

R Textbook Companion for  
Probability and Statistics for Engineering and  
the Sciences  
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# Book Description

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R numbering policy used in this document and the relation to the above book.

**Exa** Example (Solved example)

**Eqn** Equation (Particular equation of the above book)

For example, Exa 3.51 means solved example 3.51 of this book. Sec 2.3 means an R code whose theory is explained in Section 2.3 of the book.

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# Chapter 1

## Overview and descriptive statistics

R code Exa 1.1 Branches of statistics

```
1 #Ex1.1, Page 4
2
3 library(lattice)
4
5 data<-c
      (6.1,12.6,34.7,1.6,18.8,2.2,3.0,2.2,5.6,3.8,2.2,3.1,1.3,1.1,14.1,
6
7 stem(data,scale=2)
8
9 hist(data,main="Histogram for charity fundraising
      percentage data",xlab="FundRaising",col="grey",
      xlim=c(0,100),ylim=c(0,40))
```

---

R code Exa 1.5 Collecting data

```

1 #Ex1.5, Page 11
2
3 Adhesive_Type<-c(1,1,2,2)
4 Conductor_material<-c(1,2,1,2)
5 Observed_bond_strength<-c(82,75,84,78,77,87,80,90)
6 mat1<-matrix(Observed_bond_strength,nrow=4,ncol=2)
7 Average<-c(sum(mat1[1,])/2,sum(mat1[2,])/2,sum(mat1
  [3,])/2,sum(mat1[4,])/2)
8 mat1<-cbind(Adhesive_Type,Conductor_material,mat1,
  Average)
9 colnames(mat1)[3]<-"Observed bond strength"
10 print(mat1)
11
12 n<-length(Adhesive_Type)/2
13 plot(Conductor_material,Average,main="Average bond
  strengths",xlab="Conducting material",ylab="
  Average strength")
14 segments(Conductor_material[1],Average[1],Conductor_
  material[2],Average[2])
15 segments(Conductor_material[3],Average[3],Conductor_
  material[4],Average[4])

```

---

### R code Exa 1.8 Dotplots

```

1 #Ex1.8, Page 15
2
3 library(graphics)
4
5 data<-c
  (10.8,6.9,8.0,8.8,7.3,3.6,4.1,6.0,4.4,8.3,8.1,8.0,5.9,5.9,7.6,8.9
6
7 #To create a stacked dotplot
8 stripchart(data,method="stack",offset=0.5,pch=20,at
  =0,main="Dotplot of given data")

```

---

## R code Exa 1.9 Histograms

```
1 #Ex1.9 , Page 17
2
3 Hits_per_game<-0:27
4 No_of_games<-c
   (20,72,209,527,1048,1457,1988,2256,2403,2256,1967,1509,1230,834,56
5
6 sum1<-sum(No_of_games)
7 Relative_frequency<-round(No_of_games/sum1,digits=4)
8 df1<-data.frame(Hits_per_game,No_of_games,Relative_
   frequency)
9 print(df1)
10
11 #Proportion of games with atmost 2 hits
12 f1<-0
13 for(i in Hits_per_game){
14   if(i<=2){
15     f1<-f1+df1[i+1,3]
16   }
17 }
18 print(paste("Proportion of games with atmost two
   hits:",f1))
19
20 #Proportion of games with between 5 and 10 hits
21 f2<-0
22 for(j in Hits_per_game){
23   if(j>=5 && j<=10){
24     f2<-f2+df1[j+1,3]
25   }
26 }
27 print(paste("Proportion of games with between 5 and
   10 hits:",f2))
```

```

28 par(mfrow=c(1,2))
29 plot(Hits_per_game,Relative_frequency,type="h",xlim=
      c(0,20),main="Histogram of hits/game")
30 barplot(Relative_frequency,width=1,space=0,xlim=c
      (0,25),main="Histogram of hits/game",xlab="Hits/
      game",ylab="Relative frequency")

```

---

### R code Exa 1.10 Histograms

```

1 #Ex1.10, Page 18
2 #Answers may vary since different class intervals
  have been taken
3
4 #Please install and load the following packages: 1)
  HistogramTools 2)ash
5 library(HistogramTools)
6 library(ash)
7
8 data<-c
  (2.97,4.00,5.20,5.56,5.94,5.98,6.35,6.62,6.72,6.78,6.80,6.85,6.94
9
10 #Class intervals can also be created using
  classIntervals() from ClassInt library:
11 #classIntervals(data,style="pretty",intervalClosure
  ="left") which creates different intervals as
  compared to those of textbook
12
13 #To create class intervals
14 ci<-seq(1,19,2)
15
16 freq<-table(cut(data,ci,right=FALSE))
17 rel_freq<-freq/length(data)
18 print(cbind(freq,rel_freq))
19

```



```

20 #To display histogram
21 PlotRelativeFrequency(hist(data,breaks = ci,right=
    FALSE,plot=FALSE),xlab="BTUIN",main="Histogram of
    energy consumption data",ylim=c(0,0.30))

```

---

### R code Exa 1.11 Histograms

```

1 #Ex1.11, Page 20
2
3 data<-c
    (11.5,12.1,9.9,9.3,7.8,6.2,6.6,7.0,13.4,17.1,9.3,5.6,5.7,5.4,5.2,
4 #Unequal class widths
5 ci<-c(2,4,6,8,12,20,30)
6
7 #To find frequency, relative frequency and density
    of data manually
8 freq<-table(cut(data,ci,right=FALSE))
9 rel_freq<-freq/length(data)
10 density<-rel_freq/diff(ci)
11
12 print(cbind(freq,rel_freq,density))
13
14 #To create histogram of the bond strength data
15 hist(data,breaks=ci,freq=FALSE,right=FALSE,main="
    Histogram for bond strength data",xlab="Bond
    Strength")

```

---

### R code Exa 1.13 Qualitative data

```

1 #Ex1.13, Page 23
2 #Answers may vary slightly due to rounding off of
    values

```

```

3
4 Rating<-c("A","B","C","D","F","Don't know")
5 Frequency<-c(478,893,680,178,100,172)
6 Relative_frequency<-Frequency/sum(Frequency)
7 df1<-data.frame(Rating,Frequency,Relative_frequency)
8 print(df1)
9
10 print(paste("Total frequency:",sum(Frequency)))
11 print(paste("Total relative frequency:",sum(Relative
    _frequency)))
12
13 barplot(df1$Relative_frequency,space=1,names.arg=df1
    $Rating,ylim=c(0,0.4),xlab="Rating",ylab="
    Relative Frequency",main="Chart of relative
    frequency vs. rating")

```

---

#### R code Exa 1.14 Mean

```

1 #Ex1.14, Page 29
2 #Steam and leaf plot differs since number of stem
    parts=1
3
4 library(aplpack)
5
6 x<-c
    (16.1,9.6,24.9,20.4,12.7,21.2,30.2,25.8,18.5,10.3,25.3,14.0,27.1,
7
8 sl<-stem.leaf(x,unit=0.1,m=1,style="bare")
9
10 s<-sum(x)
11 mean<-s/length(x)
12 print(paste("Mean value:",mean))
13 print(paste("The value",sl$upper,"is an outlier"))
14 print(paste("Mean value when",sl$upper,"is excluded="

```

```
”,(s-45)/(length(x)-1)))
```

---

### R code Exa 1.15 Median

```
1 #Ex1.15, Page 30
2 #Answers may vary slightly due to rounding off of
  values
3
4 data<-c
  (62.3,62.8,63.6,65.2,65.7,66.4,67.4,68.4,68.8,70.8,75.7,79.0)

5
6 dotplot(data,xlab="Duration")
7
8 n<-length(data)
9 n1<-data[n/2]
10 n2<-data[(n/2)+1]
11 med<-(n1+n2)/2
12 print(paste("Median:",med))
13
14 max<-max(data)
15 #If maximum value is omitted
16 data1<-data[1:length(data)-1]
17 mean1<-mean(data1)
18 print(paste("Mean when maximum value,",max," is
  omitted:",mean1))
```

---

### R code Exa 1.17 Measures of variability for sample data

```
1 #Ex1.17, Page 36
2
3 Car<-1:11
```

```

4  xi<-c
    (27.3,27.9,32.9,35.2,44.9,39.9,30.0,29.7,28.5,32.0,37.6)

5
6  xibar<-mean(xi)
7  s<-sd(xi)
8
9  print(paste("The size of the representative
    deviation from the mean,",round(xibar,digits=2),"
    is roughly",round(s,digits=2),"mpg"))

```

---

#### R code Exa 1.18 Measures of variability

```

1  #Ex1.18, Page 38
2  #Answers may vary slightly due to rounding off of
    values
3
4  data<-c
    (154,142,137,133,122,126,135,135,108,120,127,134,122)

5
6  print(paste("s:",sd(data)))

```

---

#### R code Exa 1.19 Boxplots

```

1  #Ex1.19, Page 40
2
3  obs<-c
    (40,52,55,60,70,75,85,85,90,90,92,94,94,95,98,100,115,125,125)

4
5  #To find five-number summary
6  print(summary(fivenum(obs)))

```

```
7
8 boxplot(obs,horizontal=TRUE,xlab="Depth",main="
  Boxplot of the corrosion data")
```

---

**R code Exa 1.20** Box plots that show outliers

```
1 #Ex1.20, Page 41
2
3 data<-c
  (9.69,13.16,17.09,18.12,23.70,24.07,24.29,26.43,30.75,31.54,35.07
4
5 #To find relevant summary quantities
6 print(summary(fivenum(data)))
7
8 c<-seq(0,1600,by=200)
9
10 boxplot(data,horizontal=TRUE,xlab="Daily nitrogen
  load",main="Boxplot of daily nitrogen load data
  showing mild and extreme outliers")
```

---

# Chapter 2

## Probability

**R code Exa 2.8** Relations from set theory

```
1 #Ex2.8, Page 53
2
3 A<-c(0,1,2,3,4)
4 B<-c(3,4,5,6)
5 C<-c(1,3,5)
6 U<-union(A,B)
7
8 cat("A'=",setdiff(U,A),"\\n")
9 cat("A U B =",union(A,B),"\\n")
10 cat("A U C =",union(A,C),"\\n")
11 cat("A n B =",intersect(A,B),"\\n")
12 cat("A n C =",intersect(A,C),"\\n")
13 cat("(A n C)'=",setdiff(U,intersect(A,C)))
```

---

**R code Exa 2.11** Properties of probability

```
1 #Ex2.11, Page 56
2
```

```

3 #S={U,D}
4
5 p<-seq(0,1,by=0.25)
6
7 for(i in p){
8   pu<-i
9   pd<-1-i
10  cat("P(U) :",pu,"\n")
11  cat("P(D) :",pd,"\n\n")
12 }
13 cat("p value can represent any fixed value between 0
      and 1")

```

---

#### R code Exa 2.13 Properties of probability

```

1 #Ex2.13, Page 59
2 #Answers may vary slightly due to rounding off of
  values
3
4 #For A to occur, at least one of the individual
  components must fail
5 #To find P(A')=SSSSS
6 p<-90/100
7 pa_dash<-p^5
8 cat("P(A') :",pa_dash,"\n")
9
10 pa<-1-pa_dash
11 cat("P(A) :",1-pa_dash,"\n\n")
12 cat("Roughly",round(pa*100,digits=0),"% of the
      systems will fail")

```

---

#### R code Exa 2.14 Properties of probability

```

1 #Ex2.14, Page 60
2
3 #P(A)=>probability that a household gets internet
  service
4 pa<-60/100
5
6 #P(B)=>probability that a household gets TV service
7 pb<-80/100
8
9 #P(A n B)=>probability that a household gets both
  services
10 pab<-50/100
11
12 #To find probability that a household gets at least
  one of the services
13 p1<-pa+pb-pab
14 cat("P(at least one service)=P(A U B) =",p1,"\n")
15
16 pa_b<-p1-pa
17 pb_a<-p1-pb
18 #To find probability that a household gets exactly
  one service
19 p2<-pa_b+pb_a
20 cat("P(exactly one service)=",p2)

```

---

#### R code Exa 2.19 Product rule

```

1 #Ex2.19, Page 67
2
3 #No of appliance dealers
4 n1<-5
5
6 #No of plumbing contractors
7 n2<-12
8

```



```

9 #No of electrical contractors
10 n3<-9
11
12 cat("There are",n1*n2*n3,"ways to first choose an
    appliance dealer, then a plumbing contractor and
    finally an electrical contractor")

```

---

#### R code Exa 2.21 Permutations and combinations

```

1 #Ex2.21, Page 68
2
3 #No of teaching assistants
4 n<-10
5 #No of questions in the first exam
6 k<-4
7
8 #To find number of permutations
9 p<-factorial(n)/factorial(n-k)
10 cat("The professor could give",p,"different four-
    question exam papers without using the same
    graders")

```

---

#### R code Exa 2.22 Permutations and combinations

```

1 #Ex2.22, Page 69
2 #Answers may vary slightly due to rounding off of
    values
3
4 #Total no of songs
5 n<-100
6
7 #No of songs by the Beatles
8 m<-10

```

```

9
10 #Probability that the 1st Beatles song heard is the
    5th song played
11 t<-5
12 p1<-(factorial(n-m)/factorial(n-m-(t-1))*m/(
    factorial(n)/factorial(n-t)))
13 cat("P(1st B is 5th song played):",p1,"\n")
14
15 #Probability that the 1st Beatles song heard is the
    1st,2nd,3rd,4th or 5th song played
16 p2<-0
17 for(i in 1:5){
18     p2<-p2+(choose(n-i,m-1))/(choose(n,m))
19 }
20 cat("P(1st B is 1st,2nd,3rd,4th or 5th song played):
    ",p2,"\n")

```

---

### R code Exa 2.23 Permutations and combinations

```

1 #Ex2.23, Page 70
2 #Answers may vary slightly due to rounding off of
    values
3
4 #Total no of printers
5 n<-25
6 l<-10
7 i<-15
8
9 #Probability that 6 random printers selected will
    have 3 laser printers
10 r1<-6
11 r<-3
12 ND3<-(choose(l,r)*choose(i,r))/choose(n,r1)
13 cat("P(3 laser printers are selected among the 6
    selected):",ND3,"\n")

```

```

14
15
16 #To find the probability that at least 3 inkjet
    printers are selected
17 prob<-0
18 for(j in 3:6){
19     num<-choose(i,j)*choose(l,r1-j)
20     denom<-choose(n,r1)
21     prob<-prob+(num/denom)
22 }
23 cat("P(at least 3 inkjet printers are selected):",
    prob,"\n")

```

---

#### R code Exa 2.25 Conditional probability

```

1 #Ex2.25, Page 74
2
3 #Probability that memory card is purchased
4 pa<-0.6
5
6 #Probability that battery is purchased
7 pb<-0.4
8
9 #Probability that both memory card and battery is
    purchased
10 pab<-0.3
11
12 #Probability that an optional card was purchased
    given that individuals purchased extra battery
13 pa_b<-pab/pb
14 cat("P(A|B)=",pa_b,"\n")
15 cat("Of all individuals who purchased an extra
    battery,",pa_b*100,"% purchased an optional
    memory card\n")
16

```

```

17 #Probability that an additional battery was
    purchased given that individuals purchased an
    optional memory card
18 pb_a<-pab/pa
19 cat("P(B|A)=" ,pb_a,"\\n")
20
21 if(pa_b!=pb_a)  print(paste("P(A|B) is not equal to
    P(B|A)"))

```

---

### R code Exa 2.26 Conditional probability

```

1 #Ex2.26, Page 75
2 #Answers may vary slightly due to rounding off of
    values
3
4 #Package to be installed: VennDiagram
5 library(VennDiagram)
6
7 #A=>Arts , B=>Books , C=>Cinema
8 pa<-0.14
9 pb<-0.23
10 pc<-0.37
11 pab<-0.08
12 pbc<-0.13
13 pac<-0.09
14 pabc<-0.05
15
16 #To illustrate the Venn diagram using the given
    probabilities
17 draw.triple.venn(area1=pa,area2=pb,area3=pc,n12=pab,
    n23=pbc,n13=pac,n123=pabc,category=c("Arts","
    Books","Cinema"))
18
19 cat("With the help of Venn diagram:\\n")
20 #P(A|B)

```

```

21 pa_b<-pab/pb
22 print(paste("P(A|B)=",pa_b))
23
24 #P(A|B U C)
25 pa_bUc<-(pab+pac-pabc)/(pb+pc-pbc)
26 print(paste("P(A|B U C)=",pa_bUc))
27
28 #P(A|reads at least once)=P(A|A U B U C)
29 pa_aUbUc<-pa/(pa+pb+pc-pab-pac-pbc+pabc)
30 print(paste("P(A|reads at least once)=",pa_aUbUc))
31
32 #P(A U B|C)
33 paUb_c<-(pac+pbc-pabc)/pc
34 print(paste("P(A U B|C)=",paUb_c))

```

---

### R code Exa 2.30 Bayes Theorem

```

1 #Ex2.30 , Page 78
2
3 # % of messages which come into account 1
4 pa1<-70/100
5
6 # % of messages which come into account 2
7 pa2<-20/100
8
9 # % of messages which come into account 3
10 pa3<-10/100
11
12 # % of messages which are spam in account 1
13 pb_a1<-1/100
14
15 # % of messages which are spam in account 2
16 pb_a2<-2/100
17
18 # % of messages which are spam in account 3

```

```

19 pb_a3<-5/100
20
21 #To find the probability that a randomly selected
    message is spam
22 #Using law of total probability
23 pb<-pa1*pb_a1+pa2*pb_a2+pa3*pb_a3
24 print(paste("P(Randomly selected message is spam):",
    pb,"and thus",pb*100,"% messages will be spam"))

```

---

### R code Exa 2.36 Independence of more than two events

```

1 #Ex2.36, Page 85
2 #Answers may vary slightly due to rounding off of
    values
3
4 pai<-rep(0.9,times=6)
5
6 #To find probability that system lifetime exceeds t0
7 prob1<-pai[1]*pai[2]*pai[3]+pai[4]*pai[5]*pai[6]-
    prod(pai)
8 print(paste("P(system lifetime exceeds t0):",prob1))
9
10 #To find probability that system lifetime is at
    least t0
11 prob2<-(1-(1-pai[1])*(1-pai[2]))^3
12 print(paste("P(system lifetime is at least t0):",
    prob2))

```

---

## Chapter 3

# Discrete random variables and probability distributions

**R code Exa 3.7** Probability distributions for discrete random variables

```
1 #Example 3.7, Page 97
2 #Storing all probabilities in an array
3 prob<-c(0.05,0.1,0.15,0.25,0.20,0.15,0.10)
4 P<-array(c(prob),dim=c(1,7,1))
5
6 #P(X<=2)=P(X=0)+P(X=1)+P(X=2)
7 p1<-P[1,1,1]+P[1,2,1]+P[1,3,1]
8 print(paste("P(X<=2) =",p1))
9
10 #P(X>=3)=1-P(X<=2)
11 p2<-1-(P[1,1,1]+P[1,2,1]+P[1,3,1])
12 print(paste("P(X>=3) =",p2))
13
14 #P(2<=X<=5)=P(X=2,3,4 or 5)
15 p3<-P[1,3,1]+P[1,4,1]+P[1,5,1]+P[1,6,1]
16 print(paste("P(2<=X<=5) =",p3))
17
18 #P(2<X<5)=P(X=3,4)
19 p4<-P[1,4,1]+P[1,5,1]
```

```
20 print(paste("P(2<X<5) =",p4))
```

---

**R code Exa 3.8** Probability distributions for discrete random variables

```
1 #Example 3.8, Page 97
2 #The answers may slightly vary due to rounding off
  of values
3 p1<-c(1,0,2,2,3,0,4,1,5,2,6,0)
4 row.names<-c("Lot","Number of defectives")
5 column.names<-c("","","","","","","")
6 matrix.names<-c("Number of defectives in each lot")
7 p<-array(c(p1),dim=c(2,6,1),dimnames=list(row.names,
  column.names,matrix.names))
8 print(p)
9 #p(0)=P(X=0)
10 c<-0
11 for(i in 1:6) {
12   d<-p[2,i,1]
13   if(d==0){
14     c<-c+1
15   }
16 }
17 d<-c/6
18 print(paste("p(0)=",d))
19
20 #p(1)=P(X=1)
21 c<-0
22 for(i in 1:6){
23   d<-p[2,i,1]
24   if(d==1){
25     c<-c+1
26   }
27 }
28 d<-c/6
29 print(paste("p(1)=",d))
```



```

30
31 #p(2)=P(X=2)
32 c<-0
33 for(i in 1:6){
34     d<-p[2,i,1]
35     if(d==2){
36         c<-c+1
37     }
38 }
39 d<-c/6
40 print(paste("p(2)=",d))

```

---

### R code Exa 3.9 Probability distributions for discrete random variables

```

1 #Ex3.9, Page 98
2 #X=0 if customer purchases laptop computer
3 #X=1 if customer purchases desktop computer
4 #p(0)
5 p0<-80/100
6 print(paste("Probability that next customer
  purchases laptop model:",p0))
7 p1<-20/100
8 print(paste("Probability that next customer
  purchases desktop model:",p1))
9 p1<-c(0.8,0.2,0)
10 row.names<-c("if x=0","if x=1","if (x!=0 or 1)")
11 column.names<-c("")
12 matrix.names<-c("p(x)")
13 p<-array(c(p1),dim=c(3,1,1),dimnames=list(row.names,
  column.names,matrix.names))
14 print(p)
15 a<- 1
16 p<- dbinom(a, size = 1, prob = 0.2)
17 plot(1, p, type = 'h', xlab = 'x', ylab = 'p(x)')

```

---

**R code Exa 3.10** Probability distributions for discrete random variables

```
1 #Ex3.10, Page 98
2 #Let Y=>O+ blood type and N=>Other blood types
3 p1<-c('a','Y','b','Y','c','N','d','N','e','N')
4 row.names<-c("Blood donors","O+ blood donors")
5 column.names<-c("","","","","")
6 matrix.names<-c("Blood donation table")
7 p<-array(c(p1),dim=c(2,5,1),dimnames=list(row.names,
      column.names,matrix.names))
8 print(p)
9 #p(1)=P(X=1)
10 c<-0
11 for(i in 1:5) {
12   d<-p[2,i,1]
13   if(d=="Y"){
14     c<-c+1
15   }
16 }
17 d1<-c/5
18 print(paste("p(1)=",d1))
19
20 #p(2)=P(X=2)
21 g<-0
22 e<-0
23 for(i in 1:5) {
24   h<-p[2,i,1]
25   if(h=="Y"){
26     g<-g+1
27   }
28   else{
29     e<-e+1
30   }
31 }
```

```

32 h<-g/5
33 f<-e/4
34 d2<-h*f
35 print(paste("p(2)=",d2))
36
37 #p(3)=P(X=3)
38 d3<-(e/5)*((e-1)/4)*(c/3)
39 print(paste("p(3)=",d3))
40
41 #p(4)=P(X=4)
42 d4<-(e/5)*((e-1)/4)*((e-2)/3)
43 print(paste("p(4)=",d4))
44
45 p2<-c(1,d1,2,d2,3,d3,4,d4)
46 row.names<-c("y","p(y)")
47 column.names<-c("","","","")
48 matrix.names<-c("PMF")
49 q<-array(c(p2),dim=c(2,4,1),dimnames=list(row.names,
        column.names,matrix.names))
50 print(q)
51
52 py<- 1:4
53 y<- c(0.4,0.3,0.2,0.1)
54 plot(py,y,type="h",main="Line graph for pmf",xlab="y
    ",ylab="p(y)")

```

---

### R code Exa 3.13 Cumulative distribution function

```

1 #Ex3.13, Page 101
2
3 y<-c(1,2,4,8,16)
4 p<-c(0.05,0.1,0.35,0.4,0.1)
5
6 #F(1)=P(Y<=1)=p(1)
7 print(paste("F(1)=",cumsum(p[1])))

```

```

8
9 #F(2)=P(Y<=2)=p(2)
10 print(paste("F(2)=",max(cumsum(p[1:2]))))
11
12 #F(4)=P(Y<=4)=p(4)
13 print(paste("F(4)=",max(cumsum(p[1:3]))))
14
15 #F(8)=P(Y<=8)=p(8)
16 print(paste("F(8)=",max(cumsum(p[1:4]))))
17
18 #F(16)=P(X<=16)=p(16)
19 print(paste("F(16)=",max(cumsum(p[1:5]))))

```

---

**R code Exa 3.15** Cumulative distribution function of discrete random variable

```

1 #Ex3.15, Page 104
2 a<-c(0,0.58,1,0.72,2,0.76,3,0.81,4,0.88,5,0.94)
3 row.names<-c("X","F(X)")
4 column.names<-c("","","","","","")
5 matrix.names<-c("Number of days of sick leave taken
  by a randomly selected employee")
6 b<-array(c(a),dim=c(2,6,1),dimnames=list(row.names,
  column.names,matrix.names))
7 print(b)
8
9 #P(2<=X<=5)=P(X=2,3,4,5)
10 p<-b[2,6,1]-b[2,2,1]
11 print(paste("P(2<=X<=5)=",p))
12
13 #P(X=3)
14 q<-b[2,4,1]-b[2,3,1]
15 print(paste("P(X=3)=",q))

```

---

**R code Exa 3.16** Expected value of X

```
1 #Ex3.16, Page 107
2
3 x<-1:7
4 w<-c(0.01,0.03,0.13,0.25,0.39,0.17,0.02)
5
6 #Mean, m=sum of(xp(x))
7 m<-weighted.mean(x,w)
8 print(paste("Population mean =",m))
```

---

**R code Exa 3.17** Expected value of X

```
1 #Ex3.17, Page 107
2
3 x<-0:10
4 w<-c
   (0.002,0.001,0.002,0.005,0.02,0.04,0.18,0.37,0.25,0.12,0.01)
5
6 #Mean, m= sum of(xp(x))
7 m<-weighted.mean(x,w)
8 print(paste("Population mean, E(X) =",m))
```

---

**R code Exa 3.23** Expected value of a function

```
1 #Ex3.23, Page 110
2
3 comp<-c(0,0.1,1,0.2,2,0.3,3,0.4)
```

```

4 row.names<-c("X", "p(X)")
5 column.names<-c("", "", "", "")
6 matrix.names<-c("X->Number of computers sold")
7 p<-array(c(comp), dim=c(2,4,1), dimnames=list(row.
    names, column.names, matrix.names))
8 print(p)
9
10 x<-0:3
11 w<-c(0.1,0.2,0.3,0.4)
12 #h(x)<-profit associated with selling x units<-
    revenue-cost<-800x-900
13 print(paste("Expected profit:$", weighted.mean(800*x
    -900, w)))

```

---

#### R code Exa 3.24 The variance of X

```

1 #Ex3.24, Page 111
2 #Answers may slightly vary due to rounding off of
    values
3
4 w<-c(0.3,0.25,0.15,0.05,0.1,0.15)
5 x<-c(1,2,3,4,5,6)
6
7 #Mean, m= sum of(xp(x))
8 m<-weighted.mean(x, w)
9 print(paste("Mean =", m))
10
11 #Variance, v= sum of(x-m)p(x)
12 v<-weighted.mean((x-m)^2, w)
13 print(paste("Variance =", v))
14
15 #Standard deviation, sd= square root(variance)
16 print(paste("Standard deviation=", sqrt(v)))

```

---

**R code Exa 3.25** The variance of X

```
1 #Ex3.25, Page 112
2
3 #Alternative formula for variance,  $v = E(X^2) - (m^2)$ 
4 x<-1:6
5 w<-c(0.3,0.25,0.15,0.05,0.1,0.15)
6
7 #E(X^2)= sum of (X^2)p(X)
8 e<-weighted.mean(x^2,w)
9 print(paste("E(X^2)=" ,e))
10
11 #Variance
12 mean<-weighted.mean(x,w)
13 print(paste("Variance through alternative formula ="
              ,e-(mean^2)))
```

---

**R code Exa 3.26** Rules of variance

```
1 #Ex3.26, Page 113
2
3 x<-0:3
4 w<-c(0.1,0.2,0.3,0.4)
5
6 #Mean, m = sum of (xp(x))
7 m<-weighted.mean(x,w)
8 print(paste("Mean, E(X)=" ,m))
9
10 #To find E(X^2)
11 e<-weighted.mean(x^2,w)
12 print(paste("E(X^2)=" ,e))
13
```

```

14 #To find V(X)
15 v<-e-(m^2)
16 print(paste("V(X)=",v))
17
18 #Profit function , h(X)=800X-900
19 #Variance
20 print(paste("Variance=", (800^2)*v))
21
22 #Standard deviation
23 print(paste("Standard deviation =",sqrt((800^2)*v)))

```

---

#### R code Exa 3.29 Binomial experiment

```

1 #Ex3.29, Page 115
2 #Number of licensed restaurants
3 n<-50
4 #Number of restaurants having at least one serious
   health code violation
5 f<-15
6 #Number of restaurants having no serious health code
   violations
7 s<-35
8 #P(s on first trial)
9 p1<-s/n;
10 print(paste("P(s on first trial)=",p1))
11
12 #P(s on second trial)=P(ss)+P(fs)
13 p2<-(s/n)*((s-1)/(n-1))+(f/n)*(s/(n-1))
14 print(paste("P(s on second trial)=",p2))
15 print(paste("Similarly , P(s on ith trial) = 0.7
   for i=3,4,5"))
16
17 #P(s on fifth trial|ssss)
18 p3<-(s-4)/(n-4)
19 print(paste("P(s on fifth trial|ssss)=",p3))

```



```

20
21 #P(s on fifth trial|ffff)
22 p4<-s/(n-4)
23 print(paste("P(s on fifth trial|ffff)=",p4))

```

---

#### R code Exa 3.30 Binomial experiment

```

1 #Ex3.30, Page 116
2 #Number of licensed drivers
3 n<-500000
4 #Number of licensed drivers who are insured
5 s<-400000
6 #P(s on second trial after s on first trial)
7 p1<-(s-1)/(n-1)
8 print(paste("P(s on 2|s on 1)=",p1))
9
10 #P(s on tenth trial after s on ninth trial)
11 p2<-(s-9)/(n-9)
12 print(paste("P(s on 10|s on 9)=",p2))
13
14 print(paste("The experiment is binomial with n=10
    and p=0.8"))

```

---

#### R code Exa 3.31 Binomial random variable and distribution

```

1 #Ex3.31, Page 118
2 #Answers may slightly vary due to rounding off of
  values
3
4 #n=6, p=0.5
5
6 #P(X=3)
7 print(paste("P(X=3)=",dbinom(3,size=6,prob=0.5)))

```

```

8
9 #Probability that at least 3 three prefer S is
10 print(paste("P(X>=3)=", pbinom(3, size=6, prob=0.5)))
11
12 #Probability that at most one prefers S is
13 print(paste("P(X<=1)=", pbinom(1, size=6, prob=0.5)))

```

---

#### R code Exa 3.32.1 Cumulative distribution function

```

1 #Example 3.32.1, Page 118
2 #Binomial distribution: n=15, p=2
3 #The answers may slightly vary due to rounding off
  of values
4
5 #Probability that at most 8 copies fail the test
6 #P(X<=8)
7 a=pbinom(8, size=15, prob=0.2)
8 print(paste("The probability that at most 8 copies
  fail the test is", a))

```

---

#### R code Exa 3.32.2 Using binomial tables

```

1 #Example 3.32.2, Page 118
2 #Binomial distribution: n=15, p=0.2
3 #The answers may slightly vary due to rounding off
  of values
4
5 #Probability that exactly 8 copies fail
6 #P(X=8)=P(X<=8)-P(X<=7)
7 a<-0.999
8 b=pbinom(7, size=15, prob=0.2)
9 print(paste("The probability that exactly 8 copies
  fail the test", a-b))

```

---

**R code Exa 3.32.3** Using binomial tables

```
1 #Example 3.32.3, Page 119
2 #Binomial distribution: n=15, p=0.2
3 #The answers may slightly vary due to rounding off
  of values
4
5 #Probability that at least 8 copies fail
6 #P(X>=8)=1-P(X<=7)
7 b<-0.996
8 c=1-b
9 print(paste("The probability that at least 8 copies
  fail", c))
```

---

**R code Exa 3.32.4** Using binomial tables

```
1 #Example 3.32.4, Page 119
2 #Binomial distribution: n=15, p=0.2
3 #The answers may slightly vary due to rounding off
  of values
4
5 #Probability that between 4 to 7 copies fail
6 #P(4<=X<=7)=P(X=4)+P(X=5)+P(X=6)+P(X=7)
7 d=pbinom(7,15,0.2)-pbinom(3,15,0.2)
8 print(paste("The probability that between 4 to 7
  copies fail is", d))
```

---

**R code Exa 3.33** Using binomial tables

```

1 #Ex3.33, Page 119
2 #Answers may vary slightly due to rounding off of
  values
3
4 #n=20
5 #P(X>=5) when p=0.10
6 p1<-1-pbinom(4,size=20,prob=0.1)
7 print(paste("P(X>=5 when p=.1)=",p1))
8
9 #P(X<=4) when p=0.2
10 p2<-pbinom(4,size=20,prob=0.2)
11 print(paste("P(X<=4 when p=.2)=",p2))

```

---

#### **R code Exa 3.34** Mean and variance of X

```

1 #Ex3.34, Page 120
2 n<-10
3 p<-0.75
4
5 #Mean, E(x)=np
6 e<-n*p
7 print(paste("Mean=",e))
8
9 #Variance, V(X)=npq
10 v<-n*p*(1-p)
11 print(paste("Variance=",v))
12
13 #Standard deviation=sqrt(V(X))
14 s<-sqrt(v)
15 print(paste("Standard deviation=",s))
16
17 #P(X= 7 or 8)=P(X=7)+P(X=8)
18 prob1=dbinom(0:7,size=10,prob=0.75)
19 prob2=dbinom(0:8,size=10,prob=0.75)
20 c<-max(prob1)+max(prob2)

```

```
21 print(paste("P(X= 7 or 8)=", c))
```

---

**R code Exa 3.35** Hypergeometric distribution

```
1 #Ex3.35, Page 123
2 #Answers may slightly vary due to rounding off of
  values
3
4 N<-20
5 n<-5
6 M<-12
7
8 #P(X=2)=h(2,5,12,20)
9 x<-2
10 h<-(choose(M,x)*choose(N-M,n-x))/choose(N,n)
11 print(paste("h(2;5,12,20)=", h))
```

---

**R code Exa 3.36** Hypergeometric distribution

```
1 #Ex3.36, Page 124
2 #Answers may slightly vary due to rounding off of
  values
3
4 N<-25
5 n<-10
6 M<-5
7
8 #P(X=2)=h(2,10,5,25)
9 x<-2
10 h1<-(choose(M,X)*choose(N-M,n-x))/choose(N,n)
11 print(paste("P(X=2)=", h1))
12
13 #P(X<=2)=P(X=0)+P(X=1)+P(X=2)
```

```

14 s<-0
15 for(i in 0:2){
16   h2<-(choose(M,i)*choose(N-M,n-i))/choose(N,n)
17   s<-s+h2
18 }
19 print(paste("P(X<=2)=" , s))

```

---

**R code Exa 3.37** Mean and variance of hypergeometric distribution

```

1 #Ex3.37 , Page 125
2
3 n<-10
4 M<-5
5 N<-25
6 p<-M/N
7
8 #E(X)=np
9 e<-n*p
10 print(paste("Mean=" , e))
11
12 #V(X)=npq
13 v<-n*p*(1-p)
14 print(paste("Variance without replacement=" , v))
15
16 #Estimate of N when M=5, n=10 and x=16
17 M<-100
18 n<-40
19 x<-16
20 ne<-(M*n)/x
21 print(paste("Estimate is" , ne))

```

---

**R code Exa 3.38** Negative binomial distribution

```

1 #Ex3.38, Page 126
2 #Answers may slightly vary due to rounding off of
  values
3
4 #P(X=10)=nb(10;5,0.2)
5 prob1=dnbinom(0:10,size=5,prob=0.2)
6 print(paste("P(X=10)=",max(prob1)))
7
8 #P(X<=10)
9 s<-pnbinom(0:10,size=5,prob=0.2)
10 print(paste("P(X<=10)=",max(s)))

```

---

#### R code Exa 3.39 Poisson probability distribution

```

1 #Ex3.39, Page 129
2 #Answers may slightly vary due to rounding off of
  values
3
4 u<-4.5
5 #Poisson distribution
6 #P(X=5)
7 p1<-dpois(5,lambda=u)
8 print(paste("P(X=5)=",p1))
9
10 #P(X<=5)
11 p2<-ppois(5,lambda=u)
12 print(paste("P(X<=5)=",p2))

```

---

#### R code Exa 3.40 Poisson distribution as a limit

```

1 #Ex3.40, Page 129
2 #Answers may vary slightly due to rounding off of
  values

```

```

3
4 n<-400
5 p<-0.005
6
7 #P(X=1): b(x;n,p)->p(x;u)
8 u<-n*p
9 p1<-dpois(1,lambda=u)
10 print(paste("P(X=1)=",p1))
11
12 #P(X<=3)
13 p2<-ppois(3,lambda=u)
14 print(paste("P(X<=3)=",p2))

```

---

**R code Exa 3.41** Mean and variance in Poisson distribution

```

1 #Ex3.41, Page 130
2 #Answer may slightly vary due to rounding off of
  value
3
4 u<-4.5
5 #Standard deviation
6 sd<-sqrt(u)
7 print(paste("Standard deviation=",sd))

```

---

**R code Exa 3.42** Poisson process

```

1 #Ex3.42, Page 131
2 #Answer may slightly vary due to rounding off of
  value
3
4 alpha<-6
5 t<-0.5
6 l<-alpha*t

```



```
7
8 #P(X>=1)=1-P(X=0)
9 p<-1-dpois(0,lambda=1)
10 print(paste("P(X>=1)=",p))
```

---

## Chapter 4

# Continuous random variables and probability distributions

**R code Exa 4.4** Probability density function

```
1 #Example 4.4 , Page 139*
2 #n=10,M=5,N=25
3
4 #f(x)>=0
5 #Area under density curve
6 a<-(1/360)*360
7
8 #P(90<=X<=180)
9 f1<-function(x) {(x^0)/360}
10 p1<-integrate(f1,lower=90,upper=180)
11 print(paste("Probability that angle is between 90
    and 180 is",p1$value))
12
13 #P(0<=X<=90)+P(270<=X<360)
14 f2<-function(x) {(x^0)/360}
15 p2<-integrate(f2,lower=0,upper=90)
16 f3<-function(x) {(x^0)/360}
17 p3<-integrate(f3,lower=270,upper=360)
18 p4<-p2$value+p3$value
```

```
19 print(paste("P(angle of occurrence is within 90
    degrees of reference line)",p4))
```

---

#### R code Exa 4.5 Uniform distribution

```
1 #Example 4.5, Page 141
2 #The answers may slightly vary due to rounding off
  of values
3
4 #Taking upper limit as 1000 instead of infinity
5 integrand<-function(x) {.15*exp(-.15*(x-.5))}
6 #f(x)>=0
7 #To show that integral from negative infinity to
  infinity is 1
8 c<-integrate(integrand,lower=0.5,upper=Inf)
9 #To access the list of values of integrate by name
10 print(paste("Value of integral is",c$value))
11
12 #To find P(X<5)
13 d<-integrate(integrand,lower=0.5,upper=5)
14 print(paste("Thus, probability that headway time is
    atmost 5 sec:",d$value))
```

---

#### R code Exa 4.7 Computing probabilities

```
1 #Ex4.7, Page 145*
2
3 #if x<0,          F(X)=0
4 #if 0<=x<=2,      F(X)=(x/8)+(3/16)*(x^2)
5 #if x>2,          F(X)=1
6
7 load<-function(x){
8   f<-(x/8)+(3/16)*(x^2)
```

```

9   return(f)
10 }
11
12 #P(1<=X<=1.5)=P(X=1.5)-P(X=1)
13 p1<-load(1.5)-load(1)
14 print(paste("P(1<=X<=1.5)=",p1))
15
16 #P(X>1)=1-P(X<=1)
17 p2<-1-load(1)
18 print(paste("P(load exceeds 1)=",p2))

```

---

#### R code Exa 4.10 Expected values

```

1 #Ex4.10 , Page 148
2
3 #if 0<=x<=1      f(x)= (3/2)(1-x^2)
4 #otherwise       f(x)= 0
5
6 f1<-function(x) {(3/2)*x*(1-x^2)}
7 p1<-integrate(f1,lower=0,upper=1)
8 print(paste("E(X)=",p1$value))

```

---

#### R code Exa 4.11 Expected value of a function

```

1 #Ex4.11 , Page 149
2 #if 0<=x<=1      f(x)=1
3 #otherwise       f(x)=0
4
5 #h(X)=max(1-X,X)
6 #if 0<=X<1/2     h(X)=1-X
7 #if 1/2<=X<=1    h(X)=X
8
9 #f<-function(x) {max(x,1-x)*f(x)}

```

```

10 #integrate(f, lower=0, upper=1)
11
12 f1<-function(x) {(1-x)*1}
13 p1<-integrate(f1, lower=0, upper=1/2)
14
15 f2<-function(x) {x*1}
16 p2<-integrate(f2, lower=0.5, upper=1)
17
18 p3<-p1$value+p2$value
19 print(paste("E[h(X)]=", p3))

```

---

**R code Exa 4.12** Variance and standard deviation

```

1 #Ex4.12, Page 150
2
3 #E(X)
4 e<-3/8
5
6 #E(X^2)
7 e2<-function(x) {(3/2)*(x^2)*(1-(x^2))}
8 p1<-integrate(e2, lower=0, upper=1)
9 print(paste("E(X^2)=", p1$value))
10
11 #V(X)
12 v<-(p1$value)-(e^2)
13 print(paste("V(X)=", v))
14
15 #Standard deviation
16 sd<-sqrt(v)
17 print(paste("Standard deviation =", sd))

```

---

**R code Exa 4.13.a** Standard normal distribution

```

1 #Example 4.13.a, Page 154
2 #The answers may slightly vary due to rounding off
  of values
3
4 #P(Z<=1.25)
5 a<-pnorm(1.25,0,1)
6 print(paste("The standard normal probability is",a))
7
8 #To plot the standard normal distribution bell curve
9 curve(dnorm, xlim = c(-3, 3), ylim = c(0, 0.4), xlab
    = "z", ylab="f(z)",lwd=2, col="black")
10 z = 1.25
11 pnorm(z)
12 x = c(-3, seq(-3, z, by=.001), z)
13 #plot(x)
14 y = c(0, dnorm(seq(-3, z, by=.001)), 0)
15 #plot(y)
16 polygon(x, y, col="red")

```

---

#### R code Exa 4.13.b Standard normal distribution

```

1 #Example 4.13.b, Page 154
2 #The answers may slightly vary due to rounding off
  of values
3
4 #P(Z>1.25)
5 #Variable to store result(1-P(Z<=1.25))
6 a=1-pnorm(1.25,0,1)
7 print(paste("The standard normal probability is", a)
  )
8
9
10 #To plot the standard normal distribution bell curve
11 curve(dnorm, xlim = c(-3, 3), ylim = c(0, 0.4), xlab
    = "z", ylab="f(z)",lwd=2, col="black")

```

```

12 z = 1.25
13 pnorm(z)
14 x = c(z, seq(z, 3, by=.001), 3)
15 #plot(x)
16 y = c(0, dnorm(seq(z, 3, by=.001)), 0)
17 #plot(y)
18 polygon(x, y, col="red")

```

---

### R code Exa 4.13.c Standard normal distribution

```

1 #Example 4.13.C, Page 154
2 #The answers may slightly vary due to rounding off
  of values
3
4 #P(Z<=-1.25)=P(Z>=1.25)
5 #Variable to store result(1-P(Z<=1.25))
6 a=1-pnorm(1.25,0,1)
7 print(paste("The standard normal probability is", a)
  )
8
9 #To plot the standard normal distribution bell curve
10 x=seq(-3,3,length=200)
11 y=dnorm(x)
12 plot(x,y,type="l", lwd=2, col="black")
13 x=seq(-3,-1.25,length=200)
14 y=dnorm(x)
15 polygon(c(-3,x,-1.25),c(0,y,0),col="red")
16
17 #To plot the standard normal distribution bell curve
18 curve(dnorm, xlim = c(-3, 3), ylim = c(0, 0.4), xlab
  = "z", ylab="f(z)",lwd=2, col="black")
19 z = -1.25
20 pnorm(z)
21 x = c(-3, seq(-3, z, by=.001), z)
22 #plot(x)

```

```

23 y = c(0, dnorm(seq(-3, z, by=.001)), 0)
24 #plot(y)
25 polygon(x, y, col="red")

```

---

#### R code Exa 4.13.d Standard normal distribution

```

1 #Example 4.13.d, Page 154
2 #The answers may slightly vary due to rounding off
  of values
3
4 #P(-0.38<=Z<=1.25)
5 #Variable to store result(1-P(Z<=1.25))
6 a<-pnorm(1.25,0,1)-pnorm(-0.38,0,1)
7 print(paste("The standard normal probability is", a)
  )
8
9 #To plot the standard normal distribution bell curve
10 curve(dnorm, xlim = c(-3, 3), ylim = c(0, 0.4), xlab
  = "z", ylab="f(z)",lwd=2, col="black")
11 z1 = 1.25
12 z2 = -0.38
13 pnorm(z)
14 x = c(z2, seq(z2, z1, by=.001), z1)
15 #plot(x)
16 y = c(0, dnorm(seq(z2, z1, by=.001)), 0)
17 #plot(y)
18 polygon(x, y, col="red")

```

---

#### R code Exa 4.14 Percentiles of standard normal distribution

```

1 #Ex4.14, Page 155
2 #Answers may slightly vary due to rounding off of
  values

```



```

3
4 print(paste("99th percentile:", qnorm(0.99)))
5 print(paste("1st percentile:", qnorm(0.01)))
6
7 #Blue=> 99th percentile
8 #Red=> 1st percentile
9 plot.new()
10 title(main="Blue: 99th percentile
11 Red: 1st percentile")
12 curve(dnorm, xlim = c(-3, 3), ylim = c(0, 0.5), xlab
      = "z", ylab="f(z)")
13 prob1 = 0.99
14 x1= c(-3, seq(-3, qnorm(prob1), by=.001), qnorm(
      prob1))
15 y1= c(0, dnorm(seq(-3, qnorm(prob1), by=.001)), 0)
16 polygon(x1, y1, col="blue")
17
18 par(new=TRUE)
19
20 prob2 = 0.01
21 x2= c(-3, seq(-3, qnorm(prob2), by=.0001), qnorm(
      prob2))
22 y2= c(0, dnorm(seq(-3, qnorm(prob2), by=.0001)), 0)
23 polygon(x2, y2, col="red")

```

---

#### R code Exa 4.15 z critical values

```

1 #Ex4.15, Page 156
2 #Answers may slightly vary due to rounding off of
  values
3
4 #z(.05)
5 x<-0.05
6 z<-100*(1-x)
7

```

```

8 plot.new()
9 title(main="Red: z(0.05)
10 Blue: -z(0.05)")
11
12 #z(.05)
13 print(paste(z,"th percentile:",qnorm(z/100)))
14 curve(dnorm, xlim = c(-3, 3), ylim = c(0, 0.5), xlab
      = "z", ylab="f(z)")
15 prob = z/100
16 x = c(-3, seq(-3, qnorm(prob), by=.001), qnorm(prob)
      )
17 y = c(0, dnorm(seq(-3, qnorm(prob), by=.001)), 0)
18 polygon(x, y, col="red")
19
20 #-z(.05)
21 print(paste("-z(.05)",-qnorm(z/100)))
22 x = c(-3, seq(-3, -qnorm(prob), by=.001), -qnorm(
      prob))
23 y = c(0, dnorm(seq(-3, -qnorm(prob), by=.001)), 0)
24 polygon(x, y, col="blue")

```

---

#### R code Exa 4.16 Nonstandard normal distributions

```

1 #Ex4.16, Page 158
2 #Answers may slightly vary due to rounding off of
  values
3
4 #P(1.00<=X<=1.75)
5 #Mean=>1.25, Standard deviation=>0.46
6
7 title(main="P(1.00<=X<=1.75)")
8 a<-pnorm(1.00,mean=1.25,sd=0.46)
9 b<-pnorm(1.75,mean=1.25,sd=0.46)
10 c<-b-a
11 print(paste("P(1.00<=X<=1.75)=",c))

```

```

12
13 #To plot the standard normal distribution bell curve
14 curve(dnorm, xlim = c(-3, 3), ylim = c(0, 0.4), xlab
      = "z", ylab="f(z)",lwd=2, col="black")
15 z<-1.75
16 # Cumulative density function of the standard normal
      distribution
17 pnorm(z)
18 x<-c(1.00, seq(1.00, z, by=.001), z)
19 #plot(x)
20 y<-c(0, dnorm(seq(1.00, z, by=.001)), 0)
21 #plot(y)
22 polygon(x, y, col="red")
23
24 #P(X>2)
25 d<-1-pnorm(2,mean=1.25,sd=0.46)
26 print(paste("P(X>2)=",d))

```

---

#### R code Exa 4.17 Nonstandard normal distributions

```

1 #Ex4.17, Page 158
2 #Answers may slightly vary due to rounding off of
      values
3
4 #P(-1.00<=Z<=1.00)
5 a<-pnorm(1.00,0,1)-pnorm(-1.00,0,1)
6 print(paste("P(X is within 1 standard deviation of
      its mean):",a))
7
8 #P(-2.00<=Z<=2.00)
9 b<-pnorm(2.00,0,1)-pnorm(-2.00,0,1)
10 print(paste("P(X is within 2 standard deviation of
      its mean):",b))
11
12 #P(-3.00<=Z<=3.00)

```

```

13 c<-pnorm(3.00,0,1)-pnorm(-3.00,0,1)
14 print(paste("P(X is within 3 standard deviation of
    its mean):",c))

```

---

#### R code Exa 4.18 Percentiles of arbitrary normal distribution

```

1 #Ex4.18, Page 159
2 #Answers may slightly vary due to rounding off of
  values
3
4 m<-64
5 sd<-0.78
6 #P(X>c)=0.05
7 p<-0.005
8 #P(X<=c)=1-P(X>c)
9 s<-(1-p)*100
10 print(paste("c is the",s,"th percentile of the
    normal distribution with mean=64 and sd=0.78"))
11 q<-qnorm(s/100)
12 print(paste("Therefore",s,"th percentile:",q))
13
14 c<-m+q*sd
15 print(paste("c=",c,"oz"))
16
17 #Plotting 99.5th percentile
18 plot.new()
19 title(main="99.5th percentile",sub="Shaded area
    =>0.995")
20 curve(dnorm, xlim = c(-3, 3), ylim = c(0, 0.5), xlab
    = "z", ylab="f(z)")
21 prob = 0.995
22 x1= c(-3, seq(-3, qnorm(prob), by=.001), qnorm(prob)
    )
23 y1= c(0, dnorm(seq(-3, qnorm(prob), by=.001)), 0)
24 polygon(x1, y1, col="blue")

```

---

**R code Exa 4.20** Approximating binomial distribution

```
1 #Ex4.20 , Page 161
2 #Answers may vary slightly due to rounding off of
  values
3
4 n<-50
5 p<-25/100
6 q<-1-p
7 m<-12.5
8 sd<-3.06
9 x<-n*p
10 y<-sqrt(n*p*q)
11 z<-n*q
12 if(x>=10 && z>=10){
13   #P(X<=10)
14   a<-pnorm(10+0.5, mean=x, sd=y)
15   print(paste("P(X<=10)=", a))
16
17   #P(5<=X<=15)
18   b<-pnorm(15+0.5, mean=x, sd=y)-pnorm(5-0.5, mean=x, sd
     =y)
19   print(paste("P(5<=X<=15)=", b))
20 }
```

---

**R code Exa 4.21** Exponential distribution

```
1 #Ex4.21 , Page 166
2 #Answers may slightly vary due to rounding off of
  values
3
```

```

4 #E(X)=6MPa=1/lambda
5 e<-6
6 lambda<-1/e
7 #Probability that stress range is at most 10MPa
8 p<-pexp(10,lambda)
9 print(paste("P(stress range is at most 10MPa)=",p))
10
11 #Probability that stress range is between 5 and 10
    MPa
12 q<-pexp(10,lambda)-pexp(5,lambda)
13 print(paste("P(stress range is between 5 and 10MPa)=
    ",q))

```

---

#### R code Exa 4.22 Exponential distribution

```

1 #Ex4.22, Page 167
2 #Answer may slightly vary due to rounding off of
    value
3
4 alpha<-0.5
5
6 #P(X>2)=1-P(X<=2)
7 x<-2
8 p<-1-pexp(x,alpha)
9 print(paste("P(X>2)=",p))
10
11 t<-1/alpha
12 print(paste("The expected time between successive
    calls is",t," days"))

```

---

#### R code Exa 4.23 Gamma distribution

```

1 #Ex4.23, Page 169

```

```

2 #Answers may slightly vary due to rounding off of
  values
3
4 alpha<-2
5
6 #P(3<=X<=5)
7 a<-pgamma(5,shape=alpha)-pgamma(3,shape=alpha)
8 print(paste("P(3<=X<=5)=",a))
9
10 #P(X>4)=1-P(X<=4)
11 b<-1-pgamma(4,shape=alpha)
12 print(paste("P(reaction time is more than 4 sec)=",b
  ))

```

---

#### R code Exa 4.24 Gamma distribution

```

1 #Ex4.24, Page 169
2 #Answers may slightly vary due to rounding off of
  values
3
4 #alpha
5 a<-8
6 #beta
7 b<-15
8
9 #E(X)
10 e<-a*b
11 print(paste("E(X)=",e))
12
13 #V(X)
14 v<-a*(b^2)
15 print(paste("V(X)=",v))
16
17 #SD
18 sd<-sqrt(v)

```

```

19 print(paste("Standard deviation =",sd))
20
21 #P(60<=X<=120)=P(X<=120)-P(X<=60)
22 p1<-pgamma(120/b,shape=a)-pgamma(60/b,a)
23 print(paste("P(mouse survives between 60 and 120
    weeks)=" ,p1))
24
25 #P(X>=30)=1-P(X<30)
26 p2<-1-pgamma(30/b,a)
27 print(paste("P(mouse survives at least 30 weeks)=" ,
    p2))

```

---

#### R code Exa 4.25 Weibull distribution

```

1 #Ex4.25 , Page 173
2 #Answers may slightly vary due to rounding off of
    values
3
4 alpha<-2
5 beta<-10
6
7 #P(X<=10)
8 p1<-pweibull(10,alpha,beta)
9 print(paste("P(X<=10)=" ,p1))
10
11 #P(X<=25)
12 p2<-pweibull(25,alpha,beta)
13 print(paste("P(X<=25)=" ,p2))
14
15 #To find c value which separates 5% of all engines
    having largest NOx emissions from remaining 95%
16 per<-5/100
17 c<-sqrt((-1)*(10^2)*log(per))
18 print(paste(c,"is the 95th percentile of the
    emission distribution"))

```



---

**R code Exa 4.26** Weibull distribution

```
1 #Ex4.26 , Page 173
2 #Answers may slightly vary due to rounding off of
  values
3
4 #alpha
5 a<-1.3
6 #beta
7 b<-0.8
8 #gamma
9 g<-4
10
11 #P(5<=X<=6)
12 p1<-pweibull(6-g,a,b)-pweibull(5-g,a,b)
13 print(paste("P(air void volume of a specimen is
  between 5% and 6%)=",p1))
```

---

**R code Exa 4.27** Lognormal distribution

```
1 #Ex4.27 , Page 175
2 #Answers may slightly vary due to rounding off of
  values
3
4 m<-0.353
5 s<-0.754
6
7 e<-exp(m+(s^2)/2)
8 print(paste("E(X)=",e))
9
10 v<-exp(2*m+(s^2))*(exp(s^2)-1)
```

```

11 print(paste("V(X)=" ,v))
12
13 #P(1<=X<=2)
14 l1<-log(1,base=exp(1))
15 l2<-log(2,base=exp(1))
16 p<-pnorm(l2,m,s)-pnorm(l1,m,s)
17 print(paste("P(maximum pit depth is between 1 and 2
      mm)=" ,p))
18
19 #To find c such that only 1% of all specimens have a
      maximum pit depth excceding c
20 per<-1/100
21 c<-exp(1)^(qnorm(1-per)*s+m)
22 print(paste(c," is the 99th percentile of the maximum
      pit depth distribution"))

```

---

#### R code Exa 4.28 Beta distribution

```

1 #Ex4.28, Page 177
2
3 #Optimistic time
4 A<-2
5
6 #Pessimistic time
7 B<-5
8
9 #alpha
10 a<-2
11 #beta
12 b<-3
13
14 #E(X)
15 e<-A+(B-A)*(a/(a+b))
16 print(paste("Mean=" ,e))
17

```

```

18 c<-(1/(B-A))*((gamma(a+b))/(gamma(a)*gamma(b)))
19 f1<-function(x) {c*(((x-A)/(B-A))^(a-1))*(((B-x)/(B-
    A))^(b-1))}
20 p<-integrate(f1,lower=2,upper=3)
21 print(paste("P(it take atmost 3 days to lay the
    foundation)=",p$value))

```

---

#### R code Exa 4.29 Probability plots

```

1 #Ex4.29 , Page 181
2
3 old.par<-par(mfrow=c(1,2))
4
5 #To plot first sample
6 Percentage1<-c(5, 15, 25, 35, 45)
7 z_percentile1<-c(qnorm(5/100), qnorm(15/100), qnorm
    (25/100), qnorm(35/100), qnorm(45/100))
8 Sample_observation1<-c(-1.91, -1.25, -0.75, -0.53,
    0.2)
9 df1<-data.frame(Percentage1, z_percentile1, Sample_
    observation1)
10 print(df1)
11 plot(z_percentile1,Sample_observation1)
12
13 #To plot second sample
14 Percentage2<-c(55, 65, 75, 85, 95)
15 z_percentile2<-c(qnorm(55/100), qnorm(65/100), qnorm
    (75/100), qnorm(85/100), qnorm(95/100))
16 Sample_observation2<-c(0.35, 0.72, 0.87, 1.40, 1.56)
17 df2<-data.frame(Percentage2, z_percentile2, Sample_
    observation2)
18 print(df2)
19 plot(z_percentile2,Sample_observation2)
20
21 par(old.par)

```

---

### R code Exa 4.30 Probability plots

```
1 #Ex4.30 , Page 182
2
3 old.par<-par(mfrow=c(1,1))
4 n<-20
5 z_percentile<-c(qnorm((1-0.5)/n), qnorm((2-0.5)/n),
6                 qnorm((3-0.5)/n), qnorm((4-0.5)/n), qnorm((5-0.5)/n),
7                 qnorm((6-0.5)/n), qnorm((7-0.5)/n),
8                 qnorm((8-0.5)/n), qnorm((9-0.5)/n),
9                 qnorm((10-0.5)/n), qnorm((11-0.5)/n), qnorm((12-0.5)/n),
10                qnorm((13-0.5)/n), qnorm((14-0.5)/n), qnorm((15-0.5)/n),
11                qnorm((16-0.5)/n), qnorm((17-0.5)/n), qnorm((18-0.5)/n), qnorm((19-0.5)/n), qnorm((20-0.5)/n))
12 Sample_observation<-c(24.46, 25.61, 26.25, 26.42,
13                       26.66, 27.15, 27.31, 27.54, 27.74, 27.94, 27.98,
14                       28.04, 28.28, 28.49, 28.50, 28.87, 29.11, 29.13,
15                       29.50, 30.88)
16 df<-data.frame(z_percentile, Sample_observation)
17 print(df)
18 plot(z_percentile, Sample_observation)
19
20 par(old.par)
```

---

### R code Exa 4.31 Beyond normality

```
1 #Ex4.31 , Page 186
```

```

2
3 old.par<-par(mfrow=c(1,1))
4
5 Percentile<-c(-2.97, -1.82, -1.25, -0.84, -0.51,
  -0.23, 0.05, 0.33, 0.64, 1.10)
6 x<-c(282, 501, 741, 851, 1072, 1122, 1202, 1585,
  1905, 2138)
7 lnx<-c(log(282,base=exp(1)),log(501,base=exp(1)),
  log(741,base=exp(1)), log(851,base=exp(1)), log
  (1072,base=exp(1)),
8         log(1122,base=exp(1)), log(1202,base=exp(1)
  ), log(1585,base=exp(1)), log(1905,base=
  exp(1)), log(2138,base=exp(1)))
9 df<-data.frame(Percentile,x,lnx)
10 print(df)
11 plot(Percentile,lnx)
12
13 par(old.par)

```

---

## Chapter 5

# Joint probability distributions and random samples

**R code Exa 5.1** Joint probability mass function

```
1 #Ex5.1, Page 194
2
3 m<-matrix(c(0.20,0.05,0.10,0.15,0.20,0.30),nrow=2,
4           ncol=3)
5 rownames(m)<-c("100","250")
6 colnames(m)<-c("0","100","200")
7 print(m)
8
9 #To find P(Y>=100)
10 prob<-m[1,2]+m[1,3]+m[2,2]+m[2,3]
11 print(paste("P(Y>=100)=",prob))
```

---

**R code Exa 5.2** Marginal probability mass function

```
1 #Ex5.2, Page 195
2
```

```

3  m<-matrix(c(0.20,0.05,0.10,0.15,0.20,0.30),nrow=2,
             ncol=3)
4  rownames(m)<-c("100","250")
5  colnames(m)<-c("0","100","200")
6  print(m)
7  cat("\n")
8
9  px_100<-apply(m,1,sum)[1]
10 px_250<-apply(m,1,sum)[2]
11 cat("px(100)=",px_100,"\n")
12 cat("px(250)=",px_250,"\n\n")
13
14 cat("Marginal pmf of X:\n")
15 cat("p(x)=",px_100,"when x=100,250\n")
16 cat("p(x)=0 otherwise\n\n")
17
18 py_0<-apply(m,2,sum)[1]
19 py_100<-apply(m,2,sum)[2]
20 py_200<-apply(m,2,sum)[3]
21 cat("py(0)=",py_0,"\n")
22 cat("py(100)=",py_100,"\n")
23 cat("py(200)=",py_200,"\n\n")
24
25 cat("Marginal pmf of Y:\n")
26 cat("p(y)=",py_0,"when y=0,100\n")
27 cat("p(y)=",py_200,"when y=200\n")
28 cat("p(y)=0 otherwise\n\n")
29
30 #To find P(Y>=100)
31 prob<-py_100+py_200
32 cat("P(Y>=100)=",prob)

```

---

**R code Exa 5.3** Joint probability density function

1 #Ex5.3, Page 196

```

2 #Answers may ary slightly due to rounding off of
  values
3
4 f<-function(x,y) {(6/5)*(x+y^2)}
5
6 #To check whether pdf is legitimate
7 check_val<-integrate(function(y) {sapply(y,function(
  y) {integrate(function(x) f(x,y),0,1)$value}
  },0,1)
8 if(check_val$value==1) cat("PDF is legitimate\n\n")
9
10 #P(0<=X<=1/4,0<=Y<=1/4)
11 low1<-0
12 high1<-1/4
13 low2<-0
14 high2<-1/4
15 integral<-integrate(function(y) {sapply(y,function(y
  ) {integrate(function(x) f(x,y),low1,high1)$value
  })},low2,high2)
16 print(paste("P(0<=X<=1/4,0<=Y<=1/4)=",integral$value
  ))

```

---

#### R code Exa 5.4 Marginal probability density function

```

1 #Ex5.4 , Page 197
2
3 f<-function(x,y) {(6/5)*(x+y^2)}
4
5 #Finding P(1/4<=Y<=3/4) using marginal pdf of Y
6 xlow<-0
7 xhigh<-1
8 ylow<-1/4
9 yhigh<-3/4
10 prob<-integrate(function(y) {sapply(y,function(y) {
  integrate(function(x) f(x,y),xlow,xhigh)$value}

```



```

    }, ylow, yhigh)
11 print(paste("P(1/4<=Y<=3/4)=", prob$value))

```

---

### R code Exa 5.5 Marginal probability density function

```

1 #Ex5.5, Page 198
2
3 #Joint pdf
4 f<-function(x,y) {24*x*y}
5
6 val<-Vectorize(function(x) {sapply(x,function(z) {
7   integrate(function(y) f(x,y),0,1-z)$value})})
8 check_val<-integrate(val,0,1)$value
9 if(check_val==1) print(paste("PDF is legitimate"))
10
11 #Probability that two types of nuts together make up
12   50% of the can
13 #Taking A={(x,y):0<=x<=1,0<=y<=1,x+y<=0.5}
14 xlow<-0
15 xhigh<-0.5
16 integral<-Vectorize(function(x) {sapply(x,function(z)
17   ) {integrate(function(y) f(x,y),0,0.5-z)$value})
18   })
19 prob<-integrate(integral,xlow,xhigh)
20 print(paste("P((X,Y)EA) =", prob$value))

```

---

### R code Exa 5.8 Independent random variables

```

1 #Ex5.8, Page 200
2 #Answers may vary slightly due to rounding off of
3   values
4
5 #Expected lifetimes are 1000 and 1200 hours

```

```

5 lambda1<-(1/1000)
6 lambda2<-(1/1200)
7
8 #P(X1>=1500)
9 p1<-exp(-(lambda1)*1500)
10
11 #P(X2>=1500)
12 p2<-exp(-(lambda2)*1500)
13
14 #Joint PDF
15 #P(X1>=1500,X2>=1500)
16 p<-p1*p2
17 print(paste("P(both component lifetimes are at least
    1500 hours)=",p))

```

---

#### R code Exa 5.9 Multinomial distribution

```

1 #Ex5.9, Page 201
2 #Answer may slightly vary due to rounding off of
  value
3
4 #Respective number and probabilities
5 n<-c(2,5,3)
6 p<-c(0.25,0.5,0.25)
7
8 #Multinomial PMF
9 m<-dmultinom(n,prob=p)
10 print(paste("Multinomial PMF:",m))

```

---

#### R code Exa 5.12 Conditional distributions

```

1 #Ex5.12, Page 203

```

```

2 #Answers may slightly vary due to rounding off of
  values
3
4 #Function when X=0.8: (1/34)(24+30(y^2))
5
6 #P(Y<=0.5) given X=0.8
7 f1<-function(y) {(1/34)*(24+30*(y^2))}
8 f<-integrate(f1,lower=0,upper=0.5)
9 print(paste("P(walk-up facility is busy atmost half
  the time)=",f$value))
10
11 #E(Y) given that X=0.8
12 f2<-function(y) {y*(1/34)*(24+30*(y^2))}
13 p<-integrate(f2,lower=0,upper=1)
14 print(paste("Expected proportion of time that walk-
  up facility is busy:",p$value))

```

---

#### R code Exa 5.14 Expected values

```

1 #Ex5.14, Page 207
2
3 #Joint pdf
4 f<-function(x,y) {24*x*y}
5
6 almonds<-1
7 cashews<-1.5
8 peanuts<-0.5
9
10 #Total cost of the contents of a can
11 h<-function(x,y) {almonds*x+cashews*y+(1-x-y)*
  peanuts}
12
13 #Expected total cost
14 integral<-Vectorize(function(x) {supply(x,function(z
  ) {integrate(function(y) h(x,y)*f(x,y),0,1-z)$

```

```

    value}}))
15 prob<-integrate(integral,0,1)
16 print(paste("Expected total cost:$",prob$value))

```

---

### R code Exa 5.15 Covariance

```

1 #Ex5.15, Page 208
2
3 x<-c(100,250)
4 y<-c(0,100,200)
5
6 m1<-matrix(c(0.20,0.05,0.10,0.15,0.20,0.30),nrow=2,
7           ncol=3)
8 rownames(m1)<-c("100","250")
9 colnames(m1)<-c("0","100","200")
10 print(m1)
11 cat("\n")
12
13 m2<-matrix(c(0.5,0.5),nrow=1,ncol=2)
14 rownames(m2)<-c("p(x)")
15 colnames(m2)<-c("100","250")
16 print(m2)
17 cat("\n")
18
19 m3<-matrix(c(0.25,0.25,0.5),nrow=1,ncol=3)
20 rownames(m3)<-c("p(y)")
21 colnames(m3)<-c("0","100","200")
22 print(m3)
23 cat("\n")
24
25 x1<-c(0.5,0.5)
26 j<-1
27 mu_x<-0
28 for(i in x){

```

```

29   if(j<=length(x1)){
30       mu_x<-mu_x+i*x1[j]
31       j=j+1
32   }
33 }
34 cat("mu_X=",mu_x,"\n\n")
35
36 y1<-c(0.25,0.25,0.5)
37 k<-1
38 mu_y<-0
39
40 for(i in y){
41     if(k<=length(y1)){
42         mu_y<-mu_y+i*y1[k]
43         k=k+1
44     }
45 }
46 cat("mu_Y=",mu_y,"\n\n")
47
48 p<-c(0.2,0.1,0.20,0.05,0.15,0.30)
49 sum<-0
50 k<-1
51 for(i in x){
52     for(j in y){
53         if(k<=length(p)){
54             sum=sum+(i-175)*(j-125)*p[k]
55             k=k+1
56         }
57     }
58 }
59 print(paste("Cov(X,Y):",sum))

```

---

**R code Exa 5.18** Correlation

1 [#Ex5.18](#), Page 210

```

2
3 x<-c(-4,4,2,-2)
4 y<-c(1,-1,2,-2)
5 xy<-x*y
6 w<-c(1/4,1/4,1/4,1/4)
7 m<-weighted.mean(xy,w)
8 print(paste("E(XY)=",m))
9
10 #Cov(XY)=E(XY)-ux.uy
11 ux<-0
12 uy<-0
13 cov<-m-(ux*uy)
14 print(paste("Cov(XY)=",cov))
15
16 print(paste("Complete absence of any linear
    relationship"))
17
18 #To scatterplot
19 df<-data.frame(x,y)
20 plot(x,y)

```

---

#### R code Exa 5.19 Statistics and their distributions

```

1 #Ex5.19, Page 212
2 #Answers may vary slightly due to rounding off of
    values
3
4 #alpha
5 a<-2
6
7 #beta
8 b<-5
9
10 #Mean
11 u<-b*gamma(1+(1/a))

```

```

12 print(paste("Mean, E(X)=" ,u))
13
14 #Variance
15 v<-(b^2)*{gamma(1+(2/a))-(gamma(1+(1/a)))^2}
16 print(paste(" Variance=" ,v))
17
18 #Standard deviation
19 s<-sqrt(v)
20 print(paste(" Standard deviation=" ,s))
21
22 #Median
23 med<-b*(log(2,base=exp(1)))^(1/a)
24 print(paste(" Median=" ,med))
25
26 q<-qweibull(0.99,shape=a,scale=b)
27 curve(dweibull(x,a,b),from=0,to=q,ylab='density')
28
29 if(u>med){
30     print(paste("Mean exceeds median because of the
        distribution 's positive skew"))
31 }

```

---

#### R code Exa 5.24 Standard error of the mean

```

1 #Ex5.24, Page 223
2
3 n<-25
4 u<-28000
5 s<-5000
6
7 #E(Mean)
8 print(paste("E(Mean)=" ,u))
9
10 #E(T0)
11 t<-n*u

```

```

12 print(paste("E(T0)=",t))
13
14 #Standard deviation of mean
15 sm<-s/sqrt(n)
16 print(paste("SD of mean:",sm))
17
18 #Standard deviation of T0
19 st<-s*sqrt(n)
20 print(paste("SD of T0:",st))
21
22 #When sample size increases to n=100
23 print(paste("When n=100:"))
24 n<-100
25 #E(Mean)
26 print(paste("E(Mean)=",u))
27
28 #E(T0)
29 t<-n*u
30 print(paste("E(T0)=",t))
31
32 #Standard deviation of mean
33 sm<-s/sqrt(n)
34 print(paste("SD of mean:",sm))
35
36 #Standard deviation of T0
37 st<-s*sqrt(n)
38 print(paste("SD of T0:",st))

```

---

#### R code Exa 5.25 Normal population distribution

```

1 #Ex5.25, Page 224
2 #Answers may slightly vary due to rounding off of
  values
3
4 n<-5

```



```

5
6 #Mean( min )
7 m<-1.5
8
9 #Standard deviation( min )
10 sd<-0.35
11
12 ut<-n*m
13 print(paste("uT0:",ut))
14
15 vt<-n*(sd^2)
16 print(paste("vT0:",vt))
17
18 st<-sqrt(vt)
19 print(paste("sT0:",st))
20
21 #P(6<=T0<=8)
22 p1<-pnorm(8,mean=ut,sd=st)-pnorm(6,mean=ut,sd=st)
23 print(paste("P(6<=T0<=8)=",p1))
24
25 #Probability that the sample average time is atmost
    2 min
26 u<-1.5
27 s<-sd/sqrt(n)
28
29 #P( Mean<=2.0)
30 p2<-pnorm(2,mean=u,sd=s)
31 print(paste("P( Mean<=2)=",p2))

```

---

#### R code Exa 5.26 Central limit theorem

```

1 #Ex5.26, Page 225
2 #Answer may slightly vary due to rounding off of
    value
3

```

```

4  n<-50
5  m<-4
6  sd<-1.5
7
8  ux<-m
9  sx<-sd/sqrt(n)
10
11 #P(3.5<=Mean<=3.8)
12 p<-pnorm(3.8,mean=ux,sd=sx)-pnorm(3.5,mean=ux,sd=sx)
13 print(paste("P(3.5<=Mean<=3.8)=",p))

```

---

#### R code Exa 5.27 Central limit theorem

```

1  #Ex5.27, Page 226
2  #Answer may slightly vary due to rounding off of
   value
3
4  n<-100
5  m<-3.2
6  sd<-2.4
7
8  ux<-m
9  sx<-sd/sqrt(n)
10
11 #P(Mean>4)=1-P(Mean<=4)
12 p<-1-pnorm(4,mean=ux,sd=sx)
13 print(paste("P(Mean>4)=",p))

```

---

#### R code Exa 5.29 Linear combination

```

1  #Ex5.29, Page 231
2  #Answers may vary slightly due to rounding off of
   values

```

```

3
4 a1<-3.00
5 a2<-3.20
6 a3<-3.40
7
8 #Mean
9 u1<-1000
10 u2<-500
11 u3<-300
12
13 #Standard deviation
14 s1<-100
15 s2<-80
16 s3<-50
17
18 #Y=3.0X1+3.20X2+3.40X3
19 EY<-a1*u1+a2*u2+a3*u3
20 VY<-(a1^2)*(s1^2)+(a2^2)*(s2^2)+(a3^2)*(s3^2)
21 sY<-sqrt(VY)
22 print(paste("E(Y)=$",EY))
23 print(paste("V(Y)=$",VY))
24 print(paste("sY=$",sY))

```

---

### R code Exa 5.30 Difference between two random variables

```

1 #Ex5.30, Page 232
2 #Answers may slightly vary due to rounding off of
  values
3
4 u1<-22
5 u2<-26
6 s1<-1.2
7 s2<-1.5
8
9 #Mean

```

```

10 e<-u1-u2
11 print(paste("E(X1-X2)=" ,e))
12
13 #Variance
14 v<-(s1^2)+(s2^2)
15 print(paste("V(X1-X2)=" ,v))
16
17 #Standard deviation
18 sd<-sqrt(v)
19 print(paste("SD=" ,sd))

```

---

#### R code Exa 5.31 Normal random variables

```

1 #Ex5.31 , Page 232
2
3 #Y=3.0X1+3.2X2+3.4X3
4 u<-5620
5 s<-429.46
6
7 #P(Y>4500)
8 p<-1-pnorm(4500 ,mean=u ,sd=s)
9 print(paste("P(revenue exceeds 4500)=" ,p))

```

---

# Chapter 6

## Point estimation

**R code Exa 6.1** Some general concepts of point estimation

```
1 #Ex6.1 , Page 241
2
3 x<-15
4 n<-25
5 p<-x/n
6 print(paste("Most reasonable estimate is",p))
```

---

**R code Exa 6.2** Some general concepts of point estimation

```
1 #Ex6.2 , Page 241
2 #Answers may vary slightly due to rounding off of
  values
3
4 volt<-c
  (24.46 , 25.61 , 26.25 , 26.42 , 26.66 , 27.15 , 27.31 , 27.54 , 27.74 , 27.94 , 27.98)
5
6 #Estimator1
```

```

7 x1<-mean(volt)
8 print(paste(" Estimator (1) :",x1))
9
10 #Estimator2
11 x2<-median(volt)
12 print(paste(" Estimator (2) :",x2))
13
14 #Estimator3
15 x3<-(min(volt)+max(volt))/2
16 print(paste(" Estimator (3) :",x3))
17
18 #Estimator4
19 volt<-setdiff(volt,c(volt[1],volt[2],volt[19],volt
    [20]))
20 x4<-mean(volt)
21 print(paste(" Estimator (4) :",x4))

```

---

### R code Exa 6.3 Some general concepts of point estimation

```

1 #Ex6.3, Page 242
2 #Answers may vary slightly due to rounding off of
  values
3
4 obs<-c
    (74.33,71.07,73.82,77.42,79.35,82.27,77.75,78.65,77.19,74.69,
5
    77.25,74.84,60.90,60.75,74.09,65.36,67.84,69.97,68.83,75.09,
6
    62.54,67.47,72.00,66.51,68.21,64.46,64.34,64.93,67.33,66.08,
7
    67.31,74.87,69.40,70.83,81.73,82.50,79.87,81.96,79.51,84.12,
8
    80.61,79.89,79.70,78.74,77.28,79.97,75.09,74.38,77.67,83.73,
9
    80.39,76.90)

```

```

10
11 #Mean
12 print(paste("Mean=", mean(obs)))
13
14 #Variance
15 print(paste("Point estimate of the population
    variance:", var(obs)))
16
17 #Alternative formula
18 #To find numerator of s^2
19 s1<-sum(obs)
20 s2<-sum(obs^2)
21 Sxx<-s2-(s1^2)/length(obs)
22
23 #Standard deviation
24 print(paste("Point estimate of the population
    standard deviation:", sd(obs)))
25
26 #Alterative estimator
27 print(paste("Alternative estimator of population
    variance:", Sxx/length(obs)))

```

---

### R code Exa 6.9 Standard error of an estimator

```

1 #Ex6.9, Page 251
2 #Answers may slightly vary due to rounding off of
    values
3
4 n<-20
5 sigma<-1.5
6
7 #Standard error of mean
8 sigx<-sigma/sqrt(n)
9 print(paste("Standard error of mean:", sigx))
10

```

```

11 #Estimate of standard deviation
12 s<-1.462
13
14 sx<-s/sqrt(n)
15 print(paste("Estimated standard error:",sx))

```

---

#### R code Exa 6.10 Standard error of an estimator

```

1 #Ex6.10, Page 251
2 #Answers may vary slightly due to rounding off of
  values
3
4 n<-25
5 p<-0.6
6 q<-1-p
7
8 #To find estimated standard error
9 sp1<-sqrt((p*q)/n)
10 print(paste("Estimated standard error:",sp1))
11
12 #pq is largest when
13 p<-q<-0.5
14 sp2<-sqrt((p*q)/n)
15 print(paste("Estimated standard error when pq is
  largest:",sp2))

```

---

#### R code Exa 6.11 Standard error of an estimator

```

1 #Ex6.11, Page 252
2 #Answers may slightly vary due to rounding off of
  values
3

```



```

4  t<-c
    (41.53,18.73,2.99,30.34,12.33,117.52,73.02,223.63,4.00,26.78)

5
6  #Mean
7  m<-mean(t)
8  print(paste("Mean:",m))
9
10 #lambda(l)
11 l<-1/m
12 print(paste("Reasonable estimate of lambda:",l))

```

---

#### R code Exa 6.13 Method of moments

```

1  #Ex6.13, Page 257
2  #Answers may vary slightly due to rounding off of
    values
3
4  data<-c
    (152,115,109,94,88,137,152,77,160,165,125,40,128,123,136,101,62,1

5  n<-length(data)
6
7  #Mean
8  m<-mean(data)
9  print(paste("Mean:",m))
10
11 #Sum of x^2
12 s<-sum(data^2)/n
13
14 #alpha
15 alpha<-(m^2)/(s-(m^2))
16 print(paste("Alpha:",alpha))
17
18 #beta

```

```
19 beta<-(s-(m^2))/m
20 print(paste("Beta:",beta))
```

---

**R code Exa 6.14** Method of moments

```
1 #Ex6.14, Page 257
2 #Answers may vary slightly due to rounding off of
  values
3
4 goals<-c(0,1,2,3,4,5,6,7,8,9,10)
5 freq<-c(29,71,82,89,65,45,24,7,4,1,3)
6
7 #Mean
8 m<-weighted.mean(goals,freq)
9 print(paste("Mean=",m))
10
11 #To find sum of x^2
12 ms<-weighted.mean(goals^2,freq)
13 print(paste("Sum of x^2/frequency=",ms))
14
15 #To find alpha
16 p_cap<-m/(ms-(m^2))
17 print(paste("Alpha:",p_cap))
18
19 #To find beta
20 r_cap<-(m^2)/(ms-(m^2)-m)
21 print(paste("Beta:",r_cap))
```

---

**R code Exa 6.21** The invariance principle

```
1 #Ex6.21, Page 262
2 #Answers may slightly vary due to rounding off of
  values
```

```
3
4 alpha<-11.9731
5 beta<-77.0153
6 m<-beta*gamma(1+(1/alpha))
7 print(paste("Mean estimate:",m))
```

---

# Chapter 7

## Statistical intervals based on a single sample

R code Exa 7.2 Basic properties of confidence intervals

```
1 #Ex7.2, Page 270
2 #Answers may slightly vary due to rounding off of
  values
3
4 mean<-80
5 n<-31
6 sd<-2
7 error<-qt(0.95,df=n-1)*sd/sqrt(n)
8
9 #Left endpoint
10 l<-mean-error
11 print(paste("Left endpoint of interval:",l))
12
13 #Right endpoint
14 r<-mean+error
15 print(paste("Right endpoint of interval:",r))
```

---

### R code Exa 7.3 Other levels of confidence

```
1 #Ex7.3, Page 272
2 #Answers may slightly vary due to rounding off of
  values
3
4 mean<-5.426
5 n<-40
6 sd<-0.1
7
8 #To find alpha for (100(1-alpha)=90)
9 alpha<-solve(100,10)
10
11 error<-qnorm(1-(alpha/2))*(sd/sqrt(n))
12
13 #Left endpoint
14 l<-mean-error
15 print(paste("Left endpoint of interval:",l))
16
17 #Right endpoint
18 r<-mean+error
19 print(paste("Right endpoint of interval:",r))
```

---

### R code Exa 7.4 Confidence level and precision and sample size

```
1 #Ex7.4, Page 273
2 #Answers may slightly vary due to rounding off of
  values
3
4 #For 95% CI
5 z<-qnorm(1-(0.05/2))
6
7 sd<-25
8 w<-10
9
```

```

10 n<-(2*z*sd/w)^2
11 print(paste("Sample size necessary for CI to have
    width 10 is ",n,"and thus approx. 97"))

```

---

### R code Exa 7.6 Large sample CI for mean

```

1 #Ex7.6, Page 277–278
2 #Answers may slightly vary due to rounding off of
  values
3
4 data<-c
  (2948,2996,7197,8338,8500,8759,12710,12925,15767,20000,23247,24863
5 26210,30552,30600,35700,36466,40316,40596,41021,41234,43000,44607,45
6 45027,45442,46963,47978,49518,52000,53334,54208,56062,57000,57365,60
7 60265,60803,62851,64404,72140,74594,79308,79500,80000,80000,84000,11
8 boxplot(data)
9 print(data)
10
11 n<-50
12
13 #To find mean
14 mean<-mean(data)
15 print(paste("Mean:",mean))
16
17 #To find median
18 med<-median(data)
19 print(paste("Median:",med))
20
21 #To find standard deviation
22 s<-sd(data)
23 print(paste("Standard deviation:",s))

```

```

24
25 #To find confidence intervals
26 z<-qnorm(1-(0.05/2))
27 error<-z*(s/sqrt(n))
28
29 #Left endpoint
30 l<-mean-error
31 print(paste("Left endpoint of interval:",l))
32
33 #Right endpoint
34 r<-mean+error
35 print(paste("Right endpoint of interval:",r))

```

---

#### R code Exa 7.7 Large sample CI for mean

```

1 #Ex7.7, Page 279
2
3 t1<-320
4 t2<-440
5
6 #To find value of s
7 s<-(t2-t1)/4
8
9 w<-5
10
11 z<-qnorm(1-(0.05/2))
12
13 n<-(z*s/w)^2
14 print(paste("Sample size:",n))
15 print(paste("Integer value of sample size:",ceiling(
    n)))

```

---

#### R code Exa 7.8 CI for population proportion

```

1 #Ex7.8, Page 282
2 #Answer may vary slightly compared to textbook value
3
4 n<-48
5 n1<-16
6 p<-n1/n
7
8 alpha<-0.05
9 b<-binom.test(16,48,p,conf.level=0.95)
10 print(b)

```

---

#### R code Exa 7.10 Confidence bounds

```

1 #Ex7.10, Page 283
2 #Answer may slightly vary due to rounding off of
  value
3
4 mean<-17.17
5 z<-qnorm(0.95)
6 n<-48
7 sd<-3.28
8
9 #To find lower bound
10 c<-mean-(z*sd/sqrt(n))
11 print(paste("Lower confidence bound for mean:",c))

```

---

#### R code Exa 7.11 One sample t confidence interval

```

1 #Ex7.11, Page 288
2 #Answers may slightly vary due to rounding off of
  values

```



```

3  data<-c
    (6807.99,7637.06,6663.28,6165.03,6991.41,6992.23,6981.46,7569.75,
4      7437.88,6872.39,7663.18,6032.28,6906.04,6617.17,6984.12,7093
5      7659.50,7378.61,7295.54,6702.76,7440.17,8053.26,8284.75,7347
6      7422.69,7886.87,6316.67,7713.65,7503.33,7674.99)

7
8  n<-30
9
10 #To find mean
11 mean<-mean(data)
12 print(paste("Mean:",mean))
13
14 #To find standard deviation
15 s<-sd(data)
16 print(paste("Standard deviation:",s))
17
18 #To find confidence intervals
19 t<-qt(1-(0.05/2),df=n-1)
20 print(paste("Critical value,t:",t))
21 error<-t*(s/sqrt(n))
22
23 #Left endpoint
24 l<-mean-error
25 print(paste("Left endpoint of interval:",l))
26
27 #Right endpoint
28 r<-mean+error
29 print(paste("Right endpoint of interval:",r))
30
31 #To plot the normal probability plot
32 qqnorm(data,main="Normal Probability of MOR")

```

---

**R code Exa 7.12** A prediction interval for a single future value

```
1 #Ex7.12, Page 288
2 #Answers may slightly vary due to rounding off of
  values
3
4 data<-c
  (25.2,21.3,22.8,17.0,29.8,21.0,25.5,16.0,20.9,19.5)

5 print(data)
6
7 n<-10
8
9 #To find mean
10 mean<-mean(data)
11 print(paste("Mean:",mean))
12
13 #To find standard deviation
14 s<-sd(data)
15 print(paste("Standard deviation:",s))
16
17 #To find confidence intervals
18 t<-qt(1-(0.05/2),df=n-1)
19 print(paste("Critical value,t:",t))
20 error<-t*(s/sqrt(n))
21
22 #Left endpoint
23 l<-mean-error
24 print(paste("Left endpoint of interval:",l))
25
26 #Right endpoint
27 r<-mean+error
28 print(paste("Right endpoint of interval:",r))
```

---

### R code Exa 7.13 Prediction interval

```
1 #Ex7.13, Page 290
2 #Answers may slightly vary due to rounding off of
  values
3
4 mean<-21.9
5 s<-4.134
6 n<-10
7
8 #To find critical value
9 t<-qt(1-(0.05/2),df=n-1)
10 print(paste("Critical value ,t:",t))
11
12 error<-t*s*sqrt(1+(1/n))
13
14 #Left endpoint
15 l<-mean-error
16 print(paste("Left endpoint of prediction interval:",
  l))
17
18 #Right endpoint
19 r<-mean+error
20 print(paste("Right endpoint of prediction interval:"
  ,r))
```

---

### R code Exa 7.14 Tolerance intervals

```
1 #Ex7.14, Page 291
2 #Answers may vary slightly due to rounding off of
  values
3
```

```

4  data<-c
    (10490,16620,17300,15480,12970,17260,13400,13900,
5      13630,13260,14370,11700,15470,17840,14070,14760)

6  print(data)
7
8  n<-16
9
10 #To find mean
11 mean<-mean(data)
12 print(paste("Mean:",mean))
13
14 #To find standard deviation
15 s<-sd(data)
16 print(paste("Standard deviation:",s))
17
18 #To find tolerance intervals
19 t1<-2.903
20 error1<-t1*s
21 #Left endpoint
22 l1<-mean-error1
23 print(paste("Left endpoint of tolerance interval:",
    l1))
24 #Right endpoint
25 r1<-mean+error1
26 print(paste("Right endpoint of tolerance interval:",
    r1))
27
28 #To find confidence intervals
29 t2<-qt(1-(0.05/2),df=n-1)
30 error2<-t2*(s/sqrt(n))
31 #Left endpoint
32 l2<-mean-error2
33 print(paste("Left endpoint of confidence interval:",
    l2))
34 #Right endpoint
35 r2<-mean+error2
36 print(paste("Right endpoint of confidence interval:")

```

```

    ,r2))
37
38 #To find prediction intervals
39 t3<-qt(1-(0.05/2),df=n-1)
40 error3<-t3*s*sqrt(1+(1/n))
41 #Left endpoint
42 l3<-mean-error3
43 print(paste("Left endpoint of prediction interval:",
    l3))
44 #Right endpoint
45 r3<-mean+error3
46 print(paste("Right endpoint of prediction interval:"
    ,r3))

```

---

#### R code Exa 7.15 CI for variance and sd of normal population

```

1 #Ex7.15, Page 296
2 #Answers may slightly vary due to rounding off of
  values
3
4 data<-c
    (1470,1510,1690,1740,1900,2000,2030,2100,2190,
5     2200,2290,2380,2390,2480,2500,2580,2700)
6
7 interval<- function(data,conf.level=0.95){
8   df<-length(data)-1
9   l<-qchisq((1 - conf.level)/2, df)
10  r<-qchisq((1 - conf.level)/2, df, lower.tail =
    FALSE)
11  v<-var(data)
12  c(df* v/r,df*v/l)
13 }
14 print(paste("Confidence interval for variance of
    normal population:"))
15 print(interval(data))

```



# Chapter 8

## Test of hypotheses based on a single sample

R code Exa 8.1 Hypotheses and test procedures

```
1 #Ex8.1, Page 304
2 #Answers may vary slightly due to rounding off of
  values
3
4 #X<-number of crashes with no visible damage
5 #Reject H0 if x>=8 (x<-observed value of test
  statistic)
6
7 #When H0 is true
8 n<-20
9 p1<-0.25
10
11 #alpha=P(type I error)
12 alpha<-1-pbinom(7,n,p1)
13 print(paste("Alpha value:",alpha))
14
15 print(paste("When H0 is true, roughly",trunc(100*
  alpha,digits=2),"% of all experiments consisting
  of 20 crashes would result into a Type I error"))
```

```

16
17 #beta=P(type II error)
18 #Different beta for each different p that exceeds
    0.25, thus taking p=0.3
19 p2<-0.3
20 beta<-pbinom(7,n,p2)
21 print(paste("Beta value:",beta))
22 print(paste("When H0 is false , roughly",trunc(100*
    beta,digits=2),"% of all experiments would result
    into a Type II error"))
23
24 #Table showing variation of beta values with
    increase in value of p
25 for(i in 3:8){
26     p<-0.1*i
27     a<-pbinom(7,n,p)
28     print(paste("p-value:",p))
29     print(paste("Beta value:",a))
30 }

```

---

### R code Exa 8.2 Hypotheses and test procedures

```

1 #Ex8.2 , Page 305
2 #Answers may vary slightly due to rounding off of
    values
3
4 m<-75
5 sd<-9
6 #Hypotheses H0: mean=75, H1: mean<75
7
8 #Experimental data
9 n<-25
10 sd<-9
11 #Standard deviation of normal distribution
12 sd<-sd/sqrt(n)

```



```

13 print(paste("SD of normal distribution:",sdx))
14
15 #When H0 is true
16 mx<-75
17
18 #Consider rejection region x<=70.8
19 #To find alpha and beta
20
21 #P(type I error)
22 alpha<-pnorm(70.8,mx,sdx)
23 print(paste("Alpha value:",alpha))
24
25 #P(type II error)
26 mx<-c(72,70,67) #H0 is not rejected
    when it is false because mean= eg:72
27 for(i in mx){
28     beta<-1-pnorm(70.8,i,sdx)
29     print(paste("Beta value:",beta,"when mean is",i))
30 }
31
32 print(paste("Only",round(alpha,digits=3)*100,"% of
    all experiments will result into Type I error"))
33 print(paste("Possibility of Type II error decreases
    as mean value increases"))

```

---

### R code Exa 8.3 Hypotheses and test procedures

```

1 #Ex8.3, Page 306
2 #Answers may vary slightly due to rounding off of
    values
3
4 #R9={9,10,.....,20}
5 n<-20
6
7 #To find alpha=P(H0 rejected when p=.25)=P(X>=9 when

```

```

      X follows binomial dist.)
8  p<-0.25
9  alpha<-1-pbinom(8,n,p)
10 print(paste("Alpha value:",alpha))
11 print(paste("Type I error probability has reduced
      considerably when using R9 rather than R8
      rejection region"))
12
13 #To find beta value=P(H0 not rejected)=P(X<=8 when X
      follows binomial dist.)
14 p1<-0.3      #1st case
15 beta1<-pbinom(8,n,p1)
16 print(paste("Beta(0.3)=",beta1))
17
18 p2<-0.5      #2nd case
19 beta2<-pbinom(8,n,p2)
20 print(paste("Beta(0.5)=",beta2))
21 print(paste("P(Type II error in R9)>P(Type II error
      in R8)"))

```

---

#### R code Exa 8.4 Hypotheses and test procedures

```

1 #Ex8.4, Page 307
2 #Answers may vary slightly due to rounding off of
      values
3
4 #New rejection region x<=72
5 sdx<-1.8
6
7 #To find alpha=P(H0 rejected when true)=P(X<=72 when
      X follows normal dist)
8 mx<-75
9 alpha<-pnorm(72,mx,sdx)
10 print(paste("Alpha value:",alpha))
11 print(paste("Type I error probability is greater in

```

```

    the new rejection region"))
12
13 #To find beta=P(H0 not rejected)
14 mx<-c(72,70,67) #H0 is not rejected
    when it is false because mean= eg:72
15 for(i in mx){
16   beta<-1-pnorm(72,i,sdx)
17   print(paste("Beta(",i,"):",beta))
18 }
19 print(paste("Type II error probability is lower in
    the new rejection region"))

```

---

### R code Exa 8.5 Hypotheses and test procedures

```

1 #Ex8.5 , Page 308
2
3 #H0: mu=1.5
4 #Ha: mu>1.5
5
6 mu0<-1.5
7 n<-32
8 sd<-0.2
9 sdx<-sd/sqrt(n)
10 print(paste("Rejection region: Z>=c"))
11
12 alpha<-0.05
13 #where alpha=P(type I error)=P(Z>=c when Z follows N
    (0,1))
14 c<-qnorm(1-alpha)
15 print(paste("C value:",c))
16
17 #Z>=c
18 x<-round(mu0+sdx*c,digits=2)
19 print(paste("xbar>=",x))
20 print(paste("Beta refers to the prob. that xbar<",x,

```

"and can be calculated for any  $\mu > 1.5$ ")

---

### R code Exa 8.6 Tests about a population mean

```
1 #Ex8.6, Page 312
2 #Answers may vary slightly due to rounding off of
  values
3
4 mu0<-130
5 n<-9
6 sd<-1.5
7 xbar<-131.08
8
9 z<-(xbar-mu0)/(sd/sqrt(n))           #Test statistic
  value
10 print(paste("Test statistic value:",z))
11
12 #To determine rejection region
13 alpha<-0.01
14 a<-qnorm(1-alpha/2)
15 l<-(-a)
16 r<-a
17 print(paste("Rejection region: z<=",l," ,z>=",r))
18 if(z<=l || z>=r)print(paste(z,"falls in the
  rejection region and H0 is rejected"))else print(
  paste(z,"does not fall in the rejection region so
  H0 cannot be rejected at significance level",
  alpha))
```

---

### R code Exa 8.7 Tests about a population mean

```
1 #Ex8.7, Page 314
```

```

2 #Answers may vary slightly due to rounding off of
  values
3
4 n<-16
5 sd<-1500
6 alpha1<-0.01
7 xbar<-30000
8 mu0<-31000
9
10 zalpha1<-qnorm(1-alpha1)
11
12 z<-(xbar-mu0)/(sd/sqrt(n))
13
14 #To find beta(31000) when alpha=0.01
15 beta1<-pnorm(zalpha1+z)
16 print(paste("Beta(31000):",beta1,"when alpha=",
  alpha1))
17
18 alpha2<-0.1
19 zalpha2<-qnorm(1-alpha2)
20
21 #To find beta(31000) when alpha=0.1
22 beta2<-pnorm(zalpha2+z)
23 print(paste("Beta(31000):",round(beta2,digits=1),"
  when alpha=",alpha2))
24
25 #To find n
26 n<-(sd*(zalpha1+zalpha2)/(xbar-mu0))^2
27 print(paste("n value:",n))
28 print(paste("Sample size=",ceiling(n)))

```

---

### R code Exa 8.8 Tests about a population mean

```

1 #Ex8.8, Page 315
2 #Answers may vary slightly due to rounding off of

```

```

        values
3
4  n<-52
5  mu0<-30
6  data<-c
      (14.1,14.5,15.5,16.0,16.0,16.7,16.9,17.1,17.5,17.8,17.8,18.1,18.2
7
      19.0,19.2,19.4,20.0,20.0,20.8,20.8,21.0,21.5,23.5,27.5,27.5,
8
      30.0,31.6,31.7,31.7,32.5,33.5,33.9,35.0,35.0,35.0,36.7,40.0,
9
      47.5,50.0,51.0,51.8,54.4,55.0,57.0)
10 xbar<-mean(data)
11 sd<-sd(data)
12 print(paste("Mean:",xbar))
13 print(paste("Standard deviation:",sd))
14
15 alpha<-0.05
16 #To find zalpha
17 z1<-qnorm(alpha)
18
19 #To find z
20 z<-(xbar-mu0)/(sd/sqrt(n))
21 print(paste("z value:",z))
22
23 if(z>z1) print(paste("H0 cannot be rejected since",z
      ,">",z1)) else print(paste("H0 is rejected since"
      ,z,"<",z1))

```

---

### R code Exa 8.9 Tests about a population mean

```

1 #Ex8.9, Page 317
2 #Answers may vary slightly due to rounding off of
  values
3

```

```

4 data<-c(2.67,4.62,4.14,3.81,3.83)
5 n<-length(data)
6
7 alpha<-0.05
8 mu0<-4
9
10 #To determine rejection region
11 t1<-qt(1-alpha/2,df=n-1)
12 l<-(-t1)
13 r<-t1
14 print(paste("Rejection region: t<=",l," ,t>=",r))
15
16 #To find mean
17 print(paste("Sum of sample:",sum(data)))
18 print(paste("Mean:",mean(data)))
19
20 #To find standard deviation
21 print(paste("Standard deviation:",sd(data)))
22 print(paste("Estimated standard error of the mean:",
23             sd(data)/sqrt(n)))
24
25 #Test statistic value
26 t1<-t.test(data,alternative="two.sided",mu=mu0)
27 print(t1)

```

---

#### **R code Exa 8.11** Tests concerning a population proportion

```

1 #Ex8.11, Page 324
2 #Answers may vary slightly due to rounding off of
3   values
4
5 #H0: p=0.15
6 #Ha: p>0.15
7
8 p0<-0.15

```

```

8 q0<-1-p0
9 n<-91
10 n1<-16
11
12 #Check validity of test procedures
13 if(n*p0>10 && n*q0>10) print(paste("Large sample z
    test can be used")) else print(paste("z test
    cannot be used"))
14
15 alpha<-0.1
16 zalpha<-qnorm(1-alpha)
17 print(paste("Rejection region: z>=",zalpha))
18
19 p<-n1/n
20 print(paste("p-cap:",p))
21
22 #To find z
23 z<-(p-p0)/(sqrt(p0*q0/n))
24 print(paste("Test statistic value:",z))
25
26 if(z<zalpha)print(paste("z=",z,"is not in the
    rejection region and hence, H0 cannot be rejected
    ")) else print(paste("z is in the rejection
    region"))

```

---

#### **R code Exa 8.12** Tests concerning a population proportion

```

1 #Ex8.12, Page 326
2 #Answers may vary slightly due to rounding off of
  values
3
4 #H0: p=0.9
5 #Ha: p<0.9
6
7 n<-225

```



```

8 p0<-0.9
9 pdash<-0.8
10 alpha<-0.01
11
12 zalpha<-qnorm(1-alpha)
13
14 #To find beta(0.8)
15 num<-p0-pdash-(zalpha*sqrt((p0*(1-p0))/n))
16 denom<-sqrt(pdash*(1-pdash)/n)
17 beta<-1-pnorm(num/denom)
18 print(paste("Beta(0.8):",beta))
19
20 #P(H0 rejected)
21 prob<-1-beta
22 print(paste("P(H0 rejected)=",prob,"and thus ",
              ceiling(prob*100),"% of samples will result in
              correct H0 rejection"))
23
24 #To find n
25 n<-((zalpha*sqrt(p0*(1-p0))+zalpha*sqrt(pdash*(1-
      pdash)))/(pdash-p0))^2
26 print(paste("Sample size:",ceiling(n)))

```

---

### R code Exa 8.13 Tests concerning a population proportion

```

1 #Ex8.13, Page 327
2
3 #H0: p=0.9
4 #Ha: p<0.9
5 #Rejection region: x<=15
6
7 n<-20
8 pdash<-0.8
9 alpha<-0.05
10

```

```

11 #To find beta=P(H0 not rejected when X follows
    binomial dist)
12 beta<-1-pbinom(15,n,pdash)
13 print(paste("Beta(",pdash,")=",round(beta,digits=2))
    )
14
15 print(paste(ceiling(beta*100),"% of all samples
    causes H0 to be rejected incorrectly when p=",
    pdash))

```

---

#### R code Exa 8.14 P values

```

1 #Ex8.14, Page 329
2 #Answers may vary slightly due to rounding off of
    values
3
4 #H0: mu=2.0
5 #Ha: mu>2.0
6
7 mu0<-2
8 xbar<-2.06
9 sd<-0.141
10 n<-51
11
12 #To find z value
13 z<-(xbar-mu0)/(sd/sqrt(n))
14 print(paste("Test statistic value:",z))
15
16 #To find P-value=P(Z>=z when mu=2.0)
17 p<-1-pnorm(z)
18 print(paste("P-value:",p))
19
20 alpha<-c(0.01,0.001)
21 for(i in alpha){
22     if(p<=i)

```

```

23         print(paste("H0 rejected when significance
                        level is at",i))
24     else
25         print(paste("H0 not rejected when
                        significance level is at",i))
26 }

```

---

### R code Exa 8.15 P values

```

1  #Ex8.15, Page 330
2  #Answers may vary slightly due to rounding off of
   values
3
4  #H0: mu=15
5  #Ha: mu>1.5
6
7  z<-2.10
8
9  #To determine p-value
10 p<-1-pnorm(z)
11 print(paste("P-value:",p))
12
13 alpha<-c(0.1,0.05,0.01)
14 for(i in alpha){
15     z1<-qnorm(1-i)
16     if(z>=z1)
17         print(paste("Reject H0 at significance level",i)
                )
18     else
19         print(paste("Do not reject H0 at significance
                        level",i))
20 }
21
22 #Using p-value approach
23 print(paste("Using p-value approach:"))

```

```

24 for(j in alpha){
25   if(j>=p)
26     print(paste("Reject H0 at significance level",j,
27               "since",j,">=",p))
27   else
28     print(paste("Do not reject H0 at significance
29               level",j,"since",j,"<",p))
29 }

```

---

#### R code Exa 8.17 P values for z tests

```

1 #Ex8.17, Page 333
2 #Answers may vary slightly due to rounding off of
  values
3
4 #H0: mu=245
5 #Ha: mu!=245
6
7 xbar<-246.18
8 mu0<-245
9 n<-50
10 sd<-3.6
11 alpha<-0.01
12
13 #To find z value
14 z<-(xbar-mu0)/(sd/sqrt(n))
15 print(paste("Test statistic value:",z))
16
17 #To find p value in two tailed experiment
18 p<-2*(1-pnorm(z))
19 print(paste("P-value:",p))
20
21 if(p>alpha) print(paste("H0 not rejected at
  significance level",alpha)) else print(paste("H0
  rejected at significance level",alpha))

```

---

**R code Exa 8.18** P values for t tests

```
1 #Ex8.18 , Page 335
2
3 #H0: mu=4
4 #Ha: mu!=4
5
6 n<-5
7 t<-round(-0.594,digits=1)
8 df<-n-1
9
10 alpha<-0.05
11
12 #To find critical values
13 a<-qt(1-alpha/2,df=n-1)
14
15 #Since test is two tailed
16 p<-2*pt(t,df=n-1)
17 print(paste("P-value:",round(p,digits=2)))
18
19 b<-c(0.01,0.05,0.1)
20 for(i in b){
21   if(p>i)
22     print(paste("H0 is not rejected at significance
23                 level",i))
24   else
25     print(paste("H0 rejected at significance level",
26                 i))
27 }
```

---

**R code Exa 8.19** Interpreting P values

```

1 #Ex8.19, Page 335
2 #Answers may vary slightly due to rounding off of
  values
3 #Tests for mean=20 have only been done....
4
5 #H0: mu=20
6 #Ha: mu>20
7
8 n<-4
9 df<-n-1
10 mu0<-20
11 sd<-2
12 x<-c(20.83,22.232,20.276,17.718)
13
14 xbar<-mean(x)
15 print(paste("Mean:",xbar))
16 s<-sd(x)
17 print(paste("Standard deviation:",s))
18
19 #To find t
20 t<-(xbar-mu0)/(s/sqrt(n))
21 print(paste("Test statistic value:",t))
22
23 #To find p value
24 p<-pt(t,df=n-1,lower.tail = FALSE)
25
26 alpha<-0.05
27 if(p>alpha) print(paste("H0 is not rejected at
  significance level",alpha)) else print(paste("H0
  is rejected at significance level",alpha))

```

---

# Chapter 9

## Inferences based on two samples

**R code Exa 9.1** Test procedures for normal population with known variances

```
1 #Ex9.1 , Page 348
2 #Answers may slightly vary due to rounding off of
  values
3
4 #H0:  $\mu_1 - \mu_2 = 0$ 
5 #Ha:  $\mu_1 - \mu_2 \neq 0$ 
6
7 xbar <- 29.8
8 ybar <- 34.7
9 s1 <- 4
10 s2 <- 5
11 m <- 20
12 n <- 25
13
14 #To find percentile
15 alpha <- 0.01
16 H0 <- qnorm(1-alpha/2)
17
```

```

18 #To find test statistic value
19 z<-(xbar-ybar)/sqrt((s1^2)/m+(s2^2)/n)
20 print(paste("Test statistic value:",z))
21
22 if(z>=H0||z<=H0)print(paste("H0 is rejected at level
    ",alpha)) else print(paste("H0 is accepted at
    level",alpha))

```

---

### R code Exa 9.2 Using a comparison to identify causality

```

1 #Ex9.2, Page 349
2 #Answers may slightly vary due to rounding off of
  values
3
4 #H0:  $\mu_1 - \mu_2 = 0$ 
5 #Ha:  $\mu_1 - \mu_2 > 0$ 
6
7 xbar<-48.9
8 ybar<-43.2
9 s1<-14.6
10 s2<-14.4
11 m<-125
12 n<-90
13 alpha<-0.01
14
15 #To find test statistic value
16 z<-(xbar-ybar)/sqrt((s1^2)/m+(s2^2)/n)
17 print(paste("Test statistic value:",z))
18
19 #To find p-value
20 p<-pnorm(-abs(z))
21 print(paste("p-value:",p))
22
23 if(alpha>p) print(paste("H0 is rejected at level",
    alpha)) else print(paste("H0 is accepted at

```



```
level",alpha))
```

---

**R code Exa 9.3** Beta and the choice of sample size

```
1 #Ex9.3, Page 350
2 #Answers may slightly vary due to rounding off of
  values
3
4 t<-5
5 t1<-0
6 s<-1.34
7 n<-qnorm(1-0.01/2)
8 beta<-pnorm(n-((t-t1)/s))-pnorm(-n-((t-t1)/s))
9 print(paste("Beta(5):",beta))
```

---

**R code Exa 9.4** Large sample tests

```
1 #Ex9.4, Page 352
2 #Answers may vary slightly due to rounding off of
  values
3
4 mu1<-2258
5 mu2<-2637
6 s1<-1519
7 s2<-1138
8 m<-663
9 n<-413
10 l<-0.05
11 H0<-(-200)
12
13 #To find z0.05
14 z1<-qnorm(1-l)
15
```

```

16 #To find test statistic value
17 z<-(mu1-mu2-H0)/sqrt(((s1^2)/m+(s2^2)/n))
18 print(paste("Test statistic value:",z))
19
20 if(z<=-z1) print(paste("H0 is rejected since",z,"<="
    ,-z1))
21
22 #To find p-value
23 p<-pnorm(-abs(z))
24 print(paste("P-value:",p))
25 if(p<=1) print(paste("H0 is rejected since",p,"<=",1
    ))

```

---

#### R code Exa 9.5 Confidence intervals for difference of means

```

1 #Ex9.5, Page 354
2 #Answers may vary slightly due to rounding off
  values
3
4 c1<-c('Variable_diam', 'N', 'Mean', 'Median', '
    TrMean', 'StDev', 'SEMean', 'Min', 'Max', 'Q1', '
    Q3')
5 c2<-c('3/8'
    ,78,4.250,4.230,4.238,1.300,0.147,1.634,7.327,3.389,5.075)
6 c3<-c('1/2'
    ,88,7.140,7.113,7.150,1.680,0.179,2.450,11.343,5.965,8.447)
7 df<-data.frame(c1,c2,c3)
8 print(df)
9
10 mu1<-4.25
11 mu2<-7.14
12 z<-qnorm(1-(0.05/2))
13 s1<-1.3

```

```

14 s2<-1.68
15 m<-78
16 n<-88
17
18 #To find confidence intervals
19 l<-(mu1-mu2)-z*sqrt((s1^2)/m+(s2^2)/n)
20 r<-(mu1-mu2)+z*sqrt((s1^2)/m+(s2^2)/n)
21 print(paste("Left endpoint of interval:",l))
22 print(paste("Right endpoint of interval:",r))

```

---

#### R code Exa 9.6 Two sample t test and confidence interval

```

1 #Ex9.6 , Page 358
2 #Answers may vary slightly due to rounding off of
  values
3
4 Fabric_type<-c('Cotton','Triacetate')
5 Sample_size<-c(10,10)
6 Sample_mean<-c(51.71,136.14)
7 Sample_standard_deviation<-c(.79,3.59)
8 data<-data.frame(Sample_size,Sample_mean,Sample_
  standard_deviation,row.names=Fabric_type)
9 print(data)
10
11 #Assigning values to variables
12 s1<-Sample_standard_deviation[1]
13 s2<-Sample_standard_deviation[2]
14
15 xbar<-Sample_mean[1]
16 ybar<-Sample_mean[2]
17
18 m<-Sample_size[1]
19 n<-Sample_size[2]
20
21 #To find df

```

```

22 num<-((s1^2/m)+(s2^2/n))^2
23 denom<-((s1^2/m)^2)/(m-1)+((s2^2/n)^2)/(n-1)
24 df<-num/denom
25 print(paste("df:",df))
26
27 #To find t value
28 alpha<-0.05
29 t<-qt(1-alpha/2,df=m-1)
30 print(paste("t:",t))
31
32 #To find intervals
33 l<-xbar-ybar-t*sqrt((s1^2/m)+(s2^2/n))
34 r<-xbar-ybar+t*sqrt((s1^2/m)+(s2^2/n))
35 print(paste("Left endpoint of interval:",l))
36 print(paste("Right endpoint of interval:",r))
37
38 print(paste("The true average porosity of the
    triacetate fabric is greater than that of the
    cotton fabric by between",round(-r,digits=2),"and
    ",round(-l,digits=2),"cm^3/cm^2/sec"))

```

---

#### R code Exa 9.7 Two sample t test and confidence interval

```

1 #Ex9.7, Page 359
2 #Answers may vary slightly due to rounding off of
  values
3
4 No_fusion<-c
  (2748,2700,2655,2822,2511,3149,3257,3213,3220,2753)
5
6
7 Fused<-c(3027,3356,3359,3297,3125,2910,2889,2902)
6
7 #H0: mu1-mu2=0
8 #Ha: mu1-mu2<0
9

```

```

10 #To find test staistic value
11 t<-t.test(No_fusion,Fused)
12 print(t)
13 print(paste("t statistic value:",t$statistic))
14 print(paste("df:",t$parameter))
15
16 #To plot normal probability plots
17 par(mfrow=c(2,1))
18 qqnorm(No_fusion,datax=TRUE,ylab="Not fused",xlab="
    Probability",main="Normal probability plot for
    not fused data")
19 qqline(No_fusion,datax=TRUE)
20 qqnorm(Fused,datax=TRUE,ylab="Fused",xlab="
    Probability",main="Normal probability plot for
    fused data")
21 qqline(Fused,datax=TRUE)

```

---

### R code Exa 9.9 Paired t test

```

1 #Ex9.9, Page 367
2 #Answers may vary slightly due to rounding off of
  values
3
4 Subject<-c(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16)
5 Before<-c
  (81,87,86,82,90,86,96,73,74,75,72,80,66,72,56,82)
6 After<-c
  (78,91,78,78,84,67,92,70,58,62,70,58,66,60,65,73)
7 Difference<-Before-After
8 data<-data.frame(Before,After,Difference,row.names=
  Subject)
9 print(data)
10
11 #Normal probability plot
12 par(mfrow=c(1,2))

```

```

13 qqnorm(data$Difference,main="Normal probability plot
    of the differences",xlab="diff",ylab="
    Probability")
14 qqline(data$Difference)
15 boxplot(Difference,horizontal=TRUE,xlab="Difference"
    )
16
17 #H0: mu=0
18 #Ha: mu!=0
19 t1<-t.test(Difference,mu=0)
20 print(paste("t statistic value:",t1$statistic))
21 print(paste("p value:",t1$p.value))
22
23 alpha<-0.01
24 if(t1$p.value<alpha) print(paste("H0 rejected"))
    else print(paste("H0 not rejected"))

```

---

#### R code Exa 9.10 Paired t confidence interval

```

1 #Ex9.10, Page 369
2 #Answers may vary slightly due to rounding off of
    values
3
4 Subject<-c(1,2,3,4,5,6,7,8,9,10,11,12,13)
5 Slide<-c(30,35,40,25,20,30,35,62,40,51,25,42,33)
6 Digital<-c(25,16,15,15,10,20,7,16,15,13,11,19,19)
7 Difference<-Slide-Digital
8 data<-data.frame(Slide,Digital,Difference,row.names=
    Subject)
9 print(data)
10
11 #Normal probability plot
12 qqnorm(data$Difference,main="Normal probability plot
    of the differences",ylab="diff",xlab="
    Probability")

```

```

13 qqline(data$Difference)
14
15 #To find t critical value
16 alpha<-0.05
17 t<-qt(1-alpha/2,df=12)
18 print(paste("t statistic value:",t))
19
20 #To find confidence intervals
21 t1<-t.test(Slide,Digital,paired=TRUE)
22 print(t1)

```

---

**R code Exa 9.11** Inferences concerning difference between population proportions

```

1 #Ex9.11, Page 376
2 #Answers may slightly vary due to rounding off of
  values
3
4 alpha<-0.05
5
6 #H0:  $p_1 - p_2 = 0$ 
7 #Ha:  $p_1 - p_2 < 0$ 
8
9 a<-81
10 tot1<-549
11 p1cap<-a/tot1
12 q1cap<-1-p1cap
13
14 b<-141
15 tot2<-730
16 p2cap<-b/tot2
17 q2cap<-1-p2cap
18
19 pcap<-(a+b)/(tot1+tot2)
20 qcap<-1-pcap

```

```

21
22 #To find test statistic value
23 if((tot1*p1cap)>=10 && (tot1*q1cap)>=10 && (tot2*
    p2cap)>=10 && (tot2*q2cap)>=10)
24     z<-(p1cap-p2cap)/sqrt(pcap*qcap*(1/tot1+1/tot2))
25 print(paste("Test statistic value:",z))
26
27 #To find P-value
28 p<-pnorm(z)
29 print(paste("P-value:",p))
30
31 if(p<=alpha) print(paste("H0 can be rejected at
    significance level",alpha)) else print(paste("H0
    cannot be rejected"))

```

---

#### R code Exa 9.12 Type II error probabilities and sample sizes

```

1 #Ex9.12, Page 378
2 #Answers may vary slightly due to rounding off of
    values
3
4 #H0: p1-p2=0
5 #Ha: p1-p2>0
6
7 a<-30
8 tot1<-100000
9 p1<-a/tot1
10 print(paste("p1:",p1))
11 q1<-1-p1
12
13 b<-a/2
14 p2<-b/tot1
15 print(paste("p2:",p2))
16 q2<-1-p2
17

```



```

18 alpha<-0.05
19 beta<-0.1
20
21 zalpha<-qnorm(1-alpha)
22 zbeta<-qnorm(1-beta)
23 d<-p1-p2
24
25 #To find n value
26 n<-(zalpha*sqrt((p1+p2)*(q1+q2)/2)+zbeta*sqrt(p1*q1+
    p2*q2))^2/d^2
27 print(paste("Value of n:",n))

```

---

#### R code Exa 9.13 A large sample confidence interval

```

1 #Ex9.13, Page 380
2 #Answers may vary slightly due to rounding off of
  values
3
4 alpha<-0.01
5 a<-76      #number of people who survived through
  chemotherapy treatment
6 m<-154     #number of people who received
  chemotherapy treatment
7 p1cap<-a/m
8 q1cap<-1-p1cap
9 print(paste("p1cap:",p1cap))
10
11 b<-98      #number of people who survived through
  hybrid treatment
12 n<-164     #number of people who received hybrid
  treatment
13 p2cap<-b/n
14 q2cap<-1-p2cap
15 print(paste("p2cap:",p2cap))
16

```

```

17 #To find z
18 zalpha<-qnorm(1-alpha/2)
19
20 if(m*p1cap>=10 && m*q1cap>=10 && n*p2cap>=10 && n*
    q2cap>=10)
21   l<-p1cap-p2cap-zalpha*sqrt((p1cap*q1cap/m)+(p2cap*
    q2cap/n))
22   r<-p1cap-p2cap+zalpha*sqrt((p1cap*q1cap/m)+(p2cap*
    q2cap/n))
23
24 print(paste("Left endpoint of confidence interval:",
    l))
25 print(paste("Right endpoint of confidence interval:"
    ,r))
26 print(paste("Hence",round(l,digits=3),"<p1-p2<" ,
    round(r,digits=3)))

```

---

#### R code Exa 9.14 F test for equality of variances

```

1 #Ex9.14, Page 385
2 #Answers may vary slightly due to rounding off of
  values
3
4 s1<-52.6
5 s2<-84.2
6 m<-28
7 n<-26
8
9 #H0:  $s_1^2 = s_2^2$ 
10 #Ha:  $s_1^2 < s_2^2$ 
11
12 alpha<-0.01
13 #To find F value
14 Falpha<-qf(alpha,df1=m-1,df2=n-1)
15 print(paste("F value:",Falpha))

```

```
16
17 f<-(s1^2)/(s2^2)
18 print(paste("f value:",f))
19
20 if(f<=Falpha) print(paste("H0 is rejected at
    significance level",alpha)) else print(paste("H0
    is not rejected"))
```

---

# Chapter 10

## The analysis of variance

**R code Exa 10.1** Single factor ANOVA

```
1 #Ex10.1 , Page 392
2 #Answers may vary slightly due to rounding off of
  values
3
4 Type_of_box<-c(1,2,3,4)
5 CS1<-c(655.5,789.2,737.1,535.1)
6 CS2<-c(788.3,772.5,639.0,628.7)
7 CS3<-c(734.3,786.9,696.3,542.4)
8 CS4<-c(721.4,686.1,671.7,559.0)
9 CS5<-c(679.1,732.1,717.2,586.9)
10 CS6<-c(699.4,774.8,727.1,520.0)
11 Mean<-c(NULL)
12 SD<-c(NULL)
13
14 #To find mean of observations
15 for(i in 1:4){
16   sum<-CS1[i]+CS2[i]+CS3[i]+CS4[i]+CS5[i]+CS6[i]
17   mean<-sum/6
18   Mean[i]<-mean
19 }
20 #To find standard deviation of values
```

```

21 for(i in 1:4){
22   s<-(CS1[i]-Mean[i])^2+(CS2[i]-Mean[i])^2+(CS3[i]-
      Mean[i])^2+(CS4[i]-Mean[i])^2+(CS5[i]-Mean[i])
      ^2+(CS6[i]-Mean[i])^2
23   sd<-sqrt(s/5)
24   SD[i]<-sd
25 }
26
27 data<-data.frame(Type_of_box,CS1,CS2,CS3,CS4,CS5,CS6
      ,Mean,SD)
28 print(data)
29
30 #To find grand mean of mean values
31 tot_mean<-0
32 for(i in 1:4){
33   tot_mean<-tot_mean+Mean[i]
34 }
35 tot_mean<-tot_mean/4
36 print(paste("Grand mean:",tot_mean))
37
38 par(mfrow=c(1,2))
39 #To create boxplots of the data
40 #First element of each CS column gives us the
      compression strength of one box
41 a<-c(CS1[1],CS2[1],CS3[1],CS4[1],CS5[1],CS6[1])
42 b<-c(CS1[2],CS2[2],CS3[2],CS4[2],CS5[2],CS6[2])
43 c<-c(CS1[3],CS2[3],CS3[3],CS4[3],CS5[3],CS6[3])
44 d<-c(CS1[4],CS2[4],CS3[4],CS4[4],CS5[4],CS6[4])
45 boxplot(a,b,c,d)
46 title(main="Boxplots for original data")
47
48 #Altering data in sample 2 by adding 120 to every
      value in Sample 4
49 e<-c(NULL)
50 for(i in 1:6){
51   e[i]<-d[i]+120
52 }
53 boxplot(a,b,c,e)

```

```
54 title(main="Boxplots for altered data")
```

---

### R code Exa 10.2 F distribution and the F test

```
1 #Ex10.2, Page 397
2 #Answers may vary slightly due to rounding off of
  values
3
4 #From Ex10.1
5 Mean<-c(713.00,756.93,698.07,562.02)
6 SD<-c(46.55,40.34,37.20,39.87)
7 tot_mean<-682.50
8
9 I<-4
10 J<-6
11
12 #Numerator
13 v1<-I-1
14
15 #Denominator
16 v2<-I*(J-1)
17
18 alpha<-0.05
19
20 #H0:mu1=mu2=mu3=mu4
21 #Ha:at least two mean values will be different
22
23 #To find F critical value
24 F<-qf(p=1-alpha,df1=v1,df2=v2)
25 print(paste("Critical value F:",F))
26
27 #To find mean square for treatment
28 MStr<-(J/(I-1))*sum((Mean-tot_mean)^2)
29 print(paste("MStr:",MStr))
30
```

```

31 #To find mean square for error
32 MSE<-(1/I)*sum(SD^2)
33 print(paste("MSE:",MSE))
34
35 #To find f value
36 f<-MSTr/MSE
37 print(paste("f:",f))
38
39 if(f>=F) print(paste("H0 is rejected at
    significance level",alpha,"since f>=F")) else
    print(paste("H0 not rejected"))

```

---

### R code Exa 10.3 F distribution and the F test

```

1 #Ex10.3, Page 397
2 #Answers may vary slightly due to rounding off of
  values
3
4 Treatment<-c(1,2,3,4,5)
5 Sample_mean<-c(10.5,14.8,15.7,16.0,21.6)
6 Sample_sd<-c(4.5,6.8,6.5,6.7,6.0)
7
8 #To find total mean of mean values
9 tot_mean<-mean(Sample_mean)
10 print(paste("Grand mean:",tot_mean))
11
12 #H0: mu1=mu2=mu3=mu4=mu5
13 #Ha: at least two of the mean values are different
14
15 I<-5
16 J<-10
17 alpha<-0.01
18
19 #Numerator
20 v1<-I-1

```

```

21
22 #Denominator
23 v2<-I*(J-1)
24
25 #To find F value
26 F<-qf(1-alpha,v1,v2)
27 print(paste("Critical value F:",F))
28
29 #To find mean square for treatment
30 s<-Sample_mean-tot_mean
31 MStr<-(J/(I-1))*sum(s^2)
32 print(paste("MStr:",MStr))
33
34 #To find mean square for error
35 MSE<-mean(Sample_sd^2)
36 print(paste("MSE:",MSE))
37
38 #To find f value
39 f<-MStr/MSE
40 print(paste("f:",f))
41
42 if(f>=F) print(paste("H0 is rejected at
    significance level",alpha,"since f>=F")) else
    print(paste("H0 not rejected"))

```

---

#### R code Exa 10.4 Sums of squares

```

1 #Ex10.4, Page 400
2 #Answers may vary slightly due to rounding off of
  values
3
4 Mixture1<-c(0.56,1.12,0.90,1.07,0.94)
5 Mixture2<-c(0.72,0.69,0.87,0.78,0.91)
6 Mixture3<-c(0.62,1.08,1.07,0.99,0.93)
7

```



```

8 data1<-data.frame(Mixture1,Mixture2,Mixture3)
9
10 I<-3
11 J<-5
12
13 #Numerator
14 v1<-I-1
15
16 #Denominator
17 v2<-I*(J-1)
18
19 alpha<-0.01
20 #To find F value
21 F<-qf(1-alpha,v1,v2)
22 print(paste("Critical value , F:",F))
23
24
25 #To create ANOVA table
26 mix<-stack(data1)
27 names(mix)<-c("Degree","Treatment")
28
29 a<-aov(Degree~Treatment,data=mix)
30 print(paste("ANOVA table:"))
31 a1<-summary(a)
32 print(a1)
33 print(paste("f value:",a1[[1]]$F[1]))

```

---

### R code Exa 10.6 Multiple comparisons in ANOVA

```

1 #Ex10.6 , Page 404
2
3 Treatment1<-c(88.6,73.2,91.4,68.0,75.2)
4 Treatment2<-c(63.0,53.9,69.2,50.1,71.5)
5 Treatment3<-c(44.9,59.5,40.2,56.3,38.7)
6 Treatment4<-c(31.0,39.6,45.3,25.2,22.7)

```

```

7 data1<-data.frame(Treatment1,Treatment2,Treatment3,
  Treatment4)
8
9 Treatment<-stack(data1)
10 names(Treatment)<-c("Time","Model")
11
12 #To display ANOVA table
13 a<-aov(Time~Model,data=Treatment)
14 print(summary(a))

```

---

#### R code Exa 10.9 Unequal sample sizes

```

1 #Ex10.9, Page 412
2
3 Permanent_molding<-c
  (45.5,45.3,45.4,44.4,44.6,43.9,44.6,44.0)
4 Die_casting<-c
  (44.2,43.9,44.7,44.2,44.0,43.8,44.6,43.1)
5 Plaster_molding<-c(46.0,45.9,44.8,46.2,45.1,45.5)
6
7 #To display ANOVA table
8 data1<-data.frame(Mold=c(Permanent_molding,Die_
  casting,Plaster_molding),Treatment=factor(rep(c("
  Mold1","Mold2","Mold3"),times=c(length(Permanent_
  molding),length(Die_casting),length(Plaster_
  molding)))))
9
10 cat("ANOVA table\n")
11 a<-aov(Mold~Treatment,data=data1)
12 print(summary(a))

```

---

#### R code Exa 10.10 Unequal sample sizes

```

1 #Ex10.10 , Page 413
2
3 alpha<-0.95
4 n<-22
5 J1<-8
6 J2<-8
7 J3<-6
8 I<-3
9 J<-n-I
10 MSE<-0.316
11 Q<-qtukey(alpha,I,J)
12 xibar<-c(44.71,44.06,45.58)
13
14 #To find w12
15 w12<-Q*sqrt((MSE/2)*(1/J1+1/J2))
16 print(paste("w12:",w12))
17 w23<-Q*sqrt((MSE/2)*(1/J2+1/J3))
18 print(paste("w23:",w23))
19 w31<-Q*sqrt((MSE/2)*(1/J3+1/J1))
20 print(paste("w31:",w31))
21
22 if(xibar[1]-xibar[2]< w12){
23   print(paste("mu1 and mu2 are not significantly
                different"))
24 }
25 if(xibar[2]-xibar[3]< w23){
26   print(paste("mu2 and mu3 are not significantly
                different"))
27 }
28 if(xibar[3]-xibar[1]< w31){
29   print(paste("mu3 and mu1 are not significantly
                different"))
30 }

```

---

**R code Exa 10.11** A random effects model

```

1 #Ex.10.11, Page 415
2
3 #H0:  $\sigma_A^2=0$ 
4
5 C1<-c(55,53,54)
6 C2<-c(26,37,32)
7 C3<-c(78,91,85)
8 C4<-c(92,100,96)
9 C5<-c(49,51,50)
10 C6<-c(80,85,83)
11 data1<-data.frame(C1,C2,C3,C4,C5,C6)
12 print(paste("x..=",sum(data1)))
13
14
15 #To create ANOVA table
16 col<-stack(data1)
17 names(col)<-c("Type","Treatments")
18 a<-aov(Type~Treatments,data=col)
19 print(paste("ANOVA table:"))
20 print(summary(a))

```

---

# Chapter 11

## Multifactor analysis of variance

**R code Exa 11.1** Two factor ANOVA

```
1 #Ex11.1, Page 420
2
3 dat<-c
   (0.97,0.48,0.48,0.46,0.77,0.14,0.22,0.25,0.67,0.39,0.57,0.19)

4 mat1<-matrix(dat,nrow=3,ncol=4,byrow=TRUE)
5 rownames(mat1)<-c("Brand1","Brand2","Brand3")
6 colnames(mat1)<-c("WT1","WT2","WT3","WT4")
7 print(mat1)
8
9 print(paste("Number of levels of factor A(Brand of
   pen), I:",nrow(mat1)))
10 print(paste("Number of levels of factor B(Washing
   treatment), J:",ncol(mat1)))
```

---

**R code Exa 11.3** Fixed effects model

```
1 #Ex11.3, Page 424
```

```

2 #Answers may vary slightly due to rounding off of
  values
3
4 #To create data frame
5 brand<-rep(c(" I", " II", " III"),4)
6 WT<-rep(c(" 1", " 2", " 3", " 4"),each=3) #WT<-Washing
  treatment
7 col_change<-c
  (0.97,0.77,0.67,0.48,0.14,0.39,0.48,0.22,0.57,0.46,0.25,0.19)

8
9 df1<-data.frame(brand,WT,col_change)
10 print(df1)
11
12 #To display ANOVA table
13 a<-aov(col_change~brand+WT,data=df1)
14 print(a)
15 cat("\nSummarized table:\n")
16 print(summary(a))
17
18 #Diagnostic plots
19 model<-lm(col_change~brand+WT,data=df1)
20 par(mfrow=c(1,2))
21 qqnorm(model$residuals,datax=TRUE,main="Normal
  probability plot",ylab="Residual",xlab="Percent")
22 qqline(model$residuals,datax=TRUE)
23
24 plot(df1$col_change,resid(model),main="Residual plot
  ",xlab="Fitted value",ylab="Residual")
25 abline(0,0)

```

---

### R code Exa 11.5 Randomized block experiments

```

1 #Ex11.5, Page 427
2 #Answers may vary slightly due to rounding off of

```

```

      values
3
4 #To create data frame
5 Treatment<-rep(c(" I", " II", " III", " IV", " V"),4)
6 Block<-rep(c(" 1", " 2", " 3", " 4"),each=5)
7 Power<-c
      (685,722,733,811,828,792,806,802,888,920,838,893,880,952,978,875,

8
9 df1<-data.frame(Treatment,Block,Power)
10 print(df1)
11
12 #To display ANOVA table
13 a<-aov(Power~Treatment+Block,data=df1)
14 print(a)
15 cat("\nSummarized table:\n")
16 print(summary(a))

```

---

### R code Exa 11.6 Randomized block experiments

```

1 #Ex11.6, Page 428
2 #Answers may vary slightly due to rounding off of
      values
3
4 #To create data frame
5 Tension<-rep(c(" 210", " 235", " 260", " 285"),6)
6 Player<-rep(c(" 1", " 2", " 3", " 4", " 5", " 6"),each=4)
7 Speed<-c
      (105.7,113.3,117.2,110.0,116.6,119.9,124.4,106.8,106.6,120.5,122.1,

8
9 df1<-data.frame(Tension,Player,Speed)
10 print(df1)
11
12 #To find mean for corresponding tension data

```

```

13 cat("\nTension mean values:\n")
14 m1<-tapply(df1$Speed,list(Tension),mean)
15 print(m1)
16
17 #To find mean of corresponding player data
18 cat("\nPlayer mean values:\n")
19 m2<-tapply(df1$Speed,list(Player),mean)
20 print(m2)
21
22 #To display ANOVA table
23 a<-aov(Speed~Tension+Player,data=df1)
24 print(a)
25 print(summary(a))

```

---

#### R code Exa 11.7 Two factor ANOVA with Kij greater than 1

```

1 #Ex11.7, Page 435
2
3 AggCont<-rep(c("38","41","44"),each=6)
4 AsphGr<-rep(c("PG58","PG64","PG70"),6)
5 data<-c
      (0.835,0.855,0.815,0.845,0.865,0.825,0.822,0.832,0.800,0.826,0.83
6 df1<-data.frame(AggCont,AsphGr,data)
7 print(df1)
8
9 #To find mean for corresponding content data
10 cat("\nCoarse aggregate content(%) mean values:\n")
11 m1<-tapply(df1$data,list(AggCont),mean)
12 print(m1)
13
14 #To find mean of corresponding Asphalt binder grade
    data
15 cat("\nAsphalt binder grade mean values:\n")
16 m2<-tapply(df1$data,list(AsphGr),mean)

```



```

17 print(m2)
18
19
20 #To display ANOVA table
21 model<-lm(data~AsphGr+AggCont+AsphGr:AggCont)
22 a<-aov(data~AsphGr+AggCont+AsphGr:AggCont)
23 print(a)
24 print(summary(a))
25
26 #To display interaction plots(Thermal diffusivity
    data is not given)
27 interaction.plot(AggCont,AsphGr,response=data)
28
29 #Plots for checking normality and constant variance
    assumptions
30 par(mfrow=c(1,2))
31 qqnorm(model$residuals,datax=TRUE,main="Normal
    probability plot",ylab="Residual",xlab="Percent")
32 qqline(model$residuals,datax=TRUE)
33
34 plot(df1$data,resid(model),main="Residual plot",xlab
    ="Fitted value",ylab="Residual")
35 abline(0,0)

```

---

### R code Exa 11.9 Models with mixed and random effects

```

1 #Ex11.9, Page 439
2 #Answers may vary slightly from textbook values
3
4 #To create data frame
5 Casmater<-rep(c("Steel","Aluminium","Plastic"),10)
6 Source<-rep(c("1","2","3","4","5"),each=6)
7 Power<-c
    (13.1,15.0,14.0,13.2,14.8,14.3,16.3,15.7,17.2,15.8,16.4,16.7,13.7

```

```

8
9 df1<-data.frame(Casmater,Source,Power)
10 print(df1)
11
12 #To create ANOVA table
13 a<-aov(Power~Casmater+Source+Casmater:Source,data=
      df1)
14 print(a)
15 cat("\nSummarized table:\n")
16 print(summary(a))

```

---

#### R code Exa 11.10 Three factor ANOVA

```

1 #Ex11.10, Page 444
2 #Answers may vary slightly due to rounding off of
  values
3
4 #To create data frame
5 A<-rep(c(rep(c("A1","A2","A3","A4"),each=3)),8)
6 B<-rep(c("B1","B2"),each=48)
7 C<-rep(c("C1","C2","C3","C4","C1","C2","C3","C4"),
      each=12)
8 data<-c
      (3.6,3.8,3.9,3.8,3.6,4.0,3.7,3.9,4.2,3.6,3.5,3.8,3.4,3.7,3.9,3.8,
9
10 df1<-data.frame(A,B,C,data)
11 print(df1)
12
13 #To find cell totals for all combinations of the
    three factors
14 cat("\nCell totals:\n")
15 m1<-tapply(df1$data,list(A,B,C),sum)
16 print(m1)
17

```

```

18 #To create ANOVA table
19 a<-aov(data~A+B+C+A:B+A:C+B:C+A:B:C,data=df1)
20 print(a)
21 cat("\nSummarized table:\n")
22 print(summary(a))

```

---

### R code Exa 11.11 Latin square designs

```

1 #Ex11.11, Page 447
2 #Answers may vary from textbook values
3
4 library(nlme)
5 library(multcomp)
6
7 #Dependent variable
8 data<-c
   (7.38,7.15,6.75,8.05,5.65,6.00,5.39,8.16,5.64,6.45,5.44,6.55,5.03
9
10 #Repeated measures
11 ##B(Columns)
12 B<-factor(rep(c("1","2","3","4","5","6"),each=6))
13
14 ##A(Rows)
15 A<-factor(rep(c("I","II","III","IV","V","VI"),6))
16
17 df1<-data.frame(A,B,data)
18 print(df1)
19
20 #To create ANOVA table
21 #using aov() function
22 cat("\nUsing aov():\n")
23 a1<-aov(data~A+Error(B/A),data=df1)
24 print(summary(a1))
25

```

```

26 #using lme()
27 cat("\nUsing lme():\n")
28 model<-lme(data~A,random=~1|B/A,data=df1)
29 a2<-anova(model)
30 print(a2)
31
32 #Tukey test
33 print(summary(glht(model,linfct=mcp(A="Tukey"))))

```

---

### R code Exa 11.12 Factorial experiments with p factors at two levels

```

1 #Ex11.12, Page 452
2
3 Age<-rep(c("1","2"),each=2,4)
4 Temp<-rep(c("1","2"),8)
5 Soil<-rep(c("1","2"),each=8)
6 Comp_Stre<-c
   (471,485,712,712,413,552,637,789,385,530,770,741,434,593,705,806)
7 df1<-data.frame(Age,Temp,Soil,Comp_Stre)
8
9 #To compute cell totals
10 s1<-s2<-s3<-s4<-s5<-s6<-s7<-s8<-c(NULL)
11 for(i in 1:length(Comp_Stre)){
12   if(df1$Age[i]=="1" && df1$Temp[i]=="1" && df1$Soil
       [i]=="1"){
13     s1[i]<-df1$Comp_Stre[i]
14     x111<-sum(s1,na.rm=TRUE)
15   }
16   else if(df1$Age[i]=="1" && df1$Temp[i]=="2" && df1
       $Soil[i]=="1"){
17     s2[i]<-df1$Comp_Stre[i]
18     x121<-sum(s2,na.rm=TRUE)
19   }
20   else if(df1$Age[i]=="1" && df1$Temp[i]=="1" && df1

```

```

    $Soil[i]=="2"){
21     s3[i]<-df1$Comp_Stre[i]
22     x112<-sum(s3,na.rm=TRUE)
23   }
24   else if(df1$Age[i]=="2" && df1$Temp[i]=="1" && df1
    $Soil[i]=="1"){
25     s4[i]<-df1$Comp_Stre[i]
26     x211<-sum(s4,na.rm=TRUE)
27   }
28   else if(df1$Age[i]=="2" && df1$Temp[i]=="1" && df1
    $Soil[i]=="2"){
29     s5[i]<-df1$Comp_Stre[i]
30     x212<-sum(s5,na.rm=TRUE)
31   }
32   else if(df1$Age[i]=="1" && df1$Temp[i]=="2" && df1
    $Soil[i]=="2"){
33     s6[i]<-df1$Comp_Stre[i]
34     x122<-sum(s6,na.rm=TRUE)
35   }
36   else if(df1$Age[i]=="2" && df1$Temp[i]=="2" && df1
    $Soil[i]=="1"){
37     s7[i]<-df1$Comp_Stre[i]
38     x221<-sum(s7,na.rm=TRUE)
39   }
40   else if(df1$Age[i]=="2" && df1$Temp[i]=="2" && df1
    $Soil[i]=="2"){
41     s8[i]<-df1$Comp_Stre[i]
42     x222<-sum(s8,na.rm=TRUE)
43   }
44
45 }
46 #Parameters of the model
47 n<-2
48 alpha1_cap<-(x111+x121+x112+x122-x211-x212-x221-x222
  )/(8*n)
49 alpha2_cap<-(-1)*alpha1_cap
50
51 gamma11_cap<-(x111-x121-x211+x221+x112-x122-x212+

```

```

      x222)/(8*n)
52 gamma12_cap<-(-1)*gamma11_cap
53 gamma21_cap<-(-1)*gamma11_cap
54 gamma22_cap<-gamma11_cap
55
56 print(paste("alpha1_cap:",alpha1_cap))
57 print(paste("alpha2_cap:",alpha2_cap))
58 print(paste("gamma11_cap:",gamma11_cap))
59 print(paste("gamma12_cap:",gamma12_cap))
60 print(paste("gamma21_cap:",gamma21_cap))
61 print(paste("gamma22_cap:",gamma22_cap))

```

---

### R code Exa 11.13 Factorial experiments with p factors at two levels

```

1 #Ex11.13, Page 453
2 #Answers may vary slightly due to rounding off of
  values
3
4 Age<-factor(rep(c("1","2"),each=2,4))
5 Temp<-factor(rep(c("1","2"),8))
6 Soil<-factor(rep(c("1","2"),each=8))
7 Comp_Stre<-c
  (471,485,712,712,413,552,637,789,385,530,770,741,434,593,705,806)
8 df1<-data.frame(Age,Temp,Soil,Comp_Stre)
9
10 #To compute cell totals
11 s1<-s2<-s3<-s4<-s5<-s6<-s7<-s8<-c(NULL)
12 for(i in 1:length(Comp_Stre)){
13   if(df1$Age[i]=="1" && df1$Temp[i]=="1" && df1$Soil
      [i]=="1"){
14     s1[i]<-df1$Comp_Stre[i]
15     x111<-sum(s1,na.rm=TRUE)
16   }
17   else if(df1$Age[i]=="1" && df1$Temp[i]=="2" && df1

```

```

    $Soil[i]=="1"){
18     s2[i]<-df1$Comp_Stre[i]
19     x121<-sum(s2,na.rm=TRUE)
20   }
21   else if(df1$Age[i]=="1" && df1$Temp[i]=="1" && df1
    $Soil[i]=="2"){
22     s3[i]<-df1$Comp_Stre[i]
23     x112<-sum(s3,na.rm=TRUE)
24   }
25   else if(df1$Age[i]=="2" && df1$Temp[i]=="1" && df1
    $Soil[i]=="1"){
26     s4[i]<-df1$Comp_Stre[i]
27     x211<-sum(s4,na.rm=TRUE)
28   }
29   else if(df1$Age[i]=="2" && df1$Temp[i]=="1" && df1
    $Soil[i]=="2"){
30     s5[i]<-df1$Comp_Stre[i]
31     x212<-sum(s5,na.rm=TRUE)
32   }
33   else if(df1$Age[i]=="1" && df1$Temp[i]=="2" && df1
    $Soil[i]=="2"){
34     s6[i]<-df1$Comp_Stre[i]
35     x122<-sum(s6,na.rm=TRUE)
36   }
37   else if(df1$Age[i]=="2" && df1$Temp[i]=="2" && df1
    $Soil[i]=="1"){
38     s7[i]<-df1$Comp_Stre[i]
39     x221<-sum(s7,na.rm=TRUE)
40   }
41   else if(df1$Age[i]=="2" && df1$Temp[i]=="2" && df1
    $Soil[i]=="2"){
42     s8[i]<-df1$Comp_Stre[i]
43     x222<-sum(s8,na.rm=TRUE)
44   }
45 }
46 }
47
48 Effect_contrasts<-c(NULL)

```

```

49 #To compute effect contrasts
50 for(j in 1:7){
51   if(j==1){
52     Effect_contrasts[j]=-x111+x211-x121+x221-x112+
      x212-x122+x222
53   }
54   else if(j==2){
55     Effect_contrasts[j]=-x111-x211+x121+x221-x112-
      x212+x122+x222
56   }
57   else if(j==3){
58     Effect_contrasts[j]=x111-x211-x121+x221+x112-
      x212-x122+x222
59   }
60   else if(j==4){
61     Effect_contrasts[j]=-x111-x211-x121-x221+x112+
      x212+x122+x222
62   }
63   else if(j==5){
64     Effect_contrasts[j]=x111-x211+x121-x221-x112+
      x212-x122+x222
65   }
66   else if(j==6){
67     Effect_contrasts[j]=x111+x211-x121-x221-x112-
      x212+x122+x222
68   }
69   else if(j==7){
70     Effect_contrasts[j]=-x111+x211+x121-x221+x112-
      x212-x122+x222
71   }
72 }
73
74 SS<-(Effect_contrasts^2)/(8*n)
75 df2<-data.frame(Effect_contrasts,SS)
76 cat("\nEffect contrast and SS values:\n")
77 print(df2)
78 SSsum<-sum(SS)
79

```



```

80 #To find sum of squares value
81 x2_ijkl<-0
82 X2<-0
83 for(k in 1:length(Comp_Stre)){
84     x2_ijkl<-x2_ijkl+(Comp_Stre[k]^2)
85 }
86 x2<-(x111+x112+x211+x121+x212+x221+x122+x222)^2/16
87 print(paste("Sum of square of data:",x2_ijkl))
88 print(paste("x...^2/16:",x2))
89
90 #To find SST value
91 SST<-x2_ijkl-x2
92 print(paste("SST:",SST))
93
94 #To find SSE value
95 SSE<-SST-SSsum
96 print(paste("SSE:",SSE))
97
98 #To display corresponding ANOVA tables
99 model1<-lm(Comp_Stre~Age:Temp:Soil)
100 model2<-lm(Comp_Stre~Age*Temp*Soil)
101 a<-aov(model1,data=df1)
102 b<-aov(model2,data=df1)
103 print(summary(a))
104 print(summary(b))

```

---

## Chapter 12

# Simple linear regression and correlation

**R code Exa 12.1** The simple linear regression model

```
1 #Ex12.1 , Page 470
2
3 i<-c
   (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25)
4 xi<-c
   (0.40,0.42,0.48,0.51,0.57,0.60,0.70,0.75,0.75,0.78,0.84,0.95,0.99)
5 yi<-c
   (1.02,1.21,0.88,0.98,1.52,1.83,1.50,1.80,1.74,1.63,2.00,2.80,2.48)
6 data<-data.frame(xi,yi,row.names=i)
7 print(data)
8
9 par(mfrow=c(1,3))
10 plot(xi,yi,main="Scatterplot",xlab="Width of
    palprebal fissure",ylab="OSA")
11 dotchart(xi,labels=row.names(xi),main="Dot plot of x
    values")
```

```
12 dotchart(yi, labels=row.names(yi), main="Dot plot of y
    values")
```

---

### R code Exa 12.2 The simple linear regression model

```
1 #Ex12.2, Page 471
2
3 x<-c
    (7.01,7.11,7.12,7.24,7.94,7.94,8.04,8.05,8.07,8.90,8.94,8.95,8.97
4 y<-c
    (60,67,66,52,50,45,52,48,40,23,20,40,31,26,9,22,13,7)
5 data1<-data.frame(x,y)
6 print(data)
7
8 plot(x,y,main="Scatterplot",xlab="pH",ylab="Arsenic
    removed(%)" )
```

---

### R code Exa 12.3 A linear probabilistic model

```
1 #Ex12.3, Page 474
2 #Answers may slightly vary due to rounding off of
    probabilities
3
4 y<- function(x) (65-1.2*x)
5 sd<-8
6
7 #P(time-to-failure exceeds 50 when applied stress is
    20)
8 x1<-20
9 mu1<-y(x1)
10 q<-50
```

```

11 P1<-1-pnorm(q,mu1,sd)
12 print(paste("Probability that the time-to-failure >50
    when applied stress=20):",P1))
13
14 #P(time-to-failure exceeds 50 when applied stress is
    25)
15 x2<-25
16 mu2<-y(x2)
17 q<-50
18 P2<-1-pnorm(q,mu2,sd)
19 print(paste("Probability that the time-to-failure >50
    when applied stress=25):",P2))
20
21 #Let Y1<-observation made with x=25 and Y2<-
    observation made with x=24
22 #Mean value
23 X1<-25
24 X2<-24
25 E<-y(X1)-y(X2)
26 #Variance
27 V<-sd^2+sd^2
28 SD<-sqrt(V)
29 #Probability that Y1 exceeds Y2
30 P3<-1-pnorm(0,E,V)
31 print(paste("P(Y1>Y2):",P3))

```

---

#### R code Exa 12.4 Estimating model parameters

```

1 #Ex12.4, Page 479
2 #Answers may slightly vary due to rounding off of
    values
3
4 x<-c
    (132.0,129.0,120.0,113.2,105.0,92.0,84.0,83.2,88.4,59.0,80.0,81.5

```

```

5  y<-c
    (46.0,48.0,51.0,52.1,54.0,52.0,59.0,58.7,61.6,64.0,61.4,54.6,58.8

6  data1<-data.frame(x,y)
7
8  #To display regression model
9  model<-lm(y~x,data=data1)
10 cat("Regression model of data:\n")
11 a<-summary(model)
12 print(a)
13
14 #To display the scatterplot
15 plot(x,y,main="Scatterplot with least square line
    superimposed",xlab="iod val",ylab="cet num")
16 abline(a$coefficients[1],a$coefficients[2])

```

---

#### R code Exa 12.5 Estimating model parameters

```

1  #Ex12.5, Page 480
2
3  y<- function(x) {75.212-0.2094*x}
4  #If iodine value 100 is selected...
5  x<-100
6  print(paste("Point prediction for the resulting
    cetane number:",round(y(x),digits=2)))

```

---

#### R code Exa 12.6 Estimating model parameters

```

1  #Ex12.6, Page 482
2  #Answers may slightly vary due to rounding off of
    values
3

```

```

4  x<-c
    (125.3,98.2,201.4,147.3,145.9,124.7,112.2,120.2,161.2,178.9,159.5
5  y<-c
    (77.9,76.8,81.5,79.8,78.2,78.3,77.5,77.0,80.1,80.2,79.9,79.0,76.7

6  data1<-data.frame(x,y)
7
8  model<-lm(y~x)
9
10 #To display final table of predicted values and
    residuals
11 yb<-predict(model,data1)
12 Residual<-resid(model)
13 data1<-data.frame(x,y,yb,Residual)
14 colnames(data1)[colnames(data1)=="x"]<-"Filtrate"
15 colnames(data1)[colnames(data1)=="y"]<-"Moistcon"
16 colnames(data1)[colnames(data1)=="yb"]<-"Fit"
17 print(data1)

```

---

### R code Exa 12.7 Estimating model parameters

```

1  #Ex12.7, Page 483
2  #Answers may vary slightly due to rounding off of
    values
3
4  #From Ex12.6
5  x<-c
    (125.3,98.2,201.4,147.3,145.9,124.7,112.2,120.2,161.2,178.9,159.5
6  y<-c
    (77.9,76.8,81.5,79.8,78.2,78.3,77.5,77.0,80.1,80.2,79.9,79.0,76.7

7  data1<-data.frame(x,y)
8

```

```

9  model<-lm(y~x)
10 Residual<-resid(model)
11 n<-length(Residual)
12
13 SSE<-sum(Residual^2)
14 print(paste("SSE:",SSE))
15
16 #To find variance(sigma^2)
17 s2<-SSE/(n-2)
18 print(paste("Estimate of variance:",s2))
19
20 #To find standard deviation(sigma)
21 sd<-sqrt(s2)
22 print(paste("Estimate of standard deviation:",sd))
23
24 #To display ANOVA table
25 a<-aov(model,data=data1)
26 cat("ANOVA table:\n")
27 print(summary(a))

```

---

### R code Exa 12.8 Estimating model parameters

```

1  #Ex12.8, Page 484
2  #Answers may vary slightly due to rounding off of
   values
3
4  x<-c(12,30,36,40,45,57,62,67,71,78,93,94,100,105)
5  y<-c
   (3.3,3.2,3.4,3.0,2.8,2.9,2.7,2.6,2.5,2.6,2.2,2.0,2.3,2.1)
6  data1<-data.frame(x,y)
7
8  #To display ANOVA table
9  model<-lm(y~x)
10 print(summary(model))

```

```

11 a<-aov(model,data=data1)
12 cat("ANOVA table:\n")
13 print(summary(a))

```

---

### R code Exa 12.9 Coefficient of determination

```

1 #Ex12.9, Page 486
2 #Answers may vary slightly due to rounding off of
  values
3
4 #From Ex12.4
5 x<-c
  (132.0,129.0,120.0,113.2,105.0,92.0,84.0,83.2,88.4,59.0,80.0,81.5
6
7 y<-c
  (46.0,48.0,51.0,52.1,54.0,52.0,59.0,58.7,61.6,64.0,61.4,54.6,58.8
8
9 data1<-data.frame(x,y)
10
11 n<-14
12 beta0<-75.212432
13 beta1<-(-0.20938742)
14 yi<-779.2
15 xiyi<-71347.30
16 yi2<-43745.22
17
18 #To find SST and SSE
19 SST<-yi2-(yi^2)/n
20 print(paste("SST:",SST))
21 SSE<-yi2-beta0*yi-beta1*xiyi
22 print(paste("SSE:",SSE))
23
24 #Coefficient of determination
25 r2<-1-(SSE/SST)
26 print(paste("Coefficient of determination:",r2))

```



```

25
26 #To create regression model using ANOVA table
27 a<-lm(y~x)
28 print(summary(a))
29 a1<-aov(a,data=data1)
30 print(summary(a1))

```

---

### R code Exa 12.11 Confidence interval for beta1

```

1 #Ex12.11, Page 494
2 #Answers may vary slightly due to rounding off of
  values
3
4 x<-c
  (5.7,6.8,9.6,10.0,10.7,12.6,14.4,15.0,15.3,16.2,17.8,18.7,19.7,20.7,21.6,22.5,23.4,24.3,25.2,26.1,27.0,27.9,28.8,29.7,30.6,31.5,32.4,33.3,34.2,35.1,36.0,36.9,37.8,38.7,39.6,40.5,41.4,42.3,43.2,44.1,45.0,45.9,46.8,47.7,48.6,49.5,50.4,51.3,52.2,53.1,54.0,54.9,55.8,56.7,57.6,58.5,59.4,60.3,61.2,62.1,63.0,63.9,64.8,65.7,66.6,67.5,68.4,69.3,70.2,71.1,72.0,72.9,73.8,74.7,75.6,76.5,77.4,78.3,79.2,80.1,81.0,81.9,82.8,83.7,84.6,85.5,86.4,87.3,88.2,89.1,90.0,90.9,91.8,92.7,93.6,94.5,95.4,96.3,97.2,98.1,99.0,100.0)
5 y<-c
  (119.0,121.3,118.2,124.0,112.3,114.1,112.2,115.1,111.3,107.2,108.9,110.5,112.7,114.9,117.1,119.3,121.5,123.7,125.9,128.1,130.3,132.5,134.7,136.9,139.1,141.3,143.5,145.7,147.9,150.1,152.3,154.5,156.7,158.9,161.1,163.3,165.5,167.7,169.9,172.1,174.3,176.5,178.7,180.9,183.1,185.3,187.5,189.7,191.9,194.1,196.3,198.5,200.7,202.9,205.1,207.3,209.5,211.7,213.9,216.1,218.3,220.5,222.7,224.9,227.1,229.3,231.5,233.7,235.9,238.1,240.3,242.5,244.7,246.9,249.1,251.3,253.5,255.7,257.9,260.1,262.3,264.5,266.7,268.9,271.1,273.3,275.5,277.7,279.9,282.1,284.3,286.5,288.7,290.9,293.1,295.3,297.5,299.7,301.9,304.1,306.3,308.5,310.7,312.9,315.1,317.3,319.5,321.7,323.9,326.1,328.3,330.5,332.7,334.9,337.1,339.3,341.5,343.7,345.9,348.1,350.3,352.5,354.7,356.9,359.1,361.3,363.5,365.7,367.9,370.1,372.3,374.5,376.7,378.9,381.1,383.3,385.5,387.7,389.9,392.1,394.3,396.5,398.7,400.9,403.1,405.3,407.5,409.7,411.9,414.1,416.3,418.5,420.7,422.9,425.1,427.3,429.5,431.7,433.9,436.1,438.3,440.5,442.7,444.9,447.1,449.3,451.5,453.7,455.9,458.1,460.3,462.5,464.7,466.9,469.1,471.3,473.5,475.7,477.9,480.1,482.3,484.5,486.7,488.9,491.1,493.3,495.5,497.7,499.9,502.1,504.3,506.5,508.7,510.9,513.1,515.3,517.5,519.7,521.9,524.1,526.3,528.5,530.7,532.9,535.1,537.3,539.5,541.7,543.9,546.1,548.3,550.5,552.7,554.9,557.1,559.3,561.5,563.7,565.9,568.1,570.3,572.5,574.7,576.9,579.1,581.3,583.5,585.7,587.9,590.1,592.3,594.5,596.7,598.9,601.1,603.3,605.5,607.7,609.9,612.1,614.3,616.5,618.7,620.9,623.1,625.3,627.5,629.7,631.9,634.1,636.3,638.5,640.7,642.9,645.1,647.3,649.5,651.7,653.9,656.1,658.3,660.5,662.7,664.9,667.1,669.3,671.5,673.7,675.9,678.1,680.3,682.5,684.7,686.9,689.1,691.3,693.5,695.7,697.9,700.1,702.3,704.5,706.7,708.9,711.1,713.3,715.5,717.7,719.9,722.1,724.3,726.5,728.7,730.9,733.1,735.3,737.5,739.7,741.9,744.1,746.3,748.5,750.7,752.9,755.1,757.3,759.5,761.7,763.9,766.1,768.3,770.5,772.7,774.9,777.1,779.3,781.5,783.7,785.9,788.1,790.3,792.5,794.7,796.9,799.1,801.3,803.5,805.7,807.9,810.1,812.3,814.5,816.7,818.9,821.1,823.3,825.5,827.7,829.9,832.1,834.3,836.5,838.7,840.9,843.1,845.3,847.5,849.7,851.9,854.1,856.3,858.5,860.7,862.9,865.1,867.3,869.5,871.7,873.9,876.1,878.3,880.5,882.7,884.9,887.1,889.3,891.5,893.7,895.9,898.1,900.3,902.5,904.7,906.9,909.1,911.3,913.5,915.7,917.9,920.1,922.3,924.5,926.7,928.9,931.1,933.3,935.5,937.7,939.9,942.1,944.3,946.5,948.7,950.9,953.1,955.3,957.5,959.7,961.9,964.1,966.3,968.5,970.7,972.9,975.1,977.3,979.5,981.7,983.9,986.1,988.3,990.5,992.7,994.9,997.1,999.3,1001.5,1003.7,1005.9,1008.1,1010.3,1012.5,1014.7,1016.9,1019.1,1021.3,1023.5,1025.7,1027.9,1030.1,1032.3,1034.5,1036.7,1038.9,1041.1,1043.3,1045.5,1047.7,1049.9,1052.1,1054.3,1056.5,1058.7,1060.9,1063.1,1065.3,1067.5,1069.7,1071.9,1074.1,1076.3,1078.5,1080.7,1082.9,1085.1,1087.3,1089.5,1091.7,1093.9,1096.1,1098.3,1100.5,1102.7,1104.9,1107.1,1109.3,1111.5,1113.7,1115.9,1118.1,1120.3,1122.5,1124.7,1126.9,1129.1,1131.3,1133.5,1135.7,1137.9,1140.1,1142.3,1144.5,1146.7,1148.9,1151.1,1153.3,1155.5,1157.7,1159.9,1162.1,1164.3,1166.5,1168.7,1170.9,1173.1,1175.3,1177.5,1179.7,1181.9,1184.1,1186.3,1188.5,1190.7,1192.9,1195.1,1197.3,1199.5,1201.7,1203.9,1206.1,1208.3,1210.5,1212.7,1214.9,1217.1,1219.3,1221.5,1223.7,1225.9,1228.1,1230.3,1232.5,1234.7,1236.9,1239.1,1241.3,1243.5,1245.7,1247.9,1250.1,1252.3,1254.5,1256.7,1258.9,1261.1,1263.3,1265.5,1267.7,1269.9,1272.1,1274.3,1276.5,1278.7,1280.9,1283.1,1285.3,1287.5,1289.7,1291.9,1294.1,1296.3,1298.5,1300.7,1302.9,1305.1,1307.3,1309.5,1311.7,1313.9,1316.1,1318.3,1320.5,1322.7,1324.9,1327.1,1329.3,1331.5,1333.7,1335.9,1338.1,1340.3,1342.5,1344.7,1346.9,1349.1,1351.3,1353.5,1355.7,1357.9,1360.1,1362.3,1364.5,1366.7,1368.9,1371.1,1373.3,1375.5,1377.7,1379.9,1382.1,1384.3,1386.5,1388.7,1390.9,1393.1,1395.3,1397.5,1399.7,1401.9,1404.1,1406.3,1408.5,1410.7,1412.9,1415.1,1417.3,1419.5,1421.7,1423.9,1426.1,1428.3,1430.5,1432.7,1434.9,1437.1,1439.3,1441.5,1443.7,1445.9,1448.1,1450.3,1452.5,1454.7,1456.9,1459.1,1461.3,1463.5,1465.7,1467.9,1470.1,1472.3,1474.5,1476.7,1478.9,1481.1,1483.3,1485.5,1487.7,1489.9,1492.1,1494.3,1496.5,1498.7,1500.9,1503.1,1505.3,1507.5,1509.7,1511.9,1514.1,1516.3,1518.5,1520.7,1522.9,1525.1,1527.3,1529.5,1531.7,1533.9,1536.1,1538.3,1540.5,1542.7,1544.9,1547.1,1549.3,1551.5,1553.7,1555.9,1558.1,1560.3,1562.5,1564.7,1566.9,1569.1,1571.3,1573.5,1575.7,1577.9,1580.1,1582.3,1584.5,1586.7,1588.9,1591.1,1593.3,1595.5,1597.7,1599.9,1602.1,1604.3,1606.5,1608.7,1610.9,1613.1,1615.3,1617.5,1619.7,1621.9,1624.1,1626.3,1628.5,1630.7,1632.9,1635.1,1637.3,1639.5,1641.7,1643.9,1646.1,1648.3,1650.5,1652.7,1654.9,1657.1,1659.3,1661.5,1663.7,1665.9,1668.1,1670.3,1672.5,1674.7,1676.9,1679.1,1681.3,1683.5,1685.7,1687.9,1690.1,1692.3,1694.5,1696.7,1698.9,1701.1,1703.3,1705.5,1707.7,1709.9,1712.1,1714.3,1716.5,1718.7,1720.9,1723.1,1725.3,1727.5,1729.7,1731.9,1734.1,1736.3,1738.5,1740.7,1742.9,1745.1,1747.3,1749.5,1751.7,1753.9,1756.1,1758.3,1760.5,1762.7,1764.9,1767.1,1769.3,1771.5,1773.7,1775.9,1778.1,1780.3,1782.5,1784.7,1786.9,1789.1,1791.3,1793.5,1795.7,1797.9,1800.1,1802.3,1804.5,1806.7,1808.9,1811.1,1813.3,1815.5,1817.7,1819.9,1822.1,1824.3,1826.5,1828.7,1830.9,1833.1,1835.3,1837.5,1839.7,1841.9,1844.1,1846.3,1848.5,1850.7,1852.9,1855.1,1857.3,1859.5,1861.7,1863.9,1866.1,1868.3,1870.5,1872.7,1874.9,1877.1,1879.3,1881.5,1883.7,1885.9,1888.1,1890.3,1892.5,1894.7,1896.9,1899.1,1901.3,1903.5,1905.7,1907.9,1910.1,1912.3,1914.5,1916.7,1918.9,1921.1,1923.3,1925.5,1927.7,1929.9,1932.1,1934.3,1936.5,1938.7,1940.9,1943.1,1945.3,1947.5,1949.7,1951.9,1954.1,1956.3,1958.5,1960.7,1962.9,1965.1,1967.3,1969.5,1971.7,1973.9,1976.1,1978.3,1980.5,1982.7,1984.9,1987.1,1989.3,1991.5,1993.7,1995.9,1998.1,2000.3,2002.5,2004.7,2006.9,2009.1,2011.3,2013.5,2015.7,2017.9,2020.1,2022.3,2024.5,2026.7,2028.9,2031.1,2033.3,2035.5,2037.7,2039.9,2042.1,2044.3,2046.5,2048.7,2050.9,2053.1,2055.3,2057.5,2059.7,2061.9,2064.1,2066.3,2068.5,2070.7,2072.9,2075.1,2077.3,2079.5,2081.7,2083.9,2086.1,2088.3,2090.5,2092.7,2094.9,2097.1,2099.3,2101.5,2103.7,2105.9,2108.1,2110.3,2112.5,2114.7,2116.9,2119.1,2121.3,2123.5,2125.7,2127.9,2130.1,2132.3,2134.5,2136.7,2138.9,2141.1,2143.3,2145.5,2147.7,2149.9,2152.1,2154.3,2156.5,2158.7,2160.9,2163.1,2165.3,2167.5,2169.7,2171.9,2174.1,2176.3,2178.5,2180.7,2182.9,2185.1,2187.3,2189.5,2191.7,2193.9,2196.1,2198.3,2200.5,2202.7,2204.9,2207.1,2209.3,2211.5,2213.7,2215.9,2218.1,2220.3,2222.5,2224.7,2226.9,2229.1,2231.3,2233.5,2235.7,2237.9,2240.1,2242.3,2244.5,2246.7,2248.9,2251.1,2253.3,2255.5,2257.7,2259.9,2262.1,2264.3,2266.5,2268.7,2270.9,2273.1,2275.3,2277.5,2279.7,2281.9,2284.1,2286.3,2288.5,2290.7,2292.9,2295.1,2297.3,2299.5,2301.7,2303.9,2306.1,2308.3,2310.5,2312.7,2314.9,2317.1,2319.3,2321.5,2323.7,2325.9,2328.1,2330.3,2332.5,2334.7,2336.9,2339.1,2341.3,2343.5,2345.7,2347.9,2350.1,2352.3,2354.5,2356.7,2358.9,2361.1,2363.3,2365.5,2367.7,2369.9,2372.1,2374.3,2376.5,2378.7,2380.9,2383.1,2385.3,2387.5,2389.7,2391.9,2394.1,2396.3,2398.5,2400.7,2402.9,2405.1,2407.3,2409.5,2411.7,2413.9,2416.1,2418.3,2420.5,2422.7,2424.9,2427.1,2429.3,2431.5,2433.7,2435.9,2438.1,2440.3,2442.5,2444.7,2446.9,2449.1,2451.3,2453.5,2455.7,2457.9,2460.1,2462.3,2464.5,2466.7,2468.9,2471.1,2473.3,2475.5,2477.7,2479.9,2482.1,2484.3,2486.5,2488.7,2490.9,2493.1,2495.3,2497.5,2499.7,2501.9,2504.1,2506.3,2508.5,2510.7,2512.9,2515.1,2517.3,2519.5,2521.7,2523.9,2526.1,2528.3,2530.5,2532.7,2534.9,2537.1,2539.3,2541.5,2543.7,2545.9,2548.1,2550.3,2552.5,2554.7,2556.9,2559.1,2561.3,2563.5,2565.7,2567.9,2570.1,2572.3,2574.5,2576.7,2578.9,2581.1,2583.3,2585.5,2587.7,2589.9,2592.1,2594.3,2596.5,2598.7,2600.9,2603.1,2605.3,2607.5,2609.7,2611.9,2614.1,2616.3,2618.5,2620.7,2622.9,2625.1,2627.3,2629.5,2631.7,2633.9,2636.1,2638.3,2640.5,2642.7,2644.9,2647.1,2649.3,2651.5,2653.7,2655.9,2658.1,2660.3,2662.5,2664.7,2666.9,2669.1,2671.3,2673.5,2675.7,2677.9,2680.1,2682.3,2684.5,2686.7,2688.9,2691.1,2693.3,2695.5,2697.7,2699.9,2702.1,2704.3,2706.5,2708.7,2710.9,2713.1,2715.3,2717.5,2719.7,2721.9,2724.1,2726.3,2728.5,2730.7,2732.9,2735.1,2737.3,2739.5,2741.7,2743.9,2746.1,2748.3,2750.5,2752.7,2754.9,2757.1,2759.3,2761.5,2763.7,2765.9,2768.1,2770.3,2772.5,2774.7,2776.9,2779.1,2781.3,2783.5,2785.7,2787.9,2790.1,2792.3,2794.5,2796.7,2798.9,2801.1,2803.3,2805.5,2807.7,2809.9,2812.1,2814.3,2816.5,2818.7,2820.9,2823.1,2825.3,2827.5,2829.7,2831.9,2834.1,2836.3,2838.5,2840.7,2842.9,2845.1,2847.3,2849.5,2851.7,2853.9,2856.1,2858.3,2860.5,2862.7,2864.9,2867.1,2869.3,2871.5,2873.7,2875.9,2878.1,2880.3,2882.5,2884.7,2886.9,2889.1,2891.3,2893.5,2895.7,2897.9,2899.9,2902.1,2904.3,2906.5,2908.7,2910.9,2913.1,2915.3,2917.5,2919.7,2921.9,2924.1,2926.3,2928.5,2930.7,2932.9,2935.1,2937.3,2939.5,2941.7,2943.9,2946.1,2948.3,2950.5,2952.7,2954.9,2957.1,2959.3,2961.5,2963.7,2965.9,2968.1,2970.3,2972.5,2974.7,2976.9,2979.1,2981.3,2983.5,2985.7,2987.9,2990.1,2992.3,2994.5,2996.7,2998.9,3001.1,3003.3,3005.5,3007.7,3009.9,3012.1,3014.3,3016.5,3018.7,3020.9,3023.1,3025.3,3027.5,3029.7,3031.9,3034.1,3036.3,3038.5,3040.7,3042.9,3045.1,3047.3,3049.5,3051.7,3053.9,3056.1,3058.3,3060.5,3062.7,3064.9,3067.1,3069.3,3071.5,3073.7,3075.9,3078.1,3080.3,3082.5,3084.7,3086.9,3089.1,3091.3,3093.5,3095.7,3097.9,3100.1,3102.3,3104.5,3106.7,3108.9,3111.1,3113.3,3115.5,3117.7,3119.9,3122.1,3124.3,3126.5,3128.7,3130.9,3133.1,3135.3,3137.5,3139.7,3141.9,3144.1,3146.3,3148.5,3150.7,3152.9,3155.1,3157.3,3159.5,3161.7,3163.9,3166.1,3168.3,3170.5,3172.7,3174.9,3177.1,3179.3,3181.5,3183.7,3185.9,3188.1,3190.3,3192.5,3194.7,3196.9,3199.1,3201.3,3203.5,3205.7,3207.9,3210.1,3212.3,3214.5,3216.7,3218.9,3221.1,3223.3,3225.5,3227.7,3229.9,3232.1,3234.3,3236.5,3238.7,3240.9,3243.1,3245.3,3247.5,3249.7,3251.9,3254.1,3256.3,3258.5,3260.7,3262.9,3265.1,3267.3,3269.5,3271.7,3273.9,3276.1,3278.3,3280.5,3282.7,3284.9,3287.1,3289.3,3291.5,3293.7,3295.9,3298.1,3300.3,3302.5,3304.7,3306.9,3309.1,3311.3,3313.5,3315.7,3317.9,3320.1,3322.3,3324.5,3326.7,3328.9,3331.1,3333.3,3335.5,3337.7,3339.9,3342.1,3344.3,3346.5,3348.7,3350.9,3353.1,3355.3,3357.5,3359.7,3361.9,3364.1,3366.3,3368.5,3370.7,3372.9,3375.1,3377.3,3379.5,3381.7,3383.9,3386.1,3388.3,3390.5,3392.7,3394.9,3397.1,3399.3,3401.5,3403.7,3405.9,3408.1,3410.3,3412.5,3414.7,3416.9,3419.1,3421.3,3423.5,3425.7,3427.9,3430.1,3432.3,3434.5,3436.7,3438.9,3441.1,3443.3,3445.5,3447.7,3449.9,3452.1,3454.3,3456.5,3458.7,3460.9,3463.1,3465.3,3467.5,3469.7,3471.9,3474.1,3476.3,3478.5,3480.7,3482.9,3485.1,3487.3,3489.5,3491.7,3493.9,3496.1,3498.3,3500.5,3502.7,3504.9,3507.1,3509.3,3511.5,3513.7,3515.9,3518.1,3520.3,3522.5,3524.7,3526.9,3529.1,3531.3,3533.5,3535.7,3537.9,3540.1,3542.3,3544.5,3546.7,3548.9,3551.1,3553.3,3555.5,3557.7,3559.9,3562.1,3564.3,3566.5,3568.7,3570.9,3573.1,3575.3,3577.5,3579.7,3581.9,3584.1,3586.3,3588.5,3590.7,3592.9,3595.1,3597.3,3599.5,3601.7,3603.9,3606.1,3608.3,3610.5,3612.7,3614.9,3617.1,3619.3,3621.5,3623.7,3625.9,3628.1,3630.3,3632.5,3634.7,3636.9,3639.1,3641.3,3643.5,3645.7,3647.9,3650.1,3652.3,3654.5,3656.7,3658.9,3661.1,3663.3,3665.5,3667.7,3669.9,3672.1,3674.3,3676.5,3678.7,3680.9,3683.1,3685.3,3687.5,3689.7,3691.9,3694.1,3696.3,3698.5,3700.7,3702.9,3705.1,3707.3,3709.5,3711.7,3713.9,3716.1,3718.3,3720.5,3722.7,3724.9,3727.1,3729.3,3731.5,3733.7,3735.9,3738.1,3740.3,3742.5,3744.7,3746.9,3749.1,3751.3,3753.5,3755.7,3757.9,3760.1,3762.3,3764.5,3766.7,3768.9,3771.1,3773.3,3775.5,3777.7,3779.9,3782.1,3784.3,3786.5,3788.7,3790.9,3793.1,3795.3,3797.5,3799.7,3801.9,3804.1,3806.3,3808.5,3810.7,3812.9,3815.1,3817.3,3819.5,3821.7,3823.9,3826.1,3828.3,3830.5,3832.7,3834.9,3837.1,3839.3,3841.5,3843.7,3845.9,3848.1,3850.3,3852.5,3854.7,3856.9,3859.1,3861.3,3863.5,3865.7,3867.9,3870.1,3872.3,3874.5,
```

```

21 print(paste("Sxy:", Sxy))
22
23 #To find beta1 and beta0
24 xbar<-mean(x)
25 ybar<-mean(y)
26 beta1<-Sxy/Sxx
27 beta0<-ybar-beta1*xbar
28 print(paste("Beta1:", beta1))
29 print(paste("Beta0:", beta0))
30
31 #To find SSE
32 SSE<-yi2-beta0*yi-beta1*xiyi
33 print(paste("SSE:", SSE))
34
35 #To find SST
36 SST<-yi2-yi^2/n
37 print(paste("SST:", SST))
38
39 #To find r^2
40 r2<-1-(SSE/SST)
41 print(paste("r^2:", r2))
42
43 I<-15
44 J<-2
45 df<-I-J
46
47 #s^2
48 s2<-SSE/df
49 s<-sqrt(s2)
50
51 #Estimated standard deviation of beta1
52 sb<-s/sqrt(Sxx)
53 print(paste("Estimated standard deviation of beta1:",
54             ,sb))
54
55 alpha<-0.05
56 t<-qt(1-alpha/2, df)
57

```

```

58 #To find confidence intervals
59 l<-beta1-t*sb
60 r<-beta1+t*sb
61 print(paste("Left endpoint of confidence interval:",
62             1))
63 print(paste("Right endpoint of confidence interval:",
64             ,r))
65
66 #To display analysis of variance and parameter
67 estimates
68 model<-lm(y~x)
69 cat("\nParameter estimates\n")
70 print(summary(model))
71 cat("\nANOVA table:\n")
72 print(summary(aov(model)))
73 a<-predict(model)
74 print(paste("Predict values:"))
75 print(a)
76 print(paste("Residuals:"))
77 print(resid(model))

```

---

### R code Exa 12.12 Hypothesis testing procedures

```

1 #Ex12.12, Page 496
2 #Answers may vary slightly due to rounding off of
3 values
4
5 x<-c
6   (42.2,42.6,43.3,43.5,43.7,44.1,44.9,45.3,45.7,45.7,45.9,46.0,46.2
7
8 y<-c
9   (44,44,44,45,45,46,46,46,47,48,48,48,47,48,48,49,49,49)
10
11 data1<-data.frame(x,y)
12

```

```

8 model<-lm(y~x,data=data1)
9 cat("Regression model:\n")
10 print(summary(model))
11
12 #To display ANOVA table
13 a<-aov(model)
14 cat("ANOVA table:\n")
15 print(summary(a))

```

---

### R code Exa 12.13 Inferences concerning mean

```

1 #Ex12.13, Page 501
2 #Answers may vary slightly due to rounding off of
  values
3
4 library(ggplot2)
5
6 x<-c
  (8.0,15.0,16.5,20.0,20.0,27.5,30.0,30.0,35.0,38.0,40.0,45.0,50.0,
7
7 y<-c
  (22.8,27.2,23.7,17.1,21.5,18.6,16.1,23.4,13.4,19.5,12.4,13.2,11.4
8
8 data1<-data.frame(x,y)
9
10 #To plot data using ggplot
11 ggplot(data=data1,mapping=aes(x,y))+geom_point(size
  =2)+geom_smooth(method=lm)
12
13 #To display linear regression model
14 model<-lm(y~x)
15 print(summary(model))
16
17 #To display analysis of variance
18 cat("ANOVA table:\n")

```

```

19 a<-aov(model,data=data1)
20 print(summary(a))
21
22 #To display confidence and prediction intervals
23 d<-data.frame(x=c(35,45))
24 cat("Confidence intervals:\n")
25 print(predict(model,newdata=d,interval="confidence",
    level=0.95))
26 cat("Prediction intervals:\n")
27 print(predict(model,newdata=d,interval="predict",
    level=0.95))

```

---

#### R code Exa 12.14 Prediction interval for a future value of Y

```

1 #Ex12.14, Page 505
2 #Answers may vary slightly due to rounding off of
  values
3
4 ycap<-13.79
5 sy<-0.7582
6 s<-2.8640
7 n<-18
8 df<-n-2
9 t<-2.120
10
11 #To find confidence intervals
12 l<-ycap-t*sqrt(s^2+sy^2)
13 r<-ycap+t*sqrt(s^2+sy^2)
14 print(paste("Left endpoint of confidence interval:",
    l))
15 print(paste("Right endpoint of confidence interval:",
    r))

```

---

### R code Exa 12.15 Correlation

```
1 #Ex12.15, Page 509
2 #Answers may vary slightly due to rounding off of
  values
3
4 x<-c(2.4,3.4,4.6,3.7,2.2,3.3,4.0,2.1)
5 y<-c(1.33,2.12,1.80,1.65,2.00,1.76,2.11,1.63)
6
7 #To find sample correlation coefficient
8 r<-cor(x,y)
9 print(paste("Sample correlation coefficient ,r:",r))
```

---

### R code Exa 12.16 Inferences about the population correlation coefficient

```
1 #Ex12.16, Page 511
2 #Answers may vary slightly due to rounding off of
  values
3
4 x<-c
  (0.066,0.088,0.120,0.050,0.162,0.186,0.057,0.100,0.112,0.055,0.154)
5 y<-c
  (4.6,11.6,9.5,6.3,13.8,15.4,2.5,11.8,8.0,7.0,20.6,16.6,9.2,17.9,21.5)
6
7 #To find r(point estimate of population correlation
  coefficient)
8 r<-cor(x,y)
9 print(paste("r:",r))
```

---

### R code Exa 12.17 Inferences about the population correlation coefficient

```

1 #Ex12.17, Page 513
2 #Answers may vary slightly due to rounding off of
  values
3
4 r<-0.29
5 n<-45
6 #To find the test statistic
7 t<-r*sqrt(n-2)/sqrt(1-r^2)
8 print(paste("t:",round(t,digits=0)))
9
10 #P value for two tailed test
11 p<-2*pt(-abs(t),df=n-1)
12 print(p)

```

---

#### R code Exa 12.18 Other inferences concerning rho

```

1 #Ex12.18, Page 515
2 #Answers may vary slightly due to rounding off of
  values
3
4 x<-c
  (55.10,44.83,46.32,51.10,49.89,45.20,48.18,46.70,54.31,41.50,47.50)
5 y<-c
  (49.10,31.20,32.80,42.60,42.50,32.70,36.21,40.40,37.42,30.80,35.30)
6
7 #To find r
8 r<-cor(x,y)
9 print(paste("Correlation coefficient , r:",r))
10
11 #H0: rho=0.5
12 #Ha: rho>0.5
13 rho<-0.5
14

```

```

15 #To find v
16 v<-0.5*log((1+r)/(1-r))
17 print(paste("v:",v))
18
19 muv<-0.5*log((1+rho)/(1-rho))
20 print(paste("muv:",muv))
21
22 z<-(v-muv)*sqrt(length(x)-3)
23 print(paste("z:",z))
24
25 #To find P value
26 p<-pnorm(z,lower.tail=FALSE)
27 print(paste("p value:",p))
28
29 print(paste("H0 is rejected..."))

```

---

#### R code Exa 12.19 Other inferences concerning rho

```

1 #Ex12.19, Page 516
2 #Answers may vary slightly due to rounding off of
  values
3
4 xi<-285.90
5 xi2<-4409.55
6 yi<-690.30
7 yi2<-29040.29
8 xiyi<-10818.56
9 r<-0.733
10 v<-0.935
11 n<-20
12 alpha<-0.05
13
14 #To find z
15 zalp2<-qnorm(1-alpha/2)
16

```



```

17 #95% interval for muv
18 c1<-v-zalpha2/sqrt(n-3)
19 c2<-v+zalpha2/sqrt(n-3)
20 print(paste("Left endpoint of CI for muv:",c1))
21 print(paste("Right endpoint of CI for muv:",c2))
22
23 #95% interval for rho
24 l<-(exp(1)^(2*c1)-1)/(exp(1)^(2*c1)+1)
25 r<-(exp(1)^(2*c2)-1)/(exp(1)^(2*c2)+1)
26 print(paste("Left endpoint of CI for rho:",l))
27 print(paste("Right endpoint of CI for rho:",r))

```

---

# Chapter 13

## Nonlinear and multiple regression

R code Exa 13.2 Diagnostic plots

```
1 #Ex13.2 , Page 526
2
3 x<-c
   (100,125,125,150,150,200,200,250,250,300,300,350,400,400)
4 y<-c
   (150,140,180,210,190,320,280,400,430,440,390,600,610,670)
5 y_cap<-(-45.55)+(1.71*x)
6 m1<-lm(y~x)
7 ei<-round(resid(m1),digits=1)
8 ei_std<-round(rstandard(m1),digits=2)
9 df<-data.frame(x,y,y_cap,ei,ei_std)
10 print(df)
11
12 par(mfrow=c(3,2))
13 plot(x,y,main="y vs. x")
14 abline(m1)
15 plot(y_cap,ei_std,main="Standardized residuals vs. y
```

```

    _cap")
16 abline(0,0)
17 plot(y,y_cap,main="y_cap vs. y")
18 abline(lm(y_cap~y))
19 plot(x,ei_std,main="Standardized residuals vs. x")
20 abline(0,0)
21 qqnorm(ei_std,xlab="z percentile",ylab="e*",main="
    Normal probability plot")

```

---

#### R code Exa 13.4 Regression with transformed variables

```

1 #Ex13.4, Page 534
2 #Answers may vary slightly due to rounding off of
  values
3
4 x<-c(2,10,20,30,40,50,60,70,80,90,100)
5 y<-c(408,274,196,137,90,78,51,40,30,22,15)
6 y_dash<-log(y,base=exp(1))
7 m1<-lm(y~x)
8 m2<-lm(y_dash~x)
9 ei_std1<-rstandard(m1)
10 ei_std2<-rstandard(m2)
11
12 par(mfrow=c(2,2))
13 plot(x,y,main="Scatterplot")
14 abline(m1)
15 plot(x,ei_std1,xlab="x",ylab="e*",main="Residual
    plot from linear regression for the data")
16 abline(0,0)
17
18 #To find the coefficients
19 beta0<-m2$coefficients[1]
20 beta1<-round(m2$coefficients[2],digits=4)
21
22 y_cap<-(exp(1)^(beta0))*(exp(1)^(beta1*x))

```

```

23
24 df<-data.frame(x,y,y_dash,y_cap)
25 print(df)
26
27 plot(x,ei_std2,ylab="e*",main="Standardized
    residuals(after transforming) vs. x")
28 abline(0,0)
29 plot(y,y_cap,main="ycap vs. y")
30 abline(lm(y_cap~y))

```

---

### R code Exa 13.7 Polynomial regression

```

1 #Ex13.7, Page 544
2
3 Thickness<-c
    (220,220,220,220,370,370,370,370,440,440,440,440,680,680,680,680,
4
5 Strength<-c
    (24.0,22.0,19.1,15.5,26.3,24.6,23.1,21.2,25.2,24.0,21.7,19.2,17.0
6
7 df<-data.frame(Thickness,Strength)
8 cat("Regression model:\n")
9 relation<-lm(Strength~Thickness+I(Thickness^2))
10 print(summary(relation))
11
12 cat("Analysis of variance:\n")
13 model<-lm(Strength~poly(Thickness,2))
14 a<-aov(model,data=df)
15 print(summary(a))
16
17 #To find confidence and prediction intervals
18 cat("\nWhen thickness=500\n")
19 cat("95% CI:\n")
20 print(predict(relation,newdata=data.frame(Thickness

```

```

    =500), interval="confidence"))
20 cat("95% PI:\n")
21 print(predict(relation, newdata=data.frame(Thickness
    =500), interval="prediction"))
22
23 cat("\nWhen thickness=800\n")
24 cat("95% CI:\n")
25 print(predict(relation, newdata=data.frame(Thickness
    =800), interval="confidence"))
26 cat("95% PI:\n")
27 print(predict(relation, newdata=data.frame(Thickness
    =800), interval="prediction"))

```

---

### R code Exa 13.8 Estimating parameters

```

1 #Ex13.8, Page 546
2 #Answers may vary slightly due to rounding off of
  values
3
4 #From Ex13.7
5 Thickness<-c
  (220,220,220,220,370,370,370,370,440,440,440,440,680,680,680,680,
6
7 Strength<-c
  (24.0,22.0,19.1,15.5,26.3,24.6,23.1,21.2,25.2,24.0,21.7,19.2,17.0
8
9 df<-data.frame(Thickness,Strength)
10 relation<-lm(df$Strength~poly(df$Thickness,2), data=
  df)
11
12 a<-summary(aov(relation, data=df))
13 print(a)
14
15 n<-20
16 k<-2
17 SSE<-a[[1]][, 'Sum Sq'][2]

```

```

15 SST<-a[[1]][, 'Sum Sq'] [1]+a[[1]][, 'Sum Sq'] [2]
16 R2<-1-(SSE/SST)
17 print(paste("R-squared value:",R2))
18
19 #To find variance
20 sigma2<-SSE/(n-(k+1))
21 print(paste("sigma^2:",sigma2))
22 print(paste("sigma:",sqrt(sigma2)))

```

---

### R code Exa 13.9 Statistical intervals and test procedures

```

1 #Ex13.9, Page 548
2 #Answers may vary slightly due to rounding off of
  values
3
4 Thickness<-c
  (220,220,220,220,370,370,370,370,440,440,440,440,680,680,680,680,
5
6 Strength<-c
  (24.0,22.0,19.1,15.5,26.3,24.6,23.1,21.2,25.2,24.0,21.7,19.2,17.0
7
8
9 new<-data.frame(Thickness=500)
10 y_cap<-predict(model,new)
11 print(paste("ycap when x=500:",y_cap))
12
13 #To find confidence and prediction intervals
14 cat("95% CI:\n")
15 print(predict(model,newdata=data.frame(Thickness
  =500),interval="confidence"))
16 cat("95% PI:\n")
17 print(predict(model,newdata=data.frame(Thickness
  =500),interval="prediction"))

```

---

**R code Exa 13.10** Centering x values

```
1 #Ex13.10, Page 549
2 #Answers may vary slightly due to rounding off of
  values
3
4 x<-c(280,284,292,295,298,305,308,315)
5 x_dash<-c
  (-17.13,-13.13,-5.13,-2.13,0.87,7.87,10.87,17.87)
6 y<-c(770,800,840,810,735,640,590,560)
7 df<-data.frame(x,x_dash,y)
8
9 #To find estimated coefficients and standard
  deviations using original model
10 m1<-lm(y~x+I(x^2))
11 print(summary(m1))
12 Estimate1<-c(NULL)
13 Estimated_SD1<-c(NULL)
14 for(i in 1:3){
15   Estimate1[i]<-m1$coefficients[i]
16   Estimated_SD1[i]<-coef(summary(m1))[, "Std. Error"
    ][i]
17 }
18 cat("Estimated coefficients and standard deviations
  using original model\n")
19 Parameter1<-c("beta0","beta1","beta2")
20 df1<-data.frame(Parameter1,Estimate1,Estimated_SD1)
21 print(df1)
22
23 #To find estimated coefficients and standard
  deviations using centered model
24 Estimate2<-c(NULL)
25 Estimated_SD2<-c(NULL)
26 m2<-lm(y~x_dash+I(x_dash^2))
```

```

27 print(summary(m2))
28 for(i in 1:3){
29   Estimate2[i]<-m2$coefficients[i]
30   Estimated_SD2[i]<-coef(summary(m2))[, "Std. Error"
    ][i]
31 }
32 cat("Estimated coefficients and standard deviations
    using centered model\n")
33 Parameter2<-c("beta0*", "beta1*", "beta2*")
34 df2<-data.frame(Parameter2, Estimate2, Estimated_SD2)
35 print(df2)

```

---

### R code Exa 13.12 Estimating parameters

```

1 #Ex13.12, Page 558
2 #Answers may vary slightly due to rounding off of
  values
3
4 Observation<-1:30
5 Force<-c
  (30,40,30,40,30,40,30,40,30,40,30,40,30,40,30,40,25,45,35,35,35,30)
6 Power<-c
  (60,60,90,90,60,60,90,90,60,60,90,90,60,60,90,90,75,75,45,105,75,75)
7 Temperature<-c
  (175,175,175,175,225,225,225,225,175,175,175,175,225,225,225,225,175,175,175,175,225,225,225)
8 Time<-c
  (15,15,15,15,15,15,15,15,25,25,25,25,25,25,25,25,20,20,20,20,20,20,20)
9 Strength<-c
  (26.2,26.3,39.8,39.7,38.6,35.5,48.8,37.8,26.6,23.4,38.6,52.1,39.5,39.5,39.5,39.5,39.5,39.5,39.5,39.5,39.5,39.5)
10

```



```

11 m1<-lm(Strength~(Force+Power+Temperature+Time))
12
13 #To form estimated regression equation
14 Force<-35 #Force
15 Power<-75 #Power
16 Temperature<-200 #Temperature
17 Time<-20 #Time
18
19 new<-data.frame(Force,Power,Temperature,Time)
20 y_cap<-predict(m1,new)
21 print(paste("Point prediction of strength at
    (35,75,200,20):",y_cap,"gm"))

```

---

### R code Exa 13.13 Estimating parameters

```

1 #Ex13.13, Page 560
2 #Answers may vary slightly due to rounding off of
  values
3
4 temp1<-x1<-c(21,21,7,7,28,0,14,14,14)
5 temp2<-x2<-c
    (0.65,0.55,0.65,0.55,0.60,0.60,0.70,0.50,0.60)
6 x1x2<-x1*x2
7 Comp_str<-c
    (33.55,47.55,35.00,35.90,40.90,39.10,31.55,48.00,42.30)

8 Adsorbability<-c
    (8.42,6.26,6.74,6.59,7.28,6.90,10.80,5.63,7.43)
9 df1<-data.frame(x1,x2,x1x2,Comp_str,Adsorbability)
10
11 #To find mean and SST values
12 ybar_compstr<-mean(Comp_str)
13 ybar_Adsorb<-mean(Adsorbability)
14 SST_CompStr<-sum((Comp_str-ybar_compstr)^2)
15 SST_Adsorb<-sum((Adsorbability-ybar_Adsorb)^2)

```

```

16 cat("Mean of compression strength values:", ybar_
    compstr, "\n")
17 cat("Mean of adsorbability values:", ybar_Adsorb, "\n"
    )
18 cat("SST of compression strength:", SST_CompStr, "\n")
19 cat("SST of adsorbability:", SST_Adsorb, "\n\n")
20
21 #First-order regression model
22 m1<-lm(Comp_str~(x1+x2))
23 SSE1<-sum(resid(m1)^2)
24 cat("SSE for first-order model:", SSE1, "\n")
25 cat("R squared value:", summary(m1)$r.squared, "\n\n")
26
27 #Model including interaction predictor
28 m2<-lm(Comp_str~(x1*x2))
29 SSE2<-sum(resid(m2)^2)
30 cat("SSE for first-order model including interaction
    predictor:", SSE2, "\n")
31 cat("R squared value:", summary(m2)$r.squared, "\n\n")
32
33 #To create estimated regression function
34 x1<-14 #% limestone
35 x2<-0.6 #Water-cement ratio
36 new<-data.frame(x1, x2)
37 y_cap<-predict(m2, new)
38 cat("Prediction of compression strength at (14,0.6):
    ", y_cap, "\n")
39
40 #When adsorbability is taken as the dependent
    variable
41 #First-order regression model
42 m3<-lm(Adsorbability~(temp1+temp2))
43 cat("R squared value for first-order model:", summary
    (m3)$r.squared, "\n")
44
45 #Model including interaction predictor
46 m4<-lm(Adsorbability~(temp1*temp2))
47 cat("R squared value when using interaction

```



```
22 print(paste("F value:",a[[1]]$F[1]))
```

---

### R code Exa 13.15 Inferences in multiple regression

```
1 #Ex13.15, Page 564
2 #Answers may vary slightly due to rounding off of
  values
3
4 x1<-c
  (61,175,111,124,130,173,169,169,160,244,257,333,199)

5 x2<-c(13,21,24,23,64,38,33,61,39,71,112,88,54)
6 y<-c(4,18,14,18,26,26,21,30,28,36,65,62,40)
7 data1<-data.frame(x1,x2,y)
8
9 model<-lm(y~(x1+x2))
10 print(summary(model))
11
12 new<-data.frame(x1=160,x2=39)
13 y_cap<-predict(model,new)
14 print(paste("y_cap:",y_cap))
15
16 cat("\n99% CI:\n")
17 print(confint(model,level=0.99))
18
19 #To find confidence and prediction intervals
20 cat("95% CI:\n")
21 print(predict(model,new,interval="confidence"))
22 cat("95% PI:\n")
23 print(predict(model,new,interval="prediction"))
```

---

### R code Exa 13.16 Inferences in multiple regression

```

1 #Ex13.16, Page 566
2 #Answers may slightly vary due to rounding off of
  values
3
4 Observation<-1:30
5 x1<-c
  (8,2,7,10,7,7,7,5,4,5,8,2,4,6,10,4,4,10,5,8,10,2,6,7,5,8,4,6,4,7)
6 x2<-c
  (4,4,4,7,4,7,13,4,7,1,10,4,10,7,13,10,13,10,4,13,1,13,13,1,13,1,1
7 x3<-c
  (100,180,180,120,180,180,140,160,140,100,140,100,180,120,180,160,
8 x4<-c
  (1,7,1,5,5,1,1,7,3,7,3,3,3,7,3,5,7,7,1,1,1,1,7,7,1,7,7,1,1,7)
9 y<-c
  (1.4,2.2,4.6,4.9,4.6,4.7,4.6,4.5,4.8,1.4,4.7,1.6,4.5,4.7,4.8,4.6,
10 x12<-x1^2
11 x22<-x2^2
12 x32<-x3^2
13 x42<-x4^2
14 x1x2<-x1*x2
15 x1x3<-x1*x3
16 x1x4<-x1*x4
17 x2x3<-x2*x3
18 x2x4<-x2*x4
19 x3x4<-x3*x4
20 df1<-data.frame(Observation,x1,x2,x3,x4,y)
21 print(df1)
22
23 #To find estimate for reduced model
24 cat("\nReduced model:\n")
25 m1<-lm(y~x1+x2+x3+x4)
26 print(summary(m1))
27

```

```

28 #To find estimate for full model
29 cat(" Full model:\n")
30 m2<-lm(y~(x1+x2+x3+x4+x12+x22+x32+x42+x1x2+x1x3+x1x4
      +x2x3+x2x4+x3x4))
31 print(summary(m2))
32
33 #H0: beta5=beta6=.....=beta14=0
34 #Ha: at least one among beta5 ,....beta14 is not 0
35 n<-30
36 k<-14
37 l<-4
38 alpha<-0.01
39 #To find F value
40 Fval<-qf(1-alpha,df1=k-1,df2=n-(k+1))
41 cat("F value:",Fval,"\n")
42
43 #Unexplained variation for the full model
44 SSEk<-sum(resid(m2)^2)
45
46 #Unexplained variation for the reduced model
47 SSEl<-sum(resid(m1)^2)
48
49 #f value
50 f<-((SSEl-SSEk)/(k-1))/(SSEk/(n-(k+1)))
51 cat("f value:",f,"\n")
52
53 if(f>=Fval) cat("H0 is rejected since",f,">=",Fval)

```

---

### R code Exa 13.18 Transformations

```

1 #Ex13.18, Page 575
2
3 s<-c
      (20,20,20,20,20,20,20,20,20,20,60,60,60,60,60,60,60,60,60,60,100,100,10

```

```

4 l_1000s<-c
    (3,3,3,6,6,6,10,10,10,3,3,3,6,6,6,10,10,10,3,3,3,6,6,6,10,10,10)

5 w<-c
    (300.2,310.8,333.0,99.6,136.2,142.4,20.2,28.2,102.7,67.3,77.9,93.9)

6
7 si<-log(s,base=exp(1))
8 li<-log(l_1000s,base=exp(1))
9 wi<-log(w,base=exp(1))
10
11 #To display estimated coefficients and t ratios
12 df1<-data.frame(si,li,wi)
13 model<-lm(wi~(si+li),data=df1)
14 print(summary(model))

```

---

# Chapter 14

## Goodness of fit tests and categorical data analysis

**R code Exa 14.1** Goodness of fit tests when category probabilities are completely specified

```
1 #Ex14.1, Page 597
2 #Answers may vary slightly due to rounding off of
  value
3
4 p<-c(9/16,3/16,3/16,1/16)
5 ni<-c(926,288,293,104)
6 n<-1611
7 k<-4
8 df<-k-1
9
10 np<-round(n*p,digits=1)
11 df1<-data.frame(ni,np)
12
13 #To find contribution to chi square from each cell
14 chi<-c(NULL)
15 for(i in 1:length(ni)){
16   s<-((ni[i]-np[i])^2)/np[i]
17   chi[i]<-s
```



```

18 }
19 print(paste("Chi-square values:"))
20 print(chi)
21
22 sum<-sum(chi)
23 print(paste("Chi square value:",sum))
24
25 alpha<-0.1
26 #To find chi square value
27 chival<-qchisq(1-alpha,df)
28 print(paste("Critical value:",chival))
29
30 if(sum < chival) print(paste("H0 cannot be rejected"
    )) else print(paste("H0 can be rejected"))

```

---

#### R code Exa 14.2 P values for chi squared tests

```

1 #Ex14.2, Page 599
2 #Answers may vary slightly due to rounding off of
  values
3
4 p<-9/16
5 n<-4
6 k<-5
7 df<-k-1
8 b<-c(NULL)
9 prob<-dbinom(0:4,n,p)
10 print(prob)
11
12 npi<-269*prob
13 print(npi)
14
15 Observed<-c(16,45,100,82,26)
16 Expected<-npi
17

```

```

18 df1<-data.frame(Observed,Expected)
19
20 #To find contribution to chi square from each cell
21 chi<-c(NULL)
22 for(i in 1:length(prob)){
23   s<-((Observed[i]-Expected[i])^2)/Expected[i]
24   chi[i]<-round(s,digits=3)
25 }
26 print(paste("Chi-square values:"))
27 print(chi)
28
29 sum<-sum(chi)
30 print(paste("Chi square value:",sum))
31
32 alpha<-0.01
33 #To find chi square value
34 chival<-qchisq(1-alpha,df)
35 print(paste("Critical value:",chival))
36
37 if(sum < chival) print(paste("H0 cannot be rejected"
  )) else print(paste("H0 can be rejected"))

```

---

### R code Exa 14.3 Chi square when underlying distribution is continuous

```

1 #Ex14.3, Page 600
2 #Answers may vary slightly due to rounding off of
  values
3
4 k<-24
5 df<-k-1
6 p<-1/24
7
8 #Cell counts
9 Observed<-c
  (52,73,89,88,68,47,58,47,48,53,47,34,21,31,40,24,37,31,47,34,36,4

```

```

10 npi<-1186*p
11 Expected<-c(NULL)
12 for(i in 1:length(Observed)){
13   Expected[i]<-npi
14 }
15 chi<-c(NULL)
16 for(i in 1:length(Observed)){
17   s<-((Observed[i]-Expected[i])^2)/Expected[i]
18   chi[i]<-round(s,digits=3)
19 }
20 sum<-sum(chi)
21 print(paste("Chi square value:",sum))
22
23 alpha<-0.01
24 #To find chi square value
25 chival<-qchisq(1-alpha,df)
26 print(paste("Critical value:",chival))
27
28 if(sum < chival) print(paste("H0 cannot be rejected"
  )) else print(paste("H0 can be rejected"))

```

---

**R code Exa 14.4** Chi square when underlying distribution is continuous

```

1 #Ex14.4, Page 600
2 #Answers may vary slightly due to rounding off fo
  values
3
4 #Cell counts
5 Observed<-c(21,17,12,16,10,15,19,10)
6 p<-1/8
7 n<-120
8 k<-8
9 df<-k-1
10 npi<-n*p

```

```

11
12 Expected<-c(NULL)
13 for(i in 1:length(Observed)){
14   Expected[i]<-npi
15 }
16 chi<-c(NULL)
17 for(i in 1:length(Observed)){
18   s<-((Observed[i]-Expected[i])^2)/Expected[i]
19   chi[i]<-round(s,digits=3)
20 }
21 sum<-sum(chi)
22 print(paste("Chi square value:",sum))
23
24 alpha<-0.1
25 #To find chi square value
26 chival<-qchisq(1-alpha,df)
27 print(paste("Critical value:",chival))
28
29 if(sum < chival) print(paste("H0 cannot be rejected"
    )) else print(paste("H0 can be rejected"))

```

---

#### R code Exa 14.5 Chi square when parameters are estimated

```

1 #Ex14.5, Page 603
2
3 Type<-c("Observed")
4 M<-c(125)
5 MN<-c(225)
6 N<-c(150)
7 print(paste("Observed counts:"))
8 df<-data.frame(Type,M,MN,N)
9 print(df)
10
11 n<-500
12

```

```

13 n1<-M
14 n2<-MN
15 theta<-(2*n1+n2)/(2*n)
16 print(paste("Resulting estimator:",theta))

```

---

#### R code Exa 14.6 Chi square when parameters are estimated

```

1 #Ex14.6, Page 604
2 #Answers may slightly due to rounding off of values
3
4 k<-3
5 m<-1
6 theta<-0.475
7 n<-500
8 np1<-500*theta^2
9 np2<-n*2*theta*(1-theta)
10 np3<-n-np1-np2
11
12 Observed<-c(125,225,150)
13 Estimated_expected<-c(np1,np2,np3)
14
15 chi<-c(NULL)
16 for(i in 1:length(Observed)){
17   s<-((Observed[i]-Estimated_expected[i])^2)/
18     Estimated_expected[i]
19   chi[i]<-round(s,digits=3)
20 }
21 sum<-sum(chi)
22 print(paste("Chi square value:",sum))
23
24 alpha<-0.05
25 df<-k-m-1
26 #To find chi square value
27 chival<-qchisq(1-alpha,df)
28 print(paste("Critical value:",chival))

```

```

28
29 if(sum < chival) print(paste("H0 is rejected")) else
    print(paste("H0 can be rejected"))

```

---

### R code Exa 14.9 Goodness of fit for discrete distributions

```

1 #Ex14.9, Page 608
2 #Answers may vary slightly due to rounding off of
  values
3
4 k<-5
5 m<-1
6 mu<-2.10
7 n<-48
8 df<-k-m-1
9
10 Frequency<-c(9,9,10,14,6)
11
12 np1<-c(NULL)
13 for(i in 1:(k-1)){
14   num<-(exp(1)^((-1)*mu))*(mu^(i-1))
15   np1[i]<-n*(num/factorial(i-1))
16
17 }
18 np1[5]<-n-np1[1]-np1[2]-np1[3]-np1[4]
19 print(np1)
20
21 chi<-c(NULL)
22 for(i in 1:length(Frequency)){
23   s<-((Frequency[i]-np1[i])^2)/np1[i]
24   chi[i]<-round(s,digits=3)
25 }
26 sum<-sum(chi)
27 print(paste("Chi square value:",sum))
28

```

```

29 alpha<-0.05
30 #To find chi square value
31 chival<-qchisq(1-alpha,df)
32 print(paste("Critical value:",chival))
33
34 if(sum < chival) print(paste("H0 is not rejected
    since",sum,"<",chival)) else print(paste("H0 can
    be rejected"))

```

---

#### R code Exa 14.10 Goodness of fit for continuous distributions

```

1 #Ex14.10, Page 608
2 #Answers may vary slightly from textbook values
3
4 data<-c
    (204,108,140,152,158,129,175,146,157,174,192,194,144,152,135,223,
5
6 n<-length(data)
7 mu_cap<-round(mean(data),digits=2)
8 sigma_cap<-round(sqrt((n-1)*(sd(data)^2)/n),digits
    =2)
9 print(paste("mu_cap:",mu_cap))
10 print(paste("sigma_cap:",sigma_cap))
11
12 #To find estimated expected counts
13 interval<-c(-Inf
    ,117.9,132.9,144.6,155.4,167.1,182.1,Inf)
14 p<-c(NULL)
15 np_i<-c(NULL)
16 for(i in 1:length(interval)-1){
17     p[i]<-pnorm(interval[i+1],mean=mu_cap,sd=sigma_cap
    )-pnorm(interval[i],mean=mu_cap,sd=sigma_cap)
18     np_i[i]<-n*p[i]
19 }

```

```

20
21 Cell<-c("(-Inf,117.9)","(117.9,132.9)","
      (132.9,144.6)","(144.6,155.4)","(155.4,167.1)","
      (167.1,182.1)","(182.1,Inf)")
22 Observed<-c(5,5,11,6,6,7,9)
23 Estimated_expected<-npi
24 data1<-data.frame(Cell,Observed,Estimated_expected)
25 print(data1)
26
27 #To find chi-squared values
28 alpha<-0.95
29 df1<-length(npi)-1
30 m<-2
31 q1<-qchisq(alpha,df=df1)
32 print(paste("X^2(0.05,k-1):",q1))
33
34 df2<-df1-m
35 q2<-qchisq(alpha,df=df2)
36 print(paste("X^2(0.05,k-1-m):",q2))

```

---

#### R code Exa 14.12 A special test for normality

```

1 #Ex14.12, Page 611
2
3 #Package to be installed: DescTools
4 library(DescTools)
5
6 yi<-c
      (-1.871,-1.404,-1.127,-0.917,-.742,-0.587,-0.446,-0.313,-0.186,-0
7
8 xi<-c
      (24.46,25.61,26.25,26.42,26.66,27.15,27.31,27.54,27.74,27.94,27.9
8
9 z<-c(NULL)

```



```

10 #To find z percentiles
11 for(i in 1:length(xi)){
12   z[i]<-round(qnorm(1-((i-0.5)/length(xi)),lower.
      tail=FALSE),digits=2)
13 }
14
15 #Shapiro test is similar to Ryan-Joiner test which
    is not available in R
16 t<-shapiro.test(xi)
17 print(t)
18
19 #To create normal probability plot
20 qqnorm(xi,datax=TRUE,ylab="Voltage",xlab="
    Probability",main="Normal probability plot")
21 qqline(xi,datax=TRUE)

```

---

#### R code Exa 14.13 Testing for homogeneity

```

1 #Ex14.13, Page 615
2
3 print(paste("Reason for nonconformity:"))
4 Blemish<-c(34,23,32)
5 Crack<-c(65,52,28)
6 Location<-c(17,25,16)
7 Missing<-c(21,19,14)
8 Other<-c(13,6,10)
9 Sample_size<-c(150,125,100)
10 df1<-data.frame(Blemish,Crack,Location,Missing,Other
    ,Sample_size)
11 print(df1)
12
13
14 #Total values
15 Blemish_tot<-sum(Blemish)
16 Crack_tot<-sum(Crack)

```

```

17 Location_tot<-sum(Location)
18 Missing_tot<-sum(Missing)
19 Other_tot<-sum(Other)
20 Sample_tot<-sum(Sample_size)
21
22 #H0: production lines are homogeneous
23 #Ha: production lines are not not homogeneous
24
25 tbl<-cbind.data.frame(df1$Blemish,df1$Crack,df1$
      Location,df1$Missing,df1$Other)
26 c<-chisq.test(tbl,df1$Sample_size)
27 print(c)
28
29 print(paste("P value:",c$p.value))
30
31 print(paste("H0 should not be rejected at levels
      0.05 or 0.01"))

```

---

#### R code Exa 14.14 Testing for independence

```

1 #Ex14.14, Page 617
2 #Answers may vary slightly due to rounding off of
  values
3
4 O<-c(24,15,17,52,73,80,58,86,36)
5 E<-c
      (17.02,22.10,16.89,62.29,80.88,61.83,54.69,71.02,54.29)
6
7 print(paste("Observed pricing policy"))
8 Observed<-matrix(data=0,nrow=3,ncol=3,byrow=TRUE)
9 rownames(Observed)<-c("Substandard","Standard","
      Modern")
10 colnames(Observed)<-c("Aggressive","Neutral","
      Nonaggressive")

```

```

11 print(Observed)
12
13 print(paste("Expected pricing policy"))
14 Expected<-matrix(data=E,nrow=3,ncol=3,byrow=TRUE)
15 rownames(Expected)<-c("Substandard","Standard","
    Modern")
16 colnames(Expected)<-c("Aggressive","Neutral","
    Nonaggressive")
17 print(Expected)
18
19 chi<-c(NULL)
20 for(i in 1:length(Observed)){
21     chi[i]<-(Observed[i]-Expected[i])^2/(Expected[i
        ])
22 }
23 sum<-sum(chi)
24 print(paste("Chi square value:",sum))
25
26 alpha<-0.01
27 #To find chi square value
28 chival<-qchisq(1-alpha,df=4)
29 print(paste("Critical value (Chi square):",chival))

```

---

# Chapter 15

## Distribution free procedures

**R code Exa 15.1** The Wilcoxon signed rank test

```
1 #Ex15.1, Page 629
2
3 library(stats)
4
5 x<-c
  (494.6,510.8,487.5,493.2,502.6,485.0,495.9,498.2,501.6,497.3,492.6,
6 y<-x-500
7
8 #Performing Wilcoxon signed rank test using wilcox.
  test function
9 print(wilcox.test(y))
```

---

**R code Exa 15.2** Paired observations

```
1 #Ex15.2, Page 630
2
3 library(stats)
```

```

4
5 IF_REE<-c
    (1753.7,1604.4,1576.5,1279.7,1754.2,1695.5,1700.1,1717.0)

6 Std_REE<-c
    (1755.0,1691.1,1697.1,1477.7,1785.2,1669.7,1901.3,1735.3)

7 Difference<-IF_REE-Std_REE
8
9 #Performing wilcoxon signed rank test
10 w1<-wilcox.test(Difference)      #or wilcox.test(IF_
    REE,Std_REE,paired=TRUE)
11 print(w1)
12
13 #Along with continuity correction
14 w2<-wilcox.test(Difference,exact=FALSE)  #or wilcox
    .test(IF_REE,Std_REE,paired=TRUE,exact=FALSE)
15 print(w2)

```

---

### R code Exa 15.3 Large sample approximation

```

1 #Ex15.3, Page 631
2 #Here p value is used to make an inference regarding
    the rejection of hypotheses
3 #wilcoxsign_test() from coin package can also be
    used to find z value if needed
4
5 #H0: mu=50000
6 #Ha: mu<50000
7
8 Signed_rank<-c
    (-1,-2,+3,-4,+5,-6,-7,+8,-9,-10,+11,-12,-13,+14,-15,-16,-17,+18,-
9
10 xi_50000<-c
    (-10,-27,36,-55,73,-77,-81,90,-95,-99,113,-127,-129,136,-150,-155

```

```

10 alpha_level<-0.01
11
12 w1<-wilcox.test(xi_50000,Signed_rank,paired=T,exact=
    F,correct=F,alt="two.sided")
13
14 #Using p value:
15 if(w1$p.value<alpha_level) print(paste("H0 is
    rejected since p value,",w1$p.value,"is smaller
    than level,",alpha_level)) else print(paste("H0
    is not rejected"))

```

---

#### R code Exa 15.4 The Wilcoxon rank sum test

```

1 #Ex15.4, Page 637
2
3 Polluted<-c(21.3,18.7,23.0,17.1,16.8,20.9,19.7)
4 Unpolluted<-c(14.2,18.3,17.2,18.4,20.0)
5
6 #Performing Wilcoxon rank sum test....
7 print(wilcox.test(Polluted,Unpolluted,conf.level
    =0.99))

```

---

#### R code Exa 15.5 A normal approximation for W

```

1 #Ex15.5, Page 638
2 #Answers may vary slightly due to rounding off of
    values
3
4 Allergics<-c
    (67.6,39.6,1651.0,100.0,65.9,1112.0,31.0,102.4,64.7)

```

```

5 Nonallergics<-c
  (34.3,27.3,35.4,48.1,5.2,29.1,4.7,41.7,48.0,6.6,18.9,32.4,45.5)

6
7 m<-length(Allergics)
8 n<-length(Nonallergics)
9
10 #H0: mu1-mu2=0
11
12 new<-c(Allergics,Nonallergics)
13
14 #To find rank of combined vectors
15 r<-rank(new)
16 s<-0
17 for(i in 1:length(Allergics)){
18   s<-s+r[i]
19 }
20 print(paste("Rank sum of allergies:",s))
21
22 #To find mean and variance of W
23 mu_w<-(m+n+1)*m/2
24 print(paste("Mean of W:",mu_w))
25 var_w<-m*n*(m+n+1)/12
26 print(paste("Variance of W:",var_w))
27
28 #To find test statistic value
29 z<-(s-mu_w)/sqrt(var_w)
30 print(paste("Test statistic value:",z))
31
32 alpha<-0.01
33 z1<-qnorm(alpha/2)
34
35 if(z>=z1 || z<=-z1) print(paste("H0 is rejected"))
  else print(paste("H0 is not rejected"))

```

---

### R code Exa 15.6 The Wilcoxon signed rank interval

```
1 #Ex15.6, Page 642
2 #Interval values may vary slightly .....
3
4 #Given data
5 averages<-c
   (4.51,4.55,4.59,4.705,4.72,4.745,4.76,4.795,4.835,4.90,4.915,4.93
6
7 print(summary(averages))
8
9 #To display confidence intervals of the data
10 print(wilcox.test(averages,conf.int=TRUE,conf.level
   =0.95))
```

---

### R code Exa 15.8 The Wilcoxon rank sum interval

```
1 #Ex15.8, Page 643
2 #Answers vary from that of text book(CI in text
   =>(4830,8220))
3
4 Epoxy<-c(10860,11120,11340,12130,14380,13070)
5 Other<-c(4590,4850,6510,5640,6390,0)
6
7 dat<-data.frame(Crush_Strength=c(Epoxy,Other),Bark_
   board=rep(c("Epoxy","Other"),each=6))
8 print(dat)
9
10 #To display differences between crushing strengths
   of both bark boards
11 o<-outer(Epoxy,Other,"-")
12 print(o)
13
14 #To find corresponding CI intervals
```



```
15 print(wilcox.test(Crush_Strength~Bark_board,data=dat
    ,conf.int=TRUE,conf.level=0.95))
```

---

### R code Exa 15.9 The Kruskal Wallis test

```
1 #Ex15.9, Page 646
2 #Answers may vary slightly due to rounding off of
  values
3
4 data<-c
    (309.2,309.7,311.0,316.8,326.5,349.8,409.5,331.0,347.2,348.9,361.0)

5 m1<-matrix(data,nrow=5,dimnames=list(c("4'", "6'", "8'", "10'", "12'"),byrow=TRUE))
6 print(m1)
7
8 N<-length(data)
9
10 #To find ranks of respective data
11 r1<-matrix(rank(m1),nrow=5)
12 cat("Ranks of data\n")
13 print(r1)
14
15 I<-nrow(r1)
16 J<-ncol(r1)
17
18 #To find rank sum and mean with respect to each
  plate length type
19 ri<-c(NULL)
20 ribar<-c(NULL)
21 for(i in 1:nrow(r1)){
22   ri[i]<-sum(r1[i,])
23   ribar[i]<-round(mean(r1[i,]),digits=2)
24 }
25 df1<-data.frame(ri,ribar)
```

```

26 cat("Rank sum and mean\n")
27 print(df1)
28
29 #To find k value
30 sum<-0
31 for(j in 1:nrow(r1)){
32     sum<-sum+(ri[j])^2/J
33 }
34 k<-(12/(N*(N+1)))*sum-3*(N+1)
35 print(paste("k value:",k))
36
37 #To find chi-square value
38 alpha<-0.99
39 chival<-qchisq(alpha,df=I-1)
40 print(paste("Chi-squared value:",chival))
41
42 if(k>=chival) print(paste("H0 is not rejected since",
    ,k,">=",chival)) else print(paste("H0 is
    rejected"))

```

---

#### R code Exa 15.10 Friedman test for a randomized block experiment

```

1 #Ex15.10, Page 647
2 #Answers may vary slightly due to rounding off of
  values
3
4 data<-c
    (23.1,57.6,10.5,23.6,11.9,54.6,21.0,20.3,22.7,53.2,9.7,19.6,13.8,
5 m1<-matrix(data,nrow=4,dimnames=list(c("Fear","
    Happiness","Depression","Calmness"),c("1","2","3"
    ,"4","5","6","7","8"))),byrow=TRUE)
6 print(m1)
7
8 #To find ranks of respective data

```

```

 9  r1<-rank(m1[,1])
10  r2<-rank(m1[,2])
11  r3<-rank(m1[,3])
12  r4<-rank(m1[,4])
13  r5<-rank(m1[,5])
14  r6<-rank(m1[,6])
15  r7<-rank(m1[,7])
16  r8<-rank(m1[,8])
17  data1<-data.frame(r1,r2,r3,r4,r5,r6,r7,r8)
18  cat("\nRanks of data:\n")
19  print(data1)
20
21  I<-nrow(data1)
22  J<-ncol(data1)
23
24  #To find rank sum and mean with respect to each
    plate length type
25  ri<-c(NULL)
26  ri2<-c(NULL)
27  for(i in 1:nrow(data1)){
28    ri[i]<-sum(data1[i,])
29    ri2[i]<-(ri[i])^2
30  }
31  df1<-data.frame(ri,ri2)
32  print(df1)
33
34  #To find test statistic
35  Fr<-(12/(I*J*(I+1)))*sum(ri2)-3*J*(I+1)
36  print(paste("Test statistic value:",Fr))
37
38  #To find chi-square value
39  alpha<-0.95
40  chival<-qchisq(alpha,df=I-1)
41  print(paste("Chi-squared value:",chival))
42
43  if(Fr<chival) print(paste("H0 is not rejected since",
    ,Fr,"<",chival)) else print(paste("H0 is
    rejected"))

```



# Chapter 16

## Quality control methods

**R code Exa 16.1** X bar chart based on known parameter values

```
1 #Ex16.1 , Page 655
2 #Answers may vary slightly due to rounding off of
  values
3
4 #Package to be installed: qcc(Quality control charts
  )
5 library(qcc)
6
7 Vis_obs<-c
  (10.37,10.48,10.77,10.47,10.84,10.48,10.41,10.40,10.33,10.73,10.4
8
9 s<-c(rep(1:25,3))
10
11 q<-qcc.groups(Vis_obs,s)
12
13 #To plot X-bar chart
14 q1<-qcc(q,type="xbar",center=10.5,std.dev=0.18)
15 plot(q1)
16 print(summary(q1))
```

---

**R code Exa 16.2** X bar charts based on estimated parameters

```
1 #Ex16.2, Page 657
2 #Answers may vary slightly due to rounding off of
  values
3
4 n<-3
5 k<-25
6 xdoublebar<-261.896/k
7 sbar<-3.834/k
8 a3<-0.886
9
10 #To find control limits
11 LCL<-xdoublebar-n*sbar/(a3*sqrt(n))
12 UCL<-xdoublebar+n*sbar/(a3*sqrt(n))
13 print(paste("LCL:",LCL))
14 print(paste("UCL:",UCL))
15
16 #Old limits:
17 #mu=10.5, sigma=0.18
18
19 #New limits:
20 print(paste("mu_cap:",xdoublebar))
21 print(paste("sigma_cap:",sbar/a3))
```

---

**R code Exa 16.3** X bar charts based on estimated parameters

```
1 #Ex16.3, Page 658
2
3 n<-3
4 k<-25
5 xdoublebar<-261.896/k
```

```

6 rbar<-0.292
7 b3<-1.693
8 sigma_cap<-0.292/b3
9
10 #To find control limits
11 LCL<-xdoublebar-3*rbar/(b3*sqrt(n))
12 UCL<-xdoublebar+3*rbar/(b3*sqrt(n))
13 print(paste("LCL:",round(LCL,digits=3)))
14 print(paste("UCL:",round(UCL,digits=3)))

```

---

#### R code Exa 16.4 The S chart

```

1 #Ex16.4, Page 663
2 #Answers may vary slightly due to rounding off of
  values
3
4 #Package to be installed: qcc(Quality control charts
  )
5 library(qcc)
6
7 n<-4
8 k<-22
9 obs1<-c
  (29.7,32.2,35.9,28.8,30.9,30.6,32.3,32.0,24.2,33.7,35.3,28.1,28.7
10
11 obs2<-c
  (29.0,29.3,29.1,27.2,32.6,34.3,27.7,27.9,27.5,24.4,33.2,34.0,28.9
12
13 obs3<-c
  (28.8,32.2,32.1,28.5,28.3,34.8,30.9,31.0,28.5,34.3,31.4,31.0,25.8
14
15 obs4<-c
  (30.2,32.9,31.3,35.7,28.3,26.3,27.8,30.8,31.1,31.0,28.0,30.8,29.7
16
17 SD<-c

```

```

      (0.64,1.60,2.83,3.83,2.11,3.94,2.30,1.76,2.85,4.53,3.09,2.41,1.71,
14  Range<-c
      (1.4,3.6,6.8,8.5,4.3,8.5,4.6,4.1,6.9,9.9,7.3,5.9,3.9,4.0,4.4,5.2,

15  data1<-data.frame(obs1,obs2,obs3,obs4)
16
17  #Sum of SD values
18  si<-sum(SD)
19  print(paste("Sum of SD values:",si))
20  sbar<-mean(SD)
21  print(paste("Mean of SD values:",sbar))
22
23  a4<-0.921
24  LCL<-0
25  UCL<-sbar+3*sbar*sqrt(1-a4^2)/a4
26  print(paste("LCL:",LCL))
27  print(paste("UCL:",UCL))
28
29  #To plot S chart
30  q4<-qcc(data1,type="S")
31  print(summary(q4))

```

---

### R code Exa 16.5 R chart

```

1  #Ex16.5, Page 665
2  #Answers may vary slightly due to rounding off of
   values
3
4  #Package to be installed: qcc(Quality control charts
   )
5  library(qcc)
6
7  des_dim<-c
      (200,250,300,350,400,450,500,550,600,650,700,750,800,850,900,950,

```



```

8  obs1<-c(12,6,5,19,9,9,8,4,11,13,10,8,14,7,14,10,7)
9  obs2<-c(17,9,9,6,14,15,11,14,14,9,14,9,7,9,5,12,11)
10 obs3<-c(6,17,15,11,9,8,12,11,7,9,8,4,9,12,8,10,15)
11 mean<-c
    (11.7,10.7,9.7,12.0,10.7,10.7,10.3,9.7,10.7,10.3,10.7,7.0,10.0,9.7,10.7,10.7,10.7)

12 range<-c(11,11,10,13,5,7,4,10,7,4,6,5,7,5,9,2,8)
13 st_dev<-c
    (5.51,5.69,5.03,6.56,2.84,3.79,2.08,5.13,3.51,2.31,3.06,2.65,3.61,2.65,2.65,2.65,2.65)

14 data1<-data.frame(obs1,obs2,obs3)
15
16 n<-3
17 b3<-1.693
18 c3<-0.888
19
20 #To find sum of range values
21 ri<-sum(range)
22 rbar<-mean(range)
23
24 #To find control limits
25 UCL<-rbar+n*c3*rbar/b3
26 print(paste("LCL:",LCL))
27 print(paste("UCL:",UCL))
28
29 #To display X-bar chart (can only be displayed
    separately by using run method)
30 q1<-qcc(data1,type="xbar")
31 print(summary(q1))
32
33 #To display R chart (can only be displayed
    separately by using run method)
34 q2<-qcc(data1,type="R")
35 print(summary(q2))

```

---

### R code Exa 16.6 The p chart

```
1 #Ex16.6, Page 669
2 #Answers may vary slightly due to rounding off of
  values
3
4 #Package to be installed: qcc(Quality control charts
  )
5 library(qcc)
6
7 Dayi<-c
  (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25)
8 xi<-c
  (7,4,3,6,4,9,6,7,5,3,7,8,4,6,2,9,7,6,7,11,6,7,4,8,6)
9 pi<-c
  (0.07,0.04,0.03,0.06,0.04,0.09,0.06,0.07,0.05,0.03,0.07,0.08,0.04,0.05,0.06,0.04,0.03,0.05,0.07,0.06,0.04,0.05,0.03,0.06)
10 data1<-data.frame(xi,pi)
11
12 #Sum of pi values
13 p<-sum(pi)
14 print(paste("Sum of pi values:",p))
15 pbar<-mean(pi)
16 print(paste("Mean of p values:",pbar))
17
18 #To find control limits
19 LCL<-pbar-3*sqrt(pbar*(1-pbar)/100)
20 UCL<-pbar+3*sqrt(pbar*(1-pbar)/100)
21 print(paste("LCL:",LCL))
22 print(paste("UCL:",UCL))
23
24 #To display p chart
```

```

25 q1<-qcc(data1,sizes=length(Dayi),type="p")
26 print(summary(q1))

```

---

**R code Exa 16.7** c chart for number of defectives

```

1 #Ex16.7, Page 670
2 #Answers may slightly vary due to rounding off of
  values
3
4 #Package to be installed: qcc(Quality control charts
  )
5 library(qcc)
6
7 x<-c
  (7,10,9,12,13,6,13,7,5,11,8,10,13,9,21,10,6,8,3,12,7,11,14,10)
8
9 #To find the sum of x values
10 xi<-sum(x)
11 xbar<-mean(x)
12 print(paste("Sum of x values:",xi))
13 print(paste("Mean of x values:",xbar))
14
15 #To find control limits
16 LCL<-xbar-3*sqrt(xbar)
17 UCL<-xbar+3*sqrt(xbar)
18 print(paste("LCL:",LCL))
19 print(paste("UCL:",UCL))
20
21 #To display c chart
22 q1<-qcc(x,sizes=length(x),type="c")
23 print(summary(q1))
24
25 #From the chart, the 15th value lies above the UCL
26 #Eliminating that observation.....

```

```

27 xbar1<-(sum(x)-x[15])/(length(x)-1)
28 print(paste("Mean value when the 15th value is
    eliminated:",xbar1))
29 LCL1<-xbar1-3*sqrt(xbar1)
30 UCL1<-xbar1+3*sqrt(xbar1)
31 print(paste("Control limits when 15th value is
    eliminated:"))
32 print(paste("New LCL value:",LCL1))
33 print(paste("New UCL value:",UCL1))
34
35 #Remaining 23 observation lie within control lines (
    in-control)

```

---

#### R code Exa 16.8 CUSUM procedures

```

1 #Ex16.8, Page 674
2
3 #Package to be installed: qcc(Quality control charts
  )
4 library(qcc)
5
6 n<-4
7 mu=mu0=40
8 sigma<-0.5
9 obs<-c
    (40.77,39.95,40.86,39.21,38.94,39.70,40.37,39.88,40.43,40.27,40.9
10 sa<-c(rep(1,4),rep(2,4),rep(3,4),rep(4,4),rep(5,4),
    rep(6,4),rep(7,4),rep(8,4),rep(9,4),rep(10,4),rep
    (11,4),rep(12,4),rep(13,4),rep(14,4),rep(15,4),
    rep(16,4))
11
12 #To display X-bar chart
13 q<-qcc.groups(data=obs,sample=sa)
14 q4<-qcc(q,center=mu0,std.dev=sigma,type="xbar")

```

```
15 print(summary(q4))
```

---

### R code Exa 16.9 Computational version

```
1 #Ex16.9 , Page 676
2
3 #Package to be installed: qcc(Quality control charts
  )
4 library(qcc)
5
6 mu0<-40
7 delta<-0.3
8 k<-delta/2
9 l<-mu0-k
10 r<-mu0+k
11 obs<-c
    (40.77,39.95,40.86,39.21,38.94,39.70,40.37,39.88,40.43,40.27,40.9
12 sa<-c(rep(1,4),rep(2,4),rep(3,4),rep(4,4),rep(5,4),
    rep(6,4),rep(7,4),rep(8,4),rep(9,4),rep(10,4),rep
    (11,4),rep(12,4),rep(13,4),rep(14,4),rep(15,4),
    rep(16,4))
13 xbar<-c
    (40.20,39.72,40.42,39.98,40.06,39.76,39.65,40.41,40.32,39.84,40.4
14
15 xbar1<-xbar-r
16 xbar2<-xbar-l
17
18 #To find d and e values
19 d0<-0
20 d1<-max(0,d0+(xbar[1]-r))
21 e0<-0
22 e1<-max(0,e0-(xbar[1]-l))
23
```

```

24 d<-c(d1)
25 e<-c(e1)
26 for(i in 2:length(xbar)){
27   d[i]<-max(0,d[i-1]+(xbar[i]-r))
28   e[i]<-max(0,e[i-1]-(xbar[i]-l))
29 }
30
31 df<-data.frame(xbar,xbar1,d,xbar2,e)
32 print(df)
33
34 q<-qcc.groups(data=obs,sample=sa)
35 c<-cusum(q,sizes=4,se.shift=delta,center=mu0)
36 print(summary(c))

```

---

#### R code Exa 16.10 Designing a CUSUM procedure

```

1 #Ex16.10 , Page 680
2
3 sigma<-0.004
4 kbar<-0.74
5 delta<-0.003
6
7 n<-round((2*kbar*sigma/delta)^2,digits=0)
8 print(paste("Sample size:",n))
9
10 hbar<-3.2
11 h<-(sigma/sqrt(n))*hbar
12 print(paste("h value:",h))
13
14 print(paste("Out of control signal if d>",h,"or e>",
             h))

```

---

#### R code Exa 16.11 Acceptance sampling

```

1 #Ex16.11, Page 681
2
3 P<- function(p) {(1-p)^50+50*p*(1-p)^49+1225*p^2*(1-
  p)^48}
4
5 p<-c
  (0.01,0.02,0.03,0.04,0.05,0.06,0.07,0.08,0.09,0.10,0.12,0.15)

6
7 #Finding P(A) values
8 PA<-c(NULL)
9 for(i in 1:length(p)){
10   PA[i]<-round(P(p[i]),digits=3)
11 }
12
13 df<-data.frame(p,PA)
14 print(df)
15
16 #To plot data
17 plot(p,PA,ylab="P(A)",main="Curve plotting P(A) and
  p values")

```

---

#### R code Exa 16.12 Designing a single sample plan

```

1 #Ex16.12, Page 683
2 #Answers may vary slightly due to rounding off of
  values
3
4 #Package to be installed: AcceptanceSampling
5 library(AcceptanceSampling)
6
7 c<-0:15
8 np1<-c
  (0.051,0.355,0.818,1.366,1.970,2.613,3.285,3.981,4.695,5.425,6.160,6.881,7.601,8.321,9.041,9.761,10.481,11.201,11.921,12.641,13.361,14.081,14.801,15.521,16.241,16.961,17.681,18.401,19.121,19.841,20.561,21.281,22.001,22.721,23.441,24.161,24.881,25.601,26.321,27.041,27.761,28.481,29.201,29.921,30.641,31.361,32.081,32.801,33.521,34.241,34.961,35.681,36.401,37.121,37.841,38.561,39.281,40.001,40.721,41.441,42.161,42.881,43.601,44.321,45.041,45.761,46.481,47.201,47.921,48.641,49.361,50.081,50.801,51.521,52.241,52.961,53.681,54.401,55.121,55.841,56.561,57.281,58.001,58.721,59.441,60.161,60.881,61.601,62.321,63.041,63.761,64.481,65.201,65.921,66.641,67.361,68.081,68.801,69.521,70.241,70.961,71.681,72.401,73.121,73.841,74.561,75.281,76.001,76.721,77.441,78.161,78.881,79.601,80.321,81.041,81.761,82.481,83.201,83.921,84.641,85.361,86.081,86.801,87.521,88.241,88.961,89.681,90.401,91.121,91.841,92.561,93.281,94.001,94.721,95.441,96.161,96.881,97.601,98.321,99.041,99.761,100.481,101.201,101.921,102.641,103.361,104.081,104.801,105.521,106.241,106.961,107.681,108.401,109.121,109.841,110.561,111.281,112.001,112.721,113.441,114.161,114.881,115.601,116.321,117.041,117.761,118.481,119.201,119.921,120.641,121.361,122.081,122.801,123.521,124.241,124.961,125.681,126.401,127.121,127.841,128.561,129.281,130.001,130.721,131.441,132.161,132.881,133.601,134.321,135.041,135.761,136.481,137.201,137.921,138.641,139.361,140.081,140.801,141.521,142.241,142.961,143.681,144.401,145.121,145.841,146.561,147.281,148.001,148.721,149.441,150.161,150.881,151.601,152.321,153.041,153.761,154.481,155.201,155.921,156.641,157.361,158.081,158.801,159.521,160.241,160.961,161.681,162.401,163.121,163.841,164.561,165.281,166.001,166.721,167.441,168.161,168.881,169.601,170.321,171.041,171.761,172.481,173.201,173.921,174.641,175.361,176.081,176.801,177.521,178.241,178.961,179.681,180.401,181.121,181.841,182.561,183.281,184.001,184.721,185.441,186.161,186.881,187.601,188.321,189.041,189.761,190.481,191.201,191.921,192.641,193.361,194.081,194.801,195.521,196.241,196.961,197.681,198.401,199.121,199.841,200.561,201.281,202.001,202.721,203.441,204.161,204.881,205.601,206.321,207.041,207.761,208.481,209.201,209.921,210.641,211.361,212.081,212.801,213.521,214.241,214.961,215.681,216.401,217.121,217.841,218.561,219.281,220.001,220.721,221.441,222.161,222.881,223.601,224.321,225.041,225.761,226.481,227.201,227.921,228.641,229.361,230.081,230.801,231.521,232.241,232.961,233.681,234.401,235.121,235.841,236.561,237.281,238.001,238.721,239.441,240.161,240.881,241.601,242.321,243.041,243.761,244.481,245.201,245.921,246.641,247.361,248.081,248.801,249.521,250.241,250.961,251.681,252.401,253.121,253.841,254.561,255.281,256.001,256.721,257.441,258.161,258.881,259.601,260.321,261.041,261.761,262.481,263.201,263.921,264.641,265.361,266.081,266.801,267.521,268.241,268.961,269.681,270.401,271.121,271.841,272.561,273.281,274.001,274.721,275.441,276.161,276.881,277.601,278.321,279.041,279.761,280.481,281.201,281.921,282.641,283.361,284.081,284.801,285.521,286.241,286.961,287.681,288.401,289.121,289.841,290.561,291.281,292.001,292.721,293.441,294.161,294.881,295.601,296.321,297.041,297.761,298.481,299.201,299.921,300.641,301.361,302.081,302.801,303.521,304.241,304.961,305.681,306.401,307.121,307.841,308.561,309.281,310.001,310.721,311.441,312.161,312.881,313.601,314.321,315.041,315.761,316.481,317.201,317.921,318.641,319.361,320.081,320.801,321.521,322.241,322.961,323.681,324.401,325.121,325.841,326.561,327.281,328.001,328.721,329.441,330.161,330.881,331.601,332.321,333.041,333.761,334.481,335.201,335.921,336.641,337.361,338.081,338.801,339.521,340.241,340.961,341.681,342.401,343.121,343.841,344.561,345.281,346.001,346.721,347.441,348.161,348.881,349.601,350.321,351.041,351.761,352.481,353.201,353.921,354.641,355.361,356.081,356.801,357.521,358.241,358.961,359.681,360.401,361.121,361.841,362.561,363.281,364.001,364.721,365.441,366.161,366.881,367.601,368.321,369.041,369.761,370.481,371.201,371.921,372.641,373.361,374.081,374.801,375.521,376.241,376.961,377.681,378.401,379.121,379.841,380.561,381.281,382.001,382.721,383.441,384.161,384.881,385.601,386.321,387.041,387.761,388.481,389.201,389.921,390.641,391.361,392.081,392.801,393.521,394.241,394.961,395.681,396.401,397.121,397.841,398.561,399.281,400.001,400.721,401.441,402.161,402.881,403.601,404.321,405.041,405.761,406.481,407.201,407.921,408.641,409.361,410.081,410.801,411.521,412.241,412.961,413.681,414.401,415.121,415.841,416.561,417.281,418.001,418.721,419.441,420.161,420.881,421.601,422.321,423.041,423.761,424.481,425.201,425.921,426.641,427.361,428.081,428.801,429.521,430.241,430.961,431.681,432.401,433.121,433.841,434.561,435.281,436.001,436.721,437.441,438.161,438.881,439.601,440.321,441.041,441.761,442.481,443.201,443.921,444.641,445.361,446.081,446.801,447.521,448.241,448.961,449.681,450.401,451.121,451.841,452.561,453.281,454.001,454.721,455.441,456.161,456.881,457.601,458.321,459.041,459.761,460.481,461.201,461.921,462.641,463.361,464.081,464.801,465.521,466.241,466.961,467.681,468.401,469.121,469.841,470.561,471.281,472.001,472.721,473.441,474.161,474.881,475.601,476.321,477.041,477.761,478.481,479.201,479.921,480.641,481.361,482.081,482.801,483.521,484.241,484.961,485.681,486.401,487.121,487.841,488.561,489.281,490.001,490.721,491.441,492.161,492.881,493.601,494.321,495.041,495.761,496.481,497.201,497.921,498.641,499.361,500.081,500.801,501.521,502.241,502.961,503.681,504.401,505.121,505.841,506.561,507.281,508.001,508.721,509.441,510.161,510.881,511.601,512.321,513.041,513.761,514.481,515.201,515.921,516.641,517.361,518.081,518.801,519.521,520.241,520.961,521.681,522.401,523.121,523.841,524.561,525.281,526.001,526.721,527.441,528.161,528.881,529.601,530.321,531.041,531.761,532.481,533.201,533.921,534.641,535.361,536.081,536.801,537.521,538.241,538.961,539.681,540.401,541.121,541.841,542.561,543.281,544.001,544.721,545.441,546.161,546.881,547.601,548.321,549.041,549.761,550.481,551.201,551.921,552.641,553.361,554.081,554.801,555.521,556.241,556.961,557.681,558.401,559.121,559.841,560.561,561.281,562.001,562.721,563.441,564.161,564.881,565.601,566.321,567.041,567.761,568.481,569.201,569.921,570.641,571.361,572.081,572.801,573.521,574.241,574.961,575.681,576.401,577.121,577.841,578.561,579.281,580.001,580.721,581.441,582.161,582.881,583.601,584.321,585.041,585.761,586.481,587.201,587.921,588.641,589.361,590.081,590.801,591.521,592.241,592.961,593.681,594.401,595.121,595.841,596.561,597.281,598.001,598.721,599.441,600.161,600.881,601.601,602.321,603.041,603.761,604.481,605.201,605.921,606.641,607.361,608.081,608.801,609.521,610.241,610.961,611.681,612.401,613.121,613.841,614.561,615.281,616.001,616.721,617.441,618.161,618.881,619.601,620.321,621.041,621.761,622.481,623.201,623.921,624.641,625.361,626.081,626.801,627.521,628.241,628.961,629.681,630.401,631.121,631.841,632.561,633.281,634.001,634.721,635.441,636.161,636.881,637.601,638.321,639.041,639.761,640.481,641.201,641.921,642.641,643.361,644.081,644.801,645.521,646.241,646.961,647.681,648.401,649.121,649.841,650.561,651.281,652.001,652.721,653.441,654.161,654.881,655.601,656.321,657.041,657.761,658.481,659.201,659.921,660.641,661.361,662.081,662.801,663.521,664.241,664.961,665.681,666.401,667.121,667.841,668.561,669.281,670.001,670.721,671.441,672.161,672.881,673.601,674.321,675.041,675.761,676.481,677.201,677.921,678.641,679.361,680.081,680.801,681.521,682.241,682.961,683.681,684.401,685.121,685.841,686.561,687.281,688.001,688.721,689.441,690.161,690.881,691.601,692.321,693.041,693.761,694.481,695.201,695.921,696.641,697.361,698.081,698.801,699.521,700.241,700.961,701.681,702.401,703.121,703.841,704.561,705.281,706.001,706.721,707.441,708.161,708.881,709.601,710.321,711.041,711.761,712.481,713.201,713.921,714.641,715.361,716.081,716.801,717.521,718.241,718.961,719.681,720.401,721.121,721.841,722.561,723.281,724.001,724.721,725.441,726.161,726.881,727.601,728.321,729.041,729.761,730.481,731.201,731.921,732.641,733.361,734.081,734.801,735.521,736.241,736.961,737.681,738.401,739.121,739.841,740.561,741.281,742.001,742.721,743.441,744.161,744.881,745.601,746.321,747.041,747.761,748.481,749.201,749.921,750.641,751.361,752.081,752.801,753.521,754.241,754.961,755.681,756.401,757.121,757.841,758.561,759.281,760.001,760.721,761.441,762.161,762.881,763.601,764.321,765.041,765.761,766.481,767.201,767.921,768.641,769.361,770.081,770.801,771.521,772.241,772.961,773.681,774.401,775.121,775.841,776.561,777.281,778.001,778.721,779.441,780.161,780.881,781.601,782.321,783.041,783.761,784.481,785.201,785.921,786.641,787.361,788.081,788.801,789.521,790.241,790.961,791.681,792.401,793.121,793.841,794.561,795.281,796.001,796.721,797.441,798.161,798.881,799.601,800.321,801.041,801.761,802.481,803.201,803.921,804.641,805.361,806.081,806.801,807.521,808.241,808.961,809.681,810.401,811.121,811.841,812.561,813.281,814.001,814.721,815.441,816.161,816.881,817.601,818.321,819.041,819.761,820.481,821.201,821.921,822.641,823.361,824.081,824.801,825.521,826.241,826.961,827.681,828.401,829.121,829.841,830.561,831.281,832.001,832.721,833.441,834.161,834.881,835.601,836.321,837.041,837.761,838.481,839.201,839.921,840.641,841.361,842.081,842.801,843.521,844.241,844.961,845.681,846.401,847.121,847.841,848.561,849.281,850.001,850.721,851.441,852.161,852.881,853.601,854.321,855.041,855.761,856.481,857.201,857.921,858.641,859.361,860.081,860.801,861.521,862.241,862.961,863.681,864.401,865.121,865.841,866.561,867.281,868.001,868.721,869.441,870.161,870.881,871.601,872.321,873.041,873.761,874.481,875.201,875.921,876.641,877.361,878.081,878.801,879.521,880.241,880.961,881.681,882.401,883.121,883.841,884.561,885.281,886.001,886.721,887.441,888.161,888.881,889.601,890.321,891.041,891.761,892.481,893.201,893.921,894.641,895.361,896.081,896.801,897.521,898.241,898.961,899.681,900.401,901.121,901.841,902.561,903.281,904.001,904.721,905.441,906.161,906.881,907.601,908.321,909.041,909.761,910.481,911.201,911.921,912.641,913.361,914.081,914.801,915.521,916.241,916.961,917.681,918.401,919.121,919.841,920.561,921.281,922.001,922.721,923.441,924.161,924.881,925.601,926.321,927.041,927.761,928.481,929.201,929.921,930.641,931.361,932.081,932.801,933.521,934.241,934.961,935.681,936.401,937.121,937.841,938.561,939.281,940.001,940.721,941.441,942.161,942.881,943.601,944.321,945.041,945.761,946.481,947.201,947.921,948.641,949.361,950.081,950.801,951.521,952.241,952.961,953.681,954.401,955.121,955.841,956.561,957.281,958.001,958.721,959.441,960.161,960.881,961.601,962.321,963.041,963.761,964.481,965.201,965.921,966.641,967.361,968.081,968.801,969.521,970.241,970.961,971.681,972.401,973.121,973.841,974.561,975.281,976.001,976.721,977.441,978.161,978.881,979.601,980.321,981.041,981.761,982.481,983.201,983.921,984.641,985.361,986.081,986.801,987.521,988.241,988.961,989.681,990.401,991.121,991.841,992.561,993.281,994.001,994.721,995.441,996.161,996.881,997.601,998.321,999.041,999.761,1000.481,1001.201,1001.921,1002.641,1003.361,1004.081,1004.801,1005.521,1006.241,1006.961,1007.681,1008.401,1009.121,1009.841,1010.561,1011.281,1012.001,1012.721,1013.441,1014.161,1014.881,1015.601,1016.321,1017.041,1017.761,1018.481,1019.201,1019.921,1020.641,1021.361,1022.081,1022.801,1023.521,1024.241,1024.961,1025.681,1026.401,1027.121,1027.841,1028.561,1029.281,1030.001,1030.721,1031.441,1032.161,1032.881,1033.601,1034.321,1035.041,1035.761,1036.481,1037.201,1037.921,1038.641,1039.361,1040.081,1040.801,1041.521,1042.241,1042.961,1043.681,1044.401,1045.121,1045.841,1046.561,1047.281,1048.001,1048.721,1049.441,1050.161,1050.881,1051.601,1052.321,1053.041,1053.761,1054.481,1055.201,1055.921,1056.641,1057.361,1058.081,1058.801,1059.521,1060.241,1060.961,1061.681,1062.401,1063.121,1063.841,1064.561,1065.281,1066.001,1066.721,1067.441,1068.161,1068.881,1069.601,1070.321,1071.041,1071.761,1072.481,1073.201,1073.921,1074.641,1075.361,1076.081,1076.801,1077.521,107
```

```

9  np2<-c
    (2.30,3.89,5.32,6.68,7.99,9.28,10.53,11.77,12.99,14.21,15.41,16.6
10  p2_p1<-np2/np1
11  df1<-data.frame(c,np1,np2,p2_p1)
12  print(df1)
13  cat("\n")
14
15  AQL<-p1<-0.01
16  LTPD<-p2<-0.045
17
18  #Ratio of p2 to p1
19  r<-LTPD/AQL
20  print(paste("Ratio of p2 to p1:",r))
21
22  for(i in 1:length(c)){
23    if(r>p2_p1[i+1] && r<p2_p1[i]){
24      j<-i
25      np_1<-np1[i]
26      c1<-c[i]
27      c2<-c[i+1]
28      np_2<-np2[i+1]
29
30    }
31  }
32  cat("Ratio lies between",p2_p1[j],"and",p2_p1[j+1],"
    values corresponding to c values",c[j],"and",c[j
    +1],"\n")
33
34  #Single sample plan: 1st c value
35  cat("When c value is",c1,"\n")
36
37  ##To find sample size
38  n<-round(np_1/p1,digits=0)
39  print(paste("Sample size:",n))
40
41  ##To find alpha and beta values
42  alpha1<-1-pbinom(c1,n,p1)

```



```

43 print(paste("Alpha value:",alpha1))
44
45 beta1<-pbinom(c1,n,p2)
46 print(paste("Beta value:",beta1))
47
48 #Single sample plan: 2nd c value
49 cat("When c value is",c2,"\n")
50
51 #To find sample size
52 n<-round(np_2/p2,digits=0)
53 print(paste("Sample size:",n))
54
55 ##To find alpha and beta values
56 alpha2<-1-pbinom(c2,n,p1)
57 print(paste("Alpha value:",alpha2))
58
59 beta2<-pbinom(c2,n,p2)
60 print(paste("Beta value:",beta2))

```

---