

R Textbook Companion for  
Numerical Methods for Engineers  
by S. C. Chapra and R. P. Canale<sup>1</sup>

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# Book Description

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R numbering policy used in this document and the relation to the above book.

**Exa** Example (Solved example)

**Eqn** Equation (Particular equation of the above book)

For example, Exa 3.51 means solved example 3.51 of this book. Sec 2.3 means an R code whose theory is explained in Section 2.3 of the book.

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# Chapter 1

## Mathematical Modelling and Engineering Problem Solving

**R code Exa 1.1** Analytical Solution to Falling Parachutist Problem

```
1 g=9.8
2 #m/s^2; acceleration due to gravity
3 m=68.1
4 #kg
5 c=12.5
6 #kg/sec; drag coefficient
7 count=1
8 v = matrix(0,1)
9 for (i in (seq(0,12,2))){
10   v[count]=g*m*(1-exp(-c*i/m))/c
11   cat("v(m/s)=",v[count],"Time(s)=",i)
12   count=count+1;
13 }
14 cat("v(m/s)=",g*m/c,"Time(s)=", "infinity")
```

---

## Chapter 3

# Approximations and Round off Errors

**R code Exa 3.1** Calculations of Errors

```
1  lbm=9999
2  #cm, measured length of bridge
3  lrm=9
4  #cm, measured length of rivet
5  lbt=10000
6  #cm, true length of bridge
7  lrt=10
8  #cm, true length of rivet
9
10 #calculating true error below;
11 Etb=lbt-lbm
12 #cm, true error in bridge
13 Etr=lrt-lrm
14 #cm, true error in rivet
15
16 #calculating percent relative error below
17 etb=Etb*100/lbt
18 #percent relative error for bridge
19 etr=Etr*100/lrt
```

```

20 #percent relative error for rivet
21 cat("a. The true error is")
22 cat(Etb,"cm","for the bridge")
23 cat(Etr,"cm","for the rivet")
24 cat("b. The percent relative error is")
25 cat(etb,"%","for the bridge")
26 cat(etr,"%","for the rivet")

```

---

### R code Exa 3.2 Iterative error estimation

```

1  n=3
2  #number of significant figures
3  es=0.5*(10^(2-n))
4  #percent, specified error criterion
5  x=0.5;
6  f = matrix(0,1)
7  f[1]=1
8  #first estimate f=e^x = 1
9  ft=1.648721
10 #true value of e^0.5=f
11 et = matrix(0,1)
12 et[1]=(ft-f[1])*100/ft
13 ea = matrix(0,1)
14 ea[1]=100;
15 i=2
16 while (ea[i-1]>=es){
17   f[i]=f[i-1]+(x^(i-1))/(factorial(i-1))
18   et[i]=(ft-f[i])*100/ft
19   ea[i]=(f[i]-f[i-1])*100/f[i]
20   i=i+1
21 }
22 for (j in 1:i-1){
23   cat("term number=",j,"\n","Result=",f[j],"\n","
      True % relative error=",et[j],"\n","Approximate
      estimate of error(%)=",ea[j],"\n")

```

```

24   cat("
      n")
25 }

```

---

### R code Exa 3.3 Range of Integers

```

1  n=16
2  #no of bits
3  num=0
4  for (i in 0:(n-2)){
5    num=num+(1*(2^i))
6  }
7  cat("Thus a 16-bit computer word can store decimal
      integers ranging from",(-1*num),"to",num)

```

---

### R code Exa 3.4 Floating Point Numbers

```

1  n=7
2  #no. of bits
3  #the maximum value of exponents is given by
4  Max=1*(2^1)+1*(2^0)
5  #mantissa is found by
6  mantissa=1*(2^-1)+0*(2^-3)+0*(2^-3)
7  num=mantissa*(2^(Max*-1))
8  #smallest possible positive number for this system
9  cat("The smallest possible positive number for this
      system is",num)

```

---

### R code Exa 3.5 Machine Epsilon

```

1 b=2
2 #base
3 t=3
4 #number of mantissa bits
5 E=2^(1-t)
6 #epsilon
7 cat("value of epsilon=",E)

```

---

### R code Exa 3.6 Interdependent Computations

```

1 readinteger <- function()
2 {
3   n <- readline(prompt="Input a number: ")
4   return(as.integer(n))
5 }
6
7 num<-readinteger()
8
9 sum=0
10 for (i in 1:100000){
11   sum=sum+num
12 }
13 cat("The number summed up 100,000 times is=",sum)

```

---

### R code Exa 3.7 Subtractive Cancellation

```

1 a=1
2 b=3000.001
3 c=3
4 #the roots of the quadratic equation x^2+3000.001*x
   +3=0 are found as
5 D=(b^2)-4*a*c
6 x1=(-b+(D^0.5))/(2*a)

```

```

7 x2=(-b-(D^0.5))/(2*a)
8 cat("The roots of the quadratic equation (x^2)
    +(3000.001*x)+3=0 are = ",x1,"and",x2)

```

---

### R code Exa 3.8 Infinite Series Evaluation

```

1 f <- function(x) {
2   exp(x)
3 }
4
5 sum=1
6 test=0
7 i=0
8 term=1
9 x1= 10
10 x2= -10
11 while (sum~=test){
12   cat("sum:",sum,"\n","term:",term,"\n","i:",i,"\n",
    "-----\n")
13   i=i+1
14   term=term*x/i
15   test=sum
16   sum=sum+term
17 }
18 cat("Exact Value",f(x1))
19 cat("Exact Value",f(x2))

```

---

## Chapter 4

# Truncation Errors and the Taylor Series

R code Exa 4.1 Polynomial Taylor Series

```
1 DD <- function(expr, name, order = 1) {
2   if(order < 1) stop("'order' must be >= 1")
3   if(order == 1) D(expr, name)
4   else DD(D(expr, name), name, order - 1)
5 }
6
7 f <- function(x) {
8   return(-0.1*x^4-0.15*x^3-0.5*x^2-0.25*x+1.2)
9 }
10
11 xi=0
12 xf=1
13 h=xf-xi
14 fi=f(xi)
15 #function value at xi
16 ffa=f(xf)
17 #actual function value at xf
18
19 #for n=0, i.e, zero order approximation
```

```

20 ff=fi
21 Et = matrix(0,5)
22 Et[1]=ffa-ff
23 #truncation error at x=1
24 cat("The value of f at x=0 :",fi,"\n",
25     "The value of f at x=1 due to zero order
      approximation :",ff,"\n",
26     "Truncation error :",Et[1],"\n",
27     "-----\n")
28
29 #for n=1, i.e, first order approximation
30 f1 <- function(x) {
31     return(eval(DD(expr = expression(-0.1*x^4-0.15*x
      ^3-0.5*x^2-0.25*x+1.2),"x",1)))
32 }
33
34 f1i=f1(xi)
35 #value of first derivative of function at xi
36 f1f=fi+f1i*h
37 #value of first derivative of function at xf
38 Et[2]=ffa-f1f
39 #truncation error at x=1
40 cat("The value of first derivative of f at x=0 :",
      f1i,"\n",
41     "The value of f at x=1 due to first order
      approximation :",f1f,"\n",
42     "Truncation error :",Et[2],"\n",
43     "-----\n")
44
45
46 #for n=2, i.e, second order approximation
47 f2 <- function(x) {
48     return(eval(DD(expr = expression(-0.1*x^4-0.15*x
      ^3-0.5*x^2-0.25*x+1.2),"x",2)))
49 }
50

```



```

51
52 f2i=f2(xi)
53 #value of second derivative of function at xi
54 f2f=f1f+f2i*(h^2)/factorial(2)
55 #value of second derivative of function at xf
56 Et[3]=ffa-f2f
57 #truncation error at x=1
58 cat("The value of first derivative of f at x=0 :",
      f2i,"\n",
59      "The value of f at x=1 due to first order
      approximation :",f2f,"\n",
60      "Truncation error :",Et[3],"\n",
61      "-----\n")
62
63 #for n=3, i.e, third order approximation
64 f3 <- function(x) {
65   return(eval(DD(expr = expression(-0.1*x^4-0.15*x
      ^3-0.5*x^2-0.25*x+1.2),"x",3)))
66 }
67 f3i=f3(xi)
68 #value of third derivative of function at xi
69 f3f=f2f+f3i*(h^3)/factorial(3)
70 #value of third derivative of function at xf
71 Et[4]=ffa-f3f
72 #truncation error at x=1
73 cat("The value of first derivative of f at x=0 :",
      f3i,"\n",
74      "The value of f at x=1 due to first order
      approximation :",f3f,"\n",
75      "Truncation error :",Et[4],"\n",
76      "-----\n")
77
78 #for n=4, i.e, fourth order approximation
79 f4 <- function(x) {
80   return(eval(DD(expr = expression(-0.1*x^4-0.15*x
      ^3-0.5*x^2-0.25*x+1.2),"x",4)))

```

```

81 }
82 f4i=f4(xi)
83 #value of fourth derivative of function at xi
84 f4f=f3f+f4i*(h^4)/factorial(4)
85 #value of fourth derivative of function at xf
86 Et[5]=ffa-f4f
87 #truncation error at x=1
88 cat("The value of first derivative of f at x=0 :",
      f4i,"\n",
89      "The value of f at x=1 due to first order
      approximation :",f4f,"\n",
90      "Truncation error :",Et[5],"\n",
91      "-----\n")

```

---

#### R code Exa 4.2 Taylor Series Expansion

```

1 DD <- function(expr, name, order = 1) {
2   if(order < 1) stop("'order' must be >= 1")
3   if(order == 1) D(expr, name)
4   else DD(D(expr, name), name, order - 1)
5 }
6
7 f <- function(x) {
8   return(cos(x))
9 }
10
11 pi = 3.1415927
12 et = matrix(0,7)
13
14 xi=pi/4
15 xf=pi/3
16 h=xf-xi
17 fi=f(xi)
18 #function value at xi

```

```

19 ffa=f(xf)
20 #actual function value at xf
21
22 #for n=0, i.e, zero order approximation
23 ff=fi;
24 et[1]=(ffa-ff)*100/ffa
25 #percent relative error at x=1
26 cat("The value of f at x=1 due to zero order
    approximation :",ff,"\n",
27      "% relative error :",et[1],"\n",
28      "-----\n"
    n")
29
30
31 #for n=1, i.e, first order approximation
32 f1 <- function(x) {
33     return(eval(DD(expr = expression(cos(x)), "x", 1)))
34 }
35 f1i=f1(xi)
36 #value of first derivative of function at xi
37 f1f=fi+f1i*h
38 #value of first derivative of function at xf
39 et[2]=(ffa-f1f)*100/ffa
40 #% relative error at x=1
41 cat("The value of f at x=1 due to first order
    approximation :",f1f,"\n",
42      "% relative error :",et[2],"\n",
43      "-----\n"
    n")
44
45
46 #for n=2, i.e, second order approximation
47 f2 <- function(x) {
48     return(eval(DD(expr = expression(cos(x)), "x", 2)))
49 }
50 f2i=f2(xi)
51 #value of second derivative of function at xi
52 f2f=f1f+f2i*(h^2)/factorial(2)

```

```

53 #value of second derivative of function at xf
54 et[3]=(ffa-f2f)*100/ffa
55 #% relative error at x=1
56 cat("The value of f at x=1 due to second order
    approximation :",f2f,"\n",
57     "% relative error :",et[3],"\n",
58     "-----\n")
59
60 #for n=3, i.e, third order approximation
61 f3 <- function(x) {
62     return(eval(DD(expr = expression(cos(x)),"x",3)))
63 }
64 f3i=f3(xi)
65 #value of third derivative of function at xi
66 f3f=f2f+f3i*(h^3)/factorial(3)
67 #value of third derivative of function at xf
68 et[4]=(ffa-f3f)*100/ffa
69 #% relative error at x=1
70 cat("The value of f at x=1 due to third order
    approximation :",f3f,"\n",
71     "% relative error :",et[4],"\n",
72     "-----\n")
73
74 #for n=4, i.e, fourth order approximation
75 f4 <- function(x) {
76     return(eval(DD(expr = expression(cos(x)),"x",4)))
77 }
78 f4i=f4(xi)
79 #value of fourth derivative of function at xi
80 f4f=f3f+f4i*(h^4)/factorial(4)
81 #value of fourth derivative of function at xf
82 et[5]=(ffa-f4f)*100/ffa
83 #% relative error at x=1
84 cat("The value of f at x=1 due to fourth order
    approximation :",f4f,"\n",
85     "% relative error :",et[5],"\n",

```

```

86     "-----\
      n")
87
88
89 #for n=5, i.e, fifth order approximation
90 f5i=(f4(1.1*xi)-f4(0.9*xi))/(2*0.1)
91 #value of fifth derivative of function at xi (
    central difference method)
92 f5f=f4f+f5i*(h^5)/factorial(5)
93 #value of fifth derivative of function at xf
94 et[6]=(ffa-f5f)*100/ffa
95 #% relative error at x=1
96 cat("The value of f at x=1 due to fifth order
    approximation :",f5f,"\n",
97     "% relative error :",et[6],"\n",
98     "-----\
      n")
99
100
101 #for n=6, i.e, sixth order approximation
102 f6 <- function(x) {
103     return(eval(DD(expr = expression(cos(x)), "x", 4)))
104 }
105 f6i=(f4(1.1*xi)-2*f4(xi)+f4(0.9*xi))/(0.1^2)
106 #value of sixth derivative of function at xi (
    central difference method)
107 f6f=f5f+f6i*(h^6)/factorial(6)
108 #value of sixth derivative of function at xf
109 et[7]=(ffa-f6f)*100/ffa
110 #% relative error at x=1
111 cat("The value of f at x=1 due to sixth order
    approximation :",f6f,"\n",
112     "% relative error :",et[7],"\n",
113     "-----\
      n")

```

---

#### R code Exa 4.4 Finite divided difference approximation of derivatives

```
1 DD <- function(expr, name, order = 1) {
2   if(order < 1) stop("'order' must be >= 1")
3   if(order == 1) D(expr, name)
4   else DD(D(expr, name), name, order - 1)
5 }
6
7 f <- function(x) {
8   return(-0.1*(x^4)-0.15*(x^3)-0.5*(x^2)-0.25*(x)
9         +1.2)
10 }
11 x=0.5
12 h= 0.5
13 x1=x-h
14 x2=x+h
15 #forward difference method
16 fdx1=(f(x2)-f(x))/h
17 #derivative at x
18 et1=abs((fdx1-eval(DD(expr = expression(-0.1*(x^4)
19   -0.15*(x^3)-0.5*(x^2)-0.25*(x)+1.2), name = "x",
20   order = 1)))/eval(DD(expr = expression(-0.1*(x^4)
21   -0.15*(x^3)-0.5*(x^2)-0.25*(x)+1.2), name = "x",
22   order = 1)))*100
23 #backward difference method
24 fdx2=(f(x)-f(x1))/h
25 #derivative at x
26 et2=abs((fdx2-eval(DD(expr = expression(-0.1*(x^4)
27   -0.15*(x^3)-0.5*(x^2)-0.25*(x)+1.2), name = "x",
28   order = 1)))/eval(DD(expr = expression(-0.1*(x^4)
29   -0.15*(x^3)-0.5*(x^2)-0.25*(x)+1.2), name = "x",
30   order = 1)))*100
31 #central difference method
```

```

24 fdx3=(f(x2)-f(x1))/(2*h)
25 #derivative at x
26 et3=abs((fdx3-eval(DD(expr = expression(-0.1*(x^4)
      -0.15*(x^3)-0.5*(x^2)-0.25*(x)+1.2),name = "x",
      order = 1)))/eval(DD(expr = expression(-0.1*(x^4)
      -0.15*(x^3)-0.5*(x^2)-0.25*(x)+1.2),name = "x",
      order = 1)))*100
27 cat("For h=",h,"\n",
28      "Derivative at x by forward difference method=",
      fdx1,"and percent error=",et1,"\n",
29      "Derivative at x by backward difference method=",
      fdx2,"and percent error=",et2,"\n",
30      "Derivative at x by central difference method=",
      fdx3,"and percent error=",et3,"\n")
31
32
33 h= 0.25
34 x1=x-h
35 x2=x+h
36 #forward difference method
37 fdx1=(f(x2)-f(x))/h
38 #derivative at x
39 et1=abs((fdx1-eval(DD(expr = expression(-0.1*(x^4)
      -0.15*(x^3)-0.5*(x^2)-0.25*(x)+1.2),name = "x",
      order = 1)))/eval(DD(expr = expression(-0.1*(x^4)
      -0.15*(x^3)-0.5*(x^2)-0.25*(x)+1.2),name = "x",
      order = 1)))*100
40 #backward difference method
41 fdx2=(f(x)-f(x1))/h
42 #derivative at x
43 et2=abs((fdx2-eval(DD(expr = expression(-0.1*(x^4)
      -0.15*(x^3)-0.5*(x^2)-0.25*(x)+1.2),name = "x",
      order = 1)))/eval(DD(expr = expression(-0.1*(x^4)
      -0.15*(x^3)-0.5*(x^2)-0.25*(x)+1.2),name = "x",
      order = 1)))*100
44 #central difference method
45 fdx3=(f(x2)-f(x1))/(2*h)
46 #derivative at x

```

```

47 et3=abs((fdx3-eval(DD(expr = expression(-0.1*(x^4)
    -0.15*(x^3)-0.5*(x^2)-0.25*(x)+1.2),name = "x",
    order = 1))))/eval(DD(expr = expression(-0.1*(x^4)
    -0.15*(x^3)-0.5*(x^2)-0.25*(x)+1.2),name = "x",
    order = 1)))*100
48 cat("For h=",h,"\n",
49     "Derivative at x by forward difference method=",
    fdx1,"and percent error=",et1,"\n",
50     "Derivative at x by backward difference method=",
    fdx2,"and percent error=",et2,"\n",
51     "Derivative at x by central difference method=",
    fdx3,"and percent error=",et3,"\n")

```

---

#### R code Exa 4.5 Error propagation in function of single variable

```

1 DD <- function(expr, name, order = 1) {
2   if(order < 1) stop("'order' must be >= 1")
3   if(order == 1) D(expr, name)
4   else DD(D(expr, name), name, order - 1)
5 }
6
7 x=2.5
8 delta=0.01
9 deltafx=abs(eval(DD(expr = expression(x^3),name = "x",
    order = 1)))*delta
10 fx=f(x)
11 cat("true value is between",fx-deltafx,"and",fx+
    deltafx)

```

---

#### R code Exa 4.6 Error propagation in multivariable function

```

1 library(Deriv)
2

```



```

3 DD <- function(expr, name, order = 1) {
4   if(order < 1) stop("'order' must be >= 1")
5   if(order == 1) D(expr, name)
6   else DD(D(expr, name), name, order - 1)
7 }
8
9 f <- function(F,L,E,I) {
10  (F*(L^4))/(8*E*I)
11 }
12
13 Fbar=50
14 #lb/ft
15 Lbar=30
16 #ft
17 Ebar=1.5*(10^8)
18 #lb/ft^2
19 Ibar=0.06
20 #ft^4
21 deltaF=2
22 #lb/ft
23 deltaL=0.1
24 #ft
25 deltaE=0.01*(10^8)
26 #lb/ft^2
27 deltaI=0.0006
28 #ft^4
29 ybar=(Fbar*(Lbar^4))/(8*Ebar*Ibar)
30
31 f1 <- function(F) {
32  (F*(Lbar^4))/(8*Ebar*Ibar)
33 }
34 f_1<-Deriv(f1)
35 f2 <- function(L) {
36  (Fbar*(L^4))/(8*Ebar*Ibar)
37 }
38 f_2<-Deriv(f2)
39 f3 <- function(E) {
40  (Fbar*(Lbar^4))/(8*E*Ibar)

```

```

41 }
42 f_3<-Deriv(f3)
43 f4 <- function(I) {
44   (Fbar*(Lbar^4))/(8*Ebar*I)
45 }
46 f_4<-Deriv(f4)
47 deltay=abs(f_1(Fbar))*deltaF+
48   abs(f_2(Lbar))*deltaL+
49   abs(f_3(Ebar))*deltaE+
50   abs(f_4(Ibar))*deltaI;
51
52 cat("The value of y is between:",ybar-deltay,"and",
     ybar+deltay)
53 ymin=((Fbar-deltaF)*((Lbar-deltaL)^4))/(8*(Ebar+
     deltaE)*(Ibar+deltaI));
54 ymax=((Fbar+deltaF)*((Lbar+deltaL)^4))/(8*(Ebar-
     deltaE)*(Ibar-deltaI));
55 cat("ymin is calculated at lower extremes of F, L, E
     , I values as =",ymin)
56 cat("ymax is calculated at higher extremes of F, L,
     E, I values as =",ymax)

```

---

#### R code Exa 4.7 Condition Number

```

1 library(Deriv)
2
3 f <- function(x) {
4   tan(x)
5 }
6
7 f_ = Deriv(f)
8
9 pi = 3.1415927
10 x1bar=(pi/2)+0.1*(pi/2)
11 x2bar=(pi/2)+0.01*(pi/2)

```

```

12 #computing condition number for x1bar
13 condnum1=x1bar*f_(x1bar)/f(x1bar)
14 cat("The condition number of function for x=",x1bar,
    " is:",condnum1)
15 if (abs(condnum1)>1){
16     cat("Function is ill-conditioned for x=",x1bar)
17 }
18 #computing condition number for x2bar
19 condnum2=x2bar*f_(x2bar)/f(x2bar)
20 cat("The condition number of function for x=",x2bar,
    " is:",condnum2)
21 if (abs(condnum2)>1){
22     cat("Function is ill-conditioned for x=",x2bar)
23 }

```

---

# Chapter 5

## Bracketing Methods

R code Exa 5.1 Graphical Approach

```
1  m=68.1
2  #kg
3  v=40
4  #m/s
5  t=10
6  #s
7  g=9.8
8  #m/s^2
9
10 f <- function(c) {
11   g*m*(1-exp(-c*t/m))/c - v
12 }
13
14 cat("For various values of c and f(c) is found as:")
15 i=0
16 fc = matrix(0,5)
17 for (c in seq(4,20,4)){
18   i=i+1
19   bracket=c(c, f(c))
20   cat(bracket)
21   fc[i]=f(c)
```

```

22 }
23 c<-c(4, 8, 12, 16, 20)
24 plot(c,fc,main = 'f(c) vs c',xlab = 'c',ylab = 'f(c)
    (m/s)')
25 lines(c,fc)

```

---

### R code Exa 5.2 Computer Graphics to Locate Roots

```

1 f <- function(x) {
2   sin(10*x)+cos(3*x)
3 }
4
5 count=1
6 val = matrix(0,100)
7 func = matrix(0,100)
8 for (i in seq(1,5,0.05)){
9   val[count]=i
10  func[count]=f(i)
11  count=count+1
12 }
13 plot(val,func,main = "x vs f(x)",xlab = 'x',ylab = '
    f(x)')
14 lines(val,func)

```

---

### R code Exa 5.3 Bisection

```

1 m=68.1
2 #kg
3 v=40
4 #m/s
5 t=10
6 #s
7 g=9.8

```

```

8 #m/s^2
9
10 f <- function(c) {
11   g*m*(1-exp(-c*t/m))/c - v
12 }
13
14 x1=12
15 x2=16
16 xt=14.7802
17 #true value
18 #”enter the tolerable true percent error=”
19 e=2
20 xr=(x1+x2)/2
21 etemp=abs(xr-xt)/xt*100
22 #error
23 while (etemp>e){
24   if (f(x1)*f(xr)>0){
25     x1=xr
26     xr=(x1+x2)/2
27     etemp=abs(xr-xt)/xt*100
28   }
29   if (f(x1)*f(xr)<0){
30     x2=xr
31     xr=(x1+x2)/2
32     etemp=abs(xr-xt)/xt*100
33   }
34   if (f(x1)*f(xr)==0) {
35     break
36   }
37 }
38 cat("The result is=",xr)

```

---

#### R code Exa 5.4 Error Estimates for Bisection

```
1 m=68.1
```

```

2 #kg
3 v=40
4 #m/s
5 t=10
6 #s
7 g=9.8
8 #m/s^2
9
10 f <- function(c) {
11   g*m*(1-exp(-c*t/m))/c - v
12 }
13
14 x1=12
15 x2=16
16 xt=14.7802
17 #true value
18 #”enter the tolerable approximate error=”
19 e=0.5
20 xr=(x1+x2)/2
21 i=1
22 et=abs(xr-xt)/xt*100
23 #error
24 cat(" Iteration :",i)
25 cat(" x1:",x1)
26 cat(" x2:",x2)
27 cat(" xr:",xr)
28 cat(" et(%):" ,et)
29 cat("_____")
30 etemp=100
31
32 while (etemp>e){
33   if (f(x1)*f(xr)>0){
34     x1=xr
35     xr=(x1+x2)/2
36     etemp=abs(xr-x1)/xr*100
37     et=abs(xr-xt)/xt*100
38   }
39   if (f(x1)*f(xr)<0){

```

```

40     x2=xr
41     xr=(x1+x2)/2
42     etemp=abs(xr-x2)/xr*100
43     et=abs(xr-xt)/xt*100
44 }
45 if (f(x1)*f(xr)==0){
46     break
47 }
48 i=i+1
49 cat(" Iteration:",i)
50 cat(" x1:",x1)
51 cat(" xu:",x2)
52 cat(" xr:",xr)
53 cat(" et(%):" ,et)
54 cat(" ea(%)",etemp)
55 cat("-----")
56 }
57 cat("The result is=",xr)

```

---

### R code Exa 5.5 False Position

```

1  m=68.1
2  #kg
3  v=40
4  #m/s
5  t=10
6  #s
7  g=9.8
8  #m/s^2
9
10 f <- function(c) {
11     g*m*(1-exp(-c*t/m))/c - v
12 }
13
14 x1=12

```



```

15 x2=16
16 xt=14.7802
17 #true value
18 #"enter the tolerable true percent error="
19 e=
20 xr=x1-(f(x1)*(x2-x1))/(f(x2)-f(x1))
21 etemp=abs(xr-xt)/xt*100
22 #error
23 while (etemp>e){
24   if (f(x1)*f(xr)>0){
25     x1=xr
26     xr=x1-(f(x1)*(x2-x1))/(f(x2)-f(x1))
27     etemp=abs(xr-xt)/xt*100
28   }
29   if (f(x1)*f(xr)<0){
30     x2=xr
31     xr=x1-(f(x1)*(x2-x1))/(f(x2)-f(x1))
32     etemp=abs(xr-xt)/xt*100
33   }
34   if (f(x1)*f(xr)==0){
35     break
36   }
37 }
38 cat("The result is=",xr)

```

---

### R code Exa 5.6 Bracketing and False Position Methods

```

1 f <- function(x) {
2   x^10 - 1
3 }
4
5 x1=0
6 x2=1.3
7 xt=1
8

```

```

9 #using bisection method
10 cat("BISECTION METHOD:")
11 xr=(x1+x2)/2
12 et=abs(xr-xt)/xt*100
13 #error
14 cat("Iteration:",1,"\n", "xl:",x1,"\n", "xu:",x2,"\n",
    "xr:",xr,"\n", "et(%):",et,"\n",
    "-----\n")
15
16 for (i in 2:5){
17     if (f(x1)*f(xr)>0){
18         x1=xr
19         xr=(x1+x2)/2
20         ea=abs(xr-x1)/xr*100
21         et=abs(xr-xt)/xt*100
22     } else if (f(x1)*f(xr)<0){
23         x2=xr
24         xr=(x1+x2)/2
25         ea=abs(xr-x2)/xr*100
26         et=abs(xr-xt)/xt*100
27     }
28
29     if (f(x1)*f(xr)==0){
30         break
31     }
32 cat("Iteration:",i,"\n")
33 cat("xl:",x1,"\n")
34 cat("xu:",x2,"\n")
35 cat("xr:",xr,"\n")
36 cat("et(%):",et,"\n")
37 cat("ea(%)",ea,"\n")
38 cat("-----\n")
39 }
40
41 #using false position method
42 cat("FALSE POSITION METHOD:")
43 x1=0
44 x2=1.3

```

```

45 xt=1
46 xr=x1-(f(x1)*(x2-x1))/(f(x2)-f(x1))
47 et=abs(xr-xt)/xt*100
48 #error
49 cat("Iteration:",1,"\n","xl:",x1,"\n","xu:",x2,"\n",
    "xr:",xr,"\n","et(%):",et,"\n",
    "-----")
50
51 for (i in 2:5){
52     if (f(x1)*f(xr)>0){
53         x1=xr
54         xr=x1-(f(x1)*(x2-x1))/(f(x2)-f(x1))
55         ea=abs(xr-x1)/xr*100
56         et=abs(xr-xt)/xt*100
57     }
58     else if (f(x1)*f(xr)<0){
59         x2=xr
60         xr=x1-(f(x1)*(x2-x1))/(f(x2)-f(x1))
61         ea=abs(xr-x2)/xr*100
62         et=abs(xr-xt)/xt*100
63     }
64     if (f(x1)*f(xr)==0){
65         break
66     }
67 cat("Iteration:",i,"\n")
68 cat("xl:",x1,"\n")
69 cat("xu:",x2,"\n")
70 cat("xr:",xr,"\n")
71 cat("et(%):",et,"\n")
72 cat("ea(%)",ea,"\n")
73 cat("-----\n")
74 }

```

---

# Chapter 6

## Open Methods

**R code Exa 6.11** Newton Raphson for a nonlinear Problem

```
1 u <- function(x,y) {  
2   x^2+x*y-10  
3 }  
4  
5 v <- function(x,y) {  
6   y+3*x*y^2-57  
7 }  
8  
9 x=1.5  
10 y=3.5  
11 e<-c(100, 100)  
12 while (e[1]>0.0001 & e[2]>0.0001){  
13   J=matrix(data = c(2*x+y, x, 3*y^2, 1+6*x*y),nrow =  
14     2,ncol = 2,byrow = TRUE)  
15   deter=det(J)  
16   u1=u(x,y)  
17   v1=v(x,y)  
18   x=x-((u1*J[2,2]-v1*J[1,2])/deter)  
19   y=y-((v1*J[1,1]-u1*J[2,1])/deter)  
20   e[1]=abs(2-x)  
21   e[2]=abs(3-y)
```

```
21 }  
22 bracket<-c(x, y)  
23 cat(bracket)
```

---

# Chapter 7

## Roots of Polynomials

R code Exa 7.1 Polynomial Deflation

```
1 f <- function(x) {  
2   (x-4)*(x+6)  
3 }  
4  
5 n=2  
6 a = matrix(0,3)  
7 a[1]=-24  
8 a[2]=2  
9 a[3]=1  
10 t=4  
11 r=a[3]  
12 a[3]=0  
13 for (i in seq(n,1,-1)){  
14   s=a[i]  
15   a[i]=r  
16   r=s+r*t  
17 }  
18 cat("The quptient is a(1)+a(2)*x where :", "a(1)=", a  
    [1], "a(2)=", a[2], "remainder=", r)
```

---

### R code Exa 7.2 Mullers Method

```
1 f <- function(x) {
2   x^3 - 13*x - 12
3 }
4
5 x1t=-3
6 x2t=-1
7 x3t=4
8 x0=4.5
9 x1=5.5
10 x2=5
11
12 cat("iteration:",0,"\n","xr:",x2,"
    _____\n")
13
14 for (i in 1:4){
15   h0=x1-x0
16   h1=x2-x1
17   d0=(f(x1)-f(x0))/(x1-x0)
18   d1=(f(x2)-f(x1))/(x2-x1)
19   a=(d1-d0)/(h1+h0)
20   b=a*h1+d1
21   c=f(x2)
22   d=(b^2 - 4*a*c)^0.5
23   if (abs(b+d)>abs(b-d)){
24     x3=x2+((-2*c)/(b+d))
25   }else {
26     x3=x2+((-2*c)/(b-d))
27   }
28   ea=abs(x3-x2)*100/x3
29   x0=x1
30   x1=x2
31   x2=x3
```

```

32   cat(" iteration:",i,"\n")
33   cat(" xr:",x2,"\n")
34   cat(" ea(%):",ea,"\n")
35   cat("
                                     \n
      ")
36 }

```

---

### R code Exa 7.3 Bairstows Method

```

1  f <- function(x) {
2    x^5-3.5*x^4+2.75*x^3+2.125*x^2-3.875*x+1.25
3  }
4
5  r=-1
6  s=-1
7  es=1
8  #%
9  n=6
10 count=1
11 ear=100
12 eas=100
13 a<-c(1.25, -3.875, 2.125, 2.75, -3.5, 1)
14 b<-matrix(0,n)
15 c<-matrix(0,n)
16 while ((ear>es) & (eas>es)){
17   b[n]=a[n]
18   b[n-1]=a[n-1]+r*b[n]
19   for (i in seq(n-2,1,-1)){
20     b[i]=a[i]+r*b[i+1]+s*b[i+2]
21   }
22   c[n]=b[n]
23   c[n-1]=b[n-1]+r*c[n]
24   for (i in seq((n-2),2,-1)){
25     c[i]=b[i]+r*c[i+1]+s*c[i+2]

```



```

26   }
27   #c(3)*dr+c(4)*ds=-b(2)
28   #c(2)*dr+c(3)*ds=-b(1)
29   ds=((-b[1])+(b[2]*c[2]/c[3]))/(c[3]-(c[4]*c[2]/c[3])
      )
30   dr=(-b[2]-c[4]*ds)/c[3]
31   r=r+dr
32   s=s+ds
33   ear=abs(dr/r)*100
34   eas=abs(ds/s)*100
35   cat("Iteration:",count,"\n","delata r:",dr,"\n","
      delata s:",ds,"\n","r:",r,"\n","s:",s,"\n","Error
      in r:",ear,"\n","Error in s:",eas,"\n","
      _____\
      n")
36   count=count+1;
37   }
38   x1=(r+(r^2 + 4*s)^0.5)/2
39   x2=(r-(r^2 + 4*s)^0.5)/2
40   bracket<-c(x1, x2)
41   cat("The roots are:",bracket,"The quotient is:",x^3
      - 4*x^2 + 5.25*x - 2.5","\n","
      _____\
      n")
_____

```

#### R code Exa 7.4 Locate single root

```

1 f <- function(x) {
2   x-cos(x)
3 }
4
5 x1=0
6
7 if (f(x1)<0){
8   x2=x1+0.001

```

```

9   while (f(x2)<0){
10     x2=x2+0.001
11   }
12 } else if(f(x1)>0){
13   x2=x1+0.001
14   while (f(x2)>0){
15     x2=x2+0.001
16   }
17 } else{
18   cat("The root is=",x1)
19 }
20
21 x=x2-(x2-x1)*f(x2)/(f(x2)-f(x1))
22 cat("The root is=",x)

```

---

#### R code Exa 7.5 Solving nonlinear system

```

1 u <- function(x,y) {
2   x^2+x*y-10
3 }
4
5 v <- function(x,y) {
6   y+3*x*y^2-57
7 }
8
9 x=1
10 y=3.5
11 while (u(x,y)!=v(x,y)){
12   x=x+1
13   y=y-0.5
14 }
15 cat("x=",x)
16 cat("y=",y)

```

---

### R code Exa 7.6 Root Location

```
1 library(pracma)
2 fun <- function (x) x^10 -1
3 fzero(f = fun,x = c(0,4))
4 fzero(f = fun,x = c(0,1.3))
5 fzero(f = fun,x = c(-1.3,0))
6 fzero(f = fun,x = c(-1.28, 0.9051))
```

---

### R code Exa 7.7 Roots of Polynomials

```
1 library(pracma)
2 library(polynom)
3
4 fun <- function (x) (x^5 - (3.5*x^4) +(2.75*x^3)
   +(2.125*x^2) - (3.875*x) + 1.25)
5 fzero(f = fun,x =1)
6 Deriv::Deriv(f = fun,x = "x")
7
8 b<-c(1,0.5,-0.5)
9 a<-c(1,-3.5,2.75,2.125,-3.875,1.25)
10 answer = deconv(a,b)
11 d = answer$q
12 e = answer$r
13 polyroot(a)
14 polyroot(d)
15 conv(d,b)
16 a<-conv(d,b)
17 polyroot(a)
```

---

### R code Exa 7.8 Root Location

```
1 f <- function(x) {  
2   x-cos(x)  
3 }  
4  
5 x1=0  
6 if (f(x1)<0){  
7   x2=x1+0.00001  
8   while (f(x2)<0){  
9     x2=x2+0.00001  
10  }  
11 } else if (f(x1)>0){  
12   x2=x1+0.00001  
13   while (f(x2)>0){  
14     x2=x2+0.00001  
15   }  
16 } else {  
17   cat("The root is=",x1)  
18 }  
19  
20  
21 x=x2-(x2-x1)*f(x2)/(f(x2)-f(x1))  
22 cat("The root is=",x)
```

---

# Chapter 9

## Gauss Elimination

R code Exa 9.2 Determinants

```
1 #For fig9.1
2 a= matrix(data = c(3, 2,-1, 2),nrow = 2,ncol = 2,
  byrow = TRUE)
3 cat("The value of determinant for system represented
  in fig 9.1 =",det(a))
4 #For fig9.2 (a)
5 a= matrix(data = c(-0.5, 1,-0.5, 1),nrow = 2,ncol =
  2,byrow = TRUE)
6 cat("The value of determinant for system represented
  in fig 9.2 (a) =",det(a))
7 #For fig9.2 (b)
8 a= matrix(data = c(-0.5, 1,-1, 2),nrow = 2,ncol = 2,
  byrow = TRUE)
9 cat("The value of determinant for system represented
  in fig 9.2 (b) =",det(a))
10 #For fig9.2 (c)
11 a= matrix(data = c(-0.5, 1,-2.3/5, 1),nrow = 2,ncol
  = 2,byrow = TRUE)
12 cat("The value of determinant for system represented
  in fig 9.2 (c) =",det(a))
```

---

### R code Exa 9.3 Cramers Rule

```
1 #the matrix or the system
2 b1=-0.01
3 b2=0.67
4 b3=-0.44
5 a<-matrix(data = c(0.3, 0.52, 1,0.5, 1, 1.9,0.1,
6   0.3, 0.5),nrow = 3,ncol = 3,byrow = TRUE)
7 a1<-matrix(data = c(a[2,2], a[2,3],a[3,2], a[3,3]),
8   nrow = 2,ncol = 2,byrow = TRUE)
9 A1=det(a1)
10 a2<-matrix(data = c(a[2,1], a[2,3],a[3,1], a[3,3]),
11   nrow = 2,ncol = 2,byrow = TRUE)
12 A2=det(a2)
13 a3<-matrix(data = c(a[2,1], a[2,2],a[3,1], a[3,2]),
14   nrow = 2,ncol = 2,byrow = TRUE)
15 A3=det(a3)
16 D=a[1,1]*A1-a[1,2]*A2+a[1,3]*A3
17 p<-matrix(data = c(b1, 0.52, 1,b2, 1, 1.9,b3, 0.3,
18   0.5),nrow = 3,ncol = 3,byrow = TRUE)
19 q<-matrix(data = c(0.3, b1, 1,0.5, b2, 1.9,0.1, b3,
20   0.5),nrow = 3,ncol = 3,byrow = TRUE)
21 r<-matrix(data = c(0.3, 0.52, b1,0.5, 1 ,b2,0.1,
22   0.3, b3),nrow = 3,ncol = 3,byrow = TRUE)
23 x1=det(p)/D
24 x2=det(q)/D
25 x3=det(r)/D
26 cat("The values are:", "x1=",x1," ,x2=",x2," ,x3=",x3)
```

---

### R code Exa 9.4 Elimination of Unknowns

```

1 #the equations are:
2 #3*x1+2*x2=18
3 #-x1+2*x2=2
4 a11=3
5 a12=2
6 b1=18
7 a21=-1
8 a22=2
9 b2=2
10 x1=(b1*a22-a12*b2)/(a11*a22-a12*a21)
11 x2=(b2*a11-a21*b1)/(a11*a22-a12*a21)
12 cat("x1=",x1)
13 cat("x2=",x2)

```

---

#### R code Exa 9.5 Naive Gauss Elimination

```

1 n=3
2 b<-matrix(c(7.85,-19.3,71.4), nrow = 1, ncol = 3)
3 a<-matrix(data = c(3, -0.1, -0.2,0.1, 7 , -0.3,0.3,
  -0.2, 10),nrow = 3,ncol = 3,byrow = TRUE)
4 for (k in 1:1){
5   for (i in 2:3){
6     fact=a[i,k]/a[k,k]
7     for (j in 2:3){
8       a[i,j]=a[i,j]-fact*a[k,j]
9     }
10    b[i]=b[i]-fact*b[k]
11    print(b)
12  }
13 }
14 x<-matrix(0,3)
15 x[3]=b[3]/a[3,3]
16 for (i in seq(2,1,-1)){
17   s=b[i]
18   for (j in (i+1):3){

```

```

19     s=s-a[i,j]*x[j]
20     print(s)
21 }
22 x[i]=b[i]/a[i,i]
23 }
24 cat("x1=",x[1]," ,x2=",x[2]," ,x3=",x[3])

```

---

#### R code Exa 9.6 ill conditioned systems

```

1 a11=1
2 a12=2
3 b1=10
4 a21=1.1
5 a22=2
6 b2=10.4
7 x1=(b1*a22-a12*b2)/(a11*a22-a12*a21)
8 x2=(b2*a11-a21*b1)/(a11*a22-a12*a21)
9 cat("For the original system:", "x1=",x1," ,x2=",x2)
10 a21=1.05
11 x1=(b1*a22-a12*b2)/(a11*a22-a12*a21)
12 x2=(b2*a11-a21*b1)/(a11*a22-a12*a21)
13 cat("For the new system:", "x1=",x1," ,x2=",x2)

```

---

#### R code Exa 9.7 Effect of Scale on Determinant

```

1 #part a
2 a<-matrix(c(3, 2,-1, 2), nrow = 2, ncol = 2,byrow =
    TRUE)
3 b1=18
4 b2=2
5 cat("The determinant for part(a)=",det(a))
6 #part b

```



```

7 a<-matrix(c(1, 2,1.1, 2), nrow = 2, ncol = 2,byrow =
  TRUE)
8 b1=10
9 b2=10.4
10 cat("The determinant for part(b)=",det(a))
11 #part c
12 a1=a*10
13 b1=100
14 b2=104
15 cat("The determinant for part(c)=",det(a1))

```

---

#### R code Exa 9.8 Scaling

```

1 #part a
2 a<-matrix(c(1, 0.667,-0.5, 1), nrow = 2, ncol = 2,
  byrow = TRUE)
3 b1=6
4 b2=1
5 cat("The determinant for part(a)=",det(a))
6 #part b
7 a<-matrix(c(0.5, 1,0.55, 1), nrow = 2, ncol = 2,
  byrow = TRUE)
8 b1=5
9 b2=5.2
10 cat("The determinant for part(b)=",det(a))
11 #part c
12 b1=5
13 b2=5.2
14 cat("The determinant for part(c)=",det(a))

```

---

#### R code Exa 9.11 Solution of Linear Algebraic Equations

```

1 a<-matrix(c(70, 1, 0,60, -1, 1,40, 0 ,-1),nrow = 3,
            ncol = 3,byrow = TRUE)
2 b<-matrix(c(636,518,307),nrow = 3,ncol = 1,byrow =
            TRUE)
3 x=abs(solve(a,b))
4 cat("a=",x[1],"m/s ^2", "\n", "T=", x[2], "N", "\n", "R=", x
      [3], "N", "\n")

```

---

## Chapter 14

# Multidimensional Unconstrained Optimization

R code Exa 14.1 Random Search Method

```
1 maxf = -1e+09
2
3 n=10000
4 for (j in 1:n){
5   Rnd=runif(2)
6   x = -2 + 4 * Rnd[1]
7   y = 1 + 2 * Rnd[2]
8   fn = y - x - (2 * (x ^ 2)) - (2 * x * y) - (y ^ 2)
9   if (fn > maxf){
10     maxf = fn
11     maxx = x
12     maxy = y
13   }
14   if (mod(j,1000)==0){
15     cat(" Iteration:",j,"\n")
16     cat("x:",x,"\n")
17     cat("y:",y,"\n")
18     cat("function value:",fn,"\n")
19     cat("-----\n")
```

```

        n")
20   }
21 }

```

---

### R code Exa 14.2 Path of Steepest Descent

```

1  f <- function(x,y) {
2    x*y*y
3  }
4
5  p1<-c(2, 2)
6  elevation=f(p1[1],p1[2])
7  dfx=p1[1]*p1[1]
8  dfy=2*p1[1]*p1[2]
9  theta=atan(dfy/dfx)
10 slope=(dfx^2 + dfy^2)^0.5
11 cat("Elevation:",elevation,"Theta:",theta,"slope:",
      slope)

```

---

### R code Exa 14.3 1 D function along Gradient

```

1  f <- function(x,y) {
2    2*x*y + 2*x - x^2 - 2*y^2
3  }
4
5  x=-1
6  y=1
7  dfx=2*y+2-2*x
8  dfy=2*x-4*y
9  #the function can thus be expressed along h axis as
10 #f((x+dfx*h),(y+dfy*h))
11 cat("The final equation is=", "180*h^2 + 72*h - 7")

```

---

## R code Exa 14.4 Optimal Steepest Descent

```

1  f <- function(x,y) {
2    2*x*y + 2*x - x^2 - 2*y^2
3  }
4
5  x=-1
6  y=1
7  d2fx=-2
8  d2fy=-4
9  d2fxy=2
10
11 modH=d2fx*d2fy-(d2fxy)^2
12
13 for (i in 1:25){
14   dfx=2*y+2-2*x
15   dfy=2*x - 4*y
16   #the function can thus be expressed along h axis
17   as
18   #f((x+dfx*h),(y+dfy*h))
19   g <- function(h) {
20     2*(x+dfx*h)*(y+dfy*h) + 2*(x+dfx*h) - (x+dfx*h)
21     ^2 - 2*(y+dfy*h)^2
22   }
23   #2*dfx*(y+dfy*h)+2*dfy*(x+dfx*h)+2*dfx-2*(x+dfx*h)
24   #dfx-4*(y+dfy*h)*dfy=g'(h)=0
25   #2*dfx*y + 2*dfx*dfy*h + 2*dfy*x + 2*dfy*dfx*h + 2
26   #dfx - 2*x*dfx - 2*dfx*dfx*h - 4*y*dfy - 4*dfy*
27   #dfy*h=0
28   #h(2*dfx*dfy+2*dfy*dfx-2*dfx*dfx-4*dfy*dfy)=-(2*
29   #dfx*y+2*dfy*x-2*x*dfx-4*y*dfy)
30   h=(2*dfx*y+2*dfy*x-2*x*dfx-4*y*dfy+2*dfx)/(-1*(2*
31   dfx*dfy+2*dfy*dfx-2*dfx*dfx-4*dfy*dfy))
32   x=x+dfx*h

```

```
26     y=y+dfy*h
27 }
28 cat("The final values are:",x,"", y)
```

---

# Chapter 15

## Constrained Optimization

**R code Exa 15.1** Setting up LP problem

```
1 regular<-c(7, 10, 9 ,150)
2 premium<-c(11, 8 ,6 ,175)
3 res_avail<-c(77, 80)
4 #total profit(to be maximized)=z=150*x1+175*x2
5 #total gas used=7*x1+11*x2 (has to be less than 77 m
  ^3/week)
6 #similarly other constraints are developed
7 cat("Maximize z=150*x1+175*x2")
8 cat("subject to")
9 cat("7*x1+11*x2<=77 (Material constraint)")
10 cat("10*x1+8*x2<=80 (Time constraint)")
11 cat("x1<=9 (Regular storage constraint)")
12 cat("x2<=6 (Premium storage constraint)")
13 cat("x1,x2>=0 (Positivity constraint)")
```

---

**R code Exa 15.2** Graphical Solution

```
1 x21<-matrix(0,8)
```

```

2 x22<-matrix(0,8)
3 x23<-matrix(0,8)
4 x24<-matrix(0,8)
5 x25<-matrix(0,8)
6 x26<-matrix(0,8)
7 for (x1 in 0:8){
8   x21[x1+1]=-(7/11)*x1+7
9   x22[x1+1]=(80-10*x1)/8
10  x23[x1+1]=6
11  x24[x1+1]=-150*x1/175
12  x25[x1+1]=(600-150*x1)/175
13  x26[x1+1]=(1400-150*x1)/175
14 }
15 x1=0:8
16
17
18 plot(x1,x24,main = 'Z=0')
19 lines(x1,x25,main = 'Z=600')
20 lines(x1,x26,main = 'Z=1400')
21 plot(x1,x21,main = 'x2 vs x1')
22 plot(x1,x22,xlab = 'x1 (tonnes)')
23 plot(x1,x23,ylab = 'x2 (tonnes)')

```

---

### R code Exa 15.3 Linear Programming Problem

```

1 x1<-c(4.888889, 3.888889)
2 x2<-c(7, 11)
3 x3<-c(10, 8)
4 x4<-c(150, 175)
5 x5<-c(77, 80, 9, 6)
6 profit<-c(x1[1]*x4[1], x1[2]*x4[2])
7 total<-c(x1[1]*x3[1]+x1[2]*x3[2], x1[1]*x3[1]+x1[2]*
8           x3[2], x1[1], x1[2], profit[1]+profit[2])
9 e=1000

```



```

10 while (e>total[5]){
11     if (total[1]<=x5[1]){
12         if (total[2]<=x5[2]){
13             if (total[3]<=x5[3]){
14                 if (total[4]<=x5[4]){
15                     l=1
16                 }
17             }
18         }
19     }
20     if (l==1){
21         x1[1]=x1[1]+4.888889
22         x1[2]=x1[2]+3.888889
23         profit<-c(x1[1]*x4[1], x1[2]*x4[2])
24         total[5]=profit[1]+profit[2]
25     }
26 }
27 cat("The maximized profit is=",total[5])

```

---

#### R code Exa 15.4 Nonlinear constrained optimization

```

1 Mt=2000
2 #kg
3 g=9.8
4 #m/s^2
5 c0=200
6 # $
7 c1=56
8 #$/m
9 c2=0.1
10 #$/m^2
11 vc=20
12 #m/s
13 kc=3
14 #kg/(s*m^2)

```

```

15 z0=500
16 #m
17 t=27
18 r=2.943652
19 n=6
20 pi = 3.1415927
21 A=2*pi*r*r
22 l=(2^0.5)*r
23 c=3*A
24 m=Mt/n
25
26 f <- function(t) {
27   (z0+g*m*m/(c*c)*(1-exp(-c*t/m)))*c/(g*m)
28 }
29
30 while (abs(f(t)-t)>0.00001){
31   t=t+0.00001
32 }
33 v=g*m*(1-exp(-c*t/m))/c
34 cat("The final value of velocity=",v,"\n")
35 cat("The final no. of load parcels=",n,"\n")
36 cat("The chute radius=",r,"m","\n")
37 cat("The minimum cost ($)=", (c0+c1*l+c2*A*A)*n)

```

---

### R code Exa 15.5 One dimensional Optimization

```

1 library(neldermead)
2
3 fx <- function(x) {
4   -(2*sin(x))+x^2/10
5 }
6
7 x=fminsearch(fx,0)
8 x$output$algorithm
9 x = x$optbase$xopt

```

```

10 cat("After maximization:\n")
11 cat("x=",x)
12 cat("f(x)=",fx(x)," \n")

```

---

### R code Exa 15.6 Multidimensional Optimization

```

1 library(neldermead)
2
3 fx <- function(x) {
4   -(2*x[1]*x[2]+2*x[1]-x[1]^2-2*x[2]^2)
5 }
6
7 x=fminsearch(fun = fx,x0 = c(-1,1))
8 x = x$optbase$xopt
9 cat("After maximization:", "\n", "x=", x[1], ", ", x[2], "\n",
    "f(x)=", fx(x), "\n")

```

---

### R code Exa 15.7 Locate Single Optimum

```

1 fx <- function(x) {
2   -(2*sin(x)-x^2/10)
3 }
4
5 x=fminsearch(fx,0)
6 x = x$optbase$xopt
7 cat("After maximization:", "\n", "x=", x, "\n", "f(x)=",
    fx(x), "\n")

```

---

# Chapter 17

## Least squares regression

**R code Exa 17.3.a** linear regression using computer

```
1 s<-c(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15)
2 v<-c
    (10,16.3,23,27.5,31,35.6,39,41.5,42.9,45,46,45.5,46,49,50)

3 g = 9.8
4 #m/s^2
5 m = 68.1
6 #kg
7 c = 12.5
8 #kg/s
9 v1<-matrix(0,15)
10 v2<-matrix(0,15)
11 for (i in 1:15){
12   v1[i] = g*m*(1 - exp(-c*s[i]/m))/c
13   v2[i] = g*m*s[i]/(c*(3.75+s[i]))
14 }
15 cat("time = ",s,"\n","measured v =",v,"\n"," using
    equation (1.10) v1 = ", "\n",v1,"\n"," using
    equation ((17.3)) v2 =", "\n",v2)
16 plot(v,v1)
17 lines(v,v1,main = 'v vs v1',xlab = 'v',ylab = 'v1')
```

---

**R code Exa 17.3.b** linear regression using computer

```
1 s<-c(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15)
2 v<-c
    (10,16.3,23,27.5,31,35.6,39,41.5,42.9,45,46,45.5,46,49,50)

3 g = 9.8
4 #m/s^2
5 m = 68.1
6 #kg
7 c = 12.5
8 #kg/s
9 v1<-matrix(0,15)
10 v2<-matrix(0,15)
11 for (i in 1:15){
12   v1[i] = g*m*(1 - exp(-c*s[i]/m))/c
13   v2[i] = g*m*s[i]/(c*(3.75+s[i]))
14 }
15 cat("time = ",s,"\n","measured v =",v,"\n","using
    equation (1.10) v1 = ", "\n",v1,"\n","using
    equation ((17.3)) v2 =", "\n",v2)
16 plot(v,v2)
17 lines(v,v2,main = 'v vs v2',xlab = 'v',ylab = 'v2')
```

---

# Chapter 18

## Interpolation

**R code Exa 18.5** Error Estimates for Order of Interpolation

```
1 x<-c(1, 4, 6, 5, 3, 1.5, 2.5, 3.5)
2 y<-c(0, 1.3862944, 1.7917595, 1.6094379, 1.0986123,
      0.4054641, 0.9162907, 1.2527630)
3 n=8
4 fdd = matrix(0,nrow =n,ncol = n)
5 for (i in 1:n){
6   fdd[i,1]=y[i]
7 }
8
9 for (j in 2:n){
10   for (i in 1:(n-j+1)){
11     fdd[i,j]=(fdd[i+1,j-1]-fdd[i,j-1])/(x[i+j-1]-x[i
12       ])
13   }
14 xterm=1
15 yint<-matrix(0,1)
16 yint[1]=fdd[1,1]
17
18 order<-matrix(0,n)
19 Ea<-matrix(0,n)
```

```
20 for (order in 2:n){
21   xterm=xterm*(2-x[order-1])
22   yint2=yint[order-1]+fdd[1,order]*xterm
23   Ea[order-1]=yint2-yint[order-1]
24   yint[order]=yint2
25 }
26 cat("F(x)=",yint,"\n","Ea=",Ea)
```

---

# Chapter 19

## Fourier Approximation

R code Exa 19.1 Least Square Fit

```
1 f <- function(t) {  
2   1.7+cos(4.189*t+1.0472)  
3 }  
4  
5 deltat=0.15  
6 t1=0  
7 t2=1.35  
8 omega=4.189  
9 del=(t2-t1)/9  
10 t<-matrix(0,10)  
11 for (i in 1:10){  
12   t[i]=t1+del*(i-1)  
13 }  
14 sumy=0  
15 suma=0  
16 sumb=0  
17 y<-matrix(0,10)  
18 a<-matrix(0,10)  
19 b<-matrix(0,10)  
20 for (i in 1:10){  
21   y[i]=f(t[i])
```



```

22   a[i]=y[i]*cos(omega*t[i])
23   b[i]=y[i]*sin(omega*t[i])
24   sumy=sumy+y[i]
25   suma=suma+a[i]
26   sumb=sumb+b[i]
27 }
28 A0=sumy/10
29 A1=2*suma/10
30 B1=2*sumb/10
31 cat("The least square fit is y=A0+A1*cos(w0*t)+A2*
      sin(w0*t), where", "\n", "A0=", A0, "\n", "A1=", A1, "\n
      ", "B1=", B1, "\n")
32 theta=atan(-B1/A1)
33 C1=(A1^2 + B1^2)^0.5
34 cat("Alternatively, the least square fit can be
      expressed as", "\n", "y=A0+C1*cos(w0*t + theta),
      where", "\n", "A0=", A0, "\n", "Theta=", theta, "\n", "C1
      =", C1, "\n", "Or", "\n", "y=A0+C1*sin(w0*t + theta +
      pi/2), where", "\n", "A0=", A0, "\n", "Theta=", theta, "
      \n", "C1=", C1, "\n")

```

---

## R code Exa 19.2 Continuous Fourier Series Approximation

```

1  a0=0
2  #f(t)=-1 for -T/2 to -T/4
3  #f(t)=1 for -T/4 to T/4
4  #f(t)=-1 for T/4 to T/2
5  #ak=2/T* (integration of f(t)*cos(w0*t) from -T/2 to
      T/2)
6  #ak=2/T*((integration of f(t)*cos(w0*t) from -T/2 to
      -T/4) + (integration of f(t)*cos(w0*t) from -T/4
      to T/4) + (integration of f(t)*cos(w0*t) from T/
      4 to T/2))
7  #Therefore,
8  #ak=4/(k*pi) for k=1,5,9,.....

```

```

9 #ak=-4/(k*%pi) for k=3,7,11,.....
10 #ak=0 for k=even integers
11 #similarly we find the b's.
12 #all the b's=0
13 cat("The fourier approximtation is:", "\n", "4/(%pi)*cos
      (w)*t) - 4/(3*%pi)*cos(3*(w)*t) + 4/(5*%pi)*cos(5
      *(w)*t) - 4/(7*%pi)*cos(7*(w)*t) + .....")

```

---

#### R code Exa 19.4 Data Analysis

```

1 s<-c(0.0002, 0.0002, 0.0005, 0.0005, 0.001, 0.001)
2 r<-c(0.2, 0.5, 0.2, 0.5, 0.2, 0.5)
3 u<-c(0.25, 0.5, 0.4, 0.75, 0.5, 1)
4 logs=log10(s)
5 logr=log10(r)
6 logu=log10(u)
7 m<-matrix(0,nrow = 6,ncol = 3)
8 for (i in 1:6){
9   m[i,1]=1
10   m[i,2]=logs[i]
11   m[i,3]=logr[i]
12 }
13 a=qr.solve(m,transpose(logu))
14 cat(" alpha=",10^a[1], " sigma=",a[2], " rho=",a[3])

```

---

#### R code Exa 19.5 Curve Fitting

```

1 #install.packages(" signal",dependencies = TRUE)
2 library(signal)
3 x=0:10
4 y=sin(x)
5 xi=seq(0,10,.25)
6 #part a

```

```

7 yi=interp1(x,y,xi)
8 plot(xi,yi,main = "y vs x (part a)",xlab = "x",ylab
  ="y" )
9
10 #part b
11 #fitting x and y in a fifth order polynomial
12 p<-c(0.0008, -0.0290, 0.3542, -1.6854, 2.586,
  -0.0915)
13
14 for (i in 1:41){
15   yi[i]=p[1]*(xi[i]^5)+p[2]*(xi[i]^4)+p[3]*(xi[i]^3)
  +p[4]*(xi[i]^2)+p[5]*(xi[i])+p[6]
16 }
17 plot(xi,yi,main = "y vs x (part b)",xlab = "x",ylab
  = "y")
18
19 #part c
20 d = spline(x,y,method = "fmm",n = length(x))
21 plot(x,d$y,main = "y vs x (part c)",xlab = "x",ylab
  = "y")
22 lines(x,d$y)

```

---

### R code Exa 19.6 Polynomial Regression

```

1 x<-c(0.05, 0.12, 0.15, 0.3, 0.45, 0.7, 0.84, 1.05)
2 y<-c(0.957, 0.851, 0.832, 0.72, 0.583, 0.378, 0.295,
  0.156)
3 sx=sum(x)
4 sxx=sum(x*x)
5 sx3=sum(x*x*x)
6 sx4=sum(x*x*x*x)
7 sx5=sum(x*x*x*x*x)
8 sx6=sum(x*x*x*x*x*x)
9 n=8
10 sy=sum(y)

```

```

11 sxy=sum(x*y)
12 sx2y=sum(x*x*y)
13 sx3y=sum(x*x*x*y)
14 m<-matrix(data = c(n, sx, sxx, sx3,sx, sxx, sx3, sx4
    ,sxx, sx3, sx4, sx5,sx3, sx4, sx5, sx6),nrow = 4,
    ncol = 4,byrow = TRUE)
15 p<-matrix(data = c(sy,sxy,sx2y,sx3y),nrow = 4,ncol =
    1,byrow = TRUE)
16 a=solve(m,p)
17 cat("The cubic polynomial is y=a0 + a1*x + a2*x^2 +
    a3*x^3, where a0, a1, a2 and a3 are", "\n",a[1], "\n",
    a[2], "\n",a[3], "\n",a[4], "\n")

```

---

# Chapter 21

## Newton Cotes Integration Formulas

R code Exa 21.1 Single trapezoidal rule

```
1 f <- function(x) {  
2   (0.2+25*x-200*x^2+675*x^3-900*x^4+400*x^5)  
3 }  
4  
5 tval=1.640533  
6 a=0  
7 b=0.8  
8 fa=f(a)  
9 fb=f(b)  
10 l=(b-a)*((fa+fb)/2)  
11 Et=tval-l  
12 #error  
13 et=Et*100/tval  
14 #percent relative error  
15  
16 #by using approximate error estimate  
17  
18 #the second derivative of f  
19
```

```

20 g <- function(x) {-400+4050*x-10800*x^2+8000*x^3}
21 ans = integrate(f = g, lower = 0, upper = 0.8)
22
23 f2x = ans$value/(b-a)
24 #average value of second derivative
25
26 Ea=-(1/12)*(f2x)*(b-a)^3
27
28 cat("The Error Et=",Et,"\n","The percent relative
      error et=",et,"%","\n","The approximate error
      estimate without using the true value=",Ea)

```

---

### R code Exa 21.2 Multiple trapezoidal rule

```

1 f <- function(x) {
2   (0.2+25*x-200*x^2+675*x^3-900*x^4+400*x^5)
3 }
4
5 a=0
6 b=0.8
7 tval=1.640533
8 n=2
9 h=(b-a)/n
10 fa=f(a)
11 fb=f(b)
12 fh=f(h)
13 l=(b-a)*(fa+2*fh+fb)/(2*n)
14 Et=tval-l
15 #error
16 et=Et*100/tval
17 #percent relative error
18
19 #by using approximate error estimate
20
21 #the second derivative of f

```

```

22 g <- function(x) {
23   -400+4050*x-10800*x^2+8000*x^3
24 }
25 ans = integrate(f = g, lower = 0, upper = 0.8)
26
27 f2x = ans$value/(b-a)
28 #average value of second derivative
29
30 Ea=-(1/12)*(f2x)*(b-a)^3/(n^2);
31 cat("The Error Et=",Et,"\n","The percent relative
    error et=",et,"%","\n","The approximate error
    estimate without using the true value=",Ea)

```

---

### R code Exa 21.3 Evaluating Integrals

```

1 g=9.8
2 #m/s^2; acceleration due to gravity
3
4 m=68.1
5 #kg
6
7 c=12.5
8 #kg/sec; drag coefficient
9
10 f <- function(t) {
11   g*m*(1-exp(-c*t/m))/c
12 }
13
14 tval=289.43515
15 #m
16
17 a=0
18 b=10
19 fa=f(a)
20 fb=f(b)

```

```

21
22 for (i in seq(10,20,10)){
23     n=i
24     h=(b-a)/n
25     cat("No. of segments=",i,"\n","Segment size=",h,"\n")
26     j=a+h
27     s=0
28     while (j<b){
29         s=s+f(j)
30         j=j+h
31     }
32     l=(b-a)*(fa+2*s+fb)/(2*n)
33     Et=tval-l
34     #error
35     et=Et*100/tval
36     #percent relative error
37     cat("Estimated d=",l,"m","\n","et(%)",et,"\n",
        _____\
        n")
38 }
39
40 for (i in seq(50,100,50)){
41     n=i
42     h=(b-a)/n
43     cat("No. of segments=",i,"\n","Segment size=",h,"\n")
44     j=a+h
45     s=0
46     while (j<b){
47         s=s+f(j)
48         j=j+h
49     }
50     l=(b-a)*(fa+2*s+fb)/(2*n)
51     Et=tval-l
52     #error
53     et=Et*100/tval
54     #percent relative error

```



```

55     cat(" Estimated d=",l,"m", "\n", " et (%)", et, "\n", "
        -----\n"
        n")
56 }
57
58 for (i in seq(100,200,100)){
59     n=i
60     h=(b-a)/n
61     cat("No. of segments=",i, "\n", "Segment size=",h, "\n"
        n")
62     j=a+h
63     s=0
64     while (j<b){
65         s=s+f(j)
66         j=j+h
67     }
68     l=(b-a)*(fa+2*s+fb)/(2*n)
69     Et=tval-l
70     #error
71     et=Et*100/tval
72     #percent relative error
73     cat(" Estimated d=",l,"m", "\n", " et (%)", et, "\n", "
        -----\n"
        n")
74 }
75
76 for (i in seq(200,500,300)){
77     n=i
78     h=(b-a)/n
79     cat("No. of segments=",i, "\n", "Segment size=",h, "\n"
        n")
80     j=a+h
81     s=0
82     while (j<b){
83         s=s+f(j)
84         j=j+h
85     }
86     l=(b-a)*(fa+2*s+fb)/(2*n)

```

```

87     Et=tval-1
88     #error
89     et=Et*100/tval
90     #percent relative error
91     cat(" Estimated d=",l,"m", "\n", " et (%)", et, "\n", "
      _____\
          n" )
92 }
93 for (i in seq(1000,2000,1000)){
94     n=i
95     h=(b-a)/n
96     cat("No. of segments=",i," \n", "Segment size=",h," \
          n" )
97     j=a+h
98     s=0
99     while (j<b){
100         s=s+f(j)
101         j=j+h
102     }
103     l=(b-a)*(fa+2*s+fb)/(2*n)
104     Et=tval-1
105     #error
106     et=Et*100/tval
107     #percent relative error
108     cat(" Estimated d=",l,"m", "\n", " et (%)", et, "\n", "
      _____\
          n" )
109 }
110
111 for (i in seq(2000,5000,3000)){
112     n=i
113     h=(b-a)/n
114     cat("No. of segments=",i," \n", "Segment size=",h," \
          n" )
115     j=a+h
116     s=0
117     while (j<b){
118         s=s+f(j)

```

```

119     j=j+h
120 }
121 l=(b-a)*(fa+2*s+fb)/(2*n)
122 Et=tval-l
123 #error
124 et=Et*100/tval
125 #percent relative error
126 cat(" Estimated d=",l,"m","\n"," et(%)",et,"\n",
      _____\
      n")
127 }
128
129 for (i in seq(5000,10000,5000)){
130     n=i
131     h=(b-a)/n
132     cat("No. of segments=",i,"\n"," Segment size=",h,"\n")
133     j=a+h
134     s=0
135     while (j<b){
136         s=s+f(j)
137         j=j+h
138     }
139     l=(b-a)*(fa+2*s+fb)/(2*n)
140     Et=tval-l
141     #error
142     et=Et*100/tval
143     #percent relative error
144     cat(" Estimated d=",l,"m","\n"," et(%)",et,"\n",
      _____\
      n")
145 }
      _____

```

**R code Exa 21.4** Single Simpsons 1 by 3 rule

```

1 f <- function(x) {
2   (0.2+25*x-200*x^2+675*x^3-900*x^4+400*x^5)
3 }
4
5 a=0
6 b=0.8
7 tval=1.640533
8 n=2
9 h=(b-a)/n
10 fa=f(a)
11 fb=f(b)
12 fh=f(h)
13
14 l=(b-a)*(fa+4*fh+fb)/(3*n)
15 cat("l=",l)
16 Et=tval-l
17 #error
18 et=Et*100/tval
19 #percent relative error
20
21 #by using approximate error estimate
22
23 #the fourth derivative of f
24 g <- function(x) {
25   -21600+48000*x
26 }
27 ans = integrate(f = g,0,0.8)
28 f4x=ans$value/(b-a)
29 #average value of fourth derivative
30 Ea=-(1/2880)*(f4x)*(b-a)^5
31 cat("The Error Et=",Et,"\n","The percent relative
    error et=",et,"%","\n","The approximate error
    estimate without using the true value=",Ea)

```

---

**R code Exa 21.5** Multiple Simpsons 1 by 3 rule

```

1 f <- function(x) {
2   (0.2+25*x-200*x^2+675*x^3-900*x^4+400*x^5)
3 }
4
5 a=0
6 b=0.8
7 tval=1.640533
8 n=4
9 h=(b-a)/n
10 fa=f(a)
11 fb=f(b)
12 j=a+h
13 s=0
14 count=1
15 while (j<b){
16   if ((-1)^count==-1){
17     s=s+4*f(j)
18   } else {
19     s=s+2*f(j)
20   }
21   count=count+1
22   j=j+h
23 }
24
25 l=(b-a)*(fa+s+fb)/(3*n)
26 cat(" l=",l,"\n")
27 Et=tval-l
28 #error
29 et=Et*100/tval
30 #percent relative error
31
32 #by using approximate error estimate
33
34 #the fourth derivative of f
35
36 g <- function(x) {
37   -21600+48000*x
38 }

```

```

39 ans = integrate(f = g,0,0.8)
40
41 f4x=ans$value/(b-a)
42 #average value of fourth derivative
43 Ea=-(1/(180*4^4))*(f4x)*(b-a)^5
44 cat("The Error Et=",Et,"\n","The percent relative
      error et=",et,"%","\n","The approximate error
      estimate without using the true value=",Ea,"\n")

```

---

### R code Exa 21.6 Simpsons 3 by 8 rule

```

1 f <- function(x) {
2   (0.2+25*x-200*x^2+675*x^3-900*x^4+400*x^5)
3 }
4
5 a=0
6 b=0.8
7 tval=1.640533
8 #part a
9 n=3
10 h=(b-a)/n
11 fa=f(a)
12 fb=f(b)
13 j=a+h
14 s=0
15 count=1
16 while (j<b){
17   s=s+3*f(j)
18   count=count+1
19   j=j+h
20 }
21 l=(b-a)*(fa+s+fb)/(8)
22 cat("Part A:", "\n", "l=", l, "\n")
23 Et=tval-l
24 #error

```

```

25 et=Et*100/tval
26 #percent relative error
27
28 #by using approximate error estimate
29
30 #the fourth derivative of f
31 g <- function(x) {
32     -21600+48000*x
33 }
34
35 ans= integrate(f = g,0,0.8)
36
37 f4x=ans$value / (b-a)
38 #average value of fourth derivative
39 Ea=-(1/6480)*(f4x)*(b-a)^5
40 cat("The Error Et=",Et,"\n","The percent relative
      error et=",et,"%","\n","The approximate error
      estimate without using the true value=",Ea,"\n")
41
42 #part b
43 n=5
44 h=(b-a)/n
45 l1=(a+2*h-a)*(fa+4*f(a+h)+f(a+2*h))/6
46 l2=(a+5*h-a-2*h)*(f(a+2*h)+3*(f(a+3*h)+f(a+4*h))+fb)
      /8
47 l=l1+l2
48 cat("
      _____\n")
49 cat("Part B:","\n","l=",l,"\n")
50 Et=tval-l
51 #error
52 et=Et*100/tval
53 #percent relative error
54 cat("The Error Et=",Et,"\n","The percent relative
      error et=",et,"%")
      _____

```

### R code Exa 21.7 Unequal Trapezoidal segments

```
1 f <- function(x) {
2   (0.2+25*x-200*x^2+675*x^3-900*x^4+400*x^5)
3 }
4 func<-matrix(0,11)
5 tval=1.640533
6 x<-c(0, 0.12, 0.22, 0.32, 0.36, 0.4, 0.44, 0.54,
       0.64, 0.7, 0.8)
7 for (i in 1:11){
8   func[i]=f(x[i])
9 }
10 l=0
11 for (i in 1:10){
12   l=l+(x[i+1]-x[i])*(func[i]+func[i+1])/2
13 }
14
15 cat(" l=",l)
16 Et=tval-l
17 #error
18 et=Et*100/tval
19 #percent relative error
20 cat("The Error Et=",Et,"\n","The percent relative
     error et=",et,"%")
```

---

### R code Exa 21.8 Simpsons Uneven data

```
1 f <- function(x) {
2   (0.2+25*x-200*x^2+675*x^3-900*x^4+400*x^5)
3 }
4
5 tval=1.640533
```



```

6 x<-c(0, 0.12, 0.22, 0.32, 0.36, 0.4, 0.44, 0.54,
      0.64, 0.7 ,0.8)
7 func<-matrix(0,11)
8 for (i in 1:11){
9   func[i]=f(x[i])
10 }
11 l1=(x[2]-x[1])*((f(x[1])+f(x[2]))/2)
12 l2=(x[4]-x[2])*((f(x[4])+4*f(x[3])+f(x[2]))/6)
13 l3=(x[7]-x[4])*((f(x[4])+3*(f(x[5])+f(x[6]))+f(x[7]))
    /8)
14 l4=(x[9]-x[7])*((f(x[7])+4*f(x[8])+f(x[9]))/6)
15 l5=(x[10]-x[9])*((f(x[10])+f(x[9]))/2)
16 l6=(x[11]-x[10])*((f(x[11])+f(x[10]))/2)
17 l=l1+l2+l3+l4+l5+l6
18 cat("l=",l,"\n")
19 Et=tval-l
20 #error
21 et=Et*100/tval
22 #percent relative error
23 cat("The Error Et=",Et,"\n","The percent relative
    error et=",et,"%")

```

---

### R code Exa 21.9 Average Temperature Determination

```

1 f <- function(x,y) {
2   2*x*y+2*x-x^2-2*y^2+72
3 }
4
5 len=8
6 #m,length
7 wid=6
8 #m,width
9 a=0
10 b=len
11 n=2

```

```

12 h=(b-a)/n
13 a1=0
14 b1=wid
15 h1=(b1-a1)/n
16
17 fa=f(a,0)
18 fb=f(b,0)
19 fh=f(h,0)
20 lx1=(b-a)*(fa+2*fh+fb)/(2*n)
21
22 fa=f(a,h1)
23 fb=f(b,h1)
24 fh=f(h,h1)
25 lx2=(b-a)*(fa+2*fh+fb)/(2*n)
26
27 fa=f(a,b1)
28 fb=f(b,b1)
29 fh=f(h,b1)
30 lx3=(b-a)*(fa+2*fh+fb)/(2*n)
31
32 l=(b1-a1)*(lx1+2*lx2+lx3)/(2*n)
33
34 avg_temp=l/(len*wid)
35 cat("The average temperature is=",avg_temp)

```

---

# Chapter 23

## Numerical differentiation

R code Exa 23.4 Integration and Differentiation

```
1 f <- function(x) {  
2   0.2+25*x-200*x^2+675*x^3-900*x^4+400*x^5  
3 }  
4  
5 a=0  
6 b=0.8  
7 Q=integrate(f,0,0.8)  
8 cat("Q=",Q,"\n")  
9 x<-c(0, 0.12, 0.22, 0.32, 0.36, 0.4 ,0.44, 0.54  
    ,0.64, 0.7, 0.8)  
10 y=f(x)  
11  
12 #This algorithm uses  
13 #the formula for the area of a trapezoid: area =  
    width average of the lengths of the parallel  
    sides.  
14  
15 UseTrapezoidRule <- function(xmin, xmax, num_  
    intervals) {  
16   #Calculate the width of a trapezoid.  
17   dx = (xmax - xmin) / num_intervals
```

```

18  #Add up the trapezoids' areas.
19  total_area = 0
20  x = xmin
21  for (i in 1:num_intervals){
22      total_area = total_area + dx * (f(x) + f(x + dx)
23          ) / 2
24      x = x + dx
25  }
26  return(total_area)
27 }
28 integral = UseTrapezoidRule(0,0.8,10000)
29
30 cat("Trapezoid intergral=",integral,"\n","diff(x)=",
31     diff(x),"\n")
32 d=diff(y)/diff(x)
33 cat("d=",d)

```

---

### R code Exa 23.5 Integrate a function

```

1  f <- function(x) {
2      0.2+25*x-200*x^2+675*x^3-900*x^4+400*x^5
3  }
4
5  a=0
6  b=0.8
7  Qt=1.640533
8  Q=integrate(f,0,0.8)
9  cat("Computed=",Q$value,"\n","Error estimate=",abs(Q
10     $value-Qt)*100/Qt,"\n")

```

---

# Chapter 25

## Runga Kutta methods

R code Exa 25.4 Solving ODEs

```
1 m=68.1
2 g=9.8
3 c=12.5
4 a=8.3
5 b=2.2
6 vmax=46
7
8 f <- function(t,v,parms) {
9   list(c(g-c*v/m))
10 }
11
12 v0=0
13 t=0:15
14 sol<-ode(y = v0,times = t,func = f,parms = NULL)
15 sol <-data.frame(sol)
16 plot(t,sol$X1,main =" velocity vs time", xlab = "t (s
   )",ylab = "v (m/s)")
17 lines(t,sol$X1,col = "red")
18
19 f1 <- function(t,v,parms) {
20   list(c(g-(c/m)*(v+a*(v/vmax)^b)))
```

```

21 }
22
23 sol<-ode(y = v0,times = t,func = f1,parms = NULL)
24 sol <-data.frame(sol)
25 lines(t,sol$X1,col="blue")
26 legend(x = 10,y = 20,legend = c("Linear","Nonlinear"
    ),lty=c(1,1),col=c("red","blue"))

```

---

### R code Exa 25.11 Solving systems of ODEs

```

1 library(deSolve)
2 f <- function(x,y,parms) {
3   a = y[2]
4   b = -16.1*y[1]
5   list(c(a,b))
6 }
7
8 x=seq(0,4,0.1)
9 y0<-c(0.1, 0)
10 sol<-ode(y = y0,times = x,func = f,parms = NULL)
11 sol <-data.frame(sol)
12 plot(c(0,4),c(-4,4),main = "y vs x",xlab = "x",ylab
    = "y",type = "n")
13 lines(x,sol$X2,col = "blue")
14 lines(x,sol$X1,col = "red")
15 #legend(x = 3,y = 0.3,legend = c("y1,y3","y2,y4"),
    lty=c(1,1))
16
17 g <- function(x,y,parms) {
18   a = y[2]
19   b = -16.1*sin(y[1])
20   list(c(a,b))
21 }
22 sol<-ode(y = y0,times = x,func = g,parms = NULL)
23 sol <-data.frame(sol)

```

```

24 lines(c(0,4),c(-.5,.5),main = "y vs x",xlab = "x",
      ylab = "y",type = "n")
25 lines(x,sol$X2,col = "blue")
26 lines(x,sol$X1,col = "red")
27 #legend(x = 3,y = 0.3,legend = c("y1,y3","y2,y4"),
      lty=c(1,1))
28
29 pi = 3.1415927
30
31 y0<-c(pi/4, 0)
32 sol<-ode(y = y0,times = x,func = f,parms = NULL)
33 sol <-data.frame(sol)
34 lines(c(0,4),c(-4,4),main = "y vs x",xlab = "x",ylab
      = "y",type = "n")
35 lines(x,sol$X2,col = "blue")
36 lines(x,sol$X1,col = "red")
37 legend(x = 3,y = 3,legend = c("y1,y3","y2,y4"),lty=c
      (1,1))
38
39 sol<-ode(y = y0,times = x,func = g,parms = NULL)
40 sol <-data.frame(sol)
41 lines(c(0,4),c(-4,4),main = "y vs x",xlab = "x",ylab
      = "y",type = "n")
42 lines(x,sol$X2,col = "blue")
43 lines(x,sol$X1,col = "red")
44 legend(x = 3,y = 3,legend = c("y1,y3","y2,y4"),lty=c
      (1,1))

```

---

#### R code Exa 25.14 Adaptive Fourth order RK scheme

```

1 f <- function(x,y,parms) {
2   list(c(10*exp(-(x-2)^2/(2*(0.075^2)))-0.6*y))
3 }
4
5 x=seq(0,4,0.1)

```

```
6 y0=0.5
7 sol<-ode(y = y0,times = x,func = f,parms = NULL)
8 sol <-data.frame(sol)
9 plot(x,sol$X1,main ="y vs x",xlab = "x",ylab = "y")
10 lines(x,sol$X1)
```

---



# Chapter 26

## Stiffness and multistep methods

R code Exa 26.1 Explicit and Implicit Euler

```
1 f <- function(t,y) {  
2   -1000*y+3000-2000*exp(-t)  
3 }  
4  
5 y0=0  
6 #explicit euler  
7 h1=0.0005  
8 y1 = matrix(0,60)  
9 y1[1]=y0  
10 count=2  
11 t=seq(0,0.006,0.0001)  
12 for (i in seq(0,0.0059,0.0001)){  
13   y1[count]=y1[count-1]+f(i,y1[count-1])*h1  
14   count=count+1  
15 }  
16 h2=0.0015  
17 y2 = matrix(0,60)  
18 y2[1]=y0  
19 count=2  
20 t=seq(0,0.006,0.0001)  
21 for (i in seq(0,0.0059,0.0001)){
```

```

22     y2[count]=y2[count-1]+f(i,y2[count-1])*h2
23     count=count+1
24 }
25 plot(t,y2,main = "y vs t",xlab = "t",ylab = "y")
26 lines(t,y2,col="red")
27 lines(t,y1,col = "blue")
28 legend(x = 0.004,y = 0.5,legend = c("h=0.0005","h
    =0.0015"),lty = c(1,1))
29
30 #implicit order
31 h3=0.05
32 y3 = matrix(0,39)
33 y3[1]=y0
34 count=2;
35 t=seq(0,0.4,0.01)
36 for (j in seq(0,0.39,0.01)){
37     y3[count]=(y3[count-1]+3000*h3-2000*h3*exp(-(j
        +0.01)))/(1+1000*h3)
38     count=count+1
39 }
40 plot(t,y3,main = "y vs t",xlab = "t",ylab = "y")
41 lines(t,y3)

```

---

# Chapter 27

## Boundary Value and Eigenvalue problems

R code Exa 27.3 Finite Difference Approximation

```
1 h=0.01
2 delx=2
3 x=2+h*delx^2
4 a<-matrix(c(x, -1, 0, 0,-1, x, -1, 0, 0, -1, x, -1,
              0, 0, -1, x),nrow = 4,ncol = 4,byrow = TRUE)
5 b<-matrix(c(40.8, 0.8, 0.8, 200.8),nrow = 4,ncol =
              1,byrow = TRUE)
6 T=solve(a,b)
7 cat("The temperature at the interior nodes:",abs(T))
```

---

R code Exa 27.4 Mass Spring System

```
1 library(rootSolve)
2 m1=40
3 #kg
4 m2=40
```

```

5 #kg
6 k=200
7 #N/m
8 fun <- function (sqw) sqw^2-20*sqw+75
9 p <- uniroot.all(fun, c(0,100))
10 p<-round(data.frame(p))
11 r<-matrix(0,2)
12 r[1]<-p$p[1]
13 r[2]<-p$p[2]
14 f1=(r[1])^0.5
15 f2=(r[2])^0.5
16 pi = 3.1415927
17 Tp1=(2*pi)/f1
18 Tp2=(2*pi)/f2
19
20 #for first mode
21 cat("For first mode:", "\n", "Period of oscillation:",
      Tp1, "\n", "A1=-A2", "\n", "
      _____\n
      n")
22
23 #for first mode
24 cat("For second mode:", "\n", "Period of oscillation:"
      ,Tp2, "\n", "A1=A2")
      _____

```

### R code Exa 27.5 Axially Loaded column

```

1 E=10*10^9
2 #Pa
3 I=1.25*10^-5
4 #m^4
5 L=3
6 #m
7 pi = 3.1415927
8 for (i in 1:8){

```

```

9   p=i*pi/L
10  P=i^2*(pi)^2*E*I/(L^2*1000)
11  cat("n=",i,"\n","p=",p,"m^-2","\n","P=",P,"kN","
      n")
12 }

```

---

### R code Exa 27.6 Polynomial Method

```

1  library(rootSolve)
2  E=10*10^9
3  #Pa
4  I=1.25*10^-5
5  #m^4
6  L=3
7  #m
8  true<-c(1.0472, 2.0944, 3.1416, 4.1888)
9
10 #part a
11 h1=3/2
12 fun <- function (p) -h1^2*p^2+2
13 p <- uniroot.all(fun, c(-100,100))
14 p<-data.frame(p)
15 x<-matrix(0,2)
16 x[1]<-p$p[1]
17 x[2]<-p$p[2]
18 e=abs(abs(x[1])-true[1])*100/true[1];
19 cat("p=",x,"\n","error=",e,"
      n")
20
21 #part b
22 h2=3/3
23 fun <- function (p) (3-(4*p^2)+p^4) # = (2 - p^2)^2
      - 1

```

---

```

24 p <- uniroot.all(fun, c(-10,10))
25 p<-data.frame(p)
26 x<-matrix(0,2)
27 e<-matrix(0,2)
28 x[1]<-p$p[3]
29 x[2]<-p$p[1]
30 e[1]=abs(abs(x[1])-true[2])*100/true[2]
31 e[2]=abs(abs(x[2])-true[1])*100/true[1]
32 cat("p=",x,"\n", "error=",e,"
      n")
33
34 #part c
35 h3=3/4;
36 fun <- function (p) (2-h3^2*p^2)^3 - 2*(2-h3^2*p^2)
37 #a= #= (2 - 0.5625*p^2)^3 - 2 *(2 - 0.5625*p^2)
38 p <- uniroot.all(fun, c(-10,10))
39 p<-data.frame(p)
40 x<-matrix(0,3)
41 e<-matrix(0,3)
42 x[1]<-p$p[1]
43 x[2]<-p$p[2]
44 x[3]<-p$p[3]
45 e[1]=abs(abs(x[1])-true[3])*100/true[3]
46 e[2]=abs(abs(x[2])-true[2])*100/true[2]
47 e[3]=abs(abs(x[3])-true[1])*100/true[1]
48 cat("p=",x,"\n", "error=",e,"
      n")
49
50
51 #part d
52 h4=3/5;
53 fun <- function (p) (2-h4^2*p^2)^4 - 3*(2-h4^2*p^2)
      ^2 + 1
54 p <- uniroot.all(fun, c(-10,10))
55 p<-data.frame(p)
56 x<-matrix(0,4)

```

```

57 e<-matrix(0,4)
58 x[1]<-p$p[1]
59 x[2]<-p$p[2]
60 x[3]<-p$p[3]
61 x[4]<-p$p[4]
62 e[1]=abs(abs(x[1])-true[4])*100/true[4]
63 e[2]=abs(abs(x[2])-true[3])*100/true[3]
64 e[3]=abs(abs(x[3])-true[2])*100/true[2]
65 e[4]=abs(abs(x[4])-true[1])*100/true[1]
66 cat("p=",x,"\n", "error=",e,"

```

---

```

n")

```

---

### R code Exa 27.7 Power Method Highest Eigenvalue

```

1 a<-matrix(c(3.556, -1.668, 0, -1.778, 3.556, -1.778,
    0, -1.778, 3.556),nrow = 3,ncol = 3,byrow = TRUE
    )
2 b<-matrix(c(1.778,0,1.778),nrow = 3,ncol = 1,byrow =
    TRUE)
3 ea=100
4 count=1
5 eigen<-matrix(0,1000)
6 while (ea>0.1){
7     maxim=b[1]
8     for (i in 2:3){
9         if (abs(b[i])>abs(maxim)){
10             maxim=b[i]
11         }
12     }
13     eigen[count]=maxim
14     b=a %*%(b/maxim)
15     if (count==1){
16         ea=20
17         count=count+1

```

```

18   } else {
19       ea=abs(eigen[count]-eigen[count-1])*100/abs(
           eigen[count])
20       count=count+1
21   }
22 }
23 cat("The largest eigen value",eigen[count-1])

```

---

### R code Exa 27.8 Power Method Lowest Eigenvalue

```

1  a<-matrix(c(3.556, -1.668, 0, -1.778, 3.556, -1.778,
           0, -1.778, 3.556),nrow = 3,ncol = 3,byrow = TRUE
           )
2  b<-matrix(c(1.778,0,1.778),nrow = 3,ncol = 1,byrow =
           TRUE)
3  ea=100
4  count=1
5  eigen<-matrix(0,100)
6  ai=solve(a)
7  while (ea>4){
8      maxim=b[1]
9      for (i in 2:3){
10         if (abs(b[i])>abs(maxim)){
11             maxim=b[i]
12         }
13     }
14     eigen[count]=maxim
15     b=ai%*(b/maxim)
16     if (count==1){
17         ea=20
18         count=count+1
19     } else {
20         ea=abs(eigen[count]-eigen[count-1])*100/abs(
           eigen[count])
21         count=count+1

```



```

22   }
23 }
24 cat("The smallest eigen value", (1/eigen[count-1])
      ^0.5)

```

---

### R code Exa 27.9 Eigenvalues and ODEs

```

1  library(deSolve)
2
3  predprey <- function(t,y,parms) {
4    a = 1.2*y[1]-0.6*y[1]*y[2]
5    b = -0.8*y[2]+0.3*y[1]*y[2]
6    list(c(a,b))
7  }
8  t=seq(0,20,0.1)
9  y0<-c(2, 1)
10 sol=ode(y = y0,parms = NULL,times = t,func =
      predprey)
11 sol<-data.frame(sol)
12 plot(t,sol$X1,main = "y vs t", xlab = "t",ylab = "y"
      )
13 lines(t,sol$X1)
14 lines(t,sol$X2)
15
16 plot(sol$X1,sol$X2,main = "space-space plot (y1 vs
      y2)", xlab = "y1",ylab = "y2")
17 lines(sol$X1,sol$X2)

```

---

### R code Exa 27.11 Solving ODEs

```

1  library(deSolve)
2
3  predprey <- function(t,y,parms) {

```

```

4   a = 1.2*y[1]-0.6*y[1]*y[2]
5   b = -0.8*y[2]+0.3*y[1]*y[2]
6   list(c(a,b))
7 }
8 t=0:10
9 y0<-c(2, 1)
10 sol=ode(y = y0,parms = NULL,times = t,func =
    predprey)
11 sol<-data.frame(sol)
12
13 count=0;
14 for (i in 1:11){
15   cat(" istep=",count+1,"\n","time=",count,"\n","y1="
      ,sol$X1[i],"\n","y2=",sol$X2[i],"\n","
      -----\
      n")
16   count=count+1
17 }
-----

```

# Chapter 31

## Finite Element Method

**R code Exa 31.1** Analytical Solution for Heated Rod

```
1 #d2T/dx2=-10; equation to be solved
2 #T(0,t)=40; boundary condition
3 #T(10,t)=200; boundary condition
4 #f(x)=10; uniform heat source
5 #we assume a solution T=a*X^2 + b*x +c
6 #differentiating twice we get d2T/dx2=2*a
7 a=-10/2
8 #using first boundary condition
9 c=40
10 #using second boundary condtion
11 b=66
12 #hence final solution T=-5*x^2 + 66*x + 40
13 f <- function(x) {
14   -5*x^2 + 66*x + 40
15 }
16 T<-matrix(0,110)
17 count=1
18 for (i in seq(0,11,0.1)){
19   T[count]=f(i)
20   count=count+1
21 }
```

```

22 x<-seq(0,11,0.1)
23 plot(x,T,main = "Temperature(T) vs distance(x)",xlab
      = "x (cm)",ylab = "T (units)")
24 lines(x,T)

```

---

### R code Exa 31.2 Element Equation for Heated Rod

```

1  xf=10
2  #cm
3  xe=2.5
4  #cm
5  #T(0,t)=40; boundary condition
6  #T(10,t)=200; boundary condition
7  #f(x)=10; uniform heat source
8  f <- function(x) {
9    10*(xe-x)/xe
10 }
11 int1=integrate(f = f,lower = 0,upper = xe)
12
13 g <- function(x) {
14   10*(x-0)/xe
15 }
16 int2=integrate(f = g,lower = 0,upper = xe)
17
18 cat("The results are:", "\n", "0.4*T1-0.4*T2=-(dT/dx)*
    x1 + c1", "\n", "where c1=", int1$value, "\n", "and", "\n",
    "-0.4*T1+0.4*T2=-(dT/dx)*x2 + c2", "\n", "where
    c2=", int2$value, "\n")

```

---