R Textbook Companion for John E. Freund's Mathematical Statistics With Applications by John E. Freund¹

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Book Description

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R numbering policy used in this document and the relation to the above book.

Exa Example (Solved example)

Eqn Equation (Particular equation of the above book)

For example, Exa 3.51 means solved example 3.51 of this book. Sec 2.3 means an R code whose theory is explained in Section 2.3 of the book.

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Chapter 1

Introduction

R code Exa 1.1 Counting travel options

```
1 # page no 2
2 transportation= 3
3 no_of_places= 5
4 total_ways=transportation*no_of_places
5 total_ways
6 print(total_ways)
```

R code Exa 1.2 Rolling two dice

```
1 # page no 3
2 selection_of_red_die=6
3 selection_of_green_die=6
4 total_ways=selection_of_green_die*selection_of_red_die
5 print(total_ways)
```

R code Exa 1.3 Choosing inspection parts

R code Exa 1.4 Answering true false test

```
1 # page no 4
2 total_question=20
3 possible_ans=2
4 s=1
5 for (i in 1:20)
6 {
7    s=s*possible_ans
8
9 }
10 print(s)
```

R code Exa 1.5 Permutations of abc

```
1 # page number 4
2 total_letter= 3
3 letter_tobe_chosen=3
4 total_ways=choose(total_letter,letter_tobe_chosen)*
    factorial(letter_tobe_chosen)
5 print(total_ways)
```

R code Exa 1.6 Introducing basketball players

```
1 # page number 5
2 no_of_player=5
3 ways__wich_th_introdce=factorial(no_of_player)
4 print(ways__wich_th_introdce)
```

R code Exa 1.7 Permutations of 2 letters

```
1 # page number 5
2 num_of_lettr=4
3 selct_frm_lettr=2
4 s=1
5 for (i in 1:selct_frm_lettr)
6 {
7    s=s*num_of_lettr
8    num_of_lettr=num_of_lettr-1
9 }
10 print(s)
```

R code Exa 1.8 combination

```
1 # page number 6
2 total_member= 24
3 position_tobe_chosen=4
4 total_ways=choose(total_member,position_tobe_chosen)
    *factorial(position_tobe_chosen)
5 print(total_ways)
```

R code Exa 1.9 Scheduling speakers for meetings

```
1 # page number= 6
2 total_dates= 5
3 num_of_spekr=3
4 total_ways= choose(total_dates,num_of_spekr)*
    factorial(num_of_spekr)
5 print(total_ways)
```

R code Exa 1.10 circular permutations

```
1 # page number = 7
2 no_of_prsn_play_circlr=4
3 ways=factorial(no_of_prsn_play_circlr-1)
4 print(ways)
```

R code Exa 1.11 Permutations of the word book

```
1 #page number= 7
2 letter="book"
3 n= nchar(letter)
4 letter_counts= table(strsplit(letter,"")[[1]])
5 s=1
6 for (i in 1:length(letter_counts))
7 {    s= s*(1/factorial(letter_counts[i][[1]]))
        print(s)
9 }
10 total_ways=factorial(n)*s
11 print(total_ways)
```

R code Exa 1.12 Arranging novels on a shelf

```
1 #page number 7
2 copies_of_one_novel=3
3 novel_with_one_copy=4
4 total_novel=copies_of_one_novel+novel_with_one_copy
5 print(choose(total_novel,novel_with_one_copy)*
    factorial(novel_with_one_copy))
```

R code Exa 1.13 Hanging paintings on a museum wall

R code Exa 1.14 Selecting households for market research

```
1 #page number= 8
2 total_houshld=20
3 select_houshld=3
4
5
6
7
8 total_ways = choose(total_houshld, select_houshld)*
    factorial(select_houshld)
9 print(total_ways)
10
11 order_not_matters=total_ways/factorial(select_houshld)
12 print(order_not_matters)
```

R code Exa 1.15 Coin toss outcomes

```
1 # page number 9
2
3
4 total_tossed_coins=6
5 no.of.heads=2
6 no.of.tails=4
7
8 total_ways= choose(total_tossed_coins,no.of.heads)
9 print(total_ways)
```

R code Exa 1.16 Forming committees of chemists and physicists

```
1 #page number= 9
2
3 total_chemist=4
4 number.of.chemist.selct=2
```

R code Exa 1.17 Partitioning a set into subsets

```
1 #page no 9
2 library(iterpc)
3 total_object=4
4 first_subset=1
5 second_subset=2
6 third_subset=1
7
8 total.subset= multichoose(c(first_subset, second_subset, third_subset))
9 print(total.subset)
```

R code Exa 1.18 Assigning businessmen to hotel rooms

```
1 # page number =11
2 total_hotel_rooms=7
3 assign_first_room=3
4 assign_second_room=2
5 assign_thired_room=2
6 total.ways= multichoose(c(assign_first_room,assign_second_room,assign_thired_room))
7 print(total.ways)
```

R code Exa 1.19 finding combination for n

```
1 # page number = 12
2 print(choose(4,3))
3 print(choose(4,4))
```

${f R}$ code ${f Exa}$ 1.20 finding combination of n

```
1 # page number =12
2
3 print(choose(5,3))
4 print(choose(5,5))
```

R code Exa 1.21 finding the combination

```
1 # page number= 13
2 print(choose(20,12))
3 print(choose(17,10))
```

R code Exa 1.22 multinom

```
1 # page numebr= 15
2 m=2
3 n=3
4 k=4
5 sum=0
6 for(i in 0:k)
```

```
7 {
8    sum=sum+choose(m,i)*choose(n,k-i)
9 }
10
11 result= choose(m+n,k)
12
13 if(result==sum)
14 {
15    print(sum)
16 }
```

R code Exa 1.23 multinom

```
1 #page number= 15
2 library(iterpc)
3 n=6
4 r1=3
5 r2=1
6 r3=2
7 result= multichoose(c(r1,r2,r3))
8 print(result)
```

R code Exa 1.24 Selecting integrated circuit chips for assembly

```
1 # page number = 17
2 no.integrated.chips= 20
3 no_of_chips_must_solder=3
4 result= choose(no.integrated.chips,no_of_chips_must_solder)*factorial(no_of_chips_must_solder)
5 print(result)
```

${f R}$ code ${f Exa}$ 1.25 Selecting units for inspection

```
1 # page number = 17
2 #— The answer provided in the textbook is wrong.
3 total_units=16
4 units_selected=4
5 result=choose(total_units,units_selected)
6 print(result)
```

Chapter 2

probability

R code Exa 2.1 ace in cards

```
1 # page number 21
2 library(MASS)
3 total_ace = 4
4 total_cards= 52
5 prob_of_ace= (total_ace)/total_cards
6 print(fractions(prob_of_ace))
```

R code Exa 2.2 pair of dice

```
1 # page number 23
2 red_dice <- 6
3 green_dice <- 6
4 dice_combinations <- expand.grid(1:red_dice, 1:green_dice)
5 dice_combinations
6 sums <- rowSums(dice_combinations)
7 cat(paste(sums, collapse = ","))</pre>
```

R code Exa 2.3 divisible by 3

```
1 # page numeber 24
2 red_dice <- 6
3 k <- 1:red_dice
4 divisible_by_3 <- k[k %% 3 == 0]
5 cat(paste(divisible_by_3, collapse = ","))</pre>
```

R code Exa 2.4 sum is 7

```
1 # page number=24
2
3 red_dice <- 6
4 green_dice <- 6
5 dice_combinations <- expand.grid(red_die = 1:red_dice, green_die = 1:green_dice)
6 valid_combinations <- subset(dice_combinations, red_die + green_die == 7)
7 x_coord <- valid_combinations$red_die
8 y_coord <- valid_combinations$green_die
9 cat(paste("(", x_coord, ",", y_coord, ")", sep = "", collapse = ","))
10
11
12 plot(x_coord, y_coord, col = "blue", pch = 19, xlab = "Red_die", ylab = "Green_die")</pre>
```

R code Exa 2.5 hit the target once and miss it twice

```
1 # page number = 25
2 combinations <- expand.grid(m = 2:1, n = 2:1, l = 2:1)
3 all_hit <- subset(combinations, m == 1 & n == 1 & l == 1)
4 one_miss <- subset(combinations, (m == 2 & n == 1 & l == 1) | (m == 1 & n == 2 & l == 1) | (m == 1 & n == 1 & l == 2))
5 cat(paste("(", all_hit$m-1, ",", all_hit$n-1, ",", all_hit$l-1, ")", sep = "", collapse = ","))
6 if (nrow(one_miss) > 0) {
7 cat(paste("(", one_miss$m-1, ",", one_miss$n-1, ",", one_miss$l-1, ")", sep = "", collapse = ","))
8 }
```

R code Exa 2.7 union

```
1 # page number 28
2 prob_A <- 0.12
3 prob_B <- 0.63
4 prob_C <- 0.45
5 prob_D <- -0.20
6 condition_one <- all(c(prob_A, prob_B, prob_C, prob_
     D) > 0
7 condition_two <- sum(prob_A, prob_B, prob_C, prob_D)
8 if(!condition_one){
    print("violates Postulate 1")
9
10 }
11 if(condition_two){
    print(" violates Postulate 2.")
12
13 }
```

R code Exa 2.8 atleast one head

```
1 # page number= 29
2 library(MASS)
3 prob_HH=1/4
4 prob_HT=1/4
5 prob_TH=1/4
6 probability=prob_HH+prob_HT+prob_TH
7 print(fractions(probability))
```

R code Exa 2.9 a number greater than 3

```
1 # page nummber= 30
2 library(MASS)
3 w= 1/9
4 s=c(1,2,1)
5 sum= w*sum(s)
6 print(fractions(sum))
```

R code Exa 2.10 a probability measure

```
1 # page number= 30
2 prob_0=1/2
3 sum=prob_0/(1-prob_0)
4 print(sum)
```

R code Exa 2.11 a full house

```
1 # page number 31
2 king_to_be_selected= 3
```

```
3 full_house_card=4
4 aces_to_be_selected=2
5 way_to_slct_king=choose(full_house_card,king_to_be_selected)
6 ways_to_selct_ace=choose(full_house_card,aces_to_be_selected)
7 total_cards_type1=13
8 n= total_cards_type1*(total_cards_type1-1)*way_to_slct_king*ways_to_selct_ace
9 total_ways_for5=choose(52,5)
10 prob= n/total_ways_for5
11 print(round(prob,4))
```

R code Exa 2.12 either or both kinds of sets

```
1 # page number =34
2 tv_set=0.86
3 hdtv_set=0.35
4 both_set=0.29
5 either_of_both=tv_set+hdtv_set-both_set
6 print(either_of_both)
```

R code Exa 2.13 faulty brakes

```
1 # page number = 34
2 faulty_breaks=0.23
3 badly_worn= 0.24
4 faulty_break_or_worn=0.38
5 both= faulty_breaks+badly_worn-faulty_break_or_worn
6 print(both)
```

R code Exa 2.14 dentist

R code Exa 2.15 services under warranty

```
1 # page number= 37
2 good_sevice_under_warranty= c(16,10)
3 poor_service_under_warranty=c(4,20)
4 df = (data.frame (good_sevice_under_warranty, poor_
     service_under_warranty))
5 rownames(df)=c("In business 10 years or more", "in
      business less than 10 years")
7 good_ser_provider= sum(df$good_sevice_under_warranty
     )
8 total_sum=0
9 for (i in colnames(df))
10 {
11 total_sum= total_sum+colSums(df[i])
12 }
13
14 prob_good_service= good_ser_provider/total_sum
15 print(prob_good_service[[1]])
```

R code Exa 2.16 good service under warranty

```
1 \# page number = 38
2 library(MASS)
3 good_sevice_under_warranty= c(16,10)
4 poor_service_under_warranty=c(4,20)
5 df = (data.frame (good_sevice_under_warranty, poor_
      service_under_warranty))
6 rownames(df)=c("In business 10 years or more", "in
      business less than 10 years")
7 total_sum=0
8 total_sum = sum(sapply(df, sum))
9 less_then_10_good_ser=df["in business less than 10
     years", 'good_sevice_under_warranty']
10 prob_less_then_10_good_ser=less_then_10_good_ser/
     total_sum
11 print(prob_less_then_10_good_ser[[1]])
12 less_then_10= rowSums(df["in business less than 10
      years",])
13 prob_less_then_10=less_then_10/total_sum
14 print(prob_less_then_10[[1]])
15
16 result= prob_less_then_10_good_ser/prob_less_then_10
17 print(fractions(result[[1]]))
```

R code Exa 2.17 perfect square given that it is greater than 3

```
# page number 39
library(MASS)
prob_odd= 2/9
prob_even= 1/9
prob_dice=c(2/9,1/9,2/9,1/9,2/9,1/9)
prob_B=prob_dice[1]+prob_dice[4]
prob_a_and_b= prob_dice[4]
prob_a=prob_dice[4]+prob_dice[5]+prob_dice[6]
prob_B_by_A=prob_a_and_b/prob_a
print(fractions(prob_B))
print(fractions(prob_B_by_A))
```

R code Exa 2.18 shipment

```
1 # page number 39
2 prob_R=0.8
3 prob_r_and_d=0.72
4 prob_d_by_r=prob_r_and_d/prob_R
5 print(prob_d_by_r)
```

R code Exa 2.19 both be defective

```
1 # page number= 40
2 library(MASS)
3 total_tv=240
4 no.defective.picked<- 2
5 total_defective=15</pre>
```

```
6 result= (total_defective/total_tv)*((total_defective
        -1)/(total_tv-1))
7 print(fractions(result))
```

R code Exa 2.20 replacement

R code Exa 2.21 3 fuses are defective

```
1 # page number = 41
2 library(MASS)
3 fuses= 20
4 defective=5
5 selected= 3
6 prob_a=defective/fuses
7 prob_b_by_a=(defective-1)/(fuses-1)
8 prob_c_by_a_and_b=(defective-2)/(fuses-2)
9 result= prob_a*prob_b_by_a*prob_c_by_a_and_b
10 print(fractions(result))
```

R code Exa 2.22 independent

```
1 # page number= 42
2 prob_a=1/4
3 prob_b= 1/2
4 prob_c=3/8
5 prob_a_and_b=1/8
6 prob_b_and_c= 1/4
7 if(prob_a*prob_b==prob_a_and_b){
8  print("event A and B are independent")
9 }
10 if(!(prob_b*prob_c==prob_b_and_c)){
11  print("event B and C are not independent")
12 }
```

R code Exa 2.23 independent

```
1 # page number= 44
2 prob_a=1/2
3 prob_b= 1/2
4 prob_c=1/2
5 \text{ prob_a_and_b=1/4}
6 \text{ prob_b_and_c} = 1/4
7 prob_a_and_c=1/4
8 \text{ prob}_a_b_c=1/4
9 if(prob_a*prob_b==prob_a_and_b)
10 {
     print("P(A)*P(B)=1/4=P(A B)")
11
12 }
13
14 if(prob_a*prob_c==prob_a_and_c){
     print("P(A)*P(C)=1/4=P(A C)")
15
```

```
16 }
17
18 if(prob_b*prob_c==prob_b_and_c){
19    print("P(B)*P(C)=1/4=P(B C)")
20 }
21
22 if(!(prob_a*prob_b*prob_c==prob_a_b_c)){
23    print("P(A)*P(B)*P(C)=1/8 P(A B C)")
24 }
```

R code Exa 2.24 heads and die

```
1 # page number = 45
2
3 library(MASS)
4 prob_of_head <- 1/2
5 tossed <- 3
6 dice_prob <- 1/6
7 result1 <- prob_of_head^tossed
8 result2 <- 5 * (dice_prob)^5
9 print(fractions(result1))
10 print(fractions(result2))</pre>
```

R code Exa 2.25 construction job

R code Exa 2.26 oil change

```
1 # page number = 46
2 prob_b1=0.6
3 prob_b2=0.3
4 prob_b3=0.1
5 prob_a_given_b1=0.09
6 prob_a_given_b2=0.2
7 prob_a_given_b3= 0.06
8 prob_a= prob_b1*prob_a_given_b1+prob_b2*prob_a_given_b2+prob_b3*prob_a_given_b3
9 print(prob_a)
```

R code Exa 2.27 rental agency 2

R code Exa 2.28 disease

```
1  # page number = 48
2  prod_d=0.01
3  prod_p_d=0.98
4  prod_p_dcom=0.03
5  prod_dcom_p= (1-prod_d)*prod_p_dcom/(prod_d*prod_d + (1-prod_d)*prod_p_dcom)
6  print(round(prod_dcom_p,3))
```

R code Exa 2.29 reliability of the system

```
1 # page number = 51
2 relibility_CDE= 1-(1-0.70)^3
3 relibility_FG= 1-(1-0.75)^2
4 relibility_H= 0.90
5 relibility_a=0.95
6 relibility_b=0.99
7 result= relibility_a*relibility_b*relibility_CDE*
    relibility_FG*relibility_H
8 print(round(result,3))
```

Chapter 3

Probability Distributions and Probability Densities

R code Exa 3.1 brown socks selected

```
1 # page number 63
2 library(MASS)
3 total_brown <- 5
4 total_green <- 3
6 possible_outcomes <- expand.grid(sock1 = c("B", "G")
7
                                     sock2 = c("B", "G")
9 possible_outcomes$brown <- rowSums(possible_outcomes
      == "B")
10
11 probabilities <- table(possible_outcomes$brown) /</pre>
     nrow(possible_outcomes)
12
13 prob <- function(n) {
     (choose(total_brown, n)*factorial(n) * choose(
       total_green, 2 - n)*factorial(2-n) )/ (choose(
```

```
total_brown + total_green, 2)*factorial(2))

15 }

16

17 print(paste("P(BB)=", fractions(prob(2))))
18 print(paste("P(BG)=", fractions(prob(1))))
19 print(paste("P(GB)=", fractions(prob(1))))
20 print(paste("P(GG)=", fractions(prob(0))))
```

R code Exa 3.2 number of heads

```
1 #page number= 63
2 library(MASS)
3 \text{ coin\_tossed} \leftarrow 4
4 possible_outcomes <- expand.grid(</pre>
     coin1 = c("H", "T"),
     coin2 = c("H", "T"),
6
     coin3 = c("H", "T"),
     coin4 = c("H", "T")
8
9)
10 possible_outcomes$combined <- paste(
11
     possible_outcomes$coin1,
12
     possible_outcomes$coin2,
13
     possible_outcomes$coin3,
14
     possible_outcomes$coin4,
     sep = ""
15
16 )
17 possible_outcomes$sum_H <- rowSums(possible_outcomes
       == "H")
18 possible_outcomes$probability <- (1 / 16)
19 possible_outcomes$fraction_probability <- as.
      character(MASS::fractions(possible_outcomes$
      probability))
20 colnames (possible_outcomes) [5] <- "element of Sample
       Space"
21 colnames (possible_outcomes) [8] <- "Probability"
```

```
22 colnames (possible_outcomes) [6] <- "x"
23 print (possible_outcomes [, c(5, 8, 6)])
```

R code Exa 3.3 number of heads

```
1 # page number = 66
2 library(MASS)
3 coin_tossed <- 4
4 for( i in 0:coin_tossed)
5 {
6    print(paste("P(X=",i,")=",fractions(choose(coin_tossed,i)/2^coin_tossed)))
7    cat("\n")
8 }</pre>
```

R code Exa 3.4 probability distribution

```
1 # page number = 66
2
3 x=c(1,2,3,4,5)
4 fun= function(n)
5 {
6   return ((n+2)/25)
7 }
8 result=fun(x[1])+fun(x[2])+fun(x[3])+fun(x[4])+fun(x[5])
9 print(result)
```

R code Exa 3.5 number of heads

```
1 # page number =69
2 library(MASS)
3 pdf=c(1/16,4/16,6/16,4/16,1/16)
4 sum=0
5 cdf=c()
6 for(i in pdf){
7  cdf=c(cdf,sum)
8  sum=sum+i
9
10 }
11 cdf=c(cdf,sum)
12 print(paste(fractions(cdf)))
```

R code Exa 3.6 distribution function

R code Exa 3.7 probability distribution

```
1 # page number = 71
2 library(MASS)
3 f=c(0,1/36,3/36,6/36,10/36,15/36,21/36,26/36,30/
        36,33/36,35/36,1)
4 f1=c(f[1])
5 for (i in 2:length(f))
6 {
7     sum=f[i]-f[i-1]
8     f1=c(f1,sum)
9 }
10
11 for( i in 2:(length(f1)))
12 {
13     print(paste("f(",i,")=",fractions(f1[i])))
14 }
```

R code Exa 3.9 Probability

R code Exa 3.10 distribution function

```
1 # page number = 78
2 f <- function(x) {
3   exp(-3*x)
4 }
5
6 result <- integrate(f, lower = 0, upper = Inf)
7
8 k = 1 / as.numeric(result$value)
9
10
11 print(k)
12
13 result1<- integrate(f,lower=0, upper= 1)
14 result2<- integrate(f,lower=0,upper=0.5)
15 result=k*(result1$value-result2$value)
print(round(result,3))</pre>
```

R code Exa 3.12 Two caplets are selected

```
10
11 for (i in 1:ncol(combinations)) {
     combination <- combinations[,i]</pre>
12
     X <- sum (combination == "Aspirin")
13
14
     Y <- sum (combination == "Sedative")
     count_table [X+1,Y+1] <-count_table [X+1,Y+1]+1</pre>
15
16 }
17
18 total_combinations <- ncol(combinations)
19 prob_table <- count_table / total_combinations
20 prob_table<- t(prob_table)</pre>
21
22 print(fractions(prob_table))
```

R code Exa 3.13 a joint probability distribution

```
1 # page number 83
2 library(MASS)
3 fun= function(x,y){
4   return (x*y)}
5 var <- expand.grid(1:3,1:3)
6 sum=sum(fun(var[1],var[2]))
7 k=fractions(1/sum)
8 print(k)</pre>
```

R code Exa 3.14 find F

```
1 # page number = 84
2 library(MASS)
3 f=c(1/6,2/9,1/3,1/6)
4 print(fractions(sum(f)))
```

R code Exa 3.15 Probability

R code Exa 3.17 Probability

```
1 # page number= 87
2
3 f <- function(x, y) {
4   return(exp(-x) * exp(-y))
5 }
6
7 x_lower <- 1
8 x_upper <- 3
9 y_lower <- 1
10 y_upper <- 2
11
12 P <- integrate(function(y) {
    sapply(y, function(y_val) {</pre>
```

R code Exa 3.18 Probability

```
1 # page nummber =89
2 f <- function(x, y, z) {
3   return((x + y) * z / 63)
4 }
5
6 P <- f(2, 1, 1) + f(2, 1, 2) + f(2, 2, 1)
7 print(fractions(P))</pre>
```

R code Exa 3.19 trivariate probability density

```
1 # page number 90
2 f <- function(x1, x2, x3) {
    return((x1 + x2) * exp(-x3))
4 }
5 result <- integrate(function(x3) {sapply(x3,</pre>
     function(x3){
     integrate(function(x2) {sapply(x2, function(x2){
6
7
       integrate(function(x1) {
         f(x1, x2, x3)
8
9
       \}, 0, 1/2)$value
10
     )), 1/2, 1)$value
11 })}, 0, 1)$value
12 cat(round(result,3))
```

R code Exa 3.23 conditional distribution

```
1 # page number = 95
2 f_0_1= 2/9
3 h_1=7/18
4 f_1_1=1/6
5 f_2_2=0
6
7 f0by1=f_0_1/h_1
8 f1by1=f_1_1/h_1
9 f2by2=f_2_2/h_1
10 print(fractions(f0by1))
11 print(fractions(f1by1))
12 print(fractions(f2by2))
```

R code Exa 3.24 conditional density

```
1 # page number = 96
2 fun_at_y= function(x){
3   return ((2*x+2)/3)
4 }
5
6 result= integrate(fun_at_y,0,1/2)$value
7 fractions(result)
```

R code Exa 3.27 r joint probability density

```
1 # page number = 99
2
```

```
3 fun = function(x1, x2, x3){
    return (6*exp(-x1-2*x2-3*x3))
5 }
6
    result <- integrate(function(x3) {sapply(x3,
        function(x3){
       integrate(function(x2) {sapply(x2,function(x2){
8
         integrate(function(x1) {
           fun(x1, x2, x3)
9
         \}, 0, 1-x2)$value
10
       })}, 0, 1)$value
11
     })}, 1, Inf)$value
12
13 print(round(result,3))
```

R code Exa 3.28 Construct a frequency distribution

```
1 \#page number = 103
2
3 data <- c(4890, 4830, 5490, 4820, 5230, 4860, 5040,
      5060, 4500, 5260,
             4610, 5100, 4730, 5250, 5540, 4910, 4430,
4
                4850, 5040, 5000,
5
             4600, 4630, 5330, 5160, 4950, 4480, 5310,
                4730, 4700, 4390,
6
             4710, 5160, 4970, 4710, 4430, 4260, 4890,
                5110, 5030, 4850,
             4820, 4550, 4970, 4740, 4840, 4910, 5200,
7
                4880, 5150, 4890,
8
             4900, 4990, 4570, 4790, 4480, 5060, 4340,
                4830, 4670, 4750)
10 class_intervals <- seq(4200, 5600, by = 200)
12 binned_data <- cut(data, breaks = class_intervals,</pre>
     right = FALSE)
13
```

```
14 frequency_table <- table(t(binned_data))
15 for (i in 1:length(frequency_table))
16 print(frequency_table[[i]])
17 print(sum(frequency_table))</pre>
```

R code Exa 3.29 frequency distribution

```
1 #page number 104
2 # Given data
3 data <- c(5771, 5839, 5840, 5864, 5880, 5890, 5892, 5902, 5908, 5912, 5914, 5918, 5924, 5926, 5928, 5932, 5933, 5934, 5936, 5938, 5942, 5944, 5946, 5948, 5950, 5952, 5954, 5956, 5958, 5960)
4
5 upper_limit= 4400
6 lower_limit= 4200
7 class_interval= upper_limit-lower_limit
8 print(class_interval)</pre>
```

R code Exa 3.30 a histogram

Chapter 4

Mathematical Expectation

R code Exa 4.1 expect

```
1 #page number= 114
2 library(MASS)
3 N <- 12
4 W <- 2
5 k <- 3
6 probs <- dhyper(0:2, W, N - W, k)
7 expected_value <- sum(0:2 * probs)
8 print(fractions(expected_value))</pre>
```

R code Exa 4.2 expected value

```
1 # page number= 115
2 fun <- function(x) {
3   return (4*x / (pi * (1 + x^2)))
4 }
5 result <- integrate(fun, 0, 1)
6 print(round(result$value,4))</pre>
```

R code Exa 4.3 expected value

```
1 # page number=116
2 library(MASS)
3 fun <- function(x) {return ((2 * x * x + 1) / 6)}
4 sum <- sum(sapply(1:6, fun))
5 print(fractions(sum))</pre>
```

R code Exa 4.4 expected value

```
1 # page number = 116
2 fun= function(x){
3   return (exp(3*x/4)*exp(-x))
4 }
5 result= integrate(fun,lower= 0,upper=100)
6 fractions(result$value)
```

R code Exa 4.5 expected value

```
1 # page number= 118
2 library(MASS)
3 e_x_square=91/6
4 sum=2*e_x_square+1
5 print(fractions(sum))
```

R code Exa 4.6 expected value

```
1 # page number = 118
2 library(MASS)
3 fun <- function(r) {return (2 / ((r + 2) * (r + 1)))
     }
4 sum <- sum(4 * fun(c(2, 1))) + 1
5 print(fractions(sum))</pre>
```

R code Exa 4.8 expected value

R code Exa 4.9 expected value

```
12
13 print(fractions(result))
```

R code Exa 4.10 calculate the variance

```
1 #page number =124
2 library(MASS)
3 prob_each_dice=1/6
4 x=c(1,2,3,4,5,6)
5 expect_x=prob_each_dice*(sum(x))
6 expect_x_square=prob_each_dice*sum(x^2)
7 var=expect_x_square-(expect_x)^2
8 fractions(var)
```

R code Exa 4.11 standard deviation

```
1 # page number= 124
2
3 expect_x=0.4413
4 fun <- function(x) {
5    return ((4*x^2 / (pi * (1 + x^2))))
6 }
7 result <- integrate(fun, 0, 1)
8
9 result=round(result$value,4)
10 var=result-expect_x^2
11 var=round(var,4)
12 sd=sqrt(var)
13 print(round(sd,4))</pre>
```

R code Exa 4.12 Chebyshev theorem

```
1 # page number 127
2 library(stats)
3 library(MASS)
5 f \leftarrow function(x) {
    ifelse(x > 0 & x < 1, 630 * x^4 * (1 - x^4, 0)
8 mean_val \leftarrow integrate(function(x) x * f(x), 0, 1)$
      value
9 var_val <- integrate(function(x) (x - mean_val)^2 *
      f(x), 0, 1) $value
10 sd_val <- sqrt(var_val)</pre>
11 lower_bound <- mean_val - 2 * sd_val
12 upper_bound <- mean_val + 2 * sd_val
13 prob_within_interval <- integrate(f, lower_bound,
      upper_bound)$value
14 chebyshev_bound \leftarrow 1 - 1 / (2^2)
15 sprintf(paste(round(prob_within_interval,2)))
```

R code Exa 4.14 moment generating function

```
1 # page number 129
2 library(MASS)
3 fun= function(x){
4   return ((3*(1+exp(x))^2*exp(x))/8)
5 }
6 fun1= function(x){
7   return ((3*(1+exp(x))*exp(2*x))/4)
8 }
9 moment1=fractions(fun(0))
10 moment2=fractions(fun1(0))+fractions(fun(0))
11 print(moment1)
12 print(moment2)
```

R code Exa 4.15 covariance of X and Y

```
1 # page number= 133
2 library(MASS)
3 p=matrix(c(1/6,1/3,1/12,2/9,1/6,0,1/36,0,0),nrow=3,
      ncol = 3, byrow = TRUE)
4 fractions(colSums(p))
5 mu1_1=0
6 \text{ x\_values=0:2}
7 \text{ y_values=0:2}
8 for (x in x_values) {
     for (y in y_values) {
10
        mu1_1 \leftarrow mu1_1 + x * y * p[x+1, y+1]
11
     }
12 }
13 print(fractions(mu1_1))
14 mu_x=fractions(sum(c(0:2)*colSums(p)))
15 mu_y=fractions(sum(c(0:2)*rowSums(p)))
16 print(mu_x)
17 print(mu_y)
18 \text{ sigma}_xy=\text{mu1}_1-\text{mu}_x*\text{mu}_y
19 print(sigma_xy)
```

R code Exa 4.16 covariance

```
sapply(x, function(x_val) {
6
         adaptIntegrate(function(y) {
           inner_fun(x_val, y)
8
         }, lowerLimit = lower_limit(y), upperLimit =
9
            upper_limit(x_val))$integral
10
11
     }, lowerLimit = lower_limit, upperLimit = upper_
        limit) $ integral
12 }
13 fun1 \leftarrow function(x, y) { 2 * y }
14 fun2 \leftarrow function(x, y) { 2 * x }
15 fun3 <- function(x, y) { 2 * x * y }
16 lower_limit <- function(x) 0</pre>
17 upper_limit <- function(x) 1 - x
18 result1 <- nested_integrate(fun1, fun1, 0, 1)
19 result2 <- nested_integrate(fun2, fun2, 0, 1)</pre>
20 result3 <- nested_integrate(fun3, fun3, 0, 1)
21 print(fractions(result1))
22 print(fractions(result2))
23 print(fractions(result3))
24 final <- result3 - result1 * result2
25 print(fractions(final))
```

R code Exa 4.17 covariance

```
1 # page number = 135
2 joint_dist <- matrix(c(1/6, 1/3, 1/6, 0, 0, 0, 1/6, 0, 1/6), nrow = 3, ncol = 3, byrow = TRUE)
3 sum <- sum(sapply(c(-1, 0, 1), function(i) {sum(sapply(c(-1, 0, 1), function(j) {(i * j) * joint_dist[i + 2, j + 2]}))}))
4 print(sum)</pre>
```

R code Exa 4.18 mean and the variance

```
1 # page number= 137
2 mu <- c(2, -3, 4)
3 var <- c(1, 5, 2)
4 cov <- c(-2, -1, 1)
5 expect <- sum(c(3, -1, 2) * mu)
6 variance <- sum(c(9, 1, 4) * var) + sum(c(-6, 12, -4) * cov)
7 print(expect)
8 print(variance)</pre>
```

R code Exa 4.19 covariance

```
1 # page number = 138
2 mu <- c(3, 5, 2)
3 var <- c(8, 12, 18)
4 cov <- c(1, -3, 2)
5 covariance <- sum(c(3, -4, -2) * var) + sum(c(11, 5, -6) * cov)
6 print(covariance)</pre>
```

R code Exa 4.20 conditional mean and the conditional variance

```
1 # page number = 139
2 cond_x=function(x){
3    return ((2/3)*x*(x+1))
4 }
5
6 cond_x_sq=function(x){
7    return ((2/3)*x*x*(x+1))
8 }
9
```

```
10 expect_x=integrate(cond_x,0,1)$value
11 expect_x_sq=integrate(cond_x_sq,0,1)$value
12 var= expect_x_sq-expect_x^2
13 print(fractions(var))
```

R code Exa 4.21 mean length and its standard deviation

Chapter 5

Special Probability Distributions

R code Exa 5.1 binomial distribution

```
1 # page number= 147
2 x=5
3 n=12
4 thita=1/2
5 result=dbinom(x, size=n, prob=thita)
6 print(round(result,2))
```

R code Exa 5.2 binomial distribution

```
1 # page number = 147
2 x=7
3 n=10
4 thita=0.8
5 result=dbinom(x, size=n, prob=thita)
6 print(round(result,2))
```

R code Exa 5.3 negative binomial distribution

```
1 # page number = 154
2 size <- 3
3 prob <- 0.40
4 failures <- 7
5 probability <- dnbinom(failures, size = size, prob = prob)
6 print(round(probability,4))</pre>
```

R code Exa 5.4 negative binomial distribution

```
1 # page number = 154
2 x <- 10
3 k <- 3
4 theta <- 0.40
5 binom_prob <- dbinom(k, size = x, prob = theta)
6 b_star <- (k / x) * binom_prob
7 print(round(b_star,4))</pre>
```

R code Exa 5.5 geometric distribution

```
1 #page number = 155
2 p <- 0.75
3 k <- 4
4 probability <- dgeom(k - 1, prob = p)
5 print(round(probability,4))</pre>
```

R code Exa 5.6 hypergeometric distribution

```
1 # page number = 156
2 x <- 0
3 m <- 4
4 n <- 20
5 k <- 6
6 probability <- dhyper(x, m, n, k)
7 print(round(probability, 4))</pre>
```

R code Exa 5.7 hyper and binomial

```
1 # page number = 158
2 x_hyper <- 2
3 m_hyper <- 80
4 n_hyper <- 40
5 k_hyper <- 5
6 probability_hyper <- dhyper(x_hyper, m_hyper, n_hyper, k_hyper)
7 print(round(probability_hyper,3))
8 x_binom <- 2
9 n_binom <- 5
10 theta_binom <- 2/3
11 probability_binom <- dbinom(x_binom, size = n_binom, prob = theta_binom)
12 print(round(probability_binom,3))</pre>
```

R code Exa 5.8 Poisson distribution

```
1 # page number = 160
2 n <- 150
3 theta <- 0.05
4 lambda <- n * theta</pre>
```

```
5 x_values <- 5:15
6 poisson_probs <- dpois(x_values, lambda)
7 binomial_probs <- dbinom(x_values, size = n, prob = theta)
8 errors <- poisson_probs - binomial_probs
9 max_error_index <- which.max(abs(errors))
10 x_max_error <- x_values[max_error_index]
11 max_error <- errors[max_error_index]
12 cat(round(max_error,4))
13 cat(x_max_error)</pre>
```

R code Exa 5.9 Poisson distribution

```
1 # page number = 160
2
3 n <- 400
4 theta <- 0.02
5 lambda <- n * theta
6 x_values <- 5
7 poisson_probs <- dpois(x_values, lambda)
8 print(round(poisson_probs,3))
9 #- The answer may slightly vary due to rounding off values.</pre>
```

R code Exa 5.10 binomial probabilities

```
1 # page number = 160
2 n <- 10000
3 theta <- 0.00005
4 lambda <- n * theta
5 x_a <- 2
6 prob_a <- dpois(x_a, lambda)
7 cat(round(prob_a,4))</pre>
```

```
8 x_b <- c(0:2)
9 prob_b <- dpois(x_b, lambda)
10 print(sum(round( prob_b,4)))</pre>
```

R code Exa 5.11 Poisson distribution

R code Exa 5.12 Poisson distribution

```
1 # page number = 163
2 lambda <- 12
3 prob_less_than_9 <- ppois(8, lambda)
4 print(round(prob_less_than_9,4))</pre>
```

R code Exa 5.13 a Poisson distribution

```
1 # page number = 164
2 lambda <- 7.5
3 x <- 5
4 prob_at_most_5 <- ppois(x, lambda)
5 prob_at_least_6 <- 1 - prob_at_most_5
6 cat(round(prob_at_least_6,4))</pre>
```

R code Exa 5.14 MULTINOMIAL DISTRIBUTION

R code Exa 5.15 MULTINOMIAL DISTRIBUTION

```
1 # page number = 168
2 N1 <- c(6,3,7,4)
3 N <- sum(N1)
4 n <- 12
5 k <- c(4, 1, 5, 2)
6 result <- prod(choose(N1, k)) / choose(N, n)
7 print(round(result, 4))</pre>
```

R code Exa 5.16 producer and consumer risks

```
1 # page number = 171
2 p_AQL <- 0.05
3 p_LTPD <- 0.20
4 L_AQL <- 0.7358
5 L_LTPD <- 0.0692
6 producer_risk <- 1 - L_AQL
7 consumer_risk <- L_LTPD
8 cat(producer_risk)
9 cat(consumer_risk)
10</pre>
```

Chapter 6

Special Probability Densities

R code Exa 6.1 a Poisson distribution

R code Exa 6.2 standard normal distribution

```
1 #page number = 188
2 prob_a <- pnorm(1.72)
3 prob_b <- pnorm(-0.88)
4 prob_c <- pnorm(1.75) - pnorm(1.30)
5 prob_d <- pnorm(0.45) - pnorm(-0.25)</pre>
```

```
6 cat("(a) P(Z < 1.72) =", round(prob_a,4), "\n") 
7 cat("(b) P(Z < -0.88) =", round(prob_b,4), "\n") 
8 cat("(c) P(1.30 < Z < 1.75) =", round(prob_c,4), "\n ") 
9 cat("(d) P(-0.25 < Z < 0.45) =", round(prob_d,4), "\n")
```

R code Exa 6.4 cosmic radiation

R code Exa 6.5 chisquare distribution

```
1 # page number = 191
2 df <- 25
3 threshold <- 30
4 prob_chi_square <- 1 - pchisq(threshold, df)
5 cat(round(prob_chi_square,4))
6
7 mean_normal <- 18.7
8 sd_normal <- 9.1
9 lower <- 10.6
10 upper <- 24.8</pre>
```

R code Exa 6.6 binomial distribution

```
1 # page number = 193
2 n <- 16
3 p <- 0.5
4 k <- 6
5 mean_binomial <- n * p
6 sd_binomial <- sqrt(n * p * (1 - p))
7 z_lower <- (k-(1/2) - mean_binomial) / sd_binomial
8 z_upper <- ((k + (1/2)) - mean_binomial) / sd_binomial
9 prob_normal_approx <- pnorm(z_upper) - pnorm(z_lower_)
10 cat(round(prob_normal_approx,4))</pre>
```

R code Exa 6.7 normal scores

```
1 # page number = 199
2
3 observations <- c(3, 2, 7, 4, 3, 5)
4 ordered_observations <- sort(observations)
5 n <- length(ordered_observations)
6 normal_scores <- round(qnorm((1:n) / (n + 1)),2)
7 data <- data.frame(Ordered_Observations = ordered_observations, Normal_Scores = normal_scores)</pre>
```

```
8 print(data)
9 #- The answer may slightly vary due to rounding off
    values.
```

R code Exa 6.8 normality

```
1 \# page number = 200
2 original_data <- c(54.9, 8.3, 5.2, 32.4, 15.5)
3 print(original_data)
4 normal_scores \leftarrow c(-0.95, -0.44, 0, 0.44, 0.95)
5 ordered_original_data <- sort(original_data)</pre>
6 plot(normal_scores, ordered_original_data, xlab = "
     Normal Scores", ylab = "Ordered Observations",
     main = "Normal-Scores Plot for Original Data")
7 abline(lm(ordered_original_data ~ normal_scores),
     col = "red")
8 shapiro_test_original <- shapiro.test(original_data)</pre>
9 shapiro_test_original
10 transformed_data <- log10(original_data)</pre>
11 ordered_transformed_data <- sort(transformed_data)</pre>
12 plot(normal_scores, ordered_transformed_data, xlab =
      "Normal Scores", ylab = "Ordered Log-Transformed
       Observations", main = "Normal-Scores Plot for
      Transformed Data")
13 abline(lm(ordered_transformed_data ~ normal_scores),
       col = "red")
14 shapiro_test_transformed <- shapiro.test(transformed
     data)
15 shapiro_test_transformed
```

Chapter 7

Functions of Random Variables

R code Exa 7.4 probability distribution

```
1 \# page number = 211
2 library(MASS)
3 n <- 4
4 p <- 0.5
5 \text{ x\_values} \leftarrow 0:n
6 x_probabilities <- dbinom(x_values, size = n, prob =
7 x_probabilities=fractions(x_probabilities)
8 \text{ y\_values} \leftarrow 1 / (1 + \text{x\_values})
9 probability_distribution <- data.frame(
     X = fractions(x_values),
     P_X = fractions(x_probabilities),
11
12
     Y = y_values
13 )
14 print((probability_distribution))
15 #— The answer may vary due to difference in
      representation
```

R code Exa 7.5 probability distribution

```
1 \# page number = 212
2 n <- 4
3 p <- 0.5
4 x_values <- 0:n
5 x_probabilities <- dbinom(x_values, size = n, prob =
       p)
6 \text{ z_values} \leftarrow (\text{x_values} - 2)^2
7 z_{probabilities} \leftarrow rep(0, max(z_{values}) + 1)
8 for (i in seq_along(z_values)) {
     z_probabilities[z_values[i] + 1] <- z_</pre>
        probabilities[z_values[i] + 1] + x_
        probabilities[i]
10 }
11 probability_distribution_Z <- data.frame(</pre>
12
     Z = seq_along(z_probabilities) - 1,
     P_Z = z_probabilities
13
14 )
15 print(probability_distribution_Z)
```

R code Exa 7.11 probability density

```
1 # page number = 219
2 fun = function(u){
3   return ((u)*exp(-u))
4 }
5 result=integrate(fun,0,Inf)$value
6 print(result)
```

R code Exa 7.12 marginal density

```
1 # page number = 221
2 fun = function(y1){
3 return ((y1)*exp(-y1))
```

```
4 }
5
6 result=integrate(fun,0,Inf)$value
7 print(result)
```

R code Exa 7.19 probability

```
1 # page number = 230
2 lambda <- 5
3 n <- 2
4 gamma_cdf <- function(x, alpha, beta) {
5    pgamma(x, shape = alpha, rate = beta)
6 }
7 prob_T_leq_3 <- gamma_cdf(3, alpha = n, beta = 1/lambda)
8
9
10 cat("P(T <= 3) =", prob_T_leq_3, "\n")
11 #- The answer may vary due to difference in representation</pre>
```

Chapter 8

Sampling Distributions

R code Exa 8.1 central limit theorem

```
#page number = 238
mean_dispensed <- 200
sd_dispensed <- 15
sample_size <- 36
sample_mean_threshold <- 204
sd_sample_mean <- sd_dispensed / sqrt(sample_size)
z_score <- (sample_mean_threshold - mean_dispensed)
    / sd_sample_mean
cumulative_prob <- pnorm(z_score)
prob_at_least_threshold <- 1 - cumulative_prob
cat(round(prob_at_least_threshold,4))</pre>
```

R code Exa 8.2 Chisquare distribution

```
1 # page number = 245
2 n <- 20
3 sigma <- 0.60
4 s <- 0.84
```

```
5 alpha <- 0.01
6 test_statistic <- (n - 1) * (s^2) / (sigma^2)
7 cat(test_statistic)
8 critical_value <- qchisq(1 - alpha, df = n - 1)
9 cat(round(critical_value,3))
10 if (test_statistic > critical_value) {
11   cat("The process is declared out of control.\n")
12 } else {
13   cat("The process is in control.\n")
14 }
```

R code Exa 8.3 t distribution

```
1 \# page number = 248
2 n <- 16
3 mu_hypothesis <- 12.0
4 x_bar <- 16.4
5 s <- 2.1
6 t_statistic <- (x_bar - mu_hypothesis) / (s / sqrt(n
      ))
7 \text{ df} < - n - 1
8 p_value <- 1 - pt(t_statistic, df)</pre>
9 cat(round(t_statistic,2))
10 cat(df)
11 cat(p_value)
12 alpha <- 0.05
13 if (p_value < alpha) {
14 cat ("Reject the null hypothesis\n")
15 } else {
     cat ("Fail to reject the null hypothesis: There is
        not enough evidence to conclude that the true
        average hourly gasoline consumption exceeds
        12.0 \text{ gallons.} \ n")
17 }
```

Chapter 9

Decision Theory

R code Exa 9.1 deciding

```
1 \# page number = 261
2 prob_recession <- 2 / 3
3 prob_good <- 1 / 3
6 profit_expand_good <- 164000
7 loss_expand_recession <- -40000
8 profit_wait_good <- 80000
9 profit_wait_recession <- 8000
10 expected_profit_expand <- (profit_expand_good * prob</pre>
     _good) + (loss_expand_recession * prob_recession)
11 expected_profit_wait <- (profit_wait_good * prob_</pre>
     good) + (profit_wait_recession * prob_recession)
12 cat(expected_profit_expand)
13 cat(expected_profit_wait)
14 if (expected_profit_expand > expected_profit_wait) {
    cat("Decision: Expand now\n")
16 } else {
17 cat ("Decision: Wait at least another year\n")
18 }
```

R code Exa 9.2 decide

```
1 \# page number = 262
2 profit_expand_good <- 164000
3 loss_expand_recession <- -40000
4 profit_wait_good <- 80000
5 profit_wait_recession <- 8000
7 min_profit_expand <- loss_expand_recession</pre>
8 min_profit_wait <- profit_wait_recession</pre>
10 cat(min_profit_expand, "\n")
11 cat(min_profit_wait, "\n")
12
13 if (min_profit_expand > min_profit_wait) {
     cat ("Decision: Expand now\n")
14
15 } else {
16 \operatorname{cat}("\operatorname{Decision}: \operatorname{Wait} \operatorname{another} \operatorname{year} \ ")
17 }
```

R code Exa 9.3 PAYOFF MATRIX

R code Exa 9.4 optimum strategies

```
1 \# page number = 264
2 payoff_matrix \leftarrow matrix(c(-4, 1, 7, 4, 3, 5), nrow =
       2, byrow = TRUE)
3 rownames(payoff_matrix) <- c("A_I", "A_II")</pre>
4 colnames(payoff_matrix) <- c("B_1", "B_2", "B_3")
5 reduced_matrix <- payoff_matrix[, c("B_1", "B_2")]
6 print ("Reduced matrix after removing dominated
      strategies for Player A:")
7 print(reduced_matrix)
8 final_matrix <- reduced_matrix[, "B_2", drop = FALSE
9 print ("Final matrix after removing dominated
      strategies for Player B:")
10 print(final_matrix)
11 optimal_strategy_A <- rownames(final_matrix)[which.
     min(final_matrix)]
12 value_of_game <- min(final_matrix)</pre>
13 cat ("Player A's optimal strategy:", optimal_strategy
      _A, "\n")
14 cat ("Player B's optimal strategy: B_2\n")
15 cat("Value of the game:", value_of_game, "\n")
```

R code Exa 9.5 minimax strategies

```
1 payoff_matrix <- matrix(c(8, -5, 2, 6), nrow = 2,
      byrow = TRUE)
2 rownames(payoff_matrix) <- c("A_I", "A_II")</pre>
3 colnames(payoff_matrix) <- c("B_1", "B_2")
5 max_losses_A <- apply(payoff_matrix, 1, max)</pre>
6 minimax_strategy_A <- which.min(max_losses_A)</pre>
8 max_gains_B <- apply(payoff_matrix, 2, max)</pre>
9 minimax_strategy_B <- which.min(max_gains_B)</pre>
10
11 cat("Player A's minimax strategy:", rownames(payoff_
      \label{eq:matrix} \verb|matrix|| [\verb|minimax_strategy_A|], "\n"|
12 cat("Player B's minimax strategy:", colnames(payoff_
      matrix) [minimax_strategy_B], "\n")
13
14 if (minimax_strategy_A == 2 && minimax_strategy_B ==
       2) {
     cat ("Player A switches to Strategy I, reducing
15
        loss to 2.\n")
16
     cat ("Player B switches to Strategy 1, increasing
        gain to 8.\n")
17 }
```

Point Estimation

R code Exa 10.19 binomial trials

```
1 # page number = 308
2 x <- 42
3 n <- 120
4 alpha_prior <- 40
5 beta_prior <- 40
6 alpha_posterior <- alpha_prior + x
7 beta_posterior <- beta_prior + (n - x)
8 posterior_mean <- alpha_posterior / (alpha_posterior + beta_posterior)
9 cat(posterior_mean)
10 thita= x/n
11 print(thita)</pre>
```

R code Exa 10.20 soft drink vending machines

```
1 # page number = 309
2 mu1 <- 715
3 sigma1 <- 9.5
```

```
4 lower_bound <- 700
5 upper_bound <- 720
6 z_low <- (lower_bound - mu1) / sigma1
7 z_up <- (upper_bound - mu1) / sigma1
8 p_low <- pnorm(z_low)
9 p_up <- pnorm(z_up)
10 probability <- p_up - p_low
11 cat(mu1)
12 cat(sigma1)
13 cat(round(z_low,2))
14 cat(round(z_up,2))
15 cat(round(probability,3))</pre>
```

R code Exa 10.21 voters

```
1 # page number = 311
2 z_alpa_by_2=1.96
3 E=0.03
4
5 n= z_alpa_by_2^2/(4*E^2)
6 print(ceiling(n))
```

Interval Estimation

R code Exa 11.1 efficiency experts

```
1 # page number= 318
2
3 n <- 150
4 sigma <- 6.2
5 z_value <- 2.575
6
7 max_error <- z_value * sigma / sqrt(n)
8
9 cat(round(max_error,2))</pre>
```

R code Exa 11.2 confidence interval

```
1 # page number= 319
2 n <- 20
3 x_bar <- 64.3
4 sigma <- 15
5 z_value <- 1.96
6 margin_of_error <- z_value * sigma / sqrt(n)</pre>
```

```
7
8 lower_bound <- x_bar - margin_of_error
9 upper_bound <- x_bar + margin_of_error
10
11 cat(round(lower_bound,1), "< <",round(upper_bound,1))</pre>
```

R code Exa 11.3 confidence interval

R code Exa 11.4 paint manufacturer

```
1 # page number = 321
2 x_bar <- 66.3
3 s <- 8.4
4 n <- 12
5 t_value <- 2.201
6 margin_of_error <- t_value * s / sqrt(n)
7 lower_bound <- x_bar - margin_of_error
8 upper_bound <- x_bar + margin_of_error</pre>
```

```
9 cat(round(lower_bound,1))
10 cat(round(upper_bound,1))
```

R code Exa 11.5 confidence interval

```
1 # page number = 322
2 x1_bar <- 418
3 x2_bar <- 402
4 n1 <- 40
5 n2 <- 50
6 sigma1 <- 26
7 sigma2 <- 22
8 z_value <- 1.88
9 se_diff <- sqrt((sigma1^2 / n1) + (sigma2^2 / n2))
10 margin_of_error <- z_value * se_diff
11 difference_means <- x1_bar - x2_bar
12 lower_bound <- difference_means - margin_of_error
13 upper_bound <- difference_means + margin_of_error
14 cat(round(lower_bound,1))
15 cat(round(upper_bound,1))</pre>
```

R code Exa 11.6 two brands of cigarettes

```
1 # page number= 324
2 n1 <- 10
3 n2 <- 8
4 x1_bar <- 3.1
5 x2_bar <- 2.7
6 s1 <- 0.5
7 s2 <- 0.7
8 t_value <- 2.120
9
10 se_diff <- sqrt(s1^2 / n1 + s2^2 / n2)</pre>
```

R code Exa 11.7 a flu vaccine experienced

```
1 # page number = 326
2 n <- 400
3 x <- 136
4 confidence_level <- 0.95
5
6 binom_result <- binom.test(x, n, conf.level = confidence_level)
7
8 cat( round(binom_result$conf.int[1], 2), "\n")
9 cat(round(binom_result$conf.int[2], 2))</pre>
```

R code Exa 11.8 voters

```
1 # page number = 327
2 n <- 400
3 x <- 140
4 confidence_level <- 0.99
5 binom_result <- binom.test(x, n, conf.level = confidence_level)
6 cat(round(binom_result$conf.int[2] - p_hat, 2))</pre>
```

R code Exa 11.9 running for governor of Illinois

R code Exa 11.10 gasoline consumption

```
1 # page number = 330
2 n <- 16
3 s <- 2.2
4 chi_sq_low <- 4.601
5 chi_sq_high <- 32.801
6 lower_bound <- (n - 1) * (s^2) / chi_sq_high
7 upper_bound <- (n - 1) * (s^2) / chi_sq_low
8 cat(round(lower_bound,2), "< 2 < ", round(upper_bound,2))</pre>
```

R code Exa 11.11 confidence interval

```
1 # page number = 331
2 n1 <- 10
3 n2 <- 8
4 s1 <- 0.5
```

```
5 s2 <- 0.7
6 f0.01_9_7 <- 6.72
7 f0.01_7_9 <- 5.61
8 lower_bound <- (s1^2 / s2^2) / f0.01_9_7
9 upper_bound <- (s1^2 / s2^2) * f0.01_7_9
10 cat(round(lower_bound,3), " < 2^2/ 2^2< ", round(upper_bound,3))</pre>
```

R code Exa 11.12 car crossings

Hypothesis Testing

R code Exa 12.1 new medication

```
1 # page number = 339
2 n <- 20
3 theta_0 <- 0.90
4 theta_1 <- 0.60
5 critical_value <- 14
6 alpha <- pbinom(critical_value, n, theta_0, lower.
        tail = TRUE)
7 beta <- pbinom(critical_value, n, theta_1, lower.
        tail = FALSE)
8 cat(round(alpha,4))
9 cat(round(beta,4))
10 #- The answer may slightly vary due to rounding off values.</pre>
```

R code Exa 12.7 likelihood ratio technique

```
1 # page number= 353
2 library(MASS)
```

```
3 f_x \leftarrow c(1/12, 1/12, 1/12, 1/4, 1/6, 1/6, 1/6)
4 \text{ g_x} \leftarrow c(1/3, 1/3, 1/3, 2/3, 0, 0, 0)
5 x_values <- 1:7
6 lambda \leftarrow ifelse(g_x != 0, f_x / g_x, 1)
7 cat("Lambda values for each x: n")
8 data.frame(x = x_values, lambda = lambda)
9 critical_region_lrt <- x_values[lambda == 1/4]
10 alpha_lrt <- sum(f_x[critical_region_lrt])</pre>
11 beta_lrt <- sum(g_x[c(4,5,6)])
12 print(fractions(beta_lrt))
13 critical_region_alt <- c(4)</pre>
14 cat(critical_region_alt)
15 alpha_alt \leftarrow sum(f_x[c(4)])
16 print(fractions(alpha_alt))
17 beta_alt <- sum(g_x[c(1,2,3)])
18 if(beta_lrt>alpha_lrt){
19 cat ("null hypothesis is rejected")
20 }
```

Tests of Hypothesis Involving Means Variances and Proportions

R code Exa 13.1 packages of cookies

```
1 \# page number = 364
2 x_bar <- 8.091
3 mu_0 <- 8
4 sigma <- 0.16
5 n <- 25
6 alpha <- 0.01
7 z <- (x_bar - mu_0) / (sigma / sqrt(n))
8 z_critical_lower <- qnorm(alpha / 2)</pre>
9 z_critical_upper <- qnorm(1 - alpha / 2)
10 \operatorname{cat}(\operatorname{round}(z,2))
11 cat(round(z_critical_lower,3))
12 cat(round(z_critical_upper,3))
13 if (z < z_critical_lower || z > z_critical_upper) {
     cat(" the null hypothesis must be rejected \n")
15 } else {
     cat ("Fail to reject the null hypothesis\n")
17 }
```

R code Exa 13.2 high performance tires

```
1 \# page number = 365
2 x_bar <- 21819
3 mu_0 <- 22000
4 sigma <- 1295
5 n <- 100
6 alpha <- 0.05
7 z <- (x_bar - mu_0) / (sigma / sqrt(n))
8 z_critical <- qnorm(alpha)</pre>
9 cat("Test Statistic (z):", round(z,1), "\n")
10 cat("Critical Value:", round(z_critical,3), "\n")
11 if (z < z_{critical}) {
12
     cat ("Reject the null hypothesis.")
13 } else {
    cat ("Fail to reject the null hypothesis.")
14
15 }
```

R code Exa 13.3 ribbon call

```
1 # page number= 365
2 sample_data <- c(171.6, 191.8, 178.3, 184.9, 189.1)
         # sample breaking strengths
3 mu_0 <- 185
4 alpha <- 0.05
5 sample_mean <- mean(sample_data)
6 sample_sd <- sd(sample_data)
7 n <- length(sample_data)
8 t_statistic <- (sample_mean - mu_0) / (sample_sd / sqrt(n))
9 critical_value <- qt(alpha, df = n - 1)</pre>
```

```
10 p_value <- pt(t_statistic, df = n - 1)
11 cat(round(t_statistic,2))
12 if (t_statistic < critical_value) {
13   cat("Reject the null hypothesis\n")
14 } else {
15   cat("the null hypothesis cannot be rejected\n")
16 }</pre>
```

R code Exa 13.4 nicotine

```
1 \# page number = 367
2 \times (-c(2.61, 2.38))
3 \text{ s} \leftarrow c(0.12, 0.14)
4 \text{ n} < -c(50,40)
5 delta <- 0.20
6 z \leftarrow (x[1] - x[2] - delta) / sqrt((s[1]^2 / n[1]) +
       (s[2]^2 / n[2])
7 p_value \leftarrow 1 - p_value(z)
8 \operatorname{cat}(\operatorname{round}(z,2))
9 alpha <- 0.05
10 if (p_value < alpha) {
     cat("Reject the null hypothesis.\n")
12 } else {
13
      cat("Fail to reject the null hypothesis.\n")
14 }
```

R code Exa 13.5 consumer testing service

```
1 # page number = 368

2 x <- c(546,492)

3 s <- c(31,26)

4 n <- c(4,4)
```

R code Exa 13.6 a semiconductor

```
# page number = 370
2 s2 <- 0.68
3 sigma0 <- 0.36
4 n <- 18
5 chi_square <- (n - 1) * s2 / sigma0
6 print(round(chi_square,2))
7 critical_value <- qchisq(0.05, df = n - 1)
8 if (chi_square > critical_value) {
9 "Reject null hypothesis"
10 } else {
11 "Fail to reject null hypothesis"
12 }
```

R code Exa 13.7 structural steel

```
1 # page numebr= 371
2 s_squared <- c(19.2,3.5)
3 alpha <- 0.02
4 F_statistic <- s_squared[1] / s_squared[2]</pre>
```

```
5 critical_value <- qf(1 - alpha, df1 = 12, df2 = 15)
6 cat(round(F_statistic, 2))
7 if (F_statistic > critical_value) {
8  "Reject null hypothesis"
9 } else {
10  "Fail to Reject null hypothesis"
11 }
```

R code Exa 13.8 patients suffering

R code Exa 13.9 oil company claims

```
1 # page number = 374
2 x <- 22
3 n <- 200
4 theta0 <- 0.20
5 alpha <- 0.01
6 theta_hat <- x / n</pre>
```

R code Exa 13.10 shoppers favoring detergent

```
1 # page number= 376
2 observed <- matrix(c(232, 168, 260, 240, 197, 203),
     nrow = 3, byrow = TRUE,
                       dimnames = list(c("Los Angeles",
3
                          "San Diego", "Fresno"),
                                        c ("Detergent A",
4
                                           "Detergent B")
                                           ))
5 chi_square_test <- chisq.test(observed)</pre>
6 cat(round(chi_square_test$statistic, 2), "\n")
7 cat(round(qchisq(0.95, chi_square_test$parameter),
     3), "\n")
8
9 if (chi_square_test$p.value < 0.05) {</pre>
     cat("Reject the null hypothesis\n")
10
11 } else {
12
     cat ("Fail to reject the null hypothesis\n")
13 }
14 #- The answer may slightly vary due to rounding off
      values.
```

R code Exa 13.11 persons ability

```
1 # page numeber= 380
2 observed <- matrix(c(63, 42, 15, 58, 61, 31, 14, 47,
      29), nrow = 3, byrow = TRUE,
3
                      dimnames = list(c("Low", "Average
                         ", "High"),
                                       c("Low", "Average
4
                                          ", "High")))
5 chi_square <- chisq.test(observed)$statistic
6 df <- chisq.test(observed) $parameter
7 critical_value <- qchisq(0.99, df)
8 cat(round(chi_square,2))
9 if (chi_square > critical_value) {
    cat ("Reject the null hypothesis")
10
11 } else {
    cat ("Fail to reject the null hypothesis")
12
13 }
```

R code Exa 13.12 a Poisson distribution

R code Exa 13.13 measurements of the heat producing capacity

```
1 # page number = 382
2 mine1 <- c(8400, 8230, 8380, 7860, 7930)
3 mine2 <- c(7510, 7690, 7720, 8070, 7660)
4 t_test_result <- t.test(mine1, mine2, var.equal = TRUE)
5 print(t_test_result)</pre>
```

Regression and Correlation

R code Exa 14.4 equation of the least squares line

```
1 # page number = 400
2 x <- c(4, 9, 10, 14, 4, 7, 12, 22, 1, 17)
3 y <- c(31, 58, 65, 73, 37, 44, 60, 91, 21, 84)
4 model <- lm(y ~ x)
5 alpha_hat <- coef(model)[1]
6 beta_hat <- coef(model)[2]
7 cat("y =", round(alpha_hat, 2), "+", round(beta_hat, 3), "x\n")
8 x_new <- 14
9 y_hat <- predict(model, newdata = data.frame(x = x_new))
10 cat(floor(y_hat))</pre>
```

R code Exa 14.5 amount of time that 10 persons studied

```
1 # page number = 404
2
3 x <- c(4, 9, 10, 14, 4, 7, 12, 22, 1, 17)</pre>
```

```
4 \text{ y} \leftarrow c(31, 58, 65, 73, 37, 44, 60, 91, 21, 84)
5 \mod \text{c} \pmod{y \sim x}
6 summary_model <- summary(model)</pre>
7 beta_hat <- coef(model)[2]</pre>
8 se_beta_hat <- summary_model$coefficients[2, "Std.
      Error"]
9 beta_null <- 3
10 t_stat <- (beta_hat - beta_null) / se_beta_hat
11 alpha <- 0.01
12 df <- summary_model$df[2]
13 t_{critical} \leftarrow qt(1 - alpha, df)
14 cat(round(t_stat,2), "\n")
15 if (t_stat > t_critical) {
16
   cat ("Reject the null hypothesis")
17 } else {
     cat ("Fail to reject the null hypothesis")
18
19 }
```

R code Exa 14.6 confidence interval

R code Exa 14.7 complete a certain form

```
1 # page number = 410
```

R code Exa 14.8 complete a certain form

```
1 # page number = 411
2 n=10
3 r=0.936
4 critical= 2.575
5 z=sqrt((n-3))/2*log((r+1)/(1-r))
6 cat(round(z,1))
7 if(z>critical)
8 {
9  cat("null hypothesis rejected")
10 }
```

R code Exa 14.9 one family houses sold

```
1 # page number = 414
2 bedrooms <- c(3, 2, 4, 2, 3, 2, 5, 4)
3 baths <- c(2, 1, 3, 1, 2, 2, 3, 2)
4 price <- c(292000, 264600, 317500, 265500, 302000, 275500, 333000, 307500)
5 housing_data <- data.frame(bedrooms, baths, price)</pre>
```

R code Exa 14.10 sales price of a threebedroom

```
1 # page number= 414
2 intercept= 224929
3 bedroom_coef = 15314
4 bathroom_coef = 10957
5 x1=3
6 x2=2
7 result= intercept+bedrooms_coef*x1+bathroom_coef*x2
8 cat(floor(result))
```

R code Exa 14.11 least squares estimates

```
1 # page number = 417
2 bedrooms <- c(3, 2, 4, 2, 3, 2, 5, 4)
3 baths <- c(2, 1, 3, 1, 2, 2, 3, 2)
4 price <- c(292000, 264600, 317500, 265500, 302000, 275500, 333000, 307500)
5 housing_data <- data.frame(bedrooms, baths, price)
6 model <- lm(price ~ bedrooms + baths, data = housing __data)
7 coefficients <- coef(model)
8 intercept <- coefficients[1]</pre>
```

```
9 bedrooms_coef <- coefficients[2]
10 baths_coef <- coefficients[3]
11 cat("y^cap =", round(intercept,0), "+", round(
         bedrooms_coef,0), " x1 +", round(baths_coef,0),"
          x2\n")</pre>
```

R code Exa 14.12 least squares estimates

R code Exa 14.13 least squares estimates using hypothesis

```
1 # page number =419
2 critical= 2.015
3 bita_cap= 15314
4 bita=9500
5 c11=32/84
6 n=8
7 sigma= 3546
8 t=(bita_cap-bita)/(sigma*sqrt(n*c11/5))
9 cat(round(t,3))
10 if(critical < t)
11 {</pre>
```

```
12 cat("null hypothesis must be rejected")
13 }
14 #— The answer provided in the textbook is wrong.
```