

R Textbook Companion for
Statistics and Probability Theory
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R numbering policy used in this document and the relation to the above book.

Exa Example (Solved example)

Eqn Equation (Particular equation of the above book)

For example, Exa 3.51 means solved example 3.51 of this book. Sec 2.3 means an R code whose theory is explained in Section 2.3 of the book.

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Chapter 1

Probability

R code Exa 1.1 probability of selecting ticket 10 in first 2 draws from 50 tickets

```
1 #Example 1 Chapter 1
2 n=50      #n is no. of tickets
3 r=2       #no of tickets to be choosen
4 ways_of_selection = length(combn(n,r))/r
5 selecting_t10_among_first2 = length(combn(1,1)) *
  length(combn(n-1,1))
6 P = selecting_t10_among_first2/ways_of_selection
7 print(P)
```

R code Exa 1.2 probability of getting no defective and only 1 defective from 2 draws

```
1 #Example 2 Chapter 1
2 screw=10
3 defective=2
4 r=2       #no of screws drawn from box
5 ways_drawing_r_screw = length(combn(screw,r))/r
```

```

6 ways_drawing_good_screw = length(combn(screw-
    defective,r))/r
7 P_no_defective = ways_drawing_good_screw/ways_
    drawing_r_screw
8 print(P_no_defective)           #answer given is wrong in
    the book
9 P_1_defective = length(combn(2,1))*length(combn(8,1)
    ) / ways_drawing_r_screw
10 print(P_1_defective)

```

R code Exa 1.3 probability of selecting a lot of all defective and only one defective

```

1 #Example 3 chapter 1
2 lot = 25
3 r = 5           #takes 5 motors for sample
4 ways_selecting_good_motors = length(combn(20,5))/5
    #lot containing 5 defective
5 total_ways = length(combn(25,5))/5
6 P_selecting_5defective_lot = ways_selecting_good_
    motors/total_ways
7 print(P_selecting_5defective_lot)
8 P_rejecting_1defective_lot = length(combn(1,1))*
    length(combn(24,4))/4 / total_ways
9 print(P_rejecting_1defective_lot)

```

R code Exa 1.5 probability of sum 7 or 8 and probability of sum not 7 and not 11 on rolling 2 dice

```

1 #Example 5 Chapter 1
2 d1 = c( 1,2,3,4,5,6 )
3 d2 = c( 1,2,3,4,5,6 )
4 P<-function(d){

```

```

5   a=expand.grid(d1,d2)
6   b=rowSums(a)
7   count=0
8   for(i in (1:length(b)))
9   {
10      if(b[i]==d)
11      {
12         count=count+1
13      }
14   }
15   return (count)
16 }
17 Psum7or8 = (P(7)+P(8))/36
18 print(Psum7or8)
19 Psum7or11 = (P(7)+P(11))/36
20 Pnot7not11 = 1-Psum7or11
21 print(Pnot7not11)

```

R code Exa 1.6 probability of 11th draw to be second blue ball

```

1 #Example 6 Chapter 1
2 W=10
3 B=9
4 R=8
5 Blue=3
6 s=W+B+R+Blue
7 ways_drawing_10_balls = length(combn(s,10))/10
8 P_2blue_at_11th_draw = 2/20
9 P_2blue_till_11th_draw = length(combn(Blue,1)) *
    length(combn(s-Blue,9))/9 * P_2blue_at_11th_draw
    / ways_drawing_10_balls
10 print(P_2blue_till_11th_draw)

```

R code Exa 1.7 probability of students to fail in maths when fail in physics and probability to fail in either

```
1 #Example 7 Chapter 1
2 P_Student_fail_in_Physics=0.3
3 P_Student_fail_in_Maths=0.25
4 P_fail_inboth=0.1
5 P_fail_in_M_failed_in_P = P_fail_inboth/P_Student_
   fail_in_Physics
6 print(P_fail_in_M_failed_in_P)
7 P_fail_in_M_or_P = P_Student_fail_in_Maths + P_
   Student_fail_in_Physics -P_fail_inboth
8 print(P_fail_in_M_or_P)
```

R code Exa 1.8 probability of integer from 1 to 200 divisible by 6 or 8

```
1 #Example 8 Chapter 1
2 P<-function(d){
3   v=c(1:200)
4   count=0
5   for(i in (1:length(v)))
6   {
7     if(v[i]%%d==0)
8     {
9       count=count+1
10    }
11  }
12  return (count/200)
13 }
14 P_div_6=P(6)
15 P_div_8=P(8)
16 P_6_And_8=P(24)
17 P_6_or_8= P(6)+P(8)-P(24)
18 print(P_6_or_8)
```

R code Exa 1.10 probability that no element in right position

```
1 #Example 10 Chapter 1
2 #8 letters on 8 envelope
3 ways_8_letters = factorial(8)
4 probb_all_true = 1/factorial(8)
5 probb_not_all_true = 1-prob_all_true
6 print(probb_not_all_true)
```

R code Exa 1.11 probability to miss a test

```
1 #Example 11 Chapter 1
2 ptest=1/5
3 P_T1bar=1-ptest
4 P_T2bar=1-ptest
5 P_testmiss_atleast1 = 1-P_T1bar*P_T2bar
6 print(P_testmiss_atleast1)
```

R code Exa 1.12 probability of getting a spade court or a jack card from a pack of true 52 cards

```
1 #Example 12 chapter 1 page no. 21
2 P_A= 1/4      #prob of spade
3 P_B= 12/52    #prob of court
4 P_C= 4/52     #prob of jack
5 P_AB= 3/52
6 P_BC= 4/52
7 P_CA= 1/52
8 P_ABC= 1/52
```

```

9 P_AorBorC= P_A+P_B+P_C-P_AB-P_BC-P_CA+P_ABC
10 print(P_AorBorC)

```

R code Exa 1.13 probability function

```

1 #Example 13 chapter 1 page no. 22
2 P_A= 3/8
3 P_B= 1/2
4 P_AB= 1/4
5 P_AcBc= 1-(P_A + P_B - P_AB)
6 print( P_AcBc)

```

R code Exa 1.15 probability of win of A B C

```

1 #Example 15 chapter 1 page no. 23
2 p= 1/2
3 q= 1-p
4 P_A= p/(1-(q^3)) #prob of A to win
5 P_B= (q*p)/(1-(q^3)) #prob of B to win
6 P_C= (q^2)*p/(1-(q^3)) #prob of C to win
7 cat("prob of A to win",P_A,"\nprob of B to win",P_B,
    "\nprob of C to win",P_C)

```

R code Exa 1.24 probability that defective is from A

```

1 #Example 24 chapter 1 page no 28
2 PABC=c(500,300,200)
3 P_A = PABC[1]/sum(PABC)
4 P_B = PABC[2]/sum(PABC)
5 P_C = PABC[3]/sum(PABC)

```

```

6 P_DwhenA=1/100
7 P_DwhenB=2/100
8 P_DwhenC=4/100
9 P_AwhenD=P_DwhenA*P_A/(P_DwhenA*P_A+P_DwhenB*P_B+P_
  DwhenC*P_C)
10 print(P_AwhenD)

```

R code Exa 1.27 probability of getting sum 5 before 7 or rolling 2 dice

```

1 #Example 27 chapter 1 page no 30
2 P_A = 4/36      # prob of sum=5 on rolling 2 dice
3 P_B = 6/36      #prob of sum=7 on rolling 2 dice
4 lemda = 1-(P_A+P_B)  # prob neither 5 nor 7 as the
  sum
5 cat("prob of sum=5 to come before",P_A/(1-lemda))

```

R code Exa 1.29 probability that none only A and both A and B hits target

```

1 #Example 29 chapter 1 page no 30
2 P_A=1/3      # prob that A hits the target
3 P_B=1/5
4 cat("prob that both hits the target",P_A*P_B)
5 cat("prob of atleast one hit the target",P_A+P_B-P_A
  *P_B)
6 cat("prob of no one hit the target",1-(P_A+P_B-P_A*P_
  _B))

```

R code Exa 1.31 probailty of 1 white from bag A and B

```

1 #Example 31 chapter 1 page no 32
2 P_A = 2/3      # prob of white ball from bag1
3 P_B = 2/4      #prob of white ball from bag 1
4 P_AorB = 1 - (1-P_A)*(1-P_B)
5 cat("prob of atleast 1 white",P_AorB)

```

R code Exa 1.34 probability that only either A or B speak the truth

```

1 #Example 34 chapter 1 page no 33
2 P_A = 0.7      # prob of A speak truth
3 P_B = 0.65     #prob of B speak truth
4 P_AandB = P_A*P_B
5 P_AandBbar = P_A*(1-P_B)
6 P_AbarandB = (1-P_A)*P_B
7 P_AbarandBbar = (1-P_A)*(1-P_B)
8 cat("prob of A and B in contradiction state",P_
    AandBbar+P_AbarandB)

```

R code Exa 1.52 probability that correct answer is given by skill

```

1 #Example 52 chapter 1 page no. 45
2 P_G = 1/3      #prob of guess
3 P_U = 1/6      # prob unfair means
4 P_S = 1 - P_G -P_U  #answer bu skill
5 P_c_when_G = 1/4
6 P_c_when_U = 1/8
7 P_c_when_S = 1
8 cat("prob that answer by skill",P_S*P_c_when_S/(P_c_
    when_U*P_U + P_c_when_G*P_G +
9

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R code Exa 1.54 probability of receiving 1 and 0 and getting 1 and 0 when 1 and zero transmitted and error probability

```
1 #Example 54 chapter 1 page no 47
2 P_0rec0sent = 0.94
3 P_1rec1sent = 0.91
4 P_1rec0sent = 1-P_0rec0sent
5 P_0rec1sent = 1-P_1rec1sent
6 p1=0.55
7 p0=1-p1
8 cat("prob of 1 recieved",P_1rec0sent*p0 + P_1
    rec1sent*p1)
9 cat("prob of 0 recieved",P_0rec0sent*p0 + P_0
    rec1sent*p1)
10 P_1sent1rec = P_1rec1sent*p1/(P_1rec1sent*p1 + P_1
    rec0sent*p0)
11 cat("prob that 1 was transmitted when 1 was recieved
    ",P_1sent1rec)
12 P_0sent0rec = P_0rec0sent*p0/(P_0rec1sent*p1 + P_0
    rec0sent*p0)
13 cat("prob that 0 was transmitted when 0 was recieved
    ",P_0sent0rec)
14 cat("prob of error",P_0rec1sent*p1 + P_1rec0sent*p0)
```

R code Exa 1.55 probability of defective when reported defective

```
1 #Example 55 chapter 1 Page no 48
2 defective = 0.02
3 P_good_when_good = 0.95
4 P_def_when_def = 0.94
5 good=1-defective
6 P_def_when_good = 1-P_good_when_good
7 P_good_when_def = 1 - P_def_when_def
8 P_actuallydefective_reported_def = P_def_when_def*
    defective/(P_def_when_def*defective +
9
10 print(P_actuallydefective_reported_def)
```

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Chapter 2

Random Variables

R code Exa 2.11 probability density function

```
1 #Example 11 chapter 2 page no 69
2 func<-function(x){
3   x*(2-x)
4 }
5 k=1/integrate(func,lower=0,upper=2)$value
6 print(k)
```

R code Exa 2.13 probability density function

```
1 #Example 13 chapter 2 page no. 71
2 x1<-function(a){
3   0
4 }
5 x2<-function(a){
6   1+a
7 }
8 lemnda=1/(integrate(x2,1,4)$value)
9 P_x3 = lemnda*integrate(x2,1,3)$value
10 print(P_x3)
```

R code Exa 2.14 plotting distribution function and density function

```
1 #Example 14 Chapter 2
2 rm(list = ls())
3
4 Fx<-function(x){
5   y=c()
6   for (i in (1:length(x))) {
7     if(x[i]<=0){y[i]=0}
8     else if(x[i]>=1){y[i]=1}
9     else {y[i]=x[i]}
10  }
11  return(y)
12 }
13 fx<-function(x){
14   y=c()
15   for (i in (1:length(x))) {
16     if(x[i]<=0){y[i]=0}
17     else if(x[i]>=1){y[i]=0}
18     else {y[i]=1}
19   }
20   return(y)
21 }
22 plot(Fx,-2, 2)    #distribution function
23 plot(fx,-2, 2)    #probability density function
```

R code Exa 2.16 probability of insufficient supply

```
1 #Example 16 chapter 2 page no. 73
2 fx<-function(a){                                     #prob density
   function for daily consumption
```

```

3   1/9 *a*exp(-a/3)
4 }
5 P_x12 = integrate(fx,0,12)$value      #probabilty
      that consumption is in supply limit of 12 MkwH
6 print(1-P_x12)    #prob of X greater than 12 supply
      insufficient

```

R code Exa 2.17 probability density function

```

1 #Example 17 chapter 2 page no. 74
2 fx<-function(a){                      #prob density
      function
3   1/2 *a
4 }
5 P_x = integrate(fx,1/2,3/4)$value
6 print(P_x)    #prob of area under line y=x/2 from x=1
      /2 to x=3/4

```

R code Exa 2.18 probability density function integrate function

```

1 #Example 18 chapter 2 page no. 74
2 fx<-function(a){                      #prob density
      function
3   2*exp(-2*a)
4 }
5 P_x1to3 = integrate(fx,1,3)$value
6 print(P_x1to3)    #prob of 1<=x<=3
7 px1by2 = integrate(fx,1/2,Inf)$value
8 cat("prob of X more than 0.5 is",px1by2)

```

R code Exa 2.19 probability of x less than 3 and between 4 and 5

```
1 #Example 19 chapter 2 page no. 76
2 Fx<-function(a){                                #prob density
3   function
4   return(1 - 4/(a^2))
5 }
6 P_x3 = Fx(3)
7 print(P_x3)   #prob of X less than equal to 3
8 cat("prob of 4<=x<=5 is",Fx(5)-Fx(4))
```

R code Exa 2.20 finding k value by integrate function

```
1 #Example 20 chapter 2 page no 76
2 func<-function(x){
3   1/(1+x^2)
4 }
5 k=1/integrate(func,lower=-Inf,upper=Inf)$value
6 print(k)
```

R code Exa 2.25 prob of x in a given range when given x greater than 2

```
1 #Example 25 chapter 2 page no.85
2 X=c(0:7)
3 a=10      #for euation 10k^2 + 9k -1 = 0
4 b=9
5 c=-1
6 k1<-(-b + sqrt(b^2 - 4*a*c))/(2*a)
7 k2<-(-b - sqrt(b^2 - 4*a*c))/(2*a)
8 print(k1)   #selecting k1 as the value of k
9 k=k1
10 P_X = c(0,k,2*k,2*k,3*k,k*k,2*k*k,7*k*k+k)
```

```
11 cat("prob of  $1.5 < x < 4.5$  when prob of  $x > 2$ ", (P_X[4]+P_X[5]) / (1-P_X[1]-P_X[2]-P_X[3]))
```

R code Exa 2.26 conditional probability

```
1 #Example 26 Chapter2 page no. 86
2 P<-function(x){
3   a=c(1,2,3,4,5)
4   for(i in (1:length(a)))
5     {
6       if(x==a[i]){return(x/15)}
7     }
8   return(0)
9 }
10 P_X_1_or_2 = P(1)+P(2)
11 print(P_X_1_or_2)
12 #prob of x bw 1/2 and 5/2 when given x greater than
13   1
14 p = P(2)/(1-P(1))
15 print(p)
```

Chapter 3

Expectations Probability Generating Functions and Random Process

R code Exa 3.1 expected value

```
1 #Example 1 Chapter 3
2 x=c(1/6,1/6,1/6,1/6,1/6,1/6)
3 g_x=c(10,20,10,20,50,70)
4 cat("expected value is ",x*%g_x)
```

R code Exa 3.2 expected value of heads in 3 coin tosses

```
1 #Example 2 Chapter3
2 x=c(0,1,2,3)
3 p_x=c(1/8,3/8,3/8,1/8)
4 cat("expected value of no of h in 3 coin toss ",x*%
    p_x)
```

R code Exa 3.3 expected no of defective with and without replacement

```
1 #Example 3 Chapter 3
2 n=25
3 d=5      #defective items
4 r=4      #4 are choosen
5 x=c(0,1,2,3,4)    #no of defectives
6 t= ncol(combn(n,r))
7 P_x=c(ncol(combn(n-d,r))/t,ncol(combn(5,1))*ncol(
      combn(20,3))/t,
8       ncol(combn(20,2))*ncol(combn(5,2))/t,ncol(
      combn(20,1))*ncol(combn(5,3))/t,
9       ncol(combn(5,4))/t)
10 cat("expected no of defective ",x*%P_x) #without
      replacement
11
12 p=5/25      #prob of getting defective
13 q = 1 - p   #prob of not defective
14 P_x_withreplace = dbinom(x,r,p)
15 cat("expected no of defective ",x*%P_x_withreplace)
```

R code Exa 3.4 expected value of defectives

```
1 #Example 4 Chapter 3
2 r=3      #sample size
3 n=12
4 d=3      #defective items
5 x=c(0,1,2,3)    #x denotes no of defectives in
      sample
6 t= ncol(combn(n,r))
7 P_x=c(ncol(combn(n-d,r))/t,ncol(combn(d,1))*ncol(
      combn(n-d,r-1))/t,
```

```

8      ncol(combn(n-d,1))*ncol(combn(d,r-2))/t,ncol(
      combn(d,r))/t)
9  cat("expected no of defective ",x%*%P_x) #without
      replacement

```

R code Exa 3.7 dbinom binomial distribution for expected value of heads

```

1  #Example 7 Chapter 3 Page no. 106
2  n=4 # no. of toss
3  x=c(0,1,2,3,4) # no of heads can be
4  p=1/2 #Prob of getting head
5  p_x = dbinom(x,n,p)
6  cat("expected no of head ",x%*%p_x)

```

R code Exa 3.8 expected points on drawing cards

```

1  #Example 8 Chapter 3 page no 107
2  p=4/52
3  x=c(1,2,3,4,5,6,7,8,9,10,10,10,10)
4  px=rep(p,times=13)
5  cat("expected points ",x%*%px)

```

R code Exa 3.11 expected money drawn

```

1  #Example 11 chapter 3 page no 108
2  X=c(1,1.5,2) # amount that can be drawn taking 2
      coins from bag
3  P_x=c(3/10,6/10,1/10)
4  cat("expected amount that can be drawn",X%*%P_x)

```

R code Exa 3.15 expected value

```
1 #Example 15 chapter 3 page no. 110
2 x=c(-2:3)
3 k=0.4/6
4 px=c(0.1,k,0.2,2*k,0.3,3*k)
5 Ex=x%%px
6 print(Ex)          # expected value of X
```

R code Exa 3.17 Expected lifetime and variance

```
1 #Example 17 chapter 3 page no 111
2 Fx<-function(x){
3   (1/6)*x*exp(-x/6)
4 }
5 En=integrate(Fx,0,Inf)$value
6 variance<-function(x){
7   (1/6)*((x-En)^2)*exp(-x/6)
8 }
9 cat("expected lifetime",En)
10 v=integrate(variance,0,Inf)
11 cat("variance of X",v$value)
```

R code Exa 3.18 expected profit

```
1 #Example 18 chapter 3 page no 113
2 X=c(3:6)
3 PX=c(1/8,5/8,2/8,0)
4 gx=c(10000,2500,-7000,0)
```

```
5 EgX=PX*%*%gx
6 cat("expected profit of rs",EgX)
```

R code Exa 3.40 poison distribution

```
1 #Example 40 Chapter 3 page no. 155
2 lemda=1    #mean rate 1 per week
3 t=3
4 m=0
5 cat("prob no failure in 3 weeks ",ppois(m,t,lemda))
6 cat("prob lessthan 5 failure in 10 weeks ",ppois
      (5,10,lemda))
7 #ppois funtion for poisson distribution
```

Chapter 4

Discrete Probability Distribution

R code Exa 4.1 binomial distribution

```
1 #Example 1 Chapter 4 page no.167
2 n=10
3 x=c(0:10)
4 f=c(6,20,28,12,8,6,0,0,0,0,0)
5 mean=sum(x*f)/sum(f)
6 p=mean/n
7 q=1-p
8 data.frame(x,sum(f)*dbinom(x,n,p))
9 #this dataframe shows the frequency in binomial
   distribution
```

R code Exa 4.2 frequency of binomial distribution

```
1 #Example 2 Chapter 4 page no.167
2 n=5
3 x=c(0:n)
```

```

4 f=c(2,14,20,34,22,8)
5 mean=sum(x*f)/sum(f)
6 p=mean/n
7 q=1-p
8 data.frame(x,sum(f)*dbinom(x,n,p))
9 #this dataframe shows the frequency in binomial
  distribution

```

R code Exa 4.4 samples reporting less than 3 literates in a binomial distribution

```

1 #Example 4 Chapter 4 page no. 171
2 p=2/10
3 n=10
4 x=c(0:n)
5 px=dbinom(x,n,p)
6 Pxlessthan_equal3 = px[1]+px[2]+px[3]+px[4]
7 print(Pxlessthan_equal3)
8 cat("for 100 samples ",100*Pxlessthan_equal3)

```

R code Exa 4.5 probability of at least 1 pen defective

```

1 #Example 5 Chapter 4 page no. 171
2 p=10/100
3 x=c(0:5)
4 P=dbinom(x,5,p)
5 Pxmorethan_equal1=1-P[1]
6 print(Pxmorethan_equal1)

```

R code Exa 4.6 probability of getting 5 or 6 on dice 3 times out of 8 throws

```

1 #Example 6 Chapter 4 page no 171
2 p=2/6
3 n=8
4 x=c(0:n)
5 P=dbinom(x,n,p)
6 cat("percentage of getting 3 success",100*P[4])

```

R code Exa 4.7 probability of exactly 2 at least 2 and no defective pen

```

1 #Example 7 Chapter 4 page no, 171
2 n=12
3 x=c(0:n)
4 p=0.1
5 P=dbinom(x,n,p)
6 cat("prob of exactly 2 defective", P[3])
7 cat("prob of at least 2 defective", 1-P[1]-P[2])
8 cat("prob of no defective", P[1])

```

R code Exa 4.8 probability that 7 out of 10 60 years aged men will live till 70 years

```

1 #Example 8 Chapter 4 page no. 172
2 P=65/100
3 n=10
4 X=c(1:n)
5 PX=dbinom(X,n,P)
6 Pxmoredthan_equal7= PX[7] + PX [8] + PX [9] +PX [10]
7 print(Pxmoredthan_equal7)

```

R code Exa 4.13 probability of true claim

```

1 #Example 13 chapter 4 page no176
2 p=0.1
3 n=20
4 x=c(0:20)
5 P=dbinom(x,n,p)
6 P_xmorethan5=1-(P[1]+P[2]+P[3]+P[4]+P[5])      #prob
   of 5 or more machine damage
7 print(P_xmorethan5)

```

R code Exa 4.16 poisson distribution

```

1 #example 16 chapter 4 page no 186
2 m=2
3 px4=dpois(4,m)
4 print(px4)

```

R code Exa 4.17 probability of 5 defective in poisson distribution

```

1 #example 17 chapter 4 page no 186
2 m=400*0.02
3 px4=dpois(5,m)
4 print(px4)      #book has an error of 0.002

```

R code Exa 4.18 probability of exactly and at most 2 flat tires in 10000 cars with poisson distribution of flat tires

```

1 #example 18 chapter 4 page no 186
2 m=0.00005*10000
3 p<-function(x){return(dpois(x,m))}
4 cat("prob of 2 flat tyre among 10000",p(2))

```



```
5 cat("prob of atmost 2 flat tyre among 10000",p(0)+p(1)+p(2))
```

R code Exa 4.19 poisson distribution for 5 defective and 5 or less defective

```
1 #example 19 chapter 4 page no 186
2 m=0.02*200
3 p<-function(x){return(dpois(x,m))}
4 cat("prob of 5 defective in 200",p(5))
5 cat("prob of 5 or less defective",p(0)+p(1)+p(2)+p(3)+p(4)+p(5))
```

R code Exa 4.20 probability of 5 success

```
1 #example 20 chapter 4 page no 186
2 m=1600*(1/2^5)
3 px5=dpois(5,m)
4 print(px5)
```

R code Exa 4.22 poisson distribution for 0 1 and 2 defective blades in a pack of 10000

```
1 #Example 22 chapter 4 page no 188
2 n=10
3 p=1/100
4 m=n*p
5 p0=round(dpois(0,m),3)
6 cat("prob of no defective blade",p0)
7 p1=round(dpois(1,m),4)
8 cat("prob of one defective blade",p1)
```

```

9 p2=round(dpois(2,m),4)
10 cat("prob of two defective blade",p2)
11 cat("packets with 0 defective blade",10000*p0)
12 cat("packets with 1 defective blade",10000*p1)
13 cat("packets with 2 defective blade",10000*p2)

```

R code Exa 4.27 fitting poisson distribution

```

1 #Example 27 chapter 4 page no 190
2 x=c(0:4)
3 days=c(21,18,7,3,1)
4 xbar=x%*%days/sum(days)
5 px=dpois(x,xbar)
6 expectedfreq=round(px*sum(days),2)
7 df=data.frame(x,expectedfreq)
8 print("expected freq are")
9 print(df)

```

R code Exa 4.28 fitting poisson distribution

```

1 #Example 28 chapter 4 page no 191
2 x=c(0:10)
3 f=c(1,4,15,22,21,20,8,6,2,0,1)
4 xbar=x%*%f/sum(f)
5 px=dpois(x,xbar)
6 expectedfreq=round(px*sum(f),0)
7 df=data.frame(x,expectedfreq)
8 print("expected freq are")
9 print(df)

```

Chapter 5

Continuous Probability Distribution

R code Exa 5.3 uniform distribution

```
1 #Example 3 chapter 5 page no. 201
2 min=0    #arrival at 9:00 AM
3 max=30    #arrival at 9:30 AM
4 px10to15 = punif(15-10,min,max)
5 px25to30 = punif(30-25,min,max)
6 pwait5min = px10to15 + px25to30
7 print(pwait5min)
8 px0to5 = punif(5-0,min,max)
9 px15to20 = punif(20-15,min,max)
10 pwait10min = px0to5 +px15to20
11 print(pwait10min)
```

R code Exa 5.5 uniform distribution

```
1 #Example 5 Chapter 5 page no. 203
2 min=0
```

```

3 max=10
4 #given equa has solution (-infi,-1) and (2,infi)
5 #4x^2 + 4(lemda)x + lemda +2
6 Prootsreal=punif(10-2,min,max)
7 print(Prootsreal)

```

R code Exa 5.6 probability of x less than 0 in uniform distribution

```

1 #Example 6 chapter 5 page no 203
2 u=1      #mean of uniform distribution
3 v=4/3    #variance of uniform distribution
4 b = (2*u+sqrt(12*v))/2
5 a = 2*u-b
6 pxlessthan0 = dunif(1,a,b)
7 print(pxlessthan0)

```

R code Exa 5.7 normal distribution

```

1 #Example 7 chapter 5 page no. 216
2 n=1000   #sample of 1000 cases
3 u=14    #mean
4 sd=2.5
5 p<-function(x){return(pnorm(x,u,sd))}
6 px12to15 = p(15)-p(12)
7 cat("candidates with score 12 to 15 ",as.integer(n*
  px12to15))
8 px10 = p(10)
9 cat("candidates with score less than 10 ",as.integer
  (n*px10))
10 px20=p(20)
11 cat("candidates with score more than 20 ",as.integer
  (n*(1-px20)))

```

R code Exa 5.8 probability of wages more than 90 and probability of wages less than 45

```
1 #Example 8 chapter5 page no 217
2 n=500    #500 workers
3 u=75     #mean 75 rs
4 sd=15    #sd 15 rs
5 p<-function(x){return(pnorm(x,u,sd))}
6 cat("workers getting more than Rs.90 are ",as.
      integer(n*(1-p(90))))
7 cat("workers getting less than Rs.45 are ",as.
      integer(n*(p(45))))
```

R code Exa 5.9 mean and deviation in normal distribution using qnorm

```
1 #Example 9 chapter 5 page no 218
2 A=0.58    # under 75
3 B=0.38    # from 75 to 80
4 C=0.04    # above 80
5 z1=qnorm(A)
6 z2=0-qnorm(C) #for z greater than z2
7 v=(80-75)/(z2-z1)
8 u=75-z1*v
9 cat("mean=",u," and standard deviation = ",v)
10 #book has variance error of 0.27 and mean has value
    0.06 more
```

R code Exa 5.10 frequency of occurrence between a period of normal distribution

```

1 #Example 10 chapter 5 page no 219
2 x40=0.3      #for x<=40
3 x50=0.3+0.33  # for x<=50
4 xmorethan50 = 0.37
5 z1=qnorm(x40)
6 z2=qnorm(x50)
7 v=(50-40)/(z2-z1)
8 u=40-z1*v
9 px50to60=pnorm(60,u,v)-pnorm(50,u,v)
10 cat("frequency in interval 50 to 60 is ",round(100*
    px50to60,digits = 0))
11 #book has error in answer.

```

R code Exa 5.11 normal distribution of heights among 300 students

```

1 #Example 11 Chapter 5 Page no 220
2 u=64.5      #mean height in inches of 300 students
3 n=300
4 sd=3.3
5 cat("students with height less than 5 feet",round(n*
    pnorm(60,u,sd),digits = 0))
6 p = (pnorm(69,u,sd)-pnorm(60,u,sd))
7 cat("students having height bw 5 ft to 5ft 9 inch ",
    as.integer(n*p))
8 x=1-0.99
9 z1=qnorm(x/2)
10 z2=0-z1
11 x1=sd*z1+u
12 x2=sd*z2+u
13 cat("99% students are bw height(in inches)",round(x1
    ,0),round(x2,0))

```

R code Exa 5.12 mean and deviation in normal distribution using qnorm

```

1 #Example 12 chapter 5 page no 221
2 x45=0.31 #30%people under 45
3 x64=1-0.08
4 z1=qnorm(x45)
5 z2=qnorm(x64)
6 v=(64-45)/(z2-z1)
7 u=45-z1*v
8 cat("mean=",u," and standard deviation = ",v)
9 #answer given approximate in book

```

R code Exa 5.13 probability of 2 out of 3 students having more than 70 marks with normal distribution of marks

```

1 #Example_13 chapter 5 page no. 222
2 u= 65
3 sd= sqrt(25)
4 q = pnorm(70,u, sd)
5 p=1-q
6 cat("prob 2 out of 3 have more than 70 marks",dbinom
    (2,3,p))
7 #book has error in the answer

```

R code Exa 5.14 probability in normal distribution

```

1 #Example 14 chapter 5 page no. 227
2 u=0
3 sd=3
4 v=9*9+16*9 #3x+4y=5 and 3x+4y=10
5 px5to10 = pnorm(10,u,sqrt(v)) - pnorm(5,u,sqrt(v))
6 print(px5to10)
7 # value of answer is 0.0023 more in book

```

R code Exa 5.17 probability in exponential distribution

```
1 #Example 17 Chapter 5 page no 229
2 lemda=1/2
3 px2=1-pexp(2,lemda)
4 print(px2)      #prob repair time exceed 2 hr
5 px10whenx9 = (1-pexp(10,lemda))/(1-pexp(9,lemda))
6 print(px10whenx9)  #prob repair takes at least 10
   hr if had taken 9hr
```

R code Exa 5.19 fitting normal distribution

```
1 #Example 19 chapter 5 page no 230
2 n=12      #coin tossed 12 times
3 p=1/2
4 q=1-p
5 u=n*p     #mean of binomial distribution
6 sd=sqrt(n*p*q)
7 patmost4heads = pnorm(4.5,u,sd)
8 pexact4heads = pnorm(4.5,u,sd)-pnorm(3.5,u,sd)
9 cat("prob exact 4 head ",pexact4heads,"\nprob of
   atmost 4 head ",patmost4heads)
```

Chapter 6

Correlation and Regression

R code Exa 6.2 coefficient of correlation

```
1 #Example 2 chapter 6 page no. 240
2 X=c(-10,-5,0,5,10)
3 y=c(5,9,7,11,13)
4 print(cor(X,y))
```

R code Exa 6.3 coefficient of correlation

```
1 #Example 3 chapter 6 page no. 240
2 X=c(1:9)
3 y=c(9,8,10,12,11,13,14,16,15)
4 print(cor(X,y))
```

R code Exa 6.4 coefficient of correlation

```
1 #Example 4 chapter 6 page no. 240
2 n=11
```

```

3 X=c(45,55,56,58,60,65,68,70,75,80,85)
4 u=X-rep(65,11)
5 y=c(56,50,48,60,62,64,65,70,74,82,90)
6 v=y-rep(65,11)
7 ubar=mean(u)
8 vbar=mean(v)
9 var_u=(sum(u*u))/n - (ubar^2)
10 var_v=(sum(v*v))/n - (vbar^2)
11 var_uv=(sum(u*v))/n - ubar*vbar
12 r = var_uv / (sqrt(var_u*var_v))
13 cat("corr coeff is ",r)      #answer given in book is
                                wrong (calculation error)

```

R code Exa 6.5 correlation between age of fathers and son

```

1 #Example 5 chapter 6 page no. 240
2 X=c(65,66,67,67,68,69,70,72)
3 y=c(67,68,65,68,72,72,69,71)
4 print(cor(X,y))

```

R code Exa 6.7 coefficient of correlation

```

1 #Example 2 chapter 6 page no. 240
2 X=c(1,3,5,7,8,10)
3 y=c(8,12,15,17,18,20)
4 print(cor(X,y))

```

R code Exa 6.8 coefficient of correlation

```

1 #Example 8 chapter 6 page no 245

```

```

2 rankx=c(2,1,4,3)
3 ranky=c(1,3,2,4)
4 name=c("ram","shyam","hari","sohan")
5 a=data.frame(name,rankx,ranky)
6 cor(a["rankx"],a["ranky"])

```

R code Exa 6.9 correlation in rank given by 3 judges

```

1 #Example 9 chapter 6 page no 247
2 rankj1=c(1,6,5,10,3,2,4,9,7,8)
3 rankj2=c(6,4,9,8,1,2,3,10,5,7)
4 rankj3=c(3,5,8,4,7,10,2,1,6,9)
5 name=c("c1","c2","c3","c4","c5","c6","c7","c8","c9",
        "c10")
6 a=data.frame(name,rankj1,rankj2,rankj3)
7 r12 = cor(a["rankj1"],a["rankj2"])
8 r23 = cor(a["rankj2"],a["rankj3"])
9 r13 = cor(a["rankj1"],a["rankj3"])
10 cat(r12,r23,r13)
11 #book has a calcultion error in r23 value

```

R code Exa 6.10 rank correlation

```

1 #Example 10 chapter 6 page no 247
2 maths=c(10,15,12,17,13,16,24,14,22)
3 rxi = rank(maths)
4 statistics=c(30,42,45,46,33,34,40,35,39)
5 ryi = rank(statistics)
6 cor(rxi,ryi)

```

R code Exa 6.13 fitting second degree parabola

```
1 #Example 13 chapter 6 page no 253
2 x=c(0:4)
3 y=c(1,1.8,1.3,2.5,6.3)
4 fit<-lm(y~poly(x,2,raw = "True"))
5 plot(x,y,main = "scatterplot")
6 parabola=fit$coefficient[3]*x^2 + fit$coefficient[2]
   *x +fit$coefficient[1]
7 lines(x,parabola,col="red")
8 cat("y =",fit$coefficient[3],"x^2 +",fit$coefficient
     [2],"x +",fit$coefficient[1])
```

R code Exa 6.14 fitting parabola on given data

```
1 #Example 14 chapter 6 page no 254
2 x=seq(1,4.0,0.5)
3 y=c(1.1,1.3,1.6,2.6,2.7,3.4,4.1)
4 fit<-lm(y~poly(x,2,raw = "True"))
5 plot(x,y,main = "scatterplot")
6 parabola=fit$coefficient[3]*x^2 + fit$coefficient[2]
   *x +fit$coefficient[1]
7 lines(x,parabola,col="red")
8 cat("y =",fit$coefficient[3],"x^2 +",fit$coefficient
     [2],"x +",fit$coefficient[1])
```

R code Exa 6.15 fitting a line

```
1 #Example 15 chapter 6 page no 257
2 x=c(-5:4)
3 y=c(45,52,54,63,62,68,75,76,92,88)
4 fit<-lm(y~poly(x,1,raw = "True"))
5 plot(x,y,main = "scatterplot")
```

```

6 best_line=fit$coefficient[2]*x + fit$coefficient[1]
7 lines(x,best_line,col="red")
8 cat("y =",fit$coefficient[2],"x +",fit$coefficient
    [1])

```

R code Exa 6.16 fitting a line

```

1 #Example 16 chapter 6 page no 254
2 x=seq(20,380,40)    #air velocity
3 y=c
    (0.18,0.37,0.35,0.78,0.56,0.75,1.18,1.36,1.17,1.65)
    #Evaporation coefficient
4 fit<-lm(y~poly(x,1,raw = "True"))
5 plot(x,y,main = "scatterplot")
6 line1= fit$coefficient[2]*x +fit$coefficient[1]
7 lines(x,line1,col="red")
8 cat("y =",fit$coefficient[2],"x +",fit$coefficient
    [1])
9 y=predict(fit,data.frame(x=190))
10 cat(y,"mm^2/sec is evaporation coeff for air
    velocity 190 cm/sec")

```

R code Exa 6.17 line of regression for estimation of value

```

1 #Example 17 chapter 6 page no 263
2 x=c(1:9)
3 y=c(9,8,10,12,11,13,14,16,15)
4 fit<-lm(y~poly(x,1,raw = "True"))
5 plot(x,y,main = "scatterplot")
6 line1= fit$coefficient[2]*x +fit$coefficient[1]
7 lines(x,line1,col="red")
8 cat("y =",fit$coefficient[2],"x +",fit$coefficient
    [1])

```

```
9 y=predict(fit,data.frame(x=6.2))
10 cat(y,"is the estimated value for x=6.2")
```

R code Exa 6.18 covariance

```
1 #Example 18 chapter 6 page no 264
2 n=10
3 xbar = 50/n
4 ybar = -30/n
5 sum_xy = -115
6 cov_xy = sum_xy/n - xbar*ybar
7 print(cov_xy)
```

R code Exa 6.22 fit a parabola then find values

```
1 #Example 22 chapter 6 page no 271
2 x=c(0:8)
3 y=c(12,10.5,10,8,7,8,7.5,8.5,9)
4 fit<-lm(y~poly(x,2,raw = "True"))
5 plot(x,y,main = "scatterplot")
6 parabola=fit$coefficient[3]*x^2 + fit$coefficient[2]
   *x +fit$coefficient[1]
7 lines(x,parabola,col="red")
8 cat("y =",fit$coefficient[3],"x^2 +",fit$coefficient
   [2],"x +",fit$coefficient[1])
9 y=predict(fit,data.frame(x=6.5))
10 cat(y,"predicted drying time in hours for x=6.5gms")
```

R code Exa 6.27 rank correlation coefficient

```

1 #Example 27 chapter 6 page no 248
2 x=c(68,64,75,50,64,80,75,40,55,64)
3 rxi = rank(x)
4 y=c(62,58,68,45,81,60,68,48,50,74)
5 ryi = rank(y)
6 cor(rxi,ryi)           #answer vary slightly from book

```

R code Exa 6.28 error of estimate

```

1 #Example 28 chapter 6 page no. 276
2 n=5
3 x=c(1:n)
4 y=c(2,5,3,8,7)
5 fit<-lm(y~poly(x,1,raw = "True"))
6 yi=predict(fit,data.frame(x))
7 error=sqrt(sum((y-yi)^2)/n)
8 cat("error of estimates",error)

```

Chapter 7

Queuing Theory

R code Exa 7.1 Model 1 M M 1 infinite FIFO

```
1 #Example 1 chapter 7 page no 295
2 l = 6          #arrival_rate persons per hour
3 u = 60/3       # mean service rate per hour
4 ls = 1/(u-1)   # average number of person in the
                system
5 cat(ls,"average no of persons in the system")
6 pts10=exp(-(u-1)*10/60)
7 cat(pts10,"prob to spent more than 10min in queue")
8 ptq10=(1/u)*exp(-(u-1)*10/60)
9 cat(ptq10,"prob to waits more than 10min in queue")
10 cat(1/u, "prob that an arrival has to wait")
11 cat(1/u*100,"% of the day phone is busy")
12 cat(u/(u-1),"avg length of non empty queues")
```

R code Exa 7.2 Model 1 M M 1 infinite FIFO

```
1 #Example 2 chapter 7 page no 297
2 l = 4          #arrival_rate persons per hour
```



```

3 u = 60/12 # mean service rate per hour
4 ls = 1/(u-1) # average number of person in the
  system
5 cat(ls,"average no of persons in the system")
6 rho = 1/u
7 cat(rho/((1-rho)^2), "variance of queue length")
8 cat((1-rho)*8,"hrs expected idle time in a day of 8
  hrs")
9 cat(rho^5,"prob of more than 5 customers")

```

R code Exa 7.3 Model 1 M M 1 infinite FIFO

```

1 #Example 3 chapter 7 page no 297
2 l = 30 / (60*24) #arrival_rate trains per minute
3 u = 1/36 # mean service rate per minute
4 ls = 1/(u-1) # average number of trains in the
  system
5 cat(ls,"average no of trains in yard")
6 rho = 1/u
7 cat(rho^11, "prob trains exceed 10") #answer
  given is wrong in the book
8 lnew=33/(60*24)
9 rhonew = lnew/u
10 lsnew = lnew/(u-lnew) # average number of
  trains in the system
11 cat(round(lsnew,digits = 0),"average no of trains in
  yard")
12 cat(rhonew^11, "prob trains exceed 10")
13 #answer given is wrong in the book

```

R code Exa 7.4 Model 1 M M 1 infinite FIFO

```

1 #Example 4 chapter 7 page no 298

```

```

2 l = 10      #arrival_rate sets per day
3 u = 16      # mean service sets per day
4 rho = l/u
5 cat((1-rho)*8,"hrs expected idle time in a day of 8
    hrs")
6 ls = 1/(u-1)      # average number of sets in the
    system
7 cat(round(ls,digits = 0),"average number of sets in
    the system")

```

R code Exa 7.6 Model 1 M M 1 infinite FIFO

```

1 #ExAMPLE 6 chapter 7 page no 300
2 l=30      #arrival rate customer per hour
3 u=40      #services per hour
4 cat("avg line length/customer in the system",1/(u-1)
    )
5 cat("avg waiting line length",((l*l)/(u*(u-1))))
6 cat("avg waiting time in the queue in minutes",(1/(u
    *(u-1)))*60) #book has calculation error
7 cat("avg waiting time in the system in minutes",1/(u
    -1)*60)

```

R code Exa 7.7 Model 1 M M 1 infinite FIFO

```

1 #Example7 chapter 7 page no 301
2 l= 5      #break downs per hout
3 u=7      #repairs per hour
4 cat("avg units per hour",1/(u-1))
5 cat("loss due to machines idle",8*15*1/(u-1))
6 cat("repair man charge",8*8)
7 costA=8*15*1/(u-1)+8*8
8 u2=9

```

```

9 cat("avg units per hour for B",1/(u2-1))
10 cat("loss due to machines idle for B",8*15*1/(u2-1))
11 costB =8*15*1/(u2-1) +10*8
12 cat("cost of A",costA,"\ncost of B",costB)

```

R code Exa 7.9 Model 2 M M 1 N FIFO

```

1 #Example 9 chapter 7 page no 309
2 l =3      # arrival rate per hour
3 u=60/36   #service rate customers per hour
4 N=4      # maximum 4 customers in the queue
5 rho = l/u
6 p0=(1-rho)/(1-(rho^(N+1)))
7 cat("Prob of no customer",p0)
8 En=(rho/(1-rho))*(1 - ((N+1)*(rho^(N+1))*(1-rho)/
   (1-(rho^(N+1)))))
9 cat("avg no of customer in the system",round(En,0))
10 #book has calculation mistake

```

R code Exa 7.10 Model 2 M M 1 N FIFO

```

1 #Example 9 chapter 7 page no 309
2 l =6      # arrival rate trains per hour
3 u=12      #service rate trains per hour
4 N=3      # maximum 3 in the queue
5 rho = l/u
6 p0=(1-rho)/(1-(rho^(N+1)))
7 cat("Prob of no train",p0)
8 p1=p0*rho
9 cat("Prob of 1 train",p0*rho)
10 p2=p0*rho^2
11 cat("Prob of 2 train",p0*rho^2)
12 p3=p0*rho^3

```

```

13 cat("Prob of 3 train",p0*rho^3)
14 En=p1+(2*p2)+(3*p3)
15 cat("avg no of train in the system",round(En,0))
16 ws = En/(u*(1-p0))
17 cat("avg waiting time in minutes",ws*60)

```

R code Exa 7.13 Model 3 M M C infinite FIFO

```

1 #Example 13 chapter 7 page no 319
2 l1=14      #avg arrival rate for withdrawers
3 u1=20      #avg service rate for withdrawers
4 rho=l1/u1
5 Ew = l1/(u1*(u1-l1))
6 cat("avg waiting time in the queue in minutes",Ew*
    60)
7 l2=16
8 Ew = l2/(u1*(u1-l2))
9 cat("avg waiting time in the queue for depositers in
    minutes",Ew*60)
10 l=l1+l2
11 c=2
12 P0=1/(1+ l/u1 + (((l/u1)^2)/2)*c*u1/(c*u1-1) )
13 print(P0)
14 EW=u1*((l/u1)^c)*P0/((c*u1-1)^2)
15 cat("avg waiting time in the queue in minutes for a
    withdrawer+depositer",EW*60)
16 u2=120/7
17 P0=1/(1+ l/u2 + (((l/u2)^2)/2)*c*u2/(c*u2-1) )
18 print(P0)
19 EW=u2*((l/u2)^c)*P0/((c*u2-1)^2)
20 cat("avg waiting time in the queue in minutes for a
    withdrawer+depositer",EW*60)

```

R code Exa 7.15 Model 3 M M C infinite FIFO

```
1 #Example 15 chapter 7 page no 322
2 l=10      #arrival rate vehicles per hour
3 c=2
4 u1=60/4    #service rate vehicles per hour
5 rho=l/u1
6 P0=1/(1+ l/u1 + (((l/u1)^2)/2)*c*u1/(c*u1-1) )
7 print(P0)
8 P_C=P0*(rho^2)*2*u1/(2*(2*u1-1))
9 cat("prob vehicle has to wait",P_C)
10 busy=l/(u1*c)
11 cat("prob of the day busy",l/(u1*c))
12 cat("pumps are idle for percentage = ",100*(1-busy))
```

R code Exa 7.16 Model 3 M M C infinite FIFO

```
1 #Example 16 chapter 7 page no 323
2 l=12      #arrival rate customers per hour
3 c=2
4 u1=60/6    #service rate customers per hour
5 rho=l/(u1*c)
6 P0=1/(1+ l/u1 + (((l/u1)^2)/2)*c*u1/(c*u1-1) )
7 print(P0)
8 P_C=P0*((l/u1)^2)*2*u1/(2*(2*u1-1))
9 cat("prob customer has to wait",P_C)
10 Em=P0*((l/u1)^2)*l*u1/((2*u1-1)^2)
11 cat("avg queue length",Em)
12 En=(Em+l/u1)
13 cat("avg customers in the system",round(En,0))
14 Ev=En/l
15 cat("avg time spent in the system in minutes",Ev*60)
```

Chapter 8

Discrete Parameter Markov Chains and Queuing Model M G 1 infinite General Discipline

R code Exa 8.1 general discipline queuing model

```
1 #Example 1 chapter 8 page no 371
2 rho=0.75
3 u=60/10.5 #avg service rate per hour
4 l=rho*u
5 sd=8.8 #in minutes
6 var=(8.8/60)^2 # in hours square
7 EWq1=(1/l)*(((rho^2) +((l^2)*var))/(2*(1-rho)))
8 cat("avg waiting time in minutes",EWq1*60)
9 u = 60/8 #new service rate per hour
10 rho=l/u
11 var=(6/60)^2 #in hours square
12 EWq2=(1/l)*(((rho^2) +((l^2)*var))/(2*(1-rho)))
13 cat("avg waiting time is reduced now in minutes by",
      EWq1*60-EWq2*60)
14 cat("percentage reduction in waiting time",(EWq1*60-
      EWq2*60)/(EWq1*60)*100)
```

R code Exa 8.2 general discipline queuing model

```
1 #Example 2 chapter 8 page no 372
2 u=60/10 #avg service rate cars per hour
3 l=4 #arrival rate in cars per hour
4 ws=1/(u-1)
5 print("for case (a) exponential distribution")
6 cat("time spent in the system in minutes",ws*60)
7 wq = ws - (1/u)
8 cat("time spent in the system in minutes",wq*60)
9 print("for case (b) constant distribution")
10 ws = 1/u + 1/(2*u*(u-1))
11 cat("time spent in the system in minutes",ws*60)
12 wq=ws - 1/u
13 cat("time spent in the system in minutes",wq*60)
14 print("hense constant distribution is more
       favourable due to reduced waiting time")
```

R code Exa 8.3 probability in markov chain

```
1 #Example 3 chapter 8 page no. 348
2 p0 = matrix(c(0.4 , 0.6) , 1,2)
3 p=matrix(c(0.7,0.4,0.3,0.6),2,2)
4 p1=p0**p
5 cat("prob distribution after step 1",p1)
6 p2=p1**p
7 cat("prob distribution after step 2 ",p2)
```

R code Exa 8.4 Markov chain and transition probability matrix

```

1 #Example 4 chapter 8 page no 349
2 ABC = matrix(c(400,500,600),1,3)
3 tpm = matrix(c(325/400,1/10,25/600,2/16,7/10,0,1/
  16,2/10,23/24),3,3)
4 cat("no. of clients of A B C after 1 year",round(ABC
  %*%tpm,digits = 0))

```

R code Exa 8.9 transition probability matrix

```

1 #Example 9 chapter 8 page no. 355
2 P=matrix(c(0,1/2,1,1/2),2,2)
3 P1=matrix(c(5/6,1/6),1,2)
4 P3 = (P1 %*% P) %*% P
5 cat("prob that he takes bus on third day=",P3[1])
6 b=matrix(c(0,1),2,1)
7 A=matrix(c(-1,0.5,1,1),2,2,byrow = TRUE)
8 cat("prob of bus and scooter in steady state
  condition",round(solve(A,b),2))

```

R code Exa 8.10 transition between 2 Brands customers

```

1 #Example 10 chapter 8 page no 356
2 P=matrix(c(0.8,0.6,0.2,0.4),2,2)
3 p0 = matrix(c(0.5,0.5),1,2)
4 p1=p0%*%P
5 cat("prob of A B after 1 year",p1)
6 p2=p1%*%P
7 cat("prob of A B after 2 year",p2)
8 #answer vary from the book as book has wrong
  calculation

```

R code Exa 8.12 joint probability in transition probability matrix

```
1 #Example 12 chapter 8 page no 358
2 P=matrix(c(0.1,0.6,0.3,0.5,0.2,0.4,0.4,0.2,0.3),3,3)
3 p0 = matrix(c(0.7,0.2,0.1),1,3)
4 P2=P%%P
5 p2=p0%%P2
6 cat("prob of X2 = C after 2 year",p2[3])
7 cat("prob of x3=B X2=C x1=C x0=B",p0[2]*P[2,3]*P
    [3,3]*P[3,2])
```

R code Exa 8.14 steady state condition in transition probability

```
1 #Example 14 chapter 8 page no. 359
2 P_ABC = matrix(c(200,500,300),1,3)
3 tpm = matrix(c(160/200,20/200,20/200,35/500,450/
    500,15/500,25/300,20/300,255/300),3,3,TRUE)
4 feb1=P_ABC%%tpm
5 cat("on 1st Feb",P_ABC%%tpm)
6 mar1=feb1%%tpm
7 cat("on 1st March",round(mar1,0))
8 apr1=mar1%%tpm
9 cat("on 1st April",round(apr1))
10 b=matrix(c(0,0,1),3,1)
11 A=matrix(c(-0.2,0.07,0.083,0.1,-0.1,0.067,1,1,1),
    ,3,3,byrow = TRUE)
12 cat("customers in steady state condition",round(
    solve(A,b),3)*1000)
```

R code Exa 8.15 probability of having 2 books after 3 weeks and on long run

```
1 #Example 15 chapter 8 page no 361
```

```

2 P = matrix(c(0,1,0,1/6,1/2,1/3,0,2/3,1/3),3,3,TRUE)
3 p0 = matrix(c(1,0,0),1,3)      # for 0,1,2 books
    initially 0 book
4 p1= p0%%P
5 p2= p1%%P
6 p3= p2%%P
7 cat("prob of 2 books after 3 weeks",p3[3])
8 b=matrix(c(0,0,1),3,1)
9 A=matrix(c(-1,1/6,0,1,-1/2,2/3,1,1,1),3,3,byrow =
    TRUE)
10 cat("prob of 2 book in long run",round(solve(A,b),3)
    [3])

```

R code Exa 8.16 expected cost of maintenance for different states of machine

```

1 #Example 16 chapter 8 page no. 363
2 tpm=matrix(c(0,3/4,1/4,0,0,1/2,1/2,0,0,0,1/2,1/
    2,1,0,0,0),4,4,TRUE)
3 b=matrix(c(0,0,0,1),4,1)
4 A=matrix(c(-1,0,0,1,3/4,-1/2,0,0,0,0,1/2,-1,1,1,1)
    ,4,4,byrow = TRUE)
5 px=(solve(A,b))
6 x=c(125,0,0,75)
7 cat("expected maintenance per day",x%%px)

```

R code Exa 8.17 transition probability matrix and markov chain

```

1 #Example 17 chapter 8 page no. 364
2 p0=matrix(c(0,0,1,0,0),1,5)      #5 states for 0 5 10
    15 20 rs
3 tpm=matrix(c(1,0,0,0,0,1/2,0,1/2,0,0,0,1/2,0,1/
    2,0,0,0,1/2,0,1/2,0,0,0,0,1),5,5,TRUE)

```

```

4 p1=p0%%tpm
5 p2=p1%%tpm
6 p3=p2%%tpm
7 p4=p3%%tpm
8 p5=p3%%tpm
9 cat("prob gambler loses all money at 5th game",p5
    [1])
10 p6=p5%%tpm
11 p7=p6%%tpm
12 cat("prob that game last more than 7 terms",p7[2]+p7
    [3]+p7[4])

```

R code Exa 8.18 probability in steady state markov chain

```

1 #Example 18 chapter 8 page no. 366
2 tpm=matrix(c(0.9,0.1,0.5,0.5),2,2,TRUE)
3 b=matrix(c(0,1),2,1)
4 A=matrix(c(-0.1,0.5,1,1),2,2,byrow = TRUE)
5 cat("prob in steady state condition",round(solve(A,b
    ),3))

```
