

R Textbook Companion for
Matrices and Linear Transformations
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Book Description

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R numbering policy used in this document and the relation to the above book.

Exa Example (Solved example)

Eqn Equation (Particular equation of the above book)

For example, Exa 3.51 means solved example 3.51 of this book. Sec 2.3 means an R code whose theory is explained in Section 2.3 of the book.

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Chapter 1

Matrices and Linear Systems

Chapter 2

Vector Spaces

R code Exa 2.1 Subspaces

```
1 #page - 74
2 #section - 2.2 SUBSPACES
3 #example 1
4
5 #let us represent 'a' with a value as we cannot
  multiply character matrix with numerical matrix
6 a = 7
7
8 #let unit vectors of F4*1 be E1, E2, E3, E,4
9 E1 <- matrix(c(1,0,0,0), 4, 1)
10 E2 <- matrix(c(0,1,0,0), 4, 1)
11 E3 <- matrix(c(0,0,1,0), 4, 1)
12 E4 <- matrix(c(0,0,0,1), 4, 1)
13
14 #thus vector A is
15 A = (E1 %*% a) + (E2 %*% a) + (E3 %*% a) + (E4 %*% a
  )
16 A
```

R code Exa 2.2 Subspaces

```
1 #page - 75
2 #section - 2.2 SUBSPACES
3 #example 2
4
5 #included package - matlib
6
7 #for echelon function
8 library(matlib)
9
10 #matrix A
11 A <- matrix(c(1,-1,-3,-1,2,5,1,2,6), 3, 3, byrow=
    TRUE)
12
13 #column matrix k
14 K <- c(-6,10,15)
15
16 #reduced row-echelon form
17 echelon(A, K, reduced=FALSE, verbose=TRUE, fractions
    =TRUE)
```

R code Exa 2.3 Echelon

```
1 #page - 80
2 #section - 2.3 LINEAR INDEPENDENCE AND BASES
3 #example 3
4
5 #included package - matlib
6
7 #for echelon function
8 library(matlib)
9
10 #matrix A
11 A <- matrix(c(1,-2,1,2,-5,0,-1,3,1,2,0,10), 4, 3,
```

```

        byrow=TRUE)
12
13 #column matrix k
14 K <- c(0,0,0,0)
15
16 #reduced row-echelon form
17 echelon(A, K, reduced=FALSE, verbose=TRUE, fractions
        =TRUE)

```

R code Exa 2.4 Echelon

```

1 #page - 81
2 #section - 2.3 LINEAR INDEPENDENCE AND BASES
3 #example 4
4
5 #included package - matlib
6
7 #for echelon function
8 library(matlib)
9
10 #matrix A
11 A <- matrix(c(-1,-2,2,0,1,3,1,1,1), 3, 3, byrow=TRUE
        )
12
13 #column matrix k
14 K <- c(0,0,0)
15
16 #reduced row-echelon form
17 echelon(A, K, reduced=FALSE, verbose=TRUE, fractions
        =TRUE)

```

R code Exa 2.5 Echelon

```

1 #page - 82
2 #section - 2.3 LINEAR INDEPENDENCE AND BASES
3 #example 5
4
5 #included package - matlab
6
7 #for echelon function
8 library(matlab)
9
10 #matrix A
11 A <- matrix(c(1,2,3,3,2,1,0,2,4,1,1,1), 4, 3, byrow=
    TRUE)
12
13 #column matrix k
14 K <- c(1,1,1,1)
15
16 #reduced row-echelon form
17 B = echelon(A, K, reduced=TRUE, verbose=TRUE,
    fractions=TRUE)

```

R code Exa 2.6 Echelon

```

1 #page - 82
2 #section - 2.3 LINEAR INDEPENDENCE AND BASES
3 #example 6
4
5 #included package - matlab
6
7 #for echelon function
8 library(matlab)
9
10 #matrix A
11 A <- matrix(c(3,-1,-1,-2,2,-2,-1,-1,3), 3, 3, byrow=
    TRUE)
12

```

```

13 #column matrix k
14 K <- c(-3,2,1)
15
16 #reduced row-echelon form
17 B = echelon(A, K, reduced=TRUE, verbose=TRUE,
    fractions=TRUE)

```

R code Exa 2.7 Rank of a matrix

```

1 #page - 88
2 #section - 2.4 THE RANK OF A MATRIX
3 #example 7
4
5 #included package - matlib
6 #included package - matrixcalc
7
8 #for echelon function
9 library(matlib)
10
11 #for rank calculation
12 library(matrixcalc)
13
14 #matrix A
15 A <- matrix(c(2,1,1,1,-2,1,0,5,-1), 3, 3, byrow=TRUE
    )
16
17 #column matrix k
18 K <- c(2,-3,8)
19
20 #reduced row-echelon form
21 B = echelon(A, K, reduced=TRUE, verbose=TRUE,
    fractions=TRUE)
22
23 #rank of A
24 matrix.rank(A)

```


Chapter 3

Determinants

R code Exa 3.1 Determinant

```
1 #page - 103
2 #section - 3.1 DEFINITION OF THE DETERMINANT
3 #example 1
4
5 #matrix A
6 A <- matrix(c(1,2,3,2,4,1,1,3,0), 3, 3, byrow=TRUE)
7 A
8
9 #determinant of A
10 det(A)
```

R code Exa 3.2 Minor and cofactor

```
1 #page - 106
2 #section - 3.2 THE LAPLACE EXPANSION
3 #example 2
4
5 #matrix A
```

```

6 A <- matrix(c(1,2,3,4,5,6,7,8,9), 3, 3, byrow=TRUE)
7 A
8
9 # Minor and cofactor functions
10 minor <- function(A, i, j) A[-i,-j]
11 cofactor <- function(A, i, j) (-1)^(i+j) * det(minor
      (A,i,j))
12
13 #calculating Minor and cofactor
14
15 minor(A, 1, 2)
16 cofactor(A, 1, 2)
17
18 minor(A, 3, 3)
19 cofactor(A, 3, 3)

```

R code Exa 3.3 Determinant

```

1 #page - 108
2 #section - 3.2 THE LAPLACE EXPANSION
3 #example 3
4
5 #matrix A
6 A <- matrix(c(2,4,6,1,2,3,1,4,9), 3, 3, byrow=TRUE)
7 A
8
9 #determinant of A
10 det(A)

```

R code Exa 3.4 Determinant

```

1 #page - 108
2 #section - 3.2 THE LAPLACE EXPANSION

```

```

3 #example 4
4
5 #matrix A
6 A <- matrix(c(1,-1,2,3,2,2,0,2,4,1,-1,-1,1,2,3,0),
7             4, 4, byrow=TRUE)
8
9 #determinant of A
10 det(A)

```

R code Exa 3.5 Adjoints

```

1 #page - 111
2 #section - 3.3 ADJOINTS AND INVERSES
3 #example 5
4
5 #matrix A
6 A <- matrix(c(2,-1,1,4,2,4,6,3,9), 3, 3)
7
8 # Minor and cofactor functions
9 minor <- function(A, i, j) det(A[-i,-j])
10 cofactor <- function(A, i, j) (-1)^(i+j) * minor(A,i
11             ,j)
12
13 #Adjoint functions
14 adjoint <- function(x) {
15   n <- nrow(x)
16   B <- matrix(NA, n, n)
17   for( i in 1:n )
18     for( j in 1:n )
19       B[j,i] <- cofactor(x, i, j)
20 }
21 adjoint(A)

```

R code Exa 3.6 Laplace expansion

```
1 #page - 114
2 #section - 3.2 THE LAPLACE EXPANSION
3 #example 6
4
5 #column matrices
6 c1 <- matrix(c(1,1,2), 3, 1, byrow=TRUE)
7 c2 <- matrix(c(2,-1,3), 3, 1, byrow=TRUE)
8 c3 <- matrix(c(1,1,-1), 3, 1, byrow=TRUE)
9 c4 <- matrix(c(4,5,1), 3, 1, byrow=TRUE)
10
11 A <- matrix(c(c1,c2,c3), 3, 3)
12 B <- matrix(c(c4,c2,c3), 3, 3)
13 C <- matrix(c(c1,c4,c3), 3, 3)
14 D <- matrix(c(c1,c2,c4), 3, 3)
15 A
16 B
17 C
18 D
19 #solution of r,s and t
20 r = det(B)/det(A)
21 r
22
23 s=det(C)/det(A)
24 s
25
26 t=det(D)/det(A)
27 t
```

R code Exa 3.7 Rank of a matrix

```

1 #page - 115
2 #section - 3.4 DETERMINANTS AND RANK
3 #example 7
4
5 #included package - matrixcalc
6
7 #for rank calculation
8 library(matrixcalc)
9
10 #matrix A
11 A <- matrix(c
      (1,-1,1,1,1,2,-1,-1,2,-2,1,-1,0,-3,-1,-1), 4, 4,
      byrow=TRUE)
12
13 #matrix N
14 N <- matrix(c(0,0,0,0,1,0,0,0,0,1,0,0,0,0,1,0), 4,
      4, byrow=TRUE)
15
16 #rank of A
17 matrix.rank(A)
18
19 #rank of N
20 matrix.rank(N)

```

Chapter 4

Linear Transformations

R code Exa 4.1 w coordinate matrices

```
1 #page - 123
2 #section - 4.2 MATRIX REPRESENTATION
3 #example 1
4
5 #w-coordinate matrices
6 Crdwb1 <- matrix(c(-2,5,6,-4), 4, 1, byrow=TRUE)
7 Crdwb2 <- matrix(c(-5,12,11,-5), 4, 1, byrow=TRUE)
8 Crdwb3 <- matrix(c(3,-5,-5,3), 4, 1, byrow=TRUE)
9
10 #matrix T
11 Tmat <- matrix(c(Crdwb1,Crdwb2,Crdwb3), 4, 3)
12 Tmat
13
14 Crdwv <- matrix(c(3,-5,7), 3, 1)
15 Crdwv
16
17 CrdwTv = Tmat %*% Crdwv
18 CrdwTv
```

R code Exa 4.2 Change of basis and similarity

```
1 #page - 131
2 #section - 4.4 CHANGE OF BASIS AND SIMILARITY
3 #example - 2
4
5 #included package - matlib
6
7 #for inverse functions
8 library(matlib)
9
10 #matrix M
11 M <- matrix(c(2,4,6,-1,2,3,1,4,9), 3, 3, byrow=TRUE)
12 M
13
14 #matrix P
15 P <- matrix(c(1,0,1,1,1,1,1,1,0), 3, 3, byrow=TRUE)
16 P
17
18 #inverse of matrix P, PI
19 (PI <- inv(P))
20
21 Mtx = PI %*% M %*% P
22 Mtx
```

R code Exa 4.3 Change of basis and similarity

```
1 #page - 132
2 #section - 4.4 CHANGE OF BASIS AND SIMILARITY
3 #example - 3
4
5 #included Package - matlib
6
7 #for inverse functions
8 library(matlib)
```

```

9
10 #matrix M
11 M <- matrix(c(17,12,18,-16,-9,-24,-5,-4,-4), 3, 3,
               byrow=TRUE)
12 M
13
14 #matrix Q
15 Q <- matrix(c(10,-3,-3,-8,3,2,-3,1,1), 3, 3, byrow=
               TRUE)
16 Q
17
18 #inverse of matrix Q, QI
19 (QI <- inv(Q))
20
21 Mtx = QI %*% M %*% Q
22 Mtx

```

R code Exa 4.4 Eigenvalues and eigenvectors

```

1 #page - 135
2 #section - 4.5 CHARACTERISTIC VECTORS AND
  CHARACTERISTIC VALUES
3 #example - 4
4
5 #matrix A
6 A <- matrix(c(7,-8,-8,9,-16,-18,-5,11,13), 3, 3,
               byrow=TRUE)
7 A
8
9 #matrix X
10 X <- matrix(c(1,3,-2), 3, 1, byrow=TRUE)
11 X
12
13 A %*% X
14

```

```
15 #eigenvalues and eigenvectors
16 eigen(A)
```

R code Exa 4.5 Characteristic polynomial

```
1 #page - 137
2 #section - 4.5 CHARACTERISTIC VECTORS AND
  CHARACTERISTIC VALUES
3 #example - 5
4
5 #included package - pracma
6
7 #for charpoly function
8 library(pracma)
9
10 #matrix A
11 A <- matrix(c(2,1,1,2,3,2,1,1,2), 3, 3, byrow=TRUE)
12 A
13
14 #characteristic polynomial
15 charpoly(A, info = FALSE)
```

R code Exa 4.6 Characteristic polynomial

```
1 #page - 156
2 #section - 4.8 SCHUR'S THEOREM AND NORMAL MATRICES
3 #example - 6
4
5 #included package - pracma
6
7 #for charpoly function
8 library(pracma)
9
```

```
10 #matrix A
11 B <- matrix(c(4,-1,1,-1,4,-1,1,-1,4), 3, 3, byrow=
    TRUE)
12 B
13
14 #characteristic polynomial
15 charpoly(B, info = FALSE)
```

Chapter 5

Similarity Part I

R code Exa 5.1 Minimum polynomial

```
1 #page - 167
2 #section - 5.1 THE CAYLEY-HAMILTON THEOREM
3 #example - 1
4
5 #included package - polynom
6
7 #for minimum polynomial function
8 library(polynom)
9
10 #matrix B
11 B <- matrix(c
      (17,-8,-12,14,46,-22,-35,41,-2,1,4,-4,4,-2,-2,3),
      4, 4, byrow = TRUE)
12 B
13
14 eigVals <- eigen(B)$values
15 multEig <- table(eigVals)
16 k <- length(multEig)
17 minPoly <- 1
18 for(i in 1:k){
19   poly.i <- polynomial(c(-as.numeric(names(multEig)[
```



```

        i]), 1))
20   minPoly <- (minPoly*poly.i)
21 }
22
23 #minimum polynomial
24 minPoly

```

R code Exa 5.2 Characteristic polynomial

```

1 #page - 168
2 #section - 5.1 THE CAYLEY-HAMILTON THEOREM
3 #example - 2
4
5 #included packages - pracma, polynom
6
7 #for charpoly function
8 library(pracma)
9
10 #for minimum polynomial function
11 library(polynom)
12
13 #matrix S
14 S <- matrix(c(4,-1,1,-1,4,-1,1,-1,4), 3, 3, byrow =
      TRUE)
15 S
16
17 eigVals <- eigen(S)$values
18 multEig <- table(eigVals)
19 k <- length(multEig)
20 minPoly <- 1
21 for(i in 1:k){
22   poly.i <- polynomial(c(-as.numeric(names(multEig)[
      i]), 1))
23   minPoly <- (minPoly*poly.i)
24 }

```

```
25
26 #characteristic polynomial
27 charpoly(S, info = FALSE)
28
29 #minimum polynomial
30 minPoly
```

Chapter 6

Polynomials and Polynomial Matrices

R code Exa 6.1 Left and right functional value

```
1 #page - 208
2 #section - 6.4 MATRICES WITH POLYNOMIAL ELEMENTS
3 #example 1
4
5 #included package - expm
6
7 #for power of a matrix
8 library(expm)
9
10 #constant matrices
11 c1 <- matrix(c(0,0,1,1), 2, 2, byrow=TRUE)
12 c2 <- matrix(c(0,1,0,1), 2, 2, byrow=TRUE)
13 c3 <- matrix(c(1,1,0,1), 2, 2, byrow=TRUE)
14
15 #matrix A
16 A <- matrix(c(1,0,0,2), 2, 2)
17 A
18
19 #square of A
```

```

20 A2 = A %^% 2
21 A2
22
23 #right functional value
24 Pr = c1%*%A2 + c2%*%A + c3
25 Pr
26
27 #left functional value
28 Pi = A2%*%c1 + A%*%c2 + c3
29 Pi
30
31 #function to compare two matrices
32 matequal <- function(x, y)
33   is.matrix(x) && is.matrix(y) && dim(x) == dim(y) &
34     & all(x == y)
35 matequal(Pr, Pi)

```

Chapter 8

Matrix Analysis

R code Exa 8.1 Primary functions

```
1 #page - 242
2 #section - 8.2 PRIMARY FUNCTIONS
3 #example 1
4
5
6 #first matrix A
7 A <- matrix(c(2,1,1,2,3,2,1,1,2), 2, 3, byrow=TRUE)
8 A
9
10 fn <- function(z)
11   sin((pi/2)*z)
12
13 fn(A)
```

Chapter 9

Numerical Methods

R code Exa 9.1 Echelon

```
1 #page - 253
2 #section - 9.2 EXACT METHODS FOR SOLVING  $AX = K$ 
3 #example 1
4
5 #included package - matlib
6
7 #for echelon function
8 library(matlib)
9
10 #matrix A
11 A <- matrix(c(2,1,-1,2,1,3,2,-3,-1,2,1,-1,2,-3,-1,4),
12             , 4, 4, byrow=TRUE)
13
14 #column matrix k
15 K <- c(1,0,1,0)
16
17 #reduced row-echelon form
18 echelon(A, K, reduced=TRUE, verbose=TRUE, fractions=
19         FALSE)
```

R code Exa 9.2 Echelon

```
1 #page - 254
2 #section - 9.2 EXACT METHODS FOR SOLVING  $AX = K$ 
3 #example 2
4
5 #included package - matlib
6
7 #for echelon function
8 library(matlib)
9
10 #matrix A
11 A <- matrix(c
12             (7,9,2,-1,4,-5,-7,2,3,-2,-5,-1,1,6,-4,-3), 4, 4,
13             byrow=TRUE)
14
15 #column matrix k
16 K <- c(1,2,4,3)
17
18 #reduced row-echelon form
19 echelon(A, K, reduced=TRUE, verbose=TRUE, fractions=
20         TRUE)
```

R code Exa 9.3 Inverse of matrix

```
1 #page - 259
2 #section - 9.2 EXACT METHODS FOR SOLVING  $AX = K$ 
3 #example 3
4
5 #included package - matlib
6
7 #for inverse functions
```

```

8  library(matlib)
9
10 #matrix A
11 A <- matrix(c
      (1,0,2,-1,4,5,3,-1,0,1,8,5,-3,1,4,6,2,0,0,1,0,1,4,2,0)
      , 5, 5, byrow=TRUE)
12 A
13
14 #determinant of matrix A
15 det(A)
16
17 #inverse of matrix A, AI
18 (AI <- inv(A))

```

R code Exa 9.4 Eigenvalues and eigenvectors

```

1  #page - 265
2  #section - 9.4 CHARACTERISTIC VALUES AND VECTORS
3  #example - 4
4
5  #matrix A
6  A <- matrix(c(1,1,3,1,-2,1,3,1,3), 3, 3, byrow=TRUE)
7  A
8
9  #matrix X
10 X <- matrix(c(1,1,1), 3, 1, byrow=TRUE)
11 X
12
13 #eigenvalues and eigenvectors
14 eigen(A)

```
