FUNDAMENTALS OF WIRELESS COMMUNICATION

Multiuser Capacity and Opportunistic Communication

EMMANUEL OBENG FRIMPONG

9th Feb 2022

HANBAT NATIONAL UNIVERSITY

OUTLINE OF PRESENTATION

- INTRODUCTION
- UPLINK CHANNEL (AWGN, FADING)
- DOWNLINK CHANNEL (AWGN, FADING)
- FREQUENCY-SELECTIVE FADING CHANNELS
- MULTIUSER DIVERSITY

INTRODUCTION

- After studying chapter 4 (multiple access techniques), a natural question is: what are the "optimal" multiple access schemes?
- To address this question, one must now step back and take a fundamental look at the multiuser channels themselves.

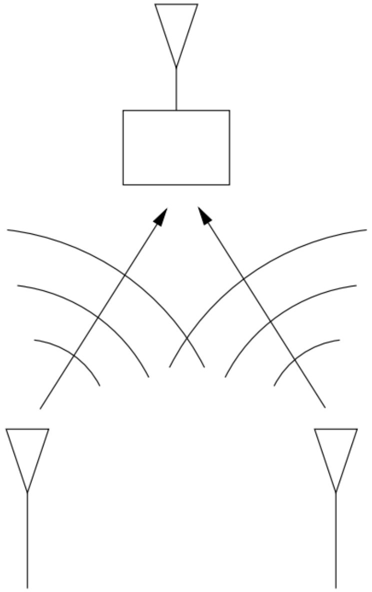
UPLINK AWGN CHANNEL

- The baseband discrete-time model for the uplink AWGN channel with two users:

$$y[m] = x_1[m] + x_2[m] + w[m]$$

- Point-to-point case: reliable communication is attained at any rate $R < C$

In the multiuser case, we should extend this concept to a capacity region C: the set of all pairs $(R1, R2)$ such that simultaneously $user\ 1\ and\ 2$ can reliably communicate at rate $R1$ and $R2$, respectively.



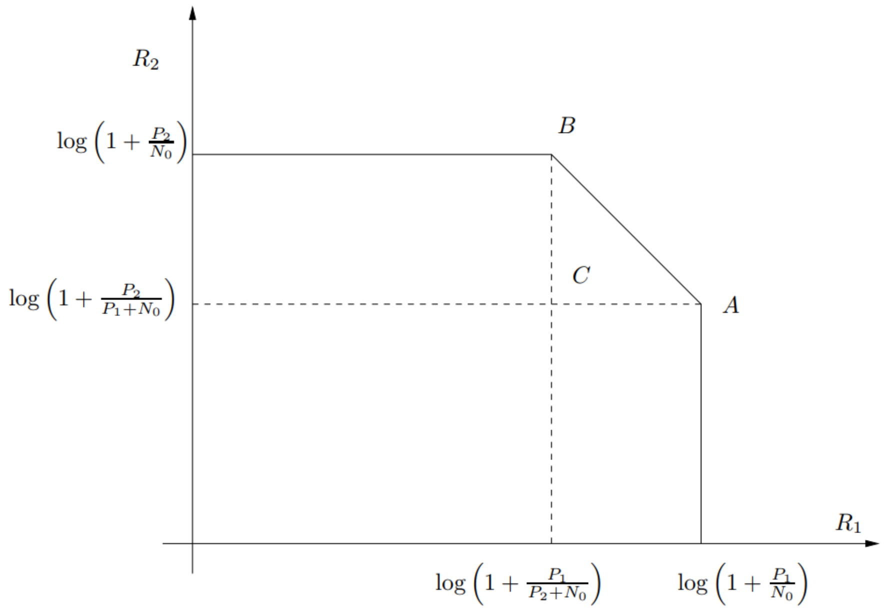
UPLINK AWGN CHANNEL

Symmetric capacity: is the maximum common rate at which both the users can simultaneously reliably communicate. $C_{sym} = \max_{(R,R)\in C} R$

Sum capacity: the maximum total throughput that c an be achieved.

$$C_{sym} = \max_{(R,R)\in C} R_1 + R_2$$

$$R_2^* = \log\left(1 + \frac{P_1 + P_2}{N_0}\right) - \log\left(1 + \frac{P_1}{N_0}\right) = \log\left(1 + \frac{P_2}{P_1 + N_0}\right)$$



$$\begin{aligned} R_1 &< \log\left(1 + \frac{P_1}{N_0}\right) \\ R_2 &< \log\left(1 + \frac{P_2}{N_0}\right) \\ R_1 + R_2 &< \log\left(1 + \frac{P_1 + P_2}{N_0}\right). \end{aligned}$$

General K-user Uplink Capacity

- The K-user capacity region is described by $2^K - 1$ constraints, one for each possible non-empty subset S of users:

$$\sum_{k \in \mathcal{S}} R_k < \log\left(1 + \frac{\sum_{k \in \mathcal{S}} P_k}{N_0}\right) \qquad \text{for all } \mathcal{S} \subset \{1, \ldots, K\}$$

R_k is the maximum sum rate that can be achieved by a single transmitter with the total power of the users in S and with no other users in the system.

The sum capacity

$$C_{\text{sum}} = \log\left(1 + \frac{\sum_{k=1}^{K} P_k}{N_0}\right) \quad \text{bits/s/Hz}$$

General K-user Uplink Capacity

- The equal received power case $(P_1 = \ldots = P_K = P)$ is particularly simple.

- The sum capacity is

$$C_{\text{sum}} = \log\left(1 + \frac{KP}{N_0}\right)$$

- The symmetric capacity is

$$C_{\text{sym}} = 1/K \cdot \log\left(1 + \frac{KP}{N_0}\right)$$

Downlink AWGN Channel

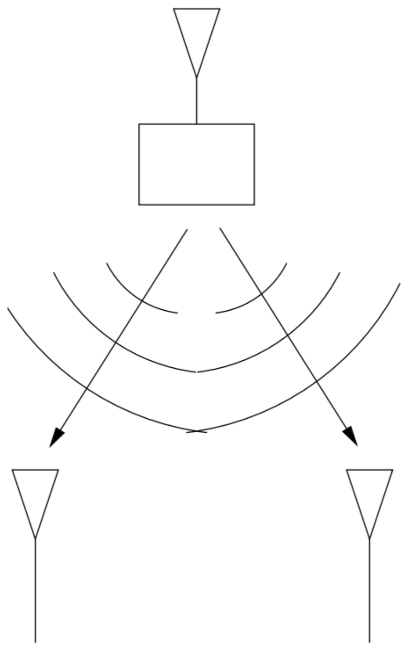
- The baseband downlink AWGN channel with two users is:

$$y_k[m] = h_k x[m] + w_k[m], \quad k = 1, 2,$$

- We assume that h_k is known to both the transmitter and the user k $(for\ k = 1, 2)$

$$R_k < \log\left(1 + \frac{P|h_k|^2}{N_0}\right), \quad k = 1, 2.$$

Upper bound, R_k, can be attained by using all the power and degrees of freedom to communicate to $user\ k$ (with the other user getting zero rate).



Downlink AWGN Channel: Symmetric Case

- Two coding schemes are both optimal:
- single-user codes followed by orthogonalization of the degrees of freedom among the users, and the superposition coding scheme.
- first consider the symmetric case where $|h_1| = |h_2|$.
- This means that if user 1 can successfully decode its data, then user 2 with same SNR should also be able to decode successfully the data of user 1 (and vice versa).
- Sum rate is bounded by the single user capacity:

$$R_1 + R_2 < \log\left(1 + \frac{P|h_1|^2}{N_0}\right)$$

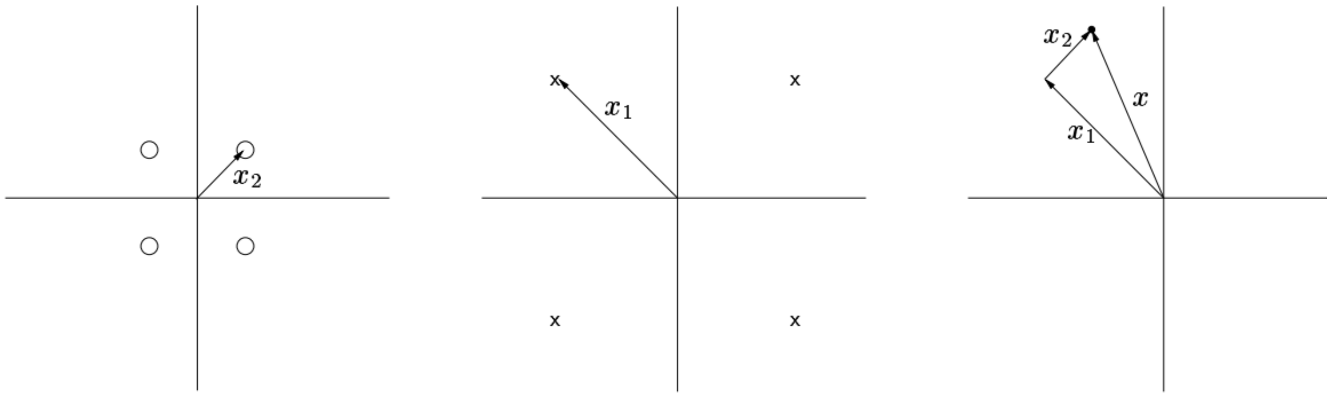
Downlink AWGN Channel: Symmetric Case

- The symmetry between the two channels suggests a natural, and alternative, idea that if $user\ 1$ can successfully decode its data from y_1, then $user\ 2$ which has the same SNR should also be able to decode the data of $user\ 1$ from y_2. Then user 2 can subtract the codeword of $user\ 1$ from its received signal y_2 to better decode its own data, i.e., It can perform successive interference cancellation.

Downlink AWGN Channel: Symmetric Case

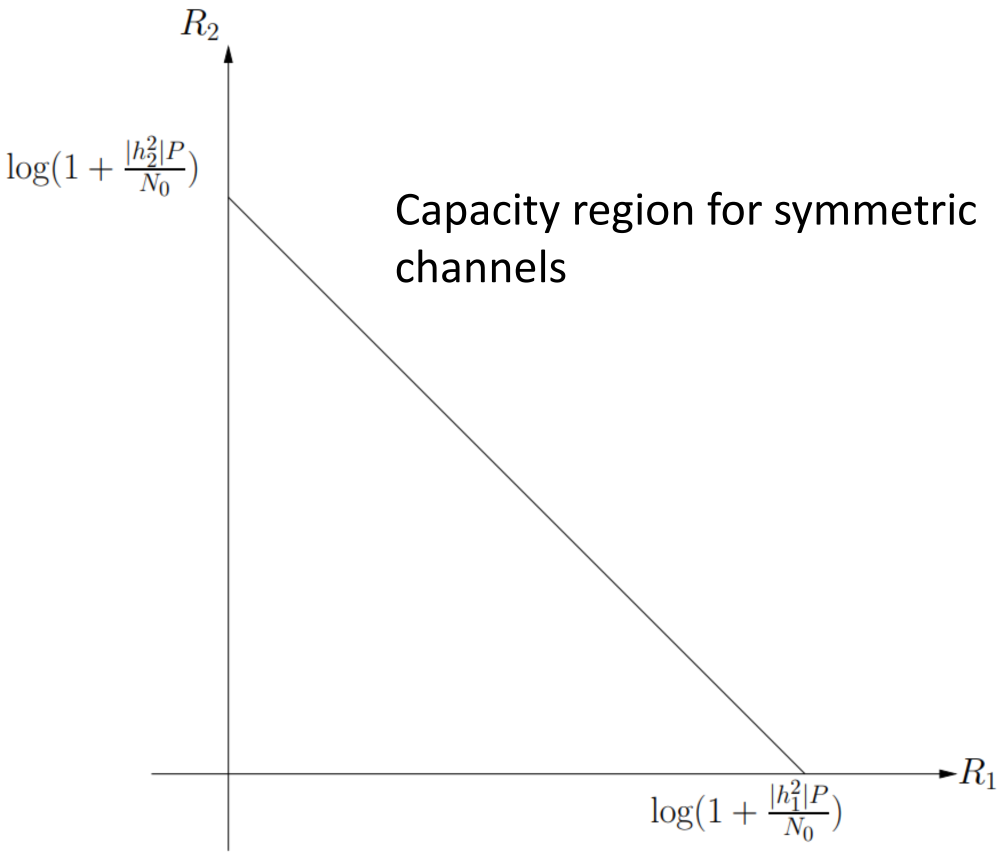
- Superposition coding scheme: The transmit signal is the sum of the two signals from $user\ 1\ and\ user\ 2$.

$$x[m] = x_1[m] + x_2[m],$$



(a) (b) (c)

The QPSK constellation of user 2 is superimposed on top of that of user 1



Capacity region for symmetric channels

Downlink AWGN Channel: General Case

- General case: Take $|h_1| < |h_2|$ (user 2 has a better channel than user 1).

Superposition coding

$$R_1 = \log\left(1 + \frac{P_1|h_1|^2}{P_2|h_1|^2 + N_0}\right) \quad \text{bits/s/Hz}$$
$$R_2 = \log\left(1 + \frac{P_2|h_2|^2}{N_0}\right) \quad \text{bits/s/Hz.}$$

user 1 treats the signal of user 2 as noise

user 2 which has the better channel performs SIC

Orthogonal Schemes

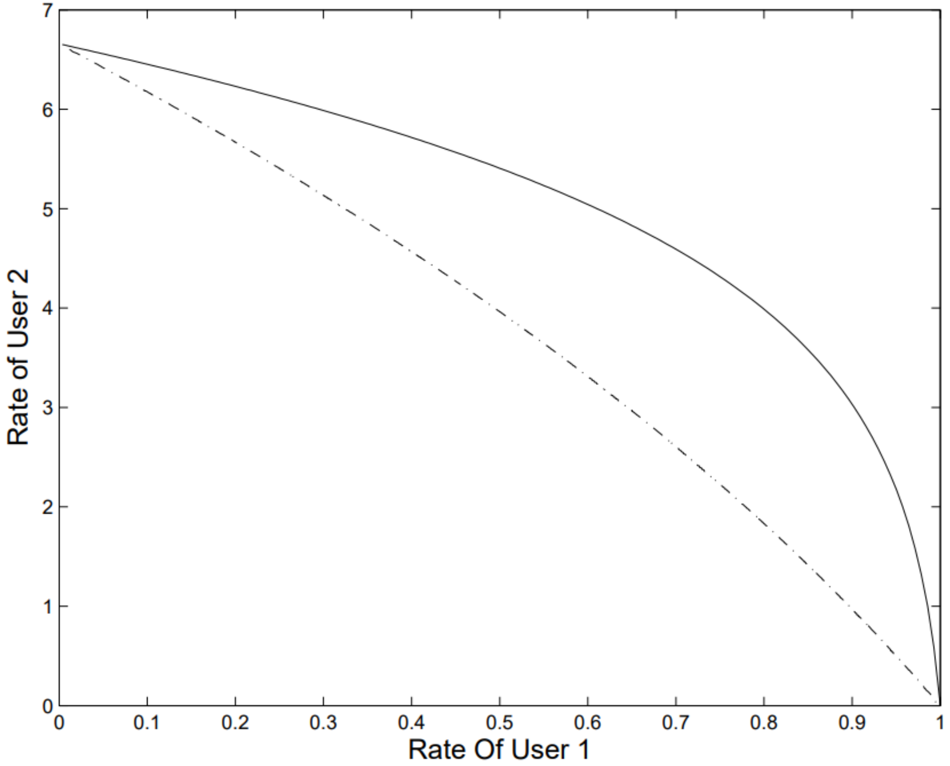
$$R_1 = \alpha \log\left(1 + \frac{P_1|h_1|^2}{\alpha N_0}\right) \quad \text{bits/s/Hz,}$$
$$R_2 = (1-\alpha) \log\left(1 + \frac{P_2|h_2|^2}{(1-\alpha) N_0}\right) \quad \text{bits/s/Hz.}$$

degree-of-freedom split $\alpha \in [0, 1]$

Downlink AWGN Channel: General Case

- Boundaries of the rate regions achievable with superposition coding (solid line) and optimal orthogonal schemes (dashed line) for the asymmetric downlink AWGN channel

SNRs for both users equal to 0 and 20 dB. In the orthogonal schemes, both the power split $P = P1 + P2$ and split in degrees of freedom α are jointly optimized to compute the boundary



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UPLINK FADING CHANNEL

- Consider the complex baseband representation of the uplink flat fading channel with $K\ users$:

$$y[m] = \sum_{k=1}^{K} h_k[m]x_k[m] + w[m], \qquad \{h_k[m]\}_m \ - \ fading\ process\ user\ k$$

we focus on the symmetric case when each user is subject to the same average power constraint, P, and the fading processes are identically distributed

To understand the effect of the channel fluctuations, we make the simplifying assumption that the base station (receiver) can perfectly track the fading processes of all the users.

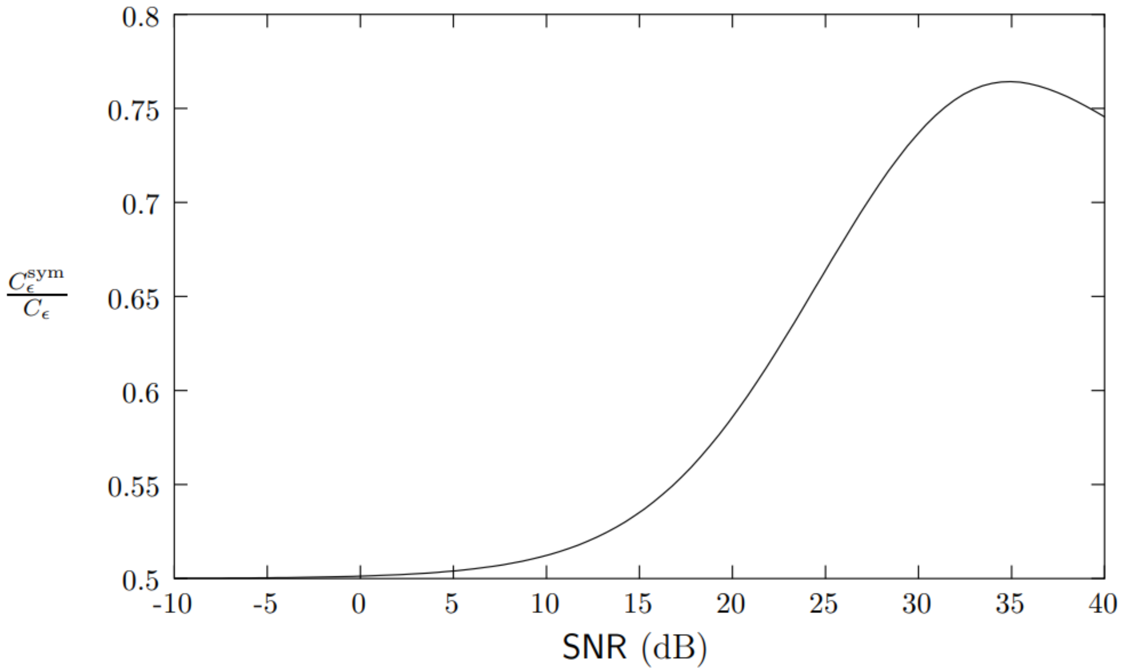
SLOW FADING CHANNEL

- If the symmetric capacity of this $K - user$ uplink AWGN channel is less than R, then the probability of the outage event can be written as:

$$p_{out}^{ul} = p\left\{log\left(1 + SNR\sum_{k\epsilon S}^{K}|h_k|^2\right) < \ |S|R, \qquad s \subset \{1, .., K\}\right\}$$

The corresponding $\epsilon - outage$ symmetric capacity, C_{ϵ}^{sym}, is then, the largest rate R such that the outage probability p_{out}^{ul} is smaller than or equal to ϵ.

SLOW FADING CHANNEL



Plot of the symmetric ϵ-outage capacity of the 2-user Rayleigh slow fading uplink as compared to C_ϵ, the corresponding performance of a point-to-point Rayleigh slow fading channel.

As SNR increases, the ratio of C_ϵ^{sym} to C_ϵ increases; thus, the effect of the inter-user interference is becoming smaller. However, as SNR becomes very large, the ratio starts to decrease; the inter-user interference begins to dominate.

FAST FADING CHANNEL

- Each $\{h_k[m]\}_m$ is modelled as a time-varying ergodic process.
- With only receiver CSI, there is no dynamic power allocation, and the sum capacity of the uplink fast fading channel is:

$$C_{sum} = E\left[\log\left(1 + \frac{\sum_{k=1}^{K} |h_k|^2 P}{N_o}\right)\right]$$

- Hence, without channel state information at the transmitter, fading always hurts, just as in the point-to-point case.
- However, when the number of users become large, the penalty due to fading vanishes.

FAST FADING CHANNEL

- For K $users$ the rate that $user$ k gets is

$$R_k = \mathbb{E}\left[\log\left(1 + \frac{|h_k|^2 P}{\sum_{i=k+1}^{K} |h_i|^2 P + N_0}\right)\right]$$

$$R_k \approx \mathbb{E}\left[\frac{|h_k|^2 P}{\sum_{i=k+1}^{K} |h_i|^2 P + N_0}\right] \log_2 e$$

- Since there are many users sharing the spectrum, the SINR for user k is low.
- Thus, the capacity penalty due to the fading of user k is small

FAST FADING CHANNEL: FULL CSI

- Fast-fading channel with tracking of the channels of all the users at the receiver and all the transmitters.
- For a given realization of the channel gains $h_{k,l},\ k = 1, \ldots, K,\ l = 1, \ldots, L$, the sum capacity (in bits/symbol) of this parallel channel is

$$\max_{P_{k,\ell}:k=1,\ldots,K,\ell=1,\ldots,L} \frac{1}{L}\sum_{\ell=1}^{L} \log\left(1 + \frac{\sum_{k=1}^{K} P_{k,\ell}|h_{k,\ell}|^2}{N_0}\right)$$

- Subject to the average power constraint on each user: $\frac{1}{L}\sum_{\ell=1}^{L} P_{k,\ell} = P, \qquad k = 1, \ldots, K.$
- The solution to this optimization problem as $L \to \infty$ yields the appropriate power allocation policy to be followed by the users.

Downlink Fading Channel

CSI at Receiver Only

$$\sum_{k=1}^{K} R_k < \mathbb{E}\left[\log\left(1 + \frac{|h|^2 P}{N_0}\right)\right]$$

Analogous to AWGN downlink analysis

The sum capacity of the downlink is

$$\mathbb{E}\left[\log\left(1 + \frac{P^*(\mathbf{h})\left(\max_{k=1\ldots K} |h_k^2|\right)}{N_0}\right)\right]$$

Full Channel Side Information

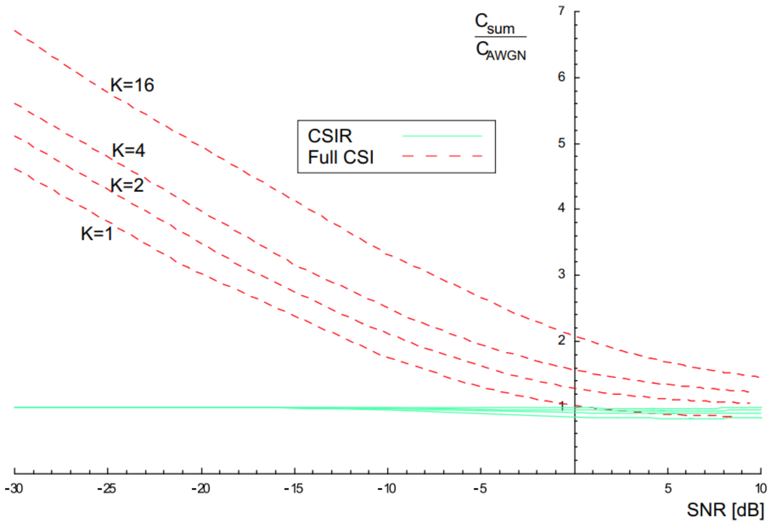
The optimal power allocation is the waterfilling solution

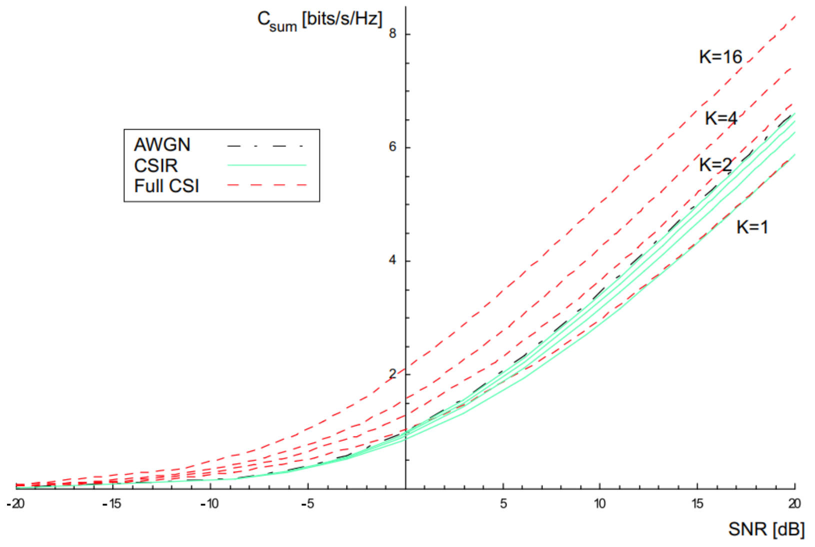
$$P^*(\mathbf{h}) = \left(\frac{1}{\lambda} - \frac{N_0}{\max_{k=1\ldots K} \ |h_k|^2}\right)^+$$

where $\boldsymbol{h} = (h_1, \ldots, h_k)$ is the joint fading state and $\lambda > 0$ is chosen such that the average power constraint is met.

Multiuser Diversity

- Compared to a system with a single transmitting user, the multiuser gain comes from two effects:
- the increase in total transmit power in the case of the uplink
- the effective channel gain at time m that is improved from $|h_1[m]|^2$ *to* $\max_{1 \le k \le K} |h_k[m]|^2$





Multiuser Diversity: System Aspects

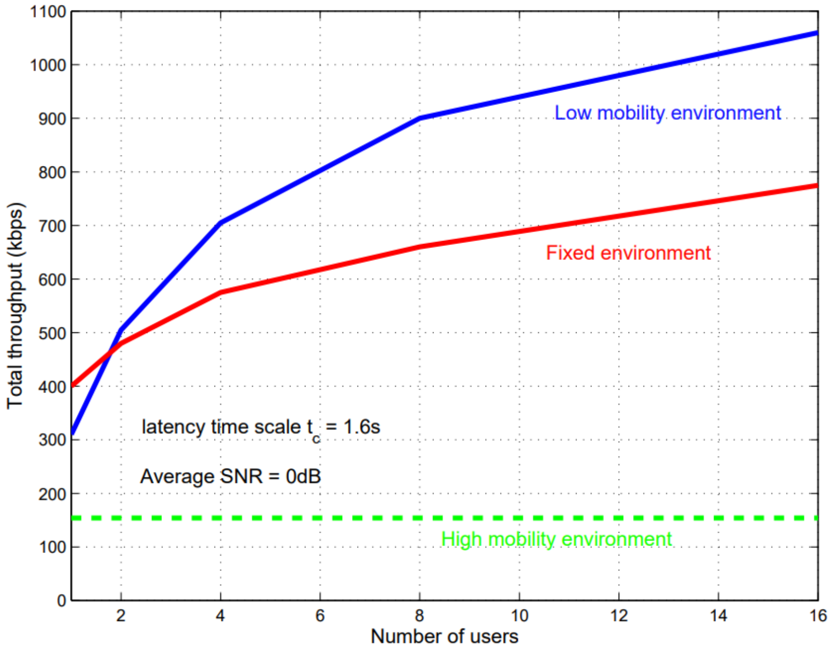
- Three main hurdles towards a system implementation of the multiuser diversity idea and some prominent ways of addressing these issues.

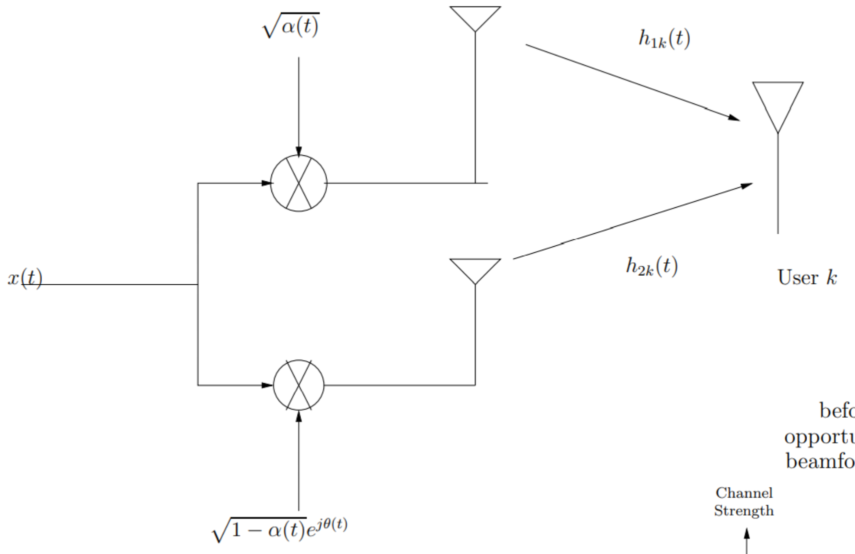
Fairness and Delay - Proportional fair scheduler: data is transmitted to a user when its channel is near its own peaks.

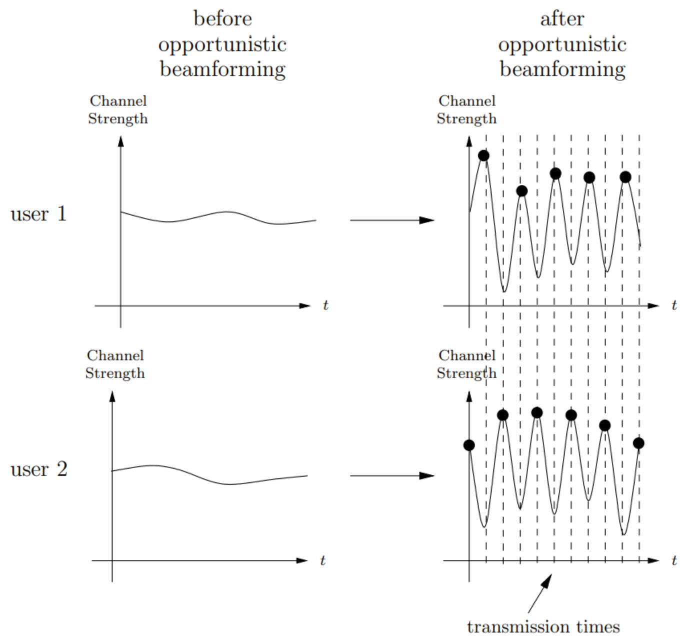
Channel Measurement and Feedback Errors - the error in measuring the channel from the pilot and the delay in feeding back the information to the base station.

Slow and Limited Fluctuations:
In environments where the channel fluctuations are small, a natural idea comes to mind: why not amplify the multiuser diversity gain by inducing faster and larger fluctuations?

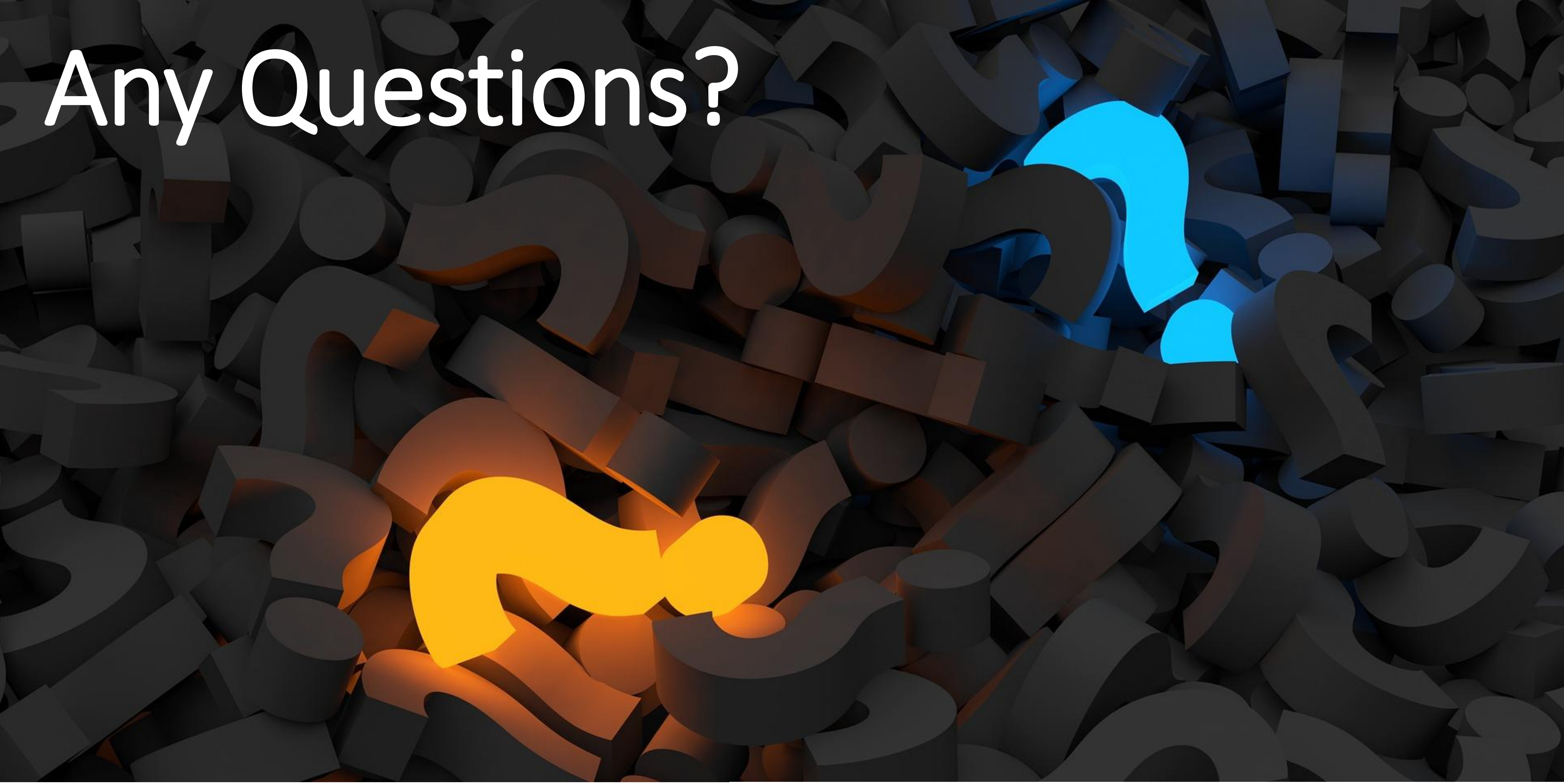
Multiuser Diversity: System Aspects







Any Questions?



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E-mail: frimpongemmanuelobeng@gmail.com