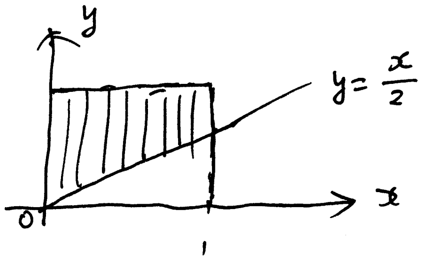
14-15-02 概率统计(A)期末试卷(A卷)答案

1. 解：(1) $P(X<2Y)=\iint\limits_{x<2y} f(x,y)\,dx\,dy$



$$=\int_0^1\int_{\frac{x}{2}}^1 (x+y)\,dy\,dx. \quad (2分)$$

$$=\int_0^1\left[x\left(1-\frac{x}{2}\right)+\frac{1-\left(\frac{x}{2}\right)^2}{2}\right]dx$$

$$=\int_0^1\left(\frac{1}{2}+x-\frac{5}{8}x^2\right)dx$$

$$=\frac{19}{24} \quad (3分)$$

(2) $f_X(x)=\int_{-\infty}^{\infty} f(x,y)\,dy$ (1分) $=\begin{cases}\int_0^1 (x+y)\,dy, & 0<x<1\\ 0, & 其它.\end{cases}$

$$=\begin{cases}x+\frac{1}{2}, & 0<x<1 \quad (3分)\\ 0, & 其它. \quad (1分)\end{cases}$$

(3) $f_{Y|X}(y|\frac{1}{2})=\dfrac{f(\frac{1}{2},y)}{f_X(\frac{1}{2})}=\begin{cases}\frac{1}{2}+y, & 0<y<1\\ 0, & 其它.\end{cases}$ (2分)

$$P\left(Y>\frac{1}{4}\,\middle|\,X=\frac{1}{2}\right)=\int_{\frac{1}{4}}^{\infty} f_{Y|X}(y|\tfrac{1}{2})\,dy$$

$$=\int_{\frac{1}{4}}^1\left(\frac{1}{2}+y\right)dy=\frac{27}{32} \quad (2分)$$

(1分)

(4) $E(X^2Y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^2y\,f(x,y)\,dx\,dy.$

$$=\int_0^1\int_0^1 x^2y(x+y)\,dx\,dy. \quad (2分)$$

$$=\int_0^1\left(\frac{y}{4}+\frac{y^2}{3}\right)dy=\frac{17}{72} \quad (3分)$$

2. 解:

X	0	1	2
p	0.35	0.25	0.4

Y	0	10	20
p	0.35	0.3	0.35

$EX = 1\times0.25 + 2\times0.4 = 1.05$ (1分), $E(X^2) = 1^2\times0.25 + 2^2\times0.4 = 1.85$ (1分)

$EY = 10\times0.3 + 20\times0.35 = 10$ (1分), $E(Y^2) = 10^2\times0.3 + 20^2\times0.35 = 170$. (1分)

$E(XY) = 1\times10\times0.15 + 1\times20\times0.05 + 2\times10\times0.1 + 2\times20\times0.25$
$= 14.5$ (1分)

$D(X) = E(X^2) - (EX)^2 = 1.85 - (1.05)^2 = 0.7475$. (1分)

$D(Y) = E(Y^2) - (EY)^2 = 170 - 10^2 = 70$. (1分)

$\mathrm{cov}(X,Y) = E(XY) - EX\cdot EY = 14.5 - 1.05\times10 = 4$. (1分)

$\rho = \dfrac{\mathrm{cov}(X,Y)}{\sqrt{DX\cdot DY}} = \dfrac{4}{\sqrt{0.7475\times70}} \approx 0.5530$ (2分)

3. 解:(1) 被盗索赔数 $X \sim b(100, 0.2)$. (3分)

(2) $EX = 100\times0.2 = 20$, $DX = 100\times0.2\times0.8 = 16$

由中心极限定理: $\dfrac{X-20}{4} \sim N(0,1)$. (3分)

故 $P(14 \le X \le 30) = P(-1.5 \le \dfrac{X-20}{4} \le 2.5)$

$\approx \Phi(2.5) - \Phi(-1.5)$ (2分)

$= \Phi(2.5) + \Phi(1.5) - 1$

$= 0.9938 + 0.9332 - 1 = 0.9270$ (2分)

4. 解：(1) $\bar{X} \sim N(1, 0.04)$，$\dfrac{\bar{X}-1}{0.2} \sim N(0,1)$ (2分)

故 $p(\bar{X} > 1.2) = p\left(\dfrac{\bar{X}-1}{0.2} > 1\right) = 1-\Phi(1) = 1-0.8413 = 0.1587$ (2分)

(2) $\dfrac{9S^2}{0.4} \sim \chi^2(9)$ (2分) 故由 $\chi^2_{0.975}(9) = 2.7$ 知

$p(S^2 > 0.12) = p\left(\dfrac{9S^2}{0.4} > 2.7\right) = p(\chi^2(9) > 2.7) = 0.975$. (2分)

(3). 由于 $\bar{X}$ 与 S^2 独立，故 $\dfrac{\bar{X}-1}{S/\sqrt{10}} \sim t(9)$ (2分)，于是由 $t_{0.15}(9) = 1.1$，

$p\left(\bar{X} - \dfrac{1.1}{\sqrt{10}}S > 1\right) = p\left(\dfrac{\bar{X}-1}{S/\sqrt{10}} > 1.1\right) = p(t(9) > 1.1) = 0.15$ (2分)

5. 解：(1) 设 $x_1, x_2, \cdots x_n$ 为样本值，则

$L(\theta) = \prod_{i=1}^{n} f(x_i) = \prod_{i=1}^{n}\left(\theta^{-1} x_i^{-(\theta^{-1}+1)}\right) = \theta^{-n} \cdot \prod_{i=1}^{n} x_i^{-(\theta^{-1}+1)}$ (2分)

$\ln L(\theta) = -n\ln\theta - (\theta^{-1}+1)\sum_{i=1}^{n}\ln x_i$.

令 $\dfrac{\partial \ln L(\theta)}{\partial\theta} = 0$ 得：$-\dfrac{n}{\theta} + \dfrac{1}{\theta^2}\sum_{i=1}^{n}\ln x_i = 0$ (2分)

解得. $\hat{\theta} = \dfrac{1}{n}\sum_{i=1}^{n}\ln x_i$. 故

θ 的最大似然估计量为 $\hat{\theta} = \dfrac{1}{n}\sum_{i=1}^{n}\ln(X_i)$ (1分)

(2) $E(\ln X) = \int_{-\infty}^{\infty}(\ln x)\cdot f(x)dx = \int_{1}^{\infty}(\ln x)\cdot\theta^{-1}x^{-(\theta^{-1}+1)}dx$.

$= -\int_{1}^{\infty}\ln x \, d x^{-\theta^{-1}} = -(\ln x)\cdot x^{-\theta^{-1}}\Big|_1^{\infty} + \int_1^{\infty} x^{-(\theta^{-1}+1)}dx$.

$= -\dfrac{x^{-\theta^{-1}}}{\theta^{-1}}\Big|_1^{\infty} = \theta$ (3分) (积分要有过程)

故 $E(\hat{\theta}) = E(\ln X) = \theta$. (2分)

(3) $E((\ln X)^2) = \int_{-\infty}^{\infty} (\ln x)^2 f(x)\,dx$.

$= \int_1^{\infty} (\ln x)^2 \cdot \theta^{-1} \cdot x^{-(\theta^{-1}+1)}\,dx$.

$= -\int_1^{\infty} (\ln x)^2 \, d\, x^{-\theta^{-1}}$

$= -(\ln x)^2 x^{-\theta^{-1}} \Big|_1^{\infty} + 2\int_1^{\infty} (\ln x) \cdot x^{-(\theta^{-1}+1)}\,dx$.

$= 2\theta^2$ (3分)(积分要有过程)

故 $D(\ln X) = E((\ln X)^2) - (E(\ln X))^2 = 2\theta^2 - \theta^2 = \theta^2$.(1分)

$D(\hat{\theta}) = D(\frac{1}{n}\sum_{i=1}^{n} \ln(X_i)) = \frac{1}{n} D(\ln X) = \frac{\theta^2}{n}$.(1分)

$E[(\hat{\theta})^2] = D(\hat{\theta}) + (E(\hat{\theta}))^2 = (1+\frac{1}{n})\theta^2$ (2分)

故当 $c = \frac{n}{n+1}$ 时 $E(c\hat{\theta}^2) = \theta^2$.
(1分)

6. 解: (1) $\sigma = 2$, $\alpha = 0.05$, $n=9$, $z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$

μ 的置信水平为0.95的置信区间为:

$(\bar{X} \pm z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}) = (59 \pm 1.96 \times \frac{2}{\sqrt{9}}) = (57.69, 60.31)$
(2分) (3分)

(2) $\alpha = 0.05$, $n=9$, $t_{\frac{\alpha}{2}}(n-1) = t_{0.025}(8) = 2.306$

μ 的置信水平为0.95的置信区间为:

$(\bar{X} \pm t_{\frac{\alpha}{2}}(n-1) \cdot \frac{s}{\sqrt{n}}) = (59 \pm 2.306 \times \frac{3}{\sqrt{9}}) = (56.694, 61.306)$
(3分)
(2分)

7. 解：(1) $n_1=n_2=10$，$\alpha=0.05$.

拒绝域 $\frac{S_1^2}{S_2^2}<F_{1-\frac{\alpha}{2}}(n_1-1,n_2-1)=F_{0.975}(9,9)=\frac{1}{F_{0.025}(9,9)}=\frac{1}{4.03}\approx 0.248$

或 $\frac{S_1^2}{S_2^2}>F_{\frac{\alpha}{2}}(n_1-1,n_2-1)=F_{0.025}(9,9)=4.03$ （3分）

而 $\frac{S_1^2}{S_2^2}=\frac{80^2}{60^2}\approx 1.78$ （1分） 不落在拒绝域中，故

不拒绝 H_0，即认为 $\sigma_1^2=\sigma_2^2$.（1分）

(2) 设 $\sigma_1^2=\sigma_2^2$.

$\alpha=0.05$，$n_1=n_2=10$，

$S_w=\sqrt{\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}}=\sqrt{\frac{9\times 80^2+9\times 60^2}{18}}=\sqrt{\cancel{400}\,5000}\approx\cancel{20}\,70.71$.

拒绝域 $\frac{\bar{x}-\bar{y}}{S_w\cdot\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}>t_\alpha(n_1+n_2-2)=t_{0.05}(18)=1.7341$ （2分）

而 $\frac{\bar{x}-\bar{y}}{S_w\cdot\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}=\frac{320-240}{70.71\cdot\sqrt{\frac{1}{10}+\frac{1}{10}}}\approx 2.53>1.7341$. （2分）

故拒绝 H_0，即认为 $\mu_1>\mu_2$.（1分）

8. 解：(1) $\sum_{i=1}^{5} x_i = 600$，$\sum_{i=1}^{5} y_i = 284$

$\sum_{i=1}^{5} x_i^2 = 73000$，$\sum_{i=1}^{5} y_i^2 = 16472$

$\sum_{i=1}^{5} x_i y_i = 34660$，$\bar{x} = 120$，$\bar{y} = 56.8$

$S_{xx} = \sum_{i=1}^{5} x_i^2 - 5\times\bar{x}^2 = 1000$ (1分)，$\cancel{S_{yy} = \sum_{i=1}^{5} y_i^2 - 5\times\bar{y}^2 = 340.8}$

$S_{xy} = \sum_{i=1}^{5} x_i y_i - 5\bar{x}\bar{y} = 580$ (1分)

故 $\hat{b} = \frac{S_{xy}}{S_{xx}} = 0.58$ (2分)，$\hat{a} = \bar{y} - \hat{b}\bar{x} = -12.8$ (2分)

回归方程：$\hat{y} = -12.8 + 0.58x$

(2) $S_{yy} = \sum_{i=1}^{n} y_i^2 - 5\times\bar{y}^2 = 340.8$ (1分)，$n=5$.

$Q_e = S_{yy} - \hat{b}S_{xy} = 4.4$. (2分)

$\hat{\sigma^2} = \frac{Q_e}{n-2} = \frac{4.4}{3} \approx 1.47$ (1分)