FUNDAMENTALS OF WIRELESS COMMUNICATION

THE WIRELESS CHANNEL

EMMANUEL OBENG FRIMPONG

18th MARCH 2021

HANBAT NATIONAL UNIVERSITY

OUTLINE OF PRESENTATION

- **Introduction**
- **Physical Modeling for Wireless Channels**
- **Input/Output Model of the Wireless Channel**
- **Time and Frequency Coherence**
- **Statistical Channel Model**

INTRODUCTION

- In past years and currently, there exist many types of communication systems – ranging from television transmission systems, wireless radio transmitters, point to point microwave systems, wired distribution network, telephony.
- All these technologies, in some way, were replaced due to the availability of new technologies.
- Presentation focuses on cellular networks – They are of great interest.
- Features of many other wireless systems can be easily understood as special cased or simple generalizations of the features of cellular networks.

INTRODUCTION

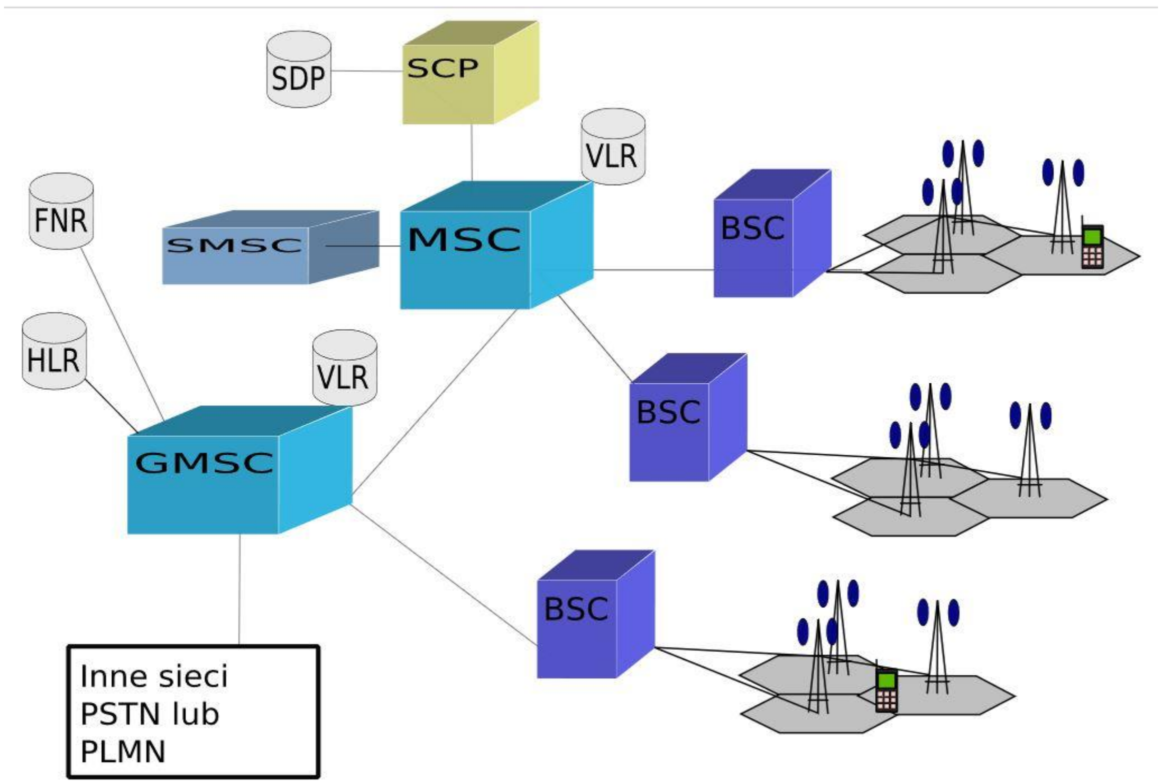


Fig.1: https://commons.wikimedia.org/wiki/File:GSM_network_architecture_01-pl.svg

INTRODUCTION

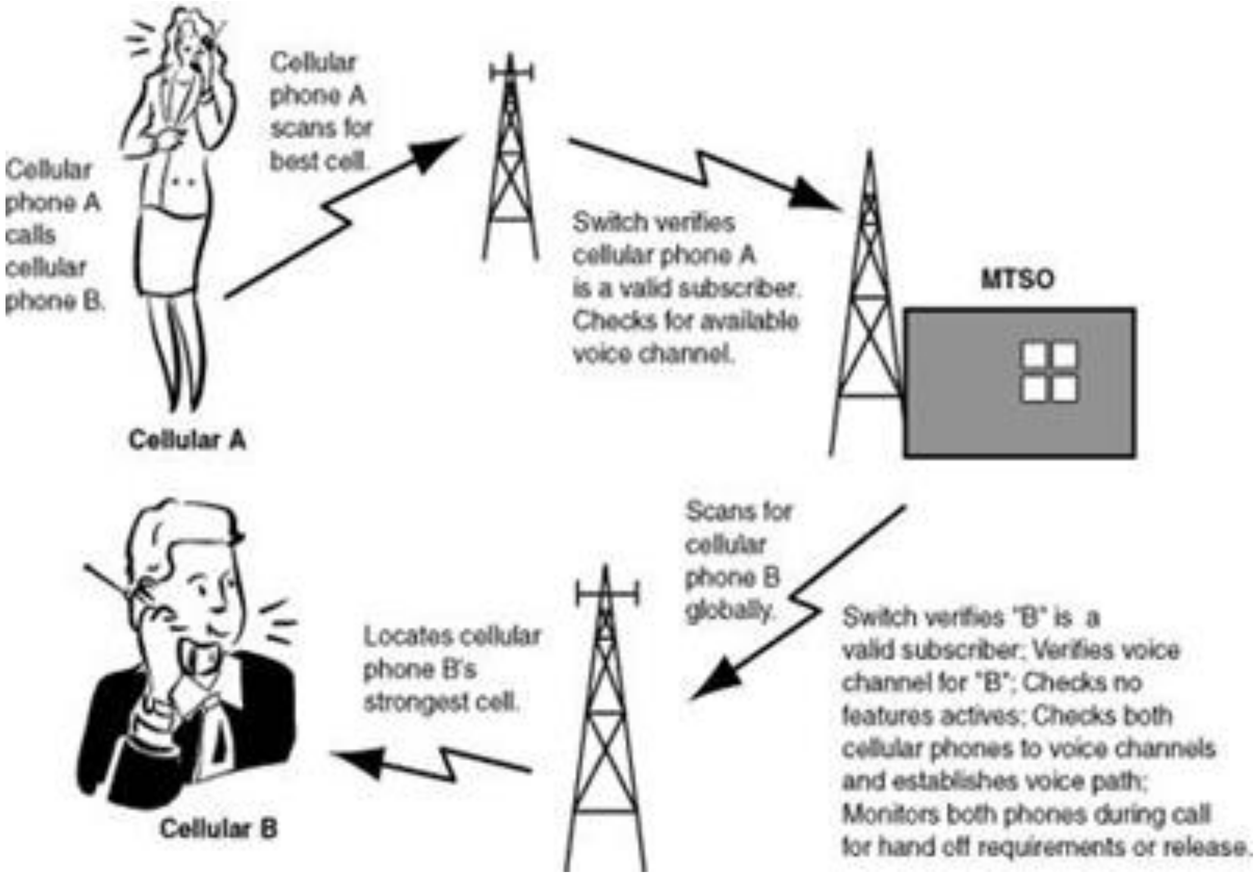


Fig.2: https://www.electroschematics.com/mobile-phone-how-it-works/

INTRODUCTION

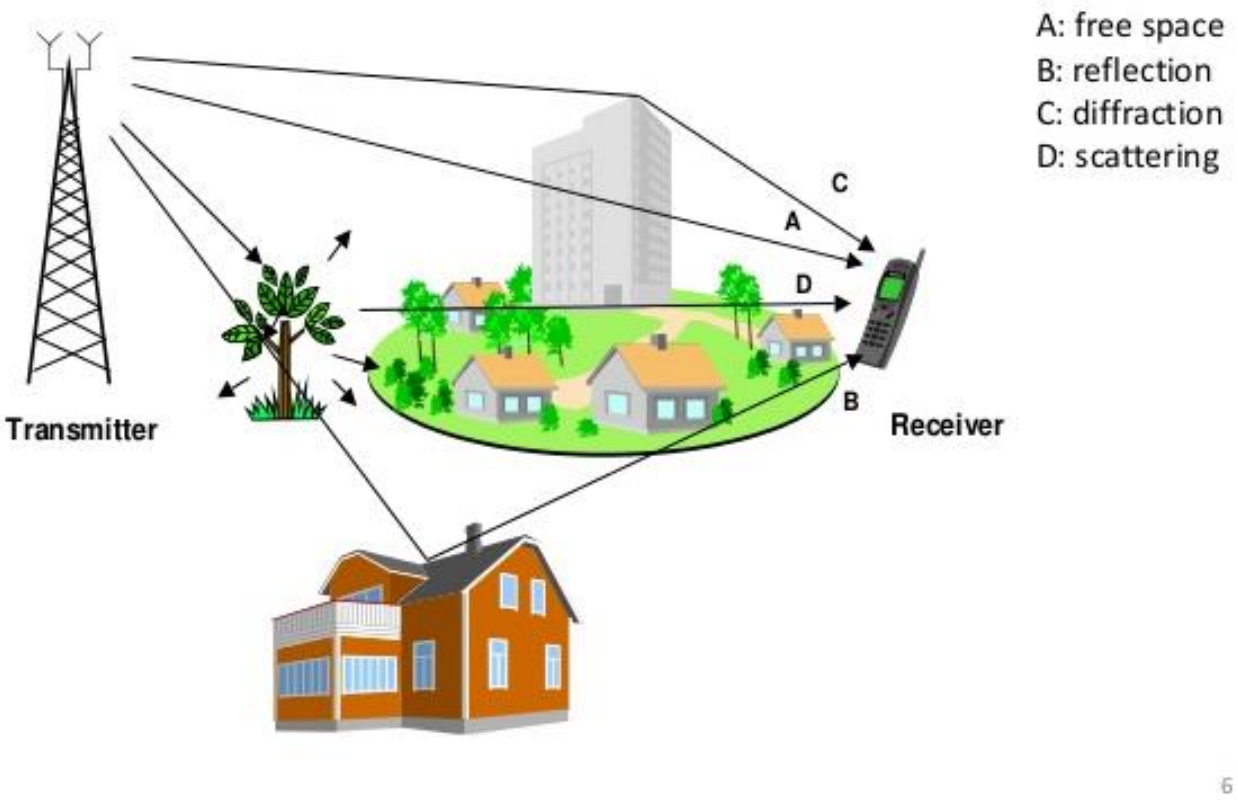


Fig 4: https://www.slideshare.net/ShreeKrupa1/multichannel-fading

INTRODUCTION

- A defining characteristic of the mobile wireless channel is the variations of the channel strength over time and over frequency. The variations can be roughly divided into two types:
- Large-scale Fading - due to path loss of signal as a function of distance and shadowing by large objects such as buildings and hills. This occurs as the mobile moves through a distance of the order of the cell size and is typically frequency independent.
- Small-scale Fading - due to the constructive and destructive interference of the multiple signal paths between the transmitter and receiver. This occurs at the spatial scale of the order of the carrier wavelength and is frequency dependent.

PHYSICAL MODELING FOR WIRELESS CHANNEL

- Idea is to use EM field equations in conjunction with the transmitted signals, to find EM field imping on the receiver antenna. EM field equations are complex.
- One of the important questions is where to choose to place the base stations, and what range of power levels are then necessary on the downlink and uplink channels.
- Another major question is what types of modulation and detection techniques look promising.
- To address this, we will construct stochastic models of the channel, assuming different channel behaviors appear with different probabilities, and change over time.

FREE SPACE, FIXED TRANSMITTING AND RECEIVING ANTENNA

- First consider a fixed antenna radiating into free space in the far field.
- In response to a transmitted sinusoid $cos 2\pi f t$, we can express the electric far field at time t as

$$E(f, t, (r, \theta, \psi)) = \frac{\alpha_s(\theta, \psi, f) \cos 2\pi f (t - \frac{r}{c})}{r}.$$

- (r, Θ, ϕ) represents the point u in space at which the electric field is being measured,
- $a_s(\Theta, \phi, f)$ is the radiation pattern of the sending antenna at frequency f in the direction (Θ, ϕ);
- Note that the phase of the field varies with fr/c; delay caused by the radiation.

FREE SPACE, FIXED TRANSMITTING AND RECEIVING ANTENNA

- Next, suppose there is a fixed receive antenna at the location $u = (r, \theta, \phi)$.
  The received waveform (in the absence of noise) in response to the above tran smitted sinusoid is then

$$E_r(f,t,u) = \frac{\alpha(\theta,\psi,f)\cos 2\pi f(t-\frac{r}{c})}{r}$$

- where $\alpha(\theta, \phi, f)$ is the product of the antenna patterns of transmitting and receive antennas in the given direction.

FREE SPACE, FIXED TRANSMITTING AND RECEIVING ANTENNA

- Now suppose, for the given u, that we define

$$H(f) := \frac{\alpha(\theta, \psi, f)e^{-j2\pi fr/c}}{r}.$$

- The above equations are both linear at the input and hence $H(f)$ is the system function for LTI channel.
- The need for understanding electromagnetism is to determine what this system function is.
- We will find in what follows that linearity is a good assumption for all the wireless channels we consider, but that the time invariance does not hold when either the antennas or obstructions are in relative motion.

FREE SPACE, MOVING ANTENNA

- Consider the fixed antenna and free space model above with a receive antenna that is moving with speed v in the direction of increasing distance from the transmitting antenna.
- Assume that the receive antenna is at a moving location described as $u(t) = (r(t), \theta, \phi)$ with $r(t) = r_0 + vt$.

$$E(f, t, (r_0 + vt, \theta, \psi)) = \frac{\alpha_s(\theta, \psi, f) \cos 2\pi f (t - \frac{r_0}{c} - \frac{vt}{c})}{r_0 + vt}.$$

- Free space e-field at the moving point $u(t)$ (Assume no receive antenna)
- The sinusoid at frequency f has been converted to a sinusoid of frequency $f(1 - v/c)$; there has been a Doppler shift of $-fv/c$ due to the motion of the observation point.

FREE SPACE, MOVING ANTENNA

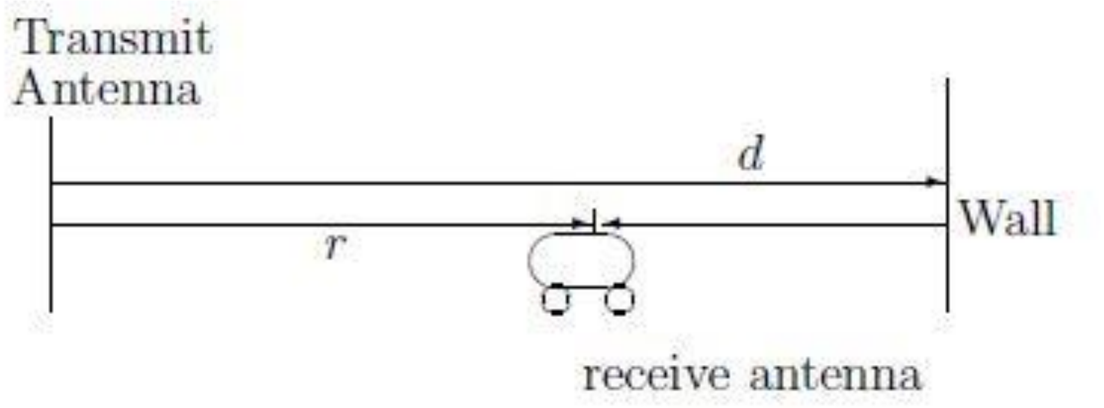
- An antenna is now placed at $u(t)$, and the change of field due to the antenna presence is again represented by the received antenna pattern, the received waveform,

$$E_r(f,t,(r_0+vt,\theta,\psi)) = \frac{\alpha(\theta,\psi,f)\cos 2\pi f\left[(1-\frac{v}{c})t - \frac{r_0}{c}\right]}{r_0+vt}.$$

- This channel cannot be represented as an LTI channel.

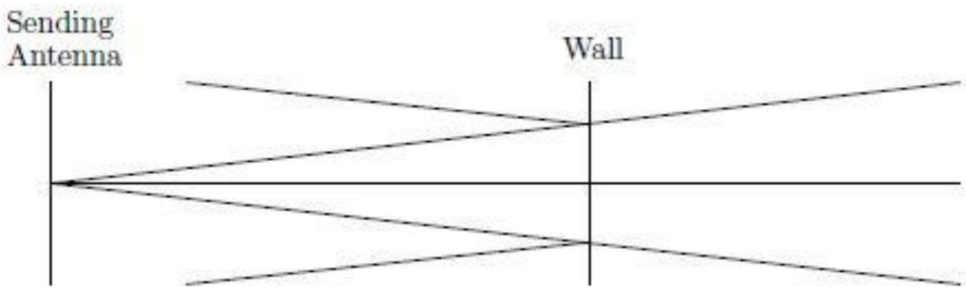
REFLECTING WALL AND FIXED ANTENNA

- Assume there is a fixed antenna transmitting the sinusoid $cos2\pi f t$, a fixed receive antenna, and a single perfectly reflecting large fixed wall.



REFLECTING WALL AND FIXED ANTENNA

- We assume that the wall is very large, the reflected wave at a given point is the same (except for a sign change) as the free space wave that would exist on the opposite side of the wall if the wall were not present



- Reflected wave from the wall has the intensity of a free space wave at a distance equal to the distance to the wall and then back to the receive antenna; $2d - r$

REFLECTING WALL AND FIXED ANTENNA

- For both direct and the reflected wave, and assuming the same antenna gain α for both waves; we have

$$E_r(f,t) = \frac{\alpha\cos 2\pi f\left(t-\frac{r}{c}\right)}{r} - \frac{\alpha\cos 2\pi f\left(t-\frac{2d-r}{c}\right)}{2d-r}.$$

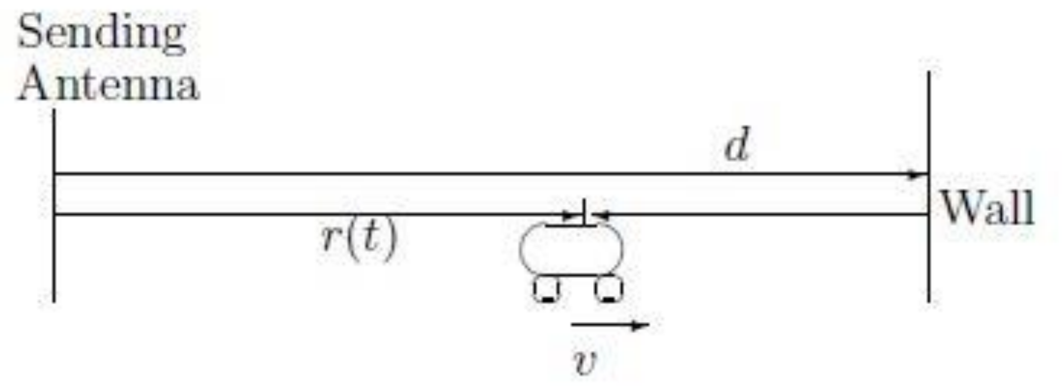
$$T_d := \frac{2d-r}{c} - \frac{r}{c}$$

REFLECTING WALL AND FIXED ANTENNA

- T_d is called the delay spread of the channel: it is the difference between the propagation delays along the two signal paths.
- The constructive and destructive interference pattern changes significantly if the frequency changes by an amount of the order of $\frac{1}{T_d}$.
- This parameter is called the coherence bandwidth.

REFLECTING WALL AND MOVING ANTENNAS

- The receive antenna is now moving at a velocity v. As it moves through the pattern of constructive and destructive interference created by the two waves, the strength of the received signal increases and decreases.
- This is the phenomenon of multipath fading.



REFLECTING WALL AND MOVING ANTENNAS

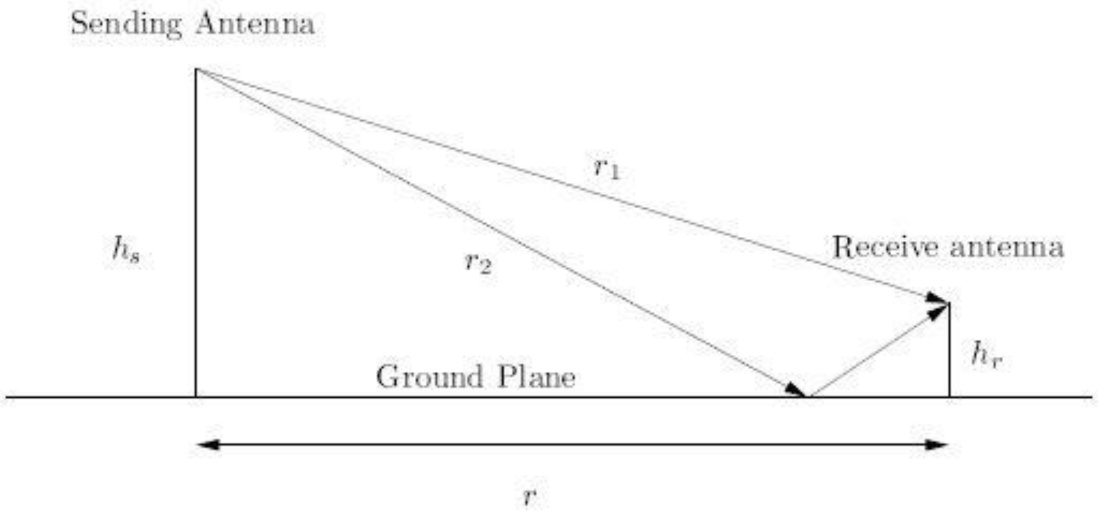
- Suppose the receive antenna is at location r_o at time 0. Taking $r = r_o + vt$
- We have;

$$E_r(f,t) = \frac{\alpha \cos 2\pi f[(1-\frac{v}{c})t - \frac{r_0}{c}]}{r_0 + vt} - \frac{\alpha \cos 2\pi f\left[(1+\frac{v}{c})t + \frac{r_0 - 2d}{c}\right]}{2d - r_0 - vt}.$$

- The first term, the direct wave, is a sinusoid of slowly decreasing magnitude at frequency $f(1 - v/c)$, experiencing a Doppler shift $D_1 = -\frac{fv}{c}$. The second is a sinusoid of smaller but increasing magnitude at frequency $f(1 + v/c)$, with a Doppler shift $D_2 = +\frac{fv}{c}$. The parameter $D_s = D_1 - D_2$ is the Doppler spread.

REFLECTION FROM A GROUND PLANE

- Consider a transmitting and a receive antenna, both above a plane surface.



- $P_R = P_t G_t G_r [\frac{h_s h_r}{r^2}]^2$
- Attenuation = r^{-2}

POWER DECAY WITH DISTANCE AND SHADOWING

- The previous example with reflection from a ground plane suggests that the received power can decrease with distance faster than r^{-2} in the presence of disturbances to free space. In practice, there are several obstacles between the transmitter and the receiver and, further, the obstacles might also absorb some power while scattering the rest.
- Thus, one expects the power decay to be considerably faster than r^{-2}. Indeed, empirical evidence from experimental field studies suggests that while power decay near the transmitter is like r^{-2}, at large distances the power decays exponentially with distance.
- Randomness in the environment is captured by modeling the density of obstacles and their absorption behavior as random numbers; the overall is called shadowing.

MOVING ANTENNA, MULTIPLE REFLECTORS

- Dealing with multiple reflectors, using the technique of ray tracing, is in principle simply a matter of modeling the received waveform as the sum of the responses from the different paths rather than just two paths.
- Consider another type of reflection called scattering which can occur in the atmosphere or in reflections from very rough objects. There are a very large number of individual paths, and the received waveform is better modeled as an integral over paths with infinitesimally small differences in their lengths, rather than as a sum.
- We are not interested in the amplitude of the reflected field from any reflector, or any detailed response on each path. We want to understand the nature of the aggregate received waveform given a representation for each reflected wave.
- This leads to modeling the input/output behavior of a channel.

THE WIRELESS CHANNEL AS A LINEAR TIME-VARYING SYSTEM

- Previously the received signal of the transmitted $cos2\pi ft$ can be written as

$$\sum_i a_i(f,t)\phi(t-\tau_i(f,t)).$$

- We have described the channel effect at a particular frequency f.
- If we further assume that the $a_i(f,t)'s$ and the $\tau_i(f,t)'s$ do not depend on the frequency f, then we can use the principle of superposition to generalize the above input-output relation to an arbitrary input $x(t)$

$$y(t)=\sum_i a_i(t)x(t-\tau_i(t)).$$

THE WIRELESS CHANNEL AS A LINEAR TIME-VARYING SYSTEM

- It should however be noted that although the individual attenuations and delays are assumed to be independent of the frequency, the overall channel response can still vary with frequency since different paths have different delays.
- Since the channel is linear, it can be described by the response $h(\tau, t)$ at time t to an impulse transmitted at time $t - \tau$. In terms of $h(\tau, t)$, the input-output relationship is

$$y(t) = \int_{-\infty}^{\infty} h(\tau, t) x(t - \tau) d\tau.$$

HANBAT NATIONAL UNIVERSITY

THE WIRELESS CHANNEL AS A LINEAR TIME-VARYING SYSTEM

- Comparing the equations, the impulse response for the fading multipath channel is

$$h(\tau, t) = \sum_i a_i(t)\delta(\tau - \tau_i(t)).$$

- In the special case when the transmitter, receiver and the environment are all stationary, the attenuations $a_i(t)'s$ and propagation delays $\tau_i(t)'s$ do not depend on time t, and we have the usual linear time-invariant channel with an impulse response.

$$h(\tau) = \sum_i a_i\delta(\tau - \tau_i).$$

THE WIRELESS CHANNEL AS A LINEAR TIME-VARYING SYSTEM

- The time taken for the channel to change significantly is of the order milliseconds while the delay spread is of the order of microseconds. (Coherence time is greater than the delay spread) – Fading channels with this characteristics are sometimes called underspread channels.

BASEBAND EQUIVALENT MODEL

- Wireless communication occurs in passband of specified bandwidth around the center frequency. However, most of the processing, such as coding/decoding, modulation/demodulation, synchronization, is done at the baseband.

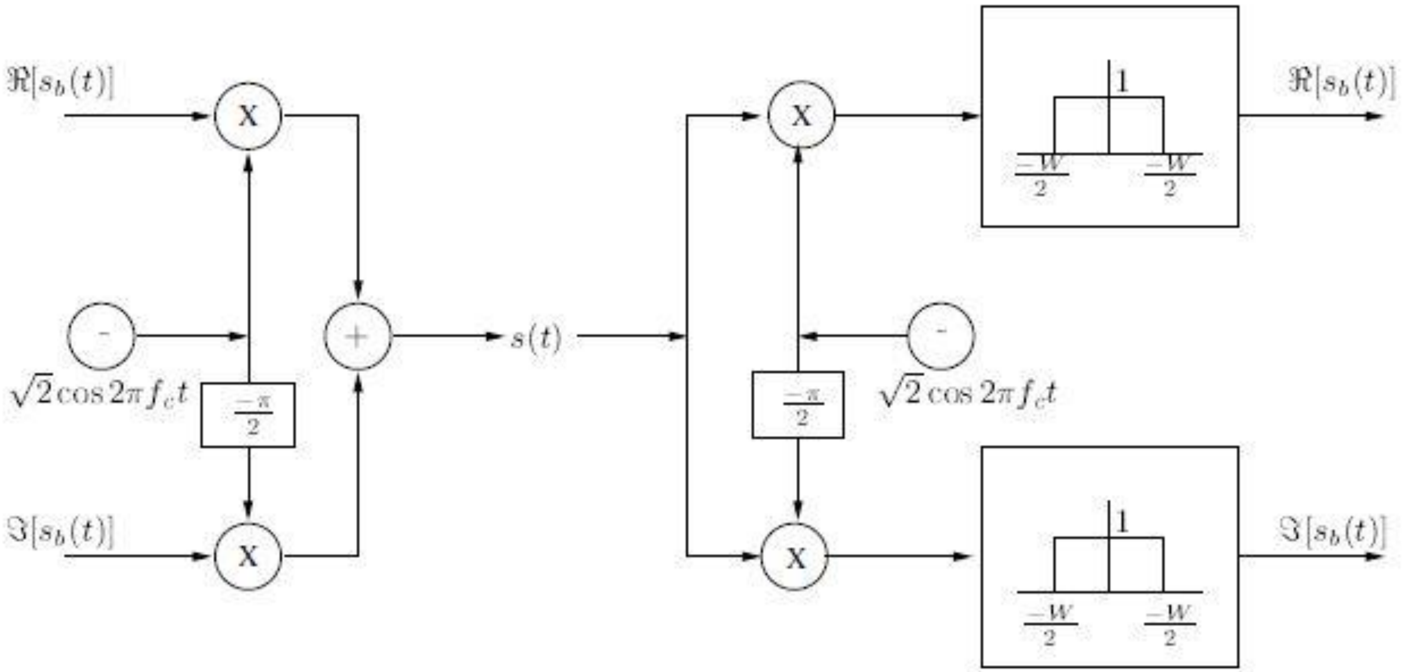
- It is important to have a baseband equivalent representation of the system.

- Consider a real signal $s(t)$ with Fourier transform $S(f)$, bandlimited in $[fc - W/2, fc + W/2]$ with $W < 2fc$. Define its complex baseband equivalent $s_b(t)$ as the signal having Fourier transform:

$$S_b(f) = \begin{cases} \sqrt{2}S(f+f_c) & f+f_c > 0 \\ 0 & f+f_c \leq 0 \end{cases}$$

BASEBAND EQUIVALENT MODEL

$$s(t) = \frac{1}{\sqrt{2}} \left\{ s_b(t) e^{j2\pi f_c t} + s_b^*(t) e^{-j2\pi f_c t} \right\} = \sqrt{2} \Re \left[s_b(t) e^{j2\pi f_c t} \right]$$



BASEBAND EQUIVALENT MODEL

- The baseband equivalent channel is

$$y_b(t) = \sum_i a_i^b(t) x_b(t - \tau_i(t)),$$

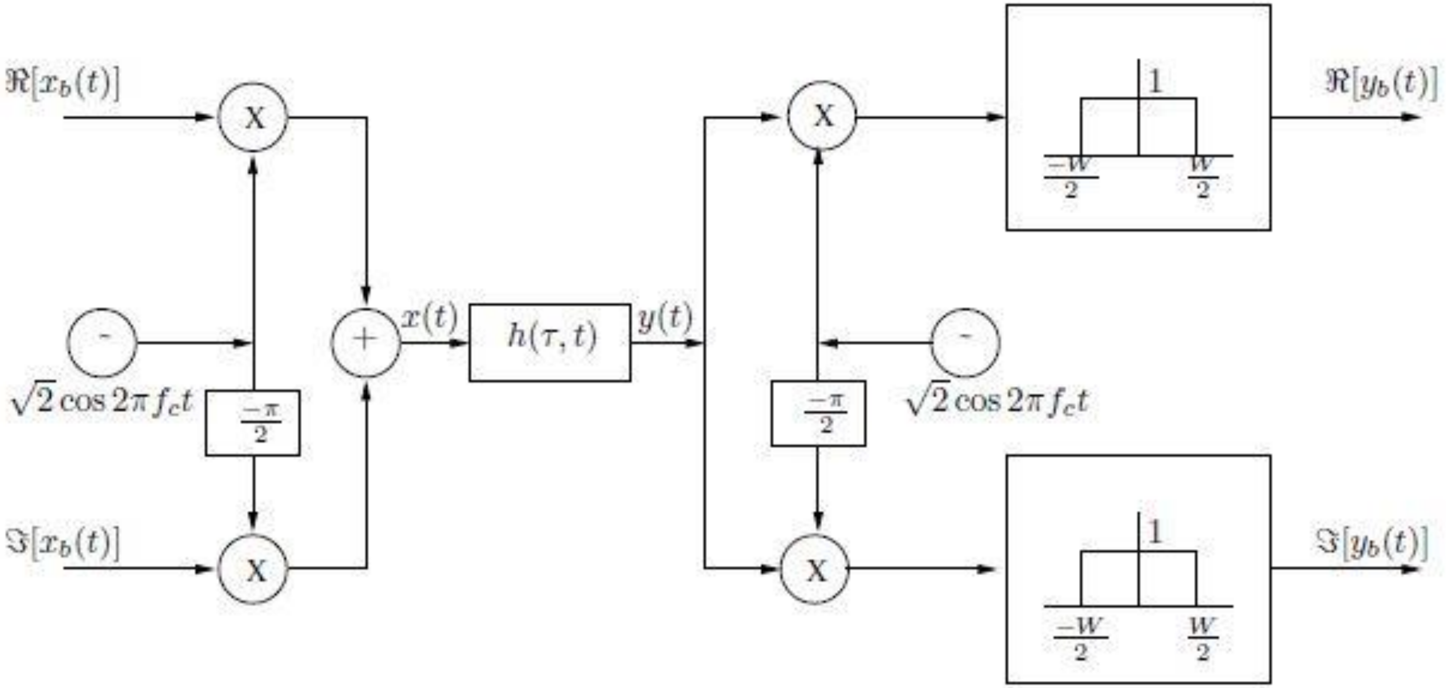
where

$$a_i^b(t) := a_i(t) e^{-j2\pi f_c \tau_i(t)}.$$

- The input and output relationship above is linear and time varying, and the baseband equivalent response is

$$h_b(\tau, t) = \sum_i a_i^b(t) \delta(\tau - \tau_i(t)).$$

BASEBAND EQUIVALENT MODEL



A DISCRETE TIME BASEBAND MODEL

- Convert the continuous time channel to a discrete time channel using sampling theorem. Assume that the input waveform $x(t)$ is bandlimited to W. The baseband equivalent is then limited to W/2 and can be represented as

$$x_b(t) = \sum_n x[n]\mathrm{sinc}(Wt - n),$$

where $x[n]$ is given by $x_b(n/W)$ and $\mathrm{sinc}(t)$ is defined as

$$\mathrm{sinc}(t) := \frac{\sin(\pi t)}{\pi t}.$$

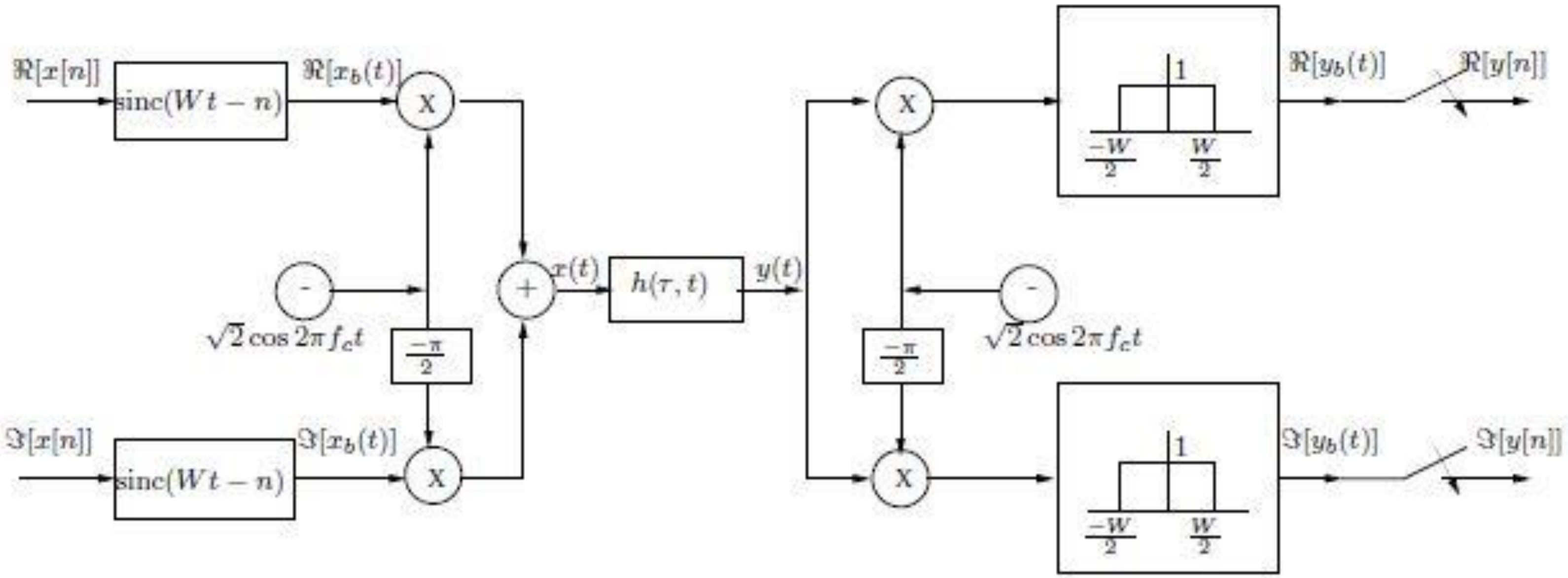
- The baseband output is

$$y_b(t) = \sum_n x[n] \sum_i a_i^b(t)\mathrm{sinc}\left(Wt - W\tau_i(t) - n\right).$$

The sampled outputs at multiples of $1/W$, $y[m] := y_b(m/W)$, are then given by

$$y[m] = \sum_n x[n] \sum_i a_i^b(m/W)\mathrm{sinc}[m - n - \tau_i(m/W)W].$$

A DISCRETE TIME BASEBAND MODEL



ADDITIVE WHITE NOISE

- As a last step, we include additive noise in our input/output model. We make the standard assumption that $w(t)$ is zero-mean additive white Gaussian noise (AWGN) with power spectral density $\frac{N_o}{2}$
- The Linear Time-Varying System can be modified as

$$y(t) = \sum_i a_i(t)x(t - \tau_i(t)) + w(t).$$

- Also, the discrete-time baseband-equivalent model becomes;

$$y[m] = \sum_\ell h_\ell[m]x[m - \ell] + w[m],$$

ADDITIVE WHITE NOISE

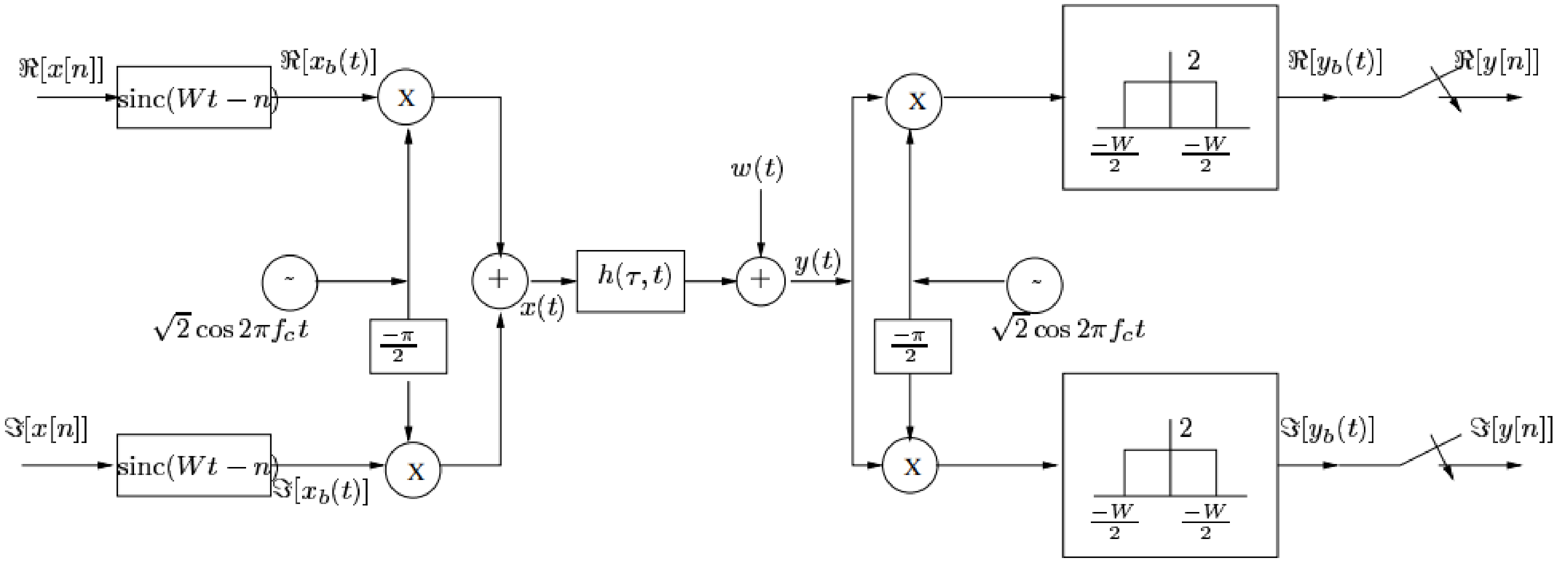
- Where w[m] is the low-pass filtered noise at the sampling instant m/W. Just like the signal, the white noise w(t) is down-converted, filtered at the baseband and ideally sampled.
- Thus,

$$\begin{aligned}\Re(w[m]) &= \int_{-\infty}^{\infty} w(t)\psi_{m,1}(t)dt,\\ \Im(w[m]) &= \int_{-\infty}^{\infty} w(t)\psi_{m,2}(t)dt,\end{aligned}$$

- Where,

$$\psi_{m,1}(t) := \sqrt{2W}\cos\left(2\pi f_c t\right)\operatorname{sinc}(Wt-m), \qquad \phi_{m,2}(t) := -\sqrt{2W}\sin\left(2\pi f_c t\right), \operatorname{sinc}(Wt-m).$$

ADDITIVE WHITE NOISE



- The assumption of AWGN essentially means that we are assuming that the primary source of the noise is at the receiver or is radiation impinging on the receiver that is independent of the paths over which the signal is being received.

DOPPLER SPREAD AND COHERENCE TIME

- An important channel parameter is the time-scale of the variation of the channel.
- How fast do the taps $h_l[m]$ vary as a function of time m?

$$\begin{aligned} h_\ell[m] &= \sum_i a_i^b(m/W)\mathrm{sinc}\left[\ell - \tau_i(m/W)W\right], \\ &= \sum_i a_i(m/W)e^{-j2\pi f_c \tau_i(m/W)}\mathrm{sinc}\left[\ell - \tau_i(m/W)W\right]. \end{aligned}$$

- Different paths contributing to the l^{th} tap may have different doppler shifts and may significantly change the magnitude of $h_l[m]$.
- This is happening at the time-scale inversely proportional to the largest difference between the Doppler shifts. The Doppler spread Ds: $D_s := \max_{i,j} f_c|\tau_i'(t) - \tau_j'(t)|,$

DOPPLER SPREAD AND COHERENCE TIME

- The coherence time, Tc, of a wireless channel is defined as the interval over which hl[m] changes significantly as a function of m.
- At coherence time; a propagating wave may be considered coherent, meaning that its phase is, on average, predictable.

$$T_c = \frac{1}{4D_s}.$$

- In the wireless communication literature, channels are often categorized as fast fading and slow fading.
- We will call a channel fast fading if the coherence time Tc is much shorter than the symbol period (delay requirement of the application), and slow fading if Tc is longer.

DELAY SPREAD AND COHERENCE BANDWIDTH

- Another important general parameter of a wireless system is the multipath delay spread, Td, defined as the difference in propagation time between the longest and shortest path, counting only the paths with significant energy.
- This is given by

$$T_d := \max_{i,j} |\tau_i(t) - \tau_j(t)|.$$

- The delay spread of the channel dictates its frequency coherence. Wireless channels change both in time and frequency. The time coherence shows us how quickly the channel changes in time, and similarly, the frequ-ency coherence shows how quickly it changes in frequency.

DELAY SPREAD AND COHERENCE BANDWIDTH

- Coherence bandwidth frequency interval over which two frequencies of a signal are likely to experience comparable or correlated amplitude fading
- W_c, coherence bandwidth is expressed as

$$W_c = \frac{1}{2T_d}.$$

- When the bandwidth of the input is considerably less than W_c, the channel is referred to as flat fading. In this case, the delay spread T_d is much less than the symbol time $\frac{1}{W}$, and a single channel filter tap is sufficient to represent the channel.
- When the bandwidth is much larger than W_c, the channel is said to be frequency-selective, and it must be represented by multiple taps.

A SUMMARY OF THE TYPES OF WIRELESS CHANNELS AND THEIR DEFINING CHARACTERISTICS.

Types of Channel	Defining Characteristic
fast fading	$T_c \ll$ delay requirement
slow fading	$T_c \gg$ delay requirement
flat fading	$W \ll W_c$
frequency-selective fading	$W \gg W_c$
underspread	$T_d \ll T_c$

MODELLING PHILOSOPHY

- We defined Doppler spread and multipath spread in the previous section as quantities associated with a given receiver at a given location, velocity, and time. However, we are interested in a characterization that is valid over some range of conditions.
- Without models, systems are designed using experience and experimentation, and creativity becomes somewhat stifled. Even with highly over-simplified models, we can compare different system approaches an get a sense of what types of approaches are worth pursuing.
- To a certain extent, all analytical work is done with simplified models.

RAYLEIGH AND RICIAN FADING

- The simplest probabilistic model for the channel filter taps assumes that there are many statistically independent reflected and scattered paths with random amplitudes in the delay window corresponding to a single tap.
- Since the reflectors and scatterers are far away relative to the carrier wavelength, i.e., di λ, it is reasonable   to assume that the phase for each path is uniformly distributed between 0 and 2π and that the phases of different paths are independent.
- The contribution of each path in the tap gain h[m] is

$$a_i(m/W)e^{-j2\pi f_c \tau_i(m/W)}\operatorname{sinc}\left[\ell - \tau_i(m/W)W\right]$$

- This can be modeled as a circular symmetric complex random variable.

RAYLEIGH AND RICIAN FADING

- It follows that R(h[m]) is the sum of many small independent real random variables, and so by the Central Limit Theorem, it can reasonably be modeled as a zero-mean Gaussian random variable.
- With this assumed Gaussian probability density, we know that the magnitude |h[m]| of the l^{th} tap is a Rayleigh random variable.
- This model, which is called Rayleigh fading, is quite reasonable for scattering mechanisms where there are many small reflectors but is adopted primarily for its simplicity in typical cellular situations with a relatively small number of reflectors. The word Rayleigh is almost universally used for this model, but the assumption is circularly symmetric complex Gaussian random variables.

RAYLEIGH AND RICIAN FADING

- There is a frequently used alternative model in which the line-of-sight path (often called a specular path) is large and has a known magnitude, and that there are also many independent paths.
- The magnitude of such a random variable is said to have a Rician distribution.

$$h_\ell[m] = \sqrt{\frac{\kappa}{\kappa+1}}\sigma_\ell e^{j\theta} + \sqrt{\frac{1}{\kappa+1}}\mathcal{CN}(0, \sigma_\ell^2)$$

- the first term corresponding to the specular path arriving with uniform phase and the second term corresponding to the aggregation of the large number of reflected and scattered paths, independent of .
- The parameter (K-factor) is the ratio of the energy in the specular path to the energy in the scattered paths; the larger is, the more deterministic is the channel.

TAP GAIN AUTOCORRELATION FUNCTION

- Modeling each $h_l[m]$ as a complex random variable provides part of the statistical description that we need, but this is not the most important part. The more important issue is how these quantities vary with time.

- The rate of channel variation has significant impact on several aspects of the communication problem. A statistical quantity that models this relationship is known as the tap gain autocorrelation function, $R_l[n]$.

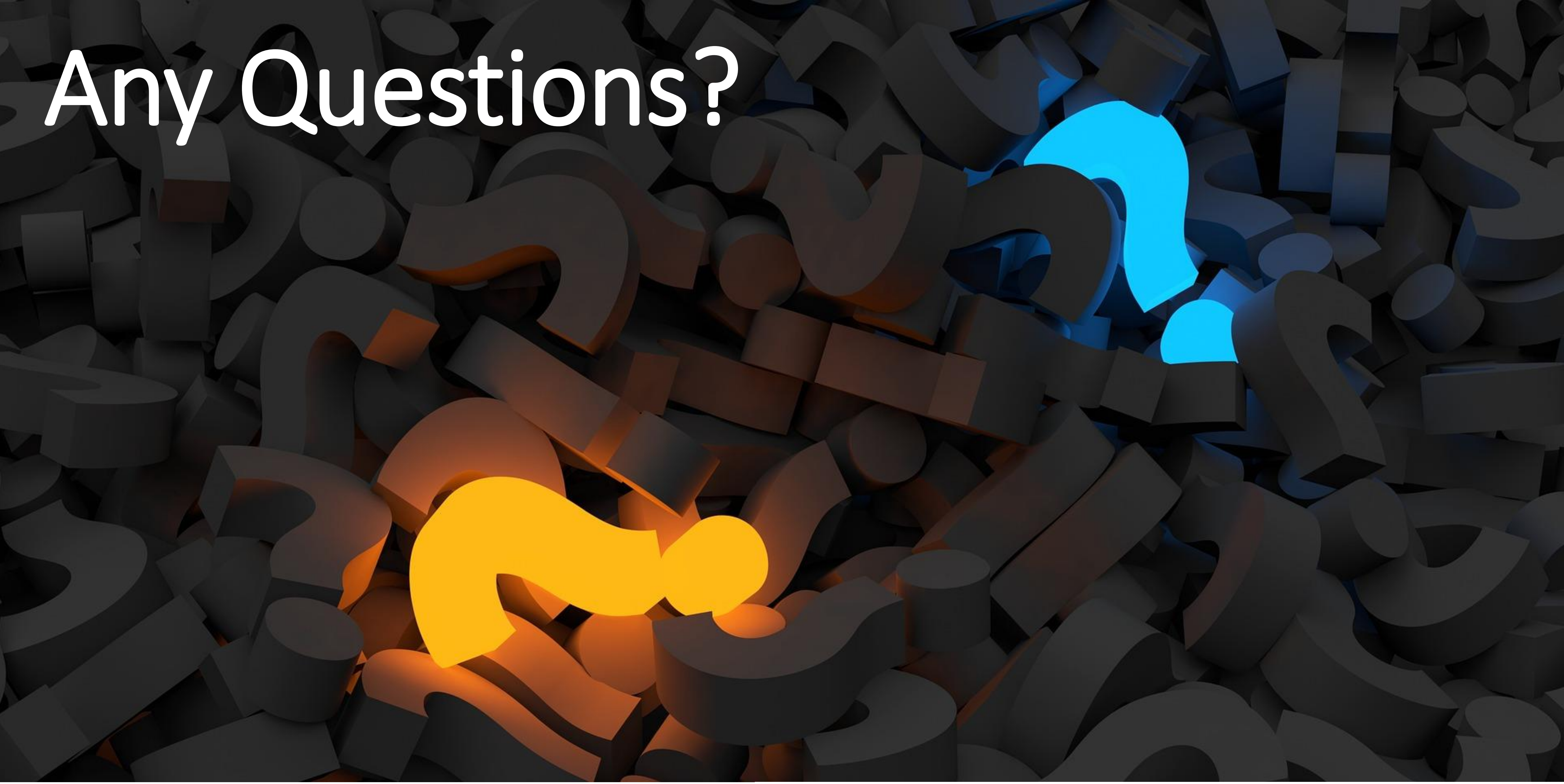
$$R_\ell[n] := \mathbb{E}\left\{h_\ell^*[m]h_\ell[m+n]\right\}.$$

HANBAT NATIONAL UNIVERSITY

TAP GAIN AUTOCORRELATION FUNCTION

- The coefficient $R_l[0]$ is proportional to the energy received in the l^{th} tap. The multipath spread T_d can be defined as the product of $1/W$ times the range of which contains most of the total energy $\sum_{l=0}^{\infty} R_l[0]$
- The tap gain autocorrelation function is useful as a way of expressing the statistics for how tap gains change given a particular bandwidth W but gives little insight into questions related to the choice of a bandwidth for communication.

Any Questions?



HANBAT NATIONAL UNIVERSITY

E-mail: frimpongemmanuelobeng@gmail.com