## R Textbook Companion for Introduction to Probability and Statistics for Engineers and Scientists by Sheldon M. Ross<sup>1</sup>

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May 22, 2020

<sup>1</sup>Funded by a grant from the National Mission on Education through ICT - http://spoken-tutorial.org/NMEICT-Intro. This Textbook Companion and R codes written in it can be downloaded from the "Textbook Companion Project" section at the website - https://r.fossee.in.

# **Book Description**

Title: Introduction to Probability and Statistics for Engineers and Scientists

Author: Sheldon M. Ross

Publisher: Elsevier Academic Press, USA

Edition: 3

**Year:** 2004

**ISBN:** 0-12-598057-4

R numbering policy used in this document and the relation to the above book.

Exa Example (Solved example)

Eqn Equation (Particular equation of the above book)

For example, Exa 3.51 means solved example 3.51 of this book. Sec 2.3 means an R code whose theory is explained in Section 2.3 of the book.

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## Chapter 2

## Descriptive Statistics

#### R code Exa 2.2.a Relative Frequency

```
1 starting_salary<-c(47, 48, 49,50,51, 52, 53, 54,
      56, 57, 60)
2 frequency <-c(4, 1, 3, 5, 8, 10, 0, 5, 2, 3, 1)
3 total = sum(frequency)
4 relative_frequency = frequency/total
5 cat("The relative frequencies are ",relative_frequency)</pre>
```

#### R code Exa 2.2.b Pie Chart

#### R code Exa 2.3.a Sample mean

#### R code Exa 2.3.b Sample mean of age

```
1 age <-c(15, 16, 17, 18, 19, 20)
2 frequencies <-c(2, 5, 11, 9, 14, 13)
3 product = age * frequencies
4 total_people = sum(frequencies)
5 mean_age = sum(product)/total_people
6 cat("The sample mean of the ages is", mean_age)</pre>
```

#### R code Exa 2.3.c Sample Median

```
1 age <-c(15, 16, 17, 18, 19, 20)
2 frequencies <-c(2, 5, 11, 9, 14, 13)
3 final_age = matrix(0,1,length(age))
4 i=1
5 for (j in 1:6){
6   for (k in 1:frequencies[j]){
7    final_age[i] = age[j]
8   i = i +1
9  }
10 }</pre>
```

```
11 final_median = median(final_age)
12 final_median
```

#### R code Exa 2.3.d Mean and Median

#### R code Exa 2.3.e Mean Median and Mode

```
1 value<-c(1, 2, 3, 4, 5, 6)
2 frequencies<-c(9, 8, 5, 5, 6, 7)
3 final_value = matrix(0,1,length(value))
4 i=1
5 for (j in 1:6){
6   for (k in 1:frequencies[j]){
7    final_value[i] = value[j]
8    i = i +1
9  }
10 }</pre>
```

```
product = value*frequencies;
product
sum(product)

total_value = sum(frequencies);
mean_value = sum(product)/total_value #the answer in the textbook is incorrect

which(frequencies==max(frequencies))

m1= max(frequencies);
n= which(frequencies==max(frequencies))

cat("The sample mean is", mean_value)
cat("The median is", median(final_value))
cat("The mode is", value[n])
```

#### R code Exa 2.3.f sample variance

```
1 A <-c(3, 4, 6, 7, 10)
2 B <-c(-20, 5, 15, 24)
3 cat("The sample variance of A is", var(A))
4 cat("The sample variance of B is", var(B))</pre>
```

#### R code Exa 2.3.g sample variance of accidents

#### R code Exa 2.3.h Percentile

#### R code Exa 2.3.i Quartiles

```
1 noise<-c(82, 89, 94, 110, 74, 122, 112, 95, 100,
        78, 65, 60, 90 ,83 ,87 ,75 ,114 ,85 ,69 ,94 ,124
        ,115 ,107 ,88 ,97 ,74 ,72 ,68 ,83 ,91 ,90 ,102,
        77, 125, 108 ,65)
2 cat("The quartiles are",quantile(noise))</pre>
```

#### R code Exa 2.4.a Chebyshev Inequality

#### R code Exa 2.5.a Empirical Rule

#### R code Exa 2.6.a Sample Correlation Coefficient

```
1 temp<-c(24.2, 22.7, 30.5, 28.6, 25.5, 32.0, 28.6,
      26.5, 25.3, 26.0, 24.4 ,24.8 ,20.6, 25.1, 21.4,
      23.7, 23.9, 25.2, 27.4, 28.3, 28.8, 26.6)
2 \text{ defects} < -c(25, 31, 36, 33, 19, 24, 27, 25, 16, 14)
      ,22 ,23 ,20 ,25 ,25 ,23 ,27 ,30 ,33 ,32 ,35, 24)
3 temp_new = temp- mean(temp)
4 defects_new = defects - mean(defects)
5 \text{ num} = 0
6 \text{ s1} = 0;
7 s2=0:
8 for (i in 1:22){
     num = num + (temp_new[i]*defects_new[i])
     s1 = s1 + (temp_new[i]*temp_new[i])
     s2 = s2 + (defects_new[i]*defects_new[i])
11
12 }
13 coefficient = num/sqrt(s1*s2)
14 coefficient
```

#### R code Exa 2.6.b Sample Correlation Coefficient

```
1 year<-c(12, 16, 13, 18, 19, 12, 18, 19, 12, 14)
2 pulserate<-c(73, 67, 74, 63, 73, 84, 60, 62, 76, 71)
3 year_new = year- mean(year)
4 pulserate_new = pulserate - mean(pulserate)
5 num=0
6 s1 =0
7 s2=0
8 for (i in 1:10){
9    num = num + (year_new[i]*pulserate_new[i])
10    s1 = s1 + (year_new[i]*year_new[i])
11    s2 = s2 + (pulserate_new[i]*pulserate_new[i])
12 }
13 coefficient = num/sqrt(s1*s2)
14 coefficient</pre>
```

## Chapter 3

## Elements Of Probability

#### R code Exa 3.4.a Union

```
1 cigarette = 0.28;
2 cigar = 0.07;
3 cigar_and_cigarette = 0.05;
4 cigar_or_cigarette = cigarette + cigar - cigar_and_cigarette;
5 cat((1-cigar_or_cigarette)*100, "% of the males smoke neither cigar nor cigarette")
```

#### R code Exa 3.5.a Basic Principle of Counting

```
white_balls = 6;
black_balls = 5;
total = white_balls + black_balls;
probability_whiteandblack = white_balls*black_balls/
    (total*(total-1));
probability_blackandwhite = white_balls*black_balls/
    (total*(total-1));
reqd_probability = probability_whiteandblack +
    probability_blackandwhite;
```

#### R code Exa 3.5.b Basic Principle of Counting

```
1 maths = 4;
2 chemistry = 3;
3 history = 2;
4 language = 1;
5 total_arrangements = factorial(4)*factorial(maths)*
        factorial(chemistry)*factorial(history)*factorial
        (language);
6 cat("The total number of possible arrangements is ",
        total_arrangements)
```

#### R code Exa 3.5.c Basic Principle of Counting

#### R code Exa 3.5.d Committee Probability

```
1 men = 6;
2 women = 9;
```

```
3 reqd_size =5;
4 total =factorial(men+women)/(factorial(reqd_size)*
          factorial(men+women-reqd_size));
5 given_committee = factorial(men)*factorial(women)/(
          factorial(3)*factorial(2)*factorial(men-3)*
          factorial(women-2));
6 prob = given_committee/total;
7 cat("Probability that the committee consists of 3
          men and 2 women is",prob)
```

#### R code Exa 3.5.f Pairing Probability

```
1 \text{ black_p} = 6;
2 \text{ white_p} = 6;
3 \text{ pair} = 2;
4 total_p = black_p + white_p;
6 total_pairs = 1;
7 while(total_p >0){
     total_pairs = total_pairs*factorial(total_p)/(
        factorial(pair) * factorial(total_p - pair) )
9
     total_p = total_p -2
10 }
11 total_pairs = total_pairs/factorial(6);
12 black_pairs = 1;
13 while(black_p >0){
     black_pairs = black_pairs*factorial(black_p)/((
14
        factorial(pair) * factorial(black_p - pair) ))
     black_p = black_p -2
15
16 }
17 black_pairs = black_pairs/factorial(3);
18 white_pairs = black_pairs;
19 allowed_pairs = black_pairs * white_pairs;
20 probb = allowed_pairs/ total_pairs;
21 cat (" Probability that a random pairing will not
```

```
result in any of the white and black players rooming together is ",probb)
```

#### R code Exa 3.6.a Acceptable Transistor

#### R code Exa 3.6.b Both Boys

#### R code Exa 3.6.c Branch Manager

```
1 prob_phoenix = 0.3;
2 prob_manager = 0.6;
3 cat("Probability that Perez will be a Phoenix branch office manager is",prob_phoenix*prob_manager)
```

#### R code Exa 3.7.a Accident Probability

```
1 accident_prone= 0.4;
2 nonaccident_prone= 0.2;
3 pop_accident = 0.3;
4 prob = pop_accident*accident_prone + (1-pop_accident        )*nonaccident_prone;
5 cat("The required probability is ",prob)
```

#### R code Exa 3.7.b Accident within a year

```
1 accident_prone= 0.4;
2 nonaccident_prone= 0.2;
3 pop_accident = 0.3;
4 prob_of_accident = pop_accident*accident_prone + (1-pop_accident)*nonaccident_prone;
5 prob = pop_accident * accident_prone /prob_of_accident;
6 cat("The required probability is",prob)
```

#### R code Exa 3.7.c Multiple Choice Test

```
1 m = 5;
2 p =1/2;
3 cat("The required probability is", (m*p)/(1+((m-1)*p)))
```

#### R code Exa 3.7.d blood test

```
1 detect_present = 0.99;
```

#### R code Exa 3.7.e Criminal Investigation

```
1 criminal_char = 0.9
2 convinced= 0.6;
3 pop_char = 0.2;
4 prob = (convinced*criminal_char) /((convinced*criminal_char) + (pop_char*(1-convinced)));
5 cat("The required probability is",prob)
```

#### R code Exa 3.7.f Missing Plane

```
1 alpha1 = 0.4;
2 plane_in_region1 = 1/3;
3 plane_in_region2 = 1/3;
4 plane_in_region3 = 1/3;
5 prob1 = (alpha1*plane_in_region1)/((alpha1*plane_in_region1)+ 1*plane_in_region2 + 1*plane_in_region3
);
6 prob2 = (1*plane_in_region2)/((alpha1*plane_in_region3);
7 cat("The probability that the planes is in region 1 given that the search of region 1 did not uncover it ",prob1)
```

8 cat("The probability that the planes is in region 2/3 given that the search of region 1 did not uncover it ",prob2)

#### R code Exa 3.8.a Independent Events

```
1 prob_A = 4/52;
2 prob_H = 13/52;
3 cat("P(AH) is",prob_A*prob_H)
```

## Chapter 4

# Random Variables And Expectation

R code Exa 4.1.a sum of two fair dice

```
1 #install.packages("IPSUR")
2 #install.packages("prob")
3 library(IPSUR)
4 #read (IPSUR)
5 library(prob)
6 #read (prob)
8 #First we set up the sample space with the rolldie
      function
9 S <- rolldie(2, makespace = TRUE)
10 #Next, we add random variables U and V with the
      addry function.
11 S \leftarrow addrv(S, FUN = \max, invars = c("X1", "X2"),
     name = "U")
12 S \leftarrow addrv(S, FUN = sum, invars = c("X1", "X2"),
     name = "V")
13 head(S)
14 UV \leftarrow marginal(S, vars = c("U", "V"))
15 xtabs(round(probs, 3) ~ U + V, data = UV)
```

```
16 marginal(UV, vars = "U")
17 marginal(UV, vars = "V")
18
19 temp <- xtabs(probs ~ U + V, data = UV)
20 rowSums(temp)
21 colSums(temp)
22 colSums(temp)
23
24
25 data = sample(1:6,10000000, replace=TRUE)+sample
      (1:6, 100000000, replace=TRUE)
26 table (data)/length (data)
27 sum (table (data) / length (data))
28
   table (sample (1:6,10000000, replace=TRUE))/length(
29
      sample (1:6,10000000, replace=TRUE))
30
```

#### R code Exa 4.1.c X exceeds 1

```
1 #install.packages("IPSUR")
2 #install.packages("prob")
3 library(IPSUR)
4 #read(IPSUR)
5 library(prob)
6 #read(prob)
7
8 "
9 F(x) = 0 -- { x <= 0}
10 F(x) = 1 - exp(-x^2) -- {x >0}
11
12 P{X>1} = ?
13 "
14 P = 1 - (1 - exp(-(1^2)))
15 P
```

#### R code Exa 4.2.a sum of pmf

```
1 #install.packages("IPSUR")
2 #install.packages("prob")
3 library(IPSUR)
4 #read(IPSUR)
5 library(prob)
6 #read(prob)
7
8 P1 = 1/2
9 P2 = 1/3
10 P3 = 1 - P1 - P2
11 P3
```

#### R code Exa 4.2.b pdf

```
16 P = integrate(integrand, lower = 1, upper = 2)
17 P
```

#### R code Exa 4.3.a Joint distribution of batteries

```
1 #install.packages("IPSUR")
2 #install.packages("prob")
3 library(IPSUR)
4 #read (IPSUR)
5 library(prob)
6 #read(prob)
7 #install.packages("combinat")
8 library(combinat)
9
10
p_0_0 = (length(combn(5, 3))/3)/(length(combn(12, 3))
     )/3)
12 p_0_1 = (length(combn(4, 1))/1)*(length(combn(5, 2))
     /2)/(length(combn(12, 3))/3)
13 p_0_1 = (length(combn(4, 2))/2)*(length(combn(5, 1))
     /1)/(length(combn(12, 3))/3)
14 p_0_3 = (length(combn(4, 3))/3)/(length(combn(12, 3))
     )/3)
15 p_1_0 = (length(combn(3, 1))/1)*(length(combn(5, 2))
     /2)/(length(combn(12, 3))/3)
16 p_1_1 = (length(combn(3, 1))/1)*(length(combn(4, 1))
     /1)*(length(combn(5, 1))/1)/(length(combn(12, 3))
17 p_1_2 = (length(combn(3, 1))/1)*(length(combn(4, 2))
     /2)/(length(combn(12, 3))/3)
18 p_2_0 = (length(combn(3, 2))/2)*(length(combn(5, 1))
     /1)/(length(combn(12, 3))/3)
19 p_2_1 = (length(combn(3, 2))/2)*(length(combn(4, 1))
     /1)/(length(combn(12, 3))/3)
20 p_3_0 = (length(combn(3, 3))/3)/(length(combn(12, 3))
```

#### R code Exa 4.3.b Joint distribution of boys and girls

```
1 #install.packages("IPSUR")
2 #install.packages("prob")
3 library (IPSUR)
4 #read (IPSUR)
5 library(prob)
6 #read (prob)
7 #install.packages("combinat")
8 library(combinat)
9 library(discreteRV)
10 '
11 library (discreteRV)
12 d <-RV(c("Boy", "Girl"), odds=c(0.5,0.5))
13 e \leftarrow RV(c("0","1","2","3"), odds = c
       (0.15, 0.20, 0.35, 0.3))
14 f <- joint (d, e)
15 probs (f)
16
17
18 \#P\{B=0,G=0\} = P\{\text{no children}\}\
19 P = .15
20 P
21 \#P\{B=0,G=1\} = P\{1 \text{ Girl and total of 1 child}\} = P\{1 \text{ Girl and total of 1 child}\}
       child \ P{1 girl | 1 child \}
22 P = .20 * (1/2)
23 P
24 \#P\{B=0,G=2\} = P\{2 \text{ Girls and total of 2 children}\} =
      P{2 children} P{2 girls | 2 children}
25 P = .35*(1/2)^2
26 P
27 \#P\{B=0,G=3\} = P\{3 \text{ Girls and total of 3 children}\} =
      P{3 \text{ children}} P{3 \text{ girls } | 3 \text{ children}}
```

```
28 P = .3*(1/2)^3
29 P
30 \#P\{B=1,G=0\} = P\{1 \text{ Boy and total of 1 child}\} = P\{1\}
       child \ P\langle 1 boy | 1 child \}
31 P = .2*(1/2)
32 P
33 \#P\{B=1,G=1\} = P\{1 \text{ Boy}, 1 \text{ Girl and total of } 2 \text{ children}
       P = P\{2 \text{ children}\} P\{1 \text{ boy } | 2 \text{ children}\}
34 P = .35*.5
35 \#P = .35 * ((1/2)^2 + (1/2)^2)
36 P
37 \#P\{B=1,G=2\} = P\{1 \text{ Boy}, 2 \text{ Girls and total of } 3
       children = P{2 children} P{2 Girls | 2 children
        + 1 \text{ boy} | 1 \text{ child} \}
38 P = .3*((1/2)^3+(1/2)^2)
39 P
```

#### R code Exa 4.3.c Joint Density Function

```
1 #install.packages("IPSUR")
2 #install.packages("prob")
3 library(IPSUR)
4 #read (IPSUR)
5 library(prob)
6 #read (prob)
7 #install.packages("combinat")
8 library (combinat)
9 library (discreteRV)
10 require(pracma)
11 library (cubature)
12 #1
13 fun0 = function(x,y,z) { exp(-x)*exp(-2*y)*z }
14 fun01 <- function(z) {
15
     integrate(function(y) {
16
       sapply(y, function(y) {
```

```
integrate(function(x) fun0(x,y,z), 1, Inf)$
17
            value
       })
18
     }, 0, 1)$value
19
20 }
21
22 fun01(2)
23 \#(1-\exp(-2)) * \exp(-1)
24
25 #2
26 fun0 = function(x,y,z) { exp(-x)*exp(-2*y)*z }
27 fun01 <- function(z) {
28
     integrate(function(y) {
       sapply(y, function(y) {
29
         integrate(function(x) fun0(x,y,z), 0, y)$value
30
31
       })
     }, 0, Inf)$value
32
33 }
34
35 fun01(2)
36
37
38
39 library (cubature)
40 f \leftarrow function (x) 2*\exp(-x[1])*\exp(-2*x[2])
   adaptIntegrate(f, lowerLimit = c(1,0), upperLimit = c(
41
      Inf ,1))
42
43 require (pracma)
44 f <- function (x, y) 2*exp(-x)*exp(-2*y)
45 quad2d(f, 0, 1, 1, Inf)
46
  http://stackoverflow.com/questions/23901682/double-
      integration-in-r-with-additional-argument
48
```

#### R code Exa 4.3.d Density of Independent Random Variables

```
1 #install.packages("IPSUR")
2 #install.packages("prob")
3 #install.packages("discreteRV")
4 #install.packages("pracma")
5 #install.packages("cubature")
6 library(IPSUR)
7 #read (IPSUR)
8 library(prob)
9 #read(prob)
10 #install.packages("combinat")
11 library (combinat)
12 library (discreteRV)
13 require(pracma)
14 library (cubature)
15
16 fun0 = function(x,y,a) { exp(-x)*exp(-y) }
17 fun01 \leftarrow function(z) {
     integrate(function(y) {
18
       sapply(y, function(y) {
19
         integrate(function(x) fun0(x,y,z), 0, a*y)$
20
            value
       })
21
     }, 0, Inf)$value
22
23 }
24
25 fun01(1)
26 fun01(2)
```

R code Exa 4.3.e Density of Independent Random Variables

```
#install.packages("IPSUR")
#install.packages("prob")
library(IPSUR)
#read(IPSUR)
library(prob)
#read(prob)
#install.packages("combinat")
library(combinat)
library(discreteRV)
require(pracma)
library(cubature)

#P{X1 = 1,X2 = 2, X3 = 0}
P = 0.20*0.10*0.30
```

#### R code Exa 4.3.f Conditional Probability Mass Function

```
1 #install.packages("IPSUR")
2 #install.packages("prob")
3 library(IPSUR)
4 #read (IPSUR)
5 library(prob)
6 #read (prob)
7 #install.packages("combinat")
8 library(combinat)
9 library (discreteRV)
10 require(pracma)
11 library(cubature)
12
13 \#P\{G=1\} = 0.3875
14 \quad C = 0.3875
15 \#P\{B=0|G=1\}=((P\{B=0,G=1\})/(P\{G=1\}))
16 P = 0.1/C
17 P
18 \#P\{B=1|G=1\}
```

```
19 P = .175/C

20 P

21 #P{B=2|G=1}

22 P = .1125/C

23 P

24 #P{B=3|G=1}

25 P = 0

26 P
```

#### R code Exa 4.3.g Conditional Probability Mass Function

```
1 #install.packages("IPSUR")
2 #install.packages("prob")
3 library(IPSUR)
4 #read (IPSUR)
5 library(prob)
6 #read (prob)
7 #install.packages("combinat")
8 library(combinat)
9 library(discreteRV)
10 require(pracma)
11 library(cubature)
12
13 \#P(0,0)
14 P_0_0 = .4
15 \#P(0,1)
16 P_0_1 = .2
17 #P(1,0)
18 P_1_0 = .1
19 \#P(1,1)
20 P_1_1 = .3
21
22 \#P\{Y = 1\} = P(0,1) + \#P(1,1)
23 P_Y = P_0_1 + P_1_1
24
```

```
25 #P{X = 0|Y=1} = P(0,1)/P{Y=1}

26 P1 = P_0_1 / P_Y

27 P1

28

29 #P{X = 1|Y=1} = P(1,1)/P{Y=1}

30 P2 = P_1_1 / P_Y

31 P2
```

#### R code Exa 4.4.a Expectation of a fair die

```
1 #install.packages("IPSUR")
2 #install.packages("prob")
3 library(IPSUR)
4 #read (IPSUR)
5 library(prob)
6 #read (prob)
7 #install.packages("combinat")
8 library(combinat)
9 library(discreteRV)
10 require(pracma)
11 library(cubature)
12
13
14 S <- rolldie(1, makespace = TRUE)
15 Ex = sum(S$X1*S$probs)
16 Ex
```

#### R code Exa 4.4.d Expectation of the message time

```
1 #install.packages("IPSUR")
2 #install.packages("prob")
3 library(IPSUR)
4 #read(IPSUR)
```

```
5 library(prob)
6 #read(prob)
7 #install.packages("combinat")
8 library(combinat)
9 library(discreteRV)
10 require(pracma)
11 library(cubature)
12
13
14 integrand <- function(x) {x / 1.5}
15 R = integrate(integrand, lower = 0, upper = 1.5)
16 R</pre>
```

#### R code Exa 4.5.a Expectation

```
1 #install.packages("IPSUR")
2 #install.packages("prob")
3 library(IPSUR)
4 #read (IPSUR)
5 library(prob)
6 #read(prob)
7 #install.packages("combinat")
8 library(combinat)
9 library(discreteRV)
10 require(pracma)
11 library (cubature)
12
13
14 # P {X}
15 \text{ PYO} = .2
16 \text{ PY1} = .5
17 \text{ PY2} = .3
18 \# E[X^2] = E[Y] = Y0 * PY0 + Y1 * PY1 + Y2 * PY0
19 E = 0 * PYO + 1 * PY1 + 4 * PY2
20 E
```

### R code Exa 4.5.b Expected cost of breakdown

```
#install.packages("IPSUR")
#install.packages("prob")

library(IPSUR)

#read(IPSUR)

library(prob)

#read(prob)

#install.packages("combinat")

library(combinat)

library(discreteRV)

require(pracma)

library(cubature)

integrand <- function(x) {x*(1/3)*(x^(-2/3))}

R = integrate(integrand, lower = 0, upper = 1)</pre>
```

### R code Exa 4.5.c Expectation

```
1 #install.packages("IPSUR")
2 #install.packages("prob")
3 library(IPSUR)
4 #read(IPSUR)
5 library(prob)
6 #read(prob)
7 #install.packages("combinat")
8 library(combinat)
9 library(discreteRV)
10 require(pracma)
11 library(cubature)
```

```
12
13
14 # P {X}
15 P0 = .2
16 P1 = .5
17 P2 = .3
18 # E[X^2] = X0^2*P0 + X1^2*P1 + X2^2*P2
19 E = (0^2)*P0 + (1^2)*P1 + (2^2)*P2
20 E
```

### R code Exa 4.5.d Expectation

```
#install.packages("IPSUR")
#install.packages("prob")

library(IPSUR)

#read(IPSUR)

library(prob)

#install.packages("combinat")

library(combinat)

library(discreteRV)

require(pracma)

library(cubature)

integrand <- function(x) {x^(3)}

R = integrate(integrand, lower = 0, upper = 1)</pre>
```

### R code Exa 4.5.e Expected profit

```
1 #install.packages("IPSUR")
2 #install.packages("prob")
3 library(IPSUR)
```

```
4 #read (IPSUR)
5 library(prob)
6 #read (prob)
7 #install.packages("combinat")
8 library(combinat)
9 library (discreteRV)
10 require(pracma)
11 library (cubature)
12
13
14 profits <-c(10,20,40)
15 probs <-c(.2,.8,.3)
16 E1 = profits[1]*probs[1]+0*(1-probs[1])
17 E2 = profits[2]*probs[2]+0*(1-probs[2])
18 E3 = profits [3] *probs [3] +0 * (1-probs [3])
19 E = E1 + E2 + E3
20 E
```

### R code Exa 4.5.g Different types of coupons

```
#install.packages("IPSUR")
#install.packages("prob")
library(IPSUR)
#read(IPSUR)
library(prob)
#read(prob)
#install.packages("combinat")
library(combinat)
library(discreteRV)
require(pracma)
library(cubature)

#### Additional Combinat Combin
```

### R code Exa 4.6.a Variance of a fair die

```
1 #install.packages("IPSUR")
2 #install.packages("prob")
3 library(IPSUR)
4 #read (IPSUR)
5 library(prob)
6 #read (prob)
7 #install.packages("combinat")
8 library(combinat)
9 library(discreteRV)
10 require(pracma)
11 library (cubature)
12
13
14 S <- rolldie(1, makespace = TRUE)
15 Ex = sum(S$X1*S$probs)
16 \quad \text{Ex2} = \text{sum}((S\$X1)^2*S\$probs)
17 Ex2
18 Varx = Ex2 - (Ex)^2
19 Varx
```

### R code Exa 4.7.a Variance of 10 rolls of a fair die

```
1 #install.packages("IPSUR")
2 #install.packages("prob")
3 library(IPSUR)
4 #read(IPSUR)
5 library(prob)
6 #read(prob)
7 #install.packages("combinat")
```

```
8 library(combinat)
9 library(discreteRV)
10 require(pracma)
11 library(cubature)
12
13 S <- rolldie(1, makespace = TRUE)
14 Ex = sum(S$X1*S$probs)
15 Ex2 = sum((S$X1)^2*S$probs)
16 Ex2
17 Varx = Ex2 - (Ex)^2
18 Varx
19 Varx10 = 10 * Varx
20 Varx10</pre>
```

### R code Exa 4.7.b Variance of 10 tosses of a coin

```
1 #install.packages("IPSUR")
2 #install.packages("prob")
3 library(IPSUR)
4 #read (IPSUR)
5 library(prob)
6 #read (prob)
7 #install.packages("combinat")
8 library(combinat)
9 library(discreteRV)
10 require(pracma)
11 library(cubature)
12
13 \text{ Var} = .5 * (1 - .5)
14 Var
15
16 \ Var10 = 10 * Var
17 Var10
```

### R code Exa 4.9.a Inequalities

```
1 #install.packages("IPSUR")
2 #install.packages("prob")
3 library(IPSUR)
4 #read(IPSUR)
5 library(prob)
6 #read (prob)
7 #install.packages("combinat")
8 library(combinat)
9 library(discreteRV)
10 require(pracma)
11 library(cubature)
12
13
14 # Markov
15 # P \{X > 75\}
16 \text{ meanX} = 50
17 P = meanX / 75
18 P
19
20 #Chebyshev
21 # P \{|X - 50| > = 10 \} < = (SigmaX^2 / 10^2)
22 \text{ VarX} = 25
23 P = 1 - (VarX / 10^2)
24 P
```

# Chapter 5

# Special Random Variables

### R code Exa 5.1.a Returning of disks

```
#Probability that the package will have to be
    replaced
library(IPSUR)
P = 1 - choose(10,0)*(0.01)^0*(0.99)^10-choose(10,1)
    *(0.01)^1*(0.99)^9

Probability that exactly one of the three packages
    will be returned
PP = choose(3,1)*P*(1-P)^2
PP
```

### R code Exa 5.1.b Colour of Eyes

```
1 library(IPSUR)
2 #The probability that a child recieves the blue eyed
        gene from both parents
3 Pblue = (1/2)*(1/2)
4 Pblue
```

### R code Exa 5.1.e Binomial Random Variable

```
1 library(IPSUR)
2 install.packages("binom")
3 library(binom)
4 \text{ pbinom}(q = 0, \text{ size} = 6, \text{prob} = 0.4)
5 \text{ pbinom}(q = 1, \text{ size} = 6, \text{prob} = 0.4) - \text{pbinom}(q = 0,
      size = 6, prob = 0.4)
6 pbinom(q = 2, size = 6, prob = 0.4) - pbinom(q = 1,
      size = 6, prob = 0.4)
7 pbinom(q = 3, size = 6, prob = 0.4) - pbinom(q = 2,
      size = 6, prob = 0.4)
8 pbinom(q = 4, size = 6, prob = 0.4) - pbinom(q = 3,
      size = 6, prob = 0.4)
9 pbinom(q = 5, size = 6, prob = 0.4) - pbinom(q = 4,
      size = 6, prob = 0.4)
10 pbinom(q = 6, size = 6, prob = 0.4) - pbinom(q = 5,
      size = 6, prob = 0.4)
```

### R code Exa 5.1.f Binomial Distribution

```
1 pbinom(q = 70, size = 100, prob = 0.75) - pbinom(q =
69, size = 100, prob = 0.75)
2 pbinom(q = 70, size = 100, prob = 0.75)
```

### R code Exa 5.2.a Probability of accident

```
1 #P{X>=1} = 1 - P{X = 0}
2 1-ppois(0,3)
3 #Or
4 1-dpois(0,3)
```

### R code Exa 5.2.b Defective Items

```
1 ppois(1,1)
```

### R code Exa 5.2.c Number of Alpha particles

```
1 ppois(2,3.2)
```

### R code Exa 5.2.d Claims handled by an insurance company

```
1 #Proportion of days have less than 3 claims
2 ppois(2,5)
3
4 dpois(4,lambda = 5)
5
6 #the probability that 3 of the next 5 days will have claims is
7 library(IPSUR)
8 choose(5,3) *(dpois(4,lambda = 5))^3 *(1-dpois(4, lambda = 5))^2
```

### R code Exa 5.2.f Defective stereos

```
1 i <-seq(0, 3, 1)
2 sum(dpois(i,8))</pre>
```

### R code Exa 5.3.a Functional system

```
1 dhyper(4, m = 15, n = 5, k = 6) + dhyper(5, m = 15, n = 5, k = 6) + dhyper(6, m = 15, n = 5, k = 6)
```

### R code Exa 5.3.b Determining Population Size

### R code Exa 5.3.c Conditional Probability

```
4
5 condprob <- function(n,k,p,i) {
6  bino(n,i,p)*bino(m,k-i,p)/bino(n+m,k, p);
7 }
8
9 #The function condprob will give P{X=i | X+Y=k}</pre>
```

R code Exa 5.4.a Probabilities of a uniform random variable

```
1 \text{ beta} = 10
2 alpha = 0
3 #a
4 \ a = 2
5 b = 9
6 (b-a)/(beta-alpha)
7 #b
8 \ a = 1
9 \ b = 4
10 (b-a)/(beta-alpha)
11 #c
12 \ a = 0
13 \ b = 5
14 (b-a)/(beta-alpha)
15 #d
16 \ a = 6
17 b = 10
18 (b-a)/(beta-alpha)
```

### R code Exa 5.4.b Bus Timings

```
1 beta = 30
2 alpha = 0
3 #a
```

```
4 a1 = 10
5 b1 = 15
6 a2 = 25
7 b2 = 30
8 (b1-a1)/(beta-alpha)+(b2-a2)/(beta-alpha)
9
10 #b
11 a1 = 0
12 b1 = 3
13 a2 = 15
14 b2 = 18
15 (b1-a1)/(beta-alpha)+(b2-a2)/(beta-alpha)
```

### R code Exa 5.4.c Current in a diode

```
1 I0 = 10^-6
2 a = 5
3 integrand <- function(x) {0.5*exp(a*x)}
4 R = integrate(integrand, lower = 1, upper = 3)
5 (I0*R$value)-I0</pre>
```

### R code Exa 5.5.a Normal Random Variable

```
1 #a
2 pnorm(11,3,sqrt(16))
3 #b
4 1-pnorm(-1,3,sqrt(16))
5 #c
6 pnorm(7,3,sqrt(16))-pnorm(2,3,sqrt(16))
```

### R code Exa 5.5.b Noise in Binary Message

```
1 #a
2 pnorm(-1.5)
3 #b
4 1-pnorm(2.5)
```

### ${f R}$ code ${f Exa}$ 5.5.c Power dissipation

```
1 mu = 6
2 sigma = 1
3 #a
4 E = 3*(mu^2+sigma^2) #ref page 169
5 #b
6 1-pnorm(sqrt(40),mu,sigma)
```

### R code Exa 5.5.d Yearly precipitation

```
1 #a

2 1-pnorm(25,24.16,sqrt(19.22))

3 #b

4 1-pnorm(3,0,sqrt(19.22))
```

### R code Exa 5.6.a Wearing of Battery

```
1 - pexp(5000, rate = 10000)
```

### R code Exa 5.6.b Working Machines

### R code Exa 5.6.c Series System

```
1    new <- function(lamda,n, t) {
2        newsum = 0;
3        for (i in 1:n){
4          newsum = newsum + lamda[i]
5          result=(exp(1))^(-1*newsum*t)
6        }
7     }</pre>
```

R code Exa 5.8.a Chi square random variable

```
1 pchisq(30, df = 26)
```

R code Exa 5.8.b Chi square random variable

```
1 qchisq((1-0.05), df = 15)
```

### R code Exa 5.8.c Locating a Target

```
1 - pchisq(9/4, df = 3)
```

### R code Exa 5.8.d Locating a Target in 2D space

```
1 1-pchisq(9/4, df =2)
```

2 #or

3 1-pexp(9/4,1/2)

### R code Exa 5.8.e T distribution

```
1 #a
2 pt(1.4,12)
3 #b
4 -qt(0.025,9)
```

### R code Exa 5.8.f F Distribution

```
1 pf (1.5,6,14)
```

# Chapter 6

# Distribution of Sampling Statistics

R code Exa 6.3.a Claims handled by an insurance company

```
1 1-pnorm(8300000,320*25000,540*sqrt(25000))
```

R code Exa 6.3.c Class strength

```
1 1-pnorm((150.5-(450*0.3))/sqrt(450*.3*(1-.3)))
```

R code Exa 6.3.d Weights of workers

```
1 #a
2 2*pnorm(.8889)-1
3 #b
4 2*pnorm(1.7778)-1
```

### R code Exa 6.3.e Distance of a start

```
1 qnorm(((1+.95)/2),0,1)
2 n = (1.96*4)^2
3 n
```

### R code Exa 6.5.a Processing time

```
1  n = 15
2  sigmasq = 9
3  Ssq = 12
4  1-pchisq(((n-1)*Ssq/sigmasq),n-1)
```

### R code Exa 6.6.a Candidate winning an election

```
1 favour = 0.45;
2 samplesize = 200;
3 expec= favour*samplesize;
4 sd = sqrt(samplesize*favour*(1-favour));
5 cat("The expected value is",expec)
6 cat("The standard deviation is",sd)
7
8 #b
9 prob = 1-pbinom(q = samplesize/2, size = samplesize, prob = favour)
10 cat("Probability that more than half the members of the sample favour the candidate",prob)
```

### ${f R}$ code ${f Exa}$ 6.6.b Pork consumption

```
1 meaneach = 147;
2 sdeach = 62;
3 samplesize = 25;
4 lim =150;
5 samplemean = meaneach;
6 samplesd= sdeach/sqrt(samplesize)
7 prob = 1 - pnorm(q = lim, mean = samplemean, sd = samplesd)
8 prob
```

## Chapter 7

### Parameter Estimation

R code Exa 7.2.a Maximum likelihood estimator of a bernoulli parameter

R code Exa 7.2.b Errors in a manuscript

```
1 totalerror<-function(n1, n2, n12){
2   totalerror = n1*n2/n12
3   return(totalerror)
4 }
5 totalerror(n1 = 33000, n2 = 33000, n12 = 17000)</pre>
```

R code Exa 7.2.c Maximum likelihood estimator of a poisson parameter

```
1 total_people = 857;
```

### R code Exa 7.2.d Number of traffic accidents

```
1 accidents <-c(4,0,6,5,2,1,2,0,4,3)
2 lambda= mean(accidents)
3 cat("lambda:",lambda)
4 prob = ppois(q = 2, lambda = lambda)
5 cat("Proportion of non-rainy days that had 2 or fewer accidents that year:",prob)</pre>
```

### R code Exa 7.2.e Maximum likelihood estimator in a normal population

```
1 normal <-function(X, Xmean, n){
2   u= Xmean;
3   newsum = 0;
4   for (i in 1:n){
5     newsum= newsum + (X(i)-Xmean)^2
6   }
7   sigmasquared = sqrt((newsum/n));
8   return(u, sigmasquared)
9 }</pre>
```

### R code Exa 7.2.f Kolmogorovs law of fragmentation

```
1 X <-c(2.2,3.4,1.6,0.8,2.7,3.3,1.6,2.8,2.5,1.9)
2 upperlimX = 3
3 lowerlimX = 2</pre>
```

### R code Exa 7.2.g Estimating Mean of a Uniform Distribution

```
1 unif <- function(X, n) {
2  max(X)/2
3 }</pre>
```

### R code Exa 7.3.a Error in a signal

```
1 avg = 0;
2 var = 4;
3 num = 9;
4 X<-c(5,8.5,12,15,7,9,7.5,6.5,10.5)
5 samplemean = mean(X);
6 lowerlim = samplemean - (1.96*sqrt(var/num))
7 upperlim = samplemean + (1.96*sqrt(var/num))
8 cat("The 95% confidence interval is ",lowerlim," to ", upperlim)</pre>
```

### R code Exa 7.3.b Confidence interval

```
1 avg = 0;
2 var = 4;
3 num = 9;
4 X<-c(5,8.5,12,15,7,9,7.5,6.5,10.5)
5 samplemean = mean(X);
6 lowerlim = samplemean - (1.645*sqrt(var/num))
7 upperlim = samplemean + (1.645*sqrt(var/num))
8
9 cat("The 95% upper confidence interval is ",lowerlim," to infinity")
10 cat("The 95% upper confidence interval is minus infinity to ",upperlim)</pre>
```

### R code Exa 7.3.c Confidence interval

```
1 \text{ var} = 4;
2 \text{ num} = 9;
3 \text{ X} \leftarrow c (5, 8.5, 12, 15, 7, 9, 7.5, 6.5, 10.5)
4 samplemean= mean(X);
5 \text{ samplesd} = \text{sd}(X)
6 alpha= 0.005;
7 zalpha = qnorm(p = alpha, mean = 0, sd = 1, lower.tail
      = FALSE)
8 zalpha
9 lowerlim = samplemean - (zalpha*sqrt(var/num))
10 upperlim = samplemean + (zalpha*sqrt(var/num))
11 cat ("The 95% confidence interval is ",lowerlim, "to
      ", upperlim)
12
13 alpha= 0.01;
14 zalpha = qnorm(p = alpha, mean = 0, sd = 1, lower.tail
      = FALSE)
15 lowerlim = samplemean - (zalpha*sqrt(var/num))
```

```
16 upperlim = samplemean + (zalpha*sqrt(var/num))
17 cat("The 95% upper confidence interval is ",lowerlim
        ," to infinity")
18 cat("The 95% upper confidence interval is minus
        infinity to ",upperlim)
```

### R code Exa 7.3.d Weight of a salmon

```
1 sd= 0.3;
2 lim = 0.1;
3 num = (1.96*sd/lim)^2;
4 cat("Sample size should be greater than", round(num))
```

### R code Exa 7.3.e Error in a signal

```
1 X \leftarrow c(5, 8.5, 12, 15, 7, 9, 7.5, 6.5, 10.5)
2 \text{ num} = 9;
3 meanX = mean(X);
4 X2 = X^2;
5 \text{ s2= } (sum(X2) - (num*(meanX^2)))/(num-1);
6 s= sqrt(s2);
7 \text{ tval} = qt(.975,8)
8 upperlim = meanX + (tval*s)/sqrt(num);
9 lowerlim = meanX - (tval*s)/sqrt(num);
10 cat ("The 95% confidence interval is", lowerlim,"
      to ",upperlim )
11
12 \# 7.3 a
13 \text{ s2= } 4
14 s = sqrt(s2);
15 \text{ tval} = qt(.975,8)
16 upperlim = meanX + (tval*s)/sqrt(num);
17 lowerlim = meanX - (tval*s)/sqrt(num);
```

```
18 cat("The 95\% confidence interval is ", lowerlim," to ",upperlim)
```

### R code Exa 7.3.f Average resting pulse

```
1 \text{ X} \leftarrow c(54, 63, 58, 72, 49, 92, 70, 73, 69, 104, 48,
      66, 80, 64, 77)
2 \text{ num} = 15;
3 meanX= mean(X);
4 X2 = X^2;
5 \text{ s2= } (sum(X2) - (num*(meanX^2)))/(num-1);
6 \text{ s= sqrt(s2)};
7 \text{ tval} = qt(.975, num-1)
8 upperlim = meanX + (tval*s)/sqrt(num);
9 lowerlim = meanX - (tval*s)/sqrt(num);
10 cat ("The 95% confidence interval is ", lowerlim, "to"
      ,upperlim)
11 \text{ alpha} = 0.05;
12 tval = qt(1-alpha,num-1)
13 lim = meanX + (tval*s)/sqrt(num);
14 cat ("The 95% lower confidence interval is from minus
       infinity to ", lim)
```

### R code Exa 7.3.g Evaluating integrals

```
1 #Method 1: Monte Carlo Integration
2 #install.packages("functional")
3 library(functional)
4 funct<-function(sampleXVals) {
5   oned = sqrt(1-(sampleXVals*sampleXVals))
6   return(oned)
7 }
8 MCIntegrate <- function(funct, a,b,N) {</pre>
```

```
9
     sampleXVals = matrix(0,N)
10
     set . seed (148)
     sampleXVals = a + (b-a)*runif(N)
11
     Integral = (b-a)* mean(funct(sampleXVals))
12
13
     return(Integral)
14 }
15 MCIntegrate (funct, 0, 1, 50000000)
16
17
18 #Method 2
19 \text{ meanX} = 0.786;
20 s = 0.03;
21 \text{ num} = 100;
22 \text{ alpha} = 0.05;
23 tval = qt(1-alpha,num-1)
24 upperlim = meanX + (tval*s)/sqrt(num);
25 lowerlim = meanX - (tval*s)/sqrt(num);
26 cat("The 95\% confidence interval is ",lowerlim," to "
      ,upperlim)
```

### R code Exa 7.3.h Thickness of washers

### R code Exa 7.4.a Cable insulation

```
1 \text{ A} \leftarrow c(36, 44, 41, 53, 38, 36, 34, 54, 52, 37, 51, 44,
       35, 44)
2 \text{ B} \leftarrow c(52, 64, 38, 68, 66, 52, 60, 44, 48, 46, 70, 62)
3 \text{ sigmaA} = 40;
4 sigmaB= 100;
5 \text{ alpha} = 1-0.95;
6 beta= alpha/2;
7 \text{ meanA} = \text{mean}(A);
8 meanB= mean(B);
9 zbeta = qnorm(1-beta)
10
11 lowerlim = mean(A) - mean(B) - (zbeta*sqrt((sigmaA/
      length(A)) + (sigmaB/length(B))));
12 upperlim = mean(A) - mean(B) + (zbeta*sqrt((sigmaA/
      length(A)) + (sigmaB/length(B))));
13 cat("The 95\% confidence interval is ",lowerlim, "to
      ",upperlim)
14
15 beta=alpha;
16 zbeta = qnorm(1-beta)
17
18 upperlim= mean(A) - mean(B) + (zbeta*sqrt((sigmaA/
      length(A)) + (sigmaB/length(B))));
19 cat ("A value that exceed the difference of the means
       with 95% confidence is ", upperlim)
```

### R code Exa 7.4.b Battery production

```
2 tech2<-c(144, 132, 136, 140, 128, 150, 130, 134,
      130, 146, 128, 131, 137, 135)
3 \text{ num1} = 12;
4 \text{ num2} = 14;
5 mean1= mean(tech1);
6 mean2= mean(tech2);
7 \text{ alpha} = 0.9;
8 S1 = var(tech1)
9 S2 = var(tech2)
10 Sp = (((num1-1)*S1) + ((num2-1)*S2))/(num1+ num2 -2)
11 Sp= sqrt(Sp);
12 num = (1/num1) + (1/num2);
13 betaa = (1-alpha)/2;
14 tval = qt(1-betaa,num1+num2-2)
15 upperlim = mean1-mean2 + (tval*Sp)*sqrt(num);
16 lowerlim = mean1-mean2 - (tval*Sp)*sqrt(num);
17 cat ("The 90% confidence interval is ",lowerlim, "to
      ", upperlim)
18 \text{ alpha} = 0.95
19 betaaa = 1-alpha;
20 tval = qt(1-betaa,num1+num2-2)
21 lowerlim = mean1-mean2 - (tval*Sp)*sqrt(num);
22 cat ("the upper confidence interval is", lowerlim," to
       infinity")
```

### R code Exa 7.5.a Transistors

```
1 phat = 0.8;
2 zalpha = 1.96;
3
4 samplesize = 100;
5 lowerlim = phat - (zalpha*sqrt(phat*(1-phat)/samplesize));
6 upperlim = phat + (zalpha*sqrt(phat*(1-phat)/samplesize));
```

### R code Exa 7.5.b Survey

```
1 phat = 0.52;
2 error = 0.04;
3 zalpha = 1.96;
4 samplesize = (error/zalpha)^2/(phat*(1-phat));
5 1/samplesize
```

### R code Exa 7.5.c Acceptable chips

```
1 initialsample = 30;
2 acceptable= 26;
3 phat = acceptable/initialsample;
4 error = 0.05/2;
5 zalpha = 2.58;
6
7 samplesize = (error/zalpha)^2/(phat*(1-phat));
8 finalsize = round(1/samplesize);
9 acceptablenew= 1040 + acceptable;
10 phat = acceptablenew/finalsize;
11 lowerlim = phat - (zalpha*sqrt(phat*(1-phat)/finalsize));
12 upperlim = phat + (zalpha*sqrt(phat*(1-phat)/finalsize));
13 cat("The 99% confidence interval is ",lowerlim, "to ",upperlim)
```

### R code Exa 7.6.a Life of a product

```
1 sum_lives = 1740;
2 \text{ num} = 10;
3 \text{ alpha} = (1-0.95)/2;
4 chi1= qchisq(1-alpha,2*num)
5 chi2 = qchisq(alpha, 2*num)
6 lowerlim = 2*sum_lives/chi1;
7 upperlim = 2*sum_lives/chi2;
8 cat("The 95% confidence interval is ",lowerlim, "to
     ", upperlim)
9
10
  Textbook confidence interval is from 101.847 to
      360.211
12 whereas in above solution is 101.84489 to 362.8485
      because of the difference in the value of chi-
      square (0.975, 20).
13 The textbook says the value is 9.661 whereas R
      calculates its value as 9.59
14 ,
```

### R code Exa 7.7.a Point estimator

```
1 estimator1 <- function(x) {
2   X[1]
3 }
4 estimator2 <- function(variables) {
5   mean(X)
6 }</pre>
```

### R code Exa 7.7.b Point estimator

```
1 estimate <- function(d, sigma) {
2    sigmainv = 1/sigma
3    new = d/sigma
4    result1 = sum(new)/sum(sigmainv)
5 }
6
7 mserror <- function(sigma) {
8    sigmainv = 1/sigma
9    result1 = 1/sum(sigmainv)
10 }</pre>
```

### R code Exa 7.7.c Point estimator of a uniform distribution

```
1 unbiasedestimator <- function(X, n) {
2   c=(n+2)/(n+1)
3   result = c*max(X)
4 }</pre>
```

### R code Exa 7.8.a Bayes estimator

```
1 estimator <- function(X, n) {
2   (sum(X) +1)/(n+2)
3 }</pre>
```

### R code Exa 7.8.b Bayes estimator of a normal population

```
4 }
5
6 varestimator <- function(sigma0 , sigma, n) {
7  (sigma0*sigma)/((n*sigma)+sigma0)
8 }</pre>
```

### R code Exa 7.8.d estimator of the signal value

```
1 meanestimator <- function(sigma0 , u, sigma, n, X) {</pre>
     meanX = mean(X);
     result = (n*meanX/sigma0)/((n/sigma0)+(1/sigma)) +
         (u/sigma)/((n/sigma0)+(1/sigma));
     return(result)
5 }
7 varestimator <- function(sigma0 , sigma, n) {</pre>
     result = (sigma0*sigma)/((n*sigma)+sigma0);
     return(result)
9
10 }
11
12 u = 50;
13 sigma= 100;
14 \text{ sigma0} = 60;
15 \, n = 1;
16 \ X = 40;
17 expec = meanestimator(sigma0 , u, sigma, n, X);
18 var = varestimator (sigma0, sigma,n);
19
20 \text{ zalpha} = 1.645
21 lowerlim = -1*sqrt(var)*zalpha+expec;
22 upperlim = sqrt(var)*zalpha+expec;
23 cat ("With probability 0.9, the sent signal lies
      between ",lowerlim, "to ", upperlim
```

# Chapter 8

# Hypothesis Testing

### R code Exa 8.3.a Noise in a Signal

```
1 noise_var = 4;
2 noise_mean= 0;
3 num = 5;
4 Xbar = 9.5;
5 u = 8;
6 statistic = sqrt(num/noise_var)*(Xbar - u);
7 compare = qnorm(0.975,0, 1);
8 if(statistic < compare){
9    cat("Hypothesis is accepted");
10 } else{
11    cat("Hypothesis is not accepted")
12 }</pre>
```

### R code Exa 8.3.b Error in a signal

```
1 noise_var = 4;
2 noise_mean= 0;
3 num = 5;
```

```
4 Xbar = 8.5;
5 u = 8;
6 statistic = sqrt(num/noise_var)*(Xbar - u);
7
8 prob = 2*pnorm(-1*statistic , 0,1 );
9 cat("P-value is",prob)
10
11 Xbar = 11.5;
12 statistic = sqrt(num/noise_var)*(Xbar - u);
13 prob = pnorm(-1*statistic , 0,1 );
14 cat("P-value is",prob)
```

### R code Exa 8.3.c Error in a signal

```
1  noise_var = 4;
2  num = 5;
3  Xbar = 10;
4  u = 8;
5  statistic = sqrt(num/noise_var)*(Xbar - u);
6  compare = qnorm(0.975,0, 1);
7  lim1 = statistic + compare;
8  lim2 = statistic - compare;
9  prob = pnorm(lim1 , 0,1 ) - pnorm(lim2 , 0,1 );
10  prob
```

### R code Exa 8.3.d Number of signals to be sent

```
1 alpha = 0.025;
2 betaa = 0.25;
3 u1 = 9.2;
4 u0 = 8;
5 var =4;
6 zalpha = qnorm(1-alpha,0, 1);
```

```
7 zbeta = qnorm(1-betaa,0, 1);
8 n = ((zalpha + zbeta)/(u1-u0))^2 *var;
9 cat("Required number of samples is",ceiling(n))
10 statistic = sqrt(ceiling(n)/var)*(u1 - u0);
11 lim1 = -1*statistic + zalpha;
12 lim2 = -1*statistic - zalpha;
13 prob = pnorm(lim1, 0,1) - pnorm(lim2, 0,1);
14 cat("Thus, if the message is sent the reqd number of times is , then the probability that the null hypothesis will be rejected is",1-prob)
```

### R code Exa 8.3.e Number of signals to be sent

```
1 n =5;
2 Xbar = 9.5;
3 u = 8;
4 var = 4;
5 statistic = sqrt(n/var)*(Xbar - u);
6 p = 1 - pnorm(statistic, 0, 1);
7 cat("The test would call for rejection at all significance levels greater than or equal to ",p)
```

### R code Exa 8.3.f Nicotine content in a cigarette

```
1  n =20;
2  Xbar = 1.54;
3  uo = 1.6;
4  sd = 0.8;
5  statistic = sqrt(n)*(Xbar - uo)/sd;
6  cat("Test statistic is", statistic)
7  p = pnorm(statistic, 0, 1);
8  cat("P-value is",p)
```

### R code Exa 8.3.g Blood cholesterol level

```
1  n = 50;
2  Xbar = 14.8;
3  S = 6.4;
4  T = sqrt(n)*Xbar/S;
5  cat("The T value is",T)
```

### R code Exa 8.3.h Water usage

```
1 X<-c(340, 356, 332, 362, 318, 344, 386, 402, 322,
      360, 362, 354, 340, 372, 338, 375, 364, 355, 324,
       370)
2 \text{ uo} = 350;
3 \text{ Xbar} = \text{mean}(X);
4 \text{ var} = \text{var}(X);
5 S = sqrt(var)
6 n = length(X)
7 T = sqrt(n)*(Xbar - uo)/S;
8 Tvalue = qt(0.95, n-1)
9 cat("The T value is ",T)
10 if(T<Tvalue){</pre>
     cat ("Null hypothesis is accepted at 10% level of
11
        significance")
12 }else{
     cat ("Null hypothesis is not accepted at 10% level
13
        of significance")
14 }
15 cat("P value of the test data",2*pnorm(-1*T,0,1))
```

### R code Exa 8.3.i Life of a tire

```
1 \times -c(36.1, 40.2, 33.8, 38.5, 42, 35.8, 37, 41, 36.8,
       37.2, 33, 36)
2 n = length(X);
3 \text{ uo} = 40;
4 Xbar=mean(X);
5 sd = sqrt(var(X));
6 T = sqrt(n)*(Xbar - uo)/sd;
7 Tvalue = qt(0.05, n-1);
8 cat("The T value is ",T)
9 if(T<Tvalue){</pre>
10
     cat ("Null hypothesis is rejected at 5% level of
        significance")
11 }else{
12
     cat ("Null hypothesis is accepted at 5% level of
        significance")
13 }
14
  cat("P value of the test data",pnorm(T,0,1))
16
  There is a mismatch between book answer for the P
17
      value of the test data.
18
```

### R code Exa 8.3.j Service Time

```
6 T = sqrt(n)*(Xbar - uo)/sd;
7 cat("The test statistic is ",T)
8 p = 1- pt(T, n-1);
9 cat("P-value is",p)
10 cat("A small p value indicates that the mean service time exceeds 8 minutes")
```

#### R code Exa 8.4.a Tire lives

#### R code Exa 8.4.b Medicine for cold

```
1 X <-c(5.5, 6.0, 7.0, 6.0, 7.5, 6.0, 7.5, 5.5, 7.0,
6.5)
2 Y <-c(6.5, 6.0, 8.5, 7.0, 6.5, 8.0, 7.5, 6.5, 7.5,
6.0, 8.5, 7.0)
3 n = length(X);
4 m = length(Y);
5 Xbar = mean(X);</pre>
```

```
6 \text{ Ybar} = \text{mean}(Y);
7 \text{ Sx} = \text{var}(X);
8 \text{ Sy} = \text{var}(Y);
9 Sp = ((n-1)*Sx/(n+m-2)) + ((m-1)*Sy/(n+m-2));
10 den = sqrt(Sp*((1/n)+(1/m)));
11 TS = (Xbar -Ybar)/den;
12 cat("The test statistic is", TS);
13 tvalue = qt(0.95, m+n-2)
14 if (TS < tvalue) {
     cat ("Null hypothesis is rejected at 5% level of
15
         significance")
16 }else{
17
    cat ("Null hypothesis is
                                  accepted at 5% level of
         significance")
18 }
19 cat("The Pvalue is",pt(TS,m+n-2))
```

#### R code Exa 8.4.c Unknown population variance

```
1 \text{ A} \leftarrow c(61.1, 58.2, 62.3, 64, 59.7, 66.2, 57.8, 61.4,
      62.2, 63.6)
2 \text{ B} \leftarrow c(62.2, 56.6, 66.4, 56.2, 57.4, 58.4, 57.6, 65.4)
3 uA = mean(A);
4 \text{ uB} = \text{mean}(B);
5 n = length(A);
6 m = length(B);
7 Sx = var(A);
8 \text{ Sy} = \text{var}(B);
9 Sp = ((n-1)*Sx/(n+m-2)) + ((m-1)*Sy/(n+m-2));
10 den = sqrt(Sp*((1/n)+(1/m)));
11 TS = (uA-uB)/den;
12 cat ("The test statistic is", TS);
13 pvalue = 2*(1-pt(TS,m+n-2));
14 cat ("Null hypothesis is accepted at any significance
       level less than", pvalue)
```

## R code Exa 8.4.d effectiveness of safety program

#### R code Exa 8.5.a effectiveness of machine

```
1  n =20;
2  S2= 0.025;
3  chk = 0.15;
4  compare = (n-1)*S2/(chk^2);
5  pvalue = 1- pchisq(compare, n-1);
6  cat("The p-value is", pvalue)
7  cat("Thus , the null hypothesis is accepted")
```

#### R code Exa 8.5.b Catalyst

```
1 S1 = 0.14;
2 S2 = 0.28;
```

```
3 n= 10;
4 m= 12;
5 ratio = S1/S2;
6 prob1 = pf(ratio, n-1, m-1);
7 prob2 = 1-prob1;
8 prob = min(prob1,prob2);
9 pvalue = 2*prob;
10 cat("The p value is",pvalue)
11 cat("So the hypothesis of equal variance cannot be rejected")
```

#### R code Exa 8.6.a Computer chip manufacturing

```
1 samplesize = 300;
2 p =0.02;
3 defective=9;
4 val = pbinom(defective, samplesize, p, 1-p);
5 cat("P0.02{X>10} = ",val);
6 cat("Manufacturers claim cannot be rejected at the 5% level of significance")
```

#### R code Exa 8.6.b Finding p value

```
1 samplesize = 300;
2 p =0.02;
3 defective=9;
4 compare = 10;
5 npo = samplesize*p;
6 sd = sqrt(npo*(1-p));
7 tol = 0.5;
8 pvalue = 1- pnorm(compare-tol, npo,sd);
9 cat("The pvalue is",pvalue)
```

#### R code Exa 8.6.c Change in manufacturing pattern

```
1 samplesize = 500;
2 p =0.04;
3 defective=16;
4 prob1 = 1-pbinom(defective, samplesize, p, 1-p);
5 prob2 = pbinom(defective-1, samplesize, p, 1-p);
6 pvalue = 2*min(prob1,prob2);
7 cat("The pvalue is",pvalue)
```

### R code Exa 8.7.a Mean number of defective chips

```
1 x <-c(28, 34, 32, 38, 22)
2 claim = 25;
3 total = sum(x);
4 pval = 1 - ppois(total-1, (claim*length(x)));
5 cat("The pvalue is", pval)</pre>
```

#### R code Exa 8.7.b Safety Conditions in a plant

```
1 plant1 <-c(16, 18, 9, 22, 17, 19, 24, 8)
2 plant2 <-c(22, 18, 26, 30, 25, 28)
3 X1= sum(plant1);
4 X2 = sum(plant2);
5 n =length(X1);
6 m= length(X2);
7 prob1 = 1 - pbinom(X1, X1+X2,(4/7),(3/7));
8 prob2 = pbinom(X1-1, X1+X2, 4/7, 3/7);
9 cat(prob1, prob2)</pre>
```

```
10 pvalue = 2*min(prob1,prob2);
11 cat("The pvalue is",pvalue)
```

## R code Exa 8.7.c Better proof reader

```
1  Aerror =28;
2  Berror = 18;
3  common =10;
4  N2 = Aerror - common;
5  N3 =Berror - common;
6  pval = pbinom(N2-1, N2 + N3, 0.5, 0.5);
7  cat("P-value is", pval)
```

## Chapter 9

## Regression

### R code Exa 9.1.a Scatter Diagram

#### R code Exa 9.2.a Relative humidity and moisture content

```
7 p = seq(0,65,0.1)
8 q = p*X + Y
9 lines(p, q)
```

### R code Exa 9.3.a Moisture against Density

```
1 x \leftarrow c(5, 6, 7, 10, 12, 15, 18, 20)
2 y <-c (7.4, 9.3, 10.6, 15.4, 18.1, 22.2, 24.1, 24.8)
3 plot(x,y);
4
5 \text{ xbar} = \text{mean}(x);
6 ybar = mean(y);
7 n = 8;
8 \text{ SxY} = 0;
9 for (i in (1:n)){
    SxY = SxY + (x[i]*y[i]) - (xbar*ybar)
11 }
12
13 Sxx = 0;
14 for (i in 1:n){
15
     Sxx = Sxx + (x[i]*x[i]) - (xbar*xbar);
16 }
17
18 \text{ SYY} = 0;
19 for (i in 1:n){
     SYY = SYY + (y[i]*y[i]) - (ybar*ybar);
20
21 }
22
23 B = SxY/Sxx;
24 A = ybar - (B*xbar);
25 cat("A is",A);
26 cat("B is",B);
27 p = seq(0,20,0.1)
28 q = A + B*p;
29 lines(p,q)
```

```
30
31 SSR = ((Sxx*SYY) - (SxY*SxY))/Sxx;
32 cat("The SSR is", SSR)
```

## R code Exa 9.4.a Effect of speed on mileage

```
1 x < -c(45, 50, 55, 60, 65, 70, 75)
2 \text{ y} \leftarrow c(24.2, 25.0, 23.3, 22.0, 21.5, 20.6, 19.8)
3 \text{ xbar} = \text{mean}(x);
4 ybar= mean(y);
5 n = 7;
6 \text{ SxY} = 0;
7 for (i in 1:n){
     SxY = SxY + (x[i]*y[i]) - (xbar*ybar)
9 }
10
11 Sxx = 0;
12 for (i in 1:n){
     Sxx = Sxx + (x[i]*x[i]) - (xbar*xbar);
14 }
15 \text{ SYY} = 0;
16 for (i in 1:n){
     SYY = SYY + (y[i]*y[i]) - (ybar*ybar);
17
18 }
19 B = SxY/Sxx;
20 A = ybar - (B*xbar);
21 cat("A is",A);
22 cat("B is",B);
23
24 SSR = ((Sxx*SYY) - (SxY*SxY))/Sxx;
25 cat ("The SSR is", SSR)
26
27 ts = sqrt(((n-2)*Sxx)/SSR)*abs(B);
28 cat("the test statistic is", ts);
29 tvalue= qt(0.995,5);
```

#### R code Exa 9.4.b Confidence interval estimate

```
1 x < -c (45, 50, 55, 60, 65, 70, 75)
2 \text{ y} \leftarrow c(24.2, 25.0, 23.3, 22.0, 21.5, 20.6, 19.8)
3 \text{ xbar} = \text{mean}(x);
4 \text{ ybar} = \text{mean}(y);
5 n = 7;
6 \text{ SxY} = 0;
7 for (i in 1:n){
     SxY = SxY + (x[i]*y[i]) - (xbar*ybar)
9 }
10
11 Sxx = 0;
12 for (i in 1:n){
     Sxx = Sxx + (x[i]*x[i]) - (xbar*xbar);
13
14 }
15 \text{ SYY} = 0;
16 for (i in 1:n){
17
     SYY = SYY + (y[i]*y[i]) - (ybar*ybar);
18 }
19 B = SxY/Sxx;
20 A = ybar - (B*xbar);
21 cat("A is",A);
22 cat("B is",B);
23
```

```
24 SSR = ((Sxx*SYY)- (SxY*SxY))/Sxx;
25 cat("The SSR is",SSR)
26
27 tvalue= qt(0.975, 5);
28 cat("tvalue is",tvalue);
29
30 k = sqrt(SSR/((n-2)*Sxx))*tvalue;
31 int1 = B + k;
32 int2= B-k;
33 cat("The 95% confidence interval is ",int2, "to ", int1)
```

## R code Exa 9.4.c Regression to the mean

```
1 x \leftarrow c (60, 62, 64, 65, 66, 67, 68, 70, 72, 74)
2 \text{ y} \leftarrow c (63.6, 65.2, 66, 65.5, 66.9, 67.1, 67.4, 68.3,
      70.1, 70)
3 plot(x,y,xlab = "Father's Height",ylab = "Son's
      Height")
4 \text{ xbar} = \text{mean}(x)
5 ybar= mean(y)
6 n = 10;
7 SxY = 0
8 for (i in 1:n){
     SxY = SxY + (x[i]*y[i]) - (xbar*ybar);
10 }
11
12 \text{ Sxx} = 0;
13 for (i in 1:n){
     Sxx = Sxx + (x[i]*x[i]) - (xbar*xbar);
15 }
16
17 \text{ SYY} = 0;
18 for (i in 1:n){
     SYY = SYY + (y[i]*y[i]) - (ybar*ybar);
```

```
20 }
21
22 B = SxY/Sxx;
23 A = ybar - (B*xbar);
24 cat("A is",A);
25 cat("B is",B);
26 p = seq(60,72,0.1)
27 \quad q = A + B*p;
28 lines(p,q)
29 SSR = ((Sxx*SYY) - (SxY*SxY))/Sxx;
30 \text{ ts} = \text{sqrt}(((n-2)*Sxx)/SSR)*(B-1)
31 ts
32 \text{ tvalue} = qt(0.99, n-2)
33 tvalue
34 if(ts<(-1*tvalue)){
     cat ("Null hypotheis is rejected at 1% level of
         significance")
36 } else{
     cat ("Null hypotheis is accepted at 1% level of
37
        significance")
38 }
```

#### R code Exa 9.4.d Motor vehicle deaths

```
9
10 \text{ SxY} = 0
11 for (i in 1:n){
     SxY = SxY + (x[i]*y[i]) - (xbar*ybar);
13 }
14
15 \text{ Sxx} = 0;
16 for (i in 1:n){
     Sxx = Sxx + (x[i]*x[i]) - (xbar*xbar);
17
18 }
19
20 \text{ SYY} = 0;
21 for (i in 1:n){
     SYY = SYY + (y[i]*y[i]) - (ybar*ybar);
23 }
24
25 B = SxY/Sxx;
26 A = ybar - (B*xbar);
27 cat("A is",A);
28 cat("B is",B)
```

#### R code Exa 9.4.e Confidence interval for height

```
12 \text{ Sxx} = 0;
13 for (i in 1:n){
     Sxx = Sxx + (x[i]*x[i]) - (xbar*xbar);
15 }
16
17 \text{ SYY} = 0;
18 for (i in 1:n){
19
     SYY = SYY + (y[i]*y[i]) - (ybar*ybar);
20 }
21 B = SxY/Sxx;
22 A = ybar - (B*xbar);
23 tvalue = qt(0.975, n-2);
24 \quad SSR = ((Sxx*SYY) - (SxY*SxY))/Sxx ;
25 \quad \text{intvl} = A + (B*x0);
26 change = sqrt((1/n)+(((x0-xbar)^2)/Sxx))* sqrt(SSR/(
      n-2))*tvalue;
27 intvl1 = intvl - change;
28 intvl2= intvl + change;
29 cat ("The 95\% confidence interval is ", intvl1, "to ",
       intv12)
```

#### R code Exa 9.4.f Confidence interval for height

```
1 x <-c(60, 62, 64, 65, 66, 67, 68, 70, 72, 74)
2 y <-c(63.6, 65.2, 66, 65.5, 66.9, 67.1, 67.4, 68.3, 70.1, 70)
3 x0 = 68;
4 xbar = mean(x);
5 ybar= mean(y);
6 n= 10;
7 SxY = 0
8 for (i in 1:n){
9 SxY = SxY + (x[i]*y[i]) - (xbar*ybar);
10 }
11</pre>
```

```
12 Sxx = 0;
13 for (i in 1:n){
     Sxx = Sxx + (x[i]*x[i]) - (xbar*xbar);
15 }
16
17 \text{ SYY} = 0;
18 for (i in 1:n){
19
     SYY = SYY + (y[i]*y[i]) - (ybar*ybar);
20 }
21 B = SxY/Sxx;
22 A = ybar - (B*xbar);
23 tvalue = qt(0.975, n-2);
24 \quad SSR = ((Sxx*SYY) - (SxY*SxY))/Sxx ;
25 \quad \text{intvl} = A + (B*x0);
26 change = sqrt(((n+1)/n)+(((x0-xbar)^2)/Sxx))* sqrt(
      SSR/(n-2))*tvalue;
27 intvl1 = intvl - change;
28 intvl2= intvl + change;
29 cat ("The 95\% confidence interval is ", intvl1, "to ",
       intv12)
```

#### R code Exa 9.5.a Height of son and father

```
1 x <-c(60, 62, 64, 65, 66, 67, 68, 70, 72, 74)
2 y <-c(63.6, 65.2, 66, 65.5, 66.9, 67.1, 67.4, 68.3, 70.1, 70)
3
4 xbar = mean(x);
5 ybar= mean(y);
6 n= 10;
7
8 SxY = 0
9 for (i in 1:n){
10  SxY = SxY + (x[i]*y[i]) - (xbar*ybar);
11 }</pre>
```

```
12
13 Sxx = 0;
14 for (i in 1:n){
     Sxx = Sxx + (x[i]*x[i]) - (xbar*xbar);
16 }
17
18 \text{ SYY} = 0;
19 for (i in 1:n){
     SYY = SYY + (y[i]*y[i]) - (ybar*ybar);
20
21 }
22
23 B = SxY/Sxx;
24 A = ybar - (B*xbar);
25
26 \quad SSR = ((Sxx*SYY) - (SxY*SxY))/Sxx ;
27 R2 = 1 - (SSR/SYY);
28 cat ("The coefficient of determination is", R2)
```

#### R code Exa 9.7.a Percentage of chemical used

```
15 }
16
17 \text{ Sxx} = 0;
18 for (i in 1:n){
      Sxx = Sxx + (x[i]*x[i]) - (xbar*xbar);
20 }
21
22 \text{ SYY} = 0;
23 for (i in 1:n){
     SYY = SYY + (y[i]*y[i]) - (ybar*ybar);
25 }
26
27 B = SxY/Sxx;
28 A = ybar - (B*xbar);
29 \quad SSR = ((Sxx*SYY) - (SxY*SxY))/Sxx ;
30 chat = exp(-1*A);
31 dhat = 1 - \exp(-1*B);
32 \text{ phat} = 1 - \text{chat*}(1-\text{dhat})^x
33 \text{ pres} = \text{yold} - \text{phat}
34 cat("chat is", chat);
35 cat("dhat is", dhat);
36 cat("phat is",phat);
37 cat("p - phat, i.e. the residuals are", pres)
```

#### R code Exa 9.8.b Distance vs Travel Time

```
9 q = matrix(0,2,1)
10 p[1, 1] = sum(w);
11 p[1,2] = n;
12 p[2,1] = n;
13 p[2,2] = sum(x);
14 new <-c()
15 for (i in 1:10){
     new[i] = w[i]*y[i]
16
17 }
18
19 q[1,1] = 1*sum(new);
20 q[2,1] = 1*sum(y);
21 \text{ sol } = \text{solve}(p,q);
22 A = sol[1,1];
23 B = sol[2,1];
24 cat("A is",A);
25 cat("B is",B);
26 plot(x,y)
27 \ 1 = seq(0,10,0.1)
28 k = A + B*1
29 lines(1,k)
```

## R code Exa 9.9.a Polynomial Fitting

```
11 q = matrix(0,3,1);
12 p[1,1] = n;
13 p[1,2] = sum(x);
14 p[1,3] = sum(xsquared);
15 p[2,1] = sum(x);
16 p[2,2] = sum(xsquared);
17 p[2,3] = sum(xcube);
18 p[3,1] = sum(xsquared);
19 p[3,2] = sum(xcube);
20 p[3,3] = sum(xfour);
21 q[1,1] = 1*sum(y);
22 q[2,1] = 1*sum(xy);
23 q[3,1] = 1*sum(x2y);
24 B = solve(p, q);
25 \text{ cat}("B0 \text{ is",B[1,1]});
26 cat("B1 is",B[2,1]);
27 cat("B2 is",B[3,1])
```

## R code Exa 9.10.a Multiple Linear Regression

```
1 \times 1 < -c (679, 1420, 1349, 296, 6975, 323, 4200, 633)
2 \times 2 < -c (30.4, 34.1, 17.2, 26.8, 29.1, 18.7, 32.6,
       32.5)
3 y = matrix(1,8,1);
4 n = 8
5 \text{ y} \leftarrow c(11.6, 16.1, 9.3, 9.1, 8.4, 7.7, 11.3, 8.4)
6 x = matrix(1,8,3);
7 for (i in 1:8){
     x[i,2] = x1[i];
     x[i,3] = x2[i];
9
10 }
11
12 \text{ pro1} = t(x)
13 pro2= pro1%*%x;
14 \text{ pro3} = \text{solve(pro2)};
```

```
15 pro4 = pro3%*%pro1;
16 \text{ pro5} = \text{pro4} \% \% \% \% \%
17
18 B < -matrix(0,3,8)
19 matrix(1,3,1);
20 for (i in 1:3){
21
      B[i,1] = 0;
22
      for (k in 1:8){
         B[i,1]=B[i,1]+(pro4[i, k]*y[k]);
23
24
25 }
26 cat(B);
27 \text{ SSR} = t(y)
28 SSR= SSR\%*\%y;
29 \text{ sub} = t(B);
30 \text{ sub} = \text{sub} \% * \% t(x);
31 \text{ sub} = \text{sub} \% * \% y;
32 SSR = SSR[1,1] - sub[1,1];
33 cat("SSr is", SSR)
```

#### R code Exa 9.10.b Estimate of variance

```
13 pro2= pro1%*%x;
14 \text{ pro3} = \text{solve}(\text{pro2});
15 pro4 = pro3%*%pro1;
16 \text{ pro5} = \text{pro4} \% \% \% \% \% \%
17
18 B \leftarrow matrix(0,3,8)
19 matrix(1,3,1);
20 for (i in 1:3){
       B[i,1] = 0;
21
       for (k in 1:8){
22
         B[i,1]=B[i,1]+(pro4[i, k]*y[k]);
23
       }
24
25 }
26 cat(B);
27 \text{ SSR} = t(y)
28 SSR= SSR%*%y;
29 \text{ sub} = t(B);
30 \text{ sub} = \text{sub} \% * \% t(x);
31 \text{ sub} = \text{sub} \% * \% y;
32 \text{ SSR} = \text{SSR}[1,1] - \text{sub}[1,1];
33 cat("SSr is", SSR)
34
35 k=2;
36 \text{ den} = n-k-1;
37 sigma = SSR/den;
38 cat ("The variance is", sigma)
```

#### R code Exa 9.10.c Diameter of a tree

```
4 \times 4 < -c (0.63, 0.59, 0.56, 0.55, 0.54, 0.59, 0.56,
       0.46, 0.63, 0.60, 0.63, 0.58, 0.55, 0.57, 0.62)
 5 \text{ y} < -c (18.1, 19.6, 16.6, 16.4, 16.9, 17.0, 20.0, 16.6,
        16.2, 18.5, 18.7, 19.4, 17.6, 18.3, 18.8)
 6 n = length(x1);
 7 x = matrix(1,15, 5);
8 for (i in 1:15){
9
      x[i,2] = x1[i]
10
      x[i,3] = x2[i]
11
      x[i,4] = x3[i]
      x[i,5] = x4[i]
12
13 }
14
15 \text{ pro1} = \mathbf{t}(\mathbf{x})
16 pro2= pro1%*%x
17 \text{ pro3} = \text{solve}(\text{pro2})
18 pro4 = pro3%*%pro1
19 pro5 = pro4\%*\%y
20 \ B < -matrix(0,5,15)
21 for (i in 1:5){
22
      B[i,1] = 0
23
      for (k in 1:15){
         B[i,1]=B[i,1]+(pro4[i, k]*y[k])
24
25
      }
26 }
27 \text{ SSR} = t(y)
28 SSR= SSR%*%y
29 \text{ sub} = t(B)
30 \text{ sub} = \text{sub} \% * \% t(x)
31 \text{ sub} = \text{sub} \% * \% y
32 \text{ SSR} = \text{SSR}[1,1] - \text{sub}[1,1]
33 SSR
34 \text{ xxinv} = 0.379;
35 k = 4;
36 \text{ ts} = \text{sqrt}((n-k-2)/SSR)*B[2]/0.616;
37 pvalue = 2*(1-pt(ts, n-k-2))
38 cat("The p-value is", pvalue)
39
```

40 The SSR calculated by R is 19.34 whereas the textbook gives the value as 19.26 , thus the difference in the final answer.

41

#### R code Exa 9.10.d Estimating hardness

```
1 y \leftarrow c (79.2, 64.0, 55.7, 56.3, 58.6, 84.3, 70.4, 61.3,
        51.3, 49.8)
2 \times 1 < -c (0.02, 0.03, 0.03, 0.04, 0.10, 0.15, 0.15,
      0.09, 0.13, 0.09)
3 \text{ x2} < -c (1.05, 1.20, 1.25, 1.30, 1.30, 1.00, 1.10,
       1.20, 1.40, 1.40)
4 tvalue= 2.365;
5 x = matrix(1,10,3);
6 for (i in 1:10){
     x[i,2] = x1[i]
8
     x[i,3] = x2[i]
9 }
10
11 pro1 = t(x)
12 pro2= pro1%*%x
13 \text{ pro3} = \text{solve}(\text{pro2})
14 pro4 = pro3%*%pro1
15 \text{ pro5} = \text{pro4}\% \% \text{y}
16 B= matrix(1,3,1)
17
18 for (i in 1:3){
19
     B[i,1] = 0
     for ( k in 1:10) {
20
        B[i,1] = B[i,1] + (pro4[i, k]*y[k])
21
22
      }
23 }
24
25 \text{ SSR} = t(y)
```

```
26 SSR= SSR%*%y
27 \text{ sub} = t(B)
28 \text{ sub} = \text{sub} \% * \% t(x)
29 sub= sub%*%y
30 \text{ SSR} = \text{SSR}[1,1] - \text{sub}[1,1]
31 cat("SSr is", SSR)
32
33 smallx < -c(1, 0.15, 1.15)
34 product = smallx %*% B
35 n = 10;
36 k=2;
37 val= sqrt(SSR/(n-k-1));
38
39 pro5 = smallx %*% pro3
40 pro6 = pro5 %*% (smallx)
41 pro7 = val*sqrt(pro6)*tvalue
42
43 up = product + pro7;
44 low = product - pro7;
45 cat(" 95% confidence interval is from ",low,"to",up)
```

#### R code Exa 9.11.a Animal sick falling

```
1 cancer = 84;
2 total = 111;
3 level = 250;
4 alpha= -1*log((total-cancer)/total)/level;
5 cat("Alpha is ",alpha)
```

## Chapter 10

## Analysis of Variance

R code Exa 10.3.a Dependence of mileage on gas used

```
1 Xij <-matrix(c(220, 251, 226, 246, 260, 244, 235,
      232, 242, 225, 252, 272, 250, 238, 256), nrow = 3,
      ncol = 5, byrow = TRUE)
2 \text{ Xi} = \text{matrix}(0,3,1);
3 n = 5;
4 m = 3;
5 for (i in 1:3){
     for (j in 1:5){
       Xi[i] = Xi[i] + Xij[i,j]
8
     }
9 }
10 Xi
11 Xi = Xi/n;
12 SSW= 0;
13 for (i in 1:3){
     for (j in 1:5){
15
       SSW = SSW + ((Xij[i,j]-Xi[i])^2)
16
     }
17 }
18 SSW
19 sigma1 = SSW/((n*m)-m);
```

```
20  Xdotdot = sum(Xi)/m;
21  new = (Xi - Xdotdot)^2;
22  SSb= n*sum(new);
23  sigma2 = SSb/(m-1);
24  TS = sigma2/sigma1;
25  cat("Value of the test statistic is",TS)
26  pvalue = 1 - pf(TS,m-1, ((n*m)-m))
27  cat("The p-value is",pvalue)
28  if(pvalue>0.05){
29   cat("Since the p-value is greater than .05, the null hypothesis that the mean mileage is the same for all 3 brands of gasoline cannot be rejected.")
30 }
```

#### R code Exa 10.3.b Dependence of mileage on gas used

```
1 Xijold <-matrix (c(220, 251, 226, 246, 260, 244, 235,
      232, 242, 225, 252, 272, 250, 238, 256), nrow = 3,
      ncol = 5, byrow = TRUE)
2 \text{ Xij} = \text{Xijold} - 220;
3 m=3;
4 n=5;
5 \text{ Xidot} = \text{matrix}(0,3,1);
6 for (i in 1:m){
     for (j in 1:n){
7
       Xidot[i]=Xidot[i] + Xij[i,j]
8
9
     }
10 }
11 Xidot = Xidot/n;
12 Xdotdot = sum(Xidot)/m;
13 SSb=0;
14 for (i in 1:m){
     SSb = SSb + (Xidot[i]-Xdotdot)^2
15
16 }
```

```
17 SSb = SSb*n;
18 Xijsquared = Xij^2;
19 SSW = sum(Xijsquared) - (m*n*(Xdotdot^2)) - SSb;
20 sigma1 = SSW/((n*m)-m);
21 sigma2 = SSb/(m-1);
22 TS = sigma2/sigma1;
23 cat("Value of the test statistic is",TS)
```

#### R code Exa 10.3.c Difference in GPA

```
1 Xij \leftarrow matrix(c(3.2, 3.4, 3.3, 3.5, 3.4, 3.0, 3.7,
      3.3, 2.8, 2.6, 3.0, 2.7), nrow = 3, ncol = 4, by row
      = TRUE)
2 Xi = matrix(0,3,1);
3 n = 4;
4 m = 3;
5 for (i in 1:3){
     for (j in 1:4){
       Xi[i] = Xi[i] + Xij[i,j]
8
     }
9 }
10 Xi = Xi/n;
11 SSW= 0;
12 for (i in 1:3){
13
     for (j in 1:4) {
       SSW = SSW + ((Xij[i,j]-Xi[i])^2)
14
15
     }
16 }
17 sigma1 = SSW/((n*m)-m);
18 Xdotdot = sum(Xi)/m;
19 \text{ new} = (Xi - Xdotdot)^2;
20 SSb= n*sum(new);
21 \text{ sigma2} = SSb/(m-1);
22 TS = sigma2/sigma1;
23 cat ("Value of the test statistic is", TS)
```

## R code Exa 10.4.b Estimating Parameters

```
1 X<-matrix(c(75, 73, 60, 70, 86, 78, 71, 64, 72, 90,
      80, 69, 62, 70, 85, 73, 67, 63, 80, 92), nrow = 4,
      ncol = 5, byrow = TRUE)
2 Xidot = matrix(0,4,1);
3 for (i in 1:4){
     for (j in 1:5){
       Xidot[i] = Xidot[i] + X[i,j]
6
     }
7 }
8 Xidot = Xidot/5;
9 Xjdot = matrix(0,5,1);
10 for (j in 1:5){
     for (i in 1:4){
11
12
       X j dot[j] = X j dot[j] + X[i,j]
13
     }
14 }
15 Xjdot = Xjdot/4;
16 Xdotdot = sum(Xidot)/4;
17 meanhat = Xdotdot;
18 alphahat = Xidot - meanhat;
19 betahat = Xjdot - meanhat;
```

```
20 cat("The estimator of the mean is", meanhat)
21 cat("The alphas are-", alphahat)
22 cat("The betas are-", betahat)
```

### R code Exa 10.5.a Species collected

```
1 X<-matrix(c(53, 35, 31, 37, 40, 43, 36, 34, 17, 21,
      30, 18, 47, 37, 17, 31, 45, 26, 55, 31, 17, 23,
      43, 37, 40, 32, 19, 26, 45, 37, 52, 42, 20, 27,
      26, 32, 39, 28, 21, 21, 36, 28, 40, 32, 21, 21,
      36, 35), nrow = 8, ncol = 6, byrow = TRUE
2 m = 8;
3 n = 6;
4 Xidot = matrix(0,8,1);
6 for (i in 1:8){
     for (j in 1:6){
       Xidot[i]=Xidot[i] + X[i,j]
8
     }
9
10 }
11 Xidot = Xidot/6;
12 Xjdot = matrix(0,6,1);
13
14 for (j in 1:6){
     for (i in 1:8){
15
       X j dot[j] = X j dot[j] + X[i,j]
16
     }
17
18 }
19
20 Xjdot = Xjdot/8;
21 Xdotdot = sum(Xidot)/8;
22 new = (Xidot - Xdotdot)^2;
23 SSr = n*sum(new);
24 \text{ new1} = (Xjdot - Xdotdot)^2;
25 SSc = m*sum(new1);
```

```
26 \text{ SSe} = 0;
27 for (i in 1:m){
     for (j in 1:n){
       SSe = SSe + (X[i,j]-Xidot[i]-Xjdot[j]+ Xdotdot)
29
30
       }
31 }
32
33 N = (m-1)*(n-1)
34 	TS1 = SSr*N/((m-1)*SSe)
35 TS2 = SSc*N/((n-1)*SSe)
36 \text{ pvaluec} = 1 - \text{pf}(TS1, m-1, N)
37 pvaluer = 1 - pf(TS2, n-1, N)
38 cat("The value of the F-statistic for testing that
      there is no row effect is", TS1)
39 cat ("The p-value for testing that there is no row
      effect is", pvaluec)
40 cat ("The value of the F-statistic for testing that
      there is no column effect is", TS2)
41 cat ("The p-value for testing that there is no column
       effect is", pvaluer)
```

## Chapter 11

# Goodness of Fit Tests and Categorical Data Analysis

R code Exa 11.2.a Relation between death date and birth date

```
1 X<-c(90, 100, 87, 96, 101, 86, 119, 118, 121, 114,
      113, 106)
2 pi= matrix(1,12,1);
3 pi= pi/12;
4 \text{ new} = X^2;
5 \text{ npi} = sum(X)*pi;
6 T = sum(new);
7 T = T/npi;
8 T = T - sum(X);
9 cat("When there are 12 regions")
10 cat("The test statistic is",T[1])
11 pvalue = 1- pchisq(T[1], 11);
12 cat ("The pvalue is ", pvalue)
13
14 \text{ X} < -c (277, 283, 358, 333)
15 pi = matrix(1,4,1);
16 \text{ pi= pi/4};
17 \text{ new} = X^2;
18 npi = sum(X)*pi;
```

```
19 T = sum(new);
20 T = T/npi;
21 T = T - sum(X);
22 cat("When there are 4 regions")
23 cat("The test statistic is",T[1])
24 pvalue = 1- pchisq(T[1], 3);
25 cat("The pvalue is ",pvalue)
```

### R code Exa 11.2.b Quality of bulbs

```
1  X<-c(3, 6, 9, 7, 5)
2  p<-c(0.15, 0.25, 0.35, 0.20, 0.05)
3  T= 0;
4  n3=sum(X);
5  np = p*n3;
6  Xsqu = (X-np)^2;
7  cat(Xsqu);
8  XT = Xsqu/np;
9  T = sum(XT);
10
11  cat("The test statistic is",T)
12  pvalue = 1- pchisq(T[1], 4)
13  cat("The pvalue is ",pvalue)
14  cat("Thus, the hypothesis would not be rejected at 5% level of significance")</pre>
```

#### R code Exa 11.2.d Six outcomes

```
1 X <-c(3, 3, 5, 18, 4, 7)
2 p <-c(0.1, 0.1, 0.05, 0.4, 0.2, 0.15)
3 psimu = 0.1843
4 num= 10000;
5 T= 0;</pre>
```

```
6  n=sum(X);
7  np = n*p;
8  Xsqu = X^2;
9  for (i in 1:6){
10    T = T + (Xsqu[i]/np[i])
11  }
12  T = T - sum(X)
13
14  cat("The test statistic is",T[1])
15  pvalue = 1- pchisq(T[1], 5);
16  int1 = psimu - (1.645*sqrt(psimu*(1-psimu)/num));
17  int2 = psimu + (1.645*sqrt(psimu*(1-psimu)/num));
18  cat("With 90% confidence p-value lies between ",int1 ,"and",int2)
```

#### R code Exa 11.3.a Weekly accidents

```
1 \text{ Y} \leftarrow c(8, 0, 0, 1, 3, 4, 0, 2, 12, 5, 1, 8, 0, 2, 0,
      1, 9, 3, 4, 5, 3, 3, 4, 7, 4, 0, 1, 2, 1, 2)
2 \text{ weeks} = 30;
3 lamda = sum(Y)/weeks;
4 p = matrix(0,5,1);
5 p[1] = ppois(0, lamda);
6 p[2] = ppois(1, lamda) - p[1];
7 p[3] = ppois(3, lamda) - ppois(1, lamda);
8 p[4] = ppois(5, lamda) - ppois(3, lamda);
9 p[5] = 1 - ppois(5, lamda);
11 X = matrix(0,5,1);
12 for (i in 1:30) {
13
     if (Y[i]==0) {
14
       X[1] = X[1] +1;
15
     } else if(Y[i]==1){
       X[2] = X[2] +1;
16
     } else if(Y[i]==2){
17
```

```
X[3] = X[3] +1;
18
19
     } else if(Y[i]==3){
20
       X[3] = X[3] +1;
21
     } else if(Y[i]==4){
22
       X[4] = X[4] +1;
23
     } else if(Y[i]==5){
24
       X[4] = X[4] +1;
25
     } else if(Y[i]>5){
       X[5] = X[5] +1;
26
27
     }
28 }
29
30 \text{ T = 0};
31 npi = weeks * p;
32 for (i in 1:5){
     T = T + ((X[i]-npi[i])^2)/npi[i]
33
34 }
35 cat("T is",T)
36 \text{ pvalue} = 1 - \text{pchisq}(T, 3)
37 cat("The p-value is", pvalue)
38 cat ("Hypothesis of an underlying poisson
      distribution is rejected")
```

#### R code Exa 11.4.a Political affliation and Gender

```
10 for (j in 1:3){
     for (i in 1:2){
11
12
        Mj[j] = Mj[j] + Nij[i,j]
     }
13
14 }
15 \text{ NM} = \text{matrix}(1,2,3);
16 for (i in 1:2){
17
     for (j in 1:3){
        NM[i,j] = Ni[i] * Mj[j]
18
19
20 }
21 \text{ NM} = \text{NM/n};
22 \text{ TS} = 0
23 for (i in 1:2){
24
     for (j in 1:3){
        TS = TS + ((Nij[i,j]-NM[i,j])^2)/NM[i,j]
25
26
     }
27 }
28 cat ("The test statistic is", TS)
29 compare = pchisq(2, 0.95, 0.05)
30 if(TS>compare){
     cat ("The null hypothesis is rejected at the 5%
31
         level of significance")
     } else {
32
        disp("The null hypothesis is accepted at the 5%
33
           level of significance")
34
     }
```

#### R code Exa 11.4.b Machine Breakdown and shift

```
5 for (i in 1:3){
     for (j in 1:4) {
       Ni[i] = Ni[i] + Nij[i,j]
     }
8
9 }
10 for (j in 1:4){
    for (i in 1:3){
12
       Mj[j] = Mj[j] + Nij[i,j]
13
     }
14 }
15 NM = matrix(1,3,4);
16 for (i in 1:3){
17
     for (j in 1:4) {
       NM[i,j] = Ni[i] *Mj[j]
18
     }
19
20 }
21 \text{ NM= NM/n};
22 \text{ TS} = 0
23 for (i in 1:3){
24
     for (j in 1:4) {
25
       TS = TS + ((Nij[i,j]-NM[i,j])^2)/NM[i,j]
26
     }
27 }
28 cat("The test statistic is", TS)
29 pvalue = 1 - pchisq(TS, 6)
30 cat ("the hypothesis that the machine that causes a
      breakdown is independent of the shift on which
      the breakdown occurs is accepted.")
```

#### R code Exa 11.5.a Lung cancer and smoking

```
4 \text{ Mj} = \text{matrix}(0,2,1);
5 for (i in 1:2){
     for (j in 1:2){
       Ni[i] = Ni[i] + Nij[i,j]
7
8
     }
9 }
10 Ni
11 for (j in 1:2){
12
     for (i in 1:2){
13
       Mj[j] = Mj[j] + Nij[i,j]
     }
14
15 }
16
17 \text{ NM} = \text{matrix}(1,2,2);
18 for (i in 1:2){
     for (j in 1:2){
19
       NM[i,j] = Ni[i] *Mj[j]
20
21
     }
22 }
23 NM= NM/n;
24 NM
25 \text{ TS} = 0
26 for (i in 1:2){
     for (j in 1:2) {
27
       TS = TS + ((Nij[i,j]-NM[i,j])^2)/NM[i,j]
28
     }
29
30 }
31 cat("The test statistic is", TS)
32 \text{ compare} = pchisq(1, 0.99, 0.01)
33 compare
34 if(TS>compare){
     cat ("The null hypothesis is rejected at the 1%
35
         level of significance");
36 } else{
     cat ("The null hypothesis is accepted at the 1%
        level of significance");
38 }
```

## R code Exa 11.5.b Females reporting abuse

```
1 Nij <-matrix(c(28, 30, 58, 55, 472, 470, 442, 445),
      nrow = 2, ncol = 4, byrow = TRUE
2 n = sum(Nij);
3 Ni = matrix(0,2,1);
4 Mj = matrix(0,4,1);
5 for (i in 1:2){
     for (j in 1:4) {
7
       Ni[i] = Ni[i] + Nij[i,j]
8
     }
9 }
10 for (j in 1:4){
    for (i in 1:2){
12
       Mj[j] = Mj[j] + Nij[i,j]
13
     }
14 }
15 NM = matrix(1,2,4);
16 for (i in 1:2){
17
     for (j in 1:4) {
       NM[i,j] = Ni[i] *Mj[j]
18
     }
19
20 }
21 \text{ NM} = \text{NM/n};
22
23 TS = 0
24 for (i in 1:2){
25
     for (j in 1:4){
26
       TS = TS + ((Nij[i,j]-NM[i,j])^2)/NM[i,j]
27
     }
28 }
29 cat ("The test statistic is", TS)
30
31 \text{ compare} = pchisq(3, 0.99, 0.01)
```

## R code Exa 11.6.a Testing distribution of a population

```
1 X <-c(66, 72, 81, 94, 112, 116, 124, 140, 145, 155);
2 D= 0.4831487;
3 n= 10;
4 Dgiven = 1.480;
5 Dstar = (sqrt(n) + 0.12 + (0.11/sqrt(n)))*D;
6 cat("Dstar is ",Dstar);
7 if(Dstar > Dgiven) {
8  cat("Null hypothesis is rejected at 2.5% level of significance")
9 } else{
10  cat("Null hypothesis is accepted at 2.5% level of significance")
11 }
```

## Chapter 12

# Non parametric Hypothesis Tests

R code Exa 12.2.a testing the median

```
1  n= 200;
2  v = 120;
3  p =0.5;
4  if(v < (n/2)){
5    pvalue = 2*pbinom(v, n, p)
6  } else{
7    pvalue = 2*pbinom(n-v, n, p)
8  }
9  cat("Pvalue is ",pvalue)
```

R code Exa 12.2.b testing the median

```
1  n= 80;
2  v = 28;
3  p =0.5;
```

```
5 pvalue = pbinom(v, n, p);
6 cat("Pvalue is ",pvalue);
7 cat("Thus, the null hypothesis that the median income is less than or equal to $90,000 is rejected")
```

## R code Exa 12.3.b Signed Rank Test

```
1 n = 4;
2 \text{ mo} = 2;
3 \text{ X} \leftarrow c(4.2, 1.8, 5.3, 1.7)
4 t =3; #value of the test statistic (Ex12_3a)
5 tstar= min(t, (n*(n+1)/2) - t);
6 P = matrix(0,4,4);
7 P[1,1] = 0.5;
8 P[1,2] = 1;
9 P[1,3] = 1;
10 P[1,4] = 1;
11 for (i in 2:4){
12
     for (j in 1:4) {
        if (j-i <1) {</pre>
13
          P[i,j] = 0.5*P[i-1, j]
14
15
        } else{
16
          P[i,j] = 0.5*(P[i-1,j-i]+P[i-1,j])
17
        }
     }
18
19 }
20 P
```

### R code Exa 12.3.c Determining Population Distribution

```
1  n =20;
2  t =142;
```

```
3 tstar= \min(t, (n*(n+1)/2) - t);
4 P = matrix(1,20,tstar+1);
5 P[1,1] = 0.5;
6 P[1,2] = 1;
7 for (i in 2:20){
     for (j in 1:(tstar+1)){
9
       if (j-i <1){</pre>
10
         P[i,j] = 0.5*P[i-1, j]
11
       } else{
         P[i,j] = 0.5*(P[i-1,j-i]+P[i-1,j])
12
       }
13
       }
14
15
     }
16 pvalue= 2*P[20,tstar+1];
17 cat ("Pvalue is", pvalue)
18 cat ("Thus the hypothesis that the population
      distribution is symmetric about 0 is accepted at
      the alpha = 10% level of significance")
```

### R code Exa 12.4.a Treatments against corrosion

```
1 X \leftarrow c (65.2, 67.1, 69.4, 78.2, 74, 80.3)
2 Y<-c(59.4, 72.1, 68, 66.2, 58.5)
3 Z = append(X,Y)
4 Z = sort(Z);
5 n = length(X);
6 q = length(Z)
7 T = 0;
  for (i in 1:n){
     test = X[i];
9
10
     for (j in 1:q){
11
       if(test== Z[j]){
12
          T = T + j;
13
       }
14
     }
```

```
15 }
16
17 cat("The test statistic is ",T)
```

## R code Exa 12.4.b Determining P

```
1 prob <- function(N, M, K) {</pre>
     if(N==1 & M==0){
       if(K >0){
3
4
         cat(result = 1)
       } else {
5
         cat(result =0)
6
7
8
       } else if (N==0 & M==1){
            if(K <0){
9
              cat(result = 0)
10
            } else {
11
              cat(result =1)
12
            }
13
14
         } else if (N==0 \& M==0 \& K==0) {
15
            cat(result =1)
16
         } else{
            cat(result = (prob(N-1, M, K-N-M)*(N/(N+M)))
17
                + (prob(N, M-1, K)*(M/(N+M)))
         }
18
19 }
20
21 pval <- function(n,m,t) {</pre>
22
     result = 2*\min(prob(n,m,t), 1-prob(n,m,t-1));
23 }
24
25 '> pval (2,1,3)
26 001
27 Error: C stack usage 19923984 is too close to the
      limit
```

## R code Exa 12.4.c Finding p value

```
1 prob <- function(N, M, K) {</pre>
     if(N==1 & M==0){
       if(K >0){
         cat(result = 1)
4
       } else {
6
         cat(result =0)
       }
8
     } else if (N==0 & M==1){
9
       if(K <0){
10
         cat(result = 0)
       } else {
11
12
         cat(result =1)
13
     } else if (N==0 \& M==0 \& K==0){
14
15
       cat(result =1)
     } else{
16
       cat(result = (prob(N-1, M, K-N-M)*(N/(N+M))) + (
17
          prob(N, M-1, K)*(M/(N+M))))
     }
18
19 }
20
21 pval <- function(n,m,t) {</pre>
     result = 2*\min(prob(n,m,t), 1-prob(n,m,t-1));
22
23 }
24
25 '> pval (2,1,3)
26 001
```

## R code Exa 12.4.d Comparing production methods

```
prob <- function(N, M, K) {</pre>
     if (N==1 & M==0) {
       if(K >0){
3
          cat(result = 1)
4
       } else {
5
6
          cat(result =0)
7
     } else if (N==0 & M==1){
8
       if(K <0){</pre>
9
10
          cat(result = 0)
       } else {
11
12
          cat(result =1)
13
14
     } else if (N==0 \& M==0 \& K==0){
       cat(result =1)
15
16
     } else{
        cat(result = (prob(N-1, M, K-N-M)*(N/(N+M))) + (
17
           prob(N, M-1, K)*(M/(N+M)))
18
     }
19 }
20
21 pval <- function(n,m,t) {</pre>
```

## R code Exa 12.4.e Determining p value

```
1  n1 =5;
2  m1= 6;
3
4  t1 =21;
5  num1 = n1*(n1+m1+1)/2;
6  d1=abs(t1 - num1);
7  val = d1/sqrt(n1*m1*(n1+m1+1)/12);
8  pval = 2*(1- pnorm(val, 0,1))
9  cat("The p-value for eg 12.4a is", pval)
10  n2 =9;
11  m2= 13;
12  t2 =72;
13  d2=abs(t2 - n2*(n2+m2+1)/2);
14  val = d2/sqrt(n2*m2*(n2+m2+1)/12);
15  pval = 2*(1-pnorm(val, 0,1));
16  cat("The p-value for eg 12.4d is", pval)
```

R code Exa 12.5.a Testing randomness

```
1 fact <- function(num) {</pre>
     if (num <= 0) {
        result= 1
3
     } else {
        result = factorial(num)
6
     }
7 }
8
  proba <- function(n,m,k) {</pre>
     if(k\%2==0){
       k=k/2;
11
12
        result = 2*fact(m-1)*fact(n-1)*fact(n)*fact(m)/(
           fact(k-1)^2*fact(m-k)*fact(n-k)*fact(n+m));
     } else {
13
       k = (k-1)/2;
14
        result = fact(m-1)*fact(n-1)*fact(n)*fact(m)/(
15
           fact(k-1)*fact(k)*fact(m-k)*fact(n-k-1)*fact(
           n+m)) + fact(m-1)*fact(n-1)*fact(n)/(
           fact(k-1)*fact(k)*fact(m-k-1)*fact(n-k)*fact(
           n+m));
16
     }
17 }
18
19
20 \text{ r1} = 20;
21 \text{ n1} = 20;
22 \text{ m} 1 = 10;
23 \text{ ans } 1 = 0;
24 for (i in 1:19){
25
     ans1 = ans1 + proba(n1, m1, i);
26 }
27 if (ans1<0.5) {
28
     pvalue1 = 2*ans1;
29 } else {
30
     pvalue1 = 2*(1-ans1);
31 }
32
33 cat("P-value is", pvalue1)
```

## ${f R}$ code Exa 12.5.c Determining p value

```
1  u = 61;
2  sigma = 5.454;
3  r =75;
4  val = pnorm((r-u)/sigma, 0,1)
5  if(val>0.5){
6   pvalue = 2*(1-val)
7  } else{
8   pvalue = 2*val
9  }
10  cat("P-value is",pvalue)
```

## Chapter 13

# **Quality Control**

R code Exa 13.2.a Steel shaft diameter

```
1 \text{ X} \leftarrow c(3.01, 2.97, 3.12, 2.99, 3.03, 3.02, 3.10, 3.14,
        3.09, 3.20)
2 \text{ Y} < -\text{seq}(1,10,1)
3 u = 3;
4 \text{ sigma} = 0.1;
5 n=4;
6 \text{ ucl} = u + (3*\text{sigma/sqrt}(n));
7 lcl = u - (3*sigma/sqrt(n));
8 \ Z \leq seq(0.1,10,0.1)
9 P= matrix(1,1,100)
10 Q = matrix(1,1,100)
11 P = P*ucl;
12 Q = Q*lcl;
13 plot(Y, X)
14 plot(Z, P)
15 plot(Z, Q)
16 cat('ucl is',ucl)
17 cat('lcl is',lcl)
```

#### R code Exa 13.2.b unknown mean and variance

```
1 Xbar \leftarrow c(3.01, 2.97, 3.12, 2.99, 3.03, 3.02, 3.10,
      3.14, 3.09, 3.20)
2 S<-c(0.12, 0.14, 0.08, 0.11, 0.09, 0.08, 0.15, 0.16,
       0.13, 0.16
3 \text{ c} < -\text{c} (0.7978849, 0.8862266, 0.9213181, 0.9399851,
      0.9515332, 0.9593684, 0.9650309, 0.9693103,
      0.9726596)
4 n=4;
5 Xbarbar= mean(Xbar);
6 Sbar =mean(S);
7 lcl = Xbarbar - (3*Sbar/(sqrt(n)*c(n-1)));
8 ucl = Xbarbar + (3*Sbar/(sqrt(n)*c(n-1)));
9 u = Xbarbar;
10 sigma = Sbar/c[n-1];
11 prob = pnorm(3.1, u, sigma) - pnorm(2.9, u, sigma);
12 cat ("Percentage of the items that will meet the
      specifications is ", prob*100)
```

#### R code Exa 13.3.a determining control limits

```
8 \text{ Sbar} = \text{mean}(S);
9 lclX = Xbarbar - (3*Sbar/(sqrt(n)*c[n-1]));
10 uclX = Xbarbar + (3*Sbar/(sqrt(n)*c[n-1]));
11 val1 = 1/c[n-1]
12 val1 = val1^2;
13 \text{ val1} = \text{val1} - 1;
14 val = sqrt(val1);
15 ucls = Sbar*(1+(3*val));
16 lcls = Sbar*(1-(3*val));
17 plot(Y, Xbar)
18 P<-matrix(1,1, 200)
19 Q<-matrix(1,1, 200)
20 P = P*lclX;
21 \quad Q = Q * uclX;
22 cat ('UCL(X)=', uclX)
23 cat('LCL(X)=',lclX)
24 plot(Z, P)
25 plot(Z, Q, main = 'Control Chart for X')
26 cat('UCL(S)=',uclX)
27 \operatorname{cat}('\operatorname{LCL}(S)=',\operatorname{lclX})
28 plot(Y, S)
29 P = P*lcls/lclX;
30 \quad Q = Q * ucls / uclX;
31 \text{ plot}(Z, P)
32 plot(Z, Q, main = 'Control Chart for S')
```

#### R code Exa 13.4.a Defectives Screws

```
5 n=50;
6 val = sqrt(Fbar*(1-Fbar)/n);
7 lcl = Fbar - (3*val);
8 \text{ ucl} = \text{Fbar} + (3*\text{val});
9 cat("LCL is", lc1)
10 cat("UCL is",ucl)
11 for (i in 1:20){
     if( F[i]>ucl | F[i]<lcl) {</pre>
12
        totald=sum(defect)-defect[i]
13
     }
14
15 }
16 \text{ total} = \text{total} - 50;
17 Fbar = totald/total;
18 val = sqrt(Fbar*(1-Fbar)/n);
19 cat ("After recomputation");
20 \ lcl = Fbar - (3*val);
21 ucl = Fbar + (3*val);
22 cat("LCL is",1c1)
23 cat("UCL is",ucl)
```

### R code Exa 13.5.a Control during production of cars

```
1 X<-c(141, 162, 150, 111, 92, 74, 85, 95, 76, 68, 63,
       74, 103, 81, 94, 68, 95, 81, 102, 73)
2 \text{ total} = sum(X);
3 \text{ num} = 20;
4 Xbar = mean(X);
5 lcl = Xbar - 3*sqrt(Xbar);
6 ucl = Xbar + 3*sqrt(Xbar);
7 cat("UCL is",ucl)
8 cat("LCL is", lc1)
9 for (i in 1:20){
10
     if(X[i]> ucl){
       total = total - X[i]
11
12
       num = num -1
```

```
13
     }
14 }
15 Xbar = total/num
16 lcl = Xbar - 3*sqrt(Xbar);
17 ucl = Xbar + 3*sqrt(Xbar);
18 cat ("After recomputation")
19 cat("UCL is",ucl)
20 cat("LCL is", 1c1)
21 \text{ total} = \text{total} - X[4]
22 \quad \text{num} = \text{num} - 1;
23 cat("Xbar is", Xbar)
24 cat(" is",X[4])
25 Xbar = total/num
26 \ lcl = Xbar - 3*sqrt(Xbar);
27 \text{ ucl} = Xbar + 3*sqrt(Xbar);
28 cat("After second recomputation")
29 cat("UCL is", ucl);
30 cat("LCL is", 1c1);
31 cat ("It appears that the process is in control with
      mean", Xbar);
32
   'The mean after the second recomputation is
33
      incoreectly calculated in the textbook. It should
       be
34 ((17*84.41)-111)/16 = 82.748 whereas the value
      given in the book is 82.56. The values of UCL and
       LCL
35 change accordingly.
```

#### R code Exa 13.6.b Service Time

```
4 sigma = 24;
5 alpha = 0.25;
6 W = matrix(0,17);
7 W[1] = 60;
8 for (i in 2:17){
9    W[i] = (0.25*X[i-1]) + (0.75*W[i-1])
10 }
11 cat("The values of W are", W)
12 val = 3*sigma*sqrt(alpha/(n*(2-alpha)));
13 lcl = u- val;
14 ucl = u+ val;
15 cat("LCL is",lcl)
16 cat("UCL is",ucl)
```

## R code Exa 13.6.c Exponentially weighted moving average control

```
1 \text{ X} \leftarrow c (9.617728, 10.25437, 9.867195, 10.79338,
      10.60699, 10.48396, 13.33961, 9.462969, 10.14556,
       11.66342, 11.55484, 11.26203, 12.31473,
      9.220009, 11.25206, 10.48662, 9.025091, 9.693386,
       11.45989, 12.44213, 11.18981, 11.56674,
      9.869849, 12.11311, 11.48656)
2 t < -seq(1, 26, 1)
3 \text{ alpha} = 2/9;
4 \text{ val} = 9.915051 - (alpha*9.617728);
5 \text{ val} = \text{val}/(1-\text{alpha});
6 cat("val is", val)
7 u = 10;
8 n = 5;
9 \text{ sigma} = 2;
10
11 W = matrix(0,26)
12 W[1] = 10.;
13 for (i in 2:26){
    W[i] = (alpha*X[i-1]) + ((1-alpha)*W[i-1])
```

```
15 }
16 cat ("The values of W are", W)
17 val = 3*sigma*sqrt(alpha/(n*(2-alpha)));
18 \text{ lcl} = u - val;
19 \text{ ucl} = u + val;
20 cat("LCL is", 1c1)
21 cat("UCL is",ucl)
22 plot(t, W, xlab = "t", ylab = "W")
23 nlcl = matrix(1,1, 26)
24 nlcl= nlcl* lcl;
25 lines(t, nlcl)
26 nucl = matrix(1, 1, 26);
27 nucl= nucl * ucl;
28 lines(t, nucl)
29
  'The asymptpotic lines for UCL and LCL have been
30
      plotted '
```

#### R code Exa 13.6.d Finding control limit

```
1 X \leftarrow c(29, 33, 35, 42, 36, 44, 43, 45)
2 u = 30;
3 \text{ sig} = 8;
4 d = 0.5;
5 B = 5;
6 Y = X - u - (d*sig);
7 S = matrix(0,9);
8 S[1] = 0;
9 for (i in 2:9){
     S[i] = \max(S[i-1] + Y[i-1], 0)
10
11 }
12 cat("S is",S)
13 cl = B*sig;
14 cat(cl)
15 answer =100;
```

```
16 for ( i in 1:9) {
17    if(S[i]>cl) {
18       answer = i
19    }
20 }
21 cat("The mean has increased after observing the ",
      answer-1," subgroup average")
```

## Chapter 14

# Life Testing

### R code Exa 14.3.a Lifetime of a transistor

```
1 total =50;
2 failure = 15;
3 \text{ alpha} = 0.05;
4 t = 525;
5 val1 = qchisq(alpha/2,2*failure)
6 val2 = qchisq(1-alpha/2,2*failure)
8 int1 = 2*t/val1;
9 int2 = 2*t/val2;
10 cat ("The 95% confidence interval is", int2, "to", int1)
11
12 'The confidence interval is from 22.35 to 62.17
      whereas solution in R is 22.35 to 62.53
13 because of the difference in the value of chi-square
      (0.975, 30).
14 The textbook says the value is 16.89 whereas R
      calculates its value as 16.79;
```

R code Exa 14.3.b Lifetime of Battery

```
1 t = 1800;
2 theta = 150;
3 r =20;
4 pvalue = 1 - pchisq(2*r,2*t/theta)
5 cat("P-value is ",pvalue)
```

## R code Exa 14.3.c One at a time sequential test

```
1 T = 500;
2 alpha = 0.05;
3 r = 10;
4 val1 = qchisq(1-alpha/2, 2*r)
5 val2 = qchisq(alpha/2, 2*r)
6 int1= 2*T/val1;
7 int2= 2*T/val2;
8 cat("The 95% confidence interval is",int1,"to",int2)
9
10 'The confidence interval is from 29.27 to 103.52
    whereas solution in R is 29.265774 to
11 104.26683 because of the difference in the value of chi-square(0.975, 30).
12 The textbook says the value is 9.66 whereas scilab calculates its value as 9.5907774'
```

#### R code Exa 14.3.d Lifetime of semiconductors

```
1  r = 30;
2  T = 600;
3  theta = 25;
4  val1 = pchisq(2*T/theta,2*r)
5  val2 = 1 - pchisq(2*T/theta,2*(r+1))
6  val2
7  pvalue = min(val1, val2);
```

```
8 cat("The pvalue is", pvalue)
9 cat("HO would be accepted when the significance level is 0.10")
```

## R code Exa 14.3.e Bayes estimator

#### R code Exa 14.4.a Lifetime of items produced by two plants

```
1 Xlife = 420;
2 Ylife = 510;
3 Xnum= 10;
4 Ynum =15;
5 ts = Xlife*Ynum/(Ylife*Xnum);
6 cat("The value of the test statistic is",ts)
7 val = pf(ts, Xnum, Ynum)
8 pvalue = 2*(1-val);
9 cat("The p-value is",pvalue)
10 cat("We cannot reject HO");
```