

R Textbook Companion for
A First Course in Probability
by Sheldon Ross¹

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Book Description

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R numbering policy used in this document and the relation to the above book.

Exa Example (Solved example)

Eqn Equation (Particular equation of the above book)

For example, Exa 3.51 means solved example 3.51 of this book. Sec 2.3 means an R code whose theory is explained in Section 2.3 of the book.

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Chapter 1

Combinatorial Analysis

R code Exa 2.a Mother and child of the year

```
1 # Page No. 2
2
3 no_of_women = 10
4 children_per_women = 3
5
6 no_of_choices = no_of_women * children_per_women
7
8 print(no_of_choices)
```

R code Exa 2.b Committee selection

```
1 # Page No. 2
2
3 no_of_freshmen = 3
4 no_of_sophmores = 4
5 no_of_juniors = 5
6 no_of_seniors = 2
7
```

```
8 ans = no_of_seniors * no_of_juniors * no_of_
    sophmores * no_of_freshmen
9
10 print(ans)
```

R code Exa 2.c Licence plates

```
1 # Page No. 3
2
3 no_of_letters = 26
4 no_of_digits = 10
5
6 ans = (no_of_digits^4) * (no_of_letters^3)
7
8 print(ans)
```

R code Exa 2.e Licence plates without repetitions

```
1 # Page No. 3
2
3 no_of_letters = 26
4 no_of_digits = 10
5 ans = 1
6
7 for(i in 0:3)
8 {
9     ans = ans * (no_of_digits - i);
10 }
11 for(i in 0:2)
12 {
13     ans = ans * (no_of_letters - i);
14 }
15
```

```
16 print(ans)
```

R code Exa 3.a Batting orders

```
1 # Page No. 3
2
3 no_of_players = 9
4
5 ans = factorial(no_of_players)
6
7 print(ans)
```

R code Exa 3.b.a Universal rankings

```
1 # Page No. 4
2
3 no_of_women = 4
4 no_of_men = 6
5
6 ans = factorial(no_of_men + no_of_women)
7
8 print(ans)
```

R code Exa 3.b.b Gender rankings

```
1 # Page No. 4
2
3 no_of_women = 4
4 no_of_men = 6
5
```

```
6 ans = factorial(no_of_men) * factorial(no_of_women)
7
8 print(ans)
```

R code Exa 3.c Book arrangements

```
1 # Page No. 4
2
3 no_of_math_books = 4
4 no_of_chem_books = 3
5 no_of_history_books = 2
6 no_of_lang_book = 1
7 no_of_types_of_books = 4
8
9 no_of_orderings = factorial(no_of_types_of_books)
10 ans = no_of_orderings * factorial(no_of_math_books)
    * factorial(no_of_chem_books) * factorial(no_of_
    history_books) * factorial(no_of_lang_book)
11
12 print(ans)
```

R code Exa 3.d Pepper permutations

```
1 # Page No. 4
2
3 no_of_p = 3
4 no_of_r = 1
5 no_of_e = 2
6
7 total_chars = no_of_p + no_of_r + no_of_e
8 ans = factorial(total_chars) / (factorial(no_of_p) *
    factorial(no_of_r) * factorial(no_of_e))
9
```

```
10 print(ans)
```

R code Exa 3.e Chess tournament outcomes

```
1 # Page No. 5
2
3 no_of_russians = 4
4 no_of_americans = 3
5 no_of_brits = 2
6 no_of_brazillians = 1
7
8 ans = factorial(no_of_brazillians + no_of_americans
  + no_of_brits + no_of_russians) / (factorial(no_
  of_russians) * factorial(no_of_brits) * factorial
  (no_of_americans) * factorial(no_of_brazillians))
9
10 print(ans)
```

R code Exa 3.f Flag signals

```
1 # Page No. 5
2
3 no_of_white_flags = 4
4 no_of_red_flags = 3
5 no_of_blue_flags = 2
6
7 ans = factorial(no_of_white_flags + no_of_red_flags
  + no_of_blue_flags) / (factorial(no_of_white_
  flags) * factorial(no_of_red_flags) * factorial(
  no_of_blue_flags))
8
9 print(ans)
```

R code Exa 4.a Possible committees

```
1 # Page No. 6
2
3 total_people = 20
4 committee_size = 3
5
6 ans = choose(total_people, committee_size)
7
8 print(ans)
```

R code Exa 4.b Committee selection with genders and feuds

```
1 # Page No. 6
2
3 total_men = 7
4 req_men = 3
5 total_women = 5
6 req_women = 2
7
8 ans1 = choose(total_men, req_men) * choose(total_
  women, req_women)
9
10 print(ans1)
11
12 no_of_feuding_men = 2
13
14 feuding_groups = choose(no_of_feuding_men, no_of_
  feuding_men) * choose(total_men - no_of_feuding_
  men, req_men - no_of_feuding_men)
15 ans2 = (choose(total_men, req_men) - feuding_groups)
  * choose(total_women, req_women)
```

```
16
17 print(ans2)
```

R code Exa 5.a Possible divisions

```
1 # Page No. 10
2
3 patrollers = 5
4 station_officers = 2
5 reserve = 3
6
7 ans = factorial(patrollers + station_officers +
8               reserve) / (factorial(patrollers) * factorial(
9               station_officers) * factorial(reserve))
10
11 print(ans)
```

R code Exa 5.b Division into team A and team B

```
1 # Page No. 10
2
3 teamA_size = 5
4 teamB_size = 5
5
6 ans = factorial(teamA_size + teamB_size) / (
7       factorial(teamA_size) * factorial(teamB_size))
8
9 print(ans)
```

R code Exa 5.c Basketball divisions

```

1 # Page No. 10
2
3 team_size = 5
4 no_of_teams = 2
5
6 ans = factorial(team_size * no_of_teams) / ((
      factorial(team_size)^2) * factorial(no_of_teams))
7
8 print(ans)

```

R code Exa 5.d Tournaments

```

1 # Page No. 11
2
3 no_of_players = 8
4 no_of_winners = 4
5
6 ans1 = factorial(no_of_winners) * choose(no_of_
      players, no_of_winners)
7
8 cat("Ans to a)", ans1, "\n")
9
10 ans2 = factorial(no_of_players)
11
12 cat("Ans to b)", ans2, "\n")
13
14 # The answer may vary due to difference in
      representation.

```

R code Exa 6.a Distinct non-negative integral solutions

```

1 # Page No. 13
2

```



```
3 const = 3
4 no_of_vars = 2
5
6 ans = choose(const + no_of_vars - 1, no_of_vars - 1)
7
8 print(ans)
```

R code Exa 6.b Investments

```
1 # Page No. 13
2
3 no_of_vars = 4
4 const = 20
5
6 ans1 = choose(const + no_of_vars - 1, no_of_vars -
7             1)
8 print(ans1)
9
10 no_of_vars_updated = 5
11
12 ans2 = choose(const + no_of_vars_updated - 1, no_of_
13             vars_updated - 1)
14 print(ans2)
```

Chapter 2

Axioms of Probability

R code Exa 3.b Even number on a dice roll

```
1 # Page No. 28
2
3 p_of_a_side = 1/6
4 no_of_even_sides = 3
5
6 ans = p_of_a_side * no_of_even_sides
7
8 print(ans)
9
10 # The answer may vary due to difference in
    representation.
```

R code Exa 4.a Books and vacations

```
1 # Page No. 30
2
3 p_book1 = 0.5
4 p_book2 = 0.4
```

```
5 p_both_books = 0.3
6
7 ans = 1 - (p_book1 + p_book2 - p_both_books)
8
9 print(ans)
```

R code Exa 5.a Sum of dice rolls

```
1 # Page No. 34
2
3 library(MASS)
4
5 favourable_outcomes = 6
6 total_outcomes = 36
7
8 ans = favourable_outcomes / total_outcomes
9
10 print(fractions(ans))
```

R code Exa 5.b Drawing balls from a bowl

```
1 # Page No. 34
2
3 library(MASS)
4
5 total_white_balls = 6
6 total_black_balls = 5
7 fav_no_of_white_balls = 1
8 fav_no_of_black_balls = 2
9 drawn_balls = 3
10
11 ans = (choose(total_white_balls, fav_no_of_white_
               balls) * choose(total_black_balls, fav_no_of_
```

```

        black_balls)) / choose(total_black_balls + total_
        white_balls, drawn_balls)
12
13 print(fractions(ans))

```

R code Exa 5.c Committee selection

```

1 # Page No. 35
2
3 library(MASS)
4
5 total_men = 6
6 total_women = 9
7 fav_no_of_men = 3
8 fav_no_of_women = 2
9 committee_size = 5
10
11 ans = (choose(total_women, fav_no_of_women) * choose
        (total_men, fav_no_of_men)) / choose(total_men +
        total_women, committee_size )
12
13 print(fractions(ans))

```

R code Exa 5.f Probability of a straight

```

1 # Page No. 36
2
3 hand_size = 5
4 total_cards = 52
5
6 no_of straights = 10 * (4^5 - 4)
7 ans = no_of_straights / choose(total_cards, hand_
        size)

```

```
8
9 print(ans)
```

R code Exa 5.g Probability of full house

```
1 # Page No. 37
2
3 total_hands = 5
4 total_cards = 52
5
6 total_full_houses = 13 * 12 * choose(4, 2) * choose
  (4, 3)
7 ans = total_full_houses / choose(total_cards, total_
  hands)
8
9 print(ans)
```

R code Exa 5.h.a Bridge player gets all spades

```
1 # Page No. 37
2
3 total_cards = 52
4 suite_size = 13
5
6 p_13spades = 1 / choose(total_cards, suite_size)
7 ans = 4 * p_13spades
8
9 print(ans)
```

R code Exa 5.h.b Bridge each player gets 1 spade

```

1 # Page No. 37
2
3 total_cards = 52
4 suite_size = 13
5 no_of_aces = 4
6 ace_per_suite = 1
7
8 ans = (factorial(no_of_aces) * (factorial(total_
      cards - no_of_aces) / factorial(suite_size - ace_
      per_suite)^4)) / (factorial(total_cards) /
      factorial(suite_size)^4)
9
10 print(ans)

```

R code Exa 5.i Birthday problem

```

1 # Page No. 38
2
3 days_in_a_year = 365
4 n = 0
5 p = 1
6 i = 0
7
8 while(p >= 0.5)
9 {
10   p = p * (days_in_a_year - i) / days_in_a_year
11   n = n + 1
12   i = i + 1
13 }
14
15 print(n)

```

R code Exa 5.k Football team

```

1 # Page No. 39
2
3 P_2i <- function(i, o = 20, d = 20)
4 {
5   k = (factorial(o - 2 * i) / (2^(o / 2 - i) *
6     factorial(o / 2 - i)))^2
7   number = ((dim(combn(o, 2 * i))[2])^2) * factorial
8     (2 * i) * k
9   denom = (factorial(o + d) / (2^20 * factorial(o)))
10  return(number / denom)
11 }
12
13 for(i in 0:10)
14 {
15   cat("P(", i, ") =", P_2i(i), "\n")
16 }
17
18 # The answer may vary due to difference in
19 representation.

```

R code Exa 5.1 Club sports

```

1 # Page No. 40
2
3 no_tennis = 36
4 no_squash = 28
5 no_badminton = 18
6 no_tennis_badminton = 12
7 no_tennis_squash = 22
8 no_badminton_squash = 9
9 no_all = 4
10
11 ans = no_tennis + no_squash + no_badminton - no_
12   badminton_squash - no_tennis_badminton - no_
13   tennis_squash + no_all

```

```
12
13 print(ans)
```

R code Exa 5.n Married couples

```
1 # Page No. 42
2
3 no_of_couples = 10
4 ans = 0
5
6 for(i in 1:10)
7 {
8   ans = ans + ((-1)^(i + 1)) * choose(10, i) * (2^i)
9     * factorial(19 - i) / factorial(19)
10 }
11 ans = 1 - ans
12 print(ans)
```

R code Exa 7.a Horse race

```
1 # Page No. 48
2
3 P = c(0.2, 0.2, 0.15, 0.15, 0.1, 0.1, 0.1)
4
5 v1 = P[1] + P[2] + P[3]
6
7 print(v1)
8
9 v2 = P[1] + P[5] + P[6] + P[7]
10
11 if(v1 > v2)
12 {
```



```
13     print("First  wager")
14 } else
15 {
16     print("Second  wager")
17 }
```

Chapter 3

Conditional Probability and Independence

R code Exa 2.a Student exams

```
1 # Page No. 59
2
3 P_Lx <- function(x)
4 {
5   return(x/2)
6 }
7 P_F = 1 - P_Lx(1)
8 ans = P_F / (1 - P_Lx(0.75))
9
10 print(ans)
```

R code Exa 2.b.a First coin flip heads

```
1 # Page No. 59
2
3 library(MASS)
```

```
4
5 P_B = 1/4
6 P_F = 1/2
7
8 ans = P_B / P_F
9
10 print(fractions(ans))
```

R code Exa 2.b.b At least one flip heads

```
1 # Page No. 59
2
3 library(MASS)
4
5 P_B = 1/4
6 P_A = 3/4
7
8 ans = P_B / P_A
9
10 print(fractions(ans))
```

R code Exa 2.c Bridge

```
1 # Page No. 60
2
3 total_north_south = 26
4 suite_size = 13
5
6 ans = choose(5,3) * choose(total_north_south - 5,
7                             suite_size - 3) / choose(total_north_south, suite
8                                                         _size)
9
10 print(ans)
```

R code Exa 2.e Course selection

```
1 # Page No. 61
2
3 library(MASS)
4
5 P_C = 1/2
6 P_AgC = 2/3
7
8 ans = P_C * P_AgC
9
10 print(fractions(ans))
```

R code Exa 2.f.a Drawing red balls from urn

```
1 # Page No. 62
2
3 library(MASS)
4
5 no_of_red_balls = 8
6 no_of_blue_balls = 4
7
8 P_R1 = no_of_red_balls / (no_of_blue_balls + no_of_
    red_balls)
9 P_R2gR1 = (no_of_red_balls - 1 ) / (no_of_blue_balls
    + no_of_red_balls - 1)
10 ans = P_R1 * P_R2gR1
11
12 print(fractions(ans))
```

R code Exa 2.h Each pile contains 1 ace

```
1 # Page No. 64
2
3 P_E1 = 1
4 P_E2gE1 = 39/51
5 P_E3gE1E2 = 26/50
6 P_E4gE1E2E3 = 13/49
7
8 ans = P_E1 * P_E2gE1 * P_E3gE1E2 * P_E4gE1E2E3
9
10 print(ans)
```

R code Exa 3.d Laboratory blood test

```
1 # Page No. 67
2
3 P_D = 0.005
4 P_EgD = 0.95
5 P_EgDc = 0.01
6
7 ans = P_EgD * P_D / (P_EgD * P_D + P_EgDc * (1 - P_D
8     ))
9 print(ans)
```

R code Exa 3.e Surgery dilemma

```
1 # Page No. 68
2
3 P_D = 0.6
4 P_EgD = 1
5 P_EgDc = 0.3
```

```

6
7 ans = P_D * P_EgD / (P_EgD * P_D + P_EgDc * (1 - P_D
  ))
8
9 print(ans)

```

R code Exa 3.f Criminal investigation

```

1 # Page No. 69
2
3 P_G = 0.6
4 P_CgG = 1
5 P_CgGc = 0.2
6
7 ans = P_G * P_CgG / (P_G * P_CgG + (1 - P_G) * P_
  CgGc)
8
9 print(ans)

```

R code Exa 3.i Urn and coins

```

1 # Page No. 72
2
3 library(MASS)
4
5 P_A = 2/3
6 P_headsgA = 1/4
7 P_B = 1/3
8 P_headsgB = 3/4
9
10 ans = P_A * P_headsgA / (P_B * P_headsgB)
11
12 print(fractions(ans))

```

R code Exa 3.1 Coloured cards

```
1 # Page No. 75
2
3 library(MASS)
4
5 P_RgRB = 1/2
6 P_RB = 1/3
7 P_RgRR = 1
8 P_RR = 1/3
9 P_RgBB = 0
10 P_BB = 1/3
11
12 ans = P_RgRB * P_RB / (P_RgBB * P_BB + P_RgRB * P_RB
13     + P_RgRR * P_RR)
14 print(fractions(ans))
```

R code Exa 3.n.a Flashlight gives more than 100 hours of use

```
1 # Page No. 77
2
3 P_F1 = 0.2
4 P_F2 = 0.3
5 P_F3 = 0.5
6 P_AgF1 = 0.7
7 P_AgF2 = 0.4
8 P_AgF3 = 0.3
9
10 ans = P_AgF1 * P_F1 + P_AgF2 * P_F2 + P_AgF3 * P_F3
11
12 print(ans)
```

R code Exa 3.n.b Type of flashlight

```
1 # Page No. 77
2
3 library(MASS)
4
5 P_FgA <- function(P_F, P_AgF)
6 {
7   P_A = 0.41
8   return(P_F * P_AgF / P_A)
9 }
10
11 P_AgF1 = 0.7
12 P_F1 = 0.2
13 P_AgF2 = 0.4
14 P_F2 = 0.3
15 P_AgF3 = 0.3
16 P_F3 = 0.5
17
18 cat("P(F1 | A) = ")
19 print(fractions(P_FgA(P_F1, P_AgF1)))
20 cat("P(F2 | A) = ")
21 print(fractions(P_FgA(P_F2, P_AgF2)))
22 cat("P(F3 | A) = ")
23 print(fractions(P_FgA(P_F3, P_AgF3)))
```

R code Exa 3.o Crime probability

```
1 # Page No. 77
2
3 ex_criminals = 10000
```



```

4 tot_pop = 1000000
5 P_hair_match = 10^-5
6
7 alpha = function(c)
8 {
9   c / (ex_criminals * c + tot_pop - ex_criminals)
10 }
11 P_MgG = (1 - P_hair_match)^(ex_criminals - 1)
12 P_all_aj = function(c)
13 {
14   (1 - ex_criminals * alpha(c)) / (1 - alpha(c))
15 }
16 P_MgGc = function(c)
17 {
18   P_hair_match * P_all_aj(c) * (1 - P_hair_match)^(
19     ex_criminals - 1)
20 }
21 P_G = alpha
22 P_GgM = function(c)
23 {
24   (P_MgG * P_G(c)) / (P_MgG * P_G(c) + P_MgGc(c) *
25     (1 - P_G(c)))
26 }
27 c1 = 100
28 ans1 = P_GgM(c1)
29 cat("For c =", c1, "alpha =", alpha(c1), "and P(G|M)
30   =", P_GgM(c1), "\n")
31 c2 = 10
32 ans2 = P_GgM(c2)
33
34 cat("For c =", c2, "alpha =", alpha(c2), "and P(G|M)
35   =", P_GgM(c2), "\n")
36 c3 = 1
37 ans3 = P_GgM(c3)

```

```

38
39 cat("For c =", c3, "alpha =", alpha(c3), "and P(G|M)
    =", P_GgM(c3), "\n")
40
41 # The answer may vary due to difference in
    representation.

```

R code Exa 4.h Independent die trials

```

1 # Page No. 83
2
3 library(MASS)
4
5 P_En <- function(n)
6 {
7     ans = ((13/18)^(n - 1)) * 1/9
8     return(ans)
9 }
10
11 ans = 0
12 i = 2
13
14 ix = P_En(1)
15
16 while(ix != 0)
17 {
18     ans = ans + ix
19     ix = P_En(i)
20     i = i + 1
21 }
22
23 print(fractions(ans))

```

R code Exa 5.a Conditional probability of accident prone policy holder

```
1 # Page No. 94
2
3 library(MASS)
4
5 P_A1gA = 0.4
6 P_A = 0.3
7 P_A1 = 0.26
8
9 P_AgA1 = P_A1gA * P_A / P_A1
10
11 P_A2gAA1 = 0.4
12 P_A2gAcA1 = 0.2
13
14 P_A2gA1 = P_A2gAA1 * P_AgA1 + P_A2gAcA1 * (1 - P_
    AgA1)
15
16 print(P_A2gA1)
```

R code Exa 3.a.1 Accident within a year of purchasing policy

```
1 # Page No. 66
2
3 P_A1gA = 0.4
4 P_A = 0.3
5
6 P_A1gAc = 0.2
7 ans = P_A1gA * P_A + P_A1gAc * (1 - P_A)
8
9 print(ans)
```

R code Exa 3.a.2 Policy holder is accident prone

```
1 # Page No. 66
2
3 library(MASS)
4
5 P_A = 0.3
6 P_A1 = 0.26
7 P_A1gA = 0.4
8
9 ans = P_A*P_A1gA/P_A1
10
11 print(fractions(ans))
```

Chapter 4

Random Variables

R code Exa 1.b Urn with replacement

```
1 # Page No. 118
2
3 P <- function(i)
4 {
5   x = choose(i - 1,2) / choose(20,3)
6   return(x)
7 }
8
9 ans = 0
10
11 for(i in 17:20)
12 {
13   ans = ans + P(i)
14 }
15
16 print(ans)
```

R code Exa 1.d Winnings from urn experiment

```

1 # Page No. 119
2
3 library(MASS)
4
5 P <- rep(4)
6
7 P[0] <- (choose(5,3) + choose(3,1) * choose(3,1) *
           choose(5,1)) / choose(11,3)
8 P[1] <- (choose(3,1) * choose(5,2) + choose(3,2) *
           choose(3,1)) / choose(11,3)
9 P[2] <- (choose(3,2) * choose(5,1)) / choose(11,3)
10 P[3] <- choose(3,3) / choose(11,3)
11
12 print(fractions(sum(P)))

```

R code Exa 10.a Properties of random variables

```

1 # Page No. 169
2
3 library(MASS)
4
5 sentinel = exp(-12)
6
7 F_x = function(x)
8 {
9   if(x < 0)
10   {
11     return(0)
12   }
13   if(x < 1)
14   {
15     return(x/2)
16   }
17   if(x < 2)
18   {

```

```

19     return(2/3)
20 }
21 if(x < 3)
22 {
23     return(11/12)
24 }
25 else
26 {
27     return(1)
28 }
29 }
30 P_X = function(F, b)
31 {
32     return(F(b - sentinal))
33 }
34 ans1 = P_X(F_x, 3)
35
36 cat("a: ")
37 print(fractions(ans1))
38
39 ans2 = F_x(1) - P_X(F_x, 1)
40
41 cat("b: ")
42 print(fractions(ans2))
43
44 ans3 = 1 - F_x(1/2)
45
46 cat("c: ")
47 print(fractions(ans3))
48
49 ans4 = F_x(4) - F_x(2)
50
51 cat("d: ")
52 print(fractions(ans4))

```

R code Exa 3.a Expectation of a die roll

```
1 # Page No. 126
2
3 library(MASS)
4
5 X = 1:6
6 w = rep(1/6, 6)
7
8 ans = weighted.mean(X, w)
9
10 print(fractions(ans))
```

R code Exa 3.d Expectation of number of students

```
1 # Page No. 127
2
3 P_Xe36 = 36/120
4 P_Xe40 = 40/120
5 P_Xe44 = 44/120
6 w = c(P_Xe36, P_Xe40, P_Xe44)
7 X = c(36, 40, 44)
8
9 ans = weighted.mean(X, w)
10
11 print(ans)
```

R code Exa 4.a Expectation of a random variable squared

```
1 # Page No. 128
2
3 P_Ye1 = 0.3 + 0.2
4 P_Ye0 = 0.5
```



```

5 w = c(P_Ye0, P_Ye1)
6 X = 0:1
7
8 ans = weighted.mean(X, w)
9
10 print(ans)

```

R code Exa 5.a Variance of a die roll

```

1 # Page No. 133
2
3 library(MASS)
4
5 X = 1:6
6 w = rep(1/6, 6)
7
8 E_X = weighted.mean(X, w)
9 X1 = X^2
10 E_X1 = weighted.mean(X1, w)
11 Var_X = E_X1 - E_X^2
12
13 print(fractions(Var_X))

```

R code Exa 6.a Binomial random variable coin experiment

```

1 # Page No. 135
2
3 library("MASS")
4
5 n = 5
6 p = 1/2
7 x = 0:5
8

```

```

9 ans = dbinom(x, size = n, prob = p)
10
11 for(i in x)
12 {
13   cat("P{ X =", i, " } =")
14   print(fractions(ans[i + 1]))
15 }
16
17 # The answer may vary due to difference in
   representation.

```

R code Exa 6.b Package replacement

```

1 # Page No. 135
2
3 p = 0.01
4 n = 10
5
6 ans = 1 - pbinom(1, size = n, prob = p)
7
8 print(ans)

```

R code Exa 6.c Wheel of fortune

```

1 # Page No. 136
2
3 library(MASS)
4
5 n = 3
6 p = 1/6
7 x = 0:3
8 X = c(-1, 1, 2, 3)
9

```

```
10 P_X = dbinom(x, size = n, prob = p)
11 ans = weighted.mean(X, P_X)
12
13 print(fractions(ans))
```

R code Exa 6.d Genetic traits

```
1 # Page No. 136
2
3 library(MASS)
4
5 n = 4
6 p = 3/4
7
8 ans = dbinom(3, size = n, prob = p)
9
10 print(fractions(ans))
```

R code Exa 6.h Computing binomial distribution function

```
1 # Page No. 142
2
3 n = 6
4 p = 0.4
5 x = 0:6
6
7 P_X = dbinom(x, size = n, prob = p)
8
9 for(i in 0:6)
10 {
11   cat("P{ X =", i, " } =", P_X[i + 1], "\n")
12 }
```

R code Exa 6.i Binomial distribution generation

```
1 # Page No. 143
2
3 n = 100
4 p = 0.75
5
6 ans1 = dbinom(70, size = n, prob = p)
7
8 cat("P{X = 70} = ", ans1, "\n")
9
10 ans2 = pbinom(70, size = n, prob = p)
11
12 cat("P{X <= 70} = ", ans2, "\n")
```

R code Exa 7.a Typos in a page

```
1 # Page No. 144
2
3 l = 1/2
4
5 ans = 1 - dpois(0, lambda = 1)
6
7 print(ans)
```

R code Exa 7.b Defective item

```
1 # Page No. 145
2
```

```

3 n = 10
4 p = 0.1
5 x = 1
6
7 binom_ans = pbinom(x, size = n, prob = p)
8
9 cat(binom_ans, "\n")
10
11 l = n * p
12 pois_ans = ppois(x, l)
13
14 cat(pois_ans)

```

R code Exa 7.c Radioactive particles

```

1 # Page No. 145
2
3 l = 3.2
4 q = 2
5
6 ans = ppois(q, lambda = l)
7
8 print(ans)

```

R code Exa 7.e.a Earthquake occurrence

```

1 # Page No. 154
2
3 no_of_weeks = 2
4 x = 2
5
6 l = 2 * no_of_weeks
7 ans = 1 - ppois(x, lambda = l)

```

```
8
9 print(ans)
10
11 # The answer may vary due to difference in
    representation.
```

R code Exa 7.f Generating poisson distribution

```
1 # Page No. 155
2
3 l1 = 100
4 x1 = 90
5
6 ans1 = ppois(x1, l1)
7
8 cat("a) ", ans1, "\n")
9
10 l2 = 1000
11 x2 = 1075
12
13 ans2 = ppois(x2, l2)
14
15 cat("b) ", ans2)
16
17 # The answer may slightly vary due to rounding off
    values.
```

R code Exa 8.g Throws of die required

```
1 # Page No. 160
2
3 r = 4
4 p = 1/6
```

```
5
6 E_X = r / p
7 Var_X = r * (1 - p) / p^2
8
9 cat(E_X, "\n")
10 cat(Var_X, "\n")
```

R code Exa 8.i Electrical components

```
1 # Page No. 161
2
3 lot_size = 10
4
5 P_4defectives = 0.3
6 P_1defective = 0.7
7
8 inspect_size = 3
9
10 P_acceptance = choose(4,0) * choose(6,3) * P_4
    defectives / choose(lot_size, inspect_size) +
    choose(1,0) * choose(9,3) * P_1defective / choose
    (lot_size, inspect_size)
11
12 ans = 1 - P_acceptance
13
14 print(ans*100)
```

Chapter 5

Continuous Random Variables

R code Exa 1.a.a Finding the constant in "fx"

```
1 # Page No. 187
2
3 library(MASS)
4
5 integrand = function(x)
6 {
7   4 * x - 2 * x^2
8 }
9 C = 1 / integrate(integrand, lower = 0, upper = 2)$
   value
10
11 print(fractions(C))
```

R code Exa 1.a.b Probability of a continuous random variable

```
1 # Page No. 187
2
3 library("MASS")
```



```

4
5 integrand = function(x)
6 {
7   C = 3/8
8   C * (4 * x - 2 * x^2)
9 }
10 ans = integrate(integrand, lower = 1, upper = 2)$
    value
11
12 print(fractions(ans))

```

R code Exa 1.b.a Computer functioning "a"

```

1 # Page No. 188
2
3 integrand = function(x)
4 {
5   exp(-x/100)
6 }
7 l = 1 / integrate(integrand, lower = 0, upper = Inf)
    $value
8 ans = l * integrate(integrand, lower = 50, upper =
    150)$value
9
10 print(ans)

```

R code Exa 1.b.b Computer functioning "b"

```

1 # Page No. 188
2
3 l = 0.01
4
5 integrand = function(x)

```

```

6 {
7   exp(-x/100)
8 }
9 ans = 1 * integrate(integrand, lower = 0, upper =
    100)$value
10
11 print(ans)

```

R code Exa 1.c Lifetime of radio tube

```

1 # Page No. 188
2
3 library(MASS)
4
5 n = 5
6 x = 2
7
8 integrand = function(x)
9 {
10   100 / x^2
11 }
12 P_Ei = integrate(integrand, lower = 100, upper =
    150)$value
13 ans = dbinom(x, size = n, prob = P_Ei)
14
15 print(fractions(ans))

```

R code Exa 2.a Expectation of a continuous random variable

```

1 # Page No. 190
2
3 library("MASS")
4

```

```

5 integrand = function(x)
6 {
7   2 * x^2
8 }
9 ans = integrate(integrand, lower = 0, upper = 1)$
    value
10
11 print(fractions(ans))

```

R code Exa 2.e Variance of a continuous random variable

```

1 # Page No. 194
2
3 library("MASS")
4
5 E_X = 2/3
6
7 integrand = function(x)
8 {
9   2 * x^3
10 }
11 E_X2 = integrate(integrand, lower = 0, upper = 1)$
    value
12 ans = E_X2 - E_X^2
13
14 print(fractions(ans))

```

R code Exa 3.b Uniform distribution

```

1 # Page No. 196
2
3 library("MASS")
4

```

```

5 min = 0
6 max = 10
7
8 ans1 = punif(3, min = min, max = max)
9
10 cat("Ans to a)")
11 print(fractions(ans1))
12
13 ans2 = 1 - punif(6, min = min, max = max)
14
15 cat("Ans to b)")
16 print(fractions(ans2))
17
18 ans3 = punif(8, min = min, max = max) - punif(3, min
      = min, max = max)
19
20 cat("Ans to c)")
21 print(fractions(ans3))
22
23 # The answer may vary due to difference in
    representation.

```

R code Exa 3.c Waiting time for bus

```

1 # Page No. 196
2
3 library("MASS")
4
5 uni = function(lower, upper)
6 {
7   alpha = 0
8   beta = 30
9
10   P_X = punif(upper, min = alpha, max = beta) -
      punif(lower, min = alpha, max = beta)

```

```

11   return(P_X)
12 }
13 ans1 = uni(10, 15) + uni(25, 30)
14
15 cat("Ans for a)")
16 print(fractions(ans1))
17
18 ans2 = uni(0, 5) + uni(15, 20)
19
20 cat("Ans for b)")
21 print(fractions(ans2))

```

R code Exa 4.b Normal random variable

```

1 # Page No. 202
2
3 mu = 3
4 sigma = sqrt(9)
5
6 ans1 = pnorm(5, mu, sigma) - pnorm(2, mu, sigma)
7
8 cat("Ans to a) ", ans1, "\n")
9
10 ans2 = 1 - pnorm(0, mu, sigma)
11
12 cat("Ans to b) ", ans2, "\n")
13
14 ans3 = 1 - pnorm(9, mu, sigma) + pnorm(-3, mu, sigma)
15
16 cat("Ans to c) ", ans3, "\n")

```

R code Exa 4.c Grading on the curve

```

1 # Page No. 202
2
3 cat("A:", (1 - pnorm(1)) * 100, "%\n")
4 cat("B:", (pnorm(1) - pnorm(0)) * 100, "%\n")
5 cat("C:", (pnorm(0) - pnorm(-1)) * 100, "%\n")
6 cat("D:", (pnorm(2) - pnorm(1)) * 100, "%\n")
7 cat("E:", (pnorm(-2)) * 100, "%\n")

```

R code Exa 4.d Paternity suit

```

1 # Page No. 203
2
3 mu = 270
4 sigma = sqrt(100)
5
6 ans = 1 - pnorm(290, mu, sigma) + pnorm(240, mu,
      sigma)
7
8 print(ans)

```

R code Exa 4.e Signals

```

1 # Page No. 203
2
3 P_EgOne = 1 - pnorm(1.5)
4
5 cat("P{1} =", P_EgOne, "\n")
6
7 P_EgZero = 1 - pnorm(2.5)
8
9 cat("P{0} =", P_EgZero, "\n")

```

R code Exa 4.f Normal approximation

```
1 # Page No. 204
2
3 n = 40
4 p = 1/2
5 x = 20
6
7 mu = n * p
8 sigma = sqrt(n * p * (1 - p))
9 ans1 = pnorm(x + 0.5, mean = mu, sd = sigma) - pnorm
      (x - 0.5, mean = mu, sd = sigma)
10
11 cat("Ans via approx:", ans1, "\n")
12
13 ans2 = dbinom(x, size = n, prob = p)
14
15 cat("Ans:", ans2)
16
17 # The answer may slightly vary due to rounding off
    values.
```

R code Exa 4.g Probability of attendance in a college

```
1 # Page No. 205
2
3 n = 450
4 p = 0.3
5
6 mu = n * p
7 sigma = sqrt(n * p * (1 - p))
8 ans = 1 - pnorm(150.5, mu, sigma)
```

```
9
10 print(ans)
```

R code Exa 4.h Effectiveness of diet

```
1 # Page No. 206
2
3 n = 100
4 p = 1/2
5
6 mu = n * p
7 sigma = sqrt(n * p * (1 - p))
8 ans = 1 - pnorm(64.5, mu, sigma)
9
10 print(ans)
```

R code Exa 4.i Outlawing cigarettes

```
1 # Page No. 206
2
3 P_Sn = function(n)
4 {
5   p = 0.52
6   mu = p * n
7   sigma = sqrt(n * p * (1 - p))
8   ans = 1 - pnorm(0.5 * n, mu, sigma)
9   return(ans)
10 }
11
12 N = c(11, 101, 1001)
13
14 for(i in N)
15 {
```



```

16   cat("For n =", i, "P(Sn > 0.5 * n) =", P_Sn(i), "\n")
17 }
18
19 i = 1
20
21 while(P_Sn(i) < 0.95)
22 {
23   i = i + 1
24 }
25
26 cat("For at least 95%, n =", i, "\n")

```

R code Exa 5.b Exponential random variable

```

1 # Page No. 209
2
3 lambda = 1/10
4
5 ans1 = 1 - pexp(10, lambda)
6
7 cat("Ans to a)", ans1, "\n")
8
9 ans2 = pexp(20, lambda) - pexp(10, lambda)
10
11 cat("Ans to b)", ans2, "\n")

```

R code Exa 5.e Laplace distribution

```

1 # Page No. 212
2
3 library(rmutil)
4

```

```
5 P_EgOne = plaplace(-1.5)
6 P_EgZero = plaplace(-2.5)
7
8 cat("P{1} =", P_EgOne, "\n")
9 cat("P{0} =", P_EgZero, "\n")
```

Chapter 6

Jointly Distributed Random Variables

R code Exa 1.a Joint PMF of drawing balls from urn

```
1 # Page No. 233
2
3 library(MASS)
4
5 P_XY = function(x, y)
6 {
7   blue = 5
8   red = 3
9   white = 4
10  lot_size = 3
11
12  ans = choose(blue, lot_size - x - y) * choose(red,
13           x) * choose(white, y) / choose(blue + white +
14           red, lot_size)
15 }
16
17 lot_size = 3
18
19 for(i in 0:lot_size)
```

```

18 {
19   for(j in 0:(lot_size - i))
20   {
21     cat("p(", i, ", ", j, ") =")
22     print(fractions(P_XY(i, j)))
23   }
24 }
25
26 # The answer may vary due to difference in
    representation.

```

R code Exa 1.b Joint PMF of number of boys and girls in a family

```

1 # Page No. 234
2
3 P = c(.15, .2, .35, .3)
4
5 P_BG <- function(i, j)
6 {
7   if((i + j) >= 4)
8   {
9     return(0)
10  }else
11  {
12    ans = P[(i + j) + 1] * ((factorial(i + j) / (
13      factorial(i) * factorial(j))) / 2^(i + j))
14    return(ans)
15  }
16 }
17 max_children = 3
18
19 for(i in 0:max_children)
20 {
21   for(j in 0:max_children)

```

```

22  {
23      cat("p(", i, ", ", j, ") =", P_BG(i, j), "\n")
24  }
25 }

```

R code Exa 1.c.a Joint density function "a"

```

1  # Page No. 236
2
3  myfun = function(x, y) (2 * exp(-x) * exp(-2 * y))
4  llimx = 1
5  ulimx = Inf
6  llimy = 0
7  ulimy = 1
8
9  f = function()
10 {
11     return(integrate(function(y)
12         {
13             sapply(y, function(y)
14                 {
15                     integrate(function(x) myfun(x,y), llimx, ulimx
16                         )$value
17                 }, llimy, ulimy))
18     })
19     ans = f()$value
20
21     print(ans)
22
23     # The answer may vary due to difference in
24     representation.

```

R code Exa 1.c.b Joint density function "b"

```
1 # Page No. 236
2
3 library(MASS)
4
5 myfun <- function(x,y) (2 * exp(-x) * exp(-2 * y))
6 llimx <- 0
7 llimy <- 0
8 ulimy <- Inf
9
10 f <- function()
11 {
12   return(integrate(function(y)
13     {
14       sapply(y, function(y)
15         {
16           integrate(function(x) myfun(x,y), llimx, y)$
17             value
18         })
19     }, llimy, ulimy))
20 }
21 ans = f()$value
22 print(fractions(ans))
```

R code Exa 2.c Waiting time

```
1 # Page No. 243
2
3 library(pracma)
4 library(MASS)
5
6 integrand = function(x, y)
7 {
```

```

8      (1/60)^2
9  }
10 xm = function(y)
11 {
12     (y - 10)
13 }
14 ans = 2 * integral2(integrand, xmin = 10, xmax =
      60, ymin = 0, ymax = xm)$Q
15
16 print(fractions(ans))

```

R code Exa 3.c.a Basketball wins "a"

```

1 # Page No. 257
2
3 na = 26
4 pa = 0.4
5
6 E_Xa = na * pa
7 Var_Xa = na * pa * (1 - pa)
8
9 nb = 18
10 pb = 0.7
11
12 E_Xb = nb * pb
13 Var_Xb = nb * pb * (1 - pb)
14 E_Xab = E_Xa + E_Xb
15 Var_Xab = Var_Xa + Var_Xb
16 ans = 1 - pnorm(25 - 1/2, E_Xab, sqrt(Var_Xab))
17
18 print(ans)

```

R code Exa 3.c.b Basketball wins "b"

```

1 # Page No. 257
2
3 na = 26
4 pa = 0.4
5
6 E_Xa = na * pa
7 Var_Xa = na * pa * (1 - pa)
8
9 nb = 18
10 pb = 0.7
11
12 E_Xb = nb * pb
13 Var_Xb = nb * pb * (1 - pb)
14 E_Xab = E_Xa - E_Xb
15 Var_Xab = Var_Xa + Var_Xb
16 ans = 1 - pnorm(1 - 1/2, E_Xab, sqrt(Var_Xab))
17
18 print(ans)

```

R code Exa 3.d.a Price of security "a"

```

1 # Page No. 258
2
3 mu = 0.0165
4 sigma = 0.0730
5
6 p = 1 - pnorm(0, mu, sigma)
7 ans = p^2
8
9 print(ans)

```

R code Exa 3.d.b Price of security "b"


```

1 # Page No. 258
2
3 mu = 0.0165
4 sigma = 0.0730
5
6 mu = mu * 2
7 sigma = sqrt(2 * sigma^2)
8 ans = 1 - pnorm(0, mu, sigma)
9
10 print(ans)

```

R code Exa 4.a Conditional probability on joint PMF

```

1 # Page No. 264
2
3 library(MASS)
4
5 p00 = 0.4
6 p01 = 0.2
7 p10 = 0.1
8 p11 = 0.3
9
10 py1 = p01 + p11
11 px0 = p01 / py1
12 px1 = p11 / py1
13
14 print(fractions(px0))
15 print(fractions(px1))

```

R code Exa 6.b Probability of sample median

```

1 # Page No. 272
2

```

```
3 library(MASS)
4
5 integrand = function(x)
6 {
7   x * (1 - x)
8 }
9 ans = factorial(3) / factorial(1)^2 * integrate(
10   integrand, lower = 1/4, upper = 3/4)$value
11 print(fractions(ans))
```

Chapter 7

Properties of Expectation

R code Exa 2.r Chipmunks and groves

```
1 # Page No. 312
2
3 no_of_chipmunks = 15
4 total_trees = 52
5 p = 7/52
6
7 Xi = rep(1, no_of_chipmunks)
8 E_Xi = Xi * p
9 E_X = sum(E_Xi)
10
11 print(E_X)
12
13 # The answer may vary due to difference in
    representation.
```

R code Exa 5.e.a Game of craps "a"

```
1 # Page No. 335
```

```

2
3 Pi = function(i)
4 {
5   if(i > 7)
6   {
7     i = 14 - i
8   }
9   ans = (i - 1) / 36
10 }
11 E_RgSi = function(i)
12 {
13   if(i == 2 || i == 3 || i == 7 || i == 11 || i ==
14     12)
15   {
16     return(1)
17   }
18   else
19   {
20     ans = 1 + 1 / (Pi(i) + Pi(7))
21     return(ans)
22   }
23 }
24 E_R = 0
25
26 for(i in 2:12)
27 {
28   E_R = E_R + E_RgSi(i) * Pi(i)
29 }
30
31 print(E_R)

```

R code Exa 5.e.b Game of craps "b"

1 # Page No. 335

```

2
3 Pi = function(i)
4 {
5   if(i > 7)
6   {
7     i = 14 - i
8   }
9   ans = (i - 1) / 36
10 }
11 P_Si = function(c)
12 {
13   if(i == 7 || i == 11)
14   {
15     return(Pi(i))
16   }
17   if( i == 2 || i == 3 || i == 12)
18   {
19     return(0)
20   }
21   else
22   {
23     ans = Pi(i)^2 / (Pi(i) + Pi(7))
24   }
25 }
26
27 p = 0
28
29 for(i in 2:12)
30 {
31   p = p + P_Si(i)
32 }
33
34 print(p)

```

R code Exa 5.e.c Game of craps ”c”

```

1 # Page No. 335
2
3 p = 0.493
4 E_R = 3.376
5
6 E_RgSi = function(i)
7 {
8     if(i == 2 || i == 3 || i == 7 || i == 11 || i ==
9         12)
10     {
11         return(1)
12     }
13     else
14     {
15         ans = 1 + 1 / (Pi(i) + Pi(7))
16         return(ans)
17     }
18 }
19 Pi = function(i)
20 {
21     if(i > 7)
22     {
23         i = 14 - i
24     }
25     ans = (i - 1) / 36
26 }
27 Qi = function(i)
28 {
29     if( i == 2 || i == 3 || i == 12)
30     {
31         return(0)
32     }
33     if( i == 7 || i == 11)
34     {
35         return(Pi(i) / p)
36     }
37     else
38     {

```

```

38     ans = Pi(i)^2 / (p * (Pi(i) + Pi(7)))
39     return(ans)
40 }
41 }
42
43 E_Rgwin = 0
44
45 for(i in 2:12)
46 {
47     E_Rgwin = E_Rgwin + E_RgSi(i) * Qi(i)
48 }
49
50 E_Rglose = (E_R - E_Rgwin * p) / (1 - p)
51
52 print(E_Rglose)

```

Chapter 8

Limit Theorems

R code Exa 2.b Chebyshev inequality

```
1 # Page No. 390
2
3 alpha = 0
4 beta = 10
5
6 E_X = (alpha + beta) / 2
7 Var_X = (beta - alpha)^2 / 12
8
9 ans1 = Var_X / 4^2
10
11 cat("Using inequality , the ans is ", ans1, "\n")
12
13 ans2 = punif(1, min = alpha, max = beta) + 1 - punif
      (9, min = 0, max = 10)
14
15 cat("Exact ans is ", ans2, "\n")
```

R code Exa 3.a Astronomical distances


```

1 # Page No. 393
2
3 P = function(n)
4 {
5   acc = 0.5
6   ans = pnorm(0.5 * sqrt(n) / 2, 0, 1) - pnorm(-0.5
7         * sqrt(n) / 2, 0, 1)
8   return(ans)
9 }
10 i = 1
11
12 while(P(i) < 0.95)
13 {
14   i = i + 1
15 }
16
17 print(i)

```

R code Exa 3.b Probability of a professor teaching two sections

```

1 # Page No. 396
2
3 mu = 100
4 var = mu
5
6 ans = 1 - pnorm(120 - 1/2, mu, sqrt(var))
7
8 print(ans)

```

R code Exa 3.c Sum of dice rolls

```

1 # Page No. 397

```

```

2
3 p = 1/6
4 E_X = 0
5 E_X2 = 0
6 no_of_die_rolls = 10
7
8 for(i in 1:6)
9 {
10   E_X = E_X + p * i
11   E_X2 = E_X2 + p * i^2
12 }
13
14 print(E_X)
15
16 Var_X = (E_X2 - E_X^2)
17
18 print(Var_X)
19
20 Var_X = (E_X2 - E_X^2) * no_of_die_rolls
21 E_X = E_X * no_of_die_rolls
22 ans = pnorm(40 + 1/2, E_X, sqrt(Var_X)) - pnorm(30 -
      1/2, E_X, sqrt(Var_X))
23
24 print(ans)
25
26 # The answer may vary due to difference in
    representation.

```

R code Exa 3.d Approximation of uniform random variables

```

1 # Page No. 398
2
3 E_X = 1/2
4 Var_X = 1/12
5

```

```
6 n = 10
7
8 E_X = 1/2*10
9 Var_X = 1/12*10
10
11 ans = 1 - pnorm(6, E_X, sqrt(Var_X))
12
13 print(ans)
```

R code Exa 3.e Time for grading exams

```
1 # Page No. 398
2
3 mu = 20
4 sd = 4
5 n = 25
6
7 mu = mu * n
8 sd = sd * sqrt(n)
9 ans = pnorm(450, mu, sd)
10
11 print(ans)
```

R code Exa 5.a Factory production

```
1 # Page No. 405
2
3 library(MASS)
4
5 mu = 100
6 var = 400
7 a = 20
8
```

```
9 ans = var / (var + a^2)
10
11 print(fractions(ans))
```

R code Exa 5.b Pairs

```
1 # Page No. 405
2
3 tot_men = 100
4 tot_people = 200
5
6 E_Xi = tot_men / (tot_people - 1)
7 E_X = E_Xi * tot_men
8 Var_X = E_X * (tot_men - 1) / (tot_people - 1) + 2 *
      choose(tot_men, 2) * ((tot_men*(tot_men - 1)) /
      ((tot_people - 1) * (tot_people - 3)) - (tot_men
      / (tot_people - 1))^2)
9 a = E_X - 30
10 ans = Var_X / (Var_X + a^2)
11
12 print(ans)
```
