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Book Proposed: Transport Phenomena: A Unified Approach
Total Chapters: 15
Total Examples: 155
Codable Examples: 108

Chapter 1: Introduction to Transport Phenomena
Example 1.1 – Codable
Example 1.2 –Codable

Chapter 2 : MOLECULAR TRANSPORT MECHANISMS
Example 2.1 - Codable
Example 2.2 – Codable
Example 2.3 – Codable
Example 2.4 – Non codable
(Reason : No numerical Calculation)

Example 2.4. Compare the rates of heat transfer across a sample of white pine wood when the transfer is across the grain and when it is parallel to the grain.

Answer. The thermal conductivity of this sample can be found in standard references, such as *Perry's Chemical Engineers' Handbook* [P1]. For white pine

Across the grain:

$$k = 0.087 \text{ Btu h}^{-1} \text{ ft}^{-1} \text{ °F}^{-1} \tag{i}$$

Parallel to the grain:

$$k = 0.20 \text{ Btu h}^{-1} \text{ ft}^{-1} \text{ °F}^{-1} \tag{ii}$$

For a given area and temperature gradient, the relative rates of heat transfer are given by the thermal conductivities (Eq. 2.17); thus, white pine wood conducts heat 2.3 times faster **(0.20/0.087** = 2.3) parallel to the grain than across. Note that one need not convert to **SI** units, as clearly all the conversion factors cancel out.

Example 2.5 – Codable
Example 2.6 – Codable
Example 2.7 – Codable
Example 2.8 – Codable
Example 2.9 – Codable
Example 2.10 – Non Codable
(Reason : No numerical Calculations)

Example 2.10. An incompressible fluid flows between two large plates in the x direction at steady-state. The bottom plate is flat. The top plate is divided into two flat plates by a reducer plate set at an angle to the bottom plate. The fluid flows in a 2-cm wide channel at the inlet, then into the reducer section, and out a l-cm-wide channel (see Fig. 2.12). The flow is laminar throughout the channel. In the reducer, which of the nine components of the velocity tensor VU and the stress tensor τ are non-zero?

Answer. Since both plates are large and flow is in the x and y directions only, the velocity in the z direction (perpendicular to the plane of the paper in Fig. 2.12) will be zero as will all derivatives of U_z:

$$\partial U_z/\partial x = \partial U_z/\partial y = \partial U_z/\partial z = 0 \qquad \text{(i)}$$

In the reducer the incompressible fluid must accelerate because the area is being reduced while the mass entering the reducer equals the mass exiting. Since the fluid is accelerating in the x direction:

$$\partial U_x/\partial x \neq 0 \qquad \text{(ii)}$$

The velocity U_x is a function of y everywhere between the plates (zero at both

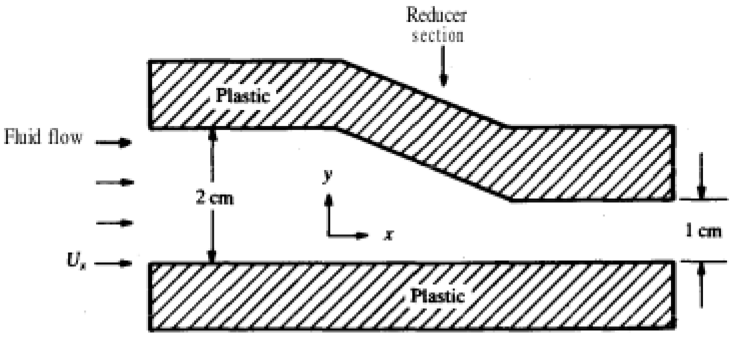

FIGURE 2.12
Convergent plate system.

walls, finite and variable in between):

$$\partial U_x/\partial y \neq 0 \qquad \text{(iii)}$$

Furthermore, the fluid must flow in the minus y direction as well as the positive x direction in order to squeeze into the l-cm-wide channel. The velocity U_y will vary in the x direction as well as in the y direction:

$$\partial U_y/\partial x \neq 0 \neq \partial U_y/\partial y \qquad \text{(iv)}$$

Since the plates are large, there is no variation of any velocity with the z direction, thus

$$\partial U_x/\partial z = 0 = \partial U_y/\partial z \qquad \text{(v)}$$

In conclusion, for the two-dimensional flow in the reducer, there are four non-zero derivatives in ∇U: $\partial U_x/\partial x, \partial U_x/\partial y, \partial U_y/\partial x,$ and $\partial U_y/\partial y$.

To find the non-zero shear stress terms, each shear stress is written in terms of the velocity derivative, as done in Eq. (2.44) through (2.46). The normal stresses are

$$\begin{aligned} \tau_{xx} &= -2\mu(\partial U_x/\partial x) \neq 0 \\ \tau_{yy} &= -2\mu(\partial U_y/\partial y) \neq 0 \\ \tau_{zz} &= -2\mu(\partial U_z/\partial z) = 0 \end{aligned} \qquad \text{(vi)}$$

The other six shear stresses are for the x direction:

$$\begin{aligned} \tau_{xy} &= -\mu[(\partial U_y/\partial x) + (\partial U_x/\partial y)] \neq 0 \\ \tau_{xz} &= -\mu[(\partial U_z/\partial x) + (\partial U_x/\partial z)] = 0 \end{aligned} \qquad \text{(vii)}$$

For the y direction:

$$\begin{aligned} \tau_{yx} &= -\mu[(\partial U_x/\partial y) + (\partial U_y/\partial x)] \neq 0 \\ \tau_{yz} &= -\mu[(\partial U_z/\partial y) + (\partial U_y/\partial z)] = 0 \end{aligned} \qquad \text{(viii)}$$

For the z direction:

$$\begin{aligned} \tau_{zx} &= -\mu[(\partial U_x/\partial z) + (\partial U_z/\partial x)] = 0 \\ \tau_{zy} &= -\mu[(\partial U_y/\partial z) + (\partial U_z/\partial y)] = 0 \end{aligned} \qquad \text{(ix)}$$

Hence, there are four non-zero shear stress terms in the reducer, i.e., $\tau_{xx}, \tau_{yy}, \tau_{xy}$, and τ_{yx}. Note the symmetry of the stress tensor:

$$\tau_{xy} = \tau_{yx} \qquad \tau_{xz} = \tau_{zx} \qquad \text{etc.} \qquad \text{(x)}$$

The stress tensor is always symmetrical under normal conditions.

Example 2.11 -Codable
Example 2.12 - Codable

Chapter 3 : The General Property Balance
Example 3.1 – Codable
Example 3.2 – Codable
Example 3.3 – Codable
Example 3.4 – Non Codable

Example 3.4. Obtain the three-dimensional equation for heat transfer in vector notation and show the form that can be obtained for constant properties. Also, express this equation completely in rectangular notation.

Answer. The vector equation is obtained by replacing ψ with $\rho c_p T$ and δ with α in Eq. (3.60). The resulting equation is Eq. (3.65):

$$\partial(\rho c_p T)/\partial t + (\boldsymbol{U}\cdot\boldsymbol{\nabla})(\rho c_p T) = \dot{\psi}_G + [\boldsymbol{\nabla}\,.\,\alpha\boldsymbol{\nabla}(\rho c_p T)] - (\rho c_p T)(\boldsymbol{\nabla}\cdot\boldsymbol{U}) \qquad (3.65)$$

For constant properties ρ, c_p, k, and thus α, Eq. (3.65) reduces to

$$\partial T/\partial t + (\boldsymbol{U}\cdot\boldsymbol{\nabla})T = \dot{\psi}_G/(\rho c_p) + \alpha(\nabla^2\,T) - T(\boldsymbol{\nabla}\,.\,\boldsymbol{U}) \qquad \text{(i)}$$

The expression in rectangular (Cartesian) coordinates may be obtained from Eqs.

(i), (3.53), (3.54), and (3.64):

$$\frac{\partial T}{\partial t} + U_x\frac{\partial T}{\partial x} + U_y\frac{\partial T}{\partial y} + U_z\frac{\partial T}{\partial z} = \frac{\dot{\psi}_G}{\rho c_p} + \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) - T\left(\frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} + \frac{\partial U_z}{\partial z}\right) \qquad \text{(ii)}$$

Example 3.5 – Non Codable
Reason: No Numerical Calculations

Example 3.5. A flow in rectangular coordinates is given by

$$U = i(x^3y) + j(2yx^2z) \qquad \text{(i)}$$

Is this flow compressible?

Answer. If the flow is incompressible, then $(\nabla \cdot U)$ will be zero as required by Eq. (3.74). 'The components of the velocity *vector from* Eq. (i) are

$$U_x = x^3y \qquad U_y = 2yx^2x \qquad U_z = o \qquad \text{(ii)}$$

Thus

$$\partial U_x/\partial x = 3x^2y \qquad \partial U_y/\partial y = 2x^2z \qquad \partial U_z/\partial z = 0 \qquad \text{(iii)}$$

These derivatives are substituted into Eq. (3.74):

$$(\nabla \cdot U) = \partial U_x/\partial x + \partial U_y/\partial y + \partial U_z/\partial z = 0 = 3x^2y + 2x^2z = (x^2)(3y + 2z) \qquad \text{(iv)}$$

The dot product $(\nabla \cdot U)$ is zero at the plane ($x = 0$). It is also zero along the plane

$$3y + 2z = 0 \qquad \text{(v)}$$

or

$$y = (-2z)/3 \qquad \text{(vi)}$$

Therefore, this flow is compressible because the dot product $(\nabla \cdot U)$ is not zero throughout the entire flow field.

Example 3.6 – Non Codable
Reason :No Numerical Calculations

Example 3.6. An incompressible flow at steady-state in rectangular coordinates is given by the vector components

$$U_x = x^3y \qquad U_y = 2yx^2z \qquad \text{(i)}$$

and U_z is unknown. Find U_z.

Answer. Equation (i) above for U_x and U_y contains two of the wmponents of the velocity given in the previous example. If the flow is incompressible, then Eq. (3.74) holds. Using the derivatives in Eq. (iii) from the previous example gives

$$(\nabla \cdot U) = 3x^2y + 2x^2z + \partial U_z/\partial z = 0 \qquad \text{(ii)}$$

After separating variables, Eq. (ii) becomes

$$\partial U_z = -(3x^2y + 2x^2z)\, \partial z \qquad \text{(iii)}$$

Next, this equation is integrated:

$$U_z = -3x^2yz - x^2z^2 + C(x, y) \qquad \text{(iv)}$$

The term $(-x^2z)$ can be factored out of the first two terms in Eq. (iv). Then the z component of velocity U_z becomes

$$U_z = (-x^2z)(3y + z) + C(x, y) \qquad \text{(v)}$$

Note that $C(x, y)$ is a constant of integration to be determined from the boundary conditions and may be a function of x and y; it cannot be determined from the information given in this problem. Hence, the **final** answer will express the vector U as follows:

$$U = i(x^3y) + j(2yx^2z) + k[(-x^2z)(3y + z) + C(x, y)] \qquad \text{(vi)}$$

The flow is incompressible for all values of x, y, z, and $C(x, y)$.

Example 3.7 – Non Codable
Reason: No Numercial Calculation

Example 3.7. Does the velocity in a one-directional incompressible flow in rectangular (Cartesian) coordinates change with the direction of flow?

Answer. For inwrnpressible flow in rectangular coordinates, **Eq.** (3.74) holds

$$(\nabla \cdot U) = \partial U_x/\partial x + \partial U_y/\partial y + \partial U_z/\partial z = 0 \qquad (3.74)$$

Now if a. one-directional flow is in the x direction, U_y and U_z must be zero

$$U_y = U_z = 0 \qquad \textbf{(i)}$$

Therefore

$$\partial U_y/\partial y = \partial U_z/\partial z = 0 \qquad \textbf{(ii)}$$

and 'from Eq. (3.74)

$$\partial U_x/\partial x = 0 \qquad \text{(iii)}$$

Integration of Eq. (iii) yields

$$U_x = C(y, z) \qquad \textbf{(iv)}$$

Therefore U_x does not change in the direction of the flow, the x direction, since $C(y, z)$ can only vary in the y or z directions. **The** importance of this result must be emphasized again. The continuity equation demands **that** in a one-dimensional incompressible flow, the velocity cannot change in the direction of flow for any reason.

Example 3.8 – Non Codable
Reason No Numerical Calculation

Example 38. Obtain the equation for x direction momentum for a general momentum transfer problem that can have velocities in all three directions. Show the equation for (a) compressible flow and (b) incompressible flow in both vector notation and in rectangular coordinates.

Answer. (a) For the x direction of momentum, the property ψ is replaced with ρU_x and δ with ν (which is μ/ρ) from Table 3.1. Equation (3.60) in vector form for compressible flow is

$$\partial(\rho U_x)/\partial t + (U \cdot \nabla)(\rho U_x) = \dot{\psi}_G + [\nabla \cdot \nu\nabla(\rho U_x)] - (\rho U_x)(\nabla \cdot U) \qquad \textbf{(i)}$$

If the density is constant, then the divergence of U is **zero** by Eq. **(3.74),** and Eq. (i) reduces to

$$\partial U_x/\partial t + (U \cdot \nabla)U_x = \dot{\psi}_G/\rho + (\nabla \cdot \nu\nabla U_x) \qquad \text{(ii)}$$

Note that Eq. (ii) has been simplified by division by p. This equation also could have been directly obtained from Eq. (3.77). Now if v is also constant, one obtains from Eq. (ii) or Eq. (3.78):

$$\partial U_x/\partial t + (U \cdot \nabla)U_x = \dot{\psi}_G/\rho + \nu(\nabla^2 U_x) \qquad \text{(iii)}$$

In rectangular coordinates, as one example, Eq. (iii) with the help of the expressions for $(U \cdot \nabla)\psi$ and $\nabla^2 \psi$ as given in Eqs. (3.53) and (3.64) respectively, becomes

$$\frac{\partial U_x}{\partial t} + U_x\frac{\partial U_x}{\partial x} + U_y\frac{\partial U_x}{\partial y} + U_z\frac{\partial U_x}{\partial z} = \frac{\dot{\psi}_G}{\rho} + \nu\left(\frac{\partial^2 U_x}{\partial x^2} + \frac{\partial^2 U_x}{\partial y^2} + \frac{\partial^2 U_x}{\partial z^2}\right) \qquad \textbf{(iv)}$$

If ν were not constant, then Eq. (ii) would have to be expanded and, if p were also not constant, then Eq. (i) would have to be expanded.

Example 3.9 – Non Codable
Reason : No numerical calculations

Example 3.9. Obtain the equation for mass transfer for a two-dimensional **flow** in the x and y directions when the density and diffusion coefficient can be considered constant and when there is no chemical reaction.

Answer. For constant-density problems, the property ψ is conveniently taken as ρ_A. Note that there must be at least two components in order to have a mass transfer problem. From the statement of the problem, the following are true:

$$U = iU_x + jU_y \qquad \psi = \rho_A \qquad \delta = D \qquad \dot{\psi}_G = 0 \tag{i}$$

Equation (3.78) with the substitutions from Eq. (i) becomes

$$\partial\rho_A/\partial t + (U \cdot \nabla)\rho_A = D(\nabla^2 \rho_A) \tag{ii}$$

The second term on the left is obtained from Eq. (3.53):

$$(U \cdot \nabla)\rho_A = U_x \frac{\partial \rho_A}{\partial x} + U_y \frac{\partial \rho_A}{\partial y} \tag{iii}$$

In Eq. (3.53) U_z for this problem is **zero** since there is no variation in the z direction for a two-dimensional flow. The **Laplacian** of ρ_A is given by Eq. (3.64):

$$\nabla^2 \rho_A = \frac{\partial^2 \rho_A}{\partial x^2} + \frac{\partial^2 \rho_A}{\partial y^2} \tag{iv}$$

where again the last term in Eq. (3.64) is zero. Equation **(3.78),** in rectangular coordinates, becomes

$$\frac{\partial \rho_A}{\partial t} + U_x \frac{\partial \rho_A}{\partial x} + U_y \frac{\partial \rho_A}{\partial y} = D\left(\frac{\partial^2 \rho_A}{\partial x^2} + \frac{\partial^2 \rho_A}{\partial y^2}\right) \tag{v}$$

Chapter 4 Molecular Transport and the General Property Balance

Example 4.1 - Codable
Example 4.2 – Codable
Example 4.3 – Codable
Example 4.4 – Codable
Example 4.5 – Codable
Example 4.6 – Non Codable
Reason : No numerical Calculations

Example 4.6. Obtain the temperature distribution for the slab shown below in which there is a uniform heat generation. The slab in Fig. 4.5 is assumed to be large in both y and z directions so that any boundary effects may be neglected.

Answer. The basic equation for this problem comes from Eq. (4.31); again Table 3.1 is used to replace ψ and δ for heat transfer. Upon assumption of constant physical properties ρ and c_p, Eq. (4.31) reduces to

$$-d[(kA_x)(dT/dx)] = \dot{T}_G\, dV \tag{i}$$

The area of transfer is constant. The volume is the area times the distance:

$$V = A_x x \tag{ii}$$

or in differential form, for constant area:

$$dV = A_x\, dx \tag{iii}$$

Equation (iii) is used to eliminate dV in Eq. (i):

$$-d[(kA_x)(dT/dx)] = \dot{T}_G A_x\, dx \tag{iv}$$

The area term can be canceled from both sides of Eq. (iv), since the transfer area is constant in Fig. 4.5. At this point, it is also convenient to assume that the slab has a constant thermal conductivity k_m as discussed before. Then Eq. (iv) can be integrated:

$$dT/dx = -(\dot{T}_G/k_m)(x) + C_1 \tag{v}$$

The problem is not symmetric. Although dT/dx might be zero somewhere, it is not known where, so C_1 cannot be determined. However, Eq. (v) can be integrated again:

$$T = -[\dot{T}_G/(2k_m)](x^2) + C_1(x) + C_2 \tag{vi}$$

There are two boundary conditions that can be used to determine the two constants of integration. These are

$$\begin{aligned} T(x = -x_o) &= T_1 \\ T(x = +x_o) &= T_2 \end{aligned} \tag{vii}$$

These are substituted into Eq. (vi) to give two equations in two unknowns:

$$\begin{aligned} T_1 &= -[\dot{T}_G/(2k_m)](x_o^2) - C_1(x_o) + C_2 \\ T_2 &= -[\dot{T}_G/(2k_m)](x_o^2) + C_1(x_o) + C_2 \end{aligned} \tag{viii}$$

The two equations can be solved for C, and C_2:

$$C_1 = -(T_1 - T_2)/(2x_o) = (T_2 - T_1)/(2x_o) \tag{ix}$$

$$C_2 = [\dot{T}_G/(2k_m)](x_o^2) + (T_1 + T_2)/2 \tag{x}$$

Combining these into Eq. (vi) gives

$$T = [\dot{T}_G/(2k_m)](x_o^2 - x^2) + \tfrac{1}{2}(T_2 - T_1)(x/x_o) + \tfrac{1}{2}(T_1 + T_2) \tag{xi}$$

At $x = x_o$, T in Eq. (xi) reduces to T_2 and at $x = -x_o$, it reduces to T_1, both of which are the given boundary conditions, Eq. (vii). One can determine the maximum temperature point from Eqs. (v) and (ix):

$$dT/dx = 0 = -(\dot{T}_G/k_m)(x) + (T_2 - T_1)/(2x_o) \tag{xii}$$

Solving for x:

$$x_{max} = [k_m/(2\dot{T}_G x_o)](T_2 - T_1) \tag{xiii}$$

When $T_2 = T_1$, x_{max} is zero; i.e., the temperature profile is symmetric about the center line with maximum temperature at $x = 0$, as already shown in Example 4.5. For the unsymmetrical distribution problem, Eq. (xi) applies. An example of such a problem is a steel wire pressed into a narrow wide slab with the conditions of Example 4.5 and with

Example 4.7 – Codable
Example 4.8 – Non Codable
Reason: No numerical Calculation

Example 4.8. Consider Fig. 4.11 in which a **fluid** of constant ρ and μ is flowing between parallel plates. The bottom plate is at rest. The top plate is moving at a constant velocity U_o. Prepare a graph of y versus U_x/U_o for various pressure gradients.

Answer. Equation (4.60) is the equation for steady-state flow in the x direction only. Equation (4.60) is integrated appropriately to obtain **Eq.** (4.89):

$$-\mu U_x = (\dot{M}_G/2)y^2 + C_1 y + C_2 \qquad (4.89)$$

The boundary conditions for this problem are

$$U_x(y = +y_o) = U_o \qquad U_x(y = -y_o) = 0 \qquad \text{(i)}$$

The **boundary conditions** are substituted into **Eq.** (4.89) to yield two equations in **two unknowns (C_1 and C_2):**

$$-\mu U_o = (\dot{M}_G/2)y_o^2 + C_1 y_o + C_2 \qquad \text{(ii)}$$

$$0 = (\dot{M}_G/2)y_o^2 - C_1 y_o + C_2 \qquad \text{(iii)}$$

After **adding these two equations, the constant** C, **is eliminated, and** C_2 **is**

$$C_2 = -(\mu U_o/2) - (\dot{M}_G/2)y_o^2 \qquad \text{(iv)}$$

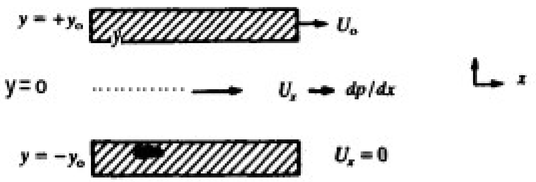

FIGURE 4.11
Flow between hvo parallel flat plates (Couette flow).

Chapter 5 Transport with a net convective Flux
Example 5.1 – Non Codable
Reason No Numerical Calculation

Example 5.1. Find the relations between shear stress and shear rate in a **fluid** of constant density and viscosity that flows in a tube of expanding diameter, as in Fig. 5.2.

Answer. The flow is incompressible (constant density) **so** that $(\nabla \cdot U)$ is 0. Cylindrical coordinates are chosen. There is no velocity in the O-direction, i.e.,

$$U_\theta = 0 \qquad \text{(i)}$$

All derivatives with respect to θ are likewise zero. Both U_z and U_r are non-zero and vary with both r and z, but not with θ. Hence, Eqs. (G) through (L) become

$$\tau_{rr} = -2\mu(\partial U_r/\partial r) \qquad \text{(ii)}$$

$$\tau_{\theta\theta} = -2\mu U_r/r \qquad \text{(iii)}$$

$$\tau_{zz} = -2\mu(\partial U_z/\partial z) \qquad \text{(iv)}$$

$$\tau_{r\theta} = \tau_{\theta r} = 0 \qquad \text{(v)}$$

$$\tau_{\theta z} = \tau_{z\theta} = 0 \qquad \text{(vi)}$$

$$\tau_{rz} = \tau_{zr} = -\mu[(\partial U_r/\partial z) + (\partial U_z/\partial r)] \qquad \text{(vii)}$$

A flow such as in Fig. 5.2 is called decelerating if the flow is left to right. Note that all three normal stresses $(\tau_{rr}, \tau_{\theta\theta}, \tau_{zz})$ are non-zero, whereas in the one-directional tube flow of Fig. 4.7 or Fig. 5.1, all three were zero as shown by Eqs. (5.3) and (5.4).

4. The Continuity Equation

Example 5.2 – Non Codable
Reason: No numerical Calculation

Example 5.2. A flow in cylindrical coordinates is given by

$$U = i_r r^3\theta + i_\theta 2r^2\theta z \tag{i}$$

Is this flow compressible?

Answer. If the flow is incompressible, then $(\nabla \cdot U)$ will be zero as required by Eq. (E) from Table 5.3

$$(\nabla \cdot U) = (1/r)[\partial(rU_r)/\partial r)] + (1/r)(\partial U_\theta/\partial\theta) + \partial U_z/\partial z = 0 \tag{E}$$

The velocity in cylindrical coordinates is

$$U = i_r U_r + i_\theta U_\theta + i_z U_z \tag{ii}$$

Thus, from the problem statement:

$$U_r = r^3\theta \tag{iii}$$

$$U_\theta = 2r^2\theta z \tag{iv}$$

$$U_z = 0 \tag{v}$$

The derivatives required in Eq. (E) are found from Eq. (iii) through Eq. (v):

$$\partial(rU_r)/\partial r = \partial(r^4\theta)/\partial r = 4r^3\theta \tag{vi}$$

$$\partial U_\theta/\partial\theta = \partial(2r^2\theta z)/\partial\theta = 2r^2 z \tag{vii}$$

$$\partial U_z/\partial z = 0 \tag{viii}$$

Now Eqs. (vi) though Eq. (viii) are inserted into $(\nabla \cdot U)$ given by Eq. (E) above:

$$(\nabla \cdot U) = (1/r)(4r^3\theta) + (1/r)(2r^2 z) + 0 = 4r^2\theta + 2rz \tag{ix}$$

Equation (ix) is seen to be zero only at $r = 0$ or along the surface

$$2\theta r = -z \tag{x}$$

Clearly the flow is compressible.

The general mass balance for species A, also called the continuity [e]quation for species A, is given by Eq. (3.66), and after substituting $\dot{\rho}_{A,G}$ for [...]$_G$, Eq. (3.55) becomes

$$\underset{\text{ACC}}{\partial\rho_A/\partial t} + \underset{\text{CONV}}{(U \cdot \nabla)\rho_A} = \underset{\text{GEN}}{\dot{\rho}_{A,G}} + \underset{\text{MOLEC}}{(\nabla \cdot D\nabla\rho_A)} - \underset{\text{CONV}}{(\rho_A)(\nabla \cdot U)} \tag{5.6}$$

Example 5.3 – Non Codable
Reason: No numerical Calculation

Example 5.3. **Consider the problem in Fig. 2.4 from Example 2.1. A copper block is subjected to a temperature difference across the face at x = 0 and at $x = 10$ cm. All other faces are insulated. Derive the differential equation to be solved using Table 5.6.**

Answer. **Rectangular coordinates are chosen; the thermal transfer is in the x direction. Since the copper is a solid in Example 2.1, all velocities are zero in this problem. Thus**

$$U_x = U_y = U_z = 0 \tag{i}$$

Equation (A) of Table 5.6 applies with the following terms being zero for the reasons given

$\partial T/\partial t = 0$	(steady-state)	(ii)
$\dot{T}_G = 0$	(no generation of any kind)	(iii)
$\partial T/\partial y = 0 = \partial T/\partial z$	(temperature varies only in the x direction)	(iv)

Thus, Eq. (A) has only one non-zero term for this problem

$$0 = \frac{\partial}{\partial x}\left(\alpha \frac{dT}{dx}\right) \tag{v}$$

This equation integrates to

$$C_1 = \alpha \frac{dT}{dx} \tag{vi}$$

where C_1 is a constant of integration and where ordinary differentials are used because the variation is only with x. Equation (2.10) is

$$\alpha = \frac{k}{\rho c_p} \tag{2.10}$$

This is **used** to replace α in Eq. (vi). For constant density and heat capacity, Eq. (vi) becomes

$$C_2 = \frac{k\,dT}{dx} \tag{vii}$$

where C_2 is a new constant equal to $(C_1 \rho c_p)$. Equation (vii) is integrated between the limits

$$x(T = T_1) = x_1$$
$$x(T = T_2) = x_2 \tag{viii}$$

The result is

$$C_2(x_2 - x_1) = k(T_2 - T_1) \tag{ix}$$

Comparison of Eq. (ix) with Eq. (ii) in Example 2.1 shows that

$$C_2 = (q/A)_x \tag{x}$$

Thus, it is shown that for the conditions of this problem, the flux $(q/A)_x$ is

Example 5.4 – Non Codable
Reason: No numerical Calculation

Example 5.4. Figure 5.3 illustrates a typical problem in laminar flow with heat transfer. Let the tube be of uniform cross sectional area so that the flow is one-directional. From Table 5.6 set up the differential equation that describes the temperature profile if the flow is steady-state and incompressible.

Answer. Cylindrical coordinates are chosen to describe the temperature distribution in the pipe, Eq. (B). Only U_z is non-zero:

$$U_r = U_\theta = 0 \tag{i}$$

Since the flow is at steady-state

$$\partial T/\partial t = 0 \tag{ii}$$

The temperature will vary with r and z, but not with the e-direction. As is typical for applications involving a gaseous fluid, or even water and oil, there is no generation of heat by viscous or other means:

$$\dot{T}_G = 0 \tag{iii}$$

Substituting the above relations into Eq. (B) in Table 5.6:

$$U_z \frac{\partial T}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\alpha \frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(\alpha \frac{\partial T}{\partial z}\right) \tag{iv}$$

Equation (iv) is a second-order partial differential equation that describes exactly **the temperature profile in laminar heat transfer.** Further assumptions are necessary to solve this equation subject to typical boundary conditions. In Example 5.6 this **specific** problem will **be** discussed further.

Example 5.5 – Non Codable
Reason: No numerical Calculation

Example **5.5. Determine the velocity distribution for the steady-state, laminar, incompressible flow of a fluid in a pipe. The flow configuration is horizontal.**

Answer. The problem is best expressed in cylindrical coordinates as shown in Fig. 5.1 or Fig. 4.7. From Table 5.7, Eqs. (D), (E), and (F) apply. The flow is one-directional (z direction), so that U_z is non-zero and

$$U_r = U_\theta = 0 \tag{i}$$

If the pipe is perfectly round, i.e., circular in cross section, U_z is symmetric in the r and θ directions. By symmetry, all derivatives with respect to θ are zero. The fluid velocity U_z is zero at the wall and a maximum at the tube center line. Thus, $\partial U_z/\partial r$ is finite and non-zero.

The continuity equation for this problem, **Eq. (E) in Table 5.3, is**

$$(1/r)[\partial(rU_r)/\partial r] + (1/r)(\partial U_\theta/\partial\theta) + \partial U_z/\partial z = 0 \tag{ii}$$

As a consequence of Eq. (i), the first two terms in Eq. (ii) are zero; hence

$$\partial U_z/\partial z = 0 \tag{iii}$$

After integrating once, Eq. (iii) becomes

$$U_z = \text{constant} \tag{iv}$$

where the constant or U_z can vary with r, but not with z. Thus, by the use of the continuity equation [Eq. (ii)] for an incompressible one-directional pipe flow, the velocity in the flow direction is invariant in that direction as was previously proved for rectangular coordinates in Example 3.7.

In **Eq. (D), for the r component of the Navier-Stokes equation, the entire**

left-hand side is zero as a result of Eq. (i). Similarly the right-hand side reduces to

$$-(1/\rho)(\partial p/\partial r) + g_r = 0 \tag{v}$$

Example 5.6 – Non Codable
Reason: No numerical Calculation

Example 5.6. Consider laminar heat transfer in a tube. Begin with Eq. (4.72) for velocity profie and Eq. (iv) in Example 5.4 and develop an equation to describe the variation of temperature with radius. Assume no viscous dissipation, no heat generation, constant physical properties, and a fully developed 'temperature profile ($\Delta T/L$ = constant). Figure 5.3 applies; in heat transfer, the inside radius of the tube is r_i.

Answer. The energy equation, Eq. (B) of Table 5.6, in cylindrical coordinates was simplified in Example 5.4; the result was a differential equation describing the variation of T with r. The answer from Eq. (iv) in that example (for constant a) is

$$U_z\frac{\partial T}{\partial z}=\alpha\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right] \qquad \text{(i)}$$

The variation of the velocity U_z with r is given by the Navier-Stokes equation, which simplified to Eq. (x) m Example 5.5:

$$\frac{\mu}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial U_z}{\partial r}\right)\right]=\frac{\Delta p}{L} \qquad \text{(ii)}$$

Equation (ii) is said to be coupled to Eq. (i) because U_z appears in both equations, and the final solution for U_z must satisfy both differential equations and all applicable boundary conditions as well. Every problem in mass or heat transport with convection results in coupled differential equations. Exact solutions are usually not possible, but a reasonably good solution is sometimes attainable if the equations can be decoupled, i.e., solved independently as Eq. (ii) was in Example 5.5.

Let us assume that the **temperature change** in this problem is relatively small. Then the physical properties may be assumed constant with little error. If the density and viscosity (the property most sensitive to temperature changes) are constant, then the fact that heat transfer is occurring will have no effect on the momentum transfer. Under these conditions, Eq. (ii) is no longer coupled to Eq. (i). Measurement of the velocity profile will prove or disprove the validity of this assumption. If the equations can be decoupled, the velocity profile is given by the solution to Example 5.5:

$$U_z=\frac{(-\Delta p)/L}{4\mu}(r_i^2-r^2) \qquad \text{(iii)}$$

where r_i is the inside tube diameter. Equations (i) and (iii) can be combined:

$$\frac{\alpha}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right]+\alpha\frac{\partial^2 T}{\partial z^2}+\frac{\Delta p/L}{4\mu}(r_i^2-r^2)\frac{\partial T}{\partial z}=0 \qquad \text{(iv)}$$

Equation (iv) is a most complex equation to solve without more assumptions. For the special case of flow far away from the entrance to the pipe and the beginning of the heat transfer section, the temperature profile will be fully developed, i.e., the following slope is constant:

$$\partial T/\partial z=\Delta T/L=\text{constant} \qquad \text{(v)}$$

Example 5.7 – Non Codable
Reason: No numerical Calculation

Example 5.7. Solve the Couette flow problem shown in Fig. 5.5. This geometry is often used for viscosity measurements. The fluid is placed in the gap between the outer and inner cylinders. The outer cylinder is rotated at a constant speed ω, which is low enough that the flow is laminar. A torque is measured at the inner cylinder by means of a calibrated spring. The apparatus is designed so that there are no end effects.

Answer. The Couette flow is interesting because the flow is in the θ direction only, so that

$$U_r = U_z = 0 \tag{i}$$

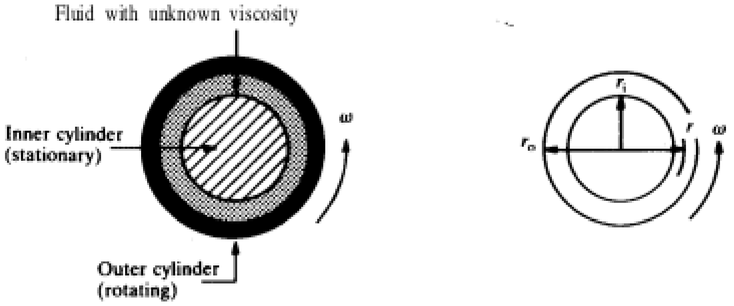

FIGURE 5.5
Flow between concentric cylinders (Couette flow).

and of course all derivatives of U_r and U_z are zero. The logical choice of coordinates is cylindrical. The angular velocity is carefully controlled so that steady-state prevails:

$$\frac{\partial U_\theta}{\partial t} = 0 \tag{ii}$$

The viscometer is long enough in the z-direction so that end effects are negligible; thus

$$\frac{\partial U_\theta}{\partial z} = 0 \tag{iii}$$

If the viscometer is manufactured carefully so that the cylinders are concentric, then the flow will be symmetric

$$\frac{\partial U_\theta}{\partial \theta} = 0 \tag{iv}$$

Equation (iv) also follows from the continuity equation (E) of Table 5.3. The viscometer configuration is such that gravity acts only in the z direction so that

$$g_r = 0 = g_\theta \tag{v}$$

Example 5.8 – Non Codable
Reason: No numerical Calculation

Example 5.8. An incompressible fluid flows in laminar flow past a flat plate. Assume that the plate is able to transfer heat and mass as well as 'momentum to the fluid. Find the non-zero terms in the appropriate balance equations. Figure 5.6 shows the geometry of system.

Answer. The flat plate experiment is perhaps the simplest explanation of boundary layer flows because there is only one surface under consideration. The fluid approaches the plate at uniform velocity U_∞, temperature T_∞, and concentration $C_{A,\infty}$. Flow is in the x direction. The z direction is perpendicular to the paper in Fig. 5.6, and there are no changes of any kind taking place in that direction. When the fluid reaches the front or leading edge of the plate (x = 0), the velocity profile must change because of the boundary condition of no slip:

$$U_x = U_y = 0 \qquad (y = 0) \tag{i}$$

The velocity must be zero at any solid surface as Eq. (i) states. As x increases, the boundary layer thickness δ increases. The fluid whose velocity has decreased must go somewhere. It moves outward and gives rise to a small but finite velocity U_y. Inside the boundary layer, therefore, the local velocity is a function of both x and y. Outside the boundary layer, the velocity is constant and equal to the free stream velocity U_∞. Clearly, both the x and y directions must be considered, but the z direction can be ignored as discussed previously. Thus

$$U_z = 0 \qquad \frac{\partial U_x}{\partial z} = \frac{\partial U_y}{\partial z} = 0 \tag{ii}$$

However, the remaining four partial derivatives of U_x and U_y are non-zero

Eqs. (iii) and (iv) reduce to

$$U_x \frac{\partial U_x}{\partial x} + U_y \frac{\partial U_x}{\partial y} = \nu \frac{\partial^2 U_x}{\partial y^2} \tag{viii}$$

$$U_x \frac{\partial U_y}{\partial x} + U_y \frac{\partial U_y}{\partial y} = \nu \frac{\partial^2 U_y}{\partial y^2} \tag{ix}$$

These equations, plus the continuity equation, are the starting point for the boundary layer analysis. However, Eq. (ix) need not be used since Eq. (viii) and the continuity equation (vi) are enough to define the system and allow a solution for U_x and U_y.

If the flat plate is maintained at a uniform temperature T_o which is different from the free stream temperature T_∞, then another equation must be added to those above. The energy equation (A) from Table 5.6 for steady-state, no generation, and constant properties reduces to

$$U_x \frac{\partial T}{\partial x} + U_y \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{x}$$

where dissipation has been neglected. The boundary layer approximation for heat transfer is $\partial^2 T/\partial x^2 \ll \partial^2 T/\partial y^2$. Equation (x) then simplifies to

$$U_x \frac{\partial T}{\partial x} + U_y \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{xi}$$

In order to solve the heat and momentum transfer problems together, Eqs. (vi), (viii), and (xi) must be solved simultaneously for U_x, U_y, and T as a function of x and y for given values of the momentum and thermal diffusivities and the given boundary conditions. Note that the temperature changes have been assumed small enough so that the physical properties can be considered constant. Even so, the "coupled" set of equations are a formidable task to solve.

Mass transfer may also be present in the problem. For example, if the plate were porous, or perhaps soluble in the fluid passing over it, so that a constant concentration $C_{A,o}$ could be maintained at the surface, then there would be a mass boundary layer formed on the plate as a result of mass transfer to or from the plate. The assumptions to be invoked have already been discussed. The mass transfer case is analogous to the heat transfer case, which resulted in Eq. (xi). The final equation is

$$U_x \frac{\partial C_A}{\partial x} + U_y \frac{\partial C_A}{\partial y} = D \left(\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} \right) \tag{xii}$$

This equation can also be obtained from Eq. (A) in Table 5.4. The boundary layer approximation for the mass transfer problem is

$$\frac{\partial^2 C_A}{\partial x^2} \ll \frac{\partial^2 C_A}{\partial y^2} \tag{xiii}$$

Thus

$$U_x \frac{\partial C_A}{\partial x} + U_y \frac{\partial C_A}{\partial y} = D \frac{\partial^2 C_A}{\partial y^2} \tag{xiv}$$

Example 5.9 – Codable
Example 5.10 – Codable
Example 5.11 – Codable
Example 5.12 – Codable
Example 5.13 – Codable

Chapter 6 : Turbulent Flow
Example 6.1 – Codable
Example 6.2 – Codable
Example 6.3 – Codable
Example 6.4 – Non Codable
Reason: No numerical Calculation

Example 6.4. For the data of Example 6.3, calculate the three r.m.s. values of the fluctuating velocities, the corresponding intensities of turbulence, and the cross turbulence term $U'_x U'_y$. Express the cross term as a ratio to the r.m.s. values.

Answer. Equation (6.22) gives the mean squared value $\overline{U'^2_z}$, and the r.m.s. value is the square root. The z component of velocity will be illustrated; Eq. (6.22) for the z component becomes

$$\overline{U'^2_z} = \frac{1}{T}\int_0^T (U'_z)^2\,dt = \frac{1}{T}\int_0^T (U_z - \bar{U}_z)^2\,dt \qquad \text{(i)}$$

The trapezoid rule as given by Eq. (viii) in Example 6.3 will be used to evaluate the integral in Eq. (i):

$$\overline{U'^2_z} = \frac{1}{T}\int_0^T (U_z - \bar{U}_z)^2\,dt$$
$$= \frac{1}{T}\frac{\Delta t}{2}\left[(U_z - \bar{U}_z)_1^2 + (U_z - \bar{U}_z)_N^2 + 2\sum_{i=2}^{12}(U_z - \bar{U}_z)_i^2\right] \qquad \text{(ii)}$$

Example 6.5 – Codable
Example 6.6 – Non Codable
Reason: No numerical Calculation

Example 6.6. Show how **the** analogy of equal eddy properties may be used to estimate the temperature distribution in water flowing in a heated pipe. Assume for this problem that $\bar{T}$ is a function only of radius and not of coordinates z or θ.

Answer. **The** equation for temperature distribution will be derived from Eq. **(6.67),** which **after** separation of variables is

$$\frac{dr}{\alpha + E_H} = -[(\rho c_p)/(q/A)_r]\,d\bar{T} \qquad \text{(i)}$$

From Eq. (6.56):

$$\tau_{rz} = \frac{r}{r_o}\tau_w \qquad (6.56)$$

It is assumed that in general

$$\Psi_r = \frac{r}{r_o}(\Psi_w) \qquad \text{(ii)}$$

For heat transfer,' Eq. (ii) becomes

$$(q/A)_r = \frac{r}{r_o}(q/A)_{r,w} \qquad \text{(iii)}$$

Eq. (i) becomes

$$\int_r^{r_o}\frac{r\,dr}{(r_o)(\alpha + E_H)} = -\frac{\rho c_p}{(q/A)_{r,w}}\int_{\bar{T}}^{\bar{T}_w} d\bar{T} \qquad \text{(iv)}$$

Upon integration of the right-hand side, Eq. (iv) is

$$\bar{T} - \bar{T}_w = \frac{(q/A)_{r,w}}{\rho c_p}\int_r^{r_o}\frac{r\,dr}{(r_o)(\alpha + E_H)} \qquad \text{(v)}$$

where α is constant $[k/(\rho c_p)]$ and E_H depends on the Reynolds number as well as the radius. Equation (6.84) is used to replace E_H in Eq. (v):

$$\bar{T} - \bar{T}_w = \frac{(q/A)_{r,w}}{\rho c_p}\int_r^{r_o}\frac{r\,dr}{(r_o)(\alpha + E_\tau)} \qquad \text{(vi)}$$

The right-hand side of Eq. (vi) can be determined for a given set of $\bar{T}_w$, $(q/A)_{r,w}$, ρ, c_p, α, and E_τ values such as those given in Table 6.4. However, Table 6.4 includes too few data points in the wall area to allow an accurate integration of **Eq.** (vi). A much better procedure is to use Eq. **(6.66),** to evaluate E_τ as a function of radius, with the *slope* $d\bar{U}_z/dr$ determined from an empirical correlation. Such a solution will be detailed in Example 6.8.

Example 6.7 – Codable
Example 6.8 – Codable
Example 6.9 – Codable

Chapter 7: Integral Methods of Analysis
Example 7.1 –Non Codable
Reason: No numerical Calculation

Example 7.1. A tank of radius r_T is filled to a height H with a liquid of density p, as shown in Fig. 7.2. The fluid drains from the bottom of the tank through a hole with a radius of r_o. The flow velocity at the exit is approximated by Torricelli's law:

$$U_{ave}^2 = 2gh \quad \text{(i)}$$

where h is the instantaneous height. What is the total time required to empty the tank?

Answer. Equation (7.11) applies. Since nothing enters the system

$$w_1 = 0 \quad \text{(ii)}$$

The mass flow exiting is the density times the velocity-times the flow area (πr_o^2), Eq. (7.10):

$$w_2 = \rho U_{2,\,ave}(\pi r_o^2) \quad \text{(iii)}$$

After substituting Torricelli's law, this equation becomes

$$w_2 = (\rho)(\pi r_o^2)(2gh)^{1/2} \quad \text{(iv)}$$

Since the term $\dot{M}$ has units of mass per second, the following unit equation is valid:

$$\dot{M} = d(\rho\pi r_T^2 h)/dt \quad \text{(v)}$$

where $\pi r_T^2 h$ is the volume of fluid in the tank at any given h. Equations (ii), (iv), and (v) are combined with Eq. (7.11):

$$d(\rho\pi r_T^2 h)/dt = 0 - (\pi r_o^2 \rho)(2gh)^{1/2} \quad \text{(vi)}$$

which rearranges to

$$\frac{dh}{h^{1/2}} = -\frac{r_o^2}{r_T^2}(2g)^{1/2}\,dt \quad \text{(vii)}$$

The following boundary conditions apply:

$$h(t = 0) = H$$
$$h\,(t = t_{total}) = 0 \quad \text{(viii)}$$

Equation (vii) is integrated with these boundary conditions to give the total time of emptying, t_{total}:

$$t_{total} = \frac{r_T^2}{r_o^2}\left(\frac{2H}{g}\right)^{1/2} \quad \text{(ix)}$$

Example 7.3 – Codable
Example 7.4 – Codable
Example 7.5 – Codable
Example 7.6 – Codable
Example 7.7 – Codable
Example 7.8 – Codable
Example 7.9 – Non Codable
Reason: No numerical Calculation

Example 7.9. Derive Torricelli's law, Eq. (i) of Example 7.1.

Answer. **Torricelli's** law related the velocity of discharge to the elevation of fluid in a tank, as depicted in Fig. 7.2. Here, the liquid is assumed incompressible. The friction loss is assumed to be negligible so that **Eq.** (7.62) can be used:

$$\frac{p_2 - p_1}{\rho} + g(z_2 - z_1) + \frac{1}{2}\left(\frac{U_{2,\,ave}^2}{\alpha_2} - \frac{U_{1,\,ave}^2}{\alpha_1}\right) = 0 \tag{7.62}$$

Following the nomenclature of Example 7.1, the **difference** $z_1 - z_2$ is h:

$$z_2 - z_1 = -h \tag{i}$$

Since the pressure on the top of the liquid is the same as the pressure of the **fluid** issuing from the drain, p_2 in the first term in Eq. (7.62) equals p_1, and that term is zero. Equation (7.13) relates the velocity exiting through the drain to the velocity with which the liquid level in the tank drops by equating the mass flow rate at point 1 with the mass flow rate that exits at the bottom at point 2. For constant density Eq. (7.13) becomes

$$U_{1,\,ave}S_1 = U_{2,\,ave}S_2 \tag{ii}$$

or in terms of diameters if both tank and drain are circular:

$$\frac{U_{1,\,ave}}{U_{2,\,ave}} = \left(\frac{d_2}{d_1}\right)^2 \tag{iii}$$

It is reasonable to assume that the diameter of the tank, d_1, is many times larger than the diameter of the drain, d_2, so that the ratio of diameters in Eq. (iii) is approximately zero. Then for a flat velocity profile ($\alpha_2 = 1$) the only non-zero terms left in Eq. (7.62) are

$$\tfrac{1}{2}U_{2,\,ave}^2 + g(-h) = 0 \tag{iv}$$

or

$$U_{2,\,ave}^2 = 2gh \tag{v}$$

This is Eq. (i) in Example 7.1. Note that for an outlet with a small diameter, the **flow** might be **laminar** and $\alpha = \frac{1}{2}$. Torricelli's law is significantly in error for this case.

Example 7.10 – Codable
Example 7.11 – Codable
Example 7.12 – Codable
Example 7.13- Codable
Example 7.14 – Codable
Example 7.15 – Codable
Example 7.16 – Codable
Example 7.17 – Non Codable
Reason: No numerical Calculation

Example 7.17. Obtain the equations for a pressure tap and Pitot[3] tube (Fig. 7.15). Summarize the assumptions **made.**[4] A pressure tap is made by drilling a hole through the tube wall with no burrs on the inside. Generally, some type of coupling is brazed over the tap so that connections can be made to a pressure measuring device. A Pitot tube is a hollow tube of small diameter inserted into the flow so the tip is exactly parallel to the flow axis. Lines from the pressure tap and the Pitot tube are connected to a pressure-measuring device such as a manometer, as shown in Fig. 7.15(*a*) and Fig. 7.15(*b*).

Answer. The pressure tap is always located at the same point in the flow as the end of the Pitot tube.. Figure 7.15(*b*) shows a side view of the Pitot tube. Figure 7.15(*c*) shows a Prandtl tube, which is similar to a Pitot tube. except the Pitot port and the pressure tap are located on a single assembly constructed of two

After the inventor, Henri de Pitot (16951771).

Film loops FM-33, FM-37, and FM-38 illustrate the various pressures that are important in this periment.

concentric tubes. In Figs. 7.15(*a*) and (*b*), two manometer legs are connected to the pressure tap and the end of the Fitot fube, so that a reading R is the pressure difference between the Fitot tube reading and the wall tap reading:

$$R = z_4 - z_1 = (z_4 - z_3) - (z_1 - z_2) \quad \text{(i)}$$

The Bernoulli balance of Eq. (7.62) or Eq. (7.64) applies:

$$\Delta p/\rho + g\,\Delta z = 0 \quad (764)$$

Considering the pressure tap first, point 2 is the pressure at the wall, which is also the static pressure across the entire cross section of the pipe. In the pressure leg from the tap to the top of the manometer leg, Eq. (7.64) becomes

$$p_2 = p_1 + \rho_f g(z_1 - z_2) \quad \text{(ii)}$$

where ρ_f is the density of the process fluid which also serves as the manometer fluid in this case.

For the Pitot tube, the pressure at the tip of the tube, p_3, is

$$p_3 = P4 + \rho_f g(z_4 - z_3) \quad \text{(iii)}$$

Since p_1 and p_4 are both equal to the pressure of the atmosphere, Eqs. (ii) and (iii) can be rearranged so these can be equated:

$$p_3 - p_2 = \rho_f g[(z_4 - z_3) - (z_1 - z_2)] = \rho_f g R \quad \text{(iv)}$$

Next, a Bernoulli balance is made in the fluid between the wall (zero velocity) and the center of the tube or the location of the Pitot tube. The Bernoulli equation is

$$\frac{p_2 - p_1}{\rho} + g(z_2 - z_1) + \frac{1}{2}\left(\frac{U^2_{2,\,ave}}{\alpha_2} - \frac{U^2_{1,\,ave}}{\alpha_1}\right) = 0 \quad (7.62)$$

Example 7.18 – Codable
Example 7.19 – Codable
Example 7.20 – Codable
Example 7.21 – Codable
Example 7.22 – Codable

Chapter 8 : METHODS OF ANALYSIS

Example 8.1 – Non Codable
Reason: No numerical Calculation

Example 8.1. The flow of fluid in a pipe has been studied experimentally, and it has been determined that the variables of importance are the following: velocity, pressure drop, density, viscosity, diameter, length, and roughness of the wall. Determine the necessary dimensionless numbers.

Answer. A tabulation of the variables is in Table 8.3. Each variable in Table 8.3 is raised to an exponent, as was done previously in Eq. (8.26). The product of these will be dimensionless if the exponents a through g are chosen according to the **Rayleigh** procedure. First, the following product is formed:

$$U^a p^b \rho^c \mu^d d_o^e L^f e^g = \text{constant} \tag{i}$$

Dimensions from Table 8.3 are substituted into Eq. (i), in a manner similar to that used in obtaining Eq. (8.28):

$$L^a\Theta^{-a}M^bL^{-b}\Theta^{-2b}M^cL^{-3c}M^dL^{-d}\Theta^{-d}L^eL^fL^g = \text{dimensionless} \tag{ii}$$

Equation (ii) will be dimensionless only if the sum of the power on any given dimension is zero:

$$L: \quad a - b - 3c - d + e + f + g = O \tag{iii}$$

$$M: \quad b + c + d = 0 \tag{iv}$$

$$\Theta: \quad -a - 2b - d = O \tag{v}$$

TABLE 8.3
Variables in pipe flow

Symbol	Exponent	Name	SI units	Dimensions
V	a	velocity	$m\ s^{-1}$	$L\Theta^{-1}$
p	b	pressure	$kg\ m^{-1}\ s^{-2}$ ($N\ m^{-2}$)	$ML^{-1}\Theta^{-2}$
ρ	c	density	$kg\ m^{-3}$	ML^{-3}
μ	d	viscosity	$kg\ m^{-1}\ s^{-1}$	$ML^{-1}\Theta^{-1}$
d_o	e	diameter	m	L
L	f	length	m	L
e	g	roughness	m	L

There are seven unknowns (a through g) and three equations, a fact which suggests that four groups are needed. Equations (iii) through (v) can be solved in terms of any four of the unknowns. Any of the unknown variables that is selected will appear only once. Let us select b, d, f, and g. Equation (v) can be solved for a:

$$a = -2b - d \tag{vi}$$

From Eq. (iv):

$$c = -b - d \tag{vii}$$

From Eq. (iii):

$$\begin{aligned} e &= -a + b + 3c + d - f - g \\ &= 2b + d + b - 3b - 3d + d - f - g \\ &= -d - f - g \end{aligned} \tag{viii}$$

Example 8.2 – Non Codable
Reason: No numerical Calculation

Example 8.2. The heat transfer coefficient h has been found to depend on the velocity, density, heat capacity, viscosity, thermal conductivity, and diameter of a rod in a specific experiment. Determine the necessary dimensionless numbers.

Answer. A tabulation of the variables is given in Table 8.4.

A common variation of the **Rayleigh** method will be illustrated here; this solution uses one less exponent than in Example 8.1. The units of h can be obtained from the defining equation:

$$(q/A)_w = h(\bar{T}_w - T_{ave}) \tag{6.86}$$

Equation (6.86) rearranges to the following:

$$h = \frac{(q/A)_w}{(\bar{T}_w - T_{ave})} \left(\frac{\mathrm{J\,m^{-2}\,s^{-1}}}{\mathrm{K}}\right) = \mathrm{J\,s^{-1}\,m^{-2}\,K^{-1}} \tag{i}$$

TABLE 8.4
Variables for the heat transfer coefficient

Symbol	Exponent	Name	SI Units	Dimensions
h	—	heat transfer coefficient	$\mathrm{J\,s^{-1}\,m^{-2}\,K^{-1}}$ or $\mathrm{kg\,s^{-3}\,K^{-1}}$	$M\Theta^{-3}T^{-1}$
V	a	velocity	$\mathrm{m\,s^{-1}}$	$L\Theta^{-1}$
ρ	b	density	$\mathrm{kg\,m^{-3}}$	ML^{-3}
c_p	c	heat capacity	$\mathrm{J\,kg^{-1}\,K^{-1}}$ or $\mathrm{m^2\,s^{-2}\,K^{-1}}$	$L^2\Theta^{-2}T^{-1}$
μ	d	viscosity	$\mathrm{kg\,m^{-1}\,s^{-1}}$	$ML^{-1}\Theta^{-1}$
k	e	thermal conductivity	$\mathrm{J\,s^{-1}\,m^{-1}\,K^{-1}}$ or $\mathrm{kg\,m\,s^{-3}\,K^{-1}}$	$ML\Theta^{-3}T^{-1}$
d_o	f	diameter	m	L

Now, one joule (J) is one kg $\mathrm{m^2\,s^{-2}}$; therefore the units of h are

$$h\,[=]\,\mathrm{J\,s^{-1}\,m^{-2}\,K^{-1}} = (\mathrm{kg\,m^2\,s^{-2}})(\mathrm{s^{-1}\,m^{-2}\,K^{-1}}) = \mathrm{kg\,s^{-3}\,K^{-1}} \tag{ii}$$

For heat capacity the units are

$$c_p\,[=]\,\mathrm{J\,kg^{-1}\,K^{-1}} = (\mathrm{kg\,m^2\,s^{-2}})(\mathrm{kg^{-1}\,K^{-1}}) = \mathrm{m^2\,s^{-2}\,K^{-1}} \tag{iii}$$

For thermal conductivity, the units are found from Fourier's law:

$$(q/A)_x = -k\frac{\partial T}{\partial x} \tag{iv}$$

The result is

$$k\,[=]\,(\mathrm{J\,m^{-2}\,s^{-1}})(\mathrm{m\,K^{-1}}) = \mathrm{kg\,m\,s^{-3}\,K^{-1}} \tag{v}$$

Now that the units have been established for all quantities in Table 8.4, the solution of this problem follows the development of Eq. (8.33)ff. The heat

Example 8.3 – Non Codable
Reason: No numerical Calculation

Example 8.3. Repeat Example 8.2 with the density and velocity combined into the mass average velocity, since in Eq. (xvii) of Example 8.2 the product ρU occurs, which is the mass average velocity G.

Answer. A tabulation of the variables is given in Table 8.5.

Here the original Rayleigh procedure is used. The equations are

$$h^a G^b c_p^c \mu^d k^e d_o^f = \text{constant} \quad \text{(i)}$$

TABLE 8.5
Variables for Example 8.3

Symbol	Exponent	Name	SI Units	Dimensions
h	a	heat transfer coefficient	J s^{-1} m^{-2} K^{-1} or kg s^{-3} K^{-1}	$M\Theta^{-3}T^{-1}$
G	b	mass average velocity	kg m^{-2} s^{-1}	$ML^{-2}\Theta^{-1}$
c_p	c	heat capacity	J kg^{-1} K^{-1} or m^2 s^{-2} K^{-1}	$L^2\Theta^{-2}T^{-1}$
μ	d	viscosity	kg m^{-1} s^{-1}	$ML^{-1}\Theta^{-1}$
k	e	thermal conductivity	J s^{-1} m^{-1} K^{-1} or kg m s^{-3} K^{-1}	$ML\Theta^{-3}T^{-1}$
d_o	f	diameter	m	L

and

$$M^a\Theta^{-3a}T^{-a}M^bL^{-2b}\Theta^{-b}L^{2c}\Theta^{-2c}T^{-c}M^dL^{-d}\Theta^{-d}M^eL^e\Theta^{-3e}T^{-e}L^f = \text{dimensionless} \quad \text{(ii)}$$

Therefore:

$$\text{M:} \quad a + b + d + e = 0 \quad \text{(iii)}$$

$$\Theta: \quad -3a - b - 2c - d - 3e = 0 \quad \text{(iv)}$$

$$L: \quad -2b + 2c - d + e + f = 0 \quad \text{(v)}$$

$$\text{T:} \quad -a - c - e = 0 \quad \text{(vi)}$$

The number of dimensionless groups of variables appears to be 6 − 4 or 2 by this procedure. Let us proceed on this basis, and solve in terms of a and c. Beginning with Eq. (vi), the exponent e is

$$e = -a - c \quad \text{(vii)}$$

From Eq. (iii):

$$b + d = -a - e = -a + a + c = c \quad \text{(viii)}$$

From Eq. (iv):

$$b + d = -3a - 2c - 3e = -3a - 2c + 3a + 3c = c \quad \text{(ix)}$$

Note that Eqs. (viii) and (ix) are identical and therefore so are Eqs. (iii) and (iv). These are clearly not independent, and one must be eliminated in the analysis.

Example 8.4 – Non Codable
Reason: No numerical Calculation

Example 8.4. Repeat Example 8.1 with G replacing ρ and U.

Answer. A tabulation of the variables is given in Table 8.6.

TABLE 8.6
Variables for Example 8.4

Symbol	Exponent	Name	SI units	Dimensions
G	a	mass average velocity	kg m^{-2} s^{-1}	$ML^{-2}\Theta^{-1}$
P	b	pressure	kg m^{-1} s^{-2} (N m^{-2})	$ML^{-1}\Theta^{-2}$
μ	c	viscosity	kg m^{-1} s^{-1}	$ML^{-1}\Theta^{-1}$
d_o	d	diameter	m	L
L	e	length	m	L
e	f	roughness	m	L

The same procedure will be followed as in Example 8.1:

$$\text{L:} \quad -2a - b - c + d + e + f = 0 \tag{i}$$

$$\text{M:} \quad a + b + c = 0 \tag{ii}$$

$$\Theta\text{:} \quad -a - 2b - c = 0 \tag{iii}$$

From Eq. (ii):

$$a + c = -b \tag{iv}$$

From Eq. (iii):

$$a + c = -2b \tag{v}$$

Equations (iv) and (v) are clearly inconsistent. The correct conclusion is that the required dimensionless numbers cannot be obtained.

Example 8.5 – Non Codable
Reason: No numerical Calculation

Example 8.5. Solve Example 8.2 by the Buckingham method.

Answer. A tabulation of the variables and units is given in Table 8.4 in Example 8.2 and will not be repeated here. In this problem

$$h = f(U,\ \rho,\ c_p,\ \mu,\ k,\ d_o) \qquad \text{(i)}$$

In Table 8.4, four dimensions are noted, L, M, Θ, and T, which suggest four possible equations. Therefore j is 4, n is 7, and from Eq. (8.36) **k** is 3. Thus, the pi theorem states that three dimensionless groups, Π_1, Π_2, and Π_3 are required. Equation (8.38) becomes

$$f(\Pi_1, \Pi_2, \Pi_3) = 0 \qquad \text{(ii)}$$

if all four possible equations are independent.

The next step in the Buckingham method is to examine Table 8.4 to locate j variables, which when taken together contain all the fundamental dimensions. Sometimes there are more than j variables available, in which case the selection of those to appear in each dimensionless group is arbitrary. From Table 8.4, d_o, μ, ρ and k are selected to appear in all Π_i. Now according to Eq. (8.40), each Π_i is a combination of these variables plus one other from the remaining three (h,

c_p, and U); i.e.

$$\Pi_1 = d_o^a \mu^b \rho^c k^d h^e \qquad \text{(iii)}$$

$$\Pi_2 = d_o^a \mu^b \rho^c k^d c_p^e \qquad \text{(iv)}$$

$$\Pi_3 = d_o^a \mu^b \rho^c k^d U^e \qquad \text{(v)}$$

At this point the solution proceeds like the **Rayleigh** method described previously. Dimensions from Table 8.4 are substituted into each of Eq. (iii) through Eq. (v). For II,, Eq. (iii) becomes

$$L^a M^b L^{-b} \Theta^{-b} M^c L^{-3c} M^d L^d \Theta^{-3d} T^{-d} M^e \Theta^{-3e} T^{-e} = \text{dimensionless} \qquad \text{(vi)}$$

The equations are

$$\text{L:} \quad 0 = a - b - 3c + d \qquad \text{(vii)}$$

$$\Theta: \quad 0 = -b - 3d - 3e \qquad \text{(viii)}$$

$$\text{M:} \quad 0 = b + c + d + e \qquad \text{(ix)}$$

$$\text{T:} \quad 0 = -d - e \qquad \text{(x)}$$

These equations can be solved in terms of e. From Eq. (x)

$$d = -e \qquad \text{(xi)}$$

From Eq. (viii):

$$b = -3d - 3e = 3e - 3e = 0 \qquad \text{(xii)}$$

Example 8.6 – Non Codable
Reason: No numerical Calculation

Example 8.6. The important variables in a two-phase gas-liquid system were found to be velocity, gravity, length, and system properties: density, viscosity, and surface tension. Suggest a set of dimensionless numbers and test the set for completeness.

Answer. The variables are given in Table 8.7. There are six unknowns and three dimensions. So by the pi theorem, Eq. **(8.36),** three groups are required. Either by dimensional analysis or simply by inspection, three groups that can be used are

$$
\begin{aligned}
N_{Re} &= LU\rho/\mu \\
N_{Fr} &= U^2/(Lg) \\
N_{We} &= U^2L\rho\,/\,\sigma
\end{aligned}
\qquad \text{(i)}
$$

These three dimensionless numbers contain all the variables. These numbers may also be expressed as a ratio of forces. **The** following forces may be **defined [L1]:**

TABLE 8.7
Variables for two-phase flow problem

Symbol	Exponent	Name	SI units	Dimensions
U	a	velocity	$m\,s^{-1}$	$L\Theta^{-1}$
g	b	gravity	$m\,s^{-2}$	$L\Theta^{-2}$
L	c	length	m	L
ρ	d	density	$kg\,m^{-3}$	ML^{-3}
μ	e	viscosity	$kg\,m^{-1}\,s^{-1}$	$ML^{-1}\Theta^{-1}$
σ	f	surface tension	$kg\,s^{-2}$	$M\Theta^{-2}$

Viscous forces: $F_\mu = \mu LU\ [(kg\ m^{-1}\ s^{-1})(m)(m\ s^{-1})]$ (ii)

Inertial forces: $F_\rho = \rho U^2 L^2\ [(kg\ m^{-3})(m^2\ s^{-2})(m^2)]$ (iii)

Surface forces: $F_\sigma = \sigma L\ [(kg\ s^{-2})(m)]$ **(iv)**

Gravitational forces: $F_g = \rho g L^3\ [(kg\ m^{-3})(m\ s^{-2})(m^3)]$ **(v)**

Each dimensionless group in Eq. (i) represents a ratio of a pair of the forces in the above equations:

$$N_{Re} = \frac{F_\rho}{F_\mu} = \frac{\rho U^2 L^2}{\mu LU} = \frac{LU\rho}{\mu} \qquad \textbf{(vi)}$$

$$N_{Fr} = \frac{F_\rho}{F_g} = \frac{\rho U^2 L^2}{\rho g L^3} = \frac{U^2}{Lg} \qquad \text{(vii)}$$

$$N_{We} = \frac{F_\rho}{F_\sigma} = \frac{\rho U^2 L^2}{\sigma L} = \frac{U^2 L\rho}{\sigma} \qquad \text{(viii)}$$

Example 8.7 – Non Codable
Reason: No numerical Calculation

Example 8.7. An investigator has proposed the following dimensionless number for an application in two phase flow (e.g., flow of suspended solids in water in a pipe):

$$N_{Re} = LU\rho/\mu$$
$$N_{Fr} = U^2/(Lg) \tag{i}$$
$$N_{We} = U^2L\rho/\sigma$$
$$N_{property} = (\rho\sigma^3)/(g\mu^4)$$

Determine whether these numbers are all independent.

Answer. Note that $N_{property}$ contains only variables that are already included in the other three groups. Hence, a figure such as Fig. **8.1(*b*)** will show that the four numbers N_{Re}, N_{Fr}, N_{We}, and $N_{property}$ lie in the same plane, and hence these are not independent.

Another solution is to find the exponents on the following equation:

$$N_{property} = N_{Re}^a N_{Fr}^b N_{We}^c \tag{ii}$$

The exponents in Eq. (ii) are easily found by inserting the correct variables from Eq. (i):

$$\rho\sigma^3 g^{-1}\mu^{-4} = (LU\rho\mu^{-1})^a(U^2L^{-1}g^{-1})^b(U^2L\rho\sigma^{-1}) \tag{iii}$$

If the above equation is to be valid, then there must be unique values for a, b, and c. From the exponents on σ:

$$3 = -c \tag{iv}$$

or

$$c = -3 \tag{v}$$

From the exponents on p:

$$1 = a + c \tag{vi}$$

or

$$a = 4 \tag{vii}$$

From the exponents on g:

$$-1 = -b \tag{viii}$$

or

$$b = 1 \tag{ix}$$

Therefore, $N_{property}$ in terms of the other three numbers is:

$$N_{property} = (N_{Re})^4(N_{Fr})(N_{We})^{-3} \tag{x}$$

Clearly, the four numbers cannot be independent, since $N_{property}$ is an exact function of the other three, as shown by Eq. (x).

Example 8.8 – Non Codable
Reason: No numerical Calculation

Example 8.8. A new pipe material is to be used in some new plant construction, but there is not enough information available to allow an exact design because of the noncircular cross sectional area of the pipe. Assume the pipe is smooth. Consider modeling if a sample is available that is **1/10** the size considered for the plant.

Answer. From the results of Example 8.1, if the pressure drop or Euler number is to remain the same, one must maintain as constant the following three dimensionless groups: the Reynolds number, the length to diameter ratio L/d_o, and the relative roughness e/d_o. Thus, the test model should have the same values for these three dimensionless numbers as the full-scale model. If the pipe is smooth, the roughness e is essentially zero; therefore, e/d_o remains constant (zero) for any diameter. The modeling will henceforth be based on the premise of keeping constant the Reynolds number and the ratio L/d_o. If d_o is to be reduced by 1/10, then L must be likewise reduced to maintain the ratio L/d_o constant. The Reynolds number is

$$N_{Re} = d_o U\rho/\mu \qquad (6.2)$$

The Reynolds number will be constant when d_o is reduced by **1/10** if any of the following changes take place:

(a) increase in U by a factor of 10
(b) increase in ρ by a factor of 10
(c) decrease in μ by a factor of 10

Of course, a combination of these changes is also feasible. Usually it is easier to increase U rather than to use a different fluid.

Chapter 9 : Agitation

Example 9.1 – Non Codable

Reason: No numerical Calculation

Example 9.1. From a survey of the literature on agitation, it has been established that the important variables are the rotational speed (N), impeller diameter (D), tank diameter (T), power input (P), and fluid properties of density (p) and viscosity (μ). Although other variables may be important, perform a dimensional analysis for these six variables.

Answer. Table 9.4 summarizes the six variables. Here the original **Rayleigh** procedure is used. The equations are

$$N^aD^bT^cP^d\rho^e\mu^f = \text{constant} \tag{i}$$

and

$$\Theta^{-a}L^bL^cM^dL^{2d}\Theta^{-3d}M^eL^{-3e}M^fL^{-f}\ \Theta^{-f} = \text{dimensionless} \tag{ii}$$

The three equations are

$$L: \quad b+c+2d-3e-f=0 \tag{iii}$$

$$M: \quad d+e+f=0 \tag{iv}$$

$$\Theta: \quad -a-3d-f=0 \tag{v}$$

TABLE 9.4
Variables for agitation, Example 9.1

Symbol	Exponent	Name	SI units	Dimensions
N	a	rotational speed	s^{-1}	Θ^{-1}
D	b	impeller diameter	m	L
T	c	tank diameter	m	L
P	d	power	$kg\,m^2\,s^{-3}$	$ML^2\Theta^{-3}$
ρ	e	density	$kg\,m^{-3}$	ML^{-3}
μ	f	viscosity	$kg\,m^{-1}\,s^{-1}$	$ML^{-1}\Theta^{-1}$

There are six unknowns and three equations. Thus, the number of dimensionless groups of variables is 6 − 3, or 3. Examining Eqs. (iii) through (v), the variables a and **b** or c appear once. Experience tells us that **b** and c cannot both be eliminated because Eq. (iii) can be used for one or the other but not both; so **b** is arbitrarily chosen. The next best candidate for elimination is e. The three variables to be eliminated (a, **b** and e) can be expressed in terms of c, ***d***, and f by using the three equations above:

$$a=-3d-f \tag{vi}$$

$$e=-d-f \tag{vii}$$

$$b=-c-2d+3e+f=-c-2d-3d-3f+f=-c-5d-2f \tag{viii}$$

Using all these results in Eq. (i) gives

$$N^{-3d-f}D^{-c-5d-2f}T^cP^d\rho^{-d-f}\mu^f = \text{constant} \tag{ix}$$

Example 9.2 – Non Codable
Reason: No numerical Calculation

Example 9.2. Uhl [U1] correlated heat transfer data in agitated, jacketed vessels with the variables listed in Table 9.5. Develop the dimensionless groups suitable for a correlation.

Answer. The exponents in Table 9.5 were chosen to avoid confusion as much as possible. A complete explanation of the dimensions of k, c_p, and h is given in Example 8.2. The Rayleigh procedure as explained in Chapter 8 will be used; the

TABLE 9.5
Variables for beat transfer in agitated vessels, Example 9.2

Symbol	Exponent	Name	SI units	Dimensions
ρ	d	density	kg m^{-3}	ML^{-3}
μ	e	viscosity	kg $m^{-1} s^{-1}$	$ML^{-1}\Theta^{-1}$
μ_w	f	wall viscosity	kg $m^{-1} s^{-1}$	$ML^{-1}\Theta^{-1}$
k	g	thermal conductivity	kg m $s^{-3} K^{-1}$	$ML\Theta^{-3}T^{-1}$
c_p	i	heat capacity	$m^2 s^{-2} K^{-1}$	$L^2\Theta^{-2}T^{-1}$
h	j	heat transfer coefficient	kg $s^{-3} K^{-1}$	$M\Theta^{-3}T^{-1}$
N	m	rotational speed	s^{-1}	Θ^{-1}
D	n	impeller diameter	m	L
T	p	tank diameter	m	L

equations are

$$\rho^d \mu^e \mu_w^f k^g c_p^i h^j N^m D^n T^p = \text{constant} \quad \text{(i}$$

and

$$M^d L^{-3d} M^e L^{-e} \Theta^{-e} M^f L^{-f} \Theta^{-f} M^g L^g \Theta^{-3g} T^{-g} L^{2i} \Theta^{-2i} T^{-i} M^j \Theta^{-3j} T^{-j} \Theta^{-m} L^n L^p$$

$$= \text{dimensionless (ii}$$

The four equations representing each dimension are

$$\Theta: \quad -e-f-3g-2i-3j-m=0 \quad \text{(iii}$$

$$L: \quad -3d-e-f+g+2i+n+p=0 \quad \text{(iv}$$

$$M: \quad d+e+f+g+j=0 \quad \text{(v}$$

$$T: \quad -g-i-j=0 \quad \text{(vi}$$

There are nine unknowns and four equations. Thus, five groups will be produced. In the above four equations, either n or p may be eliminated, and n is choser arbitrarily. Also, m from Eq. (iii) and g from Eq. (vi) are easy to eliminate:

Example 9.3 – Codable
Example 9.4 – Codable
Example 9.5 – Codable

Chapter 10: Fluid flow in Ducts

Example 10.1 – Codable
Example 10.2 – Codable
Example 10.3 – Codable
Example 10.4 – Codable
Example 10.5 – Codable
Example 10.6 – Codable
Example 10.7 – Codable
Example 10.8 – Codable
Example 10.9 – Codable
Example 10.10 – Non Codable
Reason: No numerical Calculation

Example 10.10. Find the flow rate in Example 10.9 if the three sections of pipe are connected in series as shown in Fig. 10.16(b).

Answer. In this example, the flow rate in each section is the same:

$$w = w_1 = w_2 = w_3 = \rho U_1 S_1 = \rho U_2 S_2 = \rho U_3 S_3 \qquad \text{(i)}$$

The areas of the pipes are

$$S_1 = \pi d_1^2/4 = \pi(0.04)^2/4 = 0.001257 \text{ m}^2$$
$$S_2 = 0.002\ 827 \text{ m}^2 \qquad \text{(ii)}$$
$$S_3 = 0.005\ 027 \text{ m}^2$$

The total pressure drop $(p_a - p_b)$ is the sum of the pressure drops per section:

$$-\Delta p = p_a - p_b = 1.47 \text{ x } 16 \text{ kPa} = (p_a - p_c) + (p_c - p_d) + (p_d - p_b)$$
$$(-\Delta p_1) + (-\Delta p_2) + (-\Delta p_3) = 1.47 \times 10^5 \text{ kPa} \qquad \text{(iii)}$$

The pressure drop in each section is expressible in terms of the equivalent length L, velocity, density, diameter, and friction factor by any of Eqs. (10.3), (10.14), or (10.36):

$$-\frac{\Delta p}{\rho} = 4f \frac{L}{d_0} \frac{U_{z,\text{ ave}}^2}{2} \qquad (10.14)$$

Equation (10.14) is applied to each section, with the average velocity replaced by the mass flow rate w from Eq. (i):

$$-\Delta p_1 = [(4f_1)(L_1/d_1)][w^2/(2\rho S_1^2)]$$
$$= [(4)(f_1)(50/0.04)]\{(w_1^2)/[(2)(1000)(0.001\ 257)^2]\}\left(\frac{(\text{m})(\text{kg}^2\ \text{s}^{-2})}{(\text{m})(\text{kg m}^{-3})(\text{m}^2)}\right)$$
$$= 1.583 \times 10^6 (f_1 w^2) \qquad \text{(iv)}$$
$$-\Delta p_2 = [(4f_2)(L_2/d_2)][w^2/(2\rho S_2^2)] = 6.254 \times 10^5 (f_2 w^2) \qquad \text{(v)}$$
$$-\Delta p_3 = [(4f_3)(L_3/d_3)][w^2/(2\rho S_3^2)] = 9.895 \times 10^4 (f_3 w^2) \qquad \text{(vi)}$$

Equations (iii) through (vi) constitute four equations in four unknowns, i.e., w, Δp_1, Δp_2, and Δp_3. To solve these nonlinear equations in their present form is a sophisticated task. However, a clever choice of the order of calculation yields a one-dimensional root-finding problem that can be solved by hand calculation or by a computer program.

The friction factors are functions of the Reynolds numbers [Eq. (6.2)], which may also be expressed in terms of w:

$$N_{Re,\ I} = d_1 U_1 \rho/\mu = (d_1 w)/(S_1 \mu) = [(0.04)(w)]/[(0.001\ 257)(0.001)]$$
$$= 3.183 \times 10^4 w \qquad \text{(vii)}$$
$$N_{Re,\ 2} = 2.122 \times 10^4 w \qquad \text{(viii)}$$
$$N_{Re,\ 3} = 1.592 \times 10^4 w \qquad \text{(ix)}$$

Equations (iii) through (vi) can be combined as follows:

$$(-\Delta p_1) + (-\Delta p_2) + (-\Delta p_3)$$
$$= (w^2)[(1.583 \times 10^6)(f_1) + (6.254 \times 10^5)(f_2) + (9.895 \text{ x lo''})(\&)] = 1.47 \times 10^5 \qquad \text{(x)}$$

Thus, the final answer to this problem will be the value of w that yields friction factors from Eq. (6.132) or Fig. 10.3, all of which satisfy Eq. (x). The method of successive substitution works well for this problem because the friction factors vary slowly with small changes in w (i.e., Reynolds number). The solution proceeds as follows:

1. Obtain a good initial guess for w. Let 0.0055 be the estimated value off, as was used in deriving Eq. (10.18) in the velocity head approximation. Then Eq. (x) is solved for the initial guess:

$$(0.0055w^2)[1.583 \times 10^6 + 6.254 \times 10^5 + 9.895 \times 10^4] = 1.47 \times 10^5 \qquad \text{(xi)}$$

   From this equation, the value of w is 3.40 kg s^{-1}.
2. For the value of w at hand, compute the three Reynolds numbers from Eqs. (vii), (viii), and (ix).
3. From Eq. (6.132) or from Fig. 10.3 find the friction factor for each of the three sections.

TABLE 10.7
Summary of calculations for Example10.10

Trial	Quantity	Fig. or Eq. Number	Value
0	w	Eq. (x)	3.40 kg s^{-1}
1	$N_{Re,1}$	Eq. (vii)	1.082 x 16
	$N_{Re,2}$	Eq. (viii)	7.215 x 10^4
	$N_{Re,3}$	Eq. (ix)	5.411 x 10^4
	f_1	Fig. 10.3	0.00445
	f_2	Fig. 10.3	0.00475
	f_3	Fig. 10.3	0.00513
	w	Eq. (x)	3.74 kg s^{-1}
	$N_{Re,1}$	Eq. (vii)	1.190 x 10^5
	$N_{Re,2}$	Eq. (viii)	7.937 x 10^4
	$N_{Re,3}$	Eq. (ix)	5.952 x 10^4
	f_1	Fig. 10.3	0.00440
	f_2	Fig. 10.3	0.00470
	f_3	Fig. 10.3	0.00500
	w	Eq. (x)	3.76 kg s^{-1}
3	$N_{Re,1}$	Eq. (vii)	1.197 x 16
	$N_{Re,2}$	Eq. (viii)	7.980 x 10^4
	$N_{Re,3}$	Eq. (ix)	5.984 x 10^4
	f_1	Fig. 10.3	0.00440
	f_2	Fig. 10.3	0.00470
	f_3	Fig. 10.3	0.00500
	w	Eq. (x)	3.76 kg s^{-1}

4. substitute the values of the friction factors into Eq. (x) and solve for the new estimate of w.
5. Test the new w for convergence. An appropriate test is

$$\left|\frac{w_{new} - w}{w}\right| < \text{EPS} \tag{xii}$$

where EPS is an appropriate tolerance, commonly 0.5×10^{-5}.

6. If there is no convergence, then loop to step 2. If there is convergence, then calculate the flow rate.

The calculations using this procedure are given in Table 10.7. The pressure drops and velocities can be computed from the equations given previously if desired. The flow rate at convergence is 3.76 kg s^{-1}, which is close to the initial guess. Naturally the parallel configuration of the previous example yielded a much higher flow rate for the same pressure drop.

The design equations presented so far in this text are by no means complete, in that many more equations are available for a wide variety of specific systems. For example, in Chapter 12 flow over immersed objects will be covered. The engineer often **must design systems that involve more than** just the material presented thus far. **Thus, recourse to more specific references will often be necessary.**

Figure 10.17 is an example of a complex process flow system for which it is desired to determine the overall pressure drop so that the type and size of pump and motor can be selected.

Example 10.11 – Codable
Example 10.12 – Codable
Example 10.13 – Non Codable
Reason: No numerical Calculation

Example 10.13. Determine the hydraulic radius for the following conduits: (a) square running full; (b) equilateral triangle running full; (c) equilateral triangle resting on its base and filled with running water to a depth of $\frac{1}{2}$ the height.

Answer. Equation (10.45) will be applied to each part:

$$r_H = S/L_p \qquad (10.45)$$

Put (a). For a square, Eq. (10.45) reduces to Eq. (10.47) with $L_1 = L_2$, or

$$r_H = L/4 \qquad \text{(i)}$$

where L is the length of one side.

Part (b). Since all angles of an equilateral triangle are 60°, the height is computed from

$$\sin\ 60° = h/L = (3)^{1/2}/2 \qquad \text{(ii)}$$

Equation (ii) may be solved for h in terms of L, the length of a side:

$$h = L(3)^{1/2}/2 \qquad \textit{(iii)}$$

The flow area is the area of the triangle, $\frac{1}{2}h$ times L:

$$S = [\tfrac{1}{2}(L)(3)^{1/2}/2](L) = L^2(3)^{1/2}/4 \qquad \text{(iv)}$$

The wetted perimeter of a completely filled equilateral triangle is simply $3L$:

$$r_H = S/L_p = [(L^2)(3)^{1/2}/4]/(3L) = L(3)^{1/2}/12 = L/[(4)(3)^{1/2}] \qquad \text{(v)}$$

Part (c). Determination of S and L_p for a triangular duct tilled to half the height is a simple problem in geometry. The flowing water actually occupies a **trapezoidal-**shaped area, whereas the vapor space above the water is triangular. A simple procedure is to subtract the vapor area from the total area, since the vapor area is triangular. The solution offered here will use the concept of similar triangles. The top of the water must bisect each of the top sides of the duct. Hence the wetted perimeter is the base plus the contributions from each side:

$$L_p = \tfrac{1}{2}L + L + \tfrac{1}{2}L = 2L \qquad \text{(vi)}$$

Example 10.14 – Codable
Example 10.15 – Codable
Example 10.16 – Codable
Example 10.17 – Codable

Chapter 11 : Heat and mass Transfer in Duct Flow
Example 11.1 - Codable
Example 11.2 - Codable
Example 11.3 - Codable
Example 11.4 - Non Codable
Reason: No numerical Calculation

Example 11.4. Prove that it is possible to add a layer of material to an arbitrary cylinder of radius r_2 and length L and thereby increase the amount of heat lost. The "critical" thickness is the radius r_o where the heat transfer rate is a maximum. If the convection coefficient h_o is 7.0 W m^{-2} K^{-1}, determine the critical radius for 85 percent magnesia (k = 0.07 W m^{-1} K^{-1}) and for steel (k = 45 W m^{-1} K^{-1}).

Answer. Let us assume that Fig. 11.5 applies and that the temperature of the pipe-insulation interface T_2 is constant. A heat balance now contains only two resistances: the insulation resistance and the convection resistance between the outside area of the insulation A_o and the surrounding fluid. Under this restriction, Eq. (11.28) reduces to

$$\sum R = \frac{\ln(r_o/r_2)}{2\pi L k} + \frac{1}{h_o A_o} = \frac{\ln(r_o/r_2)}{2\pi L k} + \frac{1}{h_o(2\pi L r_o)} \tag{i}$$

where k is the thermal conductivity of the insulation and r_o is the outside radius. The heat flow from Eq. (11.23) and Eq. (i) is

$$-q_r = \frac{\Delta T}{\sum R} = \frac{\Delta T}{\frac{\ln(r_o/r_2)}{2\pi L k} + \frac{1}{h_o(2\pi L r_o)}} = 2\pi L(\Delta T)\left(\frac{\ln(r_o/r_2)}{k} + \frac{1}{h_o r_o}\right)^{-1} \tag{ii}$$

where AT is the driving force for heat transfer in the radial direction (the temperature of the fluid surrounding the outside surface of the insulation minus the temperature at r_2).

The next step is to form the derivative dq_r/dr_o and set that equal to zero in order to determine whether a maximum or a minimum exists:

$$\frac{dq_r}{dr_o} = 0 = (-1)(-2\pi L)(\Delta T)\left(\frac{\ln(r_o/r_2)}{k} + \frac{1}{h_o r_o}\right)^{-2}\left(\frac{1}{r_o k} - \frac{1}{h_o r_o^2}\right) \tag{iii}$$

Equation (iii) is zero only if the following term is zero:

$$\frac{1}{r_o k} - \frac{1}{h_o r_o^2} = 0 \tag{iv}$$

Equation (iv) can be solved for the critical radius:

$$(r_o)_{\text{critical}} = \frac{k}{h_o} \tag{v}$$

Example 11.5 - Codable
Example 11.6 - Codable
Example 11.7 - Codable

Example 11.8 - Codable
Example 11.9 - Codable
Example 11.10 - Codable

Chapter 12 : Transport past Immersed Bodies
Example 12.1 – Non Codable
Reason: No numerical Calculation

Example 12.1 Set-up the boundary layer solution of Eqs. (12.11) and (12.12) for a digital computer.

Answer. Rather than a series solution (which involves finding many terms), a digital simulation that utilizes integration by numerical means will be performed. First, Eq. (12.11) is rewritten as

$$f''' + \tfrac{1}{2}ff'' = 0 \tag{i}$$

This problem is classified as a boundary value problem, since from Eq. (12.12)

$$f'(\eta = \infty) = 1 \tag{ii}$$

Hence, it will be necessary to guess values of f"(O) until Eq. (ii) is satisfied. Equation (i) is decomposed into a coupled set of differential equations:

$$z_1 = \frac{df}{d\eta} = f' \qquad f(0) = 0 \tag{iii}$$

$$z_2 = \frac{d^2f}{d\eta^2} = f'' \qquad f'(0) = 0 \tag{iv}$$

$$f''' = \frac{d^3f}{d\eta^3} = -(f/2)z_2 \qquad f''(0) = C_1 \text{ (v)}$$

where the constant C_1 is varied until Eq. (ii) is satisfied.' Since infinity is not a viable option, the value of η must also be varied until the boundary condition of Eq. (ii) is satisfied. A value of η equal to 6 is satisfactory.

The solution to Eqs. (iii) through (v) in FORTRAN or BASIC using Runge-Kutta or some other method [P1, R1] is fairly involved, since the step size, the boundary condition $\eta = \infty$, and the constant C, must all be located properly by trial and error. Since systems of ordinary differential equations are commonly encountered, several excellent simulation languages, including CSMP[2] and ACSL,[3] have been developed. These are easy to use, and the reader should consult the appropriate manuals for detailed instructions.

Figure 12.2 illustrates the solution of Eqs. (iii) through (v) with ACSL. The ACSL program is 13 lines long, whereas an analogous FORTRAN program would be quite lengthy. First, application of algebra and calculus to the appropriate definitions yields a useful pair of equations:

$$U_x/U_\infty = f' \tag{vi}$$

$$(U_y/U_\infty)(U_\infty x/\nu)^{1/2} = \tfrac{1}{2}(\eta f' - f) \tag{vii}$$

In the ACSL program, line 2 defines constants that are the boundary conditions, Eq. (iii)--FIC; Eq. (iv)-FDIC; Eq. (v)—FDDIC; and the condition for $\eta = \infty$, which is ETAT. Line 4 contains ETA, or η, since ACSL always uses time as the independent variable. Line 5 solves Eq. (i) for f'''. Then lines 6, 7 and 8 integrate (by Runge-Kutta) the system of equations in Eqs. (iii) through (v). Lines 9 and 10 compute the quantities in Eqs. (vi) and (vii). Line 11 terminates the integration when η exceeds 6.

Example 12.2 – Codable
Example 12.3 – Codable
Example 12.4 – Non Codable
Reason: No numerical Calculation

Example 12.4. Calculate the entry lengths in pipe flow for 99percent develop ment of the velocity profile at turbulent Reynolds numbers of 2100, 4000, lo', and 16. Compare these to the laminar length that might be obtained under very

TABLE U.3
Entry lengths for Example **12.4**

	L_e/d_o	
Reynolds number $N_{Re,x}$	**Laminar** $0.0567N_{Re}$	**Turbulent** $0.693(N_{Re})^{1/4}$
2100	119	4.1
4000	227	5.5
10^4	567	6.9
16	5670	12.3

unusual conditions where no instabilities exist to trigger the transition to turbulence.

Answer. At Reynolds numbers of 2100 and 4000, the flow in most equipment is really neither fully laminar nor fully turbulent. Therefore, the correct entry length is open to question. At Reynolds numbers of 10^4 and 10^5, the flow is almost always fully turbulent, and Eq. (12.26) applies.

The results from Eq. (12.19) for laminar entry and from Eq. (12.26) for turbulent entry are compared in Table 12.3. The entry lengths for turbulent flow are quite small.

Example 12.5 – Codable
Example 12.6 – Non Codable
Reason: No numerical Calculation

Example 12.6. A simple velocity potential known to be analytic is

$$\phi = U_\infty x \tag{i}$$

Determine the flow so described and the equation for the stream function. Prepare a graph of U_x and U_y as functions of x and y.

Answer. Since ϕ is an analytic function, the potential in Eq. (i) must obey Laplace's equation. Using Eq. (12.57), it is seen that $\nabla^2 \phi$ is zero, and the flow

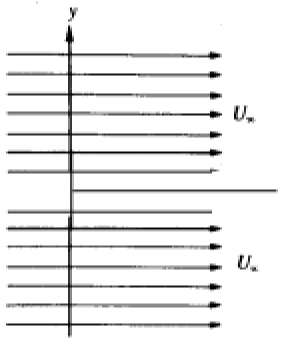

FIGURE 12.10
Uniform ideal flow in the x direction for $\phi = U_\infty x$.

therefore is incompressible by Eq. (12.55). As previously shown, for the same boundary conditions this incompressible, potential flow will be equivalent to an ideal (nonviscous), irrotational flow.

Equation (12.53), which defines the velocity potential, is used to evaluate U_x and U_y:

$$U_x = \frac{\partial \phi}{\partial x} \qquad U_y = \frac{\partial \phi}{\partial y} \qquad U_z = \frac{\partial \phi}{\partial z} \tag{12.53}$$

The derivatives of Eq. (i) yield

$$U_x = \frac{\partial \phi}{\partial x} = U_\infty \qquad U_y = \frac{\partial \phi}{\partial y} = 0 \qquad U_z = 0 \tag{ii}$$

This flow, shown in Fig. 12.10, is simply a flow in the x direction with the x velocity U_x equal to U_∞ for all values of y and z, since U_y and U_z are zero. The streamlines can be obtained from Eq. (12.60):

$$U_x = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \qquad U_y = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} \tag{12.60}$$

Each equation can be integrated in order to solve for the stream function ψ:

$$\psi = \int U_x\, dy = U_\infty y + C(x) \qquad \text{from } U_x = U_\infty = \partial\psi/\partial y \tag{iii}$$

$$\psi = C(y) \qquad \text{from } U_y = 0 = \partial\psi/\partial x \tag{iv}$$

Example 12.7 – Non Codable
Reason: No numerical Calculation

Example 12.7. Repeat Example 12.6 for the analytic velocity potential:

$$\phi = U_\infty(x^2 - y') \tag{i}$$

Answer. Since ϕ is an analytic function, the potential in Eq. (i) must obey Laplace's equation, Eq. (12.57):

$$\nabla^2\phi = 0 = 2U_\infty - 2U_\infty \tag{ii}$$

The flow therefore is incompressible by Eq. (12.55). As previously shown, for the same boundary conditions this incompressible, potential flow will be equivalent to an ideal (nonviscous), irrotational flow. From Eq. (12.53) and the derivative of Eq. (i):

$$U_x = \frac{\partial\phi}{\partial x} = 2U_\infty x, \qquad U_y = \frac{\partial\phi}{\partial y} = -2U_\infty y \qquad U_z = 0 \tag{iii}$$

The streamlines of flow can be obtained by integration with the Cauchy-Riemann conditions, Eq. (12.60):

$$\begin{aligned} \psi &= 2U_\infty xy + C(x) \qquad \text{from } 2U_\infty x = \partial\psi/\partial y \\ \psi &= 2U_\infty xy + C(y) \qquad \text{from } -2U_\infty y = -\partial\psi/\partial x \end{aligned} \tag{iv}$$

For these to be true:

$$\psi = 2U_\infty xy \tag{v}$$

The streamlines and potentials are shown in Fig. 12.11 for a negative U_∞ and a positive x. Both positive and negative values of ψ are shqwn. The flow can be pictured as occurring around the inside of a corner (upper quarter) or against a flat plate (entire picture). The actual velocities are obtained from Eq. (iii).

Example 12.8 – Non Codable
Reason: No numerical Calculation

Example **12.8.** Repeat Example 12.6 for the following analytic velocity potential in polar (cylindrical) coordinates.

$$\phi = U_\infty\left(r + \frac{1}{r}\right)(\cos\theta) \qquad \text{(i)}$$

$\phi = U_\infty(x^2 - y^2)$

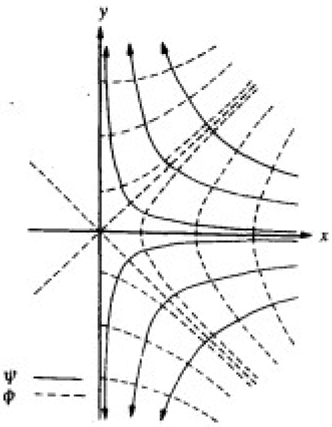

FIGURE **12.11**
Potential flow around the inside of a corner or against a plate.

Answer. The **Laplace** equation in cylindrical coordinates is given in Table 5.1:

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} = 0 \qquad \text{(ii)}$$

If the potential in Eq. (i) satisfies Eq. (ii), the **flow** is incompressible. Combining **Eqs.** (i) and (ii):

$$U_\infty(\cos\theta)(2r^{-3}) + (U_\infty/r)(\cos\theta)(1 - r^{-2}) + (U_\infty/r^2)[r + (1/r)](-\cos\theta) = 0 \qquad \text{(iii)}$$

Equation (iii) can be simplified:

$$U_\infty(\cos\theta)(2r^{-3} + r^{-1} - r^{-3} - r^{-1} - r^{-3}) = U_\infty(\cos\theta)(0) = 0 \qquad \text{(iv)}$$

Thus, Laplace's equation is satisfied. For polar coordinates, the velocities are given by

$$U_r = \frac{\partial\phi}{\partial r} = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = U_\infty\left(1 - \frac{1}{r^2}\right)\cos\theta \qquad \text{(v)}$$

$$U_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -\frac{\partial\psi}{\partial r} = -U_\infty\left(1 + \frac{1}{r^2}\right)\sin\theta \qquad \text{(vi)}$$

Example 12.9 – Non Codable
Reason: No numerical Calculation

Example 12.9. Find several representative streamlines for the potential given in Example 12.8:

$$\phi = U_\infty\left(r + \frac{1}{r}\right)\cos\theta \qquad \text{(i)}$$

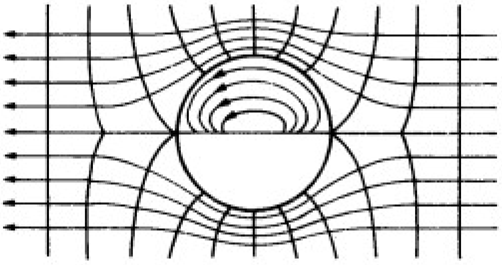

FIGURE 12.12
Potential flow outside and inside a cylindrical shape. *(From Prandtl and Tietjens, Fundamentals of Hydro- and Aeromechanics, Dover Publications, Inc., New York. Reprinted by permission of publisher.)*

Answer. The stream function ψ is Eq. (ix) in Example 12.8:

$$\psi = U_\infty\left(r - \frac{1}{r}\right)\sin\theta \qquad \text{(ii)}$$

Equations (i) and (ii) are not in convenient form for plotting. The streamline $\psi = 0$ is located by realizing that Eq. (ii) can be zero two ways:

$$r - (1/r) = 0 \qquad \text{(iii)}$$

$$\sin\theta = 0 \qquad \text{(iv)}$$

Equation (iii) is the equation for a circle of unit radius and constitutes the cylinder in Fig. 12.12. Equation (iv) is the equation for the x axis.

The remaining streamlines follow easily if Eq. (ii) is parameterized through a constant c, where

$$\psi = cU_\infty \qquad \text{(v)}$$

Equating Eqs. (ii) and (v), the result is

$$c = \left(r - \frac{1}{r}\right)\sin\theta \qquad \text{(vi)}$$

This equation rearranges to the following quadratic:

$$r^2 - \frac{cr}{\sin\theta} - 1 = 0 \qquad \text{(vii)}$$

Since r is the radius in polar coordinates, only the positive root of Eq. (vii) is meaningful. From the quadratic formula, the positive root is

$$r = \frac{c + (c^2 + 4\sin^2\theta)^{1/2}}{2\sin\theta} \qquad \text{(viii)}$$

Example 12.10 – Non Codable
Reason: No numerical Calculation

Example 12.10. Find the terminal velocity and drag force when a spherical water drop, 5 μm in diameter, falls through air at 20°C. Let $g = 9.80\ m\ s^{-2}$.

Answer. From the Appendix, Table A.1, the density of the water drop at 293.15 K is

$$\rho_p = \frac{1}{v_f} = \frac{1}{1.001 \times 10^{-3}} = 999.0\ kg\ m^{-3} \tag{i}$$

From Table A.2, the properties of air are

$$\begin{aligned} \mu &= 0.01817\ cP = 1.817 \times 10^{-5}\ kg\ m^{-1} s^{-1} \\ \rho &= 0.001205\ g\ cm^{-3} = 1.205\ kg\ m^{-3} \end{aligned} \tag{ii}$$

The particle diameter is 5 μm or 5×10^{-6} m (from Table C.11).

The terminal settling velocity is given by Eq. (12.73) in the Stokes' law region:

$$U_t = \frac{(2gr_p^2)(\rho_p - \rho)}{9\mu} \tag{12.73}$$

The procedure is to calculate the terminal settling velocity using Eq. (12.73), check to see if the Reynolds number is less than 0.5, and then find the force on the sphere. From Eq. (12.73), with r_p being half the diameter or 2.5×10^{-6} m, U_t is

$$\begin{aligned} U_t &= (2)(9.80)(2.5 \times 10^{-6})^2(999.0 - 1.205)/[(9)(1.817 \times 10^{-5})] \\ &\qquad \times [(m\ s^{-2})(m^2)(kg\ m^{-3})(kg^{-1}\ m\ s)] \\ &= 7.474 \times 10^{-4}\ m\ s^{-1} \end{aligned} \tag{iii}$$

From Eq. (12.68), the particle Reynolds number is

$$N_{Re,p} = d_p U_\infty \rho/\mu = (5 \times 10^{-6})(7.474 \times 10^{-4})(1.205)/(1.817 \times 10^{-5}) = 2.478 \times 10^{-4} \tag{iv}$$

In the above equation, U_t can be used for U_∞, since it does not matter whether the particle is stationary and the fluid moves with velocity U_∞ or the fluid is stationary and the particle moves with velocity U_t. Clearly, the flow is in the Stokes' law region at this low Reynolds number; therefore, the drag force from

Eq. (12.65) is

$$\begin{aligned} F_p &= 6\pi\mu r_p U_\infty = (6\pi)(1.817 \times 10^{-5})(2.5 \times 10^{-6})(7.474 \times 10^{-4}) \\ &\qquad \times [(kg\ m^{-1} s^{-1})(m)(m\ s^{-1})] \\ &= 6.40 \times 10^{-13}\ kg\ m\ s^{-2} = 6.40 \times 10^{-13}\ N \end{aligned} \tag{v}$$

Example 12.11 – Codable
Example 12.12 – Codable
Example 12.13 – Codable
Example 12.14 – Codable
Example 12.15 – Codable
Example 12.16 – Codable
Example 12.17 – Codable

Chapter 13 Unstedy state Transport

Example 13.1 – Codable
Example 13.2 – Non Codable
Reason: No numerical Calculation

Example 13.2. A 3-in. schedule 40 pipe is 3 ft long and contains helium at 26.03 atm and 317.2 K **(44°C),** as shown in Fig. 13.8. The ends of the pipe are initially capped by removable partitions. At time zero, the partitions are removed, and across each end of the pipe flows a stream of air plus helium at the same temperature and pressure. On the left end, the stream is 90 percent air and 10 percent He (by volume) and on the right 80 percent air and 20percent He. It may be assumed that the flow effectively maintains the helium concentration constant at the ends. If isothermal conditions are maintained and there are no end effects associated with the air flowing past the pipe, calculate the composition profile (to four decimal places) after 1.2 h at space increments of **0.5 ft.** Use Fourier series. The value of $D_{\text{He-air}}$ is 0.7652 x 10^{-4} $m^2\,s^{-1}$ ($2.965\ ft^2\ h^{-1}$) [F2].

Answer. First, note that mass transfer in Fig. 13.8 occurs in the z direction only; there is no transport in either the r- or e-directions. Since the previous equations (derived with heat transfer as the example) are in terms of the x direction, the solution to this problem **will** arbitrarily use the x direction as the direction of mass transfer.

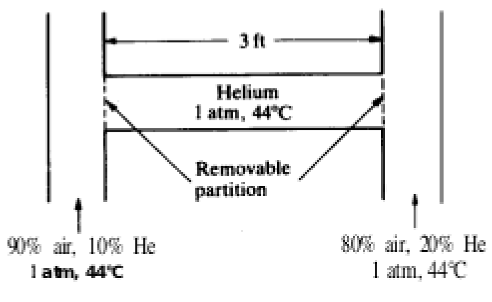

FIGURE 13.8
Transient diffusion of helium in a pipe.

Concentration is related to partial pressure by Eq. (2.37):

$$C_A = n/V = \bar{p}_A/(RT) \qquad (2.37)$$

where R is the gas constant (0.082057 atm $m^3\,kmol^{-1}\,K^{-1}$ from Table C.1). The partial pressure of species A is defined by Eq. (2.38):

$$PA = y_A p_{total} \qquad (2.38)$$

where y_A is the mole fraction of A and p_{total} is the total pressure.

Example 2.7 illustrated the method of converting partial pressures into concentrations. Initially, the partial pressure of helium in the tube equals the total pressure, 26.03 atm. When the partitions are removed, the **partial, pressures** of helium at the ends of the pipe are

$$\bar{p}_1 = y_1 p_{total} = (0.1)(26.03) = 2.603\ \text{atm}$$
$$\bar{p}_2 = y_2 p_{total} = (0.2)(26.03) = 5.206\ \text{atm} \qquad \text{(i)}$$

Example 13.3 – Non Codable
Reason: No numerical Calculation

Example 13.3. Find the **Laplace** transform of: (a) $f(t) = t$; and (b) $f(t) = \exp(at)$, where a is a constant.

Answer. For part (a), t must be nonnegative. Application of Eq. (13.68) to the function yields

$$L(f) = g(s) = \int_0^\infty [\exp(-st)][f(t)]\,dt = \int_0^\infty [\exp(-st)]t\,dt = \frac{e^{-st}}{s^2}(-st-1)\Big|_0^\infty$$

$$= \frac{e^{-\infty}}{s^2}(-\infty) - \frac{-1}{s^2} = \frac{1}{s^2} \qquad (s > 0) \qquad \textbf{(i)}$$

For part (b), with the restriction of t nonnegative

$$L(e^{at}) = \int_0^\infty [\exp(-st)][\exp(at)]\,dt = \frac{1}{a-s}\exp[-(s-a)(t)]\Big|_0^\infty$$

$$= \frac{1}{s-a} \qquad (s-a) > 0 \qquad \text{(ii)}$$

The **Laplace** transform can also be applied to dehvatives **[J2]**. If the derivative of the function f(t) is f'(t), then the **Laplace** transform of this derivative is

$$L[f'(t)] = \int_0^\infty [f'(t)][\exp(-st)]\,dt$$

$$= [f(t)][\exp(-st)]\Big|_0^\infty - s\int_0^\infty [f(t)][\exp(-st)]\,dt = sL[f(t)] - f(0) \qquad (13.70)$$

Example 13.4 – Non Codable
Reason: No numerical Calculation

Example 13.4. **A sheet of** extruded polystyrene (rigid) is 8 ft long, 4 ft wide, and 2 in. thick. Initially, its temperature is 253 K. If the temperature at each face is

TABLE 13.3
Nomenclature for transient transport charts

Symbol	Description	Heat trader	Mass transfer
Y_c	unaccomplished change at the center	$\dfrac{T_\infty - T_c}{T_\infty - T_0}$	$\dfrac{c_{A,\infty} - c_{A,c}}{c_{A,\infty} - c_{A,0}}$
Y_x	dimensionless ratio at location x	$\dfrac{T_\infty - T_x}{T_\infty - T_c}$	$\dfrac{c_{A,\infty} - c_{A,x}}{c_{A,\infty} - c_{A,c}}$
X	relative time	$\alpha t/L^2$	Dt/L^2
m	relative resistance	$k/(hL)$	$D/(k_c L)$
m_∞	relative resistance	$(h/k)(\alpha t)^{1/2}$	$(k_c/D)(Dt)^{1/2}$
n	dimensionless distance	x/L	x/L
Z	dimensionless distance	$x/[2(\alpha t)^{1/2}]$	$x/[2(Dt)^{1/2}]$

Subscripts
∞, fluid surrounding the solid
0, value at time zero
c, value at the center of the body
x, value at position x (or r)

instantaneously increased to 303 K, find the temperature at the center line after 640 s by three methods: (a) Fourier series; (b) Laplace transform; (c) Heisler charts. For this plastic, the density is 55 $kg\,m^{-3}$, the thermal conductivity 0.027 W $m^{-1}\,K^{-1}$, the heat capacity 1.21 $kJ\,kg^{-1}\,K^{-1}$, and the thermal diffusivity $4.057 \times 10^{-7}\ m^2\,s^{-1}$.

Answer. The sheet of polystyrene is large enough so that end effects may be neglected. Therefore, there is negligible error in the assumption that all heat transfer occurs in the x direction, i.e.. the coordinate corresponding to the thickness, which in SI units is

$$2L = (2)(0.0254)\ [(\text{in.})(\text{m in.}^{-1})] = 0.0508\ \text{m} \tag{i}$$

$$L = 0.0254\ \text{m} \tag{ii}$$

The boundary conditions as given correspond to those of Eq. (13.29):

$$\begin{aligned} T_0 &= 253\ \text{K} = \text{constant} \\ T_1 = T_2 = T_s &= 303\ \text{K} = \text{constant} \end{aligned} \tag{iii}$$

Example 13.5 – Non Codable
Reason: No numerical Calculation

Example **13.5.** A jacketed agitation vessel is depicted in Fig. 9.1. Consider such a vessel of inside diameter 1.3 m, the outside of which is well insulated. The jacket wall is 1.3 cm thick and is made of steel with the following properties: density 7800 kg m^{-3}, heat capacity 435 J kg^{-1} K^{-1}, thermal conductivity 84 W m^{-1} K^{-1}, and thermal diffusivity 2.476 $\times 10^{-5}$ m^2 s^{-1}. The initial temperature of the steel is 300 K. At time zero,, hot oil at 400 K is pumped through the jacket. If the heat transfer coefficient is 600 W m^{-2} K^{-1}, calculate the time for the temperature at the steel-insulation interface to reach 380 K.

Answer. The Heisler charts will be used to solve this problem. The thickness of the jacket wall is 1.3 cm (0.013 m). Since the diameter of the jacket wall in the r direction exceeds 1.3 m, the ratio of these two numbers exceeds 100. Obviously, with such a thin wall it can be assumed that the cylindrical-shaped jacket wall can be approximated as a plane wall. In Table 13.3 and the accompanying figures, L is half the thickness of the slab when the same temperature is imposed on each face in the x direction of the slab. When one face is insulated, then L becomes

tht thickness, as discussed previously:

$$L = 0.013 \text{ m} \tag{i}$$

Using the definitions in Table 13.3, the following dimensionless groups are calculated:

$$Y_c = \frac{T_\infty - T_c}{T_\infty - T_0} = \frac{400 - 380}{400 - 300} = 0.2 \tag{ii}$$

$$m = k/(hL) = (64)/[(600)(0.013)] = 8.205 \tag{iii}$$

$$n = x/L = 1 \tag{iv}$$

From Fig. 13.11 the value of Xis 14.3. With the **definition** of the Fourier number, the time is

$$t = XL^2/\alpha = (14.3)(0.013)^2/(2.476 \times 10^{-5}) = 97.6 \text{ s} \tag{v}$$

Cylinders and spheres. Graphs are also available for solids with other geometries, such as cylinders and spheres [G4, H1]. Figures 13.14 and 13.15

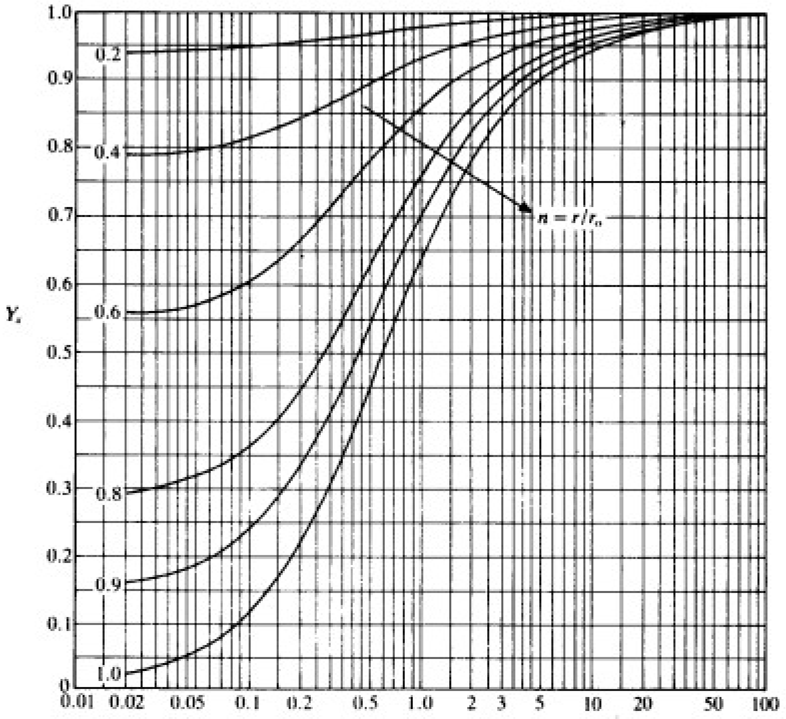

FIGURE 13.15
Position-correction chart for Figure 13.14 (cylinder). [From *Heisler, Trans. ASME* 69: *227 (1947). By permission of ASME.*]

Example 13.6 – Codable
Example 13.7 – Codable
Example 13.8 – Non Codable
Reason: No numerical Calculation

Example 13.8. Find the temperature distribution in a bar of iron (insulated on all surfaces except both ends) after 50 h if the initial temperature distribution is given by

$$T(x, 0) = 500 - 4x - 4x^2 \qquad \text{(i)}$$

where T is in K and x is in m. Let the bar be 4 m long. After the temperature distribution has been fully established, the temperature at each exposed face is instantaneously lowered to 400 K and maintained constant at that value. For iron, $\rho = 7870\ \text{kg m}^{-3}$, $c_p = 447\ \text{J kg}^{-1}\ \text{K}^{-1}$, $k = 80\ \text{W m}^{-1}\ \text{K}^{-1}$, and $\alpha = 2.274 \times 10^{-5}\ \text{m}^2\ \text{s}^{-1}$.

Answer. Let the space increment Ax be 0.1 m. If the time increment is arbitrarily

selected to be 120 s, then β is

$$\beta = \alpha\, \Delta t/(\Delta x)^2 = (2.274 \times 10^{-5})(120)/(0.1)^2 = 0.2729 \qquad \text{(ii)}$$

This value is well within the stability limit imposed by Eq. (13.108).

The first step in the solution of this problem is to form the tridiagonal system of equations. Let a one-dimensioned variable in a computer program be called a vector. The following notation will be used to avoid any two-dimensioned computer variables: the vector U contains the temperatures at time t_{j+1} (unknown, except at the boundary); the vector T contains the temperatures at time t_j (all known); the vector R contains the main diagonal; the vector C contains the upper diagonal; the vector A contains the lower diagonal; the vector D contains the constants; and N is the number of unknowns and the number of simultaneous equations to be solved. For illustration, suppose that N equals 5, and the unknowns are U(1) through U(5). Then the tridiagonal system of equations is

$$\begin{aligned}
&R(1)U(1) + C(1)U(2) + 0 + 0 + 0 = D(1)\\
&A(2)U(1) + R(2)U(2) + C(2)U(3) + 0 + 0 = D(2)\\
&0 + A(3)U(2) + R(3)U(3) + C(3)U(4) + 0 = D(3) \qquad \text{(iii)}\\
&0 + 0 + A(4)U(3) + R(4)U(4) + C(4)U(5) = D(4)\\
&0 + 0 + 0 + A(5)U(4) + R(5)U(5) = D(5)
\end{aligned}$$

The tridiagonal nature of the above system of equations is' evident. When N is large, the representation of Eq. (iii) by four vectors eliminates the need to store all the zero elements that are present.

In Eq. (13.107), the t_j temperatures are all known at each step in time. There are N unknown temperatures at time t_{j+1}, plus the two known boundary conditions:

Example 13.9 – Codable
Example 13.10 – Codable

Chapter 14 : Estimation of transport Coefficient

Example 14.1 – Codable
Example 14.2 – Codable
Example 14.3 – Codable
Example 14.4 – Codable
Example 14.5 – Codable
Example 14.6 – Codable
Example 14.7 – Codable
Example 14.8 – Codable
Example 14.9 – Codable

Chapter 15 : Non Newtonian Phenomena

Example 15.1 – Codable
Example 15.2 – Codable
Example 15.3 – Codable
Example 15.4 – Codable

Example 12.1 – Non Codable