R Textbook Companion for Probability, Random Variables, and Stochastic Processes by Athanasios Papoulis and S. Unnikrishna Pillai[1]

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Book Description

Title: Probability, Random Variables, and Stochastic Processes

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R numbering policy used in this document and the relation to the above book.

Exa Example (Solved example)

Eqn Equation (Particular equation of the above book)

For example, Exa 3.51 means solved example 3.51 of this book. Sec 2.3 means an R code whose theory is explained in Section 2.3 of the book.

Contents

List of R Codes

Chapter 1

The Meaning of Probability

R code Exa 1.2 probability that the sum of the numbers that show equals 7 when two dice are rolled

```r
#page no. 7
#LOAD PACKAGE -------->prob
#example 1-2
#function used: rolldie():This function is used to get sample space for the experiment of rolling a die repeatedly.
#                  nrow(): Gives the total number of rows

library(prob)
no_of_favourable_outcome= nrow(subset(s,X1+X2==7)) #outcomes such that their sum equals 7 when two dices are rolled
total_outcomes=nrow(s)                                    #all possible outcomes

probablity=no_of_favourable_outcome/total_outcomes
print(probablity)
```

R code Exa 1.3 probability that the length of a randomly selected cord is greater than the length r times square root of 3 of the inscribed equilateral triangle

```
#page no. 8-9
#example 1-3

# this example shows that in different cases the probablity can be different that's why conditions of probablity
# should be specified correctly

#NOTE: assuming radius r = 1 throughout the solution for simplicity

        #CASE I
area_of_circle_with_radius_r= 3.14*1*1
area_of_circle_with_radius_half_r= (3.14*1*1)/4

probablity_caseI=area_of_circle_with_radius_half_r/area_of_circle_with_radius_r
print(probablity_caseI)

        #CASE II

favourable_outcome_in_caseII=(2*3.14*1)/3
total_outcome_in_caseII=2*3.14*1

probablity_caseII=favourable_outcome_in_caseII/total_outcome_in_caseII
print(probablity_caseII)

        #CASE III

```

```
favourable_outcome_in_caseIII=1
total_outcome_in_caseIII=2

probablity_caseIII=favourable_outcome_in_caseIII/
   total_outcome_in_caseIII
print(probablity_caseIII)
```

R code Exa 1.4 probability of finding particles in boxes

```
#page no. 10-11
#example 1-4

#the solution to this problem depends on the choice
   of possible and favorable outcomes
#therefore we shall consider three celebrated cases

          #case I : maxwell-boltzmann statistics

m_caseI=6
n_caseI=2
p_caseI=factorial(n_caseI)/(m_caseI^n_caseI)
print(p_caseI)


          #case II : bose-einstein statistics

m_caseII=6
n_caseII=2
p_caseII= (factorial(m_caseII-1)*factorial(n_caseII)
   )/(factorial(n_caseII + m_caseII - 1))
print(p_caseII)

          #case III : fermi-dirac statistics

m_caseIII=6
```

```
n_caseIII=2
p_caseIII= (factorial(n_caseIII)*factorial(m_caseIII
    - n_caseIII))/factorial(m_caseIII)
print(p_caseIII)
```

Chapter 2

The Axioms of Probability

R code Exa 2.1 subsets of faces

```
#page no. 16
#example 2-1

elements=6
subsets=2^elements
print(subsets)
```

R code Exa 2.2 calculation of subsets

```
#page no. 16
#example 2-2

elements=4
subsets=2^elements
print(subsets)
```

R code Exa 2.5 probability of first 2 heads when a coin is tossed 3 times

```r
#page no. 24
#example 2-5 (b)

total_outcomes=8
three_heads=1
heads_in_first_two_tosses=2

#probablity of getting "three heads"
p_three_heads=three_heads/total_outcomes
print(p_three_heads)

#probablity of getting "two heads in first two tosses"
p_heads_in_first_two_tosses=heads_in_first_two_tosses/total_outcomes
print(p_heads_in_first_two_tosses)
```

R code Exa 2.10 conditional probability

```r
#page no. 29
#example 2-10

total_outcomes=6
favourable_A=1   #favourable outcome for event A={f2}
favourable_M=3   #favourable outcome for event M={f2,f4,f6}

p_A=favourable_A/total_outcomes    #probability of occurrence of event A
p_M=favourable_M/total_outcomes    #probability of occurrence of event M

```

```
p_AM=p_A/p_M   #condition probability of event A after even M has happened
print(p_AM)
```

R code Exa 2.11 condition probability of event A after even M has happened

```
#page no. 30
#example 2-11

alpha_t=function(t){3*10^-9*t^2*(100-t)^2}   #this is the function alpha(t)

p_A=integrate(alpha_t, lower = 60,upper = 70)     #p{60<=t<=70}
p_M=integrate(alpha_t, lower = 60,upper = 100)    #p{60<=t<=100}

p_AM=p_A$value/p_M$value        #condition probability of event A after even M has happened

print(p_AM)
```

R code Exa 2.12 probability of second red ball event when first white ball event has happened

```
#page no. 30-31
#example 2-12

#First Solution is theoritical therefore only second solution is coded here

#Second Solution
```

```
p_W=3/5      #probability of first white ball
p_R=2/4      #probability of second red ball
p_RW=p_R*p_W   # probability of second red ball
    event  when first white ball event has happened

print(p_RW)
```

R code Exa 2.13 calculation of smallest number of balls in the box

```
#page no. 31
#example 2-13

    #part (a)

b=1

cat((sqrt(3)+1)*(b/2),"< a < ",1+(sqrt(3)+1)*(b/2))
        #(2-40) in the book

#therefore
a=2

p_w2_w1= ((a-1)*a)/((a+b-1)*(a+b))       #(2-39) in
    the book

print(p_w2_w1)

print("Thus the smallest number of balls required is
     3")
```

R code Exa 2.15 calculation of probability that the person under test has that particular cancer

```
#page no. 33-34
#example 2-15

p_TC=0.95
p_C=0.02
p_TH=0.05
p_H=0.98

p_CT=(p_TC*p_C)/((p_TC*p_C)+(p_TH*p_H))        #this
    formula is obtained using Bayes' theorem

print(p_CT)

#this result states that if the test is taken by
    someone from this population without knowing
    whether that person
#has the disease or not, then even a positive test
    only suggests that there is a 27.6% chance of
    having a disease
```

R code Exa 2.16 probability that the selected component is defective

```
#page no. 34-35
#example 2-16

#(part a)

p_B1=p_B2=p_B3=p_B4=1/4

p_DB1=0.05    #given in the question
p_DB2=0.4     #given in the question
p_DB3=0.1     #given in the question
```

```
p_DB4=0.1     #given in the question

p_D=p_DB1*p_B1 + p_DB2*p_B2 + p_DB3*p_B3 + p_DB4*p_
    B4

print(p_D)

#(part b)

p_B2D=p_DB2*p_B2/p_D

print(p_B2D)
```

R code Exa 2.18 various probabilities related to arrival of trains

```
#page no. 36-38
#example 2-18

#part a

p_C_favourable=200        #area of the triange in the
    event
p_C_total=400             #total area
p_C=p_C_favourable/p_C_total
print(p_C)

#part b
p_D_favourable=159.5       #area of the region D
p_D_total=400              #total area
p_D=p_D_favourable/p_D_total
print(p_D)

#part c
p_CD_favourable=72         #area of the trapezoid CD
p_CD_total=159.5
```

```
p_CD=p_CD_favourable/p_CD_total
print(p_CD)
```

R code Exa 2.20 probability that two or more persons will have the same birthday in a group

```
#page no. 39-40
#example 2-20

N=365    #number of days in one year

    #PART (a)


n=23     #group of people
p=1-exp((-n*(n-1))/(2*N))

cat("In a group of 23 people it gives probability of
    at least one match to be",p)

n=50
p=1-exp((-n*(n-1))/(2*N))
cat("in a group of 50 person, the probability of
   birthday match is ",p)
#answer in the book is 0.97 which is due to
   approximation


    #PART (b)



n=253
p_personal=1-exp(-n/N)    #probability of personal
   match
```

```
print("For a modest 50-50 chance in this case the
    group size needs to be about 253 ")

#if n=1000
n=1000
p_personal=1-exp(-n/N)

cat("In a group of 1000 people, chances are about ",
    p_personal*100,"% that there will be someone
    sharing your birthday")
```

Chapter 3

Repeated Trials

R code Exa 3.3 probability that the ball from B1 box will be white and the ball from B2 box will be Red

```
#page no. 49
#example 3-3

w1_favourable=10    #number of white balls in box B1
w1_total=15         #total number of balls in box B1
p_w1=w1_favourable/w1_total

r2_favourable=20    #number of red balls in box B2
r2_total=40         #total number of balls in box B2
p_r2=r2_favourable/r2_total

p_w1r2=p_w1*p_r2
print(p_w1r2)
```

R code Exa 3.7 probability that six will show twice when fair die is rolled five times

```
#page no. 54
#example 3-7



#this is the probability that "six" will show twice
   when fair die is rolled five times

p5_2=(factorial(5)/(factorial(2)*factorial(3)))*(1/
   6)^2*(5/6)^3    #usig the "FUNDAMENTAL THEOREM" of
    Success or Failure of an Event A in n
   Independent Trials

print(p5_2)
```

R code Exa 3.8 probability of obtaining double six at least once when pair of dice is rolled n times

```
#page no. 54-55
#example 3-8


  #PART (b)

p_b=1/36
p_b_bar=1-p_b
p=log(2)/(log(36)-log(35))          #using the
   equation (3-15)
print(p)

print("Thus in 25 throws one is more likely to get
   double six at least once than not to get it at
   all.")
print("Also in 24 or less throws, there is greater
   chance to fail than to succeed")
```

R code Exa 3.12 kmax and k1 and k2 are calculated

```
#page no. 57
#example 3-12

#PART (a)


n=10     #(given)
p=1/3    #(given)

k_max=floor((n+1)*p)            #floor() function
   returns the largest integer not greater than the
   giving number and hence act as "greatest integer
   function" of mathematics

cat("hence Kmax=",k_max)



#PART (b)

n=11    #(given)
p=1/2   #(given)
k1=(n+1)*p
k2=k1-1
cat("hence k1=",k1,",k2=",k2)
```

R code Exa 3.13 probability that the total number of defective parts does not exceed 1100 in an order

```r
#page no. 58
#example 3-13

#sum function is used to do the summation
#choose function is used to do the nCr


p=0.1    #probability that a part is defective (given)
n=10^4   #total parts
k=0:1100 #limit of defective parts

pb=sum(choose(n,k)*(p^k)*((1-p)^(n-k)))          #this is required probability

pb

print("The value tends to infinity therefore R is taking it as NaN")
```

R code Exa 3.14 probability that a player has k matches in the lottery

```r
#page no. 61-62
#example 3-14

p_winning_the_lottery=(6*5*4*3*2*1)/(51*50*49*48*47*46)      #probability of winning the lottery when total number of balls ,n=51 and number of good balls ,m=6
print(p_winning_the_lottery)

winning_prize=4000000
average_gain=(winning_prize*p_winning_the_lottery)-1
print(average_gain)

```

```
winning_prize_5matches=15000
odds_winning_5matches=66701
average_gain_5matches=(winning_prize_5matches/odds_winning_5matches) - 1
print(average_gain_5matches)

winning_prize_4matches=200
odds_winning_4matches=1213
average_gain_4matches=(winning_prize_4matches/odds_winning_4matches) - 1
print(average_gain_4matches)
```

R code Exa 3.16 probability of winning the game

```
#page no. 67-68
#example 3-16

t_7_favourable=6            #ways in which total could be 7 when pair of dies are thrown
t_11_favourable=2           #ways in which total could be 11 when pair of dies are thrown
total_outcomes=36
p_t_7=t_7_favourable/total_outcomes     #probability of having sum equals 7 when two dies are thrown
p_t_11=t_11_favourable/total_outcomes   #probability of having sum equals 11 when two dies are thrown
p_p1=p_t_11 + p_t_7       #probability of winning the game by throwing a 7 or 11 on the first throw

print(p_p1)

          #ak_n=p_k_n/p_k_n+(p_k_n+1/6)     this formula is used to calculate ak_n which is given by equation number 3-59 in the book
```

```
p_k_4=3/36                    #probability of having sum equals 4 when two dies are thrown
ak_4=p_k_4/(p_k_4+1/6)

p_k_5=4/36                    #probability of having sum equals 5 when two dies are thrown
ak_5=p_k_5/(p_k_5+1/6)

p_k_6=5/36                    #probability of having sum equals 6 when two dies are thrown
ak_6=p_k_6/(p_k_6+1/6)

p_k_8=5/36                    #probability of having sum equals 8 when two dies are thrown
ak_8=p_k_8/(p_k_8+1/6)

p_k_9=4/36                    #probability of having sum equals 9 when two dies are thrown
ak_9=p_k_9/(p_k_9+1/6)

p_k_10=3/36                    #probability of having sum equals 10 when two dies are thrown
ak_10=p_k_10/(p_k_10+1/6)

p_p2=ak_4*p_k_4 + ak_5*p_k_6 + ak_6*p_k_6 + ak_8*p_k_8 + ak_9*p_k_9 + ak_10*p_k_10
print(p_p2)

p_winning_the_game=p_p1+p_p2
print(p_winning_the_game)

#the answer in the book is 0.492929 but by the approximation in R the answer with the code is 0.5040404
```

R code Exa 3.17 probability that how many plays should A choose to win the game

```
#page no. 68-70
#example 3-17

#taking p=0.47

p=0.47
n1=(1/(1-(2*p)))-1
n2=(1/(1-(2*p)))+1

cat(n1,"<=2n<=",n2)        # this is equation (3-71)
print("therefore if p=0.47, then 2n=16")

#when p=0.48
p=0.48
n1=(1/(1-(2*p)))-1
n2=(1/(1-(2*p)))+1

cat(n1,"<=2n<=",n2)

#when p=0
p=0
n1=(1/(1-(2*p)))-1
n2=(1/(1-(2*p)))+1

cat(n1,"<=2n<=",n2)

print("Finally if p=0, then (3-71) gives the optimum
    number of plays to be 2")
```

Chapter 4

A Concept of a Random Variable

R code Exa 4.1 function of random variable

```
#page no. 72
#example 4-1

#PART (a)

xfi=10*i

xf1=10*1
cat("x(f1)=",xf1)

xf2=10*2
cat("x(f2)=",xf2)

xf3=10*3
cat("x(f3)=",xf3)

xf4=10*4
cat("x(f4)=",xf4)

```

```
xf5=10*5
cat("x(f5)=",xf5)

xf6=10*6
cat("x(f6)=",xf6)

#part (b)


xf1=xf3=xf5=0
xf2=xf4=xf6=1

cat("x(f1)=x(f3)=x(f5)=",xf1)
cat("x(f=20=x(f4)=x(f6)=",xf2)
```

R code Exa 4.4 probabilities at different values of x

```
#page no. 76
#example 4-4


xfi=10i

f100=1    #since it contain of the fi's therefore it is a certain event
f35=3/6   #since x<=35 will only include {f1,f2,f3}
f30.1=3/6   #since x<=35 will only include {f1,f2,f3}
f30=3/6   #since x<=35 will only include {f1,f2,f3}
f29.99=2/6   #since x<=35 will only include {f1,f2}
cat("F(100)=",f100)
cat("F(35)=",f35)
cat("F(30.1)=",f30.1)
cat("F(30)=",f30)
cat("F(29.99)=",f29.99)
```

```
#distribution function of x is a staircase function
    as ploted below

x <- c(0,10,20,30,40,50,60)
y=c(0,1/6,2/6,3/6,4/6,5/6,6/6)
plot(x, y, type = "S", ylab = "F(x)")
```

R code Exa 4.9 finding the function fx for random variable x

```
#page no. 81
#example 4-9

#for x<0
fx=1/4

#for 1<=x<=2

fx=3/4


print("the function f(x) comes out to be a staircase
     like function which is ploted below")
x <- c(0,1,2,3)
y<-c(0,1/4,3/4,1)
plot(x, y, type = "S", ylab = "F(x)")
```

R code Exa 4.10 probability of the appliance that it will not fail in the next 5 years

```
#page no. 86-87
#example 4-10

```

```
p=exp(-5/10)  #probability that appliance will not fail in next  5 years

print(p)
#the answer in the book is 0.368 which seems WRONG to me
```

R code Exa 4.11 the conditional probability that the customer will spend an additional 10 minutes in the restaurant

```
#page no. 87
#example 4-11

p=exp(-10/5)    #the probability that a customer will spend more than 10 minutes in the restaurant
print(p)

p_additional_10=exp(-2)  #the conditional probability that the customer will spend an additional 10 minutes in the restaurant
print(p_additional_10)
```

R code Exa 4.13 probabilities of team winning the games

```
#page no. 97
#example 4-13

#the choose() function calculate the nCr and sum() function calculates the sumation

k=3:5
p=1/2
p_A_wins=sum(choose(k-1,2)*p^3*(1-p)^(k-3))
```

```r

cat("If p=",p," then P(A wins)=", p_A_wins)

k=4
p_4_games=choose(k-1,2)*p^3*(1-p)^(k-3)
cat("The probability that A will win in exactly four games is ", p_4_games)

k=3
p_3_games=choose(k-1,2)*p^3*(1-p)^(k-3)

cat("the probability that A will win in four games or less is ",p_4_games,"+",p_3_games,"=",p_4_games+p_3_games)

k=2:4
p_conditional=sum(choose(k-1,1)*((1/2)^2)*((1/2)^(k-2)))
cat("Given that A has won the first game, the conditional probability of A winning equals",p_conditional)
```

R code Exa 4.14 the conditional probability of random variable x of fair die experiment

```r
#page no. 98-99
#example 4-14

#probabilities are found out just by observing the different situation

# if x>=60

p_M=3/6
p_XM=p_M/p_M
```

```
print(p_XM)

#if 40 <= x < 60

p_X=2/6
p_XM=p_X/p_M
print(p_XM)

#if 20 <= x < 40

p_X=1/6
p_XM=p_X/p_M
print(p_XM)

#if x<20

p_XM=0
print(p_XM)
```

R code Exa 4.19 probability density function of a random variable x

```
#page no. 103-105
#example 4-19

k=6  #number of heads
n=10 #number of specific tosses
p_B=(k+1)/(n+2)
print(p_B)

#this example shows that if the probability density
   function of a random variable x is unknown, one
   should make
#noncommittal judgement about its a priori
   probability density function  f(x). Usually, the
   uniform distribution
```

```
#is a reasonable assumption in the absence of any other information. then experimental results (A) are obtained
#and the knowledge about x is updated reflecting this new information. Bayes' rule helps to obtain the a posteriori
#probability density function of x given A. From that point on, this a posteriori probability density functin f(x|A)
#should be used to make further predictions and calculations
```

R code Exa 4.20 probability of heads coming 500 and 510 times when the coin is tossed 1000 times

```
#page no. 107
#example 4-20

p=q=0.5 #probability of head/tail when a coin is tossed
n=1000  #number of times the coin is tossed

#part (a)
p_A=1/sqrt(2*3.14*n*p*q)
print(p_A)

#part (b)

p_B=(exp(-0.2))/(10*sqrt(5*3.14))
print(p_B)

#in book the solution of  part (b) is rounded to 0.0207
```

R code Exa 4.21 example of FUNDAMENTAL THEOREM of Success or Failure of an Event A in n Independent Trials and The normal Approximation DeMoivre Laplace Theorem

```
#page no. 107
#example 4-21

p=0.5
n=10
k=5
q=1-p

#part (a)

p_n_k=choose(n,k)*(p^k)*(q^(n-k))          #usig the "
    FUNDAMENTAL THEOREM" of Success or Failure of an
    Event A in n Independent Trials
print(p_n_k)

#part (b)

pnk=(exp((-(k-n*p)^2)/(2*n*p*q)))/sqrt(2*3.14*n*p*q)
              #using "The normal Approximation (
    DeMoivre-Laplace Theorem)
print(pnk)
```

R code Exa 4.22 probability that the number of heads is between 4900 and 5100 when coin is tossed 10000 times

```
#page no. 109
#example 4-22

```

```
4 n=10000       #number of times the coin is tossed (given)
5 p=q=0.5       #probability of getting head(or tail) in one toss
6 k1=4900
7 k2=5100
8 x2=(k2-n*p)/sqrt(n*p*q)
9 print(x2)
10 x1=(k1-n*p)/sqrt(n*p*q)
11 print(x1)
12
13 inte<-function(y){exp(-(y^2)/2)}     #these two lines (13 and 14)  gives the defination of function G(x)
14 Gx<-function(x){((1/(sqrt(2*3.14)))*integrate(inte, lower = 0,upper = x)[[1]])+0.5}     #which is given on page number 106
15
16 probability=(2*Gx(2))-1
17 cat("probability equals ",probability)
18
19 #answer in the book is 0.9545  with is by apprimation of different values
```

R code Exa 4.23 probability of random calls in four hour interval

```
#page no. 109
#example 4-23

K=50:70  #number of calls is 50 to 70


probabilitySumation=sum((exp(-((K-60)^2)/80)/(4*sqrt(5*22/7))))
```

```
cat("probability using sumation formula ",probabilitySumation)

inte<-function(y){exp(-(y^2)/2)}     #these two lines (10 and 11)  gives the defination of function G(x)
Gx<-function(x){((1/(sqrt(2*3.14)))*integrate(inte,lower = 0,upper = x)[[1]])+0.5}     #which is given on page number 106

probability=(2*Gx(sqrt(2.5)))-1
cat("probability using G(x) function (The Normal Approximation) is ",probability)

print("probability by both the methods are similar as lot of approximation comes into play in sumation formula")
```

R code Exa 4.24 probabilities for various values of n

```
#page no. 109
#example 4-23

p=q=0.5    #(given)
e=0.05


    #taking n=100
n1=100
x1=e*sqrt(n1/(p*q))

inte<-function(y){exp(-(y^2)/2)}     #these two lines  gives the defination of
```

```
function G(x)
Gx<-function(x){((1/(sqrt(2*3.14)))*integrate(inte,lower = 0,upper = x)[[1]])+0.5}      #which is given on page number 106

probability1=(2*Gx(x1))-1


#taking n=400
n2=400
x2=e*sqrt(n2/(p*q))

inte<-function(y){exp(-(y^2)/2)}      #these two lines  gives the defination of function G(x)
Gx<-function(x){((1/(sqrt(2*3.14)))*integrate(inte,lower = 0,upper = x)[[1]])+0.5}      #which is given on page number 106

probability2=(2*Gx(x2))-1

#taking n=900
n3=900
x3=e*sqrt(n3/(p*q))

inte<-function(y){exp(-(y^2)/2)}      #these two lines  gives the defination of function G(x)
Gx<-function(x){((1/(sqrt(2*3.14)))*integrate(inte,lower = 0,upper = x)[[1]])+0.5}      #which is given on page number 106

probability3=(2*Gx(x3))-1
```

```
table <- matrix(c(n1,n2,n3,x1,x2,x3,probability1,
    probability2,probability3),ncol=3,byrow=TRUE)
#colnames(table) <- c("n")
rownames(table) <- c("n","0.1sqrt(n)","2G(0.1sqrt(n)
    )-1")
table<-as.table(table)

print("table shows the probability 2G(0.1sqrt(n))-1
    that k is between 0.45n and 0.55n for various
    values of n ")

table
```

R code Exa 4.26 probability when a fair die is rolled 10 times

```
#page no. 111
#example 4-26



p1=1/6      #A1={f1}
p2=3/6      #A2={f2,f4,f6}
p3=2/6      #A3={f3,f5}

print(p1)
print(p2)
print(p3)

p_10=(factorial(10)/(factorial(3)*factorial(6)*
    factorial(1)))*((1/6)^3)*((1/2)^6)*(1/3)     #p10
    (3,6,1)
print(p_10)

#answer in the book is given 0.002 which is wrong
```

R code Exa 4.27 probability that the system of 1000 components will function at the end of one month

```
#page no. 113
#example 4−27

p=10^-3  #probability of faliure (given)
q=1-p
n=10^3   #number of components (given)

p_K=q^n



p_k=exp(-n*p)    #after applying approximation
   techniques
print(p_k)
```

R code Exa 4.28 the probability P that there will be more than five defective parts in a order of 3000 parts

```
#page no. 113−114
#example 4−28

k=0:5
p_K_lessthanequal_5=exp(-3)*sum(3^k/factorial(k))
print(p_K_lessthanequal_5)

p_K_greaterthan_5=1- p_K_lessthanequal_5
print(p_K_greaterthan_5)

```

```
#in book the answer is given 0.084 which is just
   round off
```

R code Exa 4.29 probability of an insurance company to suffer loss or make profit

```
#page no. 114-115
#example 4-29

n=10^5   #number of people (given)
p=0.001  #porobability of causality (given)
lambda=n*p


   #part (a)

n0=(50*10^6)/200000


del=(lambda*exp((1-lambda/n0)))/n0

cat("del equals",del,"so that del^250 = 0 and the
   desired probability is essentially 0")



   #part (b)

n1=((50*10^6)-(25*10^6))/200000
print(n1)

del=0.9771
deln1=del^n1
print(deln1)

```

```
p_x=1-((deln1*1)/(sqrt(2*3.14*n1))*((1-lambda)/(n1+1)))
print(p_x)

#the answer in the book is 0.9904 which is approximation which signifies that the company is assured a profit of $25
#million with almost certainty
```

R code Exa 4.30 probability of shots to hit an aircraft

```
#page no. 115
#example 4-30



#when lambda=4
lambda=4

p_not_hit=(1+lambda)*exp(-lambda)
print(p_not_hit)
#this is given as 0.0916 which is just round off


#when lambda=5
lambda=5
p_not_hit=(1+lambda)*exp(-lambda)
print(p_not_hit)

#if 5000 shots are fired at the aircraft then the probability of miss
p_miss=exp(-5)
print(p_miss)
```

R code Exa 4.31 probabilities of winning a lottery

```
#page no. 115-116
#example 4-31

#the probability of buying a winning ticket
no_of_winning_tickets=100
total_no_of_tickets=10^6
p=no_of_winning_tickets/total_no_of_tickets      #the
    probability of buying a winning ticket
print(p)

n=100  #number of ticket purchased
lambda=n*p

   #part (a)

p_win=1- exp(-lambda)
print(p_win)

   #part (b)

#in this part we have to find lambda such that the
   probability of winning is >=0.95
#for that lambda should be >=3
#for which n>=30000
p_win=1- exp(-3)
print(p_win)  #probability of winning comes out to
   be >=0.95
```

R code Exa 4.32 probability that a spacecraft mission will be danger

```
#page no. 116
#example 4-32

k=0:4
n=20000   #number of components in spacecraft (given
    )
p=10^-4   #probability of any one component
    defective (given)
lambda=n*p

p_danger_mission=1-(exp(-2)*sum((lambda^k)/factorial
    (k)))
print(p_danger_mission)
```

Chapter 5

Functions of One Random Variable

R code Exa 5.1 finding the function Fy

```
#page no. 124-125
#example 5-1


   #part (a)

#a and b are constant in the function therefore assuming b=4 and a=2 for the purpose of ploting graphs

curve((x-4)/2,from = -50, to= 50,main="part (a)",ylab ="y")

   #part(b)

curve(1-((x-4)/2),from = -50, to= 50,main="part (b)",ylab ="y")
```

R code Exa 5.2 the function y equals x square

```
#page no. 125-126
#example 5-2


# this is the cure of y=x^2
curve(x^2,from = -20,to=20,ylab = "y",main="figure
   5-3a")
```

R code Exa 5.5 the gx function

```
#page no. 127-128
#example 5-5

#the function Fy(y) is a staircase function and is
   ploted below

x<-c(-2,-1,0,1,2)
y<-c(0,0.5,0.5,1,1)
plot(x, y, type = "s", ylab = "Fy(y)",xlab = "y" ,
   axes = TRUE)
```

R code Exa 5.9 different probabilities for the function y equals x square

```
#page no. 129
#example 5-9

y=x^2   #given function
```

```
#part(a)

#for differenct values of x with probability 1/6 the
    respective values of y with probability 1/6
x=1
y=x^2
print(y)

x=2
y=x^2
print(y)

x=3
y=x^2
print(y)

x=6
y=x^2
print(y)

#part (b)

#y for different values of x with probability 1/6

x=-2
y=x^2
print(y)

x=-1
y=x^2
print(y)

x=0
y=x^2
print(y)

x=1
```

```
y=x^2
print(y)

x=2
y=x^2
print(y)

x=3
y=x^2
print(y)

print("on different values of x , y takes the values 0,1,4,9 with probabilities 1/6,2/6,2/6,1/6, respectively")
```

R code Exa 5.10 random variable for voltage

```
#page no. 131
#example 5-10

i=0.01   #(given)
ro=1000  #(given)

#if r is between 900 and 1100
r1=900
v1=i*(r1+ro)

r2=1100
v2=i*(r2+ro)


cat("If the resistence r is a random varibale uniform between",r1,"and",r2,"ohm ,then v is uniform between ",v1,"and ",v2,"V")
```

R code Exa 5.17 mean of random variable

```
#page no. 140
#example 5-17

ex=1/6*(1+2+3+4+5+6)

cat("If x takes the values 1,2,..,6 with probability
    1/6, then E{x}=",ex)
```

R code Exa 5.27 mean and variance of resultant current

```
#page no. 150-121
#example 5-27


E=120  #voltage
n=10^3
sigma_square=(100^2)/3
gn=E/n

gnd=-1*E/(n^2)

gndd=2*E/(n^3)

cat("g(n)=",gn)
cat("g'(n)=",gnd)
cat("g''(n)=",gndd)

cat("E{i}=",gn,"+",gndd*sigma_square/2)
                #using (5-85)
```

```
cat("sigma_i_square=",gnd^2*sigma_square)
                #using (5-87)
```

Chapter 7

Sequences of Random Variables

R code Exa 7.4 measurement errors

```r
#page no. 247-248
#example 7-4

x1=98.6
x2=98.8
x3=98.9
sigma1=0.20
sigma2=0.25
sigma3=0.28

E=(x1/sigma1^2 + x2/sigma2^2 + x3/sigma3^2)/(1/sigma1^2 + 1/sigma2^2 + 1/sigma3^2)

cat("estimate E obtained from (7-17) comes out to be ",E)
```

R code Exa 7.16 The probability of k heads in six tosses

```
#page no. 280
#example 7-16


pk=function(k){choose(6,k)*(1/2^6)}
nk=function(k){exp(-((k-3)^2)/3)/(sqrt(3*22/7))}


table <- matrix(c(0,1,2,3,4,5,6,pk(0),pk(1),pk(2),pk
    (3),pk(4),pk(5),pk(6),nk(0),nk(1),nk(2),nk(3),nk
    (4),nk(5),nk(6)),ncol=7,byrow=TRUE)
#colnames(table) <- c("n")
rownames(table) <- c("k","Pk","N(n,sig)")
table<-as.table(table)
table

#the values of Pk varies a little bit from the book
    because of approximation
```

Chapter 8

Statistics

R code Exa 8.1 life expectancy of the battery

```
#page no. 305
#example 8-1

#functions used
#qnorm():quantile function of the normal distribution:the quantile function maps from probabilities to values in normal distribution
#ceiling():ceiling(x) rounds to the nearest integer that's larger than x.


rho=0.05

z0.975=ceiling(qnorm(1-rho/2))

cat("we can expect with confidence coefficient 0.95 that the life expectancy of our battery will be between ",4-(z0.975*0.5),"and ",4+(z0.975*0.5)," years")
```

R code Exa 8.2 number of heads when a fair coin is tossed 100 times with given confidence coefficient

```
#page no. 306
#example 8−2

n=100 #number of times the coin is tossed (given)
p=0.5 #probability of getting head in one coin toss
    (given)
q=1-p
k1=n*p - 3*sqrt(n*p*q)
k2=n*p + 3*sqrt(n*p*q)


cat(" We predict ,therefore , with confidence
    coefficient 0.997 that the number of heads will
    be between ",k1,"and ",k2)
```

R code Exa 8.3 finding the confidence interval of the voltage

```
#page no. 309
#example 8−3

#functions used
#qnorm():quantile function of the normal
    distribution:the quantile function maps from
    probabilities to values in normal distribution
#ceiling():ceiling(x) rounds to the nearest integer
    that's larger than x.

rho=0.05
z0.975=ceiling(qnorm(1-rho/2))
```

```
  #part (a)

x_bar=112     #(given)
sigma=0.4     #(given)
n=25          #number of times the voltage is measured
i=z0.975*sigma/sqrt(n)

cat("Insetting values into (8-11),we obtain the interval ",x_bar,"+-",i,"V")

  #part (b)

s=0.6
i2=z0.975*s/sqrt(n)
cat("Insetting values into (8-14),we obtain the approximate estimate ",x_bar,"+-",i2,"V")

print("Since t0.975(25)=2.06 (from the table) the exact estimate (8-17) yields 112+-0.247 V")
```

R code Exa 8.4 finding confidence interval of light bulb

```
#page no. 311
#example 8-4

x_bar=210    # (given)

z=2
n=64      #number of bulbs
h1=x_bar/(1+(z/sqrt(n)))       #using the EXPONENTIAL DISTRIBUTION. given by equation (8-18) in the book
h2=x_bar/(1-(z/sqrt(n)))
```

```
cat("We thus expect with confidence coefficient 0.95
    that the mean time to failure of the bulb is
   between ",h1,"and ",h2,"hours.")
```

R code Exa 8.5 finding confidence interval of particles emitted from a radioactive substance

```
#page no. 312
#example 8-5

x_bar=6

cat("(z^2)/n=0.0625 then the equation (8-19) yields
   the quadratic ")
cat("(lambda-6)^2=0.0625lambda")

#from the equation and comparing coefficient with ax
   ^2+bx+c=0
a=1
b=-12.0625
c=36

#finding solution of the quadratic equation

l1=(-b-sqrt(b^2 - 4*a*c))/(2*a)
l2=(-b+sqrt(b^2 - 4*a*c))/(2*a)

cat("We can thus clain with confidence coefficient
   0.95 that ",l1,"< lambda <",l2)
```

R code Exa 8.6 confidence interval of the poll

```
#page no. 313
#example 8-6

z=2        #(given)
n=500      # total number of persons (given)
r=240      #person who reported Republican

x_bar=r/n

p=z*sqrt((x_bar*(1-x_bar))/n)      #using the
    equation (8-21)

cat("equation (8-21) yields the interval",x_bar,"+-"
    ,p)

#ansers givent in the book are approximate answers
```

R code Exa 8.7 finding interval estimate of variance

```
#page no. 314-315
#example 8-7

  #part (a)

n=6
v_cap=0.25
x1=qchisq(0.975,6)        #qchisq() is the function
    used to calculate Chi-square percentile value in
    R
x2=qchisq(0.025,6)        #qchisq() is the function
    used to calculate Chi-square percentile value in
    R
c1=n*v_cap/x1
c2=n*v_cap/x2

```

```
cat("(8-23) yields ",c1,"< sigma^2 <",c2,". The
   corresponding interval for sigma is ",sqrt(c1),"<
    sigma <",sqrt(c2),"V")
#there is slight difference in the values in the
   book and that is due to approximation

  #part (b)


n=5
s=0.6
x1=qchisq(0.975,5)       #qchisq() is the function
   used to calculate Chi-square percentile value in
   R
x2=qchisq(0.025,5)       #qchisq() is the function
   used to calculate Chi-square percentile value in
   R
c1=(n-1)*s^2/x1
c2=(n-1)*s^2/x2

cat("(8-24) yields ",c1,"< sigma^2 <",c2,". The
   corresponding interval for sigma is ",sqrt(c1),"<
    sigma <",sqrt(c2),"V")
#there is slight difference in the values in the
   book and that is due to approximation
```

R code Exa 8.8 finding confidence interval of the median of x

```
#page no. 315
#example 8-8

#functions used
#qnorm():quantile function of the normal
   distribution:the quantile function maps from
   probabilities to values in normal distribution
```

```
#ceiling():ceiling(x) rounds to the nearest integer
   that's larger than x.

rho=0.05
u=0.5
n=100   #samples of x (given)
z=ceiling(qnorm(1-rho/2))

k=n*u - z*sqrt(n*u*(1-u))                  #using the
   equation (8-26)

k_plus_r=n*u + z*sqrt(n*u*(1-u))           #using the
   equation (8-26)

print(k)
print(k_plus_r)

cat("thus we can claim with condidence coefficient
   0.95 that the median of x is between y40 and y60"
   )
```

R code Exa 8.21 testing the hypothesis related to voltage

```
#page no. 321
#example 8-21

x_bar=110.12
v0=110
n=25    #number of times V is measured

    #part (a)

sigma=0.4
z=2
q=(x_bar-v0)/(sigma/sqrt(n))
```

```
print(q)

cat("since", q, "is in the interval(-2,2), we accept
    H0")


    #part (b)

s=0.6
q=(x_bar-v0)/(s/sqrt(n))
print(q)
cat("since", q, "is in the interval(-2.06,2.06), we
    accept H0")
```

R code Exa 8.22 testing the hypothesis that a coin is fair against the hypothesis that it is loaded in favor of heads

```
#page no. 360
#example 8-22

#functions used
#qnorm():quantile function of the normal
    distribution:the quantile function maps from
    probabilities to values in normal distribution


alpha=0.05
z=qnorm(1-alpha)

q=(62-50)/sqrt(25)


cat("since q=",q,">",z,"the fair coin hypothesis is
    rejected")
```

R code Exa 8.24 Testing the hypothesis that the die is fair

```r
#page no. 362
#example 8-24

k=c(55,43,44,61,40,57)            #(given)
np0=50         #(given)
q=sum(((k-np0)^2)/np0)

x=qchisq(0.95,5)        #qchisq() is the function used to calculate Chi-square percentile value in R

cat("Since (X0.95(5))^2 =",x,">",q," We accept the fair-die hypothesis")
```

R code Exa 8.25 testing the independence hypothesis related to graduate students of a certain university

```r
#page no. 363
#example 8-25

ki=c(168,68,131,33)
poi=c(0.45,0.15,0.3,0.1)

q=sum(((ki-400*poi)^2)/(400*poi))

x=qchisq(0.95,3)        #qchisq() is the function used to calculate Chi-square percentile value in R

```

```
cat("Since (X0.95(3))^2 =",x,">",q," We accept the
   indepndence hypothesis")
```

R code Exa 8.26 testing the uniformity hypothesis related to computer generated decimal numbers

```
#page no. 364
#example 8-26

ki=c(43,56,42,38,59,61,41,57,46,57)      #(given)
m=500
poi=0.1

q=sum(((ki-m*poi)^2)/(m*poi))

x=qchisq(0.95,9)        #qchisq() is the function
   used to calculate Chi-square percentile value in
   R

cat("Since (X0.95(9))^2 =",x,">",q," We accept the
   uniformaity hypothesis")
```

Chapter 9

General Concepts

R code Exa 9.10 various probabilities related to normal process

```
#page no. 386
#example 9-10

#part (a)

inte<-function(y){exp(-(y^2)/2)}     #these two lines (6 and 7)  gives the defination of function G(x)
Gx<-function(x){((1/(sqrt(2*3.14)))*integrate(inte,lower = 0,upper = x)[[1]])+0.5}     #which is given on page number 106

p=Gx(-1/2)
print(p)

#part (b)

c=8*(1-exp(-0.6))   #variance
print(c)

```

```
p=2*Gx(1/1.9) - 1
print(p)
```

Chapter 14

Entropy

R code Exa 14.1 calculation of entropy in a fair die experiment

```
#page no. 630
#example 14-1

    #part (a)

Hu=(-1/2*log(1/2))-(1/2)*log(1/2)

print(Hu)

#which is log(2)


    #part(b)

hv=(-1/6*log(1/6))-(1/6)*log(1/2)-(1/6)*log(1/6)-(1/6)*log(1/6)-(1/6)*log(1/6)-(1/6)*log(1/6)

print(hv)

#which is log(6)
```

R code Exa 14.2 calculation of entropy in a coin experiment

```
#page no. 631
#example 14-2

hp=function(p){-p*log(p)-(1-p)*log(1-p)}

plot(hp,xlab="p",ylab = "h(p)")

print("The function h(p) is ploted for 0 <= p <= 1.
   This function is symmetrical, convex, even about
   the point p=0.5 and it reaches its maximum at
   that point. ")
print("furthermore, h(0)=h(1)=0")
```

R code Exa 14.4 relation between entropy of partitions

```
#page no. 641
#example 14-4

p=0.4        #(given)
pa=0.22      #(given)
pb=0.18      #(given)

hu=-((p*log(p))+(0.35*log(0.35))+(0.25*log(0.25)))
print(hu)

hb=-(pa*log(pa)+pb*log(pb)+0.35*log(0.35)+0.25*log
   (0.25))
print(hb)

cat("thus H(U)=",hu,"<",hb,"= H(B)")
```

```
#the answers in the book are slightly different. I
   looks to me that they are wrong because I have
   done exactly same calculation and answers are
   little different.
```

R code Exa 14.7 condition entropy of a partition in fair die experiment

```
#page no. 645
#example 14−7

p_fi_even_ieven=1/3    #if i is even
p_fi_even_iodd=0       #if i is odd
p_fi_odd_iodd =1/3     #if i is odd
p_fi_odd_ieven=0       #if i is even

hv_even=-(1/3*log(1/3) + 1/3*log(1/3) + 1/3*log(1/3)
   )    #which is log(3)
print(hv_even)   #which is log(3)

hv_b=0.5*log(3) + 0.5*log(3)   #which is log(3)

print(hv_b)     #which is log(3)

cat("Thus, in the absence of any information, our
   uncertainty about V equals H(V)=log6.")
print("If we know, however, whether at each trial '
   even' or 'odd' showed, then our uncertainty is
   reduced to H(v|B)) = log3")
```

R code Exa 14.8 information about an element partition

```
#page no. 647
```

```
#example 14-8

H_V=log(6)      #given
H_VB=log(3)     #given
H_B=log(2)      #given
H_BV=0          #given

I_VB=log(2)     #this is obtain just by observing

print("Thus the information about the element
   partition V resulting from the observation of the
    even-odd partition B equals log2")
```