R Textbook Companion for Matrices and Linear Transformations by Charles G. Cullen[1]

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Book Description

Title: Matrices and Linear Transformations

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R numbering policy used in this document and the relation to the above book.

Exa Example (Solved example)

Eqn Equation (Particular equation of the above book)

For example, Exa 3.51 means solved example 3.51 of this book. Sec 2.3 means an R code whose theory is explained in Section 2.3 of the book.

Contents

List of R Codes

Chapter 1

Matrices and Linear Systems

Chapter 2

Vector Spaces

R code Exa 2.1 Subspaces

```
#page - 74
#section - 2.2 SUBSPACES
#example 1

#let us represent 'a' with a value as we cannot
   multiply character matrix with numerical matrix
a = 7

#let unit vectors of F4*1 be E1, E2, E3, E,4
E1 <- matrix(c(1,0,0,0), 4, 1)
E2 <- matrix(c(0,1,0,0), 4, 1)
E3 <- matrix(c(0,0,1,0), 4, 1)
E4 <- matrix(c(0,0,0,1), 4, 1)

#thus vector A is
A = (E1 %*% a) + (E2 %*% a) + (E3 %*% a) + (E4 %*% a
   )
A
```

R code Exa 2.2 Subspaces

```
#page - 75
#section - 2.2 SUBSPACES
#example 2

#included package - matlib

#for echelon function
library(matlib)

#matrix A
A <- matrix(c(1,-1,-3,-1,2,5,1,2,6), 3, 3, byrow=
    TRUE)

#column matrix k
K <- c(-6,10,15)

#reduced row-echelon form
echelon(A, K, reduced=FALSE, verbose=TRUE, fractions
    =TRUE)
```

R code Exa 2.3 Echelon

```
#page - 80
#section - 2.3 LINEAR INDEPENDENCE AND BASES
#example 3

#included package - matlib

#for echelon function
library(matlib)

#matrix A
A <- matrix(c(1,-2,1,2,-5,0,-1,3,1,2,0,10), 4, 3,
```

```
    byrow=TRUE)

#column matrix k
K <- c(0,0,0,0)

#reduced row-echelon form
echelon(A, K, reduced=FALSE, verbose=TRUE, fractions
    =TRUE)
```

R code Exa 2.4 Echelon

```
#page - 81
#section - 2.3 LINEAR INDEPENDENCE AND BASES
#example 4

#included package - matlib

#for echelon function
library(matlib)

#matrix A
A <- matrix(c(-1,-2,2,0,1,3,1,1,1), 3, 3, byrow=TRUE
    )

#column matrix k
K <- c(0,0,0)

#reduced row-echelon form
echelon(A, K, reduced=FALSE, verbose=TRUE, fractions
    =TRUE)
```

R code Exa 2.5 Echelon

```
#page - 82
#section - 2.3 LINEAR INDEPENDENCE AND BASES
#example 5

#included package - matlib

#for echelon function
library(matlib)

#matrix A
A <- matrix(c(1,2,3,3,2,1,0,2,4,1,1,1), 4, 3, byrow=
    TRUE)

#column matrix k
K <- c(1,1,1,1)

#reduced row-echelon form
B = echelon(A, K, reduced=TRUE, verbose=TRUE,
    fractions=TRUE)
```

R code Exa 2.6 Echelon

```
#page - 82
#section - 2.3 LINEAR INDEPENDENCE AND BASES
#example 6

#included package - matlib

#for echelon function
library(matlib)

#matrix A
A <- matrix(c(3,-1,-1,-2,2,-2,-1,-1,3), 3, 3, byrow=
    TRUE)

```

```
#column matrix k
K <- c(-3,2,1)

#reduced row-echelon form
B = echelon(A, K, reduced=TRUE, verbose=TRUE,
    fractions=TRUE)
```

R code Exa 2.7 Rank of a matrix

```
#page - 88
#section - 2.4 THE RANK OF A MATRIX
#example 7

#included package - matlib
#included package - matrixcalc

#for echelon function
library(matlib)

#for rank calculation
library(matrixcalc)

#matrix A
A <- matrix(c(2,1,1,1,-2,1,0,5,-1), 3, 3, byrow=TRUE
    )

#column matrix k
K <- c(2,-3,8)

#reduced row-echelon form
B = echelon(A, K, reduced=TRUE, verbose=TRUE,
    fractions=TRUE)

#rank of A
matrix.rank(A)
```

Chapter 3

Determinants

R code Exa 3.1 Determinant

```
#page - 103
#section - 3.1 DEFINITION OF THE DETERMINANT
#example 1

#matrix A
A <- matrix(c(1,2,3,2,4,1,1,3,0), 3, 3, byrow=TRUE)
A

#determinant of A
det(A)
```

R code Exa 3.2 Minor and cofactor

```
#page - 106
#section - 3.2 THE LAPLACE EXPANSION
#example 2

#matrix A
```

```r
A <- matrix(c(1,2,3,4,5,6,7,8,9), 3, 3, byrow=TRUE)
A

# Minor and cofactor functions
minor <- function(A, i, j) A[-i,-j]
cofactor <- function(A, i, j) (-1)^(i+j) * det(minor(A,i,j))

#calculating Minor and cofactor

minor(A, 1, 2)
cofactor(A, 1, 2)

minor(A, 3, 3)
cofactor(A, 3, 3)
```

R code Exa 3.3 Determinant

```r
#page - 108
#section - 3.2 THE LAPLACE EXPANSION
#example 3

#matrix A
A <- matrix(c(2,4,6,1,2,3,1,4,9), 3, 3, byrow=TRUE)
A

#determinant of A
det(A)
```

R code Exa 3.4 Determinant

```r
#page - 108
#section - 3.2 THE LAPLACE EXPANSION
```

```
#example 4

#matrix A
A <- matrix(c(1,-1,2,3,2,2,0,2,4,1,-1,-1,1,2,3,0),
    4, 4, byrow=TRUE)
A

#determinant of A
det(A)
```

R code Exa 3.5 Adjoints

```
#page - 111
#section - 3.3 ADJOINTS AND INVERSES
#example 5

#matrix A
A <- matrix(c(2,-1,1,4,2,4,6,3,9), 3, 3)

# Minor and cofactor functions
minor <- function(A, i, j) det(A[-i,-j])
cofactor <- function(A, i, j) (-1)^(i+j) * minor(A,i
    ,j)

#Adjoint functions
adjoint <- function(x) {
  n <- nrow(x)
  B <- matrix(NA, n, n)
  for( i in 1:n )
    for( j in 1:n )
      B[j,i] <- cofactor(x, i, j)
  B
}
adjoint(A)
```

R code Exa 3.6 Laplace expansion

```
#page - 114
#section - 3.2 THE LAPLACE EXPANSION
#example 6

#column matrices
c1 <- matrix(c(1,1,2), 3, 1, byrow=TRUE)
c2 <- matrix(c(2,-1,3), 3, 1, byrow=TRUE)
c3 <- matrix(c(1,1,-1), 3, 1, byrow=TRUE)
c4 <- matrix(c(4,5,1), 3, 1, byrow=TRUE)

A <- matrix(c(c1,c2,c3), 3, 3)
B <- matrix(c(c4,c2,c3), 3, 3)
C <- matrix(c(c1,c4,c3), 3, 3)
D <- matrix(c(c1,c2,c4), 3, 3)
A
B
C
D
#solution of r,s and t
r = det(B)/det(A)
r

s=det(C)/det(A)
s

t=det(D)/det(A)
t
```

R code Exa 3.7 Rank of a matrix

```
#page - 115
#section - 3.4 DETERMINANTS AND RANK
#example 7

#included package - matrixcalc

#for rank calculation
library(matrixcalc)

#matrix A
A <- matrix(c(1,-1,1,1,1,2,-1,-1,2,-2,1,-1,0,-3,-1,-1), 4, 4, byrow=TRUE)

#matrix N
N <- matrix(c(0,0,0,0,1,0,0,0,0,1,0,0,0,0,1,0), 4, 4, byrow=TRUE)

#rank of A
matrix.rank(A)

#rank of N
matrix.rank(N)
```

Chapter 4

Linear Transformations

R code Exa 4.1 w coordinate matrices

```
#page - 123
#section - 4.2 MATRIX REPRESENTATION
#example 1

#w-coordinate matrices
Crdwb1 <- matrix(c(-2,5,6,-4), 4, 1, byrow=TRUE)
Crdwb2 <- matrix(c(-5,12,11,-5), 4, 1, byrow=TRUE)
Crdwb3 <- matrix(c(3,-5,-5,3), 4, 1, byrow=TRUE)

#matrix T
Tmat <- matrix(c(Crdwb1,Crdwb2,Crdwb3), 4, 3)
Tmat

Crdwv <- matrix(c(3,-5,7), 3, 1)
Crdwv

CrdwTv = Tmat %*% Crdwv
CrdwTv
```

R code Exa 4.2 Change of basis and similarity

```r
#page - 131
#section - 4.4 CHANGE OF BASIS AND SIMILARITY
#example - 2

#included package - matlib

#for inverse functions
library(matlib)

#matrix M
M <- matrix(c(2,4,6,-1,2,3,1,4,9), 3, 3, byrow=TRUE)
M

#matrix P
P <- matrix(c(1,0,1,1,1,1,1,1,0), 3, 3, byrow=TRUE)
P

#inverse of matrix P, PI
(PI  <- inv(P))

Mtx = PI %*% M %*% P
Mtx
```

R code Exa 4.3 Change of basis and similarity

```r
#page - 132
#section - 4.4 CHANGE OF BASIS AND SIMILARITY
#example - 3

#included Package - matlib

#for inverse functions
library(matlib)
```

```

#matrix M
M <- matrix(c(17,12,18,-16,-9,-24,-5,-4,-4), 3, 3, byrow=TRUE)
M

#matrix Q
Q <- matrix(c(10,-3,-3,-8,3,2,-3,1,1), 3, 3, byrow=TRUE)
Q

#inverse of matrix Q, QI
(QI  <- inv(Q))

Mtx = QI %*% M %*% Q
Mtx
```

R code Exa 4.4 Eigenvalues and eigenvectors

```
#page - 135
#section - 4.5 CHARACTERISTIC VECTORS AND CHARACTERISTIC VALUES
#example - 4

#matrix A
A <- matrix(c(7,-8,-8,9,-16,-18,-5,11,13), 3, 3, byrow=TRUE)
A

#matrix X
X <- matrix(c(1,3,-2), 3, 1, byrow=TRUE)
X

A %*% X

```

```r
#eigenvalues and eigenvectors
eigen(A)
```

R code Exa 4.5 Characteristic polynomial

```r
#page - 137
#section - 4.5 CHARACTERISTIC VECTORS AND
    CHARACTERISTIC VALUES
#example - 5

#included package - pracma

#for charpoly function
library(pracma)

#matrix A
A <- matrix(c(2,1,1,2,3,2,1,1,2), 3, 3, byrow=TRUE)
A

#characteristic polynomial
charpoly(A, info = FALSE)
```

R code Exa 4.6 Characteristic polynomial

```r
#page - 156
#section - 4.8 SCHUR'S THEOREM AND NORMAL MATRICES
#example - 6

#included package - pracma

#for charpoly function
library(pracma)

```

```
#matrix A
B <- matrix(c(4,-1,1,-1,4,-1,1,-1,4), 3, 3, byrow=
    TRUE)
B

#characteristic polynomial
charpoly(B, info = FALSE)
```

Chapter 5

Similarity Part I

R code Exa 5.1 Minimum polynomial

```
#page - 167
#section - 5.1 THE CAYLEY-HAMILTON THEOREM
#example - 1

#included package - polynom

#for minimum polynomial function
library(polynom)

#matrix B
B <- matrix(c
    (17,-8,-12,14,46,-22,-35,41,-2,1,4,-4,4,-2,-2,3),
     4, 4, byrow = TRUE)
B

eigVals <- eigen(B)$values
multEig <- table(eigVals)
k <- length(multEig)
minPoly <- 1
for(i in 1:k){
  poly.i <- polynomial(c(-as.numeric(names(multEig)[
```

```
    i]), 1))
  minPoly <- (minPoly*poly.i)
}

#minimum polynomial
minPoly
```

R code Exa 5.2 Characteristic polynomial

```
#page - 168
#section - 5.1 THE CAYLEY-HAMILTON THEOREM
#example - 2

#included packages - pracma, polynom

#for charpoly function
library(pracma)

#for minimum polynomial function
library(polynom)

#matrix S
S <- matrix(c(4,-1,1,-1,4,-1,1,-1,4), 3, 3, byrow =
    TRUE)
S

eigVals <- eigen(S)$values
multEig <- table(eigVals)
k <- length(multEig)
minPoly <- 1
for(i in 1:k){
  poly.i <- polynomial(c(-as.numeric(names(multEig)[
    i]), 1))
  minPoly <- (minPoly*poly.i)
}
```

```
#characteristic polynomial
charpoly(S, info = FALSE)

#minimum polynomial
minPoly
```

Chapter 6

Polynomials and Polynomial Matrices

R code Exa 6.1 Left and right functional value

```
#page - 208
#section - 6.4 MATRICES WITH POLYNOMIAL ELEMENTS
#example 1

#included package - exmp

#for power of a matrix
library(expm)

#constant matrices
c1 <- matrix(c(0,0,1,1), 2, 2, byrow=TRUE)
c2 <- matrix(c(0,1,0,1), 2, 2, byrow=TRUE)
c3 <- matrix(c(1,1,0,1), 2, 2, byrow=TRUE)

#matrix A
A <- matrix(c(1,0,0,2), 2, 2)
A

#square of A
```

```
A2 = A %^% 2
A2

#right functional value
Pr = c1%*%A2 + c2%*%A + c3
Pr

#left functional value
Pi = A2%*%c1 + A%*%c2 + c3
Pi

#function to compare two matrices
matequal <- function(x, y)
  is.matrix(x) && is.matrix(y) && dim(x) == dim(y) && all(x == y)

matequal(Pr,Pi)
```

Chapter 8

Matrix Analysis

R code Exa 8.1 Primary functions

```
#page - 242
#section - 8.2 PRIMARY FUNCTIONS
#example 1


#first matrix A
A <- matrix(c(2,1,1,2,3,2,1,1,2), 2, 3, byrow=TRUE)
A

fn <- function(z)
  sin((pi/2)*z)

fn(A)
```

Chapter 9

Numerical Methods

R code Exa 9.1 Echelon

```
#page - 253
#section - 9.2 EXACT METHODS FOR SOLVING AX = K
#example 1

#included package - matlib

#for echelon function
library(matlib)

#matrix A
A <- matrix(c(2,1,-1,2,1,3,2,-3,-1,2,1,-1,2,-3,-1,4)
    , 4, 4, byrow=TRUE)

#column matrix k
K <- c(1,0,1,0)

#reduced row-echelon form
echelon(A, K, reduced=TRUE, verbose=TRUE, fractions=
   FALSE)
```

R code Exa 9.2 Echelon

```
#page - 254
#section - 9.2 EXACT METHODS FOR SOLVING AX = K
#example 2

#included package - matlib

#for echelon function
library(matlib)

#matrix A
A <- matrix(c
    (7,9,2,-1,4,-5,-7,2,3,-2,-5,-1,1,6,-4,-3), 4, 4,
    byrow=TRUE)

#column matrix k
K <- c(1,2,4,3)

#reduced row-echelon form
echelon(A, K, reduced=TRUE, verbose=TRUE, fractions=
    TRUE)
```

R code Exa 9.3 Inverse of matrix

```
#page - 259
#section - 9.2 EXACT METHODS FOR SOLVING AX = K
#example 3

#included package - matlib

#for inverse functions
```

```
library(matlib)

#matrix A
A <- matrix(c(1,0,2,-1,4,5,3,-1,0,1,8,5,-3,1,4,6,2,0,0,1,0,1,4,2,0), 5, 5, byrow=TRUE)
A

#determinant of matrix A
det(A)

#inverse of matrix A, AI
(AI  <- inv(A))
```

R code Exa 9.4 Eigenvalues and eigenvectors

```
#page - 265
#section - 9.4 CHARACTERISTIC VALUES AND VECTORS
#example - 4

#matrix A
A <- matrix(c(1,1,3,1,-2,1,3,1,3), 3, 3, byrow=TRUE)
A

#matrix X
X <- matrix(c(1,1,1), 3, 1, byrow=TRUE)
X

#eigenvalues and eigenvectors
eigen(A)
```