FUNDAMENTALS OF WIRELESS COMMUNICATION

CAPACITY OF WIRELESS CHANNELS

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OUTLINE OF PRESENTATION

- INTRODUCTION
- AWGN CHANNEL CAPACITY
- CAPACITY OF LTI GAUSSIAN CHANNELS
- CAPACITY OF FADING CHANNELS

INTRODUCTION

- This chapter takes a more fundamental look at the problem of communication over wireless fading channels. We ask: what is the optimal performance achievable on a given channel and what are the techniques to achieve such optimal performance?

- The framework for studying performance limits in communication is information theory. The basic measure of performance is the capacity of a channel: the maximum rate of communication for which arbitrarily small error probability can be achieved.

AWGN CHANNEL

- For AWGN channel:

$$y[m] = x[m] + w[m],$$

- Repetitive coding: To reduce the error probability, one can repeat the same symbol N times to transmit the one bit of information.

AWGN CHANNEL

- If x_A is transmitted, the received vector is $y = x_A + w$

where $\mathbf{w} = (w[1], \ldots, w[N])^t$. Error occurs when $\mathbf{y}$ is closer to $\mathbf{x}_B$ than to $\mathbf{x}_A$, and the error probability is given by

$$Q\left(\frac{\|\mathbf{x}_A - \mathbf{x}_B\|}{2\sigma}\right) = Q\left(\sqrt{\frac{NP}{\sigma^2}}\right), \tag{5.3}$$

- decays exponentially with the block length N. The good news is that communication can now be done with arbitrary reliability by choosing a large enough N.
- The bad news is that the data rate is only 1/N bits per symbol time and with increasing N the data rate goes to zero.
- Is that the price one must pay to achieve reliable communication?

AWGN CHANNEL

- Packing Spheres

- Geometrically, repetition coding puts all the codewords in just one dimension.
- On the other hand, the signal space has many dimensions N. To communicate more efficiently, the codewords should be spread in all the N dimensions.
- The maximum number of codewords that can be packed with non-overlapping implies that the maximum number of bits per symbol that can be reliably communicated. This is the capacity of the AWGN channel.

$$\frac{1}{N}\log\left(\frac{\left(\sqrt{N(P+\sigma^2)}\right)^N}{\left(\sqrt{N\sigma^2}\right)^N}\right) = \frac{1}{2}\log\left(1+\frac{P}{\sigma^2}\right).$$

Resources of the AWGN Channel

- Consider a continuous-time AWGN channel with bandwidth W Hz, power constraint $\bar{P}$ Watts, and additive white Gaussian noise with power spectral density $\frac{N_0}{2}$.
- The capacity of the continuous-time AWGN channel is

$$C_{\text{awgn}}(\bar{P}, W) = W \log\left(1 + \frac{\bar{P}}{N_0 W}\right) \quad \text{bits/s.}$$

$$C_{\text{awgn}} = \log\left(1 + \mathsf{SNR}\right) \quad \text{bits/s/Hz.}$$

Linear Time-Invariant Gaussian Channels

Consider a SIMO channel with one transmit antenna and L receive antennas:

$$y_\ell[m] = h_\ell x[m] + w_\ell[m] \qquad \ell = 1, \ldots, L,$$

- The capacity of this channel is

$$C = \log\left(1 + \frac{P\|\mathbf{h}\|^2}{N_0}\right) \qquad \text{bits/s/Hz}.$$

Linear Time-Invariant Gaussian Channels

Consider a MISO channel with L transmit antennas and a single receive antenna :

$$y[m] = \mathbf{h}^* \mathbf{x}[m] + w[m], \qquad (5.27)$$

- where $h = [h_1, h_2, h_L]^t$. We conclude capacity of MISO channel is

$$C = \log\left(1 + \frac{P\|\mathbf{h}\|^2}{N_0}\right) \quad \text{bits/s/Hz.}$$

aligning the transmit signal in the direction of the transmit antenna array pattern", is called transmit beamforming.

Linear Time-Invariant Gaussian Channels

Consider a *time-invariant* L-tap frequency-selective AWGN channel:

$$y[m] = \sum_{\ell=0}^{L-1} h_\ell x[m-\ell] + w[m],$$

with an average power constraint P on each input symbol. Frequency-selective channel can be converted into Nc independent sub-carriers.

Each of the sub-channels is an AWGN channel.

We allocate power to each sub-channel such that the total power constraint is met.

Linear Time-Invariant Gaussian Channels

- The maximum rate of reliable communication using this scheme is

$$\sum_{n=0}^{N_c-1} \log\left(1+\frac{P_n|\tilde{h}_n|^2}{N_0}\right) \qquad \text{bits/OFDM symbol.}$$

- The power allocation can be chosen appropriately, to maximize the rate above. The "optimal power allocation", thus, is the solution to the optimization problem:

$$C_{N_c} := \max_{P_0,\ldots,P_{N_c-1}} \sum_{n=0}^{N_c-1} \log\left(1+\frac{P_n|\tilde{h}_n|^2}{N_0}\right),$$

subject to

$$\sum_{n=0}^{N_c-1} P_n = N_c P, \qquad P_n \geq 0, \quad n = 0,\ldots,N_c-1.$$

Capacity of Fading Channels

Consider the complex baseband representation of a flat fading channel:

$$y[m] = h[m]x[m] + w[m],$$

where $\{h[m]\}$ is the fading process and $\{w[m]\}$ is i.i.d. $\mathcal{CN}(0, N_0)$.

- What is the ultimate performance limit when information can be coded over a sequence of symbols?
- To answer this question, we assume that the receiver can perfectly track the fading process, i.e., coherent reception.

Slow Fading Channel

- This is a situation when the channel gain is random but remains constant for all ti-me i.e., $h[m] = h\ for\ all\ m$.
- Now suppose the transmitter encodes data at a rate R bits/s/Hz. If the channel realization h is such that $log(1 + |h|^2 SNR) < R$, then the outage probability is

$$p_{\text{out}}(R) := \mathbb{P}\left\{\log\left(1 + |h|^2 \mathsf{SNR}\right) < R\right\}$$

RECEIVE DIVERSITY

- Let us increase the diversity of the channel by having L receive antennas instead of one. For given channel gains $\boldsymbol{h} = [h_1, h_2, h_L]^t$, the capacity is calculated to be $log(1 + |\boldsymbol{h}|^2 SNR)$.
- Outage occurs whenever this is below the target rate R:

$$p_{\text{out}}^{\text{rx}}(R) := \mathbb{P}\left\{\log\left(1 + \|\mathbf{h}\|^2 \mathsf{SNR}\right) < R\right\}$$

This can be rewritten as:

$$p_{\text{out}}(R) = \mathbb{P}\left\{\|\mathbf{h}\|^2 < \frac{2^R - 1}{\mathsf{SNR}}\right\}$$

$$p_{\text{out}}(R) \approx \frac{(2^R - 1)^L}{L!\mathsf{SNR}^L}.$$

- coding cannot increase the diversity gain

TRANSMIT DIVERSITY

- Now suppose there are L transmit antennas but only one receive antenna, with a total power constraint of P.
- Unlike in the SISO and the SIMO cases, outage performance is achievable only if the transmitter knew the phases and magnitudes of the gains h so that it can perform transmit beamforming.
- When the transmitter does not know the channel gains h, it must use a fixed transmission strategy that does not depend on h.

TRANSMIT DIVERSITY

- With transmit diversity but no channel knowledge at the transmitter, the outage probability is

$$p_{\text{out}}^{\text{tx}}(R) := \mathbb{P}\left\{\log\left(1+\|\mathbf{h}\|^2\frac{\mathsf{SNR}}{L}\right) < R\right\},$$

- a loss of a factor of L in the received SNR because the transmitter has no knowledge of the channel direction and is unable to beamform in the specific channel direction.

Time and Frequency Diversity

- Outage Performance of Parallel Channels:

- An average transmit power constraint of P on the original channel translates into a total power constraint of LP on the parallel channel.

- that the maximum rate of reliable communication given the fading gains $h_l' s$ is

$$\sum_{\ell=1}^{L} \log\left(1 + |h_\ell|^2 \mathsf{SNR}\right) \qquad \text{bits/s/Hz}, \tag{5.80}$$

where $\mathsf{SNR} = P/N_0$. Hence, if the target rate is R bits/s/Hz per sub-channel, then outage occurs when

$$\sum_{\ell=1}^{L} \log\left(1 + |h_\ell|^2 \mathsf{SNR}\right) < LR. \tag{5.81}$$

Time and Frequency Diversity

- The outage probability of the time-diversity channel is

$$p_{\text{out}}(R) = \mathbb{P}\left\{\frac{1}{L}\sum_{\ell=1}^{L}\log\left(1+|h_\ell|^2\mathsf{SNR}\right) < R\right\}$$

- Without transmitter knowledge, separate coding would mean using a fixed-rate code for each sub-channel and poor diversity results: errors occur whenever one of the sub-channels is bad.
- Indeed, coding across the different coherence periods is now necessary: if the channel is in deep fade during one of the coherence periods, the information bits can still be protected if the channel is strong in other periods.

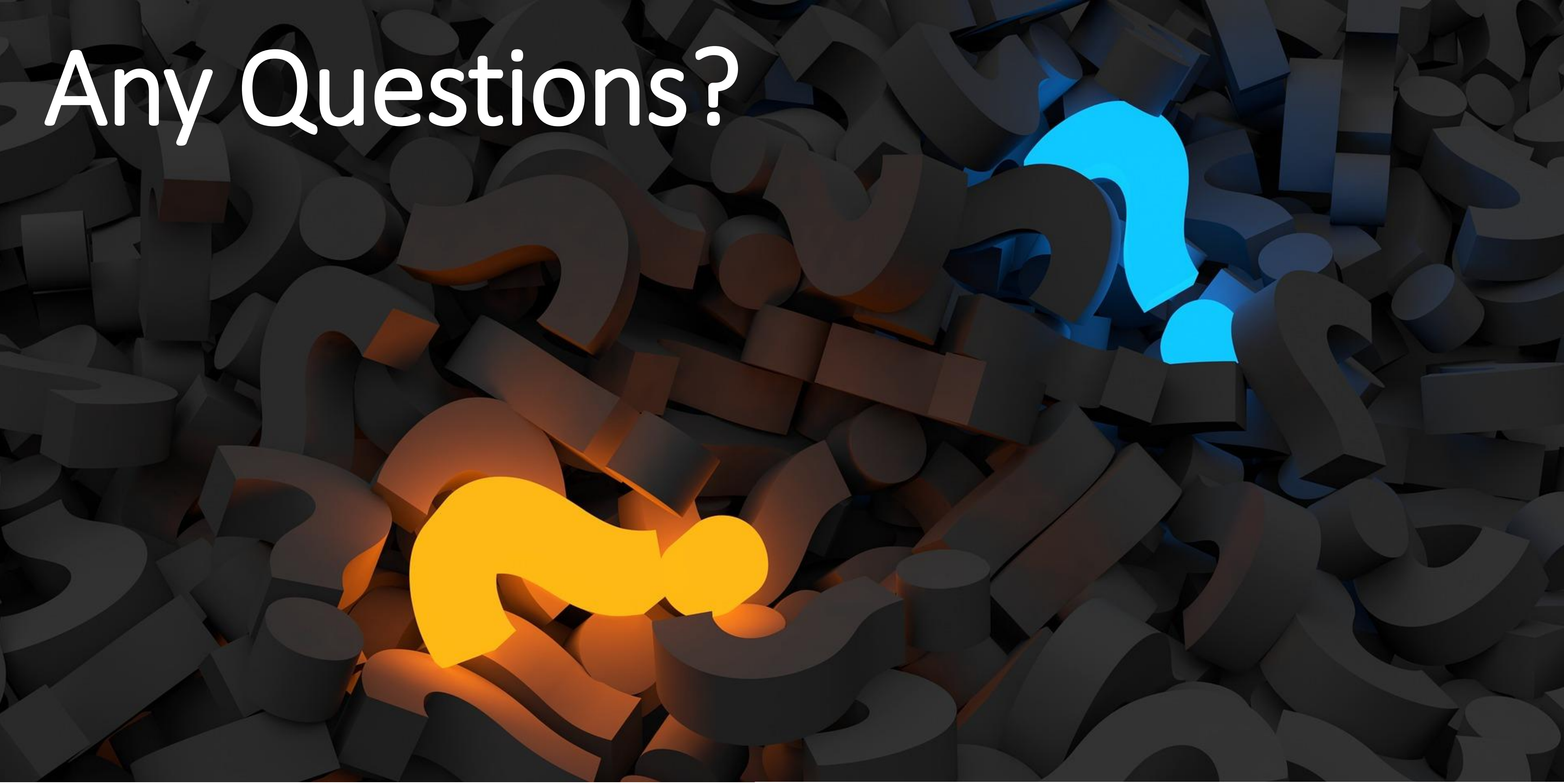
Transmitter Side Information

- Let us now consider the case when the transmitter can track the channel as well. There are several ways in which such channel information can be obtained at the transmitter.
- In a TDD (time-division duplex) system, the transmitter can exploit channel reciprocity and make channel measurements based on the signal received along the opposite link. In an FDD (frequency-division duplex) system, there is no reciprocity, and the transmitter will have to rely on feedback information from the receiver.

Slow Fading: Channel Inversion

- With transmitter knowledge, one option is now to control the transmit power such that the rate R can be delivered no matter what the fading state is. This is the channel inversion strategy.
- With exact channel inversion, there is zero outage probability.
- The price to pay is that huge power must be consumed to invert the channel when it is very bad.
- Moreover, many systems are also peak-power constrained and cannot invert the channel beyond a certain point.

Any Questions?



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