1. 化简先行

(1) 等价无穷小替换.

①$x \to 0$ 时,

$$\sin x \sim x, \tan x \sim x, \arcsin x \sim x, \arctan x \sim x,$$

$$e^x - 1 \sim x, \ln(1+x) \sim x, a^x - 1 = e^{x\ln a} - 1 \sim x\ln a (a > 0 \text{ 且 } a \neq 1),$$

$$1 - \cos x \sim \frac{1}{2}x^2, (1+x)^\alpha - 1 \sim \alpha x (\alpha \neq 0).$$

② 若 α, β 都是同一自变量变化过程下的无穷小量,且 $\alpha = o(\beta)$,则 $\alpha + \beta \sim \beta$. 见例 1.6.

(2) 恒等变形.

恒等变形:
- 提取公因式. 见例 1.6.
- 换元$\left(x = \frac{1}{t} \text{ 等}\right)$. 见例 1.7,例 1.8,例 1.18.
- 通分. 见例 1.9.
- $u^v = e^{v\ln u}$. 见例 1.6.
- 用公式:
  - 因式分解 $a^n - b^n = (a-b) \cdot (a^{n-1} + a^{n-2}b + \cdots + ab^{n-2} + b^{n-1})$. 见例 1.7.
  - 分子有理化$\left(\sqrt{a} - \sqrt{b} = \frac{a-b}{\sqrt{a} + \sqrt{b}} \text{ 等}\right)$.
- 中值定理. 见例 1.10 解法一,例 1.11.

3. 泰勒公式

(1) 熟记常用公式.

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!},$$

$$\sin x = x - \frac{1}{3!}x^3 + \cdots + (-1)^n \frac{1}{(2n+1)!}x^{2n+1} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!},$$

$$\cos x = 1 - \frac{1}{2!}x^2 + \cdots + (-1)^n \frac{1}{(2n)!}x^{2n} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!},$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \cdots + (-1)^{n-1} \frac{x^n}{n} + \cdots = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}, -1 < x \leqslant 1,$$

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + \cdots = \sum_{n=0}^{\infty} x^n, |x| < 1,$$

$$\frac{1}{1+x} = 1 - x + x^2 - \cdots + (-1)^n x^n + \cdots = \sum_{n=0}^{\infty} (-1)^n x^n, |x| < 1,$$

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2}x^2 + o(x^2) (x \to 0, \alpha \neq 0),$$

$$\tan x = x + \frac{1}{3}x^3 + o(x^3) (x \to 0),$$

$$\arcsin x = x + \frac{1}{6}x^3 + o(x^3) (x \to 0),$$

$$\arctan x = x - \frac{1}{3}x^3 + o(x^3) (x \to 0).$$

一 基本求导公式

①$(x^k)' = kx^{k-1}$（k 为任意实数）.

②$(\ln x)' = \frac{1}{x}(x > 0)$.

③$(e^x)' = e^x$；$(a^x)' = a^x \ln a, a > 0, a \neq 1$.

④$(\sin x)' = \cos x$；$(\cos x)' = -\sin x$；

$(\tan x)' = \sec^2 x$；$(\cot x)' = -\csc^2 x$；

$(\sec x)' = \sec x \tan x$；$(\csc x)' = -\csc x \cot x$；

$(\ln|\cos x|)' = -\tan x$；$(\ln|\sin x|)' = \cot x$；

$(\ln|\sec x + \tan x|)' = \sec x$；$(\ln|\csc x - \cot x|)' = \csc x$.

⑤$(\arctan x)' = \frac{1}{1+x^2}$；$(\operatorname{arccot} x)' = -\frac{1}{1+x^2}$.

⑥$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$；$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$.

⑦$[\ln(x+\sqrt{x^2+a^2})]' = \frac{1}{\sqrt{x^2+a^2}}$，常见 $a = 1$；

$[\ln(x+\sqrt{x^2-a^2})]' = \frac{1}{\sqrt{x^2-a^2}}(x > a > 0)$，常见 $a = 1$.

一 基本积分公式

① $\int x^k \mathrm{d}x = \frac{1}{k+1}x^{k+1} + C, k \neq -1$; $\begin{cases} \int \frac{1}{x^2}\mathrm{d}x = -\frac{1}{x} + C, \\ \int \frac{1}{\sqrt{x}}\mathrm{d}x = 2\sqrt{x} + C. \end{cases}$

② $\int \frac{1}{x}\mathrm{d}x = \ln|x| + C.$

③ $\int \mathrm{e}^x \mathrm{d}x = \mathrm{e}^x + C$; $\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C, a > 0$ 且 $a \neq 1$.

④ $\int \sin x\mathrm{d}x = -\cos x + C$; $\int \cos x\mathrm{d}x = \sin x + C$;

$\int \tan x\mathrm{d}x = -\ln|\cos x| + C$; $\int \cot x\mathrm{d}x = \ln|\sin x| + C$;

$\int \frac{\mathrm{d}x}{\cos x} = \int \sec x\mathrm{d}x = \ln|\sec x + \tan x| + C$;

$\int \frac{\mathrm{d}x}{\sin x} = \int \csc x\mathrm{d}x = \ln|\csc x - \cot x| + C$;

$\int \sec^2 x\mathrm{d}x = \tan x + C$; $\int \csc^2 x\mathrm{d}x = -\cot x + C$;

$\int \sec x\tan x\mathrm{d}x = \sec x + C$; $\int \csc x\cot x\mathrm{d}x = -\csc x + C$.

⑤ $\begin{cases} \int \frac{1}{1+x^2}\mathrm{d}x = \arctan x + C, \\ \int \frac{1}{a^2+x^2}\mathrm{d}x = \frac{1}{a}\arctan\frac{x}{a} + C(a > 0). \end{cases}$

⑥ $\begin{cases} \int \frac{1}{\sqrt{1-x^2}}\mathrm{d}x = \arcsin x + C, \\ \int \frac{1}{\sqrt{a^2-x^2}}\mathrm{d}x = \arcsin\frac{x}{a} + C(a > 0). \end{cases}$

⑦ $\begin{cases} \int \frac{1}{\sqrt{x^2+a^2}}\mathrm{d}x = \ln(x + \sqrt{x^2+a^2}) + C(\text{常见 } a = 1), \\ \int \frac{1}{\sqrt{x^2-a^2}}\mathrm{d}x = \ln|x + \sqrt{x^2-a^2}| + C(|x| > |a|). \end{cases}$

⑧$\int \frac{1}{x^2-a^2}\mathrm{d}x=\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+C\left(\int \frac{1}{a^2-x^2}\mathrm{d}x=\frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right|+C\right)$.

⑨$\int \sqrt{a^2-x^2}\,\mathrm{d}x=\frac{a^2}{2}\arcsin\frac{x}{a}+\frac{x}{2}\sqrt{a^2-x^2}+C(a>|x|\geqslant 0)$.

⑩$\int \sin^2 x\mathrm{d}x=\frac{x}{2}-\frac{\sin 2x}{4}+C\left(\sin^2 x=\frac{1-\cos 2x}{2}\right)$;

$\int \cos^2 x\mathrm{d}x=\frac{x}{2}+\frac{\sin 2x}{4}+C\left(\cos^2 x=\frac{1+\cos 2x}{2}\right)$;

$\int \tan^2 x\mathrm{d}x=\tan x-x+C(\tan^2 x=\sec^2 x-1)$;

$\int \cot^2 x\mathrm{d}x=-\cot x-x+C(\cot^2 x=\csc^2 x-1)$.

① **三角函数代换**——当被积函数含有如下根式时,可作三角代换,这里 $a>0$.

$$\begin{cases} \sqrt{a^2-x^2}\xrightarrow{\text{令}}x=a\sin t, & |t|<\frac{\pi}{2}, \\ \sqrt{a^2+x^2}\xrightarrow{\text{令}}x=a\tan t, & |t|<\frac{\pi}{2}, \\ \sqrt{x^2-a^2}\xrightarrow{\text{令}}x=a\sec t, & \begin{cases} \text{若 } x>0,\text{则 } 0<t<\frac{\pi}{2}, \\ \text{若 } x<0,\text{则 } \frac{\pi}{2}<t<\pi. \end{cases} \end{cases}$$

(1) 面积.

① 直角坐标系下的面积公式(如图 10-1):$S=\int_a^b |f(x)-g(x)|\mathrm{d}x$.

② 极坐标系下的面积公式(如图 10-2):$S=\int_\alpha^\beta \frac{1}{2}|r_2^2(\theta)-r_1^2(\theta)|\mathrm{d}\theta$.

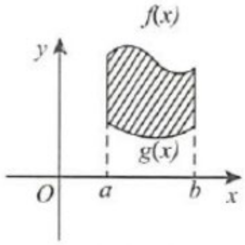

图 10-1

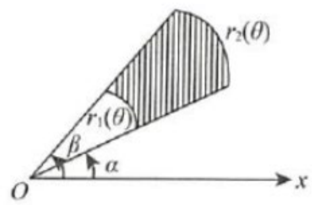

图 10-2

(2) 旋转体体积.

① 绕 x 轴(如图 10-15):$V_x=\int_a^b \pi y^2(x)\mathrm{d}x$.

② 绕 y 轴:$V_y=\int_a^b 2\pi x|y(x)|\mathrm{d}x$(柱壳法).

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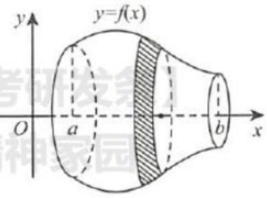

图 10-15

(4) 平面曲线的弧长.(仅数学一、数学二)

① 若平面光滑曲线由直角坐标方程 $y=y(x)(a\leqslant x\leqslant b)$ 给出,则

$$s=\int_a^b\sqrt{1+[y'(x)]^2}\,\mathrm{d}x.$$

② 若平面光滑曲线由参数方程 $\begin{cases}x=x(t),\\ y=y(t)\end{cases}(\alpha\leqslant t\leqslant\beta)$ 给出,则

$$s=\int_\alpha^\beta\sqrt{[x'(t)]^2+[y'(t)]^2}\,\mathrm{d}t.$$

③ 若平面光滑曲线由极坐标方程 $r=r(\theta)(\alpha\leqslant\theta\leqslant\beta)$ 给出,则

$$s=\int_\alpha^\beta\sqrt{[r(\theta)]^2+[r'(\theta)]^2}\,\mathrm{d}\theta.$$