### R Textbook Companion for Probability and Statistics for Engineering and the Sciences by Jay L Devore<sup>1</sup>

Created by Angela Paul B.Tech.

Computer Science and Engineering Government Engineering College, Thrissur Cross-Checked by R TBC Team

May 29, 2020

<sup>1</sup>Funded by a grant from the National Mission on Education through ICT - http://spoken-tutorial.org/NMEICT-Intro. This Textbook Companion and R codes written in it can be downloaded from the "Textbook Companion Project" section at the website - https://r.fossee.in.

### **Book Description**

Title: Probability and Statistics for Engineering and the Sciences

Author: Jay L Devore

Publisher: Richard Stratton, Boston, USA

Edition: 8

**Year:** 2012

**ISBN:** 9780538733526

R numbering policy used in this document and the relation to the above book.

Exa Example (Solved example)

Eqn Equation (Particular equation of the above book)

For example, Exa 3.51 means solved example 3.51 of this book. Sec 2.3 means an R code whose theory is explained in Section 2.3 of the book.

### Contents

Lis	st of R Codes	4
1	Overview and descriptive statistics	5
2	Probability	14
3	Discrete random variables and probability distributions	23
4	Continuous random variables and probability distributions	42
5	Joint probability distributions and random samples	62
6	Point estimation	77
7	Statistical intervals based on a single sample	84
8	Test of hypotheses based on a single sample	95
9	Inferences based on two samples	111
10	The analysis of variance	124
11	Multifactor analysis of variance	133
<b>12</b>	Simple linear regression and correlation	146
<b>13</b>	Nonlinear and multiple regression	162
14	Goodness of fit tests and categorical data analysis	176

<b>15</b>	Distribution free procedures	188
<b>16</b>	Quality control methods	197

### List of R Codes

Exa 1.1	Branches of statistics
Exa 1.5	Collecting data
Exa 1.8	Dotplots
Exa 1.9	Histograms
Exa 1.10	Histograms
Exa 1.11	Histograms
Exa 1.13	Qualitative data
Exa 1.14	Mean
Exa 1.15	Median
Exa 1.17	Measures of variability for sample data
Exa 1.18	Measures of variability
Exa 1.19	Boxplots
Exa 1.20	Box plots that show outliers
Exa 2.8	Relations from set theory
Exa 2.11	Properties of probability
Exa 2.13	Properties of probability
Exa 2.14	Properties of probability
Exa 2.19	Product rule
Exa 2.21	Permutations and combinations
Exa 2.22	Permutations and combinations
Exa 2.23	Permutations and combinations
Exa 2.25	Conditional probability
Exa 2.26	Conditional probability
Exa 2.30	Bayes Theorem
Exa 2.36	Independence of more than two events
Exa 3.7	Probability distributions for discrete random variables 23
Exa 3.8	Probability distributions for discrete random variables 24
Exa 3.9	Probability distributions for discrete random variables 25

Exa 3	3.10	Probability distributions for discrete random variables
Exa 3	3.13	Cumulative distribution function
Exa 3	3.15	Cumulative distribution function of discrete random vari-
		able
Exa 3	3.16	Expected value of X
Exa 3	3.17	Expected value of X
Exa 3	3.23	Expected value of a function
Exa 3	3.24	The variance of X
Exa 3	3.25	The variance of X
Exa 3	3.26	Rules of variance
Exa 3	3.29	Binomial experiment
Exa 3	3.30	Binomial experiment
Exa 3	3.31	Binomial random variable and distribution
Exa 3	3.32.1	Cumulative distribution function
Exa 3	3.32.2	Using binomial tables
Exa 3	3.32.3	Using binomial tables
Exa 3	3.32.4	Using binomial tables
Exa 3	3.33	Using binomial tables
Exa 3	3.34	Mean and variance of X
Exa 3	3.35	Hypergeometric distribution
Exa 3	3.36	Hypergeometric distribution
Exa 3	3.37	Mean and variance of hypergeometric distribution
Exa 3	3.38	Negative binomial distribution
Exa 3	3.39	Poisson probability distribution
Exa 3	3.40	Poisson distribution as a limit
Exa 3	3.41	Mean and variance in Poisson distribution
Exa 3	3.42	Poisson process
Exa	4.4	Probability density function
Exa 4	4.5	Uniform distribution
Exa	4.7	Computing probabilities
Exa 4	4.10	Expected values
Exa 4	4.11	Expected value of a function
Exa 4	4.12	Variance and standard deviation
Exa 4	4.13.a	Standard normal distribution
		Percentiles of standard normal distribution

Exa 4.15	z critical values	49
Exa 4.16	Nonstandard normal distributions	50
Exa 4.17	Nonstandard normal distributions	51
Exa 4.18	Percentiles of arbitrary normal distribution	52
Exa 4.20	Approximating binomial distribution	53
Exa 4.21	Exponential distribution	53
Exa 4.22		54
Exa 4.23	Gamma distribution	54
Exa 4.24		55
Exa 4.25	Weibull distribution	56
Exa 4.26	Weibull distribution	57
Exa 4.27		57
Exa 4.28	Beta distribution	58
Exa 4.29		59
Exa 4.30	Probability plots	60
Exa 4.31		60
Exa 5.1		62
Exa 5.2	Marginal probability mass function	62
Exa 5.3	Joint probability density function	63
Exa 5.4		64
Exa 5.5	Marginal probability density function	65
Exa 5.8	Independent random variables	65
Exa 5.9	Multinomial distribution	66
Exa 5.12	Conditional distributions	66
Exa 5.14	Expected values	67
Exa 5.15		68
Exa 5.18	Correlation	69
Exa 5.19	Statistics and their distributions	70
Exa 5.24	Standard error of the mean	71
Exa 5.25	Normal population distribution	72
Exa 5.26		73
Exa 5.27		74
Exa 5.29	Linear combination	74
Exa 5.30	Difference between two random variables	75
Exa 5.31	Normal random variables	76
Exa 6.1	Some general concepts of point estimation	77
Exa 6.2		77
Exa 6.3		78

Exa 6.9	Standard error of an estimator
Exa 6.10	Standard error of an estimator 80
Exa 6.11	Standard error of an estimator 80
Exa 6.13	Method of moments
Exa 6.14	Method of moments
Exa 6.21	The invariance principle 82
Exa 7.2	Basic properties of confidence intervals 84
Exa 7.3	Other levels of confidence
Exa 7.4	Confidence level and precision and sample size 85
Exa 7.6	Large sample CI for mean
Exa 7.7	Large sample CI for mean 87
Exa 7.8	CI for population proportion 87
Exa 7.10	Confidence bounds
Exa 7.11	One sample t confidence interval
Exa 7.12	A prediction interval for a single future value 90
Exa 7.13	Prediction interval
Exa 7.14	Tolerance intervals
Exa 7.15	CI for variance and sd of normal population 93
Exa 8.1	Hypotheses and test procedures 95
Exa 8.2	Hypotheses and test procedures
Exa 8.3	Hypotheses and test procedures
Exa 8.4	Hypotheses and test procedures
Exa 8.5	Hypotheses and test procedures
Exa 8.6	Tests about a population mean 100
Exa 8.7	Tests about a population mean 100
Exa 8.8	Tests about a population mean
Exa 8.9	Tests about a population mean 102
Exa 8.11	Tests concerning a population proportion 103
Exa 8.12	Tests concerning a population proportion 104
Exa 8.13	Tests concerning a population proportion 105
Exa 8.14	P values
Exa 8.15	P values
Exa 8.17	P values for z tests
Exa 8.18	P values for t tests
Exa 8.19	Interpreting P values
Exa 9.1	Test procedures for normal population with known vari-
	ances
Exa 9.2	Using a comparison to identify causality
	8

Exa 9.3	Beta and the choice of sample size		
Exa 9.4	Large sample tests		
Exa 9.5	Confidence intervals for difference of means 114		
Exa 9.6	Two sample t test and confidence interval 115		
Exa 9.7	Two sample t test and confidence interval 116		
Exa 9.9	Paired t test		
Exa 9.10	Paired t confidence interval		
Exa 9.11	Inferences concerning difference between population pro-		
	portions		
Exa 9.12	Type II error probabilities and sample sizes 120		
Exa 9.13	A large sample confidence interval		
Exa 9.14	F test for equality of variances		
Exa 10.1	Single factor ANOVA		
Exa 10.2	F distribution and the F test		
Exa 10.3	F distribution and the F test		
Exa 10.4	Sums of squares		
Exa 10.6	Multiple comparisons in ANOVA 129		
Exa 10.9	Unequal sample sizes		
Exa 10.10	Unequal sample sizes		
Exa 10.11	A random effects model		
Exa 11.1	Two factor ANOVA		
Exa 11.3	Fixed effects model		
Exa 11.5	Randomized block experiments		
Exa 11.6	Randomized block experiments 135		
Exa 11.7	Two factor ANOVA with Kij greater than 1 136		
Exa 11.9	Models with mixed and random effects 137		
Exa 11.10	Three factor ANOVA		
Exa 11.11	Latin square designs		
Exa 11.12	Factorial experiments with p factors at two levels 140		
Exa 11.13	Factorial experiments with p factors at two levels 142		
Exa 12.1	The simple linear regression model 146		
Exa 12.2	The simple linear regression model		
Exa 12.3	A linear probabilistic model		
Exa 12.4	Estimating model parameters		
Exa 12.5	Estimating model parameters		
Exa 12.6	Estimating model parameters		
Exa 12.7	Estimating model parameters		
Exa 12.8	Estimating model parameters		
9			

Exa 12.9	Coefficient of determination
Exa 12.11	Confidence interval for beta1
Exa 12.12	Hypothesis testing procedures
Exa 12.13	Inferences concerning mean
Exa 12.14	Prediction interval for a future value of Y
Exa 12.15	Correlation
Exa 12.16	Inferences about the population correlation coefficient
Exa 12.17	Inferences about the population correlation coefficient
Exa 12.18	Other inferences concerning rho
Exa 12.19	Other inferences concerning rho
Exa 13.2	Diagnostic plots
Exa 13.4	Regression with transformed variables
Exa 13.7	Polynomial regression
Exa 13.8	Estimating parameters
Exa 13.9	Statistical intervals and test procedures
Exa 13.10	Centering x values
Exa 13.12	Estimating parameters
Exa 13.13	Estimating parameters
Exa 13.14	Model utility test
Exa 13.15	Inferences in multiple regression
Exa 13.16	Inferences in multiple regression
Exa 13.18	Transformations
Exa 14.1	Goodness of fit tests when category probabilities are
	completely specified
Exa 14.2	P values for chi squared tests
Exa 14.3	Chi square when underlying distribution is continuous
Exa 14.4	Chi square when underlying distribution is continuous
Exa 14.5	Chi square when parameters are estimated
Exa 14.6	Chi square when parameters are estimated
Exa 14.9	Goodness of fit for discrete distributions
Exa 14.10	Goodness of fit for continuous distributions
Exa 14.12	A special test for normality
Exa 14.13	Testing for homogeneity
Exa 14.14	Testing for independence
Exa 15.1	The Wilcoxon signed rank test
Exa 15.2	Paired observations
Exa 15.3	Large sample approximation
Exa 15.4	The Wilcoxon rank sum test

Exa 15.5	A normal approximation for W	190
Exa 15.6	The Wilcoxon signed rank interval	191
Exa 15.8	The Wilcoxon rank sum interval	192
Exa 15.9	The Kruskal Wallis test	193
Exa 15.10	Friedman test for a randomized block experiment	194
Exa 16.1	X bar chart based on known parameter values	197
Exa 16.2	X bar charts based on estimated parameters	198
Exa 16.3	X bar charts based on estimated parameters	198
Exa 16.4	The S chart	199
Exa 16.5	R chart	200
Exa 16.6	The p chart	202
Exa 16.7	c chart for number of defectives	203
Exa 16.8	CUSUM procedures	204
Exa 16.9	Computational version	205
Exa 16.10	Designing a CUSUM procedure	206
Exa 16.11	Acceptance sampling	206
Exa 16.12	Designing a single sample plan	207

### Chapter 1

## Overview and descriptive statistics

### R code Exa 1.1 Branches of statistics

R code Exa 1.5 Collecting data

```
1 #Ex1.5, Page 11
2
3 Adhesive_Type<-c(1,1,2,2)
4 Conductor_material \langle -c(1,2,1,2) \rangle
5 Observed_bond_strength<-c(82,75,84,78,77,87,80,90)
6 mat1 <-matrix (Observed_bond_strength, nrow=4, ncol=2)
7 Average <-c(sum(mat1[1,])/2, sum(mat1[2,])/2, sum(mat1
      [3,])/2, sum(mat1[4,])/2)
8 mat1 <-cbind (Adhesive_Type, Conductor_material, mat1,</pre>
      Average)
9 colnames(mat1)[3] <- "Observed bond strength"
10 print(mat1)
11
12 n<-length(Adhesive_Type)/2
13 plot(Conductor_material, Average, main="Average bond
      strengths", xlab="Conducting material", ylab="
      Average strength")
14 segments (Conductor_material[1], Average[1], Conductor_
      material[2], Average[2])
15 segments (Conductor_material[3], Average[3], Conductor_
      material [4], Average [4])
```

#### R code Exa 1.8 Dotplots

### R code Exa 1.9 Histograms

```
1 #Ex1.9, Page 17
3 Hits_per_game <- 0:27
4 No_of_games<-c
      (20,72,209,527,1048,1457,1988,2256,2403,2256,1967,1509,1230,834,56
5 sum1 <- sum (No_of_games)
6 Relative_frequency <-round (No_of_games/sum1, digits=4)
7 df1<-data.frame(Hits_per_game, No_of_games, Relative_
      frequency)
8 print(df1)
10 #Proportion of games with atmost 2 hits
11 f1<-0
12 for(i in Hits_per_game){
     if (i <= 2) {</pre>
13
       f1<-f1+df1[i+1,3]
14
15
     }
16 }
17 print(paste("Proportion of games with atmost two
      hits:",f1))
18
19 #Proportion of games with between 5 and 10 hits
20 f2<-0
21 for(j in Hits_per_game){
     if(j >= 5 \&\& j <= 10) {
23
       f2 < -f2 + df1[j+1,3]
     }
24
25 }
26 print(paste("Proportion of games with between 5 and
      10 hits:",f2))
27
```

### R code Exa 1.10 Histograms

```
1 #Ex1.10, Page 18
2 #Answers may vary since different class intervals
      have been taken
4 #Please install and load the following packages: 1)
      Histogram Tools 2) ash
5 library(HistogramTools)
6 library(ash)
8 data<-c
      (2.97, 4.00, 5.20, 5.56, 5.94, 5.98, 6.35, 6.62, 6.72, 6.78, 6.80, 6.85, 6.94
9
10 #Class intervals can also be created using
      classIntervals() from ClassInt library:
11 #classIntervals (data, style="pretty", intervalClosure
      ="left") which creates different intervals as
      compared to those of textbook
12
13 #To create class intervals
14 \text{ ci} < -seq(1,19,2)
15
16 freq<-table(cut(data,ci,right=FALSE))</pre>
17 rel_freq<-freq/length(data)
18 print(cbind(freq,rel_freq))
19
```

```
20 #To display histogram
21 PlotRelativeFrequency(hist(data,breaks = ci,right=
      FALSE, plot=FALSE), xlab="BTUIN", main="Histogram of
       energy consumption data", ylim=c(0,0.30))
   R code Exa 1.11 Histograms
1 #Ex1.11, Page 20
2
3 data<-c
      (11.5, 12.1, 9.9, 9.3, 7.8, 6.2, 6.6, 7.0, 13.4, 17.1, 9.3, 5.6, 5.7, 5.4, 5.2,
4 #Unequal class widths
5 \text{ ci} < -c(2,4,6,8,12,20,30)
7 #To find frequency, relative frequency and density
      of data manually
8 freq<-table(cut(data,ci,right=FALSE))</pre>
9 rel_freq<-freq/length(data)
10 density <-rel_freq/diff(ci)</pre>
11
12 print(cbind(freq,rel_freq,density))
13
14 #To create histogram of the bond strength data
15 hist(data,breaks=ci,freq=FALSE,right=FALSE,main="
      Histogram for bond strength data", xlab="Bond
      Strength")
```

### R code Exa 1.13 Qualitative data

```
1 #Ex1.13, Page 23
2 #Answers may vary slightly due to rounding off of values
```

#### R code Exa 1.14 Mean

```
", (s-45)/(length(x)-1))
```

### R code Exa 1.15 Median

```
1 #Ex1.15, Page 30
2 #Answers may vary slightly due to rounding off of
      values
3
4 data<-c
      (62.3,62.8,63.6,65.2,65.7,66.4,67.4,68.4,68.8,70.8,75.7,79.0)
6 dotplot(data, xlab="Duration")
8 n<-length(data)</pre>
9 n1<-data[n/2]
10 n2 < -data[(n/2) + 1]
11 med < -(n1+n2)/2
12 print(paste("Median:",med))
13
14 max <-max(data)</pre>
15 #If maximum value is omitted
16 data1 <-data[1:length(data)-1]
17 mean1 <-mean (data1)
18 print(paste("Mean when maximum value, ", max, "is
      omitted:",mean1))
```

### R code Exa 1.17 Measures of variability for sample data

```
1 #Ex1.17, Page 36
2
3 Car<-1:11
```

```
4 xi<-c
     (27.3,27.9,32.9,35.2,44.9,39.9,30.0,29.7,28.5,32.0,37.6)
5
6 xibar <-mean(xi)
7 \text{ s} < -\text{sd}(xi)
9 print(paste("The size of the representative
     deviation from the mean, ", round (xibar, digits = 2),"
     is roughly", round(s, digits=2), "mpg"))
  R code Exa 1.18 Measures of variability
1 #Ex1.18, Page 38
2 #Answers may vary slightly due to rounding off of
     values
4 data<-c
     (154, 142, 137, 133, 122, 126, 135, 135, 108, 120, 127, 134, 122)
6 print(paste("s:",sd(data)))
  R code Exa 1.19 Boxplots
1 #Ex1.19, Page 40
2
3 obs<-c
     (40,52,55,60,70,75,85,85,90,90,92,94,94,95,98,100,115,125,125)
5 #To find five-number summary
6 print(summary(fivenum(obs)))
```

### R code Exa 1.20 Box plots that show outliers

### Chapter 2

### **Probability**

R code Exa 2.8 Relations from set theory

```
1 #Ex2.8, Page 53
2
3 A<-c(0,1,2,3,4)
4 B<-c(3,4,5,6)
5 C<-c(1,3,5)
6 U<-union(A,B)
7
8 cat("A'=",setdiff(U,A),"\n")
9 cat("A U B =",union(A,B),"\n")
10 cat("A U C =",union(A,C),"\n")
11 cat("A n B =",intersect(A,B),"\n")
12 cat("A n C =",intersect(A,C),"\n")
13 cat("(A n C)'=",setdiff(U,intersect(A,C)))</pre>
```

R code Exa 2.11 Properties of probability

```
_{2}^{1} #Ex2.11, Page 56
```

```
3 #S={U,D}
4
5 p<-seq(0,1,by=0.25)
6
7 for(i in p){
8   pu<-i
9   pd<-1-i
10   cat("P(U):",pu,"\n")
11   cat("P(D):",pd,"\n\n")
12 }
13 cat("p value can represent any fixed value between 0 and 1")</pre>
```

### R code Exa 2.13 Properties of probability

```
#Ex2.13, Page 59
#Answers may vary slightly due to rounding off of
    values

#For A to occur, at least one of the individual
    components must fail

#To find P(A')=SSSSS
p<-90/100
pa_dash<-p^5
cat("P(A'):",pa_dash,"\n")

pa<-1-pa_dash
cat("P(A):",1-pa_dash,"\n")

cat("Roughly",round(pa*100,digits=0),"% of the
    systems will fail")</pre>
```

R code Exa 2.14 Properties of probability

```
1 #Ex2.14, Page 60
3 #P(A)=>probability that a household gets internet
      service
4 pa<-60/100
6 #P(B)=>probability that a household gets TV service
7 pb<-80/100
9 #P(A n B)=>probability that a household gets both
      services
10 \text{ pab} < -50/100
11
12 #To find probability that a household gets at least
      one of the services
13 p1<-pa+pb-pab
14 cat("P(at least one service)=P(A \cup B) = ",p1," \setminus n")
15
16 pa_b<-p1-pa
17 pb_a<-p1-pb
18 #To find probability that a household gets exactly
      one service
19 p2<-pa_b+pb_a
20 cat("P(exactly one service)=",p2)
```

### R code Exa 2.19 Product rule

```
1 #Ex2.19, Page 67
2
3 #No of appliance dealers
4 n1<-5
5
6 #No of plumbing contractors
7 n2<-12</pre>
```

#### R code Exa 2.21 Permutations and combinations

```
#Ex2.21, Page 68

#No of teaching assistants

n<-10
#No of questions in the first exam

k<-4

#To find number of permutations

p<-factorial(n)/factorial(n-k)

cat("The professor could give",p," different four—question exam papers without using the same graders")</pre>
```

#### R code Exa 2.22 Permutations and combinations

```
1 #Ex2.22, Page 69
2 #Answers may vary slightly due to rounding off of values
3
4 #Total no of songs
5 n<-100
6
7 #No of songs by the Beatles
8 m<-10</pre>
```

```
9
10 #Probability that the 1st Beatles song heard is the
       5th song played
11 t<-5
12 p1 \leftarrow (factorial(n-m)/factorial(n-m-(t-1))*m/(
       factorial(n)/factorial(n-t)))
13 cat("P(1 st B is 5th song played):",p1,"\n")
14
15 #Probability that the 1st Beatles song heard is the
       1st, 2nd, 3rd, 4th or 5th song played
16 p2<-0
17 for(i in 1:5){
18
      p2 \leftarrow p2 + (choose(n-i,m-1))/(choose(n,m))
19 }
20 cat("P(1 \text{ st } B \text{ is } 1 \text{ st }, 2 \text{ nd }, 3 \text{ rd }, 4 \text{ th } \text{ or } 5 \text{ th } \text{ song } \text{ played}):
       ",p2,"\n")
```

### R code Exa 2.23 Permutations and combinations

```
1 #Ex2.23, Page 70
2 #Answers may vary slightly due to rounding off of values
3
4 #Total no of printers
5 n<-25
6 1<-10
7 i<-15
8
9 #Probability that 6 random printers selected will have 3 laser printers
10 r1<-6
11 r<-3
12 ND3<-(choose(1,r)*choose(i,r))/choose(n,r1)
13 cat("P(3 laser printers are selected among the 6 selected):",ND3,"\n")</pre>
```

### R code Exa 2.25 Conditional probability

```
1 #Ex2.25, Page 74
2
3 #Probability that memory card is purchased
4 pa<-0.6
5
6 #Probability that battery is purchased
7 \text{ pb} < -0.4
8
9 #Probability that both memory card and battery is
      purchased
10 pab<-0.3
11
12 #Probability that an optional card was purchased
      given that individuals purchased extra battery
13 pa_b<-pab/pb
14 \operatorname{cat}(\operatorname{P}(A|B)=\operatorname{pa_b},\operatorname{n})
15 cat ("Of all individuals who purchased an extra
      battery, ",pa_b*100, "% purchased an optional
      memory card\n")
16
```

### R code Exa 2.26 Conditional probability

```
1 #Ex2.26, Page 75
2 #Answers may vary slightly due to rounding off of
      values
4 #Package to be installed: VennDiagram
5 library(VennDiagram)
7 \#A => Arts, B => Books, C => Cinema
8 pa < -0.14
9 pb<-0.23
10 \text{ pc} < -0.37
11 pab<-0.08
12 pbc<-0.13
13 \text{ pac} < -0.09
14 pabc <- 0.05
15
16 #To illustrate the Venn diagram using the given
      probabilities
17 draw.triple.venn(area1=pa, area2=pb, area3=pc, n12=pab,
      n23=pbc, n13=pac, n123=pabc, category=c("Arts","
      Books", "Cinema"))
18
19 cat ("With the help of Venn diagram:\n")
20 \#P(A|B)
```

```
pa_b<-pab/pb
print(paste("P(A|B)=",pa_b))

#P(A|B U C)
pa_bUc<-(pab+pac-pabc)/(pb+pc-pbc)
print(paste("P(A|B U C)=",pa_bUc))

#P(A|reads at least once)=P(A|A U B U C)
pa_aUbUc<-pa/(pa+pb+pc-pab-pac-pbc+pabc)
print(paste("P(A|reads at least once)=",pa_aUbUc))

#P(A U B|C)
paUb_c<-(pac+pbc-pabc)/pc
print(paste("P(A U B|C)=",paUb_c))</pre>
```

### R code Exa 2.30 Bayes Theorem

```
#Ex2.30, Page 78

#Ex2.30, Page 78

## % of messages which come into account 1

## pa1<-70/100

## % of messages which come into account 2

## pa2<-20/100

## % of messages which come into account 3

## ## % of messages which are spam in account 1

## % of messages which are spam in account 1

## % of messages which are spam in account 2

## % of messages which are spam in account 2

## % of messages which are spam in account 3

## % of messages which are spam in account 3

## % of messages which are spam in account 3</pre>
```

```
19 pb_a3<-5/100
20
21 #To find the probability that a randomly selected
    message is spam
22 #Using law of total probability
23 pb<-pa1*pb_a1+pa2*pb_a2+pa3*pb_a3
24 print(paste("P(Randomly selected message is spam):",
    pb,"and thus,",pb*100,"% messages will be spam"))</pre>
```

### R code Exa 2.36 Independence of more than two events

```
#Ex2.36, Page 85
#Answers may vary slightly due to rounding off of values

pai<-rep(0.9, times=6)

#To find probability that system lifetime exceeds to prob1<-pai[1]*pai[2]*pai[3]+pai[4]*pai[5]*pai[6]-(prod(pai))

print(paste("P(system lifetime exceeds to):",prob1))

#To find probability that system lifetime is at least to prob2<-(1-(1-pai[1])*(1-pai[2]))^3

print(paste("P(system lifetime is at least to):", prob2))</pre>
```

### Chapter 3

# Discrete random variables and probability distributions

R code Exa 3.7 Probability distributions for discrete random variables

```
1 #Example 3.7, Page 97
2 #Storing all probabilities in an array
3 prob<-c(0.05,0.1,0.15,0.25,0.20,0.15,0.10)
4 P \leftarrow array(c(prob), dim = c(1,7,1))
6 \#P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)
7 p1<-P[1,1,1]+P[1,2,1]+P[1,3,1]
8 print(paste("P(X<=2) =",p1))</pre>
10 \#P(X>=3)=1-P(X<=2)
11 p2 < -1 - (P[1,1,1] + P[1,2,1] + P[1,3,1])
12 print(paste("P(X>=3) =",p2))
13
14 \#P(2 \le X \le 5) = P(X = 2, 3, 4 \text{ or } 5)
15 p3<-P[1,3,1]+P[1,4,1]+P[1,5,1]+P[1,6,1]
16 print(paste("P(2 \le X \le 5) = ",p3))
17
18 \#P(2 < X < 5) = P(X = 3, 4)
19 p4<-P[1,4,1]+P[1,5,1]
```

R code Exa 3.8 Probability distributions for discrete random variables

```
1 #Example 3.8, Page 97
2 #The answers may slightly vary due to rounding off
      of values
3 p1 < -c(1,0,2,2,3,0,4,1,5,2,6,0)
4 row.names <-c("Lot", "Number of defectives")
5 column.names<-c("","","","","","")
6 matrix.names <-c ("Number of defectives in each lot")
7 p<-array(c(p1),dim=c(2,6,1),dimnames=list(row.names,</pre>
      column.names,matrix.names))
8 print(p)
9 \#p(0)=P(X=0)
10 c<-0
11 for(i in 1:6) {
     d<-p[2,i,1]
12
     if (d==0) {
13
14
       c < -c + 1
15
     }
16 }
17 d < -c/6
18 print(paste("p(0)=",d))
19
20 \#p(1)=P(X=1)
21 c<-0
22 for(i in 1:6){
23
       d<-p[2,i,1]
       if (d==1) {
24
25
         c < -c + 1
26
     }
27 }
28 d < -c/6
29 print(paste("p(1)=",d))
```

```
30
31 #p(2)=P(X=2)
32 c<-0
33 for(i in 1:6){
34     d<-p[2,i,1]
35     if(d==2){
36         c<-c+1
37     }
38 }
39 d<-c/6
40 print(paste("p(2)=",d))
```

### R code Exa 3.9 Probability distributions for discrete random variables

```
1 #Ex3.9, Page 98
2 #X=0 if customer purchases laptop computer
3 #X=1 if customer purchases desktop computer
4 #p(0)
5 p0<-80/100
6 print(paste("Probability that next customer
      purchases laptop model:",p0))
7 p1<-20/100
8 print(paste("Probability that next customer
      purchases desktop model:",p1))
9 p1 < -c(0.8, 0.2, 0)
10 row.names < -c ("if x=0","if x=1","if (x!=0 or 1)")
11 column.names<-c("")</pre>
12 matrix.names <-c("p(x)")
13 p <- array(c(p1), dim=c(3,1,1), dimnames=list(row.names,</pre>
      column.names,matrix.names))
14 print(p)
15 a<- 1
16 \text{ p} \leftarrow \text{dbinom}(a, \text{size} = 1, \text{prob} = 0.2)
17 plot(1, p, type = 'h', xlab = 'x', ylab = 'p(x)')
```

R code Exa 3.10 Probability distributions for discrete random variables

```
1 #Ex3.10, Page 98
2 \text{ \#Let Y=>O+ blood type} and N=>Other blood types
3 p1<-c('a', 'Y', 'b', 'Y', 'c', 'N', 'd', 'N', 'e', 'N')
4 row.names<-c("Blood donors", "O+ blood donors")
5 column.names<-c("","","","","")
6 matrix.names <-c ("Blood donation table")
7 p<-array(c(p1),dim=c(2,5,1),dimnames=list(row.names,</pre>
      column.names,matrix.names))
8 print(p)
9 \#p(1)=P(X=1)
10 c<-0
11 for(i in 1:5) {
12
     d<-p[2,i,1]</pre>
13
     if (d=="Y") {
       c < -c + 1
14
15
     }
16 }
17 d1 < -c/5
18 print(paste("p(1)=",d1))
19
20 \#p(2) = P(X=2)
21 g<-0
22 e<-0
23 for(i in 1:5) {
     h < -p[2,i,1]
24
     if(h=="Y"){
25
       g < -g + 1
26
27
     }
28
     else{
29
       e<-e+1
30
     }
31 }
```

```
32 h < -g/5
33 \text{ f} < -e/4
34 d2<-h*f
35 print(paste("p(2) = ", d2))
36
37 \#p(3)=P(X=3)
38 d3 < -(e/5)*((e-1)/4)*(c/3)
39 print(paste("p(3)=",d3))
40
41 \#p(4) = P(X=4)
42 d4 < -(e/5)*((e-1)/4)*((e-2)/3)
43 print(paste("p(4)=",d4))
44
45 p2<-c(1,d1,2,d2,3,d3,4,d4)
46 row.names < -c("y", "p(y)")
47 column.names<-c("",","","","")
48 matrix.names<-c("PMF")
49 q <- array (c(p2), dim = c(2,4,1), dimnames = list(row.names,
      column.names,matrix.names))
50 print(q)
51
52 \text{ py} < -1:4
53 \text{ y} < -c (0.4, 0.3, 0.2, 0.1)
54 plot(py,y,type="h",main="Line graph for pmf",xlab="y
      ", ylab = "p(y)")
```

#### R code Exa 3.13 Cumulative distribution function

```
1 #Ex3.13, Page 101
2
3 y<-c(1,2,4,8,16)
4 p<-c(0.05,0.1,0.35,0.4,0.1)
5
6 #F(1)=P(Y<=1)=p(1)
7 print(paste("F(1)=",cumsum(p[1])))</pre>
```

```
8
9 #F(2)=P(Y<=2)=p(2)
10 print(paste("F(2)=",max(cumsum(p[1:2]))))
11
12 #F(4)=P(Y<=4)=p(4)
13 print(paste("F(4)=",max(cumsum(p[1:3]))))
14
15 #F(8)=P(Y<=8)=p(8)
16 print(paste("F(8)=",max(cumsum(p[1:4]))))
17
18 #F(16)=P(X<=16)=p(16)
19 print(paste("F(16)=",max(cumsum(p[1:5]))))</pre>
```

R code Exa 3.15 Cumulative distribution function of discrete random variable

```
1 #Ex3.15, Page 104
2 a \leftarrow c(0, 0.58, 1, 0.72, 2, 0.76, 3, 0.81, 4, 0.88, 5, 0.94)
3~\mbox{row.names}\mbox{<-c}\mbox{("X","F(X)")}
4 column.names<-c("",",",",","","","")
5 matrix.names <-c ("Number of days of sick leave taken
      by a randomly selected employee")
6 b <- array(c(a), dim=c(2,6,1), dimnames=list(row.names,
      column.names,matrix.names))
7 print(b)
9 \#P(2 \le X \le 5) = P(X = 2, 3, 4, 5)
10 p \leftarrow b[2,6,1] - b[2,2,1]
11 print(paste("P(2 \le X \le 5) = ", p))
12
13 \#P(X=3)
14 q<-b[2,4,1]-b[2,3,1]
15 print(paste("P(X=3)=",q))
```

# R code Exa 3.16 Expected value of X

```
1 #Ex3.16, Page 107
2
3 x<-1:7
4 w<-c(0.01,0.03,0.13,0.25,0.39,0.17,0.02)
5
6 #Mean, m=sum of(xp(x))
7 m<-weighted.mean(x,w)
8 print(paste("Population mean =",m))</pre>
```

# R code Exa 3.17 Expected value of X

# R code Exa 3.23 Expected value of a function

```
1 #Ex3.23, Page 110
2
3 comp<-c(0,0.1,1,0.2,2,0.3,3,0.4)
```

```
4 row.names<-c("X","p(X)")
5 column.names<-c("","","","")
6 matrix.names<-c("X->Number of computers sold")
7 p<-array(c(comp),dim=c(2,4,1),dimnames=list(row.names,column.names,matrix.names))
8 print(p)
9
10 x<-0:3
11 w<-c(0.1,0.2,0.3,0.4)
12 #h(x)<-profit associated with selling x units<-revenue-cost<-800x-900
13 print(paste("Expected profit:$",weighted.mean(800*x -900,w)))</pre>
```

# R code Exa 3.24 The variance of X

```
1 #Ex3.24, Page 111
2 #Answers may slightly vary due to rounding off of
      values
3
4 \text{ w} < -c (0.3, 0.25, 0.15, 0.05, 0.1, 0.15)
5 \text{ x} < -c(1,2,3,4,5,6)
7 #Mean, m sum of (xp(x))
8 m <-weighted.mean(x,w)</pre>
9 print(paste("Mean =",m))
10
11 #Variance, v= sum of (x-m)p(x)
12 v \leftarrow weighted.mean((x-m)^2, w)
13 print(paste("Variance =",v))
14
15 #Standard deviation, sd= square root(variance)
16 print(paste("Standard deviation=", sqrt(v)))
```

# R code Exa 3.25 The variance of X

```
#Ex3.25, Page 112

#Alternative formula for variance, v= E(X^2)-(m^2)

x<-1:6

w<-c(0.3,0.25,0.15,0.05,0.1,0.15)

#E(X^2)= sum of (X^2)p(X)

e<-weighted.mean(x^2,w)

print(paste("E(X^2)=",e))

#Variance
mean<-weighted.mean(x,w)

print(paste("Variance through alternative formula =",e-(mean^2)))</pre>
```

# R code Exa 3.26 Rules of variance

```
1 #Ex3.26, Page 113
2
3 x<-0:3
4 w<-c(0.1,0.2,0.3,0.4)
5
6 #Mean, m = sum of (xp(x))
7 m<-weighted.mean(x,w)
8 print(paste("Mean, E(X)=",m))
9
10 #To find E(X^2)
11 e<-weighted.mean(x^2,w)
12 print(paste("E(X^2)=",e))
13</pre>
```

```
#To find V(X)
v<-e-(m^2)
print(paste("V(X)=",v))

#Profit function, h(X)=800X-900
#Variance
print(paste("Variance=",(800^2)*v))

#Standard deviation
print(paste("Standard deviation =",sqrt((800^2)*v)))</pre>
```

#### R code Exa 3.29 Binomial experiment

```
1 #Ex3.29, Page 115
2 #Number of licensed restaurants
3 n < -50
4 #Number of restaurants having at least one serious
      health code violation
5 f <-15
6 #Number of restaurants having no serious health code
       violations
7 s<-35
8 #P(s on first trial)
9 p1 < -s/n;
10 print(paste("P(s on first trial)=",p1))
11
12 \#P(s \text{ on second trial})=P(ss)+P(fs)
13 p2 < -(s/n)*((s-1)/(n-1))+(f/n)*(s/(n-1))
14 print(paste("P(s on second trial)=",p2))
15 print(paste("Similarly, P(s on ith trial) = 0.7
      for i = 3, 4, 5")
16
17 #P(s on fifth trial|ssss)
18 p3 < -(s-4)/(n-4)
19 print(paste("P(s on fifth trial | ssss)=",p3))
```

```
20
21 #P(s on fifth trial | ffff)
22 p4<-s/(n-4)
23 print(paste("P(s on fifth trial | ffff)=",p4))
```

# R code Exa 3.30 Binomial experiment

```
#Ex3.30, Page 116
#Number of licensed drivers
n<-500000
#Number of licensed drivers who are insured
s<-400000
#P(s on second trial after s on first trial)
p1<-(s-1)/(n-1)
print(paste("P(s on 2|s on 1)=",p1))

#P(s on tenth trial after s on ninth trial)
p2<-(s-9)/(n-9)
print(paste("P(s on 10|s on 9)=",p2))

print(paste("The experiment is binomial with n=10 and p=0.8"))</pre>
```

# R code Exa 3.31 Binomial random variable and distribution

```
1 #Ex3.31, Page 118
2 #Answers may slightly vary due to rounding off of values
3
4 #n=6, p=0.5
5
6 #P(X=3)
7 print(paste("P(X=3)=",dbinom(3,size=6,prob=0.5)))
```

```
8
9 #Probability that at least 3 three prefer S is
10 print(paste("P(X>=3)=",pbinom(3,size=6,prob=0.5)))
11
12 #Probability that at most one prefers S is
13 print(paste("P(X<=1)=",pbinom(1,size=6,prob=0.5)))</pre>
```

#### R code Exa 3.32.1 Cumulative distribution function

```
#Example 3.32.1, Page 118
#Binomial distribution: n=15, p=2
#The answers may slightly vary due to rounding off
    of values

#Probability that at most 8 copies fail the test
#P(X<=8)
a=pbinom(8,size=15,prob=0.2)
print(paste("The probability that at most 8 copies
    fail the test is", a))</pre>
```

# R code Exa 3.32.2 Using binomial tables

```
#Example 3.32.2, Page 118
#Binomial distribution: n=15, p=0.2
#The answers may slightly vary due to rounding off
    of values

#Probability that exactly 8 copies fail
#P(X=8)=P(X<=8)-P(X<=7)
    a<-0.999

b=pbinom(7, size=15, prob=0.2)
print(paste("The probability that exactly 8 copies
    fail the test", a-b))</pre>
```

# R code Exa 3.32.3 Using binomial tables

```
#Example 3.32.3, Page 119
#Binomial distribution: n=15, p=0.2
#The answers may slightly vary due to rounding off
    of values

#Probability that at least 8 copies fail
#P(X>=8)=1-P(X<=7)
b<-0.996
c=1-b
print(paste("The probability that at least 8 copies fail", c))</pre>
```

# R code Exa 3.32.4 Using binomial tables

```
#Example 3.32.4, Page 119
#Binomial distribution: n=15, p=0.2
#The answers may slightly vary due to rounding off of values

#Probability that between 4 to 7 copies fail
#P(4<=X<=7)=P(X=4)+P(X=5)+P(X=6)+P(X=7)
d=pbinom(7,15,0.2)-pbinom(3,15,0.2)
print(paste("The probability that between 4 to 7 copies fail is", d))</pre>
```

R code Exa 3.33 Using binomial tables

```
1 #Ex3.33, Page 119
2 #Answers may vary slightly due to rounding off of values
3
4 #n=20
5 #P(X>=5) when p=0.10
6 p1<-1-pbinom(4,size=20,prob=0.1)
7 print(paste("P(X>=5 when p=.1)=",p1))
8
9 #P(X<=4) when p=0.2
10 p2<-pbinom(4,size=20,prob=0.2)
11 print(paste("P(X<=4 when p=.2)=",p2))</pre>
```

# R code Exa 3.34 Mean and variance of X

```
1 #Ex3.34, Page 120
2 n < - 10
3 p < -0.75
5 \# Mean, E(x) = np
6 e < -n*p
7 print(paste("Mean=",e))
9 \#Variance, V(X)=npq
10 \ v < -n * p * (1-p)
11 print(paste("Variance=",v))
12
13 \#Standard deviation=sqrt(V(X))
14 s <- sqrt (v)
15 print(paste("Standard deviation=",s))
16
17 \#P(X=7 \text{ or } 8) = P(X=7) + P(X=8)
18 prob1=dbinom(0:7, size=10, prob=0.75)
19 prob2=dbinom(0:8, size=10, prob=0.75)
20 \quad c \leq max(prob1) + max(prob2)
```

```
21 print(paste("P(X=7 \text{ or } 8)=",c))
```

# R code Exa 3.35 Hypergeometric distribution

```
1 #Ex3.35, Page 123
2 #Answers may slightly vary due to rounding off of values
3
4 N<-20
5 n<-5
6 M<-12
7
8 #P(X=2)=h(2,5,12,20)
9 x<-2
10 h<-(choose(M,x)*choose(N-M,n-x))/choose(N,n)
11 print(paste("h(2;5,12,20)=",h))</pre>
```

# R code Exa 3.36 Hypergeometric distribution

```
1 #Ex3.36, Page 124
2 #Answers may slightly vary due to rounding off of values
3
4 N<-25
5 n<-10
6 M<-5
7
8 #P(X=2)=h(2,10,5,25)
9 x<-2
10 h1<-(choose(M,X)*choose(N-M,n-x))/choose(N,n)
11 print(paste("P(X=2)=",h1))
12
13 #P(X<=2)=P(X=0)+P(X=1)+P(X=2)</pre>
```

```
14  s<-0
15  for(i in 0:2){
16    h2<-(choose(M,i)*choose(N-M,n-i))/choose(N,n)
17    s<-s+h2
18  }
19  print(paste("P(X<=2)=",s))</pre>
```

R code Exa 3.37 Mean and variance of hypergeometric distribution

```
1 #Ex3.37, Page 125
2
3 n<-10
4 M < -5
5 N<-25
6 p < -M/N
7
8 \#E(X)=np
9 e < -n*p
10 print(paste("Mean=",e))
11
12 \#V(X) = npq
13 \ v < -n * p * (1 - p)
14 print(paste("Variance without replacement=",v))
15
16 #Estimate of N when M=5, n=10 and x=16
17 M<-100
18 n < - 40
19 x < - 16
20 ne < -(M*n)/x
21 print(paste("Estimate is", ne))
```

R code Exa 3.38 Negative binomial distribution

```
1 #Ex3.38, Page 126
2 #Answers may slightly vary due to rounding off of values
3
4 #P(X=10)=nb(10;5,0.2)
5 prob1=dnbinom(0:10,size=5,prob=0.2)
6 print(paste("P(X=10)=",max(prob1)))
7
8 #P(X<=10)
9 s<-pnbinom(0:10,size=5,prob=0.2)
10 print(paste("P(X<=10)=",max(s)))</pre>
```

#### R code Exa 3.39 Poisson probability distribution

```
1 #Ex3.39, Page 129
2 #Answers may slightly vary due to rounding off of values
3
4 u<-4.5
5 #Poisson distribution
6 #P(X=5)
7 p1<-dpois(5,lambda=u)
8 print(paste("P(X=5)=",p1))
9
10 #P(X<=5)
11 p2<-ppois(5,lambda=u)
12 print(paste("P(X<=5)=",p2))</pre>
```

# R code Exa 3.40 Poisson distribution as a limit

```
1 #Ex3.40, Page 129
2 #Answers may vary slightly due to rounding off of
    values
```

```
3
4 n<-400
5 p<-0.005
6
7 #P(X=1):b(x;n,p)->p(x;u)
8 u<-n*p
9 p1<-dpois(1,lambda=u)
10 print(paste("P(X=1)=",p1))
11
12 #P(X<=3)
13 p2<-ppois(3,lambda=u)
14 print(paste("P(X<=3)=",p2))</pre>
```

# R code Exa 3.41 Mean and variance in Poisson distribution

```
1 #Ex3.41, Page 130
2 #Answer may slightly vary due to rounding off of value
3
4 u<-4.5
5 #Standard deviation
6 sd<-sqrt(u)
7 print(paste("Standard deviation=",sd))</pre>
```

# R code Exa 3.42 Poisson process

```
1 #Ex3.42, Page 131
2 #Answer may slightly vary due to rounding off of value
3
4 alpha<-6
5 t<-0.5
6 1<-alpha*t</pre>
```

```
7
8 #P(X>=1)=1-P(X=0)
9 p<-1-dpois(0,lambda=1)
10 print(paste("P(X>=1)=",p))
```

# Chapter 4

# Continuous random variables and probability distributions

R code Exa 4.4 Probability density function

```
1 #Example 4.4, Page 139*
2 \#n=10, M=5, N=25
4 \# f(x) > = 0
5 #Area under density curve
6 \text{ a} < -(1/360) * 360
8 \#P(90 \le X \le 180)
9 f1 < -function(x) \{(x^0)/360\}
10 p1<-integrate(f1,lower=90,upper=180)
11 print(paste("Probability that angle is between 90
      and 180 is",p1\$value))
12
13 \#P(0 \le X \le 90) + P(270 \le X \le 360)
14 f2 < -function(x) \{(x^0)/360\}
15 p2<-integrate(f2,lower=0,upper=90)
16 f3 \leftarrow function(x) \{(x^0)/360\}
17 p3<-integrate(f3,lower=270,upper=360)
18 p4 < -p2 $ value + p3 $ value
```

```
19 print(paste("P(angle of occurrence is within 90 degrees of reference line)=",p4))
```

#### R code Exa 4.5 Uniform distribution

```
1 #Example 4.5, Page 141
2 #The answers may slightly vary due to rounding off
      of values
3
4 #Taking upper limit as 1000 instead of infinity
5 integrand \leftarrow function(x) {.15*exp(-.15*(x-.5))}
6 \#f(x) >= 0
7 #To show that integral from negative infinity to
      infinity is 1
8 c<-integrate(integrand,lower=0.5,upper=Inf)</pre>
9 #To access the list of values of integrate by name
10 print(paste("Value of integral is",c$value))
11
12 #To find P(X<5)
13 d<-integrate(integrand, lower=0.5, upper=5)
14 print(paste("Thus, probability that headway time is
      atmost 5 sec:",d$value))
```

# R code Exa 4.7 Computing probabilities

```
1 #Ex4.7, Page 145*
2
3 #if x<0, F(X)=0
4 #if 0<=x<=2, F(X)=(x/8)+(3/16)(x^2)
5 #if x>2, F(X)=1
6
7 load<-function(x){
8 f<-(x/8)+(3/16)*(x^2)</pre>
```

```
9    return(f)
10 }
11
12 #P(1<=X<=1.5)=P(X=1.5)-P(X=1)
13 p1<-load(1.5)-load(1)
14 print(paste("P(1<=X<=1.5)=",p1))
15
16 #P(X>1)=1-P(X<=1)
17 p2<-1-load(1)
18 print(paste("P(load exceeds 1)=",p2))</pre>
```

# R code Exa 4.10 Expected values

# R code Exa 4.11 Expected value of a function

```
#integrate(f,lower=0,upper=1)

11

12 f1<-function(x) {(1-x)*1}
13 p1<-integrate(f1,lower=0,upper=1/2)

14

15 f2<-function(x) {x*1}
16 p2<-integrate(f2,lower=0.5,upper=1)
17

18 p3<-p1$value+p2$value
19 print(paste("E[h(X)]=",p3))</pre>
```

# R code Exa 4.12 Variance and standard deviation

```
1 #Ex4.12, Page 150
2
3 #E(X)
4 e<-3/8
5
6 #E(X^2)
7 e2<-function(x) {(3/2)*(x^2)*(1-(x^2))}
8 p1<-integrate(e2,lower=0,upper=1)
9 print(paste("E(X^2)=",p1$value))
10
11 #V(X)
12 v<-(p1$value)-(e^2)
13 print(paste("V(X)=",v))
14
15 #Standard deviation
16 sd<-sqrt(v)
17 print(paste("Standard deviation =",sd))</pre>
```

R code Exa 4.13.a Standard normal distribution

```
1 #Example 4.13.a, Page 154
2 #The answers may slightly vary due to rounding off
      of values
3
4 \#P(Z <= 1.25)
5 \text{ a} \leftarrow pnorm(1.25,0,1)
6 print(paste("The standard normal probability is",a))
8 #To plot the standard normal distribution bell curve
9 curve(dnorm, xlim = c(-3, 3), ylim = c(0, 0.4), xlab
      = "z", ylab="f(z)",lwd=2, col="black")
10 z = 1.25
11 pnorm(z)
12 x = c(-3, seq(-3, z, by=.001), z)
13 #plot(x)
14 y = c(0, dnorm(seq(-3, z, by=.001)), 0)
15 #plot(y)
16 polygon(x, y, col="red")
```

#### R code Exa 4.13.b Standard normal distribution

```
#Example 4.13.b, Page 154
2 #The answers may slightly vary due to rounding off
    of values
3
4 #P(Z>1.25)
5 #Variable to store result(1-P(Z<=1.25))
6 a=1-pnorm(1.25,0,1)
7 print(paste("The standard normal probability is", a)
    )
8
9
10 #To plot the standard normal distribution bell curve
11 curve(dnorm, xlim = c(-3, 3), ylim = c(0, 0.4), xlab
    = "z", ylab="f(z)",lwd=2, col="black")</pre>
```

```
12 z = 1.25

13 pnorm(z)

14 x = c(z, seq(z, 3, by=.001), 3)

15 #plot(x)

16 y = c(0, dnorm(seq(z, 3, by=.001)), 0)

17 #plot(y)

18 polygon(x, y, col="red")
```

#### R code Exa 4.13.c Standard normal distribution

```
1 #Example 4.13.C, Page 154
2 #The answers may slightly vary due to rounding off
      of values
3
4 \#P(Z \le -1.25) = P(Z \ge 1.25)
5 #Variable to store result (1-P(Z \le 1.25))
6 \quad a=1-pnorm(1.25,0,1)
7 print(paste("The standard normal probability is", a)
8
9 #To plot the standard normal distribution bell curve
10 x = seq(-3,3, length = 200)
11 y = dnorm(x)
12 plot(x,y,type="l", lwd=2, col="black")
13 x = seq(-3, -1.25, length = 200)
14 y = dnorm(x)
15 polygon(c(-3,x,-1.25),c(0,y,0),col="red")
17 #To plot the standard normal distribution bell curve
18 curve(dnorm, xlim = c(-3, 3), ylim = c(0, 0.4), xlab
       = "z", ylab="f(z)", lwd=2, col="black")
19 z = -1.25
20 \quad pnorm(z)
21 x = c(-3, seq(-3, z, by=.001), z)
22 #plot(x)
```

```
23 y = c(0, dnorm(seq(-3, z, by=.001)), 0)
24 #plot(y)
25 polygon(x, y, col="red")
```

#### R code Exa 4.13.d Standard normal distribution

```
1 #Example 4.13.d, Page 154
2 #The answers may slightly vary due to rounding off
      of values
3
4 \#P(-0.38 \le Z \le 1.25)
5 #Variable to store result (1-P(Z \le 1.25))
6 a \leftarrow pnorm(1.25, 0, 1) - pnorm(-0.38, 0, 1)
7 print(paste("The standard normal probability is", a)
8
9 #To plot the standard normal distribution bell curve
10 curve(dnorm, xlim = c(-3, 3), ylim = c(0, 0.4), xlab
       = "z", ylab=" f(z)", lwd=2, col=" black")
11 	 z1 = 1.25
12 	 z2 = -0.38
13 \text{ pnorm}(z)
14 x = c(z2, seq(z2, z1, by=.001), z1)
15 #plot(x)
16 y = c(0, dnorm(seq(z2, z1, by=.001)), 0)
17 #plot(y)
18 polygon(x, y, col="red")
```

# R code Exa 4.14 Percentiles of standard normal distribution

```
1 #Ex4.14, Page 155
2 #Answers may slightly vary due to rounding off of
    values
```

```
3
4 print(paste("99th percentile:",qnorm(0.99)))
5 print(paste("1st percentile:",qnorm(0.01)))
7 #Blue=> 99th percentile
8 #Red=> 1st percentile
9 plot.new()
10 title(main="Blue: 99th percentile
11 Red: 1st percentile")
12 curve(dnorm, xlim = c(-3, 3), ylim = c(0, 0.5), xlab
      = "z", ylab="f(z)")
13 \text{ prob1} = 0.99
14 x1=c(-3, seq(-3, qnorm(prob1), by=.001), qnorm(
     prob1))
15 y1 = c(0, dnorm(seq(-3, qnorm(prob1), by=.001)), 0)
16 polygon(x1, y1, col="blue")
17
18 par(new=TRUE)
19
20 \text{ prob2} = 0.01
21 	ext{ x2= c(-3, seq(-3, qnorm(prob2), by=.0001), qnorm(}
     prob2))
22 	 y2 = c(0, dnorm(seq(-3, qnorm(prob2), by=.0001)), 0)
23 polygon(x2, y2, col="red")
```

#### R code Exa 4.15 z critical values

```
1 #Ex4.15, Page 156
2 #Answers may slightly vary due to rounding off of values
3
4 #z(.05)
5 x<-0.05
6 z<-100*(1-x)</pre>
```

```
8 plot.new()
9 title (main="Red: z(0.05)
10 Blue: -z(0.05)")
11
12 #z (.05)
13 print(paste(z,"th percentile:",qnorm(z/100)))
14 curve(dnorm, xlim = c(-3, 3), ylim = c(0, 0.5), xlab
      = "z", ylab="f(z)")
15 \text{ prob} = z/100
16 \times = c(-3, seq(-3, qnorm(prob), by=.001), qnorm(prob)
17 y = c(0, dnorm(seq(-3, qnorm(prob), by=.001)), 0)
18 polygon(x, y, col="red")
19
20 #-z (.05)
21 print(paste("-z(.05)",-qnorm(z/100)))
22 \times = c(-3, seq(-3, -qnorm(prob), by=.001), -qnorm(
     prob))
23 y = c(0, dnorm(seq(-3, -qnorm(prob), by=.001)), 0)
24 polygon(x, y, col="blue")
```

# R code Exa 4.16 Nonstandard normal distributions

```
1 #Ex4.16, Page 158
2 #Answers may slightly vary due to rounding off of values
3
4 #P(1.00<=X<=1.75)
5 #Mean=>1.25, Standard deviation=>0.46
6
7 title(main="P(1.00<=X<=1.75)")
8 a<-pnorm(1.00, mean=1.25, sd=0.46)
9 b<-pnorm(1.75, mean=1.25, sd=0.46)
10 c<-b-a
11 print(paste("P(1.00<=X<=1.75)=",c))</pre>
```

```
12
13 #To plot the standard normal distribution bell curve
14 curve(dnorm, xlim = c(-3, 3), ylim = c(0, 0.4), xlab
       = "z", ylab="f(z)",lwd=2, col="black")
15 z < -1.75
16 # Cumulative density function of the standard normal
       distribution
17 \text{ pnorm}(z)
18 x < -c(1.00, seq(1.00, z, by=.001), z)
19 #plot(x)
20 y < -c(0, dnorm(seq(1.00, z, by=.001)), 0)
21 #plot(y)
22 polygon(x, y, col="red")
23
24 \#P(X>2)
25 d < 1-pnorm(2, mean = 1.25, sd = 0.46)
26 print(paste("P(X>2)=",d))
```

#### R code Exa 4.17 Nonstandard normal distributions

```
1 #Ex4.17, Page 158
2 #Answers may slightly vary due to rounding off of values
3
4 #P(-1.00<=Z<=1.00)
5 a<-pnorm(1.00,0,1)-pnorm(-1.00,0,1)
6 print(paste("P(X is within 1 standard deviation of its mean):",a))
7
8 #P(-2.00<=Z<=2.00)
9 b<-pnorm(2.00,0,1)-pnorm(-2.00,0,1)
10 print(paste("P(X is within 2 standard deviation of its mean):",b))
11
12 #P(-3.00<=Z<=3.00)</pre>
```

```
13 c<-pnorm(3.00,0,1)-pnorm(-3.00,0,1)
14 print(paste("P(X is within 3 standard deviation of
    its mean):",c))</pre>
```

# R code Exa 4.18 Percentiles of arbitrary normal distribution

```
1 #Ex4.18, Page 159
2 #Answers may slightly vary due to rounding off of
      values
3
4 \text{ m} < -64
5 \text{ sd} < -0.78
6 \#P(X>c) = 0.05
7 p<-0.005
8 \#P(X \le c) = 1 - P(X > c)
9 \text{ s} < -(1-p) * 100
10 print(paste("c is the",s,"th percentile of the
      normal distribution with mean=64 and sd=0.78"))
11 q<-qnorm(s/100)
12 print(paste("Therefore", s, "th percentile:", q))
13
14 c < -m + q * sd
15 print(paste("c=",c,"oz"))
16
17 #Plotting 99.5th percentile
18 plot.new()
19 title(main="99.5th percentile", sub="Shaded area
      =>0.995")
20 curve(dnorm, xlim = c(-3, 3), ylim = c(0, 0.5), xlab
       = "z", ylab="f(z)")
21 \text{ prob} = 0.995
22 x1 = c(-3, seq(-3, qnorm(prob), by=.001), qnorm(prob)
23 y1 = c(0, dnorm(seq(-3, qnorm(prob), by=.001)), 0)
24 polygon(x1, y1, col="blue")
```

# R code Exa 4.20 Approximating binomial distribution

```
1 #Ex4.20, Page 161
2 #Answers may vary slightly due to rounding off of
       values
3
4 n<-50
5 p<-25/100
6 q<-1-p
7 m < -12.5
8 \text{ sd} < -3.06
9 x < -n * p
10 y <-sqrt (n*p*q)</pre>
11 z < -n *q
12 if (x \ge 10 \&\& z \ge 10) {
     \#P(X <= 10)
13
      a \leftarrow pnorm(10+0.5, mean = x, sd = y)
14
      print(paste("P(X<=10)=",a))
15
16
17
     \#P(5 <= X <= 15)
      b \leftarrow pnorm(15+0.5, mean=x, sd=y) - pnorm(5-0.5, mean=x, sd
18
      print(paste("P(5<=X<=15)=",b))
19
20 }
```

# R code Exa 4.21 Exponential distribution

```
1 #Ex4.21, Page 166
2 #Answers may slightly vary due to rounding off of
     values
3
```

# R code Exa 4.22 Exponential distribution

```
1 #Ex4.22, Page 167
2 #Answer may slightly vary due to rounding off of value
3
4 alpha<-0.5
5
6 #P(X>2)=1-P(X<=2)
7 x<-2
8 p<-1-pexp(x,alpha)
9 print(paste("P(X>2)=",p))
10
11 t<-1/alpha
12 print(paste("The expected time between successive calls is",t,"days"))</pre>
```

#### R code Exa 4.23 Gamma distribution

```
1 #Ex4.23, Page 169
```

#### R code Exa 4.24 Gamma distribution

```
1 #Ex4.24, Page 169
2 #Answers may slightly vary due to rounding off of
      values
3
4 #alpha
5 a < -8
6 #beta
7 b<-15
8
9 #E(X)
10 e<-a*b
11 print(paste("E(X)=",e))
12
13 #V(X)
14 \text{ v} \leftarrow a*(b^2)
15 print(paste("V(X)=",v))
16
17 #SD
18 sd<-sqrt(v)
```

#### R code Exa 4.25 Weibull distribution

```
1 #Ex4.25, Page 173
2 #Answers may slightly vary due to rounding off of
      values
4 alpha<-2
5 beta <- 10
7 \#P(X \le 10)
8 p1<-pweibull(10,alpha,beta)
9 print(paste("P(X \le 10) = ", p1))
10
11 \#P(X \le 25)
12 p2<-pweibull(25,alpha,beta)
13 print (paste ("P(X <= 25) =", p2))
14
15 #To find c value which separates 5% of all engines
      having largest NOx emissions from remaining 95%
16 \text{ per} < -5/100
17 c < -sqrt((-1)*(10^2)*log(per))
18 print(paste(c," is the 95th percentile of the
      emission distribution"))
```

#### R code Exa 4.26 Weibull distribution

```
1 #Ex4.26, Page 173
2 #Answers may slightly vary due to rounding off of values
3
4 #alpha
5 a<-1.3
6 #beta
7 b<-0.8
8 #gamma
9 g<-4
10
11 #P(5<=X<=6)
12 p1<-pweibull(6-g,a,b)-pweibull(5-g,a,b)
13 print(paste("P(air void volume of a specimen is between 5% and 6%)=",p1))</pre>
```

# R code Exa 4.27 Lognormal distribution

```
1 #Ex4.27, Page 175
2 #Answers may slightly vary due to rounding off of values
3
4 m<-0.353
5 s<-0.754
6
7 e<-exp(m+(s^2)/2)
8 print(paste("E(X)=",e))
9
10 v<-exp(2*m+(s^2))*(exp(s^2)-1)</pre>
```

```
11  print(paste("V(X)=",v))
12
13  #P(1<=X<=2)
14  11<-log(1,base=exp(1))
15  12<-log(2,base=exp(1))
16  p<-pnorm(12,m,s)-pnorm(11,m,s)
17  print(paste("P(maximum pit depth is between 1 and 2 mm)=",p))
18
19  #To find c such that only 1% of all specimens have a maximum pit depth exceeding c
20  per<-1/100
21  c<-exp(1)^(qnorm(1-per)*s+m)
22  print(paste(c,"is the 99th percentile of the maximum pit depth distribution"))</pre>
```

#### R code Exa 4.28 Beta distribution

```
1 #Ex4.28, Page 177
2
3 #Optimistic time
4 A<-2
5
6 #Pessimistic time
7 B<-5
8
9 #alpha
10 a<-2
11 #beta
12 b<-3
13
14 #E(X)
15 e<-A+(B-A)*(a/(a+b))
16 print(paste("Mean=",e))
17</pre>
```

# R code Exa 4.29 Probability plots

```
1 #Ex4.29, Page 181
3 \text{ old.par} \leftarrow \text{par} (\text{mfrow} = c(1,2))
5 #To plot first sample
6 Percentage1<-c(5, 15, 25, 35, 45)
7 z_{percentile1} < -c(q_{norm}(5/100), q_{norm}(15/100), q_{norm}
      (25/100), qnorm(35/100), qnorm(45/100))
8 Sample_observation1<-c(-1.91, -1.25, -0.75, -0.53,
      0.2)
9 df1<-data.frame(Percentage1, z_percentile1, Sample_
      observation1)
10 print(df1)
11 plot(z_percentile1,Sample_observation1)
12
13 #To plot second sample
14 Percentage2 < -c (55, 65, 75, 85, 95)
15 z_percentile2<-c(qnorm(55/100), qnorm(65/100), qnorm
      (75/100), qnorm(85/100), qnorm(95/100))
16 Sample_observation2<-c(0.35, 0.72, 0.87, 1.40, 1.56)
17 df2<-data.frame(Percentage2, z_percentile2, Sample_
      observation2)
18 print(df2)
19 plot(z_percentile2, Sample_observation2)
20
21 par(old.par)
```

# R code Exa 4.30 Probability plots

```
1 #Ex4.30, Page 182
2
3 \text{ old.par} \leftarrow \text{par} (\text{mfrow} = c(1,1))
4 n<-20
5 z_{percentile} < -c(qnorm((1-0.5)/n), qnorm((2-0.5)/n),
      qnorm((3-0.5)/n), qnorm((4-0.5)/n), qnorm((5-0.5)
      /n),
                    qnorm((6-0.5)/n), qnorm((7-0.5)/n),
6
                       qnorm((8-0.5)/n), qnorm((9-0.5)/n
                       ), qnorm((10-0.5)/n),
                    qnorm((11-0.5)/n), qnorm((12-0.5)/n)
                       , qnorm((13-0.5)/n), qnorm
                       ((14-0.5)/n), qnorm((15-0.5)/n),
                    qnorm((16-0.5)/n), qnorm((17-0.5)/n)
8
                       , qnorm((18-0.5)/n), qnorm
                       ((19-0.5)/n), qnorm((20-0.5)/n))
9 Sample_observation <-c(24.46, 25.61, 26.25, 26.42,
      26.66, 27.15, 27.31, 27.54, 27.74, 27.94, 27.98,
      28.04, 28.28, 28.49, 28.50, 28.87, 29.11, 29.13,
      29.50, 30.88)
10 df <-data.frame(z_percentile, Sample_observation)
11 print(df)
12 plot(z_percentile, Sample_observation)
13
14 par(old.par)
```

#### R code Exa 4.31 Beyond normality

```
1 #Ex4.31, Page 186
```

```
3 \text{ old.par} \leftarrow \text{par} (\text{mfrow} = c(1,1))
5 Percentile <-c(-2.97, -1.82, -1.25, -0.84, -0.51,
      -0.23, 0.05, 0.33, 0.64, 1.10)
6 \text{ x} < -c (282, 501, 741, 851, 1072, 1122, 1202, 1585,
      1905, 2138)
7 \ln x < -c(\log(282, base = \exp(1)), \log(501, base = \exp(1)),
      log(741,base=exp(1)), log(851,base=exp(1)), log
      (1072, base = exp(1)),
             log(1122, base=exp(1)), log(1202, base=exp(1)
8
                 ), log(1585, base=exp(1)), log(1905, base=
                 exp(1)), log(2138,base=exp(1)))
9 df <-data.frame(Percentile,x,lnx)
10 print(df)
11 plot(Percentile,lnx)
12
13 par(old.par)
```

# Chapter 5

# Joint probability distributions and random samples

R code Exa 5.1 Joint probability mass function

R code Exa 5.2 Marginal probability mass function

```
1 #Ex5.2, Page 195
```

```
3 \text{ m} \leftarrow \text{matrix} (c(0.20, 0.05, 0.10, 0.15, 0.20, 0.30), \text{nrow} = 2,
      ncol=3)
4 rownames (m) <-c("100", "250")
5 colnames(m) <-c("0","100","200")
6 print(m)
7 cat("\n")
8
9 px_100<-apply(m,1,sum)[1]
10 px_250<-apply(m,1,sum)[2]
11 cat("px(100)=",px_100," \n")
12 cat("px(250)=",px_250,"\n\n")
13
14 cat ("Marginal pmf of X:\n")
15 cat("p(x)=",px_100,"when x=100,250\n")
16 cat("p(x)=0 otherwise\n\n")
17
18 py_0<-apply(m,2,sum)[1]
19 py_100<-apply(m,2,sum)[2]
20 py_200<-apply(m,2,sum)[3]
21 cat("py(0)=",py_0," n")
22 cat("py(100)=",py_100," n")
23 cat("py(200)=",py_200," \n\n")
24
25 cat ("Marginal pmf of Y:\n")
26 cat("p(y)=",py_0,"when y=0,100\n")
27 cat("p(y)=",py_200,"when y=200\n")
28 cat("p(y)=0 otherwise\n\n")
29
30 #To find P(Y>=100)
31 prob <-py_100+py_200
32 \text{ cat}("P(Y>=100)=",prob)
```

R code Exa 5.3 Joint probability density function

```
1 #Ex5.3, Page 196
```

```
2 #Answers may ary slightly due to rounding off of
      values
4 f \leftarrow function(x,y) \{(6/5)*(x+y^2)\}
6 #To check whether pdf is legitimate
7 check_val<-integrate(function(y) {sapply(y,function(</pre>
      y) {integrate(function(x) f(x,y),0,1)$value})
      },0,1)
8 if (check_valvalve==1) cat ("PDF is legitimate \ln n")
10 \#P(0 \le X \le 1/4, 0 \le Y \le 1/4)
11 low1<-0
12 high1<-1/4
13 low2<-0
14 high2<-1/4
15 integral <- integrate (function(y) {sapply(y, function(y)
      ) {integrate(function(x) f(x,y),low1,high1)$value
      })},low2,high2)
16 print (paste ("P(0 \le X \le 1/4, 0 \le Y \le 1/4) =", integral $value
      ))
```

#### R code Exa 5.4 Marginal probability density function

```
1 #Ex5.4, Page 197
2
3 f <-function(x,y) {(6/5)*(x+y^2)}
4
5 #Finding P(1/4<=Y<=3/4) using marginal pdf of Y
6 xlow<-0
7 xhigh<-1
8 ylow<-1/4
9 yhigh<-3/4
10 prob<-integrate(function(y) {sapply(y,function(y) { integrate(function(x) f(x,y),xlow,xhigh)$value})</pre>
```

### R code Exa 5.5 Marginal probability density function

```
1 #Ex5.5, Page 198
2
3 #Joint pdf
4 f \leftarrow function(x,y) \{24*x*y\}
6 val <- Vectorize (function(x) {sapply(x, function(z) {
      integrate(function(y) f(x,y),0,1-z)$value})})
7 check_val<-integrate(val,0,1)$value</pre>
8 if(check_val==1) print(paste("PDF is legitimate"))
10 #Probability that two types of nuts together make up
       50% of the can
11 #Taking A = \{(x, y): 0 < = x < =1, 0 < = y < =1, x+y < =0.5\}
12 xlow<-0
13 xhigh <- 0.5
14 integral <- Vectorize (function(x) {sapply(x, function(z)
      ) {integrate(function(y) f(x,y),0,0.5-z)$value})
      })
15 prob <- integrate (integral, xlow, xhigh)
16 print(paste("P((X,Y)EA) = ", prob$value))
```

### R code Exa 5.8 Independent random variables

```
1 #Ex5.8, Page 200
2 #Answers may vary slightly due to rounding off of values
3
4 #Expected lifetimes are 1000 and 1200 hours
```

```
5 lambda1<-(1/1000)
6 lambda2<-(1/1200)
7
8 #P(X1>=1500)
9 p1<-exp(-(lambda1)*1500)
10
11 #P(X2>=1500)
12 p2<-exp(-(lambda2)*1500)
13
14 #Joint PDF
15 #P(X1>=1500,X2>=1500)
16 p<-p1*p2
17 print(paste("P(both component lifetimes are at least 1500 hours)=",p))</pre>
```

### R code Exa 5.9 Multinomial distribution

```
#Ex5.9, Page 201
#Answer may slightly vary due to rounding off of
    value

#Respective number and probabilities
    n<-c(2,5,3)
    p<-c(0.25,0.5,0.25)

#Multinomial PMF
m<-dmultinom(n,prob=p)
print(paste("Multinomial PMF:",m))</pre>
```

### R code Exa 5.12 Conditional distributions

```
1 #Ex5.12, Page 203
```

```
#Answers may slightly vary due to rounding off of
    values

#Function when X=0.8: (1/34)(24+30(y^2))

#P(Y<=0.5) given X=0.8

f1<-function(y) {(1/34)*(24+30*(y^2))}

f<-integrate(f1,lower=0,upper=0.5)

print(paste("P(walk-up facility is busy atmost half
    the time)=",f$value))

#E(Y) given that X=0.8

f2<-function(y) {y*(1/34)*(24+30*(y^2))}

p<-integrate(f2,lower=0,upper=1)

print(paste("Expected proportion of time that walk-
    up facility is busy:",p$value))</pre>
```

### R code Exa 5.14 Expected values

```
#Ex5.14, Page 207

#Joint pdf
f<-function(x,y) {24*x*y}

almonds<-1
cashews<-1.5
peanuts<-0.5

#Total cost of the contents of a can
h<-function(x,y) {almonds*x+cashews*y+(1-x-y)*
peanuts}

#Expected total cost
integral<-Vectorize(function(x) {sapply(x, function(z)) {integrate(function(y)) h(x,y)*f(x,y),0,1-z)$</pre>
```

```
value})})
15 prob<-integrate(integral,0,1)
16 print(paste("Expected total cost:$",prob$value))</pre>
```

#### R code Exa 5.15 Covariance

```
1 #Ex5.15, Page 208
 3 \text{ x} < -c (100, 250)
4 \text{ y} < -c (0, 100, 200)
 6 m1 < -matrix(c(0.20, 0.05, 0.10, 0.15, 0.20, 0.30), nrow=2,
       ncol=3)
7 rownames (m1) <-c("100", "250")
8 colnames(m1) <-c("0","100","200")
9 print(m1)
10 cat("\n")
11
12 m2<-matrix(c(0.5,0.5),nrow=1,ncol=2)
13 rownames (m2) < -c("p(x)")
14 colnames(m2) < -c("100","250")
15 print(m2)
16 cat("\n")
17
18 m3<-matrix(c(0.25,0.25,0.5),nrow=1,ncol=3)
19 rownames (m3) <-c ("p(y)")
20 colnames(m3)<-c("0","100","200")
21 print(m3)
22 cat("\n")
23
24 \text{ x1} < -c (0.5, 0.5)
25 j <-1
26 \text{ mu} \text{\_} \text{x} \text{<-0}
27
28 \text{ for}(i \text{ in } x){
```

```
29
      if(j \le length(x1)) {
         mu_x<-mu_x+i*x1[j]
30
31
          j = j + 1
32
      }
33 }
34 \operatorname{cat}(\operatorname{"mu}_X=\operatorname{",mu}_x,\operatorname{"}\operatorname{\n})
35
36 \text{ y1} < -c (0.25, 0.25, 0.5)
37 k<-1
38 \text{ mu}_y < -0
39
40 for(i in y){
41
      if(k<=length(y1)){</pre>
         mu_y < -mu_y + i * y1[k]
42
         k=k+1
43
      }
44
45 }
46 cat("mu_Y=",mu_y,"\n")
47
48 p < -c (0.2, 0.1, 0.20, 0.05, 0.15, 0.30)
49 \text{ sum} < -0
50 k<-1
51 for(i in x){
52
      for(j in y){
          if(k<=length(p)){</pre>
53
            sum = sum + (i-175)*(j-125)*p[k]
54
55
            k=k+1
          }
56
57
      }
58 }
59 print(paste("Cov(X,Y):", sum))
```

### R code Exa 5.18 Correlation

```
1 #Ex5.18, Page 210
```

```
3 \times (-c(-4,4,2,-2))
4 y < -c (1, -1, 2, -2)
5 \text{ xy} < -x*y
6 \text{ w} < -c(1/4,1/4,1/4,1/4)
7 m < - weighted.mean(xy,w)
8 print(paste("E(XY)=",m))
10 \#Cov(XY)=E(XY)-ux.uy
11 ux<-0
12 uy<-0
13 \quad cov < -m - (ux * uy)
14 print(paste("Cov(XY)=",cov))
15
16 print(paste("Complete absence of any linear
       relationship"))
17
18 #To scatterplot
19 df <-data.frame(x,y)</pre>
20 plot(x,y)
```

### R code Exa 5.19 Statistics and their distributions

```
1 #Ex5.19, Page 212
2 #Answers may vary slightly due to rounding off of values
3
4 #alpha
5 a <-2
6
7 #beta
8 b <-5
9
10 #Mean
11 u <-b*gamma(1+(1/a))</pre>
```

```
12 print(paste("Mean, E(X)=",u))
13
14 #Variance
15 v \leftarrow (b^2) * \{gamma(1+(2/a)) - (gamma(1+(1/a)))^2\}
16 print(paste("Variance=",v))
17
18 #Standard deviation
19 s <- sqrt (v)
20 print(paste("Standard deviation=",s))
21
22 #Median
23 med < -b*(log(2,base=exp(1)))^(1/a)
24 print(paste("Median=",med))
25
26 q<-qweibull(0.99, shape=a, scale=b)
27 curve(dweibull(x,a,b),from=0,to=q,ylab='density')
28
29 \quad if(u>med){
     print(paste("Mean exceeds median because of the
30
        distribution 's positive skew"))
31 }
```

### R code Exa 5.24 Standard error of the mean

```
1 #Ex5.24, Page 223
2
3 n<-25
4 u<-28000
5 s<-5000
6
7 #E(Mean)
8 print(paste("E(Mean)=",u))
9
10 #E(T0)
11 t<-n*u</pre>
```

```
12 print(paste("E(T0)=",t))
13
14 #Standard deviation of mean
15 \text{ sm} < -s/sqrt(n)
16 print(paste("SD of mean:",sm))
17
18 #Standard deviation of T0
19 st <- s * sqrt(n)
20 print(paste("SD of T0:",st))
21
22 #When sample size increases to n=100
23 print(paste("When n=100:"))
24 n<-100
25 #E (Mean)
26 print(paste("E(Mean)=",u))
27
28 #E(T0)
29 t<-n*u
30 print(paste("E(T0)=",t))
31
32 #Standard deviation of mean
33 \text{ sm} < -s/sqrt(n)
34 print(paste("SD of mean:",sm))
35
36 #Standard deviation of T0
37 \text{ st} < -s * sqrt(n)
38 print(paste("SD of T0:",st))
```

# R code Exa 5.25 Normal population distribution

```
1 #Ex5.25, Page 224
2 #Answers may slightly vary due to rounding off of values
3
4 n<-5</pre>
```

```
5
6 #Mean(min)
7 m < -1.5
9 #Standard deviation (min)
10 sd<-0.35
11
12 \text{ ut} <-n*m
13 print(paste("uT0:",ut))
15 vt <-n*(sd^2)
16 print(paste("vT0:",vt))
17
18 st<-sqrt(vt)
19 print(paste("sT0:",st))
20
21 \#P(6 \le T0 \le 8)
22 p1<-pnorm(8,mean=ut,sd=st)-pnorm(6,mean=ut,sd=st)
23 print(paste("P(6 \le T0 \le 8) = ", p1))
24
25 #Probability that the sample average time is atmost
      2 min
26 u < -1.5
27 s<-sd/sqrt(n)
28
29 \#P(Mean <= 2.0)
30 p2 <-pnorm (2, mean=u, sd=s)
31 print(paste("P(Mean<=2)=",p2))
```

### R code Exa 5.26 Central limit theorem

```
1 #Ex5.26, Page 225
2 #Answer may slightly vary due to rounding off of value
3
```

```
4 n<-50
5 m<-4
6 sd<-1.5
7
8 ux<-m
9 sx<-sd/sqrt(n)
10
11 #P(3.5<=Mean<=3.8)
12 p<-pnorm(3.8,mean=ux,sd=sx)-pnorm(3.5,mean=ux,sd=sx)
13 print(paste("P(3.5<=Mean<=3.8)=",p))</pre>
```

### R code Exa 5.27 Central limit theorem

```
1 #Ex5.27, Page 226
2 #Answer may slightly vary due to rounding off of value
3
4 n<-100
5 m<-3.2
6 sd<-2.4
7
8 ux<-m
9 sx<-sd/sqrt(n)
10
11 #P(Mean>4)=1-P(Mean<=4)
12 p<-1-pnorm(4,mean=ux,sd=sx)
13 print(paste("P(Mean>4)=",p))
```

### R code Exa 5.29 Linear combination

```
1 #Ex5.29, Page 231
2 #Answers may vary slightly due to rounding off of
   values
```

```
3
4 a1 < -3.00
5 a2 < -3.20
6 a3 < -3.40
7
8 #Mean
9 u1<-1000
10 u2 < -500
11 u3<-300
12
13 #Standard deviation
14 s1<-100
15 s2<-80
16 s3<-50
17
18 \#Y = 3.0X1 + 3.20X2 + 3.40X3
19 EY <- a1 * u1 + a2 * u2 + a3 * u3
20 VY \leftarrow (a1^2) * (s1^2) + (a2^2) * (s2^2) + (a3^2) * (s3^2)
21 sY<-sqrt(VY)
22 print(paste("E(Y)=$",EY))
23 print(paste("V(Y)=$", VY))
24 print(paste("sY=$",sY))
```

### R code Exa 5.30 Difference between two random variables

```
1 #Ex5.30, Page 232
2 #Answers may slightly vary due to rounding off of values
3
4 u1<-22
5 u2<-26
6 s1<-1.2
7 s2<-1.5
8
9 #Mean</pre>
```

```
10  e <-u1-u2
11  print(paste("E(X1-X2)=",e))
12
13  #Variance
14  v <-(s1^2)+(s2^2)
15  print(paste("V(X1-X2)=",v))
16
17  #Standard deviation
18  sd <-sqrt(v)
19  print(paste("SD=",sd))</pre>
```

# R code Exa 5.31 Normal random variables

```
1 #Ex5.31, Page 232
2
3 #Y=3.0X1+3.2X2+3.4X3
4 u<-5620
5 s<-429.46
6
7 #P(Y>4500)
p<-1-pnorm(4500, mean=u, sd=s)
print(paste("P(revenue exceeds 4500)=",p))</pre>
```

# Chapter 6

6 #Estimator1

# Point estimation

R code Exa 6.1 Some general concepts of point estimation

```
1 #Ex6.1, Page 241
2
3 x<-15
4 n<-25
5 p<-x/n
6 print(paste("Most reasonable estimate is",p))</pre>
```

R code Exa 6.2 Some general concepts of point estimation

```
7  x1<-mean(volt)
8  print(paste("Estimator(1):",x1))
9
10  #Estimator2
11  x2<-median(volt)
12  print(paste("Estimator(2):",x2))
13
14  #Estimator3
15  x3<-(min(volt)+max(volt))/2
16  print(paste("Estimator(3):",x3))
17
18  #Estimator4
19  volt<-setdiff(volt,c(volt[1],volt[2],volt[19],volt [20]))
20  x4<-mean(volt)
21  print(paste("Estimator(4):",x4))</pre>
```

### R code Exa 6.3 Some general concepts of point estimation

```
10
11 #Mean
12 print(paste("Mean=",mean(obs)))
13
14 #Variance
15 print(paste("Point estimate of the population
      variance: ", var(obs)))
16
17 #Alternative formula
18 #To find numerator of s^2
19 s1<-sum(obs)
20 \text{ s2} < -\text{sum} (\text{obs}^2)
21 \text{ Sxx} < -s2 - (s1^2) / length (obs)
22
23 #Standard deviation
24 print(paste("Point estimate of the population
      standard deviation:",sd(obs)))
25
26 #Alterative estimator
27 print(paste("Alternative estimator of population
      variance: ",Sxx/length(obs)))
```

### R code Exa 6.9 Standard error of an estimator

```
#Estimate of standard deviation
12 s<-1.462
13
14 sx<-s/sqrt(n)
15 print(paste("Estimated standard error:",sx))</pre>
```

### R code Exa 6.10 Standard error of an estimator

```
1 #Ex6.10, Page 251
2 #Answers may vary slightly due to rounding off of
      values
3
4 n<-25
5 p<-0.6
6 q<-1-p
8 #To find estimated standard error
9 \text{ sp1} < -\text{sqrt}((p*q)/n)
10 print(paste("Estimated standard error:", sp1))
11
12 #pq is largest when
13 p < -q < -0.5
14 \text{ sp2} < -\text{sqrt}((p*q)/n)
15 print(paste("Estimated standard error when pq is
      largest:",sp2))
```

### R code Exa 6.11 Standard error of an estimator

```
1 #Ex6.11, Page 252
2 #Answers may slightly vary due to rounding off of values
3
```

### R code Exa 6.13 Method of moments

```
1 #Ex6.13, Page 257
2 #Answers may vary slightly due to rounding off of
      values
4 data<-c
      (152, 115, 109, 94, 88, 137, 152, 77, 160, 165, 125, 40, 128, 123, 136, 101, 62, 1
5 n<-length(data)
7 #Mean
8 m<-mean(data)</pre>
9 print(paste("Mean:",m))
10
11 #Sum of x^2
12 s < - sum (data ^ 2) / n
13
14 #alpha
15 alpha \langle -(m^2)/(s-(m^2))
16 print(paste("Alpha:",alpha))
17
18 #beta
```

```
19 beta<-(s-(m^2))/m
20 print(paste("Beta:",beta))</pre>
```

#### R code Exa 6.14 Method of moments

```
1 #Ex6.14, Page 257
2 #Answers may vary slightly due to rounding off of
      values
3
4 goals <-c(0,1,2,3,4,5,6,7,8,9,10)
5 freq <-c (29,71,82,89,65,45,24,7,4,1,3)
7 #Mean
8 m <-weighted.mean(goals, freq)</pre>
9 print(paste("Mean=",m))
10
11 #To find sum of x^2
12 ms <-weighted.mean(goals^2,freq)
13 print(paste("Sum of x^2/frequency=",ms))
14
15 #To find alpha
16 p_cap < -m/(ms - (m^2))
17 print(paste("Alpha:",p_cap))
18
19 #To find beta
20 r_{cap}(m^2)/(ms-(m^2)-m)
21 print(paste("Beta:",r_cap))
```

# R code Exa 6.21 The invariance principle

```
1 #Ex6.21, Page 262
2 #Answers may slightly vary due to rounding off of
    values
```

```
3
4 alpha<-11.9731
5 beta<-77.0153
6 m<-beta*gamma(1+(1/alpha))
7 print(paste("Mean estimate:",m))</pre>
```

# Chapter 7

# Statistical intervals based on a single sample

R code Exa 7.2 Basic properties of confidence intervals

```
#Ex7.2, Page 270
#Answers may slightly vary due to rounding off of values

#Ex7.2, Page 270
#Answers may slightly vary due to rounding off of values

#Ex7.2, Page 270
#Inswers may slightly vary due to rounding off of values

#Ex7.2, Page 270
#Inswers may slightly vary due to rounding off of values

#Ex7.2, Page 270
#Inswers may slightly vary due to rounding off of values

#Ex7.2, Page 270
#Inswers may slightly vary due to rounding off of values

#Inswers may slightly vary due to rounding off of values

#Inswers may slightly vary due to rounding off of values

#Inswers may slightly vary due to rounding off of values

#Inswers may slightly vary due to rounding off of values

#Inswers may slightly vary due to rounding off of values

#Inswers may slightly vary due to rounding off of values

#Inswers may slightly vary due to rounding off of values

#Inswers may slightly vary due to rounding off of values

#Inswers may slightly vary due to rounding off of values

#Inswers may slightly vary due to rounding of of the values

#Inswers may slightly vary due to rounding of of the values

#Inswers may slightly vary due to rounding off of the values

#Inswers may slightly vary due to rounding of of the values

#Inswers may slightly vary due to rounding of of of the values

#Inswers may slightly vary due to rounding of of the values

#Inswers may slightly vary due to rounding of of the values

#Inswers may slightly vary due to rounding of of of the values

#Inswers may slightly vary due to rounding of of the values

#Inswers may slightly vary due to rounding of of the value slightly vary due to rounding of of the value slightly vary due to rounding of the value slightly vary due to rounding of of the value slightly vary due to rounding of of the value slightly vary due to rounding of the value slightly vary d
```

### R code Exa 7.3 Other levels of confidence

```
1 #Ex7.3, Page 272
2 #Answers may slightly vary due to rounding off of
      values
4 \text{ mean} < -5.426
5 n<-40
6 sd<-0.1
8 #To find alpha for (100(1-alpha)=90)
9 alpha <-solve (100,10)
10
11 error <-qnorm(1-(alpha/2))*(sd/sqrt(n))</pre>
12
13 #Left endpoint
14 1 <-mean-error
15 print(paste("Left endpoint of interval:",1))
16
17 #Right endpoint
18 r < - mean + error
19 print(paste("Right endpoint of interval:",r))
```

### R code Exa 7.4 Confidence level and precision and sample size

```
1 #Ex7.4, Page 273
2 #Answers may slightly vary due to rounding off of values
3
4 #For 95% CI
5 z <-qnorm(1-(0.05/2))
6
7 sd <-25
8 w <-10</pre>
```

# R code Exa 7.6 Large sample CI for mean

```
1 #Ex7.6, Page 277-278
2 #Answers may slightly vary due to rounding off of
      values
4 data<-c
      (2948, 2996, 7197, 8338, 8500, 8759, 12710, 12925, 15767, 20000, 23247, 24863
5 26210, 30552, 30600, 35700, 36466, 40316, 40596, 41021, 41234, 43000, 44607, 45
6 45027,45442,46963,47978,49518,52000,53334,54208,56062,57000,57365,60
7 60265,60803,62851,64404,72140,74594,79308,79500,80000,80000,84000,11
8 boxplot(data)
9 print(data)
10
11 n < -50
12
13 #To find mean
14 mean <-mean (data)
15 print(paste("Mean:",mean))
16
17 #To find median
18 med <-median (data)
19 print(paste("Median:",med))
20
21 #To find standard deviation
22 s < - sd (data)
23 print(paste("Standard deviation:",s))
```

```
24
25 #To find confidence intervals
26 z <-qnorm(1-(0.05/2))
27 error <-z*(s/sqrt(n))
28
29 #Left endpoint
30 l <-mean-error
31 print(paste("Left endpoint of interval:",1))
32
33 #Right endpoint
34 r <-mean+error
35 print(paste("Right endpoint of interval:",r))
```

# R code Exa 7.7 Large sample CI for mean

```
1 #Ex7.7, Page 279
2
3 t1<-320
4 t2<-440
5
6 #To find value of s
7 s<-(t2-t1)/4
8
9 w<-5
10
11 z<-qnorm(1-(0.05/2))
12
13 n<-(z*s/w)^2
14 print(paste("Sample size:",n))
15 print(paste("Integer value of sample size:",ceiling(n)))</pre>
```

R code Exa 7.8 CI for population proportion

```
1 #Ex7.8, Page 282
2 #Answer may vary slightly compared to textbook value
3
4 n<-48
5 n1<-16
6 p<-n1/n
7
8 alpha<-0.05
9 b<-binom.test(16,48,p,conf.level=0.95)
print(b)</pre>
```

### R code Exa 7.10 Confidence bounds

```
#Ex7.10, Page 283
#Answer may slightly vary due to rounding off of
    value

mean<-17.17

<pre>z<-qnorm(0.95)</pre>
n<-48
sd<-3.28

#To find lower bound
c<-mean-(z*sd/sqrt(n))
print(paste("Lower confidence bound for mean:",c))
```

# ${f R}$ code ${f Exa}$ 7.11 One sample t confidence interval

```
1 #Ex7.11, Page 288
2 #Answers may slightly vary due to rounding off of
    values
```

```
3 data<-c
      (6807.99,7637.06,6663.28,6165.03,6991.41,6992.23,6981.46,7569.75,
           7437.88,6872.39,7663.18,6032.28,6906.04,6617.17,6984.12,7093
4
5
           7659.50,7378.61,7295.54,6702.76,7440.17,8053.26,8284.75,7347
6
           7422.69,7886.87,6316.67,7713.65,7503.33,7674.99)
8 n < -30
9
10 #To find mean
11 mean <-mean (data)</pre>
12 print(paste("Mean:",mean))
13
14 #To find standard deviation
15 s<-sd(data)
16 print(paste("Standard deviation:",s))
17
18 #To find confidence intervals
19 t < -qt(1-(0.05/2), df = n-1)
20 print(paste("Critical value, t:",t))
21 \text{ error} \leftarrow t*(s/sqrt(n))
22
23 #Left endpoint
24 1<-mean-error
25 print(paste("Left endpoint of interval:",1))
26
27 #Right endpoint
28 r <-mean+error
29 print(paste("Right endpoint of interval:",r))
30
31 #To plot the normal probability plot
32 qqnorm(data, main="Normal Probability of MOR")
```

### R code Exa 7.12 A prediction interval for a single future value

```
1 #Ex7.12, Page 288
2 #Answers may slightly vary due to rounding off of
3
4 data<-c
      (25.2,21.3,22.8,17.0,29.8,21.0,25.5,16.0,20.9,19.5)
5 print(data)
7 n<-10
9 #To find mean
10 mean <-mean (data)
11 print(paste("Mean:",mean))
12
13 #To find standard deviation
14 s<-sd(data)
15 print(paste("Standard deviation:",s))
16
17 #To find confidence intervals
18 \quad t < -qt (1 - (0.05/2), df = n - 1)
19 print(paste("Critical value, t:",t))
20 error <-t*(s/sqrt(n))
21
22 #Left endpoint
23 1 <-mean-error
24 print(paste("Left endpoint of interval:",1))
26 #Right endpoint
27 r <-mean+error
28 print(paste("Right endpoint of interval:",r))
```

### R code Exa 7.13 Prediction interval

```
1 #Ex7.13, Page 290
2 #Answers may slightly vary due to rounding off of
      values
3
4 mean <-21.9
5 s<-4.134
6 n<-10
8 #To find critical value
9 \text{ t} < -qt (1-(0.05/2), df = n-1)
10 print(paste("Critical value, t:",t))
11
12 error \leftarrow t*s*sqrt(1+(1/n))
13
14 #Left endpoint
15 1 <-mean-error
16 print(paste("Left endpoint of prediction interval:",
      1))
17
18 #Right endpoint
19 r<-mean+error
20 print(paste("Right endpoint of prediction interval:"
      (r)
```

# R code Exa 7.14 Tolerance intervals

```
1 #Ex7.14, Page 291
2 #Answers may vary slightly due to rounding off of
     values
3
```

```
4 data<-c
       (10490, 16620, 17300, 15480, 12970, 17260, 13400, 13900,
             13630, 13260, 14370, 11700, 15470, 17840, 14070, 14760)
6 print(data)
8 n < -16
10 #To find mean
11 mean <-mean (data)</pre>
12 print(paste("Mean:",mean))
13
14 #To find standard deviation
15 \text{ s} < -\text{sd}(\text{data})
16 print(paste("Standard deviation:",s))
17
18 #To find tolerance intervals
19 t1<-2.903
20 \text{ error1} \leftarrow \text{t1*s}
21 #Left endpoint
22 l1<-mean-error1
23 print(paste("Left endpoint of tolerance interval:",
      11))
24 #Right endpoint
25 \text{ r1} \leftarrow \text{mean} + \text{error1}
26 print(paste("Right endpoint of tolerance interval:",
      r1))
27
28 #To find confidence intervals
29 t2 < -qt(1-(0.05/2), df=n-1)
30 \operatorname{error2} < -t2 * (s/sqrt(n))
31 #Left endpoint
32 12 <-mean-error2
33 print(paste("Left endpoint of confidence interval:",
      12))
34 #Right endpoint
35 \text{ r2} \leftarrow \text{mean} + \text{error2}
36 print(paste("Right endpoint of confidence interval:"
```

### R code Exa 7.15 CI for variance and sd of normal population

```
1 #Ex7.15, Page 296
2 #Answers may slightly vary due to rounding off of
      values
3
  data<-c
      (1470,1510,1690,1740,1900,2000,2030,2100,2190,
              2200,2290,2380,2390,2480,2500,2580,2700)
5
6
  interval <- function (data, conf.level=0.95) {
8
     df <-length (data) -1
     1<-qchisq((1 - conf.level)/2, df)</pre>
9
10
     r \leftarrow qchisq((1 - conf.level)/2, df, lower.tail =
        FALSE)
     v<-var(data)
11
12
     c(df* v/r, df*v/l)
13 }
14 print(paste("Confidence interval for variance of
      normal population:"))
15 print(interval(data))
```

# Chapter 8

# Test of hypotheses based on a single sample

### R code Exa 8.1 Hypotheses and test procedures

```
1 #Ex8.1, Page 304
2 #Answers may vary slightly due to rounding off of
      values
4 #X<-number of crashes with no visible damage
5 #Reject H0 if x \ge 8 (x < -observed value of test
      statistic)
7 #When H0 is true
8 n < -20
9 p1<-0.25
10
11 #alpha=P(type I error)
12 alpha \leftarrow 1-pbinom(7,n,p1)
13 print(paste("Alpha value:",alpha))
14
15 print(paste("When HO is true, roughly", trunc(100*
      alpha, digits = 2), "% of all experiments consisting
      of 20 crashes would result into a Type I error"))
```

```
16
17 #beta=P(type II error)
18 #Different beta for each different p that exceeds
      0.25, thus taking p=0.3
19 p2 < -0.3
20 beta <-pbinom (7, n, p2)
21 print(paste("Beta value:",beta))
22 print(paste("When H0 is false, roughly", trunc(100*
      beta, digits=2), "% of all experiments would result
       into a Type II error"))
23
24 #Table showing variation of beta values with
      increase in value of p
  for(i in 3:8){
25
26
     p < -0.1 * i
27
     a \leftarrow pbinom(7,n,p)
     print(paste("p-value:",p))
28
     print(paste("Beta value:",a))
29
30 }
```

### R code Exa 8.2 Hypotheses and test procedures

```
1 #Ex8.2, Page 305
2 #Answers may vary slightly due to rounding off of values
3
4 m<-75
5 sd<-9
6 #Hypotheses H0: mean=75, H1: mean<75
7
8 #Experimental data
9 n<-25
10 sd<-9
11 #Standard deviation of normal distribution
12 sdx<-sd/sqrt(n)</pre>
```

```
13 print(paste("SD of normal distribution:",sdx))
14
15 #When H0 is true
16 \text{ mx} < -75
17
18 #Consider rejection region x \le 70.8
19 #To find alpha and beta
20
21 #P(type I error)
22 alpha \leftarrow pnorm (70.8, mx, sdx)
23 print(paste("Alpha value:",alpha))
24
25 #P(type II error)
26 \text{ mx} < -c (72,70,67)
                                      #HO is not rejected
      when it is false because mean= eg:72
27 for(i in mx){
     beta <-1-pnorm (70.8, i, sdx)
28
     print(paste("Beta value:",beta,"when mean is",i))
29
30 }
31
32 print(paste("Only", round(alpha, digits=3)*100, "% of
      all experiments will result into Type I error"))
33 print(paste("Possibility of Type II error decreases
      as mean value increases"))
```

# R code Exa 8.3 Hypotheses and test procedures

```
1 #Ex8.3, Page 306
2 #Answers may vary slightly due to rounding off of values
3
4 #R9={9,10,....,20}
5 n<-20
6
7 #To find alpha=P(H0 rejected when p=.25)=P(X>=9 when
```

```
X follows binomial dist.)
8 p<-0.25
9 alpha < -1-pbinom(8,n,p)
10 print(paste("Alpha value:",alpha))
11 print(paste("Type I error probability has reduced
      considerably when using R9 rather than R8
      rejection region"))
12
13 #To find beta value=P(H0 \text{ not rejected})=P(X\leq 8 \text{ when } X
       follows binomial dist.)
14 p1<-0.3
                  #1st case
15 beta1 <-pbinom(8,n,p1)</pre>
16 print(paste("Beta(0.3)=",beta1))
17
18 p2<-0.5
                  #2nd case
19 beta2 <-pbinom(8,n,p2)</pre>
20 print(paste("Beta(0.5)=",beta2))
21 print(paste("P(Type II error in R9)>P(Type II error
      in R8)"))
```

### R code Exa 8.4 Hypotheses and test procedures

```
1 #Ex8.4, Page 307
2 #Answers may vary slightly due to rounding off of values
3
4 #New rejection region x<=72
5 sdx<-1.8
6
7 #To find alpha=P(H0 rejected when true)=P(X<=72 when X follows normal dist)
8 mx<-75
9 alpha<-pnorm(72,mx,sdx)
10 print(paste("Alpha value:",alpha))
11 print(paste("Type I error probability is greater in</pre>
```

```
the new rejection region"))

12
13 #To find beta=P(H0 not rejected)
14 mx<-c(72,70,67)  #H0 is not rejected
    when it is false because mean= eg:72

15 for(i in mx){
    beta<-1-pnorm(72,i,sdx)
    print(paste("Beta(",i,"):",beta))
18 }
19 print(paste("Type II error probability is lower in the new rejection region"))</pre>
```

# R code Exa 8.5 Hypotheses and test procedures

```
1 #Ex8.5, Page 308
2
3 \# H0: mu = 1.5
4 #Ha: mu>1.5
6 \text{ mu0} < -1.5
7 n<-32
8 sd<-0.2
9 sdx<-sd/sqrt(n)
10 print(paste("Rejection region: Z>=c"))
11
12 alpha <- 0.05
13 #where alpha=P(type\ I\ error)=P(Z>=c\ when\ Z\ follows\ N
      (0,1)
14 c<-qnorm(1-alpha)
15 print(paste("C value:",c))
16
17 #Z>=c
18 x \leftarrow round(mu0 + sdx * c, digits = 2)
19 print(paste("xbar>=",x))
20 print(paste("Beta refers to the prob. that xbar<",x,
```

### R code Exa 8.6 Tests about a population mean

```
1 #Ex8.6, Page 312
2 #Answers may vary slightly due to rounding off of
      values
4 mu0<-130
5 n<-9
6 sd<-1.5
7 xbar <- 131.08
9 z \leftarrow (xbar - mu0) / (sd/sqrt(n)) #Test statistic
10 print(paste("Test statistic value:",z))
11
12 #To determine rejection region
13 alpha <- 0.01
14 \quad a \leftarrow qnorm(1-alpha/2)
15 1<-(-a)
16 r<-a
17 print(paste("Rejection region: z \le ",1,",z \ge ",r)
18 if(z<=1 || z>=r)print(paste(z, "falls in the
      rejection region and H0 is rejected"))else print(
      paste(z,"does not fall in the rejection region so
       HO cannot be rejected at significance level",
      alpha))
```

### R code Exa 8.7 Tests about a population mean

```
1 #Ex8.7, Page 314
```

```
2 #Answers may vary slightly due to rounding off of
      values
3
4 n<-16
5 sd<-1500
6 alpha1<-0.01
7 xbar<-30000
8 mu0<-31000
9
10 zalpha1 <- qnorm (1-alpha1)
11
12 z<-(xbar-mu0)/(sd/sqrt(n))</pre>
13
14 #To find beta (31000) when alpha = 0.01
15 beta1 <-pnorm (zalpha1+z)</pre>
16 print(paste("Beta(31000):",beta1,"when alpha=",
      alpha1))
17
18 alpha2<-0.1
19 zalpha2<-qnorm(1-alpha2)</pre>
20
21 #To find beta (31000) when alpha = 0.1
22 beta2 <-pnorm (zalpha2+z)
23 print(paste("Beta(31000):",round(beta2,digits=1),"
      when alpha=",alpha2))
24
25 #To find n
26 \text{ n} < -(\text{sd}*(\text{zalpha1+zalpha2})/(\text{xbar-mu0}))^2
27 print(paste("n value:",n))
28 print(paste("Sample size=",ceiling(n)))
```

# R code Exa 8.8 Tests about a population mean

```
1 #Ex8.8, Page 315
2 #Answers may vary slightly due to rounding off of
```

```
values
3
4 n<-52
5 mu0<-30
6 data<-c
      (14.1,14.5,15.5,16.0,16.0,16.7,16.9,17.1,17.5,17.8,17.8,18.1,18.2
           19.0,19.2,19.4,20.0,20.0,20.8,20.8,21.0,21.5,23.5,27.5,27.5,
7
           30.0,31.6,31.7,31.7,32.5,33.5,33.9,35.0,35.0,35.0,36.7,40.0,
8
           47.5,50.0,51.0,51.8,54.4,55.0,57.0)
10 xbar <-mean (data)
11 sd<-sd(data)</pre>
12 print(paste("Mean:",xbar))
13 print(paste("Standard deviation:",sd))
14
15 alpha <- 0.05
16 #To find zalpha
17 z1<-qnorm(alpha)
18
19 #To find z
20 z < (xbar-mu0)/(sd/sqrt(n))
21 print(paste("z value:",z))
22
23 if(z>z1) print(paste("HO cannot be rejected since",z
      ,">",z1)) else print(paste("HO is rejected since"
      ,z,"<",z1))
```

## R code Exa 8.9 Tests about a population mean

```
1 #Ex8.9, Page 317
2 #Answers may vary slightly due to rounding off of values
3
```

```
4 data<-c(2.67,4.62,4.14,3.81,3.83)
5 n<-length(data)
7 \text{ alpha} < -0.05
8 \text{ mu0} < -4
10 #To determine rejection region
11 t1 \leftarrow qt(1-alpha/2, df=n-1)
12 1<-(-t1)
13 r<-t1
14 print(paste("Rejection region: t \le ",1,",t \ge ",r)
16 #To find mean
17 print(paste("Sum of sample:", sum(data)))
18 print(paste("Mean:",mean(data)))
19
20 #To find standard deviation
21 print(paste("Standard deviation:",sd(data)))
22 print(paste("Estimated standard error of the mean:",
      sd(data)/sqrt(n)))
23
24 #Test statistic value
25 t1<-t.test(data, alternative="two.sided", mu=mu0)
26 print(t1)
```

## R code Exa 8.11 Tests concerning a population proportion

```
1 #Ex8.11, Page 324
2 #Answers may vary slightly due to rounding off of values
3
4 #H0:p=0.15
5 #Ha:p>0.15
6
7 p0<-0.15</pre>
```

```
8 q0 < -1 - p0
9 n<-91
10 n1<-16
11
12 #Check validity of test procedures
13 if(n*p0>10 && n*q0>10) print(paste("Large sample z
      test can be used")) else print(paste("z test
      cannot be used"))
14
15 alpha <- 0.1
16 zalpha <-qnorm (1-alpha)
17 print(paste("Rejection region: z>=",zalpha))
18
19 p < -n1/n
20 print(paste("p-cap:",p))
21
22 #To find z
23 z < (p-p0)/(sqrt(p0*q0/n))
24 print(paste("Test statistic value:",z))
25
26 if (z<zalpha) print (paste ("z=",z," is not in the
      rejection region and hence, HO cannot be rejected
      ")) else print(paste("z is in the rejection
      region"))
```

# R code Exa 8.12 Tests concerning a population proportion

```
1 #Ex8.12, Page 326
2 #Answers may vary slightly due to rounding off of values
3
4 #H0: p=0.9
5 #Ha: p<0.9
6
7 n<-225</pre>
```

```
8 p0 < -0.9
   9 \text{ pdash} < -0.8
10 alpha <- 0.01
11
12 zalpha <-qnorm (1-alpha)
13
14 #To find beta (0.8)
15 num \leftarrow p0-pdash-(zalpha*sqrt((p0*(1-p0))/n))
16 denom <-sqrt (pdash * (1-pdash)/n)
17 beta <- 1 - pnorm (num/denom)
18 print(paste("Beta(0.8):",beta))
19
20 #P(HO rejected)
21 prob<-1-beta
22 print(paste("P(H0 \text{ rejected})=",prob,"and thus,",
                            ceiling(prob*100),"% of samples will result in
                            correct HO rejection"))
23
24 #To find n
25 \text{ n} \leftarrow ((zalpha*sqrt(p0*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha*sqrt(pdash*(1-p0))+zalpha
                            pdash)))/(pdash-p0))^2
26 print(paste("Sample size:",ceiling(n)))
```

## R code Exa 8.13 Tests concerning a population proportion

```
1 #Ex8.13, Page 327
2
3 #H0: p=0.9
4 #Ha: p<0.9
5 #Rejection region: x<=15
6
7 n<-20
8 pdash<-0.8
9 alpha<-0.05</pre>
```

#### R code Exa 8.14 P values

```
1 #Ex8.14, Page 329
2 #Answers may vary slightly due to rounding off of
      values
3
4 \# H0: mu = 2.0
5 #Ha: mu>2.0
7 \text{ mu} 0 < -2
8 xbar<-2.06
9 sd<-0.141
10 n<-51
11
12 #To find z value
13 z <- (xbar-mu0) / (sd/sqrt(n))
14 print(paste("Test statistic value:",z))
15
16 #To find P-value=P(Z>=z \text{ when } mu=2.0)
17 p < -1 - pnorm(z)
18 print(paste("P-value:",p))
19
20 alpha <-c (0.01,0.001)
21 for(i in alpha){
22
          if (p<=i)
```

#### R code Exa 8.15 P values

```
1 #Ex8.15, Page 330
2 #Answers may vary slightly due to rounding off of
      values
3
4 #H0: mu=15
5 #Ha: mu>1.5
6
7 z < -2.10
9 #To determine p-value
10 p<-1-pnorm(z)
11 print(paste("P-value:",p))
12
13 alpha <-c (0.1,0.05,0.01)
14 for(i in alpha){
     z1<-qnorm(1-i)
15
     if(z>=z1)
16
       print(paste("Reject H0 at significance level",i)
17
18
     else
       print(paste("Do not reject H0 at significance
19
          level",i))
20 }
21
22 #Using p-value approach
23 print(paste("Using p-value approach:"))
```

# ${f R}$ code ${f Exa}$ 8.17 ${f P}$ values for z tests

```
1 #Ex8.17, Page 333
2 #Answers may vary slightly due to rounding off of
      values
4 #H0: mu=245
5 \# \text{Ha}: \text{mu!} = 245
7 xbar<-246.18
8 mu0<-245
9 n < -50
10 sd<-3.6
11 alpha <- 0.01
12
13 #To find z value
14 z <- (xbar-mu0) / (sd/sqrt(n))
15 print(paste("Test statistic value:",z))
16
17 #To find p value in two tailed experiment
18 p < -2*(1-pnorm(z))
19 print(paste("P-value:",p))
20
21 if (p>alpha) print (paste ("HO not rejected at
      significance level", alpha)) else print(paste("H0
      rejected at significance level", alpha))
```

## R code Exa 8.18 P values for t tests

```
1 #Ex8.18, Page 335
2
3 #H0: mu=4
4 #Ha: mu!=4
6 n < -5
7 t \leftarrow round(-0.594, digits=1)
8 	ext{df} < -n-1
10 \text{ alpha} < -0.05
11
12 #To find critical values
13 a < -qt (1-alpha/2, df = n-1)
14
15 #Since test is two tailed
16 p < -2 * pt (t, df = n - 1)
17 print(paste("P-value:",round(p,digits=2)))
18
19 b < -c (0.01, 0.05, 0.1)
20
  for(i in b){
21
      if(p>i)
22
        print(paste("H0 is not rejected at significance
           level",i))
23
      else
24
        print(paste("H0 rejected at significance level",
           i))
25 }
```

R code Exa 8.19 Interpreting P values

```
1 #Ex8.19, Page 335
2 #Answers may vary slightly due to rounding off of
3 #Tests for mean=20 have only been done....
5 #H0: mu=20
6 #Ha: mu>20
7
8 n < -4
9 df <-n-1
10 mu0<-20
11 sd<-2
12 x < -c (20.83, 22.232, 20.276, 17.718)
13
14 xbar <-mean(x)
15 print(paste("Mean:",xbar))
16 \text{ s} < -\text{sd}(x)
17 print(paste("Standard deviation:",s))
18
19 #To find t
20 t < -(xbar - mu0)/(s/sqrt(n))
21 print(paste("Test statistic value:",t))
22
23 #To find p value
24 p<-pt(t,df=n-1,lower.tail = FALSE)
25
26 \text{ alpha} < -0.05
27 if (p>alpha) print (paste ("HO is not rejected at
      significance level", alpha)) else print(paste("H0
       is rejected at significance level", alpha))
```

# Chapter 9

# Inferences based on two samples

 ${f R}$  code Exa 9.1 Test procedures for normal population with known variances

```
1 #Ex9.1, Page 348
2 #Answers may slightly vary due to rounding off of
      values
4 #H0: mu1-mu2=0
5 \# \text{Ha}: \text{mu1-mu2!} = 0
7 xbar <-29.8
8 ybar <-34.7
9 s1<-4
10 s2<-5
11 m<-20
12 n<-25
13
14 #To find percentile
15 \text{ alpha} < -0.01
16 HO <-qnorm (1-alpha/2)
17
```

#### R code Exa 9.2 Using a comparison to identify causality

```
1 #Ex9.2, Page 349
2 #Answers may slightly vary due to rounding off of
      values
3
4 #H0: mu1-mu2=0
5 #Ha: mu1-mu2>0
7 xbar <-48.9
8 ybar <-43.2
9 s1<-14.6
10 s2<-14.4
11 m < - 125
12 n<-90
13 alpha<-0.01
14
15 #To find test statistic value
16 z < -(xbar - ybar)/sqrt((s1^2)/m+(s2^2)/n)
17 print(paste("Test statistic value:",z))
18
19 #To find p-value
20 p < -pnorm(-abs(z))
21 print(paste("p-value:",p))
22
23 if(alpha>p) print(paste("HO is rejected at level",
      alpha)) else print(paste("HO is accepted at
```

# ${f R}$ code ${f Exa}$ 9.3 Beta and the choice of sample size

```
1 #Ex9.3, Page 350
2 #Answers may slightly vary due to rounding off of values
3
4 t<-5
5 t1<-0
6 s<-1.34
7 n<-qnorm(1-0.01/2)
8 beta<-pnorm(n-((t-t1)/s))-pnorm(-n-((t-t1)/s))
9 print(paste("Beta(5):",beta))</pre>
```

# R code Exa 9.4 Large sample tests

```
1 #Ex9.4, Page 352
2 #Answers may vary slightly due to rounding off of
      values
3
4 mu1<-2258
5 mu2<-2637
6 s1<-1519
7 s2<-1138
8 \text{ m} < -663
9 n < -413
10 1<-0.05
11 HO<-(-200)
12
13 #To find z0.05
14 z1<-qnorm(1-1)
15
```

```
#To find test statistic value
z<-(mu1-mu2-H0)/sqrt(((s1^2)/m+(s2^2)/n))
print(paste("Test statistic value:",z))

if(z<=-z1) print(paste("H0 is rejected since",z,"<=",-z1))

#To find p-value
    p<-pnorm(-abs(z))
print(paste("P-value:",p))
if(p<=1) print(paste("H0 is rejected since",p,"<=",1,"))</pre>
```

#### R code Exa 9.5 Confidence intervals for difference of means

```
1 #Ex9.5, Page 354
2 #Answers may vary slightly due to rounding off
      values
3
4 c1<-c('Variable_diam', 'N', 'Mean', 'Median', '
     TrMean', 'StDev', 'SEMean', 'Min', 'Max', 'Q1', '
     Q3 ')
5 c2 < -c ('3/8')
      ,78,4.250,4.230,4.238,1.300,0.147,1.634,7.327,3.389,5.075)
6 c3 < -c ('1/2')
      ,88,7.140,7.113,7.150,1.680,0.179,2.450,11.343,5.965,8.447)
7 df <-data.frame(c1,c2,c3)
8 print(df)
9
10 mu1<-4.25
11 mu2<-7.14
12 z < -qnorm(1-(0.05/2))
13 s1<-1.3
```

# R code Exa 9.6 Two sample t test and confidence interval

```
1 #Ex9.6, Page 358
2 #Answers may vary slightly due to rounding off of
      values
3
4 Fabric_type <-c ('Cotton', 'Triacetate')
5 Sample_size <-c(10,10)
6 Sample_mean <-c (51.71,136.14)
7 Sample_standard_deviation <-c(.79,3.59)
8 data < -data.frame(Sample_size, Sample_mean, Sample_
      standard_deviation, row.names=Fabric_type)
9 print(data)
10
11 #Assigning values to variables
12 s1 <- Sample_standard_deviation[1]
13 s2<-Sample_standard_deviation[2]
14
15 xbar <- Sample mean [1]
16 ybar <- Sample mean [2]
17
18 m<-Sample_size[1]
19 n <- Sample_size[2]
20
21 #To find df
```

```
22 num < -((s1^2/m) + (s2^2/n))^2
23 \operatorname{denom} \left( (s1^2/m)^2) / (m-1) + ((s2^2/n)^2) / (n-1) \right)
24 df <-num/denom
25 print(paste("df:",df))
26
27 #To find t value
28 \text{ alpha} < -0.05
29 t \leftarrow qt (1-alpha/2, df = m-1)
30 print(paste("t:",t))
31
32 #To find intervals
33 1 \leftarrow xbar - ybar - t * sqrt((s1^2/m) + (s2^2/n))
34 \text{ r} < -xbar - ybar + t * sqrt ((s1^2/m) + (s2^2/n))
35 print(paste("Left endpoint of interval:",1))
36 print(paste("Right endpoint of interval:",r))
37
38 print(paste("The true average porosity of the
       triacetate fabric is greater than that of the
       cotton fabric by between", round(-r, digits=2), and
      ", round(-1, digits=2), "cm<sup>3</sup>/cm<sup>2</sup>/sec"))
```

## R code Exa 9.7 Two sample t test and confidence interval

```
10 #To find test staistic value
11 t<-t.test(No_fusion, Fused)</pre>
12 print(t)
13 print(paste("t statistic value:",t$statistic))
14 print(paste("df:",t$parameter))
15
16 #To plot normal probability plots
17 par(mfrow=c(2,1))
18 qqnorm(No_fusion,datax=TRUE,ylab="Not fused",xlab="
      Probability", main="Normal probability plot for
      not fused data")
19 qqline(No_fusion,datax=TRUE)
20 qqnorm(Fused, datax=TRUE, ylab="Fused", xlab="
      Probability", main = "Normal probability plot for
      fused data")
21 qqline (Fused, datax=TRUE)
```

## R code Exa 9.9 Paired t test

```
1 #Ex9.9, Page 367
2 #Answers may vary slightly due to rounding off of
      values
3
4 Subject <-c(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16)
5 Before <-c
      (81,87,86,82,90,86,96,73,74,75,72,80,66,72,56,82)
6 After<-c
      (78,91,78,78,84,67,92,70,58,62,70,58,66,60,65,73)
7 Difference <- Before - After
8 data < -data.frame (Before, After, Difference, row.names =
      Subject)
9 print(data)
10
11 #Normal probability plot
12 par(mfrow=c(1,2))
```

## R code Exa 9.10 Paired t confidence interval

```
1 #Ex9.10, Page 369
2 #Answers may vary slightly due to rounding off of
      values
3
4 Subject <-c(1,2,3,4,5,6,7,8,9,10,11,12,13)
5 Slide <-c (30,35,40,25,20,30,35,62,40,51,25,42,33)
6 Digital <-c (25,16,15,15,10,20,7,16,15,13,11,19,19)
7 Difference <- Slide - Digital
8 data < - data . frame (Slide, Digital, Difference, row. names =
     Subject)
9 print(data)
10
11 #Normal probability plot
12 qqnorm(data$Difference, main="Normal probability plot
       of the differences", ylab="diff", xlab="
      Probability")
```

```
qqline(data$Difference)

#To find t critical value
alpha<-0.05

t<-qt(1-alpha/2,df=12)
print(paste("t statistic value:",t))

#To find confidence intervals
t1<-t.test(Slide,Digital,paired=TRUE)
print(t1)</pre>
```

 ${f R}$  code  ${f Exa}$  9.11 Inferences concerning difference between population proportions

```
1 #Ex9.11, Page 376
2 #Answers may slightly vary due to rounding off of
      values
4 alpha <- 0.05
5
6 \# H0: p1-p2=0
7 #Ha: p1-p2 < 0
8
9 a<-81
10 tot1<-549
11 p1cap<-a/tot1</pre>
12 q1cap <-1-p1cap
13
14 b<-141
15 tot2<-730
16 p2cap<-b/tot2
17 q2cap <-1-p2cap
18
19 pcap \leftarrow (a+b)/(tot1+tot2)
20 \text{ qcap} < -1 - pcap
```

# R code Exa 9.12 Type II error probabilities and sample sizes

```
1 #Ex9.12, Page 378
2 #Answers may vary slightly due to rounding off of
      values
4 \# H0: p1-p2=0
5 #Ha: p1-p2>0
7 a<-30
8 tot1<-100000
9 p1<-a/tot1
10 print(paste("p1:",p1))
11 q1<-1-p1
12
13 b<-a/2
14 p2<-b/tot1
15 print(paste("p2:",p2))
16 q2<-1-p2
17
```

```
18 alpha<-0.05
19 beta<-0.1
20
21 zalpha<-qnorm(1-alpha)
22 zbeta<-qnorm(1-beta)
23 d<-p1-p2
24
25 #To find n value
26 n<-(zalpha*sqrt((p1+p2)*(q1+q2)/2)+zbeta*sqrt(p1*q1+p2*q2))^2/d^2
27 print(paste("Value of n:",n))</pre>
```

# R code Exa 9.13 A large sample confidence interval

```
1 #Ex9.13, Page 380
2 #Answers may vary slightly due to rounding off of
      values
4 alpha <- 0.01
5 a < -76
                #number of people who survived through
      chemotherapy treatment
6 \text{ m} < -154
             #number of people who received
      chemotherapy treatment
7 p1cap<-a/m
8 q1cap <- 1 - p1cap
9 print(paste("plcap:",plcap))
10
11 b<-98
                 #number of people who survived through
      hybrid treatment
12 n<-164
             #number of people who received hybrid
      treatment
13 p2cap < -b/n
14 q2cap <-1-p2cap
15 print(paste("p2cap:",p2cap))
16
```

```
17 #To find z
18 zalpha <-qnorm (1-alpha/2)
19
20 if (m*p1cap >= 10 \&\& m*q1cap >= 10 \&\& n*p2cap >= 10 \&\& n*
      q2cap >= 10
21
     1<-p1cap-p2cap-zalpha*sqrt((p1cap*q1cap/m)+(p2cap*</pre>
        q2cap/n))
     r<-p1cap-p2cap+zalpha*sqrt((p1cap*q1cap/m)+(p2cap*
22
        q2cap/n))
23
24 print(paste("Left endpoint of confidence interval:",
25 print(paste("Right endpoint of confidence interval:"
      (r)
print(paste("Hence", round(l, digits=3), "< p1-p2<",
      round(r,digits=3)))
```

## R code Exa 9.14 F test for equality of variances

```
1 #Ex9.14, Page 385
2 #Answers may vary slightly due to rounding off of values
3
4 s1<-52.6
5 s2<-84.2
6 m<-28
7 n<-26
8
9 #H0: s1^2=s2^2
10 #Ha: s1^2<s2^2
11
12 alpha<-0.01
13 #To find F value
14 Falpha<-qf(alpha,df1=m-1,df2=n-1)
15 print(paste("F value:",Falpha))</pre>
```

```
16
17 f <-(s1^2)/(s2^2)
18 print(paste("f value:",f))
19
20 if(f <= Falpha) print(paste("H0 is rejected at significance level", alpha)) else print(paste("H0 is not rejected"))</pre>
```

# Chapter 10

# The analysis of variance

# R code Exa 10.1 Single factor ANOVA

```
1 #Ex10.1, Page 392
2 #Answers may vary slightly due to rounding off of
       values
4 Type_of_box<-c(1,2,3,4)
5 \text{ CS1} \leftarrow c (655.5, 789.2, 737.1, 535.1)
6 CS2 < -c (788.3,772.5,639.0,628.7)
7 CS3 \leftarrow c (734.3,786.9,696.3,542.4)
8 CS4 \leftarrow c (721.4,686.1,671.7,559.0)
9 CS5 \leftarrow c(679.1,732.1,717.2,586.9)
10 CS6 < -c (699.4,774.8,727.1,520.0)
11 Mean <-c (NULL)
12 \text{ SD} \leftarrow c \text{ (NULL)}
13
14 #To find mean of observations
15 for(i in 1:4){
      sum <-CS1[i]+CS2[i]+CS3[i]+CS4[i]+CS5[i]+CS6[i]</pre>
16
17
     mean <- sum / 6
     Mean[i] <-mean</pre>
18
19 }
20 #To find standard deviation of values
```

```
21 for(i in 1:4){
     s<-(CS1[i]-Mean[i])^2+(CS2[i]-Mean[i])^2+(CS3[i]-
        Mean[i])^2+(CS4[i]-Mean[i])^2+(CS5[i]-Mean[i])
        ^2+(CS6[i]-Mean[i])^2
23
     sd < -sqrt(s/5)
24
     SD[i]<-sd
25 }
26
27 data < -data . frame (Type_of_box, CS1, CS2, CS3, CS4, CS5, CS6
      , Mean , SD)
28 print(data)
29
30 #To find grand mean of mean values
31 \text{ tot_mean} < -0
32 for(i in 1:4){
     tot_mean <-tot_mean + Mean [i]</pre>
33
34 }
35 tot_mean<-tot_mean/4
36 print(paste("Grand mean:",tot_mean))
37
38 \text{ par}(\text{mfrow}=c(1,2))
39 #To create boxplots of the data
40 #First element of each CS column gives us the
      compression strength of one box
41 a <-c (CS1[1], CS2[1], CS3[1], CS4[1], CS5[1], CS6[1])
42 b <-c (CS1[2], CS2[2], CS3[2], CS4[2], CS5[2], CS6[2])
43 c<-c(CS1[3], CS2[3], CS3[3], CS4[3], CS5[3], CS6[3])
44 d <-c (CS1[4], CS2[4], CS3[4], CS4[4], CS5[4], CS6[4])
45 boxplot(a,b,c,d)
46 title(main="Boxplots for original data")
47
48 #Altering data in sample 2 by adding 120 to every
      value in Sample 4
49 e <-c (NULL)
50 for(i in 1:6){
     e[i]<-d[i]+120
51
52 }
53 boxplot(a,b,c,e)
```

## R code Exa 10.2 F distribution and the F test

```
1 #Ex10.2, Page 397
2 #Answers may vary slightly due to rounding off of
      values
4 #From Ex10.1
5 Mean <-c (713.00,756.93,698.07,562.02)
6 SD<-c(46.55,40.34,37.20,39.87)
7 tot_mean <-682.50
9 I<-4
10 J<-6
11
12 #Numerator
13 v1<-I-1
14
15 #Denominator
16 \text{ v2} < -I * (J-1)
17
18 \text{ alpha} < -0.05
19
20 #H0:mu1=mu2=mu3=mu4
21 #Ha: at least two mean values will be different
22
23 #To find F critical value
24 F \leftarrow qf (p=1-alpha, df1=v1, df2=v2)
25 print(paste("Critical value F:",F))
26
27 #To find mean square for treatment
28 MSTr \leftarrow (J/(I-1)) *sum((Mean-tot_mean)^2)
29 print(paste("MSTr:",MSTr))
30
```

```
#To find mean square for error

MSE<-(1/I)*sum(SD^2)

print(paste("MSE:",MSE))

#To find f value

f<-MSTr/MSE

print(paste("f:",f))

if(f>=F) print(paste("H0 is rejected at significance level",alpha,"since f>=F")) else
    print(paste("H0 not rejected"))
```

#### R code Exa 10.3 F distribution and the F test

```
1 #Ex10.3, Page 397
2 #Answers may vary slightly due to rounding off of
      values
4 Treatment <-c(1,2,3,4,5)
5 Sample_mean <-c (10.5,14.8,15.7,16.0,21.6)
6 Sample_sd<-c(4.5,6.8,6.5,6.7,6.0)
8 #To find total mean of mean values
9 tot_mean <-mean (Sample_mean)
10 print(paste("Grand mean:",tot_mean))
11
12 #H0: mu1=mu2=mu3=mu4=mu5
13 #Ha: at least two of the mean values are different
14
15 I<-5
16 J<-10
17 alpha <- 0.01
18
19 #Numerator
20 v1<-I-1
```

```
21
22 #Denominator
23 v2 < -I * (J-1)
24
25 #To find F value
26 F<-qf(1-alpha,v1,v2)
27 print(paste("Critical value F:",F))
28
29 #To find mean square for treatment
30 s <- Sample mean - tot mean
31 MSTr \leftarrow (J/(I-1)) * sum(s^2)
32 print(paste("MSTr:",MSTr))
33
34 #To find mean square for error
35 MSE <-mean (Sample_sd^2)
36 print(paste("MSE:", MSE))
37
38 #To find f value
39 f<-MSTr/MSE
40 print(paste("f:",f))
41
42 if(f>=F) print(paste("H0 is rejected at
      significance level",alpha, "since f>=F"))
                                                    else
      print(paste("H0 not rejected"))
```

## R code Exa 10.4 Sums of squares

```
1 #Ex10.4, Page 400
2 #Answers may vary slightly due to rounding off of values
3
4 Mixture1 <-c(0.56,1.12,0.90,1.07,0.94)
5 Mixture2 <-c(0.72,0.69,0.87,0.78,0.91)
6 Mixture3 <-c(0.62,1.08,1.07,0.99,0.93)</pre>
```

```
data1<-data.frame(Mixture1, Mixture2, Mixture3)</pre>
9
10 I<-3
11 J<-5
12
13 #Numerator
14 v1<-I-1
15
16 #Denominator
17 v2 < -I * (J-1)
18
19 alpha<-0.01
20 #To find F value
21 F<-qf(1-alpha,v1,v2)
22 print(paste("Critical value, F:",F))
23
24
25 #To create ANOVA table
26 mix <- stack (data1)
27 names(mix) <-c("Degree", "Treatment")
28
29 a <-aov (Degree Treatment, data=mix)
30 print(paste("ANOVA table:"))
31 a1 <- summary (a)
32 print(a1)
33 print(paste("f value:",a1[[1]]$F[1]))
```

## R code Exa 10.6 Multiple comparisons in ANOVA

```
1 #Ex10.6, Page 404
2
3 Treatment1 <-c(88.6,73.2,91.4,68.0,75.2)
4 Treatment2 <-c(63.0,53.9,69.2,50.1,71.5)
5 Treatment3 <-c(44.9,59.5,40.2,56.3,38.7)
6 Treatment4 <-c(31.0,39.6,45.3,25.2,22.7)</pre>
```

# R code Exa 10.9 Unequal sample sizes

```
1 #Ex10.9, Page 412
2
3 Permanent_molding<-c
      (45.5,45.3,45.4,44.4,44.6,43.9,44.6,44.0)
4 Die_casting<-c
      (44.2,43.9,44.7,44.2,44.0,43.8,44.6,43.1)
5 Plaster_molding<-c(46.0,45.9,44.8,46.2,45.1,45.5)
7 #To display ANOVA table
8 data1 <-data.frame (Mold=c(Permanent_molding, Die_
      casting,Plaster_molding),Treatment=factor(rep(c("
     Mold1", "Mold2", "Mold3"), times=c(length(Permanent_
     molding),length(Die_casting),length(Plaster_
     molding)))))
9
10 cat ("ANOVA table \n")
11 a <-aov (Mold~Treatment, data=data1)</pre>
12 print(summary(a))
```

R code Exa 10.10 Unequal sample sizes

```
1 #Ex10.10, Page 413
2
3 \text{ alpha} < -0.95
4 n<-22
5 J1<-8
6 J2<-8
7 J3<-6
8 I<-3
9 J < -n - I
10 MSE<-0.316
11 Q<-qtukey(alpha, I, J)
12 xibar <-c (44.71,44.06,45.58)
13
14 #To find w12
15 w12 < -Q * sqrt((MSE/2) * (1/J1+1/J2))
16 print(paste("w12:",w12))
17 w23 \leftarrow Q * sqrt((MSE/2) * (1/J2+1/J3))
18 print(paste("w23:",w23))
19 w31 < -Q * sqrt((MSE/2) * (1/J3+1/J1))
20 print(paste("w31:",w31))
21
22 if (xibar[1]-xibar[2] < w12) {
     print(paste("mul and mu2 are not significantly
23
        different"))
24 }
25 \text{ if}(xibar[2]-xibar[3] < w23){}
     print(paste("mu2 and mu3 are not significantly
        different"))
27 }
28 if (xibar[3]-xibar[1] < w31) {
     print(paste("mu3 and mu1 are not significantly
        different"))
30 }
```

R code Exa 10.11 A random effects model

```
1 #Ex.10.11, Page 415
 2
 3 \# H0: sigmaA^2 = 0
5 \text{ C1} < -c (55, 53, 54)
6 \quad C2 < -c (26, 37, 32)
7 C3<-c(78,91,85)
8 \quad C4 < -c (92, 100, 96)
9 \quad C5 < -c (49,51,50)
10 \quad C6 < -c (80, 85, 83)
11 data1 <-data.frame(C1,C2,C3,C4,C5,C6)</pre>
12 print(paste("x..=", sum(data1)))
13
14
15 #To create ANOVA table
16 col <- stack (data1)
17 names(col) <-c("Type", "Treatments")
18 a <-aov (Type Treatments, data=col)</pre>
19 print(paste("ANOVA table:"))
20 print(summary(a))
```

# Chapter 11

# Multifactor analysis of variance

## R code Exa 11.1 Two factor ANOVA

R code Exa 11.3 Fixed effects model

```
1 #Ex11.3, Page 424
```

```
2 #Answers may vary slightly due to rounding off of
      values
4 #To create data frame
5 brand <-rep(c("I","II","III"),4)</pre>
6 WT <-rep(c("1","2","3","4"),each=3) #WT <-Washing
      treatment
7 col_change <-c
      (0.97, 0.77, 0.67, 0.48, 0.14, 0.39, 0.48, 0.22, 0.57, 0.46, 0.25, 0.19)
8
9 df1<-data.frame(brand, WT, col_change)
10 print(df1)
11
12 #To display ANOVA table
13 a <-aov(col_change~brand+WT, data=df1)</pre>
14 print(a)
15 cat("\nSummarized table:\n")
16 print(summary(a))
17
18 #Diagnostic plots
19 model <-lm(col_change~brand+WT, data=df1)</pre>
20 par(mfrow=c(1,2))
21 qqnorm(model$residuals,datax=TRUE,main="Normal
      probability plot", ylab="Residual", xlab="Percent")
22 qqline(model$residuals,datax=TRUE)
23
24 plot(df1$col_change, resid(model), main="Residual plot
      ", xlab="Fitted value", ylab="Residual")
25 abline(0,0)
```

#### R code Exa 11.5 Randomized block experiments

```
1 #Ex11.5, Page 427
2 #Answers may vary slightly due to rounding off of
```

# R code Exa 11.6 Randomized block experiments

```
cat("\nTension mean values:\n")
m1<-tapply(df1$Speed,list(Tension),mean)
print(m1)

#To find mean of corresponding player data
cat("\nPlayer mean values:\n")
m2<-tapply(df1$Speed,list(Player),mean)
print(m2)

#To display ANOVA table
caterial and another transfer of the second second
```

# R code Exa 11.7 Two factor ANOVA with Kij greater than 1

```
1 #Ex11.7, Page 435
3 AggCont <-rep(c("38","41","44"),each=6)
4 AsphGr <-rep (c("PG58", "PG64", "PG70"),6)
5 data<-c
      (0.835,0.855,0.815,0.845,0.865,0.825,0.822,0.832,0.800,0.826,0.83
6 df1<-data.frame(AggCont, AsphGr, data)
7 print(df1)
9 #To find mean for corresponding content data
10 cat("\nCoarse aggregate content(%) mean values:\n")
11 m1<-tapply(df1$data,list(AggCont),mean)</pre>
12 print(m1)
13
14 #To find mean of corresponding Asphalt binder grade
     data
15 cat("\nAsphalt binder grade mean values:\n")
16 m2 <-tapply(df1$data,list(AsphGr),mean)</pre>
```

```
17 print(m2)
18
19
20 #To display ANOVA table
21 model <-lm(data~AsphGr+AggCont+AsphGr:AggCont)
22 a <-aov (data~AsphGr+AggCont+AsphGr:AggCont)
23 print(a)
24 print(summary(a))
25
26 #To display interaction plots (Thermal diffusivity
      data is not given)
27 interaction.plot(AggCont, AsphGr, response=data)
28
29 #Plots for checking normality and constant variance
      assumptions
30 par(mfrow=c(1,2))
31 qqnorm(model$residuals,datax=TRUE,main="Normal
      probability plot", ylab="Residual", xlab="Percent")
32 qqline(model$residuals,datax=TRUE)
33
34 plot(df1$data, resid(model), main="Residual plot", xlab
     ="Fitted value", ylab="Residual")
35 abline(0,0)
```

### R code Exa 11.9 Models with mixed and random effects

```
8
9 df1<-data.frame(Casmater, Source, Power)
10 print(df1)
11
12 #To create ANOVA table
13 a<-aov(Power~Casmater+Source+Casmater:Source, data=df1)
14 print(a)
15 cat("\nSummarized table:\n")
16 print(summary(a))</pre>
```

#### R code Exa 11.10 Three factor ANOVA

17

```
1 #Ex11.10, Page 444
2 #Answers may vary slightly due to rounding off of
      values
3
4 #To create data frame
5 A <-rep(c(rep(c("A1","A2","A3","A4"),each=3)),8)
6 B<-rep(c("B1","B2"),each=48)
7 C<-rep(c("C1","C2","C3","C4","C1","C2","C3","C4"),
      each=12)
8 data<-c
      (3.6,3.8,3.9,3.8,3.6,4.0,3.7,3.9,4.2,3.6,3.5,3.8,3.4,3.7,3.9,3.8,
9
10 df1<-data.frame(A,B,C,data)</pre>
11 print(df1)
12
13 #To find cell totals for all combinations of the
      three factors
14 cat("\nCell\ totals:\n")
15 m1<-tapply(df1$data,list(A,B,C),sum)</pre>
16 print(m1)
```

```
#To create ANOVA table
19 a <-aov(data~A+B+C+A:B+A:C+B:C+A:B:C,data=df1)
20 print(a)
21 cat("\nSummarized table:\n")
22 print(summary(a))</pre>
```

# R code Exa 11.11 Latin square designs

```
1 #Ex11.11, Page 447
2 #Answers may vary from textbook values
4 library(nlme)
5 library(multcomp)
7 #Dependent variable
8 data<-c
      (7.38,7.15,6.75,8.05,5.65,6.00,5.39,8.16,5.64,6.45,5.44,6.55,5.03
9
10 #Repeated measures
11 ##B(Columns)
12 B<-factor(rep(c("1","2","3","4","5","6"),each=6))
13
14 ##A(Rows)
15 A <-factor(rep(c("I","II","III","IV","V","V"),6))
16
17 df1<-data.frame(A,B,data)
18 print(df1)
19
20 #To create ANOVA table
21 #using aov() function
22 cat("\nUsing aov():\n")
23 a1 <-aov (data~A+Error (B/A), data=df1)
24 print(summary(a1))
25
```

```
#using lme()
cat("\nUsing lme():\n")
model<-lme(data~A,random=~1|B/A,data=df1)
a2<-anova(model)
print(a2)
#Tukey test
print(summary(glht(model,linfct=mcp(A="Tukey"))))</pre>
```

R code Exa 11.12 Factorial experiments with p factors at two levels

```
1 #Ex11.12, Page 452
2
3 Age <-rep (c("1","2"), each = 2,4)
4 Temp<-rep(c("1","2"),8)
5 Soil <-rep(c("1","2"),each=8)
6 Comp_Stre<-c
       (471, 485, 712, 712, 413, 552, 637, 789, 385, 530, 770, 741, 434, 593, 705, 806)
7 df1<-data.frame(Age, Temp, Soil, Comp_Stre)
9 #To compute cell totals
10 \text{ s1} < -\text{s2} < -\text{s3} < -\text{s4} < -\text{s5} < -\text{s6} < -\text{s7} < -\text{s8} < -\text{c} (NULL)
11 for(i in 1:length(Comp_Stre)){
      if (df1$Age[i] == "1" && df1$Temp[i] == "1" && df1$Soil
12
         [i]=="1"){
        s1[i] <-df1$Comp_Stre[i]
13
14
        x111<-sum(s1,na.rm=TRUE)
15
      }
      else if(df1$Age[i] == "1" && df1$Temp[i] == "2" && df1
16
         $Soil[i] == "1"){
17
        s2[i] <-df1$Comp_Stre[i]
        x121 <- sum (s2, na.rm = TRUE)
18
19
20
      else if (df1$Age[i] == "1" && df1$Temp[i] == "1" && df1
```

```
$Soil[i] == "2"){
21
        s3[i] <-df1$Comp_Stre[i]
       x112<-sum(s3,na.rm=TRUE)
22
23
     }
24
     else if (df1$Age[i] == "2" && df1$Temp[i] == "1" && df1
        $Soil[i] == "1"){
25
        s4[i] <-df1$Comp_Stre[i]
       x211 < -sum(s4, na.rm = TRUE)
26
27
     else if (df1$Age[i] == "2" && df1$Temp[i] == "1" && df1
28
        $Soil[i] == "2"){
29
        s5[i] <-df1$Comp_Stre[i]
30
       x212<-sum(s5, na.rm=TRUE)
31
     else if(df1$Age[i] == "1" && df1$Temp[i] == "2" && df1
32
        $Soil[i]=="2"){
        s6[i] <-df1$Comp_Stre[i]
33
34
       x122<-sum(s6,na.rm=TRUE)
35
     else if(df1$Age[i] == "2" && df1$Temp[i] == "2" && df1
36
        $Soil[i] == "1"){
        s7[i] <-df1$Comp_Stre[i]
37
       x221 < -sum(s7, na.rm = TRUE)
38
     }
39
     else if (df1$Age[i] == "2" && df1$Temp[i] == "2" && df1
40
        $Soil[i] == "2"){
41
       s8[i] <-df1$Comp_Stre[i]
       x222<-sum(s8,na.rm=TRUE)
42
43
     }
44
45 }
46 #Parameters of the model
47 n<-2
48 alpha1_cap<-(x111+x121+x112+x122-x211-x212-x221-x222
      )/(8*n)
49 alpha2_cap \leftarrow (-1)*alpha1_cap
50
51 \quad \text{gamma11\_cap} \leftarrow (x111-x121-x211+x221+x112-x122-x212+
```

```
x222)/(8*n)
52     gamma12_cap<-(-1)*gamma11_cap
53     gamma21_cap<-(-1)*gamma11_cap
54     gamma22_cap<-gamma11_cap
55
56     print(paste("alpha1_cap:",alpha1_cap))
57     print(paste("alpha2_cap:",alpha2_cap))
58     print(paste("gamma11_cap:",gamma11_cap))
59     print(paste("gamma12_cap:",gamma12_cap))
60     print(paste("gamma21_cap:",gamma21_cap))
61     print(paste("gamma22_cap:",gamma22_cap))</pre>
```

# R code Exa 11.13 Factorial experiments with p factors at two levels

```
1 #Ex11.13, Page 453
2 #Answers may vary slightly due to rounding off of
      values
4 Age <-factor(rep(c("1","2"),each=2,4))
5 Temp<-factor(rep(c("1","2"),8))
6 Soil <-factor(rep(c("1","2"),each=8))</pre>
7 Comp_Stre<-c
      (471, 485, 712, 712, 413, 552, 637, 789, 385, 530, 770, 741, 434, 593, 705, 806)
8 df1<-data.frame(Age, Temp, Soil, Comp_Stre)
9
10 #To compute cell totals
11 s1 < -s2 < -s3 < -s4 < -s5 < -s6 < -s7 < -s8 < -c (NULL)
12 for(i in 1:length(Comp_Stre)){
     if (df1$Age[i] == "1" && df1$Temp[i] == "1" && df1$Soil
13
        [i]=="1"){
14
       s1[i] <-df1$Comp_Stre[i]
       x111<-sum(s1,na.rm=TRUE)
15
16
     else if (df1$Age[i] == "1" && df1$Temp[i] == "2" && df1
17
```

```
$Soil[i] == "1"){
18
        s2[i] <-df1$Comp_Stre[i]
        x121<-sum(s2,na.rm=TRUE)
19
20
     }
21
     else if (df1$Age[i] == "1" && df1$Temp[i] == "1" && df1
        $Soil[i] == "2"){
22
        s3[i] <-df1$Comp_Stre[i]
        x112<-sum(s3,na.rm=TRUE)
23
24
     else if (df1$Age[i] == "2" && df1$Temp[i] == "1" && df1
25
        $Soil[i] == "1"){
26
        s4[i] <-df1$Comp_Stre[i]
27
        x211 <- sum (s4, na.rm = TRUE)
28
     else if(df1$Age[i] == "2" && df1$Temp[i] == "1" && df1
29
        $Soil[i] == "2"){
        s5[i] <-df1$Comp_Stre[i]
30
31
        x212 < -sum(s5, na.rm = TRUE)
32
     else if(df1$Age[i] == "1" && df1$Temp[i] == "2" && df1
33
        $Soil[i] == "2"){
        s6[i] <-df1$Comp_Stre[i]
34
        x122<-sum(s6,na.rm=TRUE)
35
     }
36
     else if (df1$Age[i] == "2" && df1$Temp[i] == "2" && df1
37
        $Soil[i]=="1"){
38
        s7[i] <-df1$Comp_Stre[i]
        x221 < -sum(s7, na.rm = TRUE)
39
40
     else if(df1$Age[i] == "2" && df1$Temp[i] == "2" && df1
41
        $Soil[i]=="2"){
        s8[i] <-df1$Comp_Stre[i]
42
43
        x222 < -sum(s8, na.rm = TRUE)
     }
44
45
46 }
47
48 Effect_contrasts<-c(NULL)
```

```
49 #To compute effect contrasts
50 for(j in 1:7){
     if(j==1){
51
       Effect_contrasts[j] = -x111+x211-x121+x221-x112+
52
          x212 - x122 + x222
53
     }
54
     else if (j==2) {
       Effect_contrasts[j]=-x111-x211+x121+x221-x112-
55
          x212+x122+x222
56
     else if(j==3){
57
58
       Effect_contrasts[j]=x111-x211-x121+x221+x112-
          x212 - x122 + x222
59
     else if(j==4){
60
       Effect_{contrasts}[j] = -x111 - x211 - x121 - x221 + x112 +
61
          x212+x122+x222
62
     else if (j==5) {
63
       Effect_contrasts[j]=x111-x211+x121-x221-x112+
64
          x212-x122+x222
     }
65
66
     else if(j==6){
       Effect_contrasts[j]=x111+x211-x121-x221-x112-
67
          x212+x122+x222
68
     }
69
     else if (j==7) {
       Effect_contrasts[j] = -x111 + x211 + x121 - x221 + x112 -
70
          x212 - x122 + x222
71
     }
72 }
73
74 SS<-(Effect_contrasts^2)/(8*n)
75 df2<-data.frame(Effect_contrasts,SS)
76 cat("\nEffect contrast and SS values:\n")
77 print(df2)
78 SSsum <-sum (SS)
79
```

```
80 #To find sum of squares value
81 x2_ijkl<-0
82 X2<-0
83 for(k in 1:length(Comp_Stre)){
84
      x2_{ijkl} < -x2_{ijkl} + (Comp_Stre[k]^2)
85 }
86 x2 < -(x111+x112+x211+x121+x212+x221+x122+x222)^2/16
87 print(paste("Sum of square of data:",x2_ijkl))
88 print(paste("x...^2/16:",x2))
89
90 #To find SST value
91 SST < -x2_ijkl - x2
92 print(paste("SST:",SST))
93
94 #To find SSE value
95 SSE <-SST-SSsum
96 print(paste("SSE:", SSE))
97
98 #To display corresponding ANOVA tables
99 model1 <-lm (Comp_Stre~Age:Temp:Soil)
100 model2<-lm(Comp_Stre~Age*Temp*Soil)
101 a <- aov (model1, data = df1)
102 b <-aov (model2, data=df1)
103 print(summary(a))
104 print(summary(b))
```

# Chapter 12

# Simple linear regression and correlation

R code Exa 12.1 The simple linear regression model

```
12 dotchart(yi, labels = row.names(yi), main = "Dot plot of y values")
```

# R code Exa 12.2 The simple linear regression model

# R code Exa 12.3 A linear probabilistic model

```
1 #Ex12.3, Page 474
2 #Answers may slightly vary due to rounding off of probabilities
3
4 y<- function(x) (65-1.2*x)
5 sd<-8
6
7 #P(time-to-failure exceeds 50 when applied stress is 20)
8 x1<-20
9 mu1<-y(x1)
10 q<-50</pre>
```

```
11 P1<-1-pnorm(q,mu1,sd)</pre>
12 print(paste("Probability that the time-to-failure >50
       when applied stress = 20): ",P1))
13
14 #P(time-to-failure exceeds 50 when applied stress is
       25)
15 x2<-25
16 \text{ mu2} \leftarrow y(x2)
17 q<-50
18 P2<-1-pnorm(q,mu2,sd)
19 print(paste("Probability that the time-to-failure >50
       when applied stress = 25): ",P2))
20
21 #Let Y1<-observation made with x=25 and Y2<-
      observation made with x=24
22 #Mean value
23 X1<-25
24 X2<-24
25 \quad E \leftarrow y(X1) - y(X2)
26 #Variance
27 \quad V \leq -sd^2 + sd^2
28 SD<-sqrt(V)
29 #Probability that Y1 exceeds Y2
30 P3<-1-pnorm(0,E,V)
31 print(paste("P(Y1>Y2):",P3))
```

# R code Exa 12.4 Estimating model parameters

```
1 #Ex12.4, Page 479
2 #Answers may slightly vary due to rounding off of
     values
3
4 x<-c
     (132.0,129.0,120.0,113.2,105.0,92.0,84.0,83.2,88.4,59.0,80.0,81.5)</pre>
```

# R code Exa 12.5 Estimating model parameters

```
1 #Ex12.5, Page 480
2
3 y - function(x) {75.212-0.2094*x}
4 #If iodine value 100 is selected...
5 x - 100
6 print(paste("Point prediction for the resulting cetane number:", round(y(x), digits=2)))
```

#### R code Exa 12.6 Estimating model parameters

```
1 #Ex12.6, Page 482
2 #Answers may slightly vary due to rounding off of
    values
3
```

# R code Exa 12.7 Estimating model parameters

```
9 \mod el < -lm(y^x)
10 Residual <-resid (model)
11 n <-length (Residual)
12
13 SSE <- sum (Residual^2)
14 print(paste("SSE:", SSE))
15
16 #To find variance (sigma^2)
17 	 s2 < -SSE/(n-2)
18 print(paste("Estimate of variance:",s2))
19
20 #To find standard deviation (sigma)
21 sd<-sqrt(s2)
22 print(paste("Estimate of standard deviation:",sd))
23
24 #To display ANOVA table
25 a <-aov (model, data=data1)
26 cat ("ANOVA table: \n")
27 print(summary(a))
```

# R code Exa 12.8 Estimating model parameters

```
11 a <-aov (model, data=data1)

12 cat ("ANOVA table:\n")

13 print (summary(a))
```

#### R code Exa 12.9 Coefficient of determination

```
1 #Ex12.9, Page 486
2 #Answers may vary slightly due to rounding off of
      values
4 #From Ex12.4
5 x < -c
      (132.0,129.0,120.0,113.2,105.0,92.0,84.0,83.2,88.4,59.0,80.0,81.5
6 y<-c
      (46.0,48.0,51.0,52.1,54.0,52.0,59.0,58.7,61.6,64.0,61.4,54.6,58.8
7 data1<-data.frame(x,y)</pre>
9 n < -14
10 beta0<-75.212432
11 beta1<-(-0.20938742)
12 yi <-779.2
13 xiyi<-71347.30
14 yi2<-43745.22
15
16 #To find SST and SSE
17 SST<-yi2-(yi^2)/n
18 print(paste("SST:",SST))
19 SSE<-yi2-beta0*yi-beta1*xiyi
20 print(paste("SSE:",SSE))
21
22 #Coefficient of determination
23 \text{ r2} \leftarrow 1 - (SSE/SST)
24 print(paste("Coefficient of determination:",r2))
```

```
25
26 #To create regression model using ANOVA table
27 a <-lm(y~x)
28 print(summary(a))
29 a1 <-aov(a,data=data1)
30 print(summary(a1))
```

#### R code Exa 12.11 Confidence interval for beta1

```
1 #Ex12.11, Page 494
2 #Answers may vary slightly due to rounding off of
      values
4 x<-c
      (5.7,6.8,9.6,10.0,10.7,12.6,14.4,15.0,15.3,16.2,17.8,18.7,19.7,20
5 y < - c
      (119.0, 121.3, 118.2, 124.0, 112.3, 114.1, 112.2, 115.1, 111.3, 107.2, 108.
  plot(x,y,main="Scatterplot",xlab="Air content",ylab=
      "Density")
9 n < -length(x)
10 \text{ xi} < -\text{sum}(x)
11 yi <- sum (y)
12 #To find sum of xi^2, yi^2 and xi*yi
13 \text{ xi2} < -\text{sum}(\text{x}^2)
14 yi2<-sum(y^2)
15 xiyi < -sum(x*y)
16
17 #To find Sxx and Sxy
18 Sxx < -xi2 - (xi^2) / length(x)
19 Sxy<-xiyi-(yi*xi)/length(y)
20 print(paste("Sxx:",Sxx))
```

```
21 print(paste("Sxy:",Sxy))
22
23 #To find beta1 and beta0
24 \text{ xbar} \leftarrow \text{mean}(x)
25 ybar <-mean(y)
26 beta1 <- Sxy/Sxx
27 beta0 <- ybar - beta1 * xbar
28 print(paste("Beta1:",beta1))
29 print(paste("Beta0:",beta0))
30
31 #To find SSE
32 SSE<-yi2-beta0*yi-beta1*xiyi
33 print(paste("SSE:", SSE))
34
35 #To find SST
36 \text{ SST} < -yi2 - yi^2/n
37 print(paste("SST:",SST))
38
39 #To find r^2
40 \text{ r2} < -1 - (SSE/SST)
41 print(paste("r^2:",r2))
42
43 I <- 15
44 J<-2
45 df <-I-J
46
47 #s^2
48 	 s2 < -SSE/df
49 s <- sqrt (s2)
50
51 #Estimated standard deviation of beta1
52 sb<-s/sqrt(Sxx)
53 print(paste("Estimated standard deviation of beta1:"
      ,sb))
54
55 \text{ alpha} < -0.05
56 t < -qt(1-alpha/2, df)
57
```

```
58 #To find confidence intervals
59 \quad 1 \leftarrow beta1 - t * sb
60 \text{ r} \leftarrow \text{beta1+t*sb}
61 print(paste("Left endpoint of confidence interval:",
62 print(paste("Right endpoint of confidence interval:"
      (r)
63
64 #To display analysis of variance and parameter
      estimates
65 \mod \text{el} < -lm(y^x)
66 cat("\nParameter estimates\n")
67 print(summary(model))
68 cat("\nANOVA table:\n")
69 print(summary(aov(model)))
70 a <-predict (model)
71 print(paste("Predict values:"))
72 print(a)
73 print(paste("Residuals:"))
74 print(resid(model))
```

# R code Exa 12.12 Hypothesis testing procedures

```
8 model <-lm(y~x,data=data1)
9 cat("Regression model:\n")
10 print(summary(model))
11
12 #To display ANOVA table
13 a <-aov(model)
14 cat("ANOVA table:\n")
15 print(summary(a))</pre>
```

# R code Exa 12.13 Inferences concerning mean

```
1 #Ex12.13, Page 501
2 #Answers may vary slightly due to rounding off of
      values
4 library(ggplot2)
5
6 x<-c
      (8.0, 15.0, 16.5, 20.0, 20.0, 27.5, 30.0, 30.0, 35.0, 38.0, 40.0, 45.0, 50.0,
7 y < - c
      (22.8,27.2,23.7,17.1,21.5,18.6,16.1,23.4,13.4,19.5,12.4,13.2,11.4
8 data1<-data.frame(x,y)</pre>
10 #To plot data using ggplot
11 ggplot(data=data1, mapping=aes(x,y))+geom_point(size
      =2) + geom_smooth(method=lm)
12
13 #To display linear regression model
14 \mod el < -lm(y^x)
15 print(summary(model))
16
17 #To display analysis of variance
18 cat("ANOVA table:\n")
```

#### R code Exa 12.14 Prediction interval for a future value of Y

```
1 #Ex12.14, Page 505
2 #Answers may vary slightly due to rounding off of
       values
4 \text{ ycap} < -13.79
5 \text{ sy} < -0.7582
6 s<-2.8640
7 n<-18
8 	 df < -n-2
9 t<-2.120
10
11 #To find confidence intervals
12 \quad 1 \leftarrow y \cdot cap - t \cdot sqrt(s^2 + sy^2)
13 r \leftarrow ycap + t * sqrt(s^2 + sy^2)
14 print(paste("Left endpoint of confidence interval:",
15 print(paste("Right endpoint of confidence interval:"
       (r)
```

#### R code Exa 12.15 Correlation

```
1 #Ex12.15, Page 509
2 #Answers may vary slightly due to rounding off of values
3
4 x<-c(2.4,3.4,4.6,3.7,2.2,3.3,4.0,2.1)
5 y<-c(1.33,2.12,1.80,1.65,2.00,1.76,2.11,1.63)
6
7 #To find sample correlation coefficient
8 r<-cor(x,y)
9 print(paste("Sample correlation coefficient,r:",r))</pre>
```

R code Exa 12.16 Inferences about the population correlation coefficient

```
#Ex12.16, Page 511
2 #Answers may vary slightly due to rounding off of
    values
3
4 x<-c
        (0.066,0.088,0.120,0.050,0.162,0.186,0.057,0.100,0.112,0.055,0.15)
5 y<-c
        (4.6,11.6,9.5,6.3,13.8,15.4,2.5,11.8,8.0,7.0,20.6,16.6,9.2,17.9,2)
6
7 #To find r(point estimate of population correlation
        coefficient)
8 r<-cor(x,y)
9 print(paste("r:",r))</pre>
```

R code Exa 12.17 Inferences about the population correlation coefficient

```
#Ex12.17, Page 513
#Answers may vary slightly due to rounding off of values

r<-0.29
for m<-45
for #To find the test statistic
for t<-r*sqrt(n-2)/sqrt(1-r^2)
for print(paste("t:",round(t,digits=0)))

#P value for two tailed test
for p<-2*pt(-abs(t),df=n-1)
for print(p)</pre>
```

# R code Exa 12.18 Other inferences concerning rho

14

```
15  #To find v
16  v<-0.5*log((1+r)/(1-r))
17  print(paste("v:",v))
18
19  muv<-0.5*log((1+rho)/(1-rho))
20  print(paste("muv:",muv))
21
22  z<-(v-muv)*sqrt(length(x)-3)
23  print(paste("z:",z))
24
25  #To find P value
26  p<-pnorm(z,lower.tail=FALSE)
27  print(paste("p value:",p))
28
29  print(paste("HO is rejected..."))</pre>
```

# R code Exa 12.19 Other inferences concerning rho

```
1 #Ex12.19, Page 516
2 #Answers may vary slightly due to rounding off of
      values
3
4 xi<-285.90
5 \text{ xi2} < -4409.55
6 yi<-690.30
7 yi2<-29040.29
8 xiyi <- 10818.56
9 r < -0.733
10 v<-0.935
11 n<-20
12 alpha <- 0.05
13
14 #To find z
15 zalpha2 <- qnorm (1-alpha/2)</pre>
16
```

```
17 #95% interval for muv
18 c1<-v-zalpha2/sqrt(n-3)
19 c2<-v+zalpha2/sqrt(n-3)
20 print(paste("Left endpoint of CI for muv:",c1))
21 print(paste("Right endpoint of CI for muv:",c2))
22
23 #95% interval for rho
24 1<-(exp(1)^(2*c1)-1)/(exp(1)^(2*c1)+1)
25 r<-(exp(1)^(2*c2)-1)/(exp(1)^(2*c2)+1)
26 print(paste("Left endpoint of CI for rho:",1))
27 print(paste("Right enpoint of CI for rho:",r))</pre>
```

# Chapter 13

# Nonlinear and multiple regression

# R code Exa 13.2 Diagnostic plots

```
1 #Ex13.2, Page 526
3 x < - c
      (100, 125, 125, 150, 150, 200, 200, 250, 250, 300, 300, 350, 400, 400)
4 y<-c
      (150, 140, 180, 210, 190, 320, 280, 400, 430, 440, 390, 600, 610, 670)
5 y_{cap} < (-45.55) + (1.71*x)
6 \text{ m1} < -1\text{m} (y^x)
7 ei<-round(resid(m1),digits=1)</pre>
8 ei_std<-round(rstandard(m1),digits=2)</pre>
9 df <-data.frame(x,y,y_cap,ei,ei_std)
10 print(df)
11
12 par(mfrow=c(3,2))
13 plot(x,y,main="y vs. x")
14 abline(m1)
15 plot(y_cap,ei_std,main="Standardized residuals vs. y
```

```
_cap")

16 abline(0,0)

17 plot(y,y_cap,main="y_cap vs. y")

18 abline(lm(y_cap~y))

19 plot(x,ei_std,main="Standardized residuals vs. x")

20 abline(0,0)

21 qqnorm(ei_std,xlab="z percentile",ylab="e*",main="Normal probability plot")
```

# R code Exa 13.4 Regression with transformed variables

```
1 #Ex13.4, Page 534
2 #Answers may vary slightly due to rounding off of
       values
4 \text{ x} < -c (2, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100)
5 \text{ y} < -c (408, 274, 196, 137, 90, 78, 51, 40, 30, 22, 15)
6 y_dash < -log(y,base = exp(1))
7 \text{ m1} < -1\text{m} (y^x)
8 \text{ m2} < -1\text{m} (y_dash^x)
9 ei_std1<-rstandard(m1)</pre>
10 ei_std2<-rstandard(m2)</pre>
11
12 par(mfrow=c(2,2))
13 plot(x,y,main="Scatterplot")
14 abline(m1)
15 plot(x,ei_std1,xlab="x",ylab="e*",main="Residual
       plot from linear regression for the data")
16 abline(0,0)
17
18 #To find the coefficients
19 beta0 <-m2 $ coefficients [1]
20 beta1 <- round (m2 $ coefficients [2], digits = 4)
21
22 y_{cap} < (exp(1)^(beta0)) * (exp(1)^(beta1*x))
```

```
23
24  df <-data.frame(x,y,y_dash,y_cap)
25  print(df)
26
27  plot(x,ei_std2,ylab="e*",main="Standardized
      residuals(after transforming) vs. x")
28  abline(0,0)
29  plot(y,y_cap,main="ycap vs. y")
30  abline(lm(y_cap~y))</pre>
```

# R code Exa 13.7 Polynomial regression

```
1 #Ex13.7, Page 544
2
3 Thickness <-c
      (220, 220, 220, 220, 370, 370, 370, 370, 440, 440, 440, 440, 680, 680, 680, 680,
4 Strength <-c
      (24.0,22.0,19.1,15.5,26.3,24.6,23.1,21.2,25.2,24.0,21.7,19.2,17.0
6 df <-data.frame(Thickness, Strength)
7 cat("Regression model:\n")
8 relation <-lm(Strength~Thickness+I(Thickness^2))</pre>
9 print(summary(relation))
10
11 cat("Analysis of variance:\n")
12 model <-lm(Strength~poly(Thickness,2))</pre>
13 a <- aov (model, data = df)</pre>
14 print(summary(a))
15
16 #To find confidence and prediction intervals
17 cat("\nWhen thickness=500\n")
18 cat("95\% CI:\n")
19 print(predict(relation, newdata=data.frame(Thickness
```

```
=500),interval="confidence"))
20 cat("95% PI:\n")
21 print(predict(relation, newdata=data.frame(Thickness =500),interval="prediction"))
22 cat("\nWhen thickness=800\n")
24 cat("95% CI:\n")
25 print(predict(relation, newdata=data.frame(Thickness =800),interval="confidence"))
26 cat("95% PI:\n")
27 print(predict(relation, newdata=data.frame(Thickness =800),interval="prediction"))
```

# R code Exa 13.8 Estimating parameters

```
1 #Ex13.8, Page 546
2 #Answers may vary slightly due to rounding off of
      values
3
4 #From Ex13.7
5 Thickness <-c
      (220, 220, 220, 220, 370, 370, 370, 370, 440, 440, 440, 440, 680, 680, 680, 680,
6 Strength <-c
      (24.0,22.0,19.1,15.5,26.3,24.6,23.1,21.2,25.2,24.0,21.7,19.2,17.0
7 df <-data.frame (Thickness, Strength)
8 relation <-lm(df$Strength~poly(df$Thickness,2),data=</pre>
      df)
9 a <- summary (aov (relation, data=df))</pre>
10 print(a)
11
12 n<-20
13 k<-2
14 SSE<-a[[1]][, 'Sum Sq'][2]
```

# ${f R}$ code ${f Exa}$ 13.9 Statistical intervals and test procedures

```
1 #Ex13.9, Page 548
2 #Answers may vary slightly due to rounding off of
      values
4 Thickness <-c
      (220, 220, 220, 220, 370, 370, 370, 370, 440, 440, 440, 440, 680, 680, 680, 680,
5 Strength <-c
      (24.0,22.0,19.1,15.5,26.3,24.6,23.1,21.2,25.2,24.0,21.7,19.2,17.0
6
7 model <-lm (Strength Thickness+I (Thickness 2))
9 new<-data.frame(Thickness=500)</pre>
10 y_cap <-predict(model, new)</pre>
11 print(paste("ycap when x=500:", y_cap))
13 #To find confidence and prediction intervals
14 cat ("95% CI:\n")
15 print(predict(model, newdata=data.frame(Thickness
      =500), interval = "confidence"))
16 cat("95% PI:\n")
17 print(predict(model, newdata=data.frame(Thickness
      =500), interval="prediction"))
```

# R code Exa 13.10 Centering x values

```
1 #Ex13.10, Page 549
2 #Answers may vary slightly due to rounding off of
      values
3
4 \text{ x} < -c (280, 284, 292, 295, 298, 305, 308, 315)
5 x_dash<-c
      (-17.13,-13.13,-5.13,-2.13,0.87,7.87,10.87,17.87)
6 y < -c (770,800,840,810,735,640,590,560)
7 df <-data.frame(x,x_dash,y)</pre>
9 #To find estimated coefficients and standard
      deviations using original model
10 m1 < -lm(y^x + I(x^2))
11 print(summary(m1))
12 Estimate1 <-c(NULL)
13 Estimated_SD1<-c(NULL)
14 for(i in 1:3){
     Estimate1[i] <-m1$coefficients[i]</pre>
15
     Estimated_SD1[i] <-coef(summary(m1))[, "Std. Error"
16
        ][i]
17 }
18 cat ("Estimated coefficients and standard deviations
      using original model\n")
19 Parameter1 <-c ("beta0", "beta1", "beta2")
20 df1<-data.frame(Parameter1, Estimate1, Estimated_SD1)
21 print(df1)
22
23 #To find estimated coefficients and standard
      deviations using centered model
24 Estimate2 <-c (NULL)
25 Estimated_SD2<-c(NULL)
26 \text{ m2} < -1\text{m} (y^x_dash + I(x_dash^2))
```

# R code Exa 13.12 Estimating parameters

```
11 m1<-lm(Strength~(Force+Power+Temperature+Time))
12
13 #To form estimated regression equation
14 Force<-35 #Force
15 Power<-75 #Power
16 Temperature<-200 #Temperature
17 Time<-20 #Time
18
19 new<-data.frame(Force,Power,Temperature,Time)
20 y_cap<-predict(m1,new)
21 print(paste("Point prediction of strength at (35,75,200,20):",y_cap,"gm"))</pre>
```

# R code Exa 13.13 Estimating parameters

```
1 #Ex13.13, Page 560
2 #Answers may vary slightly due to rounding off of
      values
3
4 temp1<-x1<-c(21,21,7,7,28,0,14,14,14)
5 \text{ temp2} < -x2 < -c
      (0.65, 0.55, 0.65, 0.55, 0.60, 0.60, 0.70, 0.50, 0.60)
6 x1x2 < -x1 * x2
7 Comp_str<-c
      (33.55,47.55,35.00,35.90,40.90,39.10,31.55,48.00,42.30)
  Adsorbability <-c
      (8.42, 6.26, 6.74, 6.59, 7.28, 6.90, 10.80, 5.63, 7.43)
9 df1<-data.frame(x1,x2,x1x2,Comp_str,Adsorbability)
10
11 #To find mean and SST values
12 ybar_compstr<-mean(Comp_str)
13 ybar_Adsorb <-mean (Adsorbability)</pre>
14 SST_CompStr <-sum ((Comp_str-ybar_compstr)^2)
15 SST_Adsorb <-sum ((Adsorbability -ybar_Adsorb)^2)
```

```
16 cat ("Mean of compression strength values:", ybar_
      compstr, " \ n")
17 cat ("Mean of adsorbability values:", ybar_Adsorb,"\n"
      )
18 cat("SST of compression strength:", SST_CompStr,"\n")
19 cat("SST of adsorbability:", SST_Adsorb," \n\n")
20
21 #First-order regression model
22 m1 < -lm(Comp_str^{(x1+x2)})
23 SSE1 \leftarrow sum(resid(m1)^2)
24 cat("SSE for first-order model:", SSE1, "\n")
25 cat("R squared value:", summary(m1) r.squared, "\n\n")
26
27 #Model including interaction predictor
28 m2 < -lm(Comp_str^{(x1*x2)})
29 SSE2 \leftarrow sum(resid(m2)^2)
30 cat ("SSE for first-order model including interaction
       predictor: ", SSE2, "\n")
31 cat("R squared value:", summary(m2) r.squared, "\n\n")
32
33 #To create estimated regression function
34 x1<-14 #% limestone
35 x2<-0.6 #Water-cement ratio
36 \text{ new} < -\text{data.frame}(x1, x2)
37 y_cap<-predict(m2,new)
38 cat ("Prediction of compression strength at (14,0.6):
      ",y_cap,"\n")
39
40 #When adsorbability is taken as the dependent
      variable
41 #First-order regression model
42 m3<-lm(Adsorbability~(temp1+temp2))
43 cat ("R squared value for first-order model:", summary
      (m3)$r.squared,"\n")
44
45 #Model including interaction predictor
46 \text{ m4} < -1 \text{m} (Adsorbability}^{\text{(temp1}*temp2)}
47 cat ("R squared value when using interaction
```

# R code Exa 13.14 Model utility test

```
1 #Ex13.14, Page 562
3 #H0: beta1=beta2=beta3=beta4=0
4 #Ha: at least one of these four betas!=0
6 Observation <-1:30
7 Force <-c
     8 Power <-c
     (60,60,90,90,60,60,90,90,60,60,90,90,60,60,90,90,75,75,45,105,75,
9 Temperature <-c
     (175, 175, 175, 175, 225, 225, 225, 225, 175, 175, 175, 175, 225, 225, 225, 225,
10 \text{ Time} < -c
     11 Strength <-c
     (26.2,26.3,39.8,39.7,38.6,35.5,48.8,37.8,26.6,23.4,38.6,52.1,39.5
12
13 Model <-cbind (Force, Power, Temperature, Time)
14
15 m1<-lm(Strength~Model)
16 print(summary(m1))
17
18 #To display analysis of variance table
19 a <- summary (aov (Strength Model))</pre>
20 print(a)
21
```

```
22 print(paste("F value:",a[[1]] $F[1]))
```

# R code Exa 13.15 Inferences in multiple regression

```
1 #Ex13.15, Page 564
2 #Answers may vary slightly due to rounding off of
      values
3
4 x1<-c
      (61, 175, 111, 124, 130, 173, 169, 169, 160, 244, 257, 333, 199)
5 \text{ x2} < -c (13, 21, 24, 23, 64, 38, 33, 61, 39, 71, 112, 88, 54)
6 y < -c (4, 18, 14, 18, 26, 26, 21, 30, 28, 36, 65, 62, 40)
7 data1 <-data.frame(x1,x2,y)</pre>
9 model < -lm(y^{(x1+x2)})
10 print(summary(model))
11
12 new < -data.frame(x1=160, x2=39)
13 y_cap<-predict(model,new)</pre>
14 print(paste("y_cap:",y_cap))
15
16 cat ("\n99\% CI:\n")
17 print(confint(model,level=0.99))
18
19 #To find confidence and prediction intervals
20 cat("95% CI:\n")
21 print(predict(model, new, interval="confidence"))
22 cat("95% PI:\n")
23 print(predict(model, new, interval="prediction"))
```

R code Exa 13.16 Inferences in multiple regression

```
1 #Ex13.16, Page 566
 2 #Answers may slightly vary due to rounding off of
       values
 3
4 Observation <- 1:30
 5 x1<-c
       (8,2,7,10,7,7,5,4,5,8,2,4,6,10,4,4,10,5,8,10,2,6,7,5,8,4,6,4,7)
 6 \times 2 < -c
       (4,4,4,7,4,7,13,4,7,1,10,4,10,7,13,10,13,10,4,13,1,13,13,1,13,1,13,1,1
       (100, 180, 180, 120, 180, 180, 140, 160, 140, 100, 140, 100, 180, 120, 180, 160,
 8 \times 4 < -c
       (1,7,1,5,5,1,1,7,3,7,3,3,3,7,3,5,7,7,1,1,1,1,7,7,1,7,7,1,1,7)
9 y<-c
       (1.4, 2.2, 4.6, 4.9, 4.6, 4.7, 4.6, 4.5, 4.8, 1.4, 4.7, 1.6, 4.5, 4.7, 4.8, 4.6,
10 x12 <- x1^2
11 \times 22 < -x2^2
12 x32<-x3^2
13 x42 < -x4^2
14 x1x2 < -x1 * x2
15 x1x3<-x1*x3
16 x1x4<-x1*x4
17 x2x3<-x2*x3
18 \quad x2x4 \leftarrow x2 * x4
19 x3x4 < -x3 * x4
20 df1 <-data.frame (Observation, x1, x2, x3, x4, y)
21 print(df1)
22
23 #To find estimate for reduced model
24 cat("\nReduced model:\n")
25 \text{ m1} < -1 \text{m} (y^x 1 + x 2 + x 3 + x 4)
26 print(summary(m1))
27
```

```
28 #To find estimate for full model
29 cat("Full model:\n")
30 \text{ m2} < -1\text{m} (y^{(x1+x2+x3+x4+x12+x22+x32+x42+x1x2+x1x3+x1x4)})
      +x2x3+x2x4+x3x4))
31 print(summary(m2))
32
33 #H0: beta5=beta6 = .... = beta14=0
34 #Ha: at least one among beta5,....beta14 is not 0
35 n < - 30
36 k<-14
37 1 < - 4
38 \text{ alpha} < -0.01
39 #To find F value
40 Fval<-qf(1-alpha,df1=k-1,df2=n-(k+1))
41 cat("F value:", Fval," \n")
42
43 #Unexplained variation for the full model
44 SSEk <- sum (resid (m2)^2)
45
46 #Unexplained variation for the reduced model
47 SSE1 <-sum (resid (m1)^2)
48
49 #f value
50 f < ((SSE1 - SSEk)/(k-1))/(SSEk/(n-(k+1)))
51 cat("f value:",f,"\n")
52
53 if (f>=Fval) cat ("HO is rejected since", f, ">=", Fval)
```

#### R code Exa 13.18 Transformations

```
1 #Ex13.18, Page 575
2
3 s<-c
     (20,20,20,20,20,20,20,20,60,60,60,60,60,60,60,60,100,100,10</pre>
```

# Chapter 14

# Goodness of fit tests and categorical data analysis

R code Exa 14.1 Goodness of fit tests when category probabilities are completely specified

```
1 #Ex14.1, Page 597
2 #Answers may vary slightly due to rounding off of
      value
4 p < -c (9/16, 3/16, 3/16, 1/16)
5 \text{ ni} < -c (926, 288, 293, 104)
6 n<-1611
7 k < -4
8 	 df < -k-1
10 npi <- round (n*p, digits=1)
11 df1<-data.frame(ni,npi)</pre>
12
13 #To find contribution to chi square from each cell
14 chi <-c (NULL)
15 for(i in 1:length(ni)){
     s<-((ni[i]-npi[i])^2)/npi[i]</pre>
16
17
     chi[i] <-s
```

```
18 }
19 print(paste("Chi-square values:"))
20 print(chi)
21
22 sum<-sum(chi)
23 print(paste("Chi square value:",sum))
24
25 alpha<-0.1
26 #To find chi square value
27 chival<-qchisq(1-alpha,df)
28 print(paste("Critical value:",chival))
29
30 if(sum < chival) print(paste("HO cannot be rejected")) else print(paste("HO can be rejected"))</pre>
```

# R code Exa 14.2 P values for chi squared tests

```
1 #Ex14.2, Page 599
2 #Answers may vary slightly due to rounding off of
      values
4 p<-9/16
5 n < -4
6 k < -5
7 	 df < -k-1
8 b < -c (NULL)
9 prob<-dbinom(0:4,n,p)</pre>
10 print(prob)
11
12 npi<-269*prob
13 print(npi)
14
15 Observed <-c (16, 45, 100, 82, 26)
16 Expected <-npi
17
```

```
18 df1<-data.frame(Observed, Expected)
19
20 #To find contribution to chi square from each cell
21 chi <-c (NULL)
22 for(i in 1:length(prob)){
23
     s<-((Observed[i]-Expected[i])^2)/Expected[i]
     chi[i] <-round(s,digits=3)</pre>
24
25 }
26 print(paste("Chi-square values:"))
27 print(chi)
28
29 sum <-sum (chi)
30 print(paste("Chi square value:", sum))
31
32 \text{ alpha} < -0.01
33 #To find chi square value
34 chival <-qchisq(1-alpha, df)
35 print(paste("Critical value:",chival))
36
37 if(sum < chival) print(paste("H0 cannot be rejected"
      )) else print(paste("H0 can be rejected"))
```

R code Exa 14.3 Chi square when underlying distribution is continuous

```
1 #Ex14.3, Page 600
2 #Answers may vary slightly due to rounding off of
    values
3
4 k<-24
5 df<-k-1
6 p<-1/24
7
8 #Cell counts
9 Observed<-c
    (52,73,89,88,68,47,58,47,48,53,47,34,21,31,40,24,37,31,47,34,36,4)</pre>
```

```
10 npi<-1186*p
11 Expected <-c (NULL)
12 for(i in 1:length(Observed)){
13
     Expected[i] <-npi</pre>
14 }
15 chi <-c (NULL)
16 for(i in 1:length(Observed)){
17
     s<-((Observed[i]-Expected[i])^2)/Expected[i]</pre>
18
     chi[i] <-round(s, digits=3)</pre>
19 }
20 sum <-sum (chi)
21 print(paste("Chi square value:", sum))
22
23 alpha <- 0.01
24 #To find chi square value
25 chival <-qchisq(1-alpha, df)
26 print(paste("Critical value:",chival))
27
28 if (sum < chival) print (paste ("HO cannot be rejected"
      )) else print(paste("H0 can be rejected"))
```

R code Exa 14.4 Chi square when underlying distribution is continuous

```
1 #Ex14.4, Page 600
2 #Answers may vary slightly due to rounding off fo
    values
3
4 #Cell counts
5 Observed<-c(21,17,12,16,10,15,19,10)
6 p<-1/8
7 n<-120
8 k<-8
9 df<-k-1
10 npi<-n*p</pre>
```

```
11
12 Expected <-c (NULL)
13 for(i in 1:length(Observed)){
     Expected[i] <-npi</pre>
14
15 }
16 chi <-c (NULL)
17 for(i in 1:length(Observed)){
     s<-((Observed[i]-Expected[i])^2)/Expected[i]</pre>
18
     chi[i] <-round(s, digits=3)</pre>
19
20 }
21 sum <-sum (chi)
22 print(paste("Chi square value:", sum))
23
24 \text{ alpha} < -0.1
25 #To find chi square value
26 chival <-qchisq(1-alpha, df)</pre>
27 print(paste("Critical value:",chival))
29 if(sum < chival) print(paste("H0 cannot be rejected"
      )) else print(paste("H0 can be rejected"))
```

# R code Exa 14.5 Chi square when parameters are estimated

```
1 #Ex14.5, Page 603
2
3 Type<-c("Observed")
4 M<-c(125)
5 MN<-c(225)
6 N<-c(150)
7 print(paste("Observed counts:"))
8 df<-data.frame(Type,M,MN,N)
9 print(df)
10
11 n<-500</pre>
```

# R code Exa 14.6 Chi square when parameters are estimated

```
1 #Ex14.6, Page 604
2 #Answers may slightly due to rounding off of values
3
4 k<-3
5 m < -1
6 \text{ theta} < -0.475
7 n < -500
8 \text{ npi1} < -500 * \text{theta}^2
9 npi2 < -n*2*theta*(1-theta)
10 npi3<-n-npi1-npi2
11
12 Observed <-c (125,225,150)
13 Estimated_expected <-c(npi1, npi2, npi3)</pre>
14
15 chi <-c (NULL)
16 for(i in 1:length(Observed)){
     s<-((Observed[i]-Estimated_expected[i])^2)/
17
        Estimated_expected[i]
     chi[i] <-round(s, digits=3)</pre>
18
19 }
20 sum <-sum (chi)
21 print(paste("Chi square value:", sum))
22
23 alpha<-0.05
24 \quad df < -k-m-1
25 #To find chi square value
26 chival <-qchisq(1-alpha, df)
27 print(paste("Critical value:",chival))
```

```
28
29 if(sum < chival) print(paste("H0 is rejected")) else
print(paste("H0 can be rejected"))
```

#### R code Exa 14.9 Goodness of fit for discrete distributions

```
1 #Ex14.9, Page 608
2 #Answers may vary slightly due to rounding off of
      values
3
4 k < -5
5 \text{ m} < -1
6 \text{ mu} < -2.10
7 n < -48
8 	 df < -k-m-1
9
10 Frequency <-c(9,9,10,14,6)
11
12 npi <-c (NULL)
13 for(i in 1:(k-1)){
14
     num \leftarrow (exp(1)^((-1)*mu))*(mu^(i-1))
15
     npi[i] <-n*(num/factorial(i-1))</pre>
16
17 }
18 npi[5] <-n-npi[1] -npi[2] -npi[3] -npi[4]
19 print(npi)
20
21 chi <-c (NULL)
22 for(i in 1:length(Frequency)){
     s<-((Frequency[i]-npi[i])^2)/npi[i]
23
24
     chi[i] <-round(s, digits=3)</pre>
25 }
26 sum <-sum (chi)
27 print(paste("Chi square value:", sum))
28
```

```
29 alpha<-0.05
30 #To find chi square value
31 chival<-qchisq(1-alpha,df)
32 print(paste("Critical value:",chival))
33
34 if(sum < chival) print(paste("H0 is not rejected since",sum,"<",chival)) else print(paste("H0 can be rejected"))</pre>
```

# R code Exa 14.10 Goodness of fit for continuous distributions

```
1 #Ex14.10, Page 608
2 #Answers may vary slightly from textbook values
3
4 data<-c
      (204, 108, 140, 152, 158, 129, 175, 146, 157, 174, 192, 194, 144, 152, 135, 223,
6 n<-length(data)
7 mu_cap <-round (mean (data), digits=2)</pre>
8 sigma_cap<-round(sqrt((n-1)*(sd(data)^2)/n),digits</pre>
9 print(paste("mu_cap:",mu_cap))
10 print(paste("sigma_cap:",sigma_cap))
11
12 #To find estimated expected counts
13 interval <-c(-Inf
      ,117.9,132.9,144.6,155.4,167.1,182.1,Inf)
14 p <- c (NULL)
15 npi <-c(NULL)
16 for(i in 1:length(interval)-1){
     p[i] <-pnorm(interval[i+1], mean=mu_cap, sd=sigma_cap</pre>
        )-pnorm(interval[i],mean=mu_cap,sd=sigma_cap)
     npi[i] <-n*p[i]
18
19 }
```

```
20
21 Cell \leftarrow c("(-Inf, 117.9)", "(117.9, 132.9)","
       (132.9,144.6)","(144.6,155.4)","(155.4,167.1)","
       (167.1,182.1)","(182.1,Inf)")
22 Observed <-c(5,5,11,6,6,7,9)
23 Estimated_expected <-npi
24 data1 <-data.frame (Cell, Observed, Estimated_expected)
25 print(data1)
26
27 #To find chi-squared values
28 \text{ alpha} < -0.95
29 	ext{ df1} \leftarrow length(npi) - 1
30 \text{ m} < -2
31 q1<-qchisq(alpha, df=df1)
32 print(paste("X^2(0.05, k-1):",q1))
33
34 df2 < -df1-m
35 \text{ q2} \leftarrow \text{qchisq} (\text{alpha}, \text{df} = \text{df2})
36 print(paste("X^2(0.05, k-1-m):",q2))
```

# R code Exa 14.12 A special test for normality

```
10 #To find z percentiles
11 for(i in 1:length(xi)){
     z[i] \leftarrow round(qnorm(1-((i-0.5)/length(xi)),lower.
12
        tail=FALSE),digits=2)
13 }
14
15 #Shapiro test is similar to Ryan-Joiner test which
      is not available in R
16 t<-shapiro.test(xi)
17 print(t)
18
19 #To create normal probability plot
20 qqnorm(xi,datax=TRUE,ylab="Voltage",xlab="
      Probability", main="Normal probability plot")
21 qqline(xi,datax=TRUE)
```

# R code Exa 14.13 Testing for homogeneity

```
1 #Ex14.13, Page 615
2
3 print(paste("Reason for nonconformity:"))
4 Blemish <-c (34,23,32)
5 Crack < -c (65, 52, 28)
6 Location <-c(17,25,16)
7 Missing <-c (21,19,14)
8 Other <-c(13,6,10)
9 Sample_size <-c(150,125,100)
10 df1<-data.frame(Blemish, Crack, Location, Missing, Other
      ,Sample_size)
11 print(df1)
12
13
14 #Total values
15 Blemish_tot<-sum(Blemish)
16 Crack_tot<-sum(Crack)
```

```
17 Location_tot <- sum (Location)
18 Missing_tot<-sum(Missing)
19 Other_tot<-sum(Other)
20 Sample_tot <-sum(Sample_size)
21
22 #HO: production lines are homogeneous
23 #Ha: production lines are not not homogeneous
24
25 tbl <-cbind.data.frame(df1$Blemish,df1$Crack,df1$
      Location, df1$Missing, df1$Other)
26 c<-chisq.test(tbl,df1\$Sample_size)
27 print(c)
28
29 print(paste("P value:",c$p.value))
30
31 print(paste("HO should not be rejected at levels
      0.05 \text{ or } 0.01"))
```

# R code Exa 14.14 Testing for independence

```
11 print(Observed)
12
13 print(paste("Expected pricing policy"))
14 Expected <-matrix (data=E, nrow=3, ncol=3, byrow=TRUE)
15 rownames (Expected) <-c("Substandard", "Standard","
      Modern")
16 colnames (Expected) <-c ("Aggressive", "Neutral","
      Nonaggressive")
17 print(Expected)
18
19 chi <-c (NULL)
20 for(i in 1:length(Observed)){
21
       chi[i] <- (Observed[i] - Expected[i]) ^2/(Expected[i</pre>
          ])
22 }
23 sum <-sum (chi)
24 print(paste("Chi square value:", sum))
25
26 \text{ alpha} < -0.01
27 #To find chi square value
28 chival <-qchisq(1-alpha, df = 4)
29 print(paste("Critical value (Chi square):", chival))
```

# Chapter 15

# Distribution free procedures

R code Exa 15.1 The Wilcoxon signed rank test

R code Exa 15.2 Paired observations

```
1 #Ex15.2, Page 630
2
3 library(stats)
```

```
5 IF_REE<-c
      (1753.7,1604.4,1576.5,1279.7,1754.2,1695.5,1700.1,1717.0)
6 Std_REE<-c
      (1755.0,1691.1,1697.1,1477.7,1785.2,1669.7,1901.3,1735.3)
7 Difference <- IF_REE-Std_REE
9 #Performing wilcoxon signed rank test
10 w1 <- wilcox.test(Difference)
                                    #or wilcox.test(IF_
     REE, Std_REE, paired=TRUE)
11 print(w1)
12
13 #Along with continuity correction
14 w2 <-wilcox.test(Difference, exact=FALSE)</pre>
                                               #or wilcox
      . test (IF_REE, Std_REE, paired=TRUE, exact=FALSE)
15 print(w2)
```

## R code Exa 15.3 Large sample approximation

```
10 alpha_level<-0.01
11
12 w1<-wilcox.test(xi_50000,Signed_rank,paired=T,exact=
        F,correct=F,alt="two.sided")
13
14 #Using p value:
15 if(w1$p.value<alpha_level) print(paste("H0 is rejected since p value,",w1$p.value,"is smaller than level,",alpha_level)) else print(paste("H0 is not rejected"))</pre>
```

#### R code Exa 15.4 The Wilcoxon rank sum test

```
1 #Ex15.4, Page 637
2
3 Polluted <-c(21.3,18.7,23.0,17.1,16.8,20.9,19.7)
4 Unpolluted <-c(14.2,18.3,17.2,18.4,20.0)
5
6 #Performing Wilcoxon rank sum test....
7 print(wilcox.test(Polluted,Unpolluted,conf.level=0.99))</pre>
```

## R code Exa 15.5 A normal approximation for W

```
1 #Ex15.5, Page 638
2 #Answers may vary slightly due to rounding off of
    values
3
4 Allergics <- c
    (67.6,39.6,1651.0,100.0,65.9,1112.0,31.0,102.4,64.7)</pre>
```

```
5 Nonallergics <-c
      (34.3,27.3,35.4,48.1,5.2,29.1,4.7,41.7,48.0,6.6,18.9,32.4,45.5)
6
7 m <-length (Allergics)</pre>
8 n<-length(Nonallergics)</pre>
10 #H0: mu1-mu2=0
11
12 new <-c (Allergics, Nonallergics)
13
14 #To find rank of combined vectors
15 r <- rank (new)
16 s<-0
17 for(i in 1:length(Allergics)){
     s<-s+r[i]
19 }
20 print(paste("Rank sum of allergics:",s))
21
22 #To find mean and variance of W
23 mu_w < -(m+n+1)*m/2
24 print(paste("Mean of W:",mu_w))
25 \text{ var_w} - \text{m*n*(m+n+1)} / 12
26 print(paste("Variance of W:", var_w))
27
28 #To find test statistic value
29 z<-(s-mu_w)/sqrt(var_w)
30 print(paste("Test statistic value:",z))
31
32 \text{ alpha} < -0.01
33 z1<-qnorm(alpha/2)
34
35 if(z>=z1 || z<=z1) print(paste("H0 is rejected"))
      else print(paste("H0 is not rejected"))
```

# R code Exa 15.6 The Wilcoxon signed rank interval

#### R code Exa 15.8 The Wilcoxon rank sum interval

#### R code Exa 15.9 The Kruskal Wallis test

```
1 #Ex15.9, Page 646
2 #Answers may vary slightly due to rounding off of
      values
3
4 data<-c
      (309.2,309.7,311.0,316.8,326.5,349.8,409.5,331.0,347.2,348.9,361.4
5 m1<-matrix(data,nrow=5,dimnames=list(c("4''","6''","
      8 ''","10''","12''")),byrow=TRUE)
6 print(m1)
7
8 N<-length(data)
10 #To find ranks of respective data
11 r1<-matrix(rank(m1), nrow=5)</pre>
12 cat ("Ranks of data\n")
13 print(r1)
14
15 \quad I \leftarrow nrow(r1)
16 \text{ J} < -ncol(r1)
17
18 #To find rank sum and mean with respect to each
      plate length type
19 ri<-c(NULL)
20 ribar <-c(NULL)
21 for(i in 1:nrow(r1)){
     ri[i] <-sum(r1[i,])
22
     ribar[i] <-round(mean(r1[i,]),digits=2)
23
24 }
25 df1<-data.frame(ri,ribar)
```

```
26 cat ("Rank sum and mean\n")
27 print(df1)
28
29 #To find k value
30 \text{ sum} < -0
31 for(j in 1:nrow(r1)){
     sum <-sum + (ri[j])^2/J</pre>
33 }
34 \text{ k} < -(12/(N*(N+1)))*sum - 3*(N+1)
35 print(paste("k value:",k))
36
37 #To find chi-square value
38 \text{ alpha} < -0.99
39 chival <-qchisq(alpha, df=I-1)
40 print(paste("Chi-squared value:",chival))
42 if (k>=chival) print (paste ("HO is not rejected since"
      ,k,">=",chival)) else print(paste("H0 is
      rejected"))
```

R code Exa 15.10 Friedman test for a randomized block experiment

```
9 r1<-rank(m1[,1])
10 r2<-rank(m1[,2])
11 r3<-rank(m1[,3])
12 r4<-rank(m1[,4])
13 r5<-rank(m1[,5])
14 r6<-rank(m1[,6])
15 r7<-rank(m1[,7])
16 r8<-rank(m1[,8])
17 data1<-data.frame(r1,r2,r3,r4,r5,r6,r7,r8)
18 cat("\nRanks of data:\n")
19 print (data1)
20
21 I<-nrow(data1)
22 J<-ncol(data1)
23
24 #To find rank sum and mean with respect to each
      plate length type
25 ri<-c(NULL)
26 \text{ ri2} < -c (NULL)
27 for(i in 1:nrow(data1)){
     ri[i] <-sum(data1[i,])
     ri2[i] <- (ri[i])^2
29
30 }
31 df1<-data.frame(ri,ri2)
32 print(df1)
33
34 #To find test statistic
35 Fr \leftarrow (12/(I*J*(I+1)))*sum(ri2)-3*J*(I+1)
36 print(paste("Test statistic value:",Fr))
37
38 #To find chi-square value
39 \text{ alpha} < -0.95
40 chival <-qchisq(alpha, df=I-1)
41 print(paste("Chi-squared value:",chival))
43 if (Fr<chival) print (paste ("HO is not rejected since"
      ,Fr,"<",chival)) else print(paste("H0 is
      rejected"))
```

# Chapter 16

# Quality control methods

R code Exa 16.1 X bar chart based on known parameter values

# R code Exa 16.2 X bar charts based on estimated parameters

```
1 #Ex16.2, Page 657
2 #Answers may vary slightly due to rounding off of
      values
3
4 n<-3
5 k < -25
6 \text{ xdoublebar} < -261.896/k
7 \text{ sbar} < -3.834/k
8 a3 < -0.886
9
10 #To find control limits
11 LCL <- xdoublebar -n*sbar/(a3*sqrt(n))</pre>
12 UCL <- xdoublebar + n * sbar / (a3 * sqrt(n))
13 print(paste("LCL:", LCL))
14 print(paste("UCL:", UCL))
15
16 #Old limits:
17 \# \text{mu} = 10.5, \text{ sigma} = 0.18
18
19 #New limits:
20 print(paste("mu_cap:",xdoublebar))
21 print(paste("sigma_cap:",sbar/a3))
```

#### R code Exa 16.3 X bar charts based on estimated parameters

```
1 #Ex16.3, Page 658
2
3 n<-3
4 k<-25
5 xdoublebar<-261.896/k</pre>
```

```
6 rbar<-0.292
7 b3<-1.693
8 sigma_cap<-0.292/b3
9
10 #To find control limits
11 LCL<-xdoublebar-3*rbar/(b3*sqrt(n))
12 UCL<-xdoublebar+3*rbar/(b3*sqrt(n))
13 print(paste("LCL:",round(LCL,digits=3)))
14 print(paste("UCL:",round(UCL,digits=3)))</pre>
```

#### R code Exa 16.4 The S chart

```
1 #Ex16.4, Page 663
2 #Answers may vary slightly due to rounding off of
      values
3
4 #Package to be installed: qcc(Quality control charts
  library(qcc)
6
7 n < -4
8 k<-22
9 obs1<-c
     (29.7,32.2,35.9,28.8,30.9,30.6,32.3,32.0,24.2,33.7,35.3,28.1,28.7
10 obs2<-c
      (29.0,29.3,29.1,27.2,32.6,34.3,27.7,27.9,27.5,24.4,33.2,34.0,28.9
11 obs3<-c
      (28.8,32.2,32.1,28.5,28.3,34.8,30.9,31.0,28.5,34.3,31.4,31.0,25.8
12 obs4<-c
      (30.2,32.9,31.3,35.7,28.3,26.3,27.8,30.8,31.1,31.0,28.0,30.8,29.7
13 SD<-c
```

```
(0.64,1.60,2.83,3.83,2.11,3.94,2.30,1.76,2.85,4.53,3.09,2.41,1.71
14 Range <-c
       (1.4, 3.6, 6.8, 8.5, 4.3, 8.5, 4.6, 4.1, 6.9, 9.9, 7.3, 5.9, 3.9, 4.0, 4.4, 5.2,
15 data1<-data.frame(obs1,obs2,obs3,obs4)
16
17 #Sum of SD values
18 si \leftarrow sum(SD)
19 print(paste("Sum of SD values:",si))
20 sbar <-mean (SD)
21 print(paste("Mean of SD values:", sbar))
22
23 \text{ a}4 < -0.921
24 LCL<-0
25 UCL<-sbar+3*sbar*sqrt(1-a4^2)/a4
26 print(paste("LCL:",LCL))
27 print(paste("UCL:", UCL))
28
29 #To plot S chart
30 \text{ q4} \leftarrow \text{qcc}(\text{data1}, \text{type} = \text{"S"})
31 print(summary(q4))
```

#### R code Exa 16.5 R chart

```
1 #Ex16.5, Page 665
2 #Answers may vary slightly due to rounding off of
     values
3
4 #Package to be installed: qcc(Quality control charts
     )
5 library(qcc)
6
7 des_dim<-c
     (200,250,300,350,400,450,500,550,600,650,700,750,800,850,900,950,</pre>
```

```
8 obs1<-c(12,6,5,19,9,9,8,4,11,13,10,8,14,7,14,10,7)
9 obs2 < -c(17,9,9,6,14,15,11,14,14,9,14,9,7,9,5,12,11)
10 obs3<-c(6,17,15,11,9,8,12,11,7,9,8,4,9,12,8,10,15)
11 mean <-c
      (11.7,10.7,9.7,12.0,10.7,10.7,10.3,9.7,10.7,10.3,10.7,7.0,10.0,9.
12 range <-c (11,11,10,13,5,7,4,10,7,4,6,5,7,5,9,2,8)
13 st_dev<-c
      (5.51,5.69,5.03,6.56,2.84,3.79,2.08,5.13,3.51,2.31,3.06,2.65,3.61
14 data1 <-data.frame(obs1,obs2,obs3)</pre>
15
16 n < - 3
17 b3<-1.693
18 c3<-0.888
19
20 #To find sum of range values
21 ri<-sum(range)
22 rbar <-mean(range)
23
24 #To find control limits
25 UCL <-rbar+n*c3*rbar/b3
26 print(paste("LCL:", LCL))
27 print(paste("UCL:", UCL))
28
29 #To display X-bar chart (can only be displayed
      separately by using run method)
30 q1<-qcc(data1,type="xbar")
31 print(summary(q1))
32
33 #To display R chart (can only be displayed
      separately by using run method)
34 q2<-qcc(data1,type="R")
35 print(summary(q2))
```

# R code Exa 16.6 The p chart

```
1 #Ex16.6, Page 669
2 #Answers may vary slightly due to rounding off of
      values
4 #Package to be installed: qcc(Quality control charts
5 library(qcc)
7 Dayi<-c
      (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,2
8 xi<-c
      (7,4,3,6,4,9,6,7,5,3,7,8,4,6,2,9,7,6,7,11,6,7,4,8,6)
9 pi<-c
      (0.07, 0.04, 0.03, 0.06, 0.04, 0.09, 0.06, 0.07, 0.05, 0.03, 0.07, 0.08, 0.04)
10 data1 <-data.frame(xi,pi)</pre>
11
12 #Sum of pi values
13 p<-sum(pi)
14 print(paste("Sum of pi values:",p))
15 pbar <-mean(pi)
16 print(paste("Mean of p values:",pbar))
17
18 #To find control limits
19 LCL <-pbar - 3 * sqrt (pbar * (1-pbar) / 100)</pre>
20 UCL<-pbar+3*sqrt(pbar*(1-pbar)/100)
21 print(paste("LCL:",LCL))
22 print(paste("UCL:", UCL))
23
24 #To display p chart
```

```
25 q1<-qcc(data1,sizes=length(Dayi),type="p")
26 print(summary(q1))</pre>
```

## R code Exa 16.7 c chart for number of defectives

```
1 #Ex16.7, Page 670
2 #Answers may slightly vary due to rounding off of
      values
4 #Package to be installed: qcc(Quality control charts
5 library(qcc)
7 x < -c
      (7,10,9,12,13,6,13,7,5,11,8,10,13,9,21,10,6,8,3,12,7,11,14,10)
9 #To find the sum of x values
10 xi < -sum(x)
11 xbar <-mean(x)
12 print(paste("Sum of x values:",xi))
13 print(paste("Mean of x values:",xbar))
14
15 #To find control limits
16 LCL <-xbar-3*sqrt(xbar)
17 UCL <- xbar + 3 * sqrt (xbar)
18 print(paste("LCL:",LCL))
19 print(paste("UCL:", UCL))
20
21 #To display c chart
22 q1 <-qcc(x, sizes=length(x), type="c")
23 print(summary(q1))
24
25 #From the chart, the 15th value lies above the UCL
26 #Eliminating that observation .....
```

```
27 xbar1 <- (sum(x) -x[15]) / (length(x) -1)
28 print(paste("Mean value when the 15th value is
        eliminated:",xbar1))
29 LCL1 <- xbar1 - 3 * sqrt(xbar1)
30 UCL1 <- xbar1 + 3 * sqrt(xbar1)
31 print(paste("Control limits when 15th value is
        eliminated:"))
32 print(paste("New LCL value:",LCL1))
33 print(paste("New UCL value:",UCL1))
34
35 #Remaining 23 observation lie within control lines (
        in - control)</pre>
```

# R code Exa 16.8 CUSUM procedures

```
1 #Ex16.8, Page 674
3 #Package to be installed: qcc(Quality control charts
4 library(qcc)
5
6 n < -4
7 \, \text{mu} = \text{mu} \, 0 = 40
8 \text{ sigma} < -0.5
9 obs<-c
      (40.77,39.95,40.86,39.21,38.94,39.70,40.37,39.88,40.43,40.27,40.9
10 sa < -c(rep(1,4), rep(2,4), rep(3,4), rep(4,4), rep(5,4),
      rep(6,4),rep(7,4),rep(8,4),rep(9,4),rep(10,4),rep
      (11,4), rep(12,4), rep(13,4), rep(14,4), rep(15,4),
      rep(16,4))
11
12 #To display X-bar chart
13 q<-qcc.groups(data=obs,sample=sa)
```

14 q4<-qcc(q,center=mu0,std.dev=sigma,type="xbar")

# R code Exa 16.9 Computational version

```
1 #Ex16.9, Page 676
2
3 #Package to be installed: qcc(Quality control charts
  library(qcc)
5
6 \text{ mu} 0 < -40
7 delta<-0.3
8 k<-delta/2
9 1<-mu0-k
10 r < - mu0+k
11 obs<-c
      (40.77,39.95,40.86,39.21,38.94,39.70,40.37,39.88,40.43,40.27,40.9
12 sa < -c(rep(1,4), rep(2,4), rep(3,4), rep(4,4), rep(5,4),
      rep(6,4),rep(7,4),rep(8,4),rep(9,4),rep(10,4),rep
      (11,4), rep(12,4), rep(13,4), rep(14,4), rep(15,4),
      rep(16,4))
13 xbar<-c
      (40.20, 39.72, 40.42, 39.98, 40.06, 39.76, 39.65, 40.41, 40.32, 39.84, 40.4
14
15 xbar1<-xbar-r
16 xbar2<-xbar-1
17
18 #To find d and e values
19 d0<-0
20 d1 < -max(0, d0 + (xbar[1] - r))
21 e0<-0
22 e1 < -max(0, e0 - (xbar[1] - 1))
23
```

```
24  d<-c(d1)
25  e<-c(e1)
26  for(i in 2:length(xbar)){
27    d[i]<-max(0,d[i-1]+(xbar[i]-r))
28    e[i]<-max(0,e[i-1]-(xbar[i]-1))
29  }
30
31  df<-data.frame(xbar,xbar1,d,xbar2,e)
32  print(df)
33
34  q<-qcc.groups(data=obs,sample=sa)
35  c<-cusum(q,sizes=4,se.shift=delta,center=mu0)
36  print(summary(c))</pre>
```

# R code Exa 16.10 Designing a CUSUM procedure

```
#Ex16.10, Page 680

sigma<-0.004
kbar<-0.74
delta<-0.003

n<-round((2*kbar*sigma/delta)^2,digits=0)
print(paste("Sample size:",n))

hbar<-3.2
h<-(sigma/sqrt(n))*hbar
print(paste("h value:",h))

print(paste("Out of control signal if d>",h,"or e>",
h))
```

# R code Exa 16.11 Acceptance sampling

```
1 #Ex16.11, Page 681
     3 \text{ P} \leftarrow \text{function}(p) \{(1-p)^50+50*p*(1-p)^49+1225*p^2*(1-p)^649+1225*p^2*(1-p)^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+1225*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125*p^649+125
                                    p) ^48}
     4
     5 p<-c
                                      (0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10, 0.12, 0.15)
     7 #Finding P(A) values
    8 PA<-c(NULL)
    9 for(i in 1:length(p)){
 10
                                PA[i] <-round(P(p[i]), digits=3)</pre>
11 }
12
13 df <-data.frame(p,PA)</pre>
14 print(df)
15
16 #To plot data
17 plot(p,PA,ylab="P(A)",main="Curve plotting P(A) and
                                    p values")
```

# R code Exa 16.12 Designing a single sample plan

```
1 #Ex16.12, Page 683
2 #Answers may vary slightly due to rounding off of
    values
3
4 #Package to be installed: AcceptanceSampling
5 library(AcceptanceSampling)
6
7 c<-0:15
8 np1<-c
    (0.051,0.355,0.818,1.366,1.970,2.613,3.285,3.981,4.695,5.425,6.16)</pre>
```

```
9 np2<-c
      (2.30,3.89,5.32,6.68,7.99,9.28,10.53,11.77,12.99,14.21,15.41,16.6
10 p2_p1<-np2/np1
11 df1<-data.frame(c,np1,np2,p2_p1)</pre>
12 print(df1)
13 cat("\n")
14
15 AQL < -p1 < -0.01
16 LTPD<-p2<-0.045
17
18 #Ratio of p2 to p1
19 r < - LTPD / AQL
20 print(paste("Ratio of p2 to p1:",r))
21
22 for(i in 1:length(c)){
23
     if(r>p2_p1[i+1] && r<p2_p1[i]){</pre>
24
       j<-i
25
       np_1<-np1[i]
26
       c1<-c[i]
27
       c2 < -c[i+1]
28
       np_2<-np2[i+1]
29
30
     }
31 }
32 cat("Ratio lies between",p2_p1[j],"and",p2_p1[j+1],"
      values corresponding to c values", c[j], and c[j]
      +1],"\n")
33
34 #Single sample plan: 1st c value
35 cat("When c value is",c1,"\n")
36
37 ##To find sample size
38 n<-round(np_1/p1,digits=0)
39 print(paste("Sample size:",n))
40
41 ##To find alpha and beta values
42 alpha1 < -1-pbinom(c1,n,p1)
```

```
43 print(paste("Alpha value:",alpha1))
44
45 beta1 \leftarrow pbinom (c1,n,p2)
46 print(paste("Beta value:",beta1))
47
48 #Single sample plan: 2nd c value
49 cat ("When c value is", c2, "\n")
50
51 #To find sample size
52 \text{ n} < -\text{round} (\text{np}_2/\text{p2}, \text{digits} = 0)
53 print(paste("Sample size:",n))
54
55 ##To find alpha and beta values
56 alpha2<-1-pbinom(c2,n,p1)
57 print(paste("Alpha value:",alpha2))
58
59 \text{ beta2} \leftarrow \text{pbinom}(c2,n,p2)
60 print(paste("Beta value:",beta2))
```