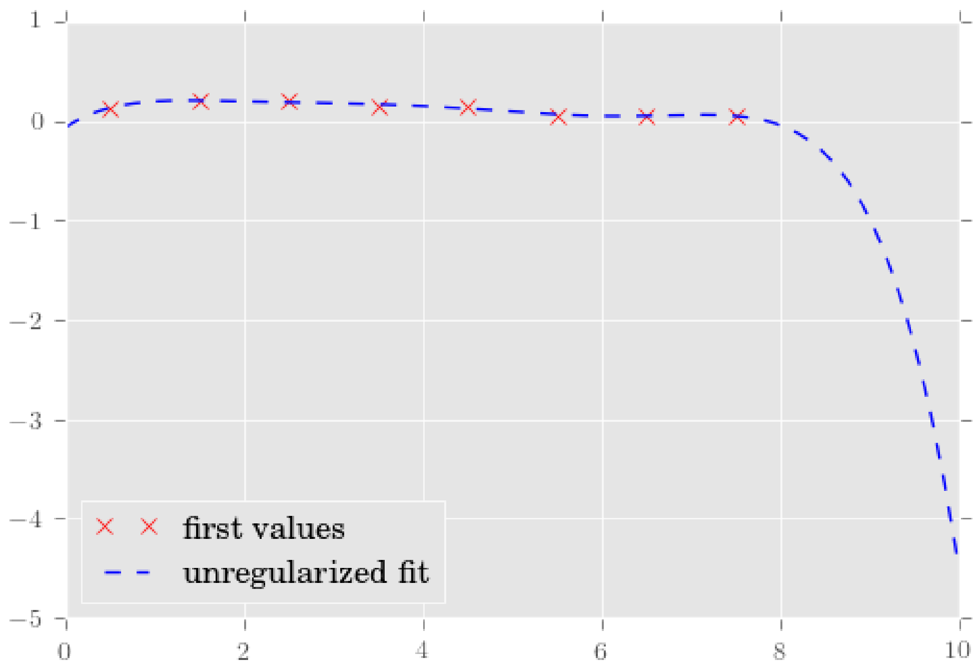
Aufgabenteil a) least squares ohne Regularisierung

In [3]:
```
def func(x,a0,a1,a2,a3,a4,a5,a6):
    return a0*x**6 + a1*x**5 + a2*x**4 + a3*x**3 + a4*x**2 + a5*x +a6
```

In [4]:
```
params , cov = curve_fit(func,df['x'],df[' "y_0"'],method='lm')
print(params)
```

```
[ -1.96287489e-04   4.78566325e-03  -4.52006131e-02   2.10565785e-01
  -5.13746564e-01   6.09607408e-01  -6.74448343e-02]
```

```
In [5]: xx=np.linspace(0,10,1000)
        plt.plot(df['x'],df[' "y_0"'],'rx',label='first values')
        plt.plot(xx,func(xx,*params),'b--',label='unregularized fit')
        plt.legend(loc='lower left')

Out[5]: <matplotlib.legend.Legend at 0x7f06a6072e80>
```

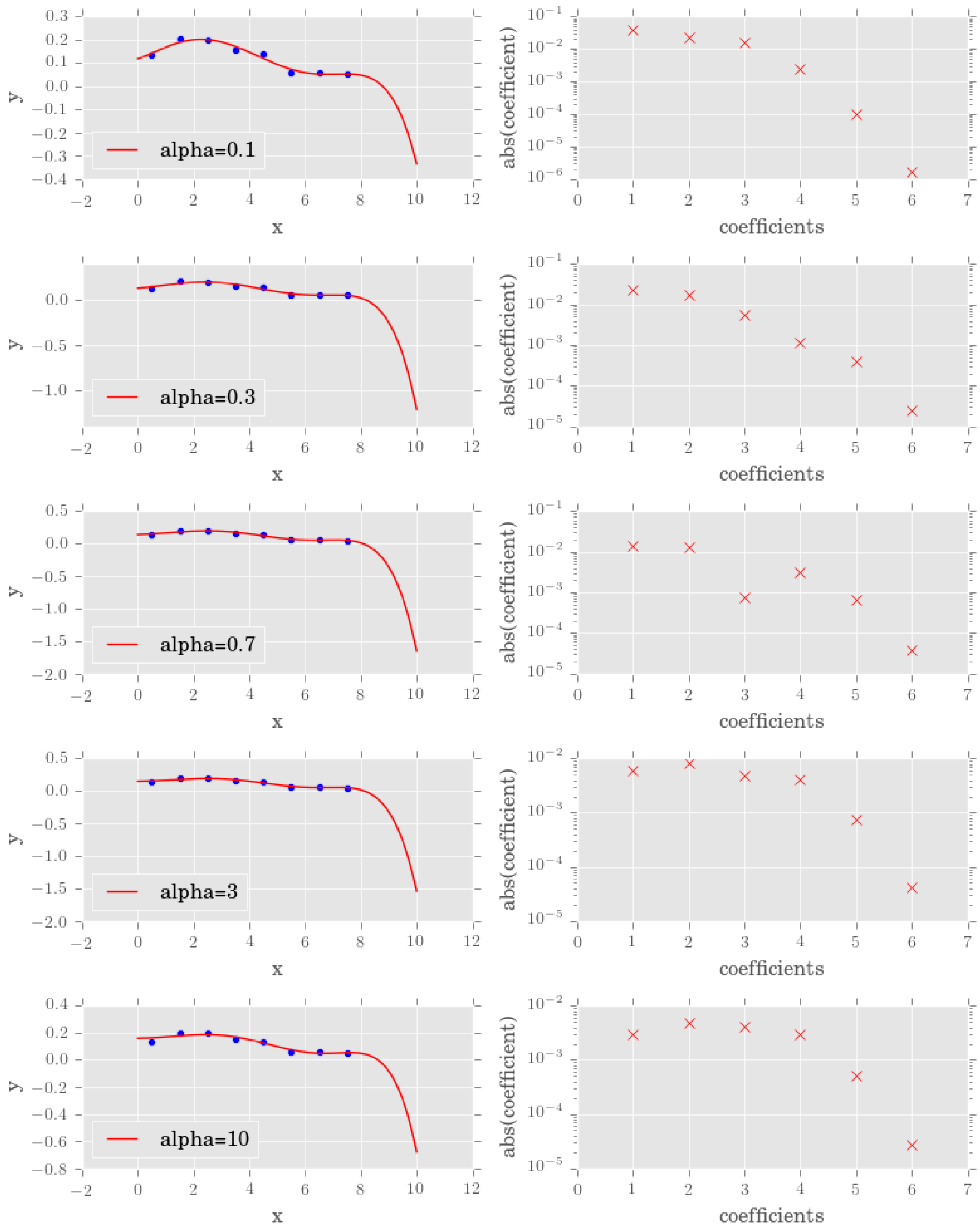


Aufgabenteil b) least squares mit Regularisierung (Ridge)

```
In [6]: X = df['x'].values
        y = df[' "y_0"'].values
        alphas = [0.1,0.3,0.7,3,10]
        xx = np.linspace(0,10,1000)

In [9]: from sklearn.linear_model import LinearRegression
        from sklearn.linear_model import Ridge
        from sklearn.pipeline import make_pipeline
        from sklearn.preprocessing import PolynomialFeatures

        def plot_approximation(est, ax, label=None):
            """Plot the approximation of ``est`` on axis ``ax``. """
            ax.scatter(X, y)
            ax.plot(xx, est.predict(xx[:, np.newaxis]), color='red', label=label)
            ax.set_ylabel('y')
            ax.set_xlabel('x')
            ax.legend(loc='lower left')   #, fontsize='small')
```

```
In [10]: fig, ax_rows = plt.subplots(5, 2, figsize=(8, 10))
         coeffs = pd.DataFrame()
         def plot_coefficients(est, ax, label=None, yscale='log'):
             coef = est.steps[-1][1].coef_.ravel()
             print(coef)
             if yscale == 'log':
                 ax.semilogy(np.abs(coef), 'rx', label=label)
             else:
                 ax.plot(np.abs(coef),'rx', label=label)
             ax.set_ylabel('abs(coefficient)')
             ax.set_xlabel('coefficients')
             ax.set_xlim((0, 7))

         degree = 6
         for alpha, ax_row in zip(alphas, ax_rows):
             ax_left, ax_right = ax_row
             est = make_pipeline(PolynomialFeatures(degree), Ridge(alpha=alpha))
             est.fit(X.reshape(-1,1), y)
             plot_approximation(est, ax_left, label='alpha=%r' % alpha)
             plot_coefficients(est, ax_right, label='Ridge(alpha=%r) coefficients'

         plt.tight_layout()
```

```
[  0.00000000e+00   3.98408576e-02   2.37780622e-02  -1.60774510e-02
   2.45036421e-03  -1.00768394e-04  -1.57905362e-06]
[  0.00000000e+00   2.29969149e-02   1.77004081e-02  -5.56852913e-03
  -1.16203148e-03   3.90150061e-04  -2.51701597e-05]
[  0.00000000e+00   1.37608196e-02   1.31475191e-02   7.69817553e-04
  -3.22846727e-03   6.62187333e-04  -3.79484647e-05]
[  0.00000000e+00   5.84671455e-03   8.15234448e-03   4.95334977e-03
  -4.17870714e-03   7.47236291e-04  -4.04580699e-05]
[  0.00000000e+00   2.95787192e-03   4.96839389e-03   4.06305987e-03
  -3.06483047e-03   5.20224266e-04  -2.68041849e-05]
```



Selbstimplementierte least squares

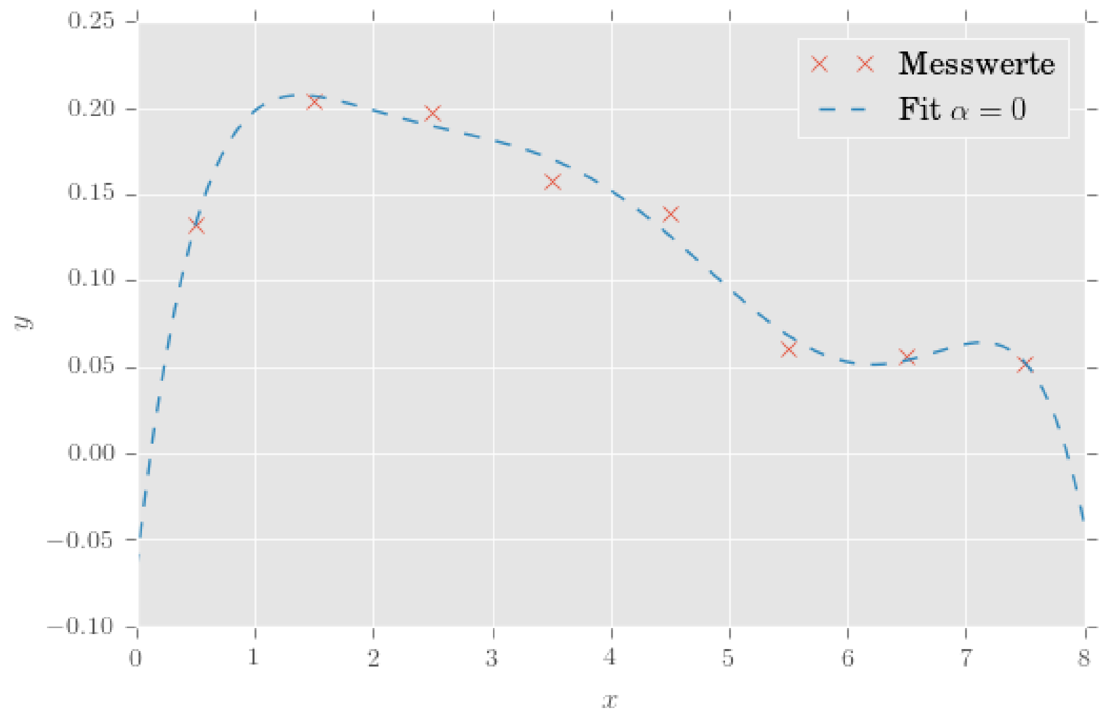
```
In [11]: def plotter(alphas,name):
             xx = np.linspace(0,8,1000)
             plt.plot(x,y,'x',label='Messwerte')
             C = np.diag(np.ones(7),-1) + np.diag(np.array([-1, -2, -2, -2, -2, -2,
```

```
        Gamma = np.dot(C,A)
        for alpha in alphas:
            a = np.dot(np.dot(np.linalg.inv(np.dot(A.T,A)+ alpha* np.dot(Gamma
            y_plot = func(xx,*a)
            plt.plot(xx,y_plot,'--',label=r'Fit $\alpha = {}$'.format(alpha))
        plt.xlabel(r'$x$')
        plt.ylabel(r'$y$')
        plt.legend(loc='best')
        plt.tight_layout()

        plt.savefig('{}.pdf'.format(name))
        plt.show()
        plt.clf()
```

```
In [12]: def get_err(y):
             return np.std(y)/np.sqrt(len(y))
         def getInvErr(y):
             return np.sqrt(len(y)) / np.std(y)
```

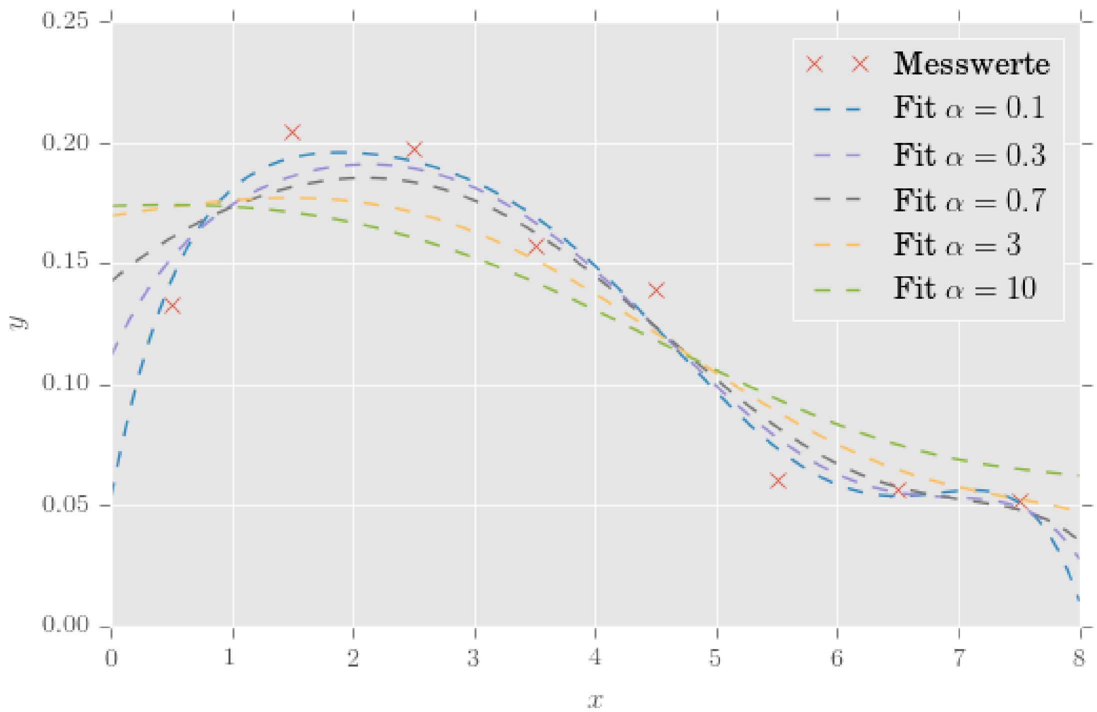
```
In [13]: def f_0(x):
             return x**0


         def f_1(x):
             return x


         def f_2(x):
             return x**2


         def f_3(x):
             return x**3


         def f_4(x):
             return x**4


         def f_5(x):
             return x**5


         def f_6(x):
             return x**6
         x = df['x']
         A = np.array([f_6(x), f_5(x), f_4(x), f_3(x), f_2(x), f_1(x), f_0(x)]).T
```

```
In [14]: plotter([0],'3a')
```




```
<matplotlib.figure.Figure at 0x7f06a1179a20>
```

```
In [15]: plotter([0.1,0.3,0.7,3,10],'3b')
```




```
<matplotlib.figure.Figure at 0x7f06a11612b0>
```

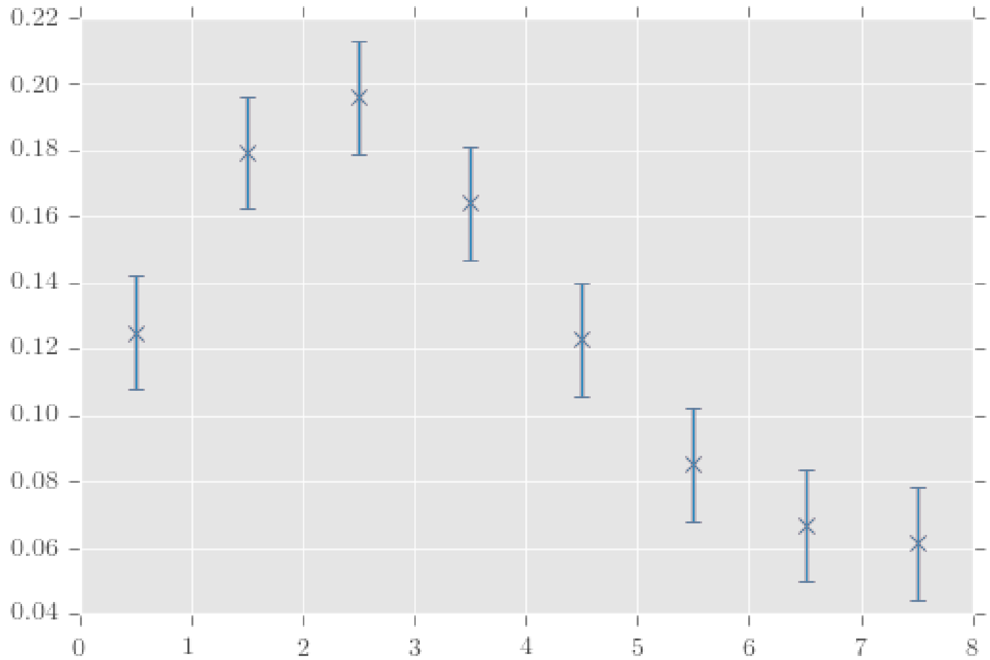
```
In [16]: df2 = pd.read_csv('aufg_c.csv')
         x2 = df2['x']
         df2
```

```
Out[16]:      x       "y_0       "y_1       "y_2       "y_3       "y_4       "y_5       "y
         0  0.5  0.103939  0.149753  0.112739  0.184439  0.145831  0.113267  0.1554
         1  1.5  0.219092  0.193544  0.160651  0.170565  0.184050  0.163337  0.2121
         2  2.5  0.177677  0.166459  0.236241  0.205570  0.197066  0.230060  0.1709
         3  3.5  0.139580  0.163861  0.167301  0.141728  0.143816  0.161717  0.1538
         4  4.5  0.115663  0.112004  0.111168  0.102507  0.108234  0.085693  0.1136
         5  5.5  0.094581  0.089400  0.079080  0.066638  0.063737  0.101826  0.0663
         6  6.5  0.086982  0.058229  0.081565  0.072236  0.088597  0.079524  0.0628
         7  7.5  0.062485  0.066751  0.051256  0.056316  0.068670  0.064577  0.0647

                "y_7       "y_8    ...        "y_40      "y_41      "y_42      "y_43  \
         0  0.155187  0.105173    ...     0.080580  0.111108  0.122147  0.146955
         1  0.166457  0.186264    ...     0.206292  0.215006  0.147765  0.164665
         2  0.181053  0.186724    ...     0.182444  0.188019  0.144067  0.239231
         3  0.131928  0.110090    ...     0.179982  0.169037  0.213696  0.157921
         4  0.148677  0.199207    ...     0.114844  0.126542  0.161769  0.111842
         5  0.092533  0.082094    ...     0.103984  0.055161  0.073740  0.091454
```

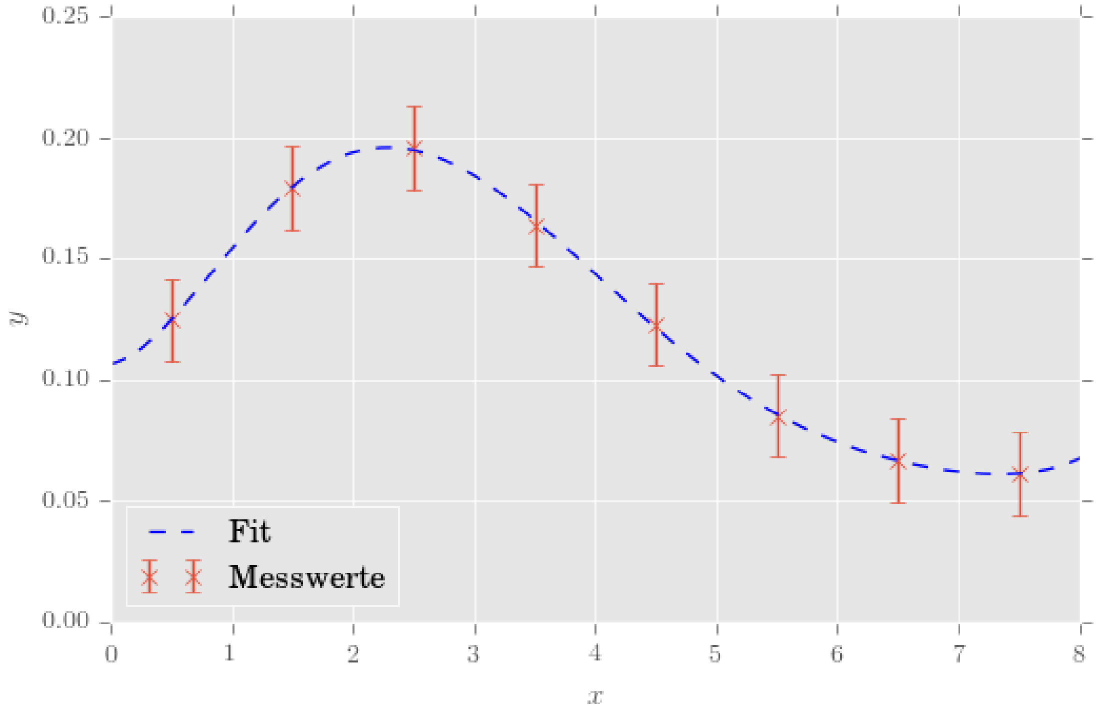
```
6  0.063917  0.060868    ...     0.065893  0.055365  0.074342  0.052247
7  0.060249  0.069581    ...     0.065981  0.079762  0.062473  0.035685

      "y_44      "y_45      "y_46      "y_47      "y_48      "y_49
0  0.076564  0.114331  0.132773  0.167562  0.135968  0.131620
1  0.238038  0.203325  0.203236  0.213183  0.223799  0.148379
2  0.103315  0.198792  0.201374  0.197383  0.126766  0.193543
3  0.229623  0.207216  0.125185  0.119271  0.212534  0.193191
4  0.142766  0.097612  0.132306  0.101199  0.085060  0.094930
5  0.069506  0.064214  0.104694  0.063192  0.094040  0.098975
6  0.070997  0.055187  0.042896  0.085384  0.053749  0.068896
7  0.069191  0.059323  0.057535  0.052826  0.068084  0.070466

[8 rows x 51 columns]
```

In [18]:
```
def getLineArrays(line):
    y = []
    for i in range(50):
        y.append(Y[i][line])
    return np.array(y)
#ekelhaft geht bestimmt auch so x, *y = ...
x, y_0, y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_10, y_11, y_12, y_1
    'aufg_c.csv', delimiter=', ', skip_header=1, unpack=True)
# Zusammenfassen der y-Werte in einer Matrix
Y = ([y_0, y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_10, y_11, y_12,
      y_26, y_27, y_28, y_29, y_30, y_31, y_32, y_33, y_34, y_35, y_36, y_
# Erzeugen von Arrays von y-Werten, die die Inhalte der Zeilen der
# csv-Tabelle enthalten
y_0 = getLineArrays(0)
y_1 = getLineArrays(1)
y_2 = getLineArrays(2)
y_3 = getLineArrays(3)
y_4 = getLineArrays(4)
y_5 = getLineArrays(5)
y_6 = getLineArrays(6)
y_7 = getLineArrays(7)
# Berechnung des Vektors y als Mittelwertvektor von y_0 bis y_7
y = np.array((np.mean(y_0), np.mean(y_1), np.mean(y_2), np.mean(
    y_3), np.mean(y_4), np.mean(y_5), np.mean(y_6), np.mean(y_7)))

# Berechnung der Gewichtungsmatrix
invErr = (getInvErr(y_0), getInvErr(y_1), getInvErr(y_2), getInvErr(
    y_3), getInvErr(y_4), getInvErr(y_5), getInvErr(y_6), getInvErr(y_7))
W = np.diag(invErr)

# Plot der Messwerte
err = get_err(y)
plt.errorbar(x, y, yerr=err, fmt='x', label='Messwerte')
```

```
plt.errorbar(x, y, yerr=err, fmt='x', label='Messwerte')

# Berechnung des Loesungsvektors a
a = np.dot(np.dot(np.dot(np.linalg.inv(np.dot(A.T, np.dot(W, A))), A.T), W
```




```
In [21]: y_plot = func(xx, *a)
         plt.errorbar(x, y, yerr=err, fmt='x', label='Messwerte')
         # Plot der Fitkurve
         plt.plot(xx, y_plot, 'b--', label=r'Fit')
         plt.xlim(0,8)
         plt.ylim(0,0.25)
         # Einstellungen
         plt.xlabel(r'$x$')
         plt.ylabel(r'$y$')
         plt.legend(loc='best')
         plt.tight_layout()
         plt.savefig('3_c.pdf')
```



In [ ]: