R Textbook Companion for Fundamentals of Mathematical Statistics by S.C. Gupta, V.K. Kapoor¹

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Book Description

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R numbering policy used in this document and the relation to the above book.

Exa Example (Solved example)

Eqn Equation (Particular equation of the above book)

For example, Exa 3.51 means solved example 3.51 of this book. Sec 2.3 means an R code whose theory is explained in Section 2.3 of the book.

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Chapter 2

Descriptive Measures

R code Exa 2.1 Arithmetic Mean

```
1 #PAGE NUMBER-2.8
2 \#Example number -2.1
3
4 #Part (a)
5 \text{ x=c}(1,2,3,4,5,6,7)
6 f=c(5,9,12,17,14,10,6)
                                      # frequency
7 fx=f*x
8 = \text{data.frame}(x,y,fx)
9 a
10 mean = (sum(fx))/sum(y)
11 mean
12
13
14
15 #Part (b)
16 Marks = c ("0-10", "10-20", "20-30", "30-40", "40-50", "
      50-60")
17 mid=c((0+10)/2,(10+20)/2,(20+30)/2,(30+40)/2,(40+50)
      /2,(50+60)/2) # mid values
18 f = c(12, 18, 27, 20, 17, 6)
                                       # frequency
19 fx=f*mid
```

```
20 b=data.frame(Marks,mid,f,fx)
21 b
22 mean=(sum(fx))/sum(f)
23 mean
```

R code Exa 2.2 Mean

```
1 #PAGE NUMBER—-2.10
2 \# \text{Example number} - -2.2
3
4 A = 28
5 h=8
6 Class_Interval=c("0-8","8-16","16-24","24-32","32-40
      ","40-48")
7 mid=c((0+8)/2,(8+16)/2,(16+24)/2,(24+32)/2,(32+40)/
      2,(40+48)/2)
8 f = c(8,7,16,24,15,7)
                                        # frequency
9 d = (mid - A)/h
10 \text{ fd=f*d}
11 a=data.frame(Class_Interval,mid,f,d,fd)
13 mean=A + h* sum(fd)/sum(f)
14 mean
```

R code Exa 2.3 Percentage Male Female Employees

```
1 #PAGE NUMBER--2.11
2 #Example number--2.3
3
4 x1=5200
5 x2=4200
6 x=5000
```

R code Exa 2.4 Age Limits

```
1 #PAGE NUMBER-2.12
2 \# \text{Example number} - -2.4
4 Age_Group=c("20-25","25-30","30-35","35-40","40-45",
      " 45 - 50" , " 50 - 55" , " 55 - 60" , " 60 - 65" )
5 \text{ n_o_p=c}(30,160,210,180,145,105,70,60,40)
                                                           #
      Number of persons
6 data.frame(Age_Group,n_o_p)
8 N = 1000
9
10 # Part(i)
11 a=15/100*N
12 sprintf("Number of persons to be retrenched from the
       lower group: %d", a)
13 sprintf("30 people from 20-25 age group")
14 sprintf("%d people from 25-30 age group", a-30)
15
16 # Part(ii)
```

```
17 b = 45 / 100 * N
18 sprintf("Number of persons to be absorbed in other
      branches: %d", b)
19 Age_Group1=c("25-30","30-35","35-40","40-45")
20 n_o_p1=c(160-120,210,180,b-(40+210+180))
21 data.frame(Age_Group1,n_o_p1)
22
23 # Part(iii)
24 c = 10 / 100 * N
25 sprintf("Number of persons to retire: %d", c)
26
27 Age_Group2=c("40-45","45-50","50-55")
28 \quad n_o_p2=c(145-20,105,70)
                                        # Number of
      persons
29 data.frame(Age_Group2,n_o_p2)
30
31 \quad A = 47.5
32 h=5
33 mid=c((40+45)/2,(45+50)/2,(50+55)/2)
34 d = (mid - A)/h; d
35 fd=n_o_p2*d
36 data.frame(Age_Group2,mid,n_o_p2,d,fd)
37 \text{ mean} = A + h* sum(fd)/sum(n_o_p2)
38 round (mean)
```

R code Exa 2.6 Median

```
1 #PAGE NUMBER--2.15
2 #Example number--2.6
3
4 x=c(1,2,3,4,5,6,7,8,9)
5 f=c(8,10,11,16,20,25,15,9,6)
6 cf=cumsum(f)
7 data.frame(x,f,cf)
8 sum(f)/2
```

```
9 sprintf("Cumulative frequence just greater than 60 is 65")
10 sprintf("So, Median = 5")
```

R code Exa 2.7 Median Wages

```
1 #PAGE NUMBER—-2.16
2 \# \text{Example number} - -2.7
4 Wages=c("2000-3000","3000-4000","4000-5000","
      5000 - 6000", "6000 - 7000")
                                       # Number of
5 \text{ n_o_e=c}(3,5,20,10,5)
      employees
6 cf=cumsum(n_o_e)
7 data.frame(Wages, n_o_e, cf)
8
9 \text{ sum}(n_o_e)/2
10 sprintf ("Cumulative frequence just greater than 21.5
       is 28")
11 sprintf("So, the median class is 4000-5000")
12
13 median = 4000 + 1000/20*(21.5-8)
14 median
```

R code Exa 2.8 Median Hours Of Work

```
1 #PAGE NUMBER--2.17
2 #Example number--2.8
3
4 Wages_hours=c("Below 3.005","3.005-4.505","
        4.505-6.005","6.005-7.505","7.505-9.005","9.005
        Above")
5 f=c(0.05*3000,580,0.3*3000,500,0.2*3000,3000-2730)
```

```
6 cf=cumsum(f)
7 data.frame(Wages_hours,f,cf)
8
9 sum(f)/2
10 sprintf("Cumulative frequence just greater than 1500 is 1630")
11 sprintf("So, the median class is 4.505-6.005")
12
13 median= 4.505 + 1.5/900*(1500-730)
14 round(median,2)
```

R code Exa 2.9 Incomplete Frequency

```
1 #PAGE NUMBER—-2.17
2 \# \text{Example number} - -2.9
4 variable=c("10-20","20-30","30-40","40-50","50-60","
      60-70", "70-80")
5 f = c(12,30,NA,65,NA,25,18)
6 data.frame(variable,f)
8 \# \text{Let frequency of } 30-40 \text{ be } f1
9 # Let frequency of 50-60 be f1
10 # sum of f1+f2=q
11 q = 229 - (12 + 30 + 65 + 25 + 18)
12 #Solving the equation ---> 46=40+(114.5-12-30-f1)/65
      *10
13 f1=round(solve(10,335))
14 f1
15 f2 = q - f1
16 f2
```

R code Exa 2.12 Mode

R code Exa 2.15 Geometric mean

```
1 #PAGE NUMBER--2.25
2 #Example number--2.15
3
4 # G'=(x1'*x2*....xn)^1/n
5 # G'=(x1*x2*....xn)1/n * (x1'/x1)^1/n
6 # G'=G* (x1'/x1)^1/n
7
8 G=16.2
9 n=10
10 x1=12.9
11 x_1=21.9
12 c_G.M=G*(x_1/x1)^(1/10) # Corrected G.M
13 c_G.M
```

R code Exa 2.17 Average Speed

```
1 #PAGE NUMBER— -2.27
```

```
2 \# Example number --2.17
4 #Average speed
6 avg.s=function(){
       #let the distance between home and college is x
          kms.
8
       x = 1
9
       #speed from home to college
10
       a=x/10
       #speed from college to home
11
12
       b=x/15
13
       AverageSpeed=2*x/(a+b)
       sprintf("Your average speed is: %s", AverageSpeed
14
          )
15 }
16 avg.s()
```

R code Exa 2.18 Average Price

```
1 #PAGE NUMBER—-2.27
2 \# Example number --2.18
3
4 #Part (a)
5 #Average price
6 avg.p=function(){
7
       #Number of varities of milk
8
9
       #Prices of milk sold at different varities
       a=c(8,10,12,15)
10
       #Average price
11
       AveragePrice=1/((1/n)*(1/a[n-3]+1/a[n-2]+1/a[n
12
          -1]+1/a[n])
       sprintf("Your average price is: %s", AveragePrice
13
```

```
14 }
15 avg.p()
16
17
18 #Part (b)
19 quality=c("A","B","C")
20 p.p.p=c(1.00, 1.50, 2.00)
                                #price per pencil
21 \text{ m.s} = c(50,30,20)
                                #money spent
22 fr.distr=data.frame(quality,p.p.p,m.s)
23 fr.distr
24 \text{ n.e=sum}(\text{m.s})
                                #net value
25 p.p= sum(m.s / p.p.p)
                                #number of pencils
      purchased
26 Averagep.p.p = n.e/p.p #Average price per pencil
27 Averagep.p.p
```

R code Exa 2.19 Average Speed

```
#PAGE NUMBER--2.28
#Example number--2.19

speed=c(60,25,350,25)
distance=c(900,3000,400,15)
wx=round(distance/speed,2)
data.frame(speed,distance,wx)

avg.sp= sum(distance)/sum(wx) #Average Speed

#Answer is little varying as of rounding off

avg.sp
```

R code Exa 2.20 Number Of Tosses Required

```
1 #PAGE NUMBER-2.29
2 \#Example number -2.20
4 x = seq(0,8,1)
5 f = c(1,9,26,59,72,52,29,7,1)
6 cf=cumsum(f)
7 data.frame(x,f,cf)
9 \text{ sum}(f)/2
10 sprintf("Cumulative frequence just greater than 128
      is 167")
11 sprintf("So, Median = 4")
12
13 Q1 = sum(f)/4; Q1
14 sprintf("Cumulative frequence just greater than 64
      is 95")
15 sprintf("So, Q1 = 3")
16 \quad Q3 = 3 * sum(f)/4; Q3
17 sprintf("Cumulative frequence just greater than 192
      is 219")
18 sprintf("So, Q3 = 5")
19 D4=4*sum(f)/10; D4
20 sprintf("Cumulative frequence just greater than
      102.4 is 167")
21 sprintf("So, D4 = 4")
22 P27 = 27 * sum(f) / 100; P27
23 sprintf("Cumulative frequence just greater than
      69.12 is 95")
24 \text{ sprintf}("So, P27 = 3")
```

R code Exa 2.21 Lower Quartile Marks

```
    #PAGE NUMBER—-2.29
    #Example number --2.21
    3
```

```
# marks more
4 \text{ m_m_t=c}(0,10,20,30,40,50)
      than
5 n_o_c=c(500,460,400,200,100,30) # number of
      candidates
6 Class_interval=c("0-10","10-20","20-30","30-40","
      40-50", "50 Above")
7 f = c(500-460, 460-400, 400-200, 200-100, 100-30, 30)
8 cf = cumsum (f)
9 data.frame(m_m_t,n_o_c,Class_interval,f,cf)
10
11 \quad sum(f)/4
12 sprintf ("Cumulative frequence just greater than 125
      is 300")
13 sprintf("So, the corresponding class is 20-30")
14 \quad Q1 = 20 + 10/200 * (125-100)
15 Q1
16
17 \ 3*sum(f)/10
18 sprintf("Cumulative frequence just greater than 150
      is 300")
19 sprintf("So, the corresponding class is 20-30")
20 \quad D3 = 20 + 10/200 * (150-100)
21 D3
```

R code Exa 2.23 Cumulative Frequency Curve

```
1 #PAGE NUMBER--2.31
2 #Example number--2.23
3
4 m.g=c("0-10","10-20","20-30","30-40","40-50","50-60","60-70") #Marks Group
5 f=c(4,8,11,15,12,6,3)
6 l.c.f=cumsum(f) #
Less than cumulative frequency
7 m.c.f=rev(cumsum(rev(f)))
```

```
8 data.distr=data.frame(m.g,f,l.c.f,m.c.f)
              #More than cumulative frequency
9 data.distr
10 #For lower quartile (Q1)
11 N = sum(f)
12 Q1=20+10/11*(N/4-1.c.f[2])
13 Q1
14 #For middle quartile (Q2)
15 Q2=30+10/15*(N/2-1.c.f[3])
     #Median
16 Q2
17 #For upper quartile (Q3)
18 Q3=40+10/12*(3*N/4-1.c.f[4])
19 Q3
20 x = seq(10,70,10)
21 plot(x,1.c.f,xlab="MARKS",ylab="No. of Students",
     main="Cumulative frequency curve", col="red")
22 sp=spline(x,1.c.f,n=20)
23 plot(x,m.c.f,xlab="MARKS",ylab="No. of Students",
     main="Cumulative frequency curve", col="red")
24 spl=spline(x,m.c.f,n=20)
25 lines(spl,col="blue")
26 lines(sp,col="green")
27 locator()
```

R code Exa 2.24 Quartile Deviation And Mean Deviation From Mean

```
1 #PAGE NUMBER--2.33
2 #Example number--2.24
3
4 marks=c("0-10","10-20","20-30","30-40","40-50","50-60","60-70")
5 n.o.s=c(6,5,8,15,7,6,3)
6 mid=seq(5,65,10)
7 d=(mid-35)/10
```

```
8 \text{ fd=n.o.s*d}
9 \text{ mean} = 35 + 10 * (sum(fd)) / sum(n.o.s)
10 l.c.f = cumsum(n.o.s)
11 a=abs(mid-mean)
12 b=n.o.s*a
13 fr.distr=data.frame(marks,mid,n.o.s,d,fd,a,b,l.c.f)
14 fr.distr
15
16 #Part (i)
17 N = sum(n.o.s)
18 Q1=20+10*(\mathbb{N}/4-11)/8
19 Q1
                #Q1 is deviated because they have taken
      N/4 = 12.75 which is 12.5
20 Q3=40+10/7*(3*N/4-34)
                #Q3 is deviated because they have taken
21 Q3
      N/4 = 12.75 which is 12.5
22
23 \quad QD = (Q3 - Q1)/2
24 sprintf("Quartile Deviation is: %s",QD)
25
26 #Part (ii)
27 MD = sum(b)/N
28 MD
```

R code Exa 2.26 Mean and Standard deviation

```
8 f=c(3,61,132,153,140,51,2)
9 d=(mid-A)/h
10 fd=f*d
11 fd2=fd*d
12
13 mean= A + h*sum(fd)/sum(f)
14 mean
15
16 s.d=sqrt(100*(765/542-0.028^2))
17 s.d # Standard Deviation
```

R code Exa 2.27 Corrected Mean And Standard Deviation

```
1 #PAGE NUMBER--2.39
2 \#Example number -2.27
3
4 n = 200
5 \text{ mean} = 40
6 \text{ s.d=15}
7
8 \text{ s_x=n*mean}
                               # sum of xi
9 s_x
10 s_x2=n*(15^2+1600)
                               # sum of xi^2
11 s_x2
12
13 \quad c_s_x = s_x - 34 - 53 + 43 + 35
14 corrected_mean=c_s_x/n
15 corrected_mean
16
17 c_s_x2 = s_x2 - 34^2 - 53^2 + 43^2 + 35^2
18 corrected_s.d= sqrt(c_s_x2/n - corrected_mean^2)
                                    # corrected Standard
19 corrected_s.d
      deviation
```

R code Exa 2.28 Original Frequency Distribution

```
1 #PAGE NUMBER—-2.39
2 \# \text{Example number} - -2.28
3
4 d = seq(-4,3,1)
5 f = c(2,5,7,13,21,16,8,3)
6 \text{ fd=f*d}
7 fd2=fd*d
8 data.frame(fd,fd2)
10 \text{ mean} = 21.9
11 s.d2=63.9725
12 h=round(sqrt(solve(193/75-(-9/75)^2,s.d2)))
13 h
14 A = 21.9 + 9*5/75
15 A
16
17 # Mid value = (x1+x2)/2=A
18 # Magnitude of class= x1-x2=h
19 a=matrix(c(1,-1,1,1),nrow=2,ncol=2)
20 b=matrix(c(A*2,h),nrow=2,ncol=1)
21 t = solve(a,b)
22 t
23 \times 1 = t[1,1]; \times 1
24 	 x2=t[2,1]; x2
25
26 # So the classes obtained
27 \text{ class\_interval=c}("0-5","5-10","10-15","15-20","20-25
      "," 25\!-\!30"," 30\!-\!35", " 35\!-\!40")
28 mid=c((0+5)/2,(5+10)/2,(10+15)/2,(15+20)/2,(20+25)/
      2,(25+30)/2,(30+35)/2,(35+40)/2)
29 data.frame(d,mid,class_interval,f)
```

R code Exa 2.29 Standard Deviation Of Second Group

```
1 #PAGE NUMBER—-2.40
2 \# \text{Example number} - -2.29
3
4 N = 250
5 n1=100
6 x1 = 15
7 \text{ s.d1} = 3
8 n2 = 250 - n1
9 \text{ mean} = 15.6
10 \text{ s.d=} \text{sqrt} (13.44)
11
12 x2 = solve(150, N*mean - 1500)
13 x2
14 d1=x1-mean; d1
15 d2=x2-mean; d2
16 s.d2=\sqrt{\text{solve}}(150, N*13.44-n1*9.36-n2*0.16))
17 s.d2
```

R code Exa 2.34 Wages By Firm A And B

```
11 # Firm B:
12 n2 = 600
                             # Number of wagers
                             # Average daily wage
13 \times 2 = 175
                                  # Total wages paid
14 \quad t_w_p2=n2*x2
15 t_w_p2
16
17 sprintf("Firm B has larger wage bill")
18
19 # Part(ii)
20 \text{ s.d1} = \text{sqrt} (81)
                                      # Standard Deviation of
      A
21 \text{ s.d2} = \text{sqrt}(100)
                                 # Standard Deviation of B
22 \text{ c.vA} = 100 * \text{s.d1/x1}
23 c. vA
24 \text{ c.vB} = 100 * \text{s.d2/x2}
25 \quad c.vB
26
27 sprintf("Firm B has greater variability in
       individual wages")
28
29 # Part(iii)
30 \text{ a\_d\_w} = (n1*x1 + n2*x2)/(n1 + n2) # Average
       daily wages
31 \quad a_d_w
32 d1=x1-a_d_w
33 d2=x2-a_d_w
34 \text{ c.v} = 1/(n1 + n2) * (n1*(81+d1^2) + n2*(100+d2^2))
35 c.v
```

R code Exa 2.35 Moments About Mean Origin Particular Point

```
1 #PAGE NUMBER--2.48
2 #Example number--2.35
3
4 A=4
```

```
5 \quad U1 = -1.5
 6 U2=17
 7 U3 = -30;
               U4=108
8 u2 = U2 - U1^2; u2
 9 u3 = U3 - 3 * U2 * U1 + 2 * U1^3; u3
10 \quad u4 = U4 - 4 * U3 * U1 + 6 * U2 * U1^2 - 3 * U1^4; u4
11
12 b1=u3^2/u2^3
13 b1
14 b2=u4/u2^2
15 b2
16 \text{ mean} = A + U1
17 mean
18
19 # Taking A=0, we get the first moment about origin
20 \text{ U1}=2.5
21 U2= u2+U1^2; U2
22 \quad U3 = u3 + 3 * u2 * U1 + U1^3; \quad U3
23 \quad U4 = u4 + 4 * u3 * U1 + 6 * u2 * U1^2 + U1^4; \quad U4
24
25 # Taking A=2, we get the first moment about x=2
26 \quad U1 = 2.5 - 2
27 U2 = u2 + U1^2; U2
28 \text{ U3} = \text{u3} + 3 * \text{u2} * \text{U1} + \text{U1}^3; \text{ U3}
29 \quad U4 = u4 + 4 * u3 * U1 + 6 * u2 * U1^2 + U1^4; \quad U4
```

R code Exa 2.36 First Four Moments

```
1 #PAGE NUMBER--2.49
2 #Example number--2.36
3
4 x=seq(0,8,1)
5 f=c(1,8,28,56,70,56,28,8,1)
6 d=x-4
7 fd=f*d
```

```
8 \text{ fd2=fd*d}
9 fd3=fd2*d
10 \text{ fd4=fd3*d}
11 data.frame(x,f,d,fd,fd2,fd3,fd4)
12
13 # Moments about point x=4
14 U1=sum(fd)/sum(f); U1
15 U2=sum(fd2)/sum(f); U2
16 U3=sum(fd3)/sum(f); U3
17 U4=sum(fd4)/sum(f); U4
18
19 # Moments about mean
20 u1=0
21 u2 = U2 - U1^2; u2
22 u3 = U3 - 3 * U2 * U1 + 2 * U1^3; u3
23 \quad u4 = U4 - 4 * U3 * U1 + 6 * U2 * U1^2 - 3 * U1^4; u4
24
25 b1=u3^2/u2^3
26 b1
27 b2=u4/u2^2
28 b2
```

R code Exa 2.40 Corrected Frequency Constants

```
1 #PAGE NUMBER--2.51
2 #Example number--2.40
3
4 N=250
5 x=54
6 s.d=3
7 b1=0; b2=3
8
9 #Wrong observation ---> 64 and 50 Correct observation ---> 62 and 52
10 #Part (i)
```

```
11 ux = N * x
12 \text{ cx=ux} - (64+50) + (62+52)
13 cx
14 \text{ cm} = \text{cx/N}
                           # Corrected mean
15 cm
16
17 # Part (ii)
18 ux2=N*s.d^2; ux2
19 cx2=ux2 - ((64-54)^2+(50-54)^2)+((62-54)^2+(52-54)^2)
      ; cx2
20 cs.d=round(sqrt(cx2/N),2) # Corrected Standard
        Deviation
21 \text{ cs.d}
22
23 # Part (iii)
24 u3=b1*u2^3; u3
25 \text{ ux3=N*u3}; \text{ ux3}
26 \text{ cx3=0} -((64-54)^3+(50-54)^3)+((62-54)^3+(52-54)^3);
       cx3
               # Answer here is wrong
27 \text{ cu3} = \text{cx3/N}; \text{ cu3}
28 cb1=(cu3)^2/(cx2/N)^3; round(cb1)
29
30 # Part (iv)
31 u4=b2*9^2
32 \text{ ux4} = N*u4; \text{ ux4}
33 cx4 = ux4 - ((64-54)^4+(50-54)^4+(62-54)^4+(52-54)
       ^4); cx4
34 \text{ cu4} = \text{cx4/N}; \text{ cu4}
35 \text{ cb2} = \text{cu4/(cx2/N)^2}; \text{ cb2}
```

R code Exa 2.42 First Four Moments About Origin

```
1 #PAGE NUMBER-2.55
2 #Example number-2.42
3
```

```
4 \text{ mean} = 10
5 u2=16
6 \text{ s.d=} \text{sqrt} (16)
7 v1 = 1
8 b1 = 4
10 # First four moments about origin
11 U1=mean
12 \ U2=u2 + U1^2; \ U2
13 u3=s.d^3; u3
14 \ U3 = u3 + 3*U2*U1 -2*U1^3; U3
15 u4= b1*u2^2; u4
                             # here we have to take value
      of b1, written wrong in book
16
17 \quad U4 = u4 + 4*U3*U1 - 6*U2*U1^2 + 3*U1^4; U4
                                                      # plus,
      minus signs are not correct in the book
18
19 sprintf("The distribution is leptokurtic")
```

R code Exa 2.43 First Four Moments About Mean Of The Distribution

```
2,(150+160)/2,(160+170)/2,
12 (170+180)/2,(180+190)/2,(190+200)/2,(200+210)/
       2,(210+220)/2,(220+230)/2)
13 d = (mid - A)/h
14 \text{ fd=f*d}
15 \text{ fd2=fd*d}
16 fd3=fd2*d
17 \text{ fd4=fd3*d}
18 data.frame(Scores, mid, f, d, fd, fd2, fd3, fd4)
19
20 # Raw Moments of variable d about origin
21 U1=sum(fd)/sum(f); U1
22 \quad U2 = sum(fd2) / sum(f); \quad U2
23 \quad U3 = sum(fd3) / sum(f); \quad U3
24 \quad U4 = sum(fd4) / sum(f); U4
25
26 # Central Moments of variable X
27 u2 = (U2 - U1^2) *h^2; u2
28 u3 = (U3 - 3 * U2 * U1 + 2 * U1^3) * h^3; u3
29 	 u4 = (U4 - 4 * U3 * U1 + 6 * U2 * U1^2 - 3 * U1^4) * h^4; u4
30
31 # Sheppard's Corrections for Moments
32
33 \quad u2b=u2-h^2/12; \quad u2b
34 u3b=u3; u3b
35 \quad u4b=u4 - h^2/2*u2 + 7/240*h^4; \quad u4b
36
37 # Moment coefficient of skewness
38 y1 = u3/(u2)^(3/2); y1
39
40 # Moment coefficient of kurtosis
41 b2= u4/u2^2; b2
42
43 sprintf("The distribution is leptokurtic")
```

Chapter 3

Probability I

R code Exa 3.2 Two Unbiased Dice

```
1 \#Page number -3.7
2 \#Example number -3.2
3 #LOAD PACKAGE---->prob
5 s=rolldie(2)
8 #Part (a)
9 subset(s, X1 == X2)
10 p=nrow(subset(s, X1==X2))/nrow(s)
11 sprintf("Probability that the two dice show the same
       number: %f",p)
12
13 #Part (b)
14 subset(s, X1 == 6)
15 p=nrow(subset(s,X1==6))/nrow(s)
16 sprintf("Probability that the first die show 6: %f",
     p)
17
18 #Part (c)
19 subset(s, X1+X2==8)
```

```
20 p=nrow(subset(s,X1+X2==8))/nrow(s)
21 sprintf("Probability that the total of numbers on
      the die is 8: \%f,p)
22
23 #Part (d)
24 subset(s, X1+X2>8)
25 p=nrow(subset(s,X1+X2>8))/nrow(s)
26 sprintf("Probability that the total of numbers on
      the die is more than 8: %f",p)
27
28 #Part (e)
29 subset(s, X1+X2==13)
30 \text{ p=nrow(subset(s,X1+X2==13))/nrow(s)}
31 sprintf("Probability: %f",p)
32 sprintf("This is an impossible event")
33
34 #Part (f)
35 sprintf("This is a certain event.")
36 sprintf("Probability: 1")
```

R code Exa 3.3 Probability That Odd Digit Selected

```
1 #Page number -- 3.8
2 #Example number -- 3.3
3
4 # Part(a)
5 N = 5 * 4
6 sprintf("Total number of cases: %d",N)
7 #Part(i)
8 # there are 12 cases -- > (1,2),(1,3),(1,4),(1,5),(3,1),(3,2),(3,4),(3,5),(5,1),(5,2),(5,3),(5,4)
9 a = 12/20
10 sprintf("Probability that the first digit drawn is odd: %f",a)
11 #Part(ii)
```

```
12 # there are 12 cases --->(2,1),(3,1),(4,1),(5,1)
      (1,3),(2,3),(4,3),(5,3),(1,5),(2,5),(3,5),(4,5)
13 b = 12/20
14 sprintf("Probability that the second digit drawn is
      odd: %f",b)
15 #Part(iii)
16 # there are 6 cases --->(1,3),(1,5),(3,1),(3,5),(5,1)
      , (5,3)
17 c = 6/20
18 sprintf("Probability that the first and second digit
       drawn are odd: %f",c)
19
20 # Part(b)
21 n = 25
22 #Number which are multiple of 5----> 5,10,15,20,25
23 \text{ d.f.c} = 5+3
                    # Distinct favourable cases
24 d=d.f.c/n
25 sprintf("Required Probability: %f",d)
26
27 #Number which are multiple of 3---->
      3,6,9,12,15,18,21,24
28 \text{ d.f.c} = 8+3-1
                  # Distinct favourable cases
29 \text{ e=d.f.c/n}
30 sprintf("Required Probability: %f",e)
```

R code Exa 3.4 Four Cards Drawn At Random

```
9 \text{ s=cards()}
10
11 #Part (i)
12 subset(s, rank == "K" | rank == "J" | rank == "Q" | rank ==
13 #there are 4 King, 4 Jack, 4 Queen, 4 Ace ---> we
      need to select one from each
14 #total there are 52 cards
15 p = (comb(4,1) * comb(4,1) * comb(4,1) * comb(4,1)) / comb
      (52,4)
16 sprintf("Probability: %f",p)
17
18 #Part (ii)
19 subset(s, rank=="K" | rank=="Q")
20 #there are 4 kings, 4 queens ----> we need to select
       two from each
21 #total there are 52 cards
22 p = (comb(4,2) * comb(4,2)) / comb(52,4)
23 sprintf("Probability: %f",p)
24
25 #Part (iii)
26 #There are 26 black cards and 26 red cards ----> we
      need to select two from each
27 #total there are 52 cards
28 p = (comb(26,2) * comb(26,2)) / comb(52,4)
29 sprintf("Probability: %f",p)
30
31 #Part (iv)
32 subset(s, suit == "Diamond" | suit == "Heart" )
33 #there are 13 heart, 13 diamonds ----> we need to
      select two from each
34 #total there are 52 cards
35 p = (comb(13,2) * comb(13,2)) / comb(52,4)
36 sprintf("Probability: %f",p)
37
38 #Part (v)
39 #4 cards can be from each suit
```

R code Exa 3.24 Probability That Letter Chosen

```
1 \#Page number -3.36
2 \# \text{Example number} - -3.24
3
4 letters
6 n=length(letters)
8 #Part (i)
9 #Probability that the letter choosen is vowel
10 n1=length(c("a","e","i","o","u"))
11 p=n1/n
12 sprintf("Probability: %f",p)
13
14 #Part (ii)
15 #Probability that the letter preceds m and is vowel
16 n2=length(c("a","e","i"))
17 p=n2/n
18 sprintf("Probability: %f",p)
19
20 #Part (iii)
21 #Probability that the letter follows m and is vowel
22 n3=length(c("o","u"))
23 p=n3/n
24 sprintf("Probability: %f",p)
```

R code Exa 3.29 Probability That Odd Number Appear

R code Exa 3.30 Probability

```
1 #Page number -- 3.39
2 #Example number -- 3.30
3 #LOAD PACKAGE -- prob
4
5 s = rolldie(2)
6
7 #S denotes sum of faces of two die
8
9 #Part (a)
10 subset(s, X1 + X2 > 8)
11 #Sample Space
12 #S = 9 -- -- -- > (3,6), (6,3), (4,5) (5,4)
13 #S = 10 -- -- > (4,6), (6,4), (5,5)
14 #S = 11 -- -- > (5,6), (6,5)
15 #S = 12 -- -- > (6,6)
16 p = nrow(subset(s, X1 + X2 = 9)) / nrow(s) + nrow(subset(s, X1 + X2 = 10)) / nrow(s) + nrow(subset(s, X1 + X2 = 11)) / nrow(s) + nrow(subset(s, X1 + X2 = 12)) / nrow(s)
```

```
17 sprintf("Probability that sum greater than 8: %f",p)
18
19 #Part(b)
20 subset(s,X1+X2==7 | X1+X2==11)
21 #Sample Space
22 #S=7----->(1,6),(6,1),(2,5),(5,2),(3,4),(4,3)
23 #S=11---->(5,6),(6,5)
24 p1=nrow(subset(s,X1+X2==7))/nrow(s)+nrow(subset(s,X1+X2==11))/nrow(s)
25 sprintf("Required Probability : %f",1-p1)
```

R code Exa 3.37 Probability

```
1 \#Page number -3.41
2 \# Example number - -3.37
3 #LOAD PACKAGE---->prob
4
5 \text{ s=cards()}
6 s
7
8 #A= Probability that the card drawn is king
9 #B= Probability that the card drawn is heart
10 #C= Probability that the card drawn is red card
11 A=nrow(subset(s,rank=="K"))/nrow(s)
12 B=nrow(subset(s,suit == "Heart"))/nrow(s)
13 C=(nrow(s)/2)/nrow(s)
                                        #Total red cards
      =26(i.e. half of deck)
14 #D=Probability that the card drawn is king of hearts
15 #E=Probability that the card drawn is heart
16 #F=Probability that the card drawn is red king
17 #G=Probability that the card drawn is king of hearts
18 D=1/nrow(s)
19 E=13/nrow(s)
20 F=2/nrow(s)
21 G=1/nrow(s)
```

```
22 #P=Probability that the card drawn is KING OR HEART OR RED CARD
```

- 23 P = A + B + C D E F + G
- 24 P

Chapter 4

Probability II

R code Exa 4.1 Factories Production

```
1 \#Page number -4.6
2 \# \text{Example number} - -4.1
4 #There are 3 factories --->X,Y,Z
5 #Product produced respectively --->3n,n,n
6 #Where n is constant, let suppose n=1
8 #E1, E2, E3 denotes event to the item produced in X,Y,
      \mathbf{Z}
9 #A be the event that item is deffective
10 PE1=3*n/(3*n+n+n)
11 PE2=n/(5*n)
12 PE3=n/(5*n)
13 \#GIVEN——>P(A | E1) = P(A | E3) = 0.03 AND P(A | E2)
      =0.05
                                   \#P(A \mid E1)
14 P.AE1=0.03
15 P.AE3=0.03
                                  \#P(A \mid E3)
16 P.AE2=0.05
                                  \#P(A \mid E2)
17
18 #Part (i)
19 PA=PE1*P.AE1+PE2*P.AE2+PE3*P.AE3
```

R code Exa 4.2 Probability

```
1 #Page number -4.7
2 \# \text{Example number} --4.2
3
principal
7 \text{ PE1} = 4/9
8 \text{ PE2} = 2/9
9 \text{ PE3} = 3/9
                                   \#P(A \mid E1)
10 P.AE1=0.3
11 P.AE2=0.5
                                       \#P(A \mid E2)
12 P.AE3=0.8
                                   \#P(A \mid E3)
13
14 #Part (i)
15 PA=PE1*P.AE1+PE2*P.AE2+PE3*P.AE3
16 sprintf ("The required probability is: %f", PA)
17
18 #Part (ii)
19 #Using bayes rule
20 P.E3A = (PE3*P.AE3)/PA
                                        \#P(E3 | A)
```

R code Exa 4.3 Probability of X Y Z Becoming Managers

```
1 #Page number -4.7
2 \# \text{Example number} - -4.3
3
4 \#X,Y,Z\longrightarrow managers
5 #E1, E2, E3---->Are the events of becoming X, Y, Z
      managers
7 \text{ PE1} = 4/9
8 \text{ PE2} = 2/9
9 \text{ PE3}=1/3
                                     \#P(A \mid E1)
10 P.AE1=0.3
11 P.AE2=0.5
                                     \#P(A \mid E2)
12 P.AE3=0.8
                                     \#P(A \mid E3)
13
14 #Part (i)
15 PA=PE1*P.AE1+PE2*P.AE2+PE3*P.AE3
17 sprintf("The probability that Bonus scheme is
      introduced: %f ",PA)
18
19 #Part (ii)
20 #Using bayes rule
21 P.E1A = (PE1 * P.AE1) / PA
                                             \#P(E1|A)
22 sprintf("The probability that X will be manager: %f
      ",P.E1A)
```

R code Exa 4.4 Probability By Machines I And II And III

```
1 \#Page number -4.8
```

```
2 \# \text{Example number} - -4.4
3
4\ \#A ——>Output is defective 5\ \#E1\,,E2\,,E3 ——>Output produced by machines I\,,II\,,III
7 PE1=3000/10000
8 PE2=2500/10000
9 PE3=4500/10000
10 P.AE1=0.01
                                       \#P(A \mid E1)
11 P.AE2=0.012
                                            \#P(A \mid E2)
12 P.AE3=0.02
                                        \#P(A \mid E3)
14 PA=PE1*P.AE1+PE2*P.AE2+PE3*P.AE3
15 PA
16 sprintf("The probability that output is deffective:
      \% f ",PA)
17
18
19 #Using bayes rule
20 P.E1A = (PE1*P.AE1)/PA
                                                    \#P(E1|A)
21 P.E2A = (PE2*P.AE2)/PA
                                                    \#P(E2|A)
22 P.E3A = (PE3*P.AE3) / PA
                                                    #P(E3|A)
23 sprintf("The probabilities of machine I, II, III are:
      \%f ,\%f ,\%f", P.E1A, P.E2A, P.E3A)
```

R code Exa 4.9 Vessel Containing Balls

```
1 #Page number -- 4.11
2 #Example number -- 4.9
3
4 #E --------> Drawing a white ball from second
    vessel
5 #E1, E2, E3, E4 -----> Transfer of (0,4),(1,3),(2,2)
    ,(3,1) white and black balls
6
```

```
7 PE1=choose (5,4)/choose (8,4)
8 PE2=choose(3,1)*choose(5,3)/choose(8,4)
9 PE3=choose(3,2)*choose(5,2)/choose(8,4)
10 PE4 = choose(3,3) * choose(5,1) / choose(8,4)
11 P.EE1=0
                              #P(E|E1)
12 P.EE2=1/4
                                       #P(E|E2)
                                  #P(E|E3)
13 P.EE3=2/4
14 P.EE4=3/4
                                    #P(E|E4)
15
16 PE=PE1*P.EE1+PE2*P.EE2+PE3*P.EE3+PE4*P.EE4
17
18 #Using bayes rule
19 P.E4E = (PE4*P.EE4)/PE
                                        #P(E4|E)
20 sprintf("The required probability is: %f ",P.E4E)
```

R code Exa 4.10 Probability That Answer Is Correct

```
1 \#Page number -4.11
2 \#Example number -4.10
3
4 #E———>They get same answer
5 #E1———>Both A and B solve correctly
6 #E2———>Exactly one of them sovles correctly
7 #E3———No one sovies correctly
8
9
10 \text{ PE1=1/6*1/8}
11 PE2=1/6*7/8+5/6*1/8
12 PE3=5/6*7/8
13
14 P.EE1=1
                                     \#P(E|E1)
15 P.EE2=0
                                          #P(E|E2)
16 P.EE3=1/525
                                          \#P(E \mid E3)
17
18 PE=PE1*P.EE1+PE2*P.EE2+PE3*P.EE3
```

```
19
20 #Using bayes rule
21 P.E1E=(PE1*P.EE1)/PE #P(E1|E)
22 sprintf("The required probability is: %f", P.E1E)
```

R code Exa 4.16 Probability That A Wrote Plus

```
1 \#Page number -4.15
2 \# \text{Example number} --4.16
3
was changed exactly twice
8 \text{ PE1} = 1/3
9 PE2=1-PE1
10
11 \#A1, A2, A3 \longrightarrow B, C, D change sign on slip
12 PA1=2/3; PA2=2/3; PA3=2/3
13 PA1b=1/3; PA2b=1/3; PA3b=1/3
                              #P(Ai) bar
14
15 PE3 = (1/3)^3
16 PE4=PA1*PA2*PA3b+PA1*PA2b*PA3+PA1b*PA2*PA3
17
18 P.EE1=PE3+PE4
                            \#P(E|E1)
19 sprintf("The required probability is: %f ",P.EE1)
20
21 #E5, E6----->Minus sign change once, Minus sign
     change thrice
22 PE5=PA1*PA2b*PA3b+PA1b*PA2*PA3b+PA1b*PA2b*PA3
23 PE6=PA1*PA2*PA3
24 P.EE2=PE5+PE6
                           \#P(E|E2)
25 sprintf("The required probability is: %f ",P.EE2)
26
```

R code Exa 4.18 Probability Of Drawing White Ball

```
1 \#Page number -4.17
2 \# \text{Example number} - -4.18
4 #E---->Event of drawing a white ball from
      the second urn in the fiest draw
5 #E0, E1, E2, E3, E4---->Drawing of (0,4),(1,3),(2,2)
      (3,1),(4,0) white and black balls
6
7 PE0 = \frac{\text{choose}(5,4)}{\text{choose}(10,4)}
8 PE1=choose(5,1)*choose(5,3)/choose(10,4)
9 PE2 = choose(5,2) * choose(5,2) / choose(10,4)
10 PE3=choose (5,3)*choose (5,1)/choose (10,4)
11 PE4=choose (5,4)/choose (10,4)
                                \#P(E \mid E0)
12 P.EE0=0
13 P.EE1=1/4
                                         \#P(E \mid E1)
14 P.EE2=2/4
                                     \#P(E \mid E2)
                                       #P(E|E3)
15 P.EE3=3/4
16 P.EE4=4/4
                                       #P(E|E4)
17
18 #C---->Future event
19
20 P.CEOE=0
                                       #P(C|intersection(E0
      , E)
                                       #P(C|intersection(E1
21 P.CE1E=0
      ,E))
22 P.CE2E=1/3
                                       #P(C|intersection(E2
      , E)
23 P.CE3E = 2/3
                                       #P(C|intersection(E3
      , E)
```

R code Exa 4.20 Die A And B

```
1 \#Page number -4.19
2 \# \text{Example number} - -4.20
3
4 #A-
             ----->Getting a red face in each of 1st n
      throws
5 #E1, E2, E3--->Event that Die A is used, Die B is
     used, Getting a red face
7 #Part (i)
8 P.E3E1=4/6
                      #P(E3 | E1)
9 P.E3E2=2/6
                       #P(E3 | E2)
10
11 PE1=1/2; PE2=1/2
12
13 PE3=PE1*P.E3E1+PE2*P.E3E2
14 sprintf("The probability of getting a red face in
     any throw: %f ",PE3)
15
16 #Part (ii)
17 #Using the Law Of Succession
18 sprintf("The probability of getting a red face at
      the 3rd throw when 1st two gave red faces: %f ",
      (1/2+1)/(1/2+2)
```

Chapter 5

Random Variables And Distribution Functions

R code Exa 5.1 Random Variable X

```
1 \#Page number -5.6
2 \# \text{Example number} --5.1
4 # Constructing Quadratic Formula
5 result <- function(a,b,c){</pre>
     if(delta(a,b,c) > 0) \{ \# first case D>0 \}
            x_1 = (-b + sqrt(delta(a,b,c)))/(2*a)
8
            x_2 = (-b-sqrt(delta(a,b,c)))/(2*a)
9
            result = c(x_1, x_2)
10
11
     else if (delta(a,b,c) == 0){ # second case D=0
12
            x = -b/(2*a)
13
14
     else {"There are no real roots."} # third case D<0
15 }
16
17 # Constructing delta
18 delta <-function(a,b,c){
19
         b^2-4*a*c
```

```
20 }
21
22 #Values of X
23 x=c(0,1,2,3,4,5,6,7)
24 #Respective probabilities
25 px = c("0","k","2k","2k","3k","k^2","2k^2","7k^2+k")
26
27 data.frame(x,px)
28
29 #Part(i)
30 #We know that total probability =1
31 \#k+2k+2k+3k+k^2+2k^2+7k^2+k=1—
                                                     ->We get
      10*k^2+9*k-1=0
32 \text{ a=result}(10,9,-1)
33 a
34 #Since, k cannot be negative
35 \text{ k=a[1]}
36 k
37 #Substituting values of k in p(x), we get
38 px=c(0,k,2*k,2*k,3*k,k^2,2*k^2,7*k^2+k)
39
40 \text{ b=data.frame}(x,px)
41 b
42
43 #Part (ii)
44 \#Evaluate P(X<6)
45 PX6 = sum(b[1:6,2])
46 sprintf("The probability X<6 is: %f",PX6)
47 sprintf("The probability X>=6 is: \%f",1-PX6)
48 POX5 = sum(b[2:5,2])
49 sprintf("The probability 0 < X < 5 is: \%f", POX5)
50
51 #Part(iii)
52 #BY trail we get a=4
53
54 #Part (iv)
55 #Cummulative probability function
56 \text{ FX=c}(\text{sum}(b[1,2]), \text{sum}(b[1:2,2]), \text{sum}(b[1:3,2]), \text{sum}(b[1:3,2]), \text{sum}(b[1:3,2]))
```

```
[1:4,2]), sum(b[1:5,2]), sum(b[1:6,2]), sum(b
[1:7,2]), sum(b[1:8,2]))

57 data.frame(x,FX)
```

R code Exa 5.2 Probability

```
1 \#Page number -5.7
2 \#Example number -5.2
3
4 x=c(1,2,3,4,5)
5 \text{ px}=c()
6 i = 1
7 while(i<=5){</pre>
8
       px[i]=i/15
       i=i+1
9
10 }
11 a=data.frame(x,px)
12 a
13
14 #Part (i)
15 PX12 = sum(a[1:2,2])
16 sprintf("The probability X=1 or X=2 is: \%f", PX12)
17
18 #Part (ii)
19 P=a[2,2]/(1-a[1,2])
20 sprintf("The required probability is: %f",P)
```

R code Exa 5.3 Distribution Function Of X

```
1 #Page number -- 5.7
2 #Example number -- 5.3
3 #LOAD PACKAGE---->prob
```

```
5 s=rolldie(2)
7 X=c(1,2,3,4,5,6,7,8,9,10,11,12)
8 a=0
9 for(i in 1:12){
10
       a[i]=nrow(subset(s,X1+X2==i))/nrow(s)
11 }
12 #Probability chart
13 plot(X,a,type="h",xlim=c(0,12),ylim=c(0,0.17),ylab="
     Prob.", xlab="Sum of two faces")
14
15 #Cummulative probability distribution
16 FX=c(sum(a[1]),sum(a[1:2]),sum(a[1:3]),sum(a[1:4]),
     sum(a[1:5]),sum(a[1:6]),sum(a[1:7]),sum(a[1:8]),
     sum(a[1:9]),sum(a[1:10]),sum(a[1:11]),sum(a
      [1:12]))
17 data.frame(X,FX)
```

R code Exa 5.4 Probability Function

```
16
17 #Part (i)
18 #Probability distribution of X
19 P3=1/8; P2=3/8; P1=3/8; P0=1/8
20 x = seq(0,3,1)
21 \text{ px} = c(P3, P2, P1, P0)
22 plot(x,px,type="h",xlim=c(0,3),ylim=c(0,0.4),ylab="
      Prob.", xlab="Number of heads")
23 points (0:3, px, pch=16, cex=2)
24
25 #Part (ii)
26 #Probability distribution of Y
27 P1=3/8; P0=5/8
y = seq(0,1,1)
29 py = c(P1, P0)
30 plot(y,py,type="h",xlim=c(0,1),ylim=c(0,0.7),ylab="
      Prob.", xlab="Number of heads runs")
31 points (0:1, py, pch=16, cex=2)
32
33 #Part (iii)
34 #Probability distribution of Z
35 P3=1/8; P2=2/8; P1=0; P0=5/8
36 z = seq(0,3,1)
37 \text{ pz} = c(P0, P1, P2, P3)
38 plot(z,pz,type="h",xlim=c(0,3),ylim=c(0,0.7),ylab="
      Prob.")
39 points (0:3, pz, pch=16, cex=2)
40
41 #Part (iv)
42 #Probability distribution of U
43 P4=1/8; P3=2/8; P2=1/8; P1=3/8; P0=1/8
44 u = seq(0,4,1)
45 \text{ pu} = \text{c}(P2, P1, P0, P3, P4)
46 plot(u,pu,type="h",xlim=c(0,4),ylim=c(0,0.4),ylab="
47 points (0:4, pu, pch=16, cex=2)
49 #Part (v)
```

```
50 #Probability distribution of V
51 P3=1/8; P2=2/8; P1=0; P0=5/8
52 v=seq(0,3,1)
53 pv=c(P0,P1,P2,P3)
54 plot(v,pv,type="h",xlim=c(0,3),ylim=c(0,0.7),ylab="Prob.")
55 points(0:3,pv,pch=16,cex=2)
```

R code Exa 5.5 Probability Density Function

```
1 \#Page number -5.13
2 \# \text{Example number} --5.5
3
4 # Constructing Quadratic Formula
5 result <- function(a,b,c){</pre>
     if (delta(a,b,c) > 0) { \# first case D>0
6
            x_1 = (-b + sqrt(delta(a,b,c)))/(2*a)
            x_2 = (-b-sqrt(delta(a,b,c)))/(2*a)
8
9
            result = c(x_1, x_2)
10
     else if(delta(a,b,c) == 0){ \# second case D=0
11
12
            x = -b/(2*a)
13
14
     else {"There are no real roots."} # third case D<0
15 }
16
17 # Constructing delta
18 delta <-function(a,b,c){
19
         b^2-4*a*c
20 }
21
22 #Part (i)
23 \#Checking f(x) is p.d.f or not
24 integrand=function(x) {6*x*(1-x)}
25 a=integrate(integrand, lower=0, upper=1)
```

```
26 a
27 sprintf("f(x) is p.d.f")
28
29 #Part (ii)
30 #Determining b
31 #integrand=function(x) \{6*x*(1-x)\}
32 #integrate(integrand, lower=0, upper=b) = integrate(
      integrand, lower=b, upper=1)
33 #Solving above integral we get quadratic equation
     ---->(2*b-1)(2*b^2-2*b-1)=0
34 \text{ b=result}(2,-2,-1)
35 b
36 #Other solution of b
37 b=1/2
38 #Since, probability cannot be negative or greater
39 sprintf("The onle real value of b: %f", b)
```

R code Exa 5.12 Relative Frequency Density

```
1 \#Page number -5.17
2 \# \text{Example number} --5.12
3 #LOADED PACKAGE---->cubature
4 # Constructing Quadratic Formula
5 result <- function(a,b,c){</pre>
     if (delta(a,b,c) > 0) { \# first case D>0
6
7
            x_1 = (-b + sqrt(delta(a,b,c)))/(2*a)
8
            x_2 = (-b-sqrt(delta(a,b,c)))/(2*a)
9
            result = c(x_1, x_2)
10
     else if(delta(a,b,c) == 0){ # second case D=0
11
12
            x = -b/(2*a)
13
14
     else {"There are no real roots."} # third case D<0</pre>
15 }
```

```
16
17 # Constructing delta
18 delta <-function(a,b,c){
         b^2-4*a*c
19
20 }
21
22
23 \#Total probability = 1
24 integrand=function(x)\{x*(2-x)\}
25 a=integrate(integrand,lower=0,upper=2)
26 a
27 y0=1/1.333
28
29 \quad u1d = (3*2^2)/(3*4)
                                         #Mean
30 \quad u2d = (3*2^3)/(4*5)
31 \quad u3d = (3*2^4) / (5*6)
32 \quad u4d = (3*2^5) / (6*7)
33 #Variance
34 \#u2 = u2d - u1d^2
35 u2=6/5-1
36 \#u3 = u3d - 3*u2d*u1d + 2*u1d^3
37 \quad u3=8/5-3*6/5*1+2
38 \#u4 = u4d - 4*u3d*u1d + 6*u2d*u1d^2 - 3*u1d^4
39 \quad u4=16/7-4*8/5*1+6*6/5*1-3*1
40 beta1=u3^2/u2^3
41 beta1
42 beta2=u4/u2^2
43 beta2
44 \#Since beta1 =0, symmetrical distribution
45 #Mean deviation about mean
46 integrand=function(x) \{3/4*(1-x)*x*(2-x)\}
47 f1=integrate(integrand, lower=0, upper=1)
48 f1
49 integrand=function(x) \{3/4*(x-1)*x*(2-x)\}
50 f2=integrate(integrand, lower=1, upper=2)
51 f2
52 #Mean deviation abt mean=f1+f2
53 M=0.1875+0.1875
```

```
54
55 #Harmonic Mean
integrand=function(x)\{3/4*(2-x)\}
57 a=integrate(integrand,lower=0,upper=2)
58 a
59 \text{ H.M} = 3/2
60
61 #Median
62 #integrand=function(x)\{3/4*x*(2-x)\}
63 #integrate (integrand, lower=0, upper=M)=1/2
64 #Solving above integral we get quadratic equation
      ---->(M-1)*(M^2-2*M-2)=0
65 \text{ b=result}(1,-2,-2)
66 b
67 #Other solution of b
68 b = 1
69 \#Since, M lying in [0,2]
70 sprintf("The median is: \%f", b)
```

R code Exa 5.13 Standard Deviation And Mean Deviation

```
#Page number -- 5.18
#Example number -- 5.13

#Mean
integrand=function(x) {x*(3+2*x)/18}
a1=integrate(integrand,lower=2,upper=4)
a1
u1d=83/27

integrand=function(x) {x^2*(3+2*x)/18)}
a2=integrate(integrand,lower=2,upper=4)
a1
u2d=88/9
```

```
15 #Varience
16 v=u2d-u1d^2
17
18 #Standard Deviation
19 s.d=sqrt(v)
20
21 #Mean Deviation
22 integrand=function(x) {(83/27-x)*(3+2*x)/18}
23 f1=integrate(integrand,lower=2,upper=83/27)
24 f1
25 integrand=function(x) {(x-83/27)*(3+2*x)/18}
26 f2=integrate(integrand,lower=83/27,upper=4)
27 f2
28 #Mean deviation=f1+f2
29 M.D=0.247264+0.247264
```

R code Exa 5.19 Probability Density Function

```
1 #Page number -- 5.21
2 #Example number -- 5.19
3
4 #Part (i)
5 integrand = function(x) {100/x^2}
6 a = integrate(integrand, lower = 100, upper = 150)
7 a
8 #Probability that tubes be replaced in first 150 hrs
.
9 p = (1/3)^3
10
11 #Part (ii)
12 #Probability that none of tubes be replaced in first 150 hrs.
13 p = (1-1/3)^3
14
15 #Part (iii)
```

```
16 #Probability that tube last more than 150 but less
      than 200 hrs.
17 integrand=function(x) \{100/x^2\}
18 a1=integrate(integrand,lower=150,upper=200)
19 a1
20 integrand=function(x) \{100/x^2\}
21 a2=integrate(integrand, lower=150, upper=Inf)
22 a2
23 \#a1 = 0.1666667
                   a2 = 0.6666667
24 p=0.1666667/0.6666667
25
26 #Part (iv)
27 #Maximum number of tubes
28 n = \log(0.5) / \log(0.6667)
29 n
```

${f R}$ code ${f Exa}$ 5.22 Probability That Tyres Will Last

```
1 \# \text{Page number} --5.24
2 \# \text{Example number} --5.22
3
4 #Part (i)
5 \#P(X \le 10)
6 integrand=function(x)\{1/20*exp(-x/20)\}
7 a=integrate(integrand,lower=0,upper=10)
8 a
9
10 #Part (ii)
11 \#P(16 <= X <= 24)
12 integrand=function(x)\{1/20*exp(-x/20)\}
13 al=integrate(integrand, lower=16, upper=24)
14 a1
15
16 #Part (iii)
17 \#P(X>=30)
```

```
18 integrand=function(x){1/20*exp(-x/20)}
19 a3=integrate(integrand,lower=30,upper=Inf)
20 a3
```

R code Exa 5.24 Probability Density Function

```
1 \#Page number -5.27
2 \# \text{Example number} --5.24
3 #LOAD PACKAGE---->ploynom
4
5 #Part (i)
6 integrand=function(x)\{6*x*(1-x)\}
7 a=integrate(integrand,lower=0,upper=1)
9 sprintf("The function p.d.f")
10
11 #Part (ii)
12 #The expression is
13 \# 0, if x \le 0
14 \#(3*x^2-2*x^3), if 0 < x < 1
15 \# 1, if x > 1
16
17 #Part (iii)
18 integrand=function(x) \{6*x*(1-x)\}
19 al=integrate(integrand, lower=1/3, upper=1/2)
20 a1
21 integrand=function(x)\{6*x*(1-x)\}
22 a2=integrate(integrand, lower=1/3, upper=2/3)
23 a2
24 \#a1 = 0.2407407
                      a2 = 0.4814815
25 p=0.2407407/0.4814815
                       #Answer may vary because of
26 p
      rounding off values
27
28 #Part (iv)
```

Chapter 6

Mathematical Expectations

R code Exa 6.1 Laws of Expectation

```
1 \#Page number -6.11
2 \# \text{Example number} - -6.1
 3
4 x = c(-3, 6, 9)
5 \text{ px} = c(1/6, 1/2, 1/3)
6 a=data.frame(x,px)
 7 a
8 #E(X)
9 EX = sum(a[1:3,1]*a[1:3,2])
10 EX
11 #E(X<sup>2</sup>)
12 EX2 = sum((a[1:3,1])^2*a[1:3,2])
13 EX2
14 \#E(2X+1)^2=4*E(X^2)+4*E(X)+1
15 EX3 = 4 * EX2 + 4 * EX + 1
16 EX3
```

R code Exa 6.2 Expectations Results

```
1 \#Page number -6.11
2 \# \text{Example number} - -6.2
3 #LOAD PACKAGE---->prob
5 s=rolldie(2,makespace=TRUE)
6 s
7
8 #Part (a)
9 #E(X)
10 EX = sum(1/6 * seq(1,6,,))
11 EX
12
13 #Part (b)
14 #Probability distribution on basis on sum of two
      faces of die
15 x=c(seq(2,12,1))
16 px=c(1/36,2/36,3/36,4/36,5/36,6/36,5/36,4/36,3/36,2/
      36,1/36)
17 a=data.frame(x,px)
18 a
19 EX=sum(a[1:11,1]*a[1:11,2])
20 EX
```

R code Exa 6.3 Expected Value Of X

```
#Page number -- 6.12
#Example number -- 6.3
#LOAD PACKAGE ----> prob

s=tosscoin(4)
s
n.o.h=c(4,3,3,3,3,2,2,2,2,2,2,1,1,1,1,0)
data.frame(s,n.o.h)
PX0=1/6; PX1=4/16; PX2=6/16; PX3=4/16; PX4=1/16
```

```
11
12 #Probability Distribution
13 x=c(0,1,2,3,4)
14 px=c(PX0,PX1,PX2,PX3,PX4)
15 data.frame(x,px)
16 EX=sum(x*px)
17 EX
```

R code Exa 6.4 Urn Example

```
#Page number -- 6.13
#Example number -- 6.4

#PX0 = choose (3,2) / choose (10,2)
FX1 = choose (7,1) * choose (3,1) / choose (10,2)

PX2 = choose (7,2) / choose (10,2)

x = c(0,1,2)
px = c(PX0,PX1,PX2)
#Probability Distribution
1 a = data.frame(x,px)
1 a
    EX = sum(x*px)
1 EX
```

R code Exa 6.5 Expectation Of Player

```
1 #Page number -- 6.13
2 #Example number -- 6.5
3
4 Event=c("Lucky number", "Special lucky no.", "Other numbers")
```

R code Exa 6.29 Joint Distribution Of Y And X1

```
1 \#Page number -6.36
2 \# \text{Example number} - -6.29
3 #LOAD PACKAGE---->prob
5 s=rolldie(2)
6 s
8 x = c(1,2,3,4,5,6)
9 y1=c(1/36,0,0,0,0,0)
10 y2=c(1/36,2/36,0,0,0,0)
11 y3=c(1/36,1/36,3/36,0,0,0)
12 y4=c(1/36,1/36,1/36,4/36,0,0)
13 y5=c(1/36,1/36,1/36,1/36,5/36,0)
14 y6=c(1/36,1/36,1/36,1/36,1/36,6/36)
15 Marginal_Totals_y=c(1/36,3/36,5/36,7/36,9/36,11/36)
16 Marginal_Totals_x=c(6/36,6/36,6/36,6/36,6/36,6/36)
17 a=data.frame(y1,y2,y3,y4,y5,y6,Marginal_Totals_x,
     Marginal_Totals_y)
18 a
19
20 EY=1*1/36+2*3/36+3*5/36+4*7/36+5*9/36+6*11/36
22 EY2=1^2*1/36+2^2*3/36+3^2*5/36+4^2*7/36+5^2*9/36+6^2
```

```
*11/36
23 EY2
24 VY=EY2-EY^2
25 VY
26 EX1=6/36*(1+2+3+4+5+6)
27 EX1
28 EX1Y=(1*1/36+2*1/36+3*1/36+4*1/36+5*1/36+6*1/36)+(4*2/36+6*1/36+8*1/36+10*1/36+12*1/36)+(9*3/36+12*1/36+15*1/36+18*1/36)+(16*4/36+20*1/36+24*1/36)+(25*5/36+30*1/36)+36*6/36

29 EX1Y
30 COV=EX1Y-EX1*EY
31 COV
```

R code Exa 6.30 Joint Probability Distribution

```
1 \#Page number -6.37
2 \# \text{Example number} - -6.30
 3
4 #Part (i)
 5 EY = -1 * 2 + 0 * 6 + 1 * 2
6 EY
 7 EX = -1 * 2 + 0 * 4 + 1 * 4
8 EX
9
10 #Part (ii)
11 EXY = -1 * -1 * 0 + 0 * -1 * 1 + 1 * -1 * 1 + 0 * -1 * 2 + 0 * 0 * 2 + 0 * 1 * 2 + 1 * -1 *
       0+1*0*1+1*1*1
12 EXY
13 COV = EXY - EX * EY
14 COV
15
16 #Part (iii)
17 EY2 = -1^2 \cdot 0.2 + 0 \cdot 0.6 + 1^2 \cdot 0.2
18 EY2
```

```
19 VY=EY2-EY^2
20 VY
21 \quad \text{EX2} = -1^2 \times 0.2 + 0 \times 0.4 + 1^2 \times 0.4
22 EX2
23 \quad VX = EX2 - EX^2
24 VX
25
26 #Part (iv)
                                          \#P(X=-1|Y=0)
27 \text{ PXY} = 0.2 / 0.6
28 \text{ PXY} = 0.2 / 0.6
                                          \#P(X=0|Y=0)
                                          \#P(X=1|Y=0)
29 \text{ PXY} = 0.2 / 0.6
30
31 #Part (v)
32 \quad \text{EYX} = -1 * 0 + 0 * 2 + 1 * 0
33 EYX2=1*0+0*2+0
34 \quad VYX = EYX2 - EYX^2
35 VYX
```

Chapter 7

Generating Functions And Law Of Large Numbers

R code Exa 7.12 Probability Of Getting 80 to 120 Sixes

```
1 \#Page number -7.31
2 \#\text{Example number} - 7.12
4 #S be total number of successes
                  #Number of times die thrown
5 n = 600
6 p=1/6; q=5/6
7 ES=n*p
8 VS=n*p*q
10 #Using Chebychev's inequality
11 \#P\{|S-ES|< k*s.d\}>=1-1/k^2 \#s.d--->Standard
      deviation
12 \#P\{100-k*sqrt(500/6)<S<100+k*sqrt(500/6)\}>=1-1/k^2
13 #comparing lower limit with 80
14 \text{ k=solve(sqrt(500/6),20)}
15 k
16 \#P\{80 \le \le \le 120\}
17 P=1-1/k^2
18 sprintf("Probabilities of getting 80 to 120 sixes: \%
```

R code Exa 7.13 Use Chebychev Inequality

```
#Page number -- 7.32
#Example number -- 7.13

#By proof of Bernoulli's law of large numbers
#P{|X/n-p|<e}<=1-1/(4*n*e^2)

p=0.5
#X/n lie between 0.4 and 0.6

e=0.1

#We want the probability to be atleast 0.9

n=solve(0.1,1/0.04)

sprintf("Required number of tosses: %f",n)</pre>
```

R code Exa 7.14 How Large Is Sample

```
1 #Page number -- 7.32
2 #Example number -- 7.14
3
4 #Part (i)
5 n = solve (0.05, 1/(4*0.02**2))
6 sprintf("The true proportions: %f",n)
7
8 #Part (ii)
9 n = solve (0.05, 0.16/0.02^2)
10 n
```

R code Exa 7.17 Discrete Variate With Density

```
1 \#Page number -7.17
2 \# \text{Example number} - -7.33
4 x = c(-1,0,1)
5 \text{ px} = c(1/8, 6/8, 1/8)
6 data.frame(x,px)
7 EX = sum(x*px)
                           #Mean
8 EX
9 EX2 = sum(abs(x*px))
10 EX2
11 VarX=EX2-(EX)^2
                                  #Variance
12 VarX
13 s.d=sqrt(VarX)
                                     #Standard Deviation
14 \text{ s.d}
15 P=1-px[2]
16 P
17 px [2]
```

R code Exa 7.18 Two Unbiased Dice

```
11 prob = c(1/36, 2/36, 3/36, 4/36, 5/36, 6/36, 5/36, 4/36, 3/36, 4/36, 3/36, 4/36, 5/36, 6/36, 5/36, 4/36, 3/36, 4/36, 3/36, 4/36, 5/36, 6/36, 5/36, 4/36, 3/36, 4/36, 5/36, 6/36, 5/36, 4/36, 3/36, 4/36, 5/36, 6/36, 5/36, 4/36, 3/36, 4/36, 5/36, 6/36, 5/36, 4/36, 3/36, 4/36, 5/36, 6/36, 5/36, 4/36, 3/36, 4/36, 5/36, 6/36, 5/36, 4/36, 5/36, 6/36, 5/36, 4/36, 5/36, 6/36, 5/36, 4/36, 5/36, 6/36, 5/36, 4/36, 5/36, 6/36, 5/36, 4/36, 5/36, 5/36, 6/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36, 5/36
                                                36,2/36,1/36)
12 a=data.frame(X, cases, prob)
13 a
14 EX = sum(X*prob)
15 EX
16 EX2 = sum(X^2 * prob)
17 EX2
18 \quad VarX = EX2 - EX^2
19 VarX
20 #By Chebychev's inequality
21 P = (35/6)/9
22 P
23 #Actual Probability
24 P1=1-(4+5+6+5+4)/36
25 P1
```

R code Exa 7.19 Chebychev Inequality

```
1 #Page number -- 7.34
2 #Example number -- 7.19
3
4 EX = (1+2+3+4+5+6)/6
5 EX
6 EX2 = (1^2+2^2+3^2+4^2+5^2+6^2)/6
7 EX2
8 VarX = EX2 - EX^2
9 VarX
10 #By Chebychev's inequality
11 k = 2.5
12 #P{|X<u|>2.5}<T
13 T = VarX/k^2
14 T
15 #But since the number lies outside the limits
16 #It cannot lie outside the limits of 1 and 6</pre>
```

17 #Actual probability = 0

Chapter 8

Special Discrete Probability Distributions

R code Exa 8.1 Probability Of Getting At Least 7 Heads

```
#Page number -- 8.5
#Example number -- 8.1

#Probability of getting head
p=1/2
#Probability of not getting head
q=1/2
#Probability of getting x heads
#P=choose(10,x)*(1/2)^10
#Probability of getting atleast 7 heads
P=(choose(10,7)+choose(10,8)+choose(10,9)+choose(10,10))*(1/2)^10
P
```

R code Exa 8.2 A And B Play A Game

```
1 #Page number -- 8.5
2 #Example number -- 8.2
3 n = 5
4 #Probability A wins the game
5 p = 3/5
6 q = 1 - p
7 #Probability of getting A wins
8 #P = choose (5,x)*(3/5)^x*(2/5)^(5-x)
9 #Probability of getting 3 wins
10 P = (choose (5,3)*2**2+choose (5,4)*3*2+1*3^2*1)*3^3/5^5
11 P
```

R code Exa 8.3 Chances Of Having The Claim Accepted Or Rejected

```
1 \#Page number --8.6
2 \# \text{Example number} - -8.3
3 n = 6
4 #Probability of correct distinction between coffee
5 p = 3/4
6 q = 1 - p
7 #Probability of getting x correct distinction
      between coffee
8 #P=choose (6,x)*(3/4)^x*(1/4)^(6-x)
9 #Probability of getting atleast 5 correct
      distinction between coffee
10 P = choose(6,5)*(3/4)**5*(1/4) + choose(6,6)*(3/4)^6
11 P
12 #Probability that the claim is rejected
13 \quad Q = 1 - P
14 Q
```

R code Exa 8.4 Probability That Student Secures Distinction

```
1 #Page number -- 8.6
2 #Example number -- 8.4
3 n = 8
4 #Probability of getting answer correct
5 p = 1/3
6 q = 1 - p
7 #Probability of getting x answer correct
8 #P = choose (8,x)*(1/3)^x*(2/3)^(8-x)
9 #Probability of getting 6 answer correct
10 P = choose (8,6)*(1/3)^6*(2/3)^2+choose (8,7)*(1/3)^7*(2 /3)+choose (8,8)*(1/3)^8
11 P
```

R code Exa 8.6 Probability

```
1 \#Page number -8.7
2 \# \text{Example number} - -8.6
4 #Probability that old machine need adjustment
5 p1=1/11
6 q1 = 1 - p
7 #Probability that new machine need adjustment
8 p2=1/21
9 q2=1-p
10 #Probability that x old machine need adjustment
11 #P=choose (3,x)*(1/11)^x*(10/11)^(3-x)
12 #Probability that x new machine need adjustment
13 #P=choose (7,x)*(1/21)^x*(20/21)^(7-x)
14
15 #Part (i)
16 #Probability that 2 old machine and no new machine
      need adjustment
17 P = choose(3,2)*(1/11)^2*(10/11)*(20/21)^7
18 P
19
```

R code Exa 8.7 Probability Man Hitting A Target

```
1 \#Page number -8.8
2 \# \text{Example number} - -8.7
4 #Probability of hitting target
5 p=1/4
6 q = 1 - p
7 #Probability that x hits in 7 shots
8 #P=choose (7,x)*(1/4)^x*(3/4)^7-x
9
10 #Part (i)
11 #Probability of hitting atleast 2 hits
12 P=1-(choose(7,0)*(1/4)^0*(3/4)^7-0+choose(7,1)*(1/4)
      ^1*(3/4)^(7-1))
13 P
14
15 #Part (ii)
16 #Probability of atleast 1 hit in n shots
17 p=solve(log(3/4), log(1/3))
18 p
19 round(p)
20 sprintf("The required number of shots: %d",round(p))
```

R code Exa 8.9 Find Parameter P

```
1 \#Page number -8.9
```

```
2 #Example number -- 8.9
3
4 n=5
5 p1=0.4096
6 p2=0.2048
7 #Probability distribution
8 #P=choose(5,x)*(p)^x*(1-p)^(5-x)
9 #p1=choose(5,1)*(p)^1*(1-p)^(5-1)
10 #p2=choose(5,2)*(p)^x*(1-p)^(5-2)
11 #The ratio of p1 and p2
12 sprintf("The parameter p is: %f", solve(5,1))
```

R code Exa 8.10 P For Binomial Variate X

```
1 \#Page number --8.9
2 \# \text{Example number} - -8.10
3
4 # Constructing Quadratic Formula
  result <- function(a,b,c){
     if(delta(a,b,c) > 0){\# first case D>0}
6
7
            x_1 = (-b + sqrt(delta(a,b,c)))/(2*a)
8
            x_2 = (-b-sqrt(delta(a,b,c)))/(2*a)
9
            result = c(x_1, x_2)
10
     else if(delta(a,b,c) == 0){ # second case D=0
11
            x = -b/(2*a)
12
13
14
     else {"There are no real roots."} # third case D<0
15 }
16
17 # Constructing delta
18 delta <-function(a,b,c){
         b^2-4*a*c
19
20 }
21
```

```
#Probability Distribution
#P=choose(6,x)*(p)^x*(1-p)^6-x
#9*P(X=4)=P(X=2)
#Solving above equation
#We get quadratic equation---->8*p^2+2*p-1=0
a=result(8,2,-1)
a
#Probability cannot be negative
sprintf("p: %f",a[1])
```

R code Exa 8.12 Probability

```
1 #Page number -- 8.11
2 #Example number -- 8.12
3
4 # Mean
5 M=4
6
7 #Variance
8 Var=4/3
9
10 q=Var/M
11 p=1-q
12 n=M/p
13
14 # P(X>=1) = 1 - P(X=0)
15 P=1-q^6
16 P
```

R code Exa 8.21 Fit Binomial Distribution

```
    #Page number -- 8.19
    #Example number -- 8.21
```

```
3
4 n=5
5 N = 4096
6 #Probability of success throw 4,5,6
7 p=1/2
8 q = 1 - p
9 #Probability distribution
10 #P=choose (12,x)*(1/2)^x*(1/2)^(12-x)
11 \#f(x) = choose(12, x)
12 success=c(0,1,2,3,4,5,6,7,8,9,10,11,12)
13 Expected_Frequency=c()
14 for(i in success){
15
       Expected_Frequency[i+1] = choose(12,i)
16 }
17 data.frame(success, Expected_Frequency)
18 Total=sum(Expected_Frequency)
19 Total
```

R code Exa 8.22 Fit A Binomial Distribution

```
1 \#Page number -8.20
2 \# \text{Example number} - -8.22
3
4 n=7
5 N = 128
6 #CASE 1: When the coin is unbaised
7 p=1/2
8 q = p
9 x = c(seq(0,7,1))
10 f=c(7,6,19,35,30,23,7,1)
11 fx=f*x
12 \#a = (n-x)/(x+1)
13 \ a=c()
14 for(i in x){
       a[i+1]=(n-i)/(i+1)
15
```

```
16 }
17 \#b = (n-x)/(x+1)*p/q
18 b = c()
19 for(i in x){
20
        b[i+1] = (n-i)/(i+1)*p/q
21 }
22 Expected_Frequency=c(1,1*7,7*3,21*5/3,35*1,35*3/5,21
      *1/3,7*1/7)
23 data.frame(x,f,fx,a,b,Expected_Frequency)
24
25 #CASE 2: When nature of coin is not known
26 \text{ mean} = 433/128
27 p=mean/n
28 q = 1 - p
29 p/q
30 \text{ x=c}(\text{seq}(0,7,1))
31 \#a = (n-x)/(x+1)
32 a = c()
33 for(i in x){
        a[i+1]=(n-i)/(i+1)
34
35 }
36 \#b = (n-x)/(x+1)*p/q
37 \, b = c \, ()
38 \text{ for}(i \text{ in } x){
        b[i+1]=(n-i)/(i+1)*p/q
39
40 }
41 Expected_Frequency=c(1.2593, 1.259*6.546, 2.805*
      8.243, 1.558*23.129, 0.935*36.05, .5611*33.715, .3117
      *18.918,.1336*5.897)
42 data.frame(x,f,fx,a,b,Expected_Frequency,round(
      Expected_Frequency))
43
44 #Part (iii)
45 \text{ f} = c (128 * .0625 * .1663, 8.591, 128 * .283, 128 * .184, 128 *
       .260,128*.146,128*.043,128*.0056)
46 data.frame(x,f,round(f))
47 Total=sum(round(f))
48 Total
                                          #Note that total is
```

R code Exa 8.33 Proportion Of Days

```
#Page number -- 8.35
#Example number -- 8.33

#For poisson distribution
#Part (i)
a = dpois(0,1.5)
sprintf("Proportion of days on which neither car is used: %f",a)

#Part (ii)
b = ppois(2,1.5,lower = FALSE)
sprintf("Proportion on which some demand is refused: %f",b)
```

R code Exa 8.34 Probability That More Than 3 Collect Policy In A Year

```
#Page number -- 8.35
#Example number -- 8.34

n=4000
#Probability of loss of both eyes
p=10/100000
#For poisson distribution
lambda=n*p
a=ppois(3,lambda,lower=FALSE)
printf("Probability that more than 3 will collect the policy: %f",a)
```

R code Exa 8.37 Random Sample Of 5 Page With No Error

```
1 #Page number -- 8.37
2 #Example number -- 8.37
3
4 lambda = 390/520
5 lambda
6 #For poisson distribution
7 a = dpois(0, lambda)
8 sprintf("Sample of 5 pages with no error: %f",a^5)
```

R code Exa 8.41 Probability That X Have More Than On Value

```
1 #Page number -- 8.38
2 #Example number -- 8.41
3
4 lambda=2
5 #For poisson distribution
6 a=dpois(1,lambda)
7 b=dpois(2,lambda)
8 sprintf("Required probability: %f",a+b)
```

R code Exa 8.55 Fit A Poisson Distribution

```
1 #Page number - -8.46
2 #Example number - -8.55
3
4 n_m_p=c(0,1,2,3,4) #Number of mistakes per page
```

```
#Number of pages
5 \text{ n_o_p=c}(109,65,22,3,1)
6 data.frame(n_m_p,n_o_p)
8 #Poission distribution parameter
9 lambda=sum(n_m_p*n_o_p)/200
10 lambda
11 Expected_Frequency=c()
12 \ a=c()
13 for(i in n_m_p){
       a[i+1] = dpois(i, lambda)
15 }
16 for(i in n_m_p){
17
       Expected_Frequency[i+1]=200*a[i+1]
18 }
19 data.frame(n_m_p, Expected_Frequency, round(Expected_
      Frequency))
```

R code Exa 8.56 Errors In 1000 Pages

```
1 \#Page number --8.46
2 \# \text{Example number} - -8.56
4 \quad lambda=2/5
5 \text{ n_o_e=c}(0,1,2,3,4)
                                  #Number of errors
6 prob=c()
7 for(i in n_o_e){
8
       prob[i+1] = dpois(i,lambda)
9 }
10 Expected_Frequency=c()
11 for(i in n_o_e){
       Expected_Frequency[i+1]=1000*prob[i+1]
12
13 }
14 data.frame(n_o_e,prob,Expected_Frequency,round(
      Expected_Frequency))
```

R code Exa 8.57 Fit A Poisson Distribution

```
1 \#Page number -8.47
2 \# \text{Example number} - -8.57
4 \text{ n_o_d=c}(0,1,2,3,4,5,6,7,8)
                                                #Number of
      doddens
5 \text{ o\_f} = c (56, 156, 132, 92, 37, 22, 4, 0, 1)
                                                #Observed
      frequency
6 data.frame(n_o_d,o_f)
7 Mean=sum(n_o_d*o_f)/500
8 lambda=Mean
9 a=c()
10 for(i in n_o_d){
       a[i+1] = lambda/(i+1)
11
12 }
13 prob=c()
14 for(i in n_o_d){
15
       prob[i+1] = dpois(i, lambda)
16 }
17 Expected_Frequency=c()
18 for(i in n_o_d){
19
       Expected_Frequency[i+1]=500*prob[i+1]
20 }
21 data.frame(n_o_d,a,prob,Expected_Frequency,round(
      Expected_Frequency))
```

R code Exa 8.63 Fit A Negative Binomial Distribution

```
1 #Page number -- 8.54
2 #Example number -- 8.63
3
```

```
#Number of cells
4 n_o_c=c(0,1,2,3,4,5)
                                    #frequency
5 \text{ f=c}(213,128,37,18,3,1)
6 Mean=sum(n_o_c*f)/400
7 u2=sum(f*n_o_c^2)/400
8 u2
9 Var=u2-Mean^2
10 p=0.6825/0.8117
11 q = 1 - p
12 r = round(p*0.6825/q)
13 a=c(p^4,0.5738*0.5,2.5*0.1592*0.2869,2*0.1592*
     0.1142,7/4*0.1592*0.0364,8/5*0.1592*0.0101)
14 Expected_Frequency=c()
15 for(i in n_o_c){
       Expected_Frequency[i+1]=400*a[i+1]
16
17 }
18 round(Expected_Frequency)
```

Chapter 9

Special Continuous Probability Distributions

R code Exa 9.1 Equation Of Normal Curve

```
1 \#Page number --9.14
2 \# \text{Example number} --9.1
3 #Load Package—>cubature
5 library(cubature)
6 N = 1000
7 u = 79.945
8 \text{ s.d=} 5.545
9 class=c("Below 60","60-65","65-70","70-75","75-80","
      80-85", "85-90", "90-95", "95-100", "100 and over")
10 \ l_c_b = c(-Inf, 60, 65, 70, 75, 80, 85, 90, 95, 100)
11 z = c()
12 for(i in c(seq(1,10,1))){
13
        z[i]=(l_c_b[i]-u)/s.d
14 }
15 z
16 #Equation of normal curve
17 \#f(x) = 1000 / sqrt(2*pi) *exp(-1/2*((x-u)/s.d)^2)
18 \text{ oZ} = \text{c}()
```

```
19 for(i in c(seq(1,10,1))){
                                 # define the integrated function
20
                                  f \leftarrow function(x) \{exp(-x^2/2)\}
21
                                  a=adaptIntegrate(f, lowerLimit =-Inf, upperLimit
 22
                                                     =z[i]
                                   oZ[i]=1/sqrt(2*pi)*a$integral
23
24 }
25 \text{ oZ}
26 deltaZ=c
                             (0.000112, 0.002914, 0.031044, 0.147870, 0.322050, 0.319300, 0.144072, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.000112, 0.0
                            NA)
27 Expected_Frequency=c()
28 for(i in c(seq(1,10,1))){
                                   Expected_Frequency[i]=N*deltaZ[i]
 29
30 }
31
32 data.frame(class,l_c_b,z,oZ,deltaZ,Expected_
                            Frequency,round(Expected_Frequency))
 33 Total=sum(round(Expected_Frequency[1:9]))
 34 Total
```

R code Exa 9.3 Probability

```
1 #Page number -- 9.15
2 #Example number -- 9.3
3
4 Mean = 12
5 s.d = 4 #Standard Deviation
6
7 #Part (i)
8 pnorm(Inf, 12, 4) - pnorm(20, 12, 4)
9 x = seq(0, 30, length = 200)
10 y = dnorm(x, 12, 4)
11 plot(x, y, type = "l")
12 x = seq(20, 25, length = 100)
```

```
13 y = dnorm(x, 12, 4)
14 polygon(c(20,x,25),c(0,y,0),col="red")
15
16 #Part (ii)
17 p=1-(pnorm(Inf,12,4)-pnorm(20,12,4))
18 p
19
20 #Part (iii)
21 pnorm (12,12,4)-pnorm (0,12,4)
22 x = seq(0,30,length=200)
23 y = dnorm(x, 12, 4)
24 plot(x,y,type="l")
25 \text{ x=seq}(0,12,length=100)
26 \ y = dnorm(x, 12, 4)
27 polygon(c(0,x,12),c(0,y,0),col="red")
28
29 #Part (a)
30 #Taking value of z1 from Normal Tables
31 z1=0.71
32 a = 12 + 4 * z1
33 a
34 #Part (b)
35 #Taking value of z1 from Normal Tables
36 z1=0.67
37 \times 1 = 12 + 4 \times z1
38 x1
39 \times 0 = 12 - 4 \times z1
40 x0
```

R code Exa 9.4 X Is A Normal Variate

```
1 #Page number -- 9.16
2 #Example number -- 9.4
3
4 Mean = 30
```

```
#Standard Deviation
5 \text{ s.d=} 5
6
7 #Part (i)
8 pnorm (40,30,5)-pnorm (26,30,5)
9 \text{ x=seq}(0,60,length=200)
10 y = dnorm(x, 30, 5)
11 plot(x,y,type="l")
12 x = seq(26,40,length=100)
13 y = dnorm(x, 30, 5)
14 polygon(c(26,x,40),c(0,y,0),col="red")
15
16 #Part (ii)
17 pnorm(Inf,30,5)-pnorm(45,30,5)
18 x = seq(0,50, length = 200)
19 y = dnorm(x, 30, 5)
20 plot(x,y,type="l")
21 x = seq(45,50, length = 100)
22 \ y = dnorm(x, 30, 5)
23 polygon(c(45,x,50),c(0,y,0),col="red")
24
25 #Part (iii)
26 \#P(|X-30|>5)=1-P(|X-30|<=5)
27 P = 1 - 2 * 0.3413
28 P
```

R code Exa 9.5 Batch Of 1000 Plots

```
1 #Page number -- 9.17
2 #Example number -- 9.5
3
4 Mean = 662
5 s.d = 32  #Standard Deviation
6
7 #Part (i)
8 P1 = pnorm(Inf, 662, 4) - pnorm(700, 662, 32)
```

```
9 sprintf("Probability that plots has a yield over 700
       kilos : %f",P1)
10 #Batch off 1000 plots
11 sprintf("Expected number of plots with yield over
      700 kilos: \%f", as.integer(1000*P1))
12
13 #Part (ii)
14 P2=pnorm (650,662,32)
15 sprintf("Probability that plots has a yield less
      than 650 kilos : %f", P2)
16 #Batch off 1000 plots
17 sprintf("Expected number of plots with yield less
      than 650 \text{ kilos}: \%f", as.integer(1000*P2))
18 #Answer vary due to rounding of values
19
20 #Part (iii)
21 \#P(X>x1)=100/1000
p=100/1000
                         #From Normal Table
23 z1=1.28
24 \times 1 = 662 + 32 \times z1
25 sprintf("Best 100 plots have yield: %f kilos",x1)
```

R code Exa 9.7 Number Of Lamps

```
11 sprintf("Expected number of bulb fail in less than
      800 hrs.: \%f", round(10000*P1))
12
13 #Part(ii)
14 P2=pnorm(1200,1000,200)-pnorm(800,1000,200)
15 sprintf("Expected number of bulb fail in 800-1200 hrs
      .: \%f", as.integer(10000*P2))
16 x = seq(0,2000, length = 200)
17 y = dnorm(x, 1000, 200)
18 plot(x,y,type="l")
19 x = seq(800, 1200, length = 100)
20 y = dnorm(x, 1000, 200)
21 polygon(c(800,x,1200),c(0,y,0),col="red")
22 #Part (a)
23 #Taking value of z1 from Normal Tables
24 z1=1.28
25 \times 1 = 1000 - 200 \times z1
26 x1
27 #Part (b)
28 #Taking value of z1 from Normal Tables
29 z2=1.28
30 x2 = 1000 + 200 * z1
31 x2
```

R code Exa 9.8 Probability Of Marks

```
#Page number -- 9.20
#Example number -- 9.8

Mean = 65
s.d = 5  #Standard Deviation

P1 = pnorm(Inf, 65, 5) - pnorm(70, 65, 5)
sprintf("Probability that marks are over 70: %f", P1)
#### students selected at random, exactly 2 will get
```

```
more than 70
10 choose(3,2)*P1^2*(1-P1)
```

R code Exa 9.9 Probability

```
1 \#Page number --9.20
 2 \# \text{Example number} --9.9
 3
4 #Part (a)
 5 \text{ Y1} = \log(1.202)
 6 Y2 = log(7.92)
7 \#For Y1=0.08
8 z1 = (0.08 - 4)/2
9 \#For Y2=7.92
10 \quad z2 = (7.92 - 4) / 2
11 \#P(0.08 < Y < 7.92)
12 p = 2 * 0.4750
13 sprintf("Required Probability: %f",p)
14
15 #Part (b)
16 y1 = log(1.202)
17 y2 = log(83180000)
18 \#U = \log(X) - \log(Y)
19 \#P(0.08 < U < 7.92)
20 p = 2 * 0.4750
21 sprintf("Required Probability: %f",p)
```

R code Exa 9.12 Mean And Standard Deviation Of The Distribution

```
1 #Page number -- 9.22
2 #Example number -- 9.12
3
4 #P(X<25)=0.1003
```

```
5 p1=0.1003
6 \#P(X < 70) = 0.8997
7 p2=0.8997
8 #When X=25, Z=(25-u)/s. d=-z1
9 #When X=70, Z=(70-u)/s. d=z2
10
11 \#P(0 < Z < z^2) = 0.3997
                      #From Normal Table
12 z2=1.28
13
14 \#P(0 < Z < z1) = 0.5 - 0.1003 = 0.3997
15 z1=1.28
                          #From Normal Table
16
17 #Solving two variables equation
18 a=matrix(c(1,1,-1.28,1.28),nrow=2,ncol=2)
19 b=matrix(c(25,70),nrow=2,ncol=1)
20 c = solve(a,b)
21 c
22 sprintf("The mean: \%f Kg", c[1,1])
23 sprintf("The standard deviation: \%f Kg", c[2,1])
```

R code Exa 9.13 Mean And Standard Deviation

```
1 #Page number --9.23

2 #Example number --9.13

3 

4 #P(X<75)=0.58

5 #P(X>80)=0.04

6 #When X=75, Z=(75-u)/s.d=z1

7 #When X=80, Z=(80-u)/s.d=z2

8 

9 #P(0<Z<z2)=1.75 #From Normal Table 11 

12 #P(0<Z<z1)=0.20 #From Normal Table
```

```
14
15 #Solving two variables equation
16 a=matrix(c(1,1,z1,z2),nrow=2,ncol=2)
17 b=matrix(c(75,80),nrow=2,ncol=1)
18 c=solve(a,b)
19 c
20 sprintf("The mean: %f ",c[1,1])
21 sprintf("The standard deviation: %f ",c[2,1])
```

R code Exa 9.14 Percentage Of Students Placed In Second Division

```
1 \#Page number --9.24
2 \# \text{Example number} --9.14
3
4 \#P(X<30)=0.10
5 \#P(X>80)=0.05
6 #When X=30, Z=(30-u)/s. d=-z1
7 #When X=80, Z=(80-u)/s. d=z2
9 \#P(0 < Z < z2) = 0.5 - 0.05
10 z2=0.45
                           #From Normal Table
11
12 \#P(0 < Z < z1) = 0.50 - 0.10
13 z1 = 0.40
                           #From Normal Table
14
15
16 #Solving two variables equation
17 a=matrix(c(1,1,-1.28,1.64),nrow=2,ncol=2)
18 b=matrix(c(30,80),nrow=2,ncol=1)
19 c = solve(a,b)
20 c
21 sprintf("The mean: %f ",c[1,1])
22 sprintf("The standard deviation: %f ",c[2,1])
23
24 \#P(45 < X < 60)
```

R code Exa 9.15 Probability Of the Defectives In The Sample

```
1 \#Page number --9.25
2 \# \text{Example number} --9.15
3
                  # Total items
4 n = 100
5 p = 0.4
6 \quad q = 0.6
                  # Mean
7 u=n*p
8 s.d=sqrt(n*p*q) # Standard Deviation
10 #Normal Distribution
11 #Part (i)
12 \#P(43.5 < X < 100.5)
13 a=pnorm(100.5,u,s.d)-pnorm(43.5,u,s.d)
14 a
15
16 #Part (ii)
17 \#P(X=44) \longrightarrow P(43.5 < X < 44.5)
18 b = pnorm(44.5, u, s.d) - pnorm(43.5, u, s.d)
19 b
20
21 #Binomial Distribution
22 \#P(X>=44)
23 sum(dbinom(44:100,100,0.4))
24 \#P(X=44)
25 dbinom (44,100,0.4)
26
27 sprintf("We can see that both Normal and Binomial
      Distribution are close to each other")
```

R code Exa 9.21 Probability

```
1 \#Page number --9.31
2 \# \text{Example number} --9.21
3 #Load Package—>cubature
5 library(cubature)
7 \text{ mean=1}
8 \text{ variance} = 4/3
9 \#mean=(a+b)/2=1
10 \#b+a=2
11 \# variance = (b-a)^2/12 = 4/3
12 \# b - a = 4
13
14 #Solving two variables equation
15 x=matrix(c(1,-1,1,1),nrow=2,ncol=2)
16 y=matrix(c(2,4),nrow=2,ncol=1)
17 z = solve(x,y)
18 z
19
                      \# -1 < x < 3
20 p = 1/4
21
22 # define the integrated function
23 f \leftarrow function(x){p}
24 a=adaptIntegrate(f, lowerLimit =-1, upperLimit =0)
25 a
```

R code Exa 9.22 Probability

```
1 \# Page number --9.31
```

```
2 #Example number -- 9.22
3 #Load Package --> cubature
4
5 library(cubature)
6
7 #P(X>=20)
8
9 fx=1/30  # 0<x<30
10 # define the integrated function
11 f <- function(x){fx}
12 a=adaptIntegrate(f, lowerLimit =20, upperLimit =30)
13 a
14 sprintf("Probability that he has to wait at least 20 min: %f",a$integral)</pre>
```

Chapter 10

Correlation

R code Exa 10.1 Correlation Coefficient

```
1 \#Page number -10.7
2 \# \text{Example number} - -10.1
3
4 X = c(65, 66, 67, 67, 68, 69, 70, 72)
5 Y=c(67,68,65,68,72,72,69,71)
6 meanX=sum(X)/length(X)
8 meanY=sum(Y)/length(Y)
9 meanY
10 \, \text{U=X-meanX}
11 V=Y-meanY
12 data.frame(X,Y,U,V,U^2,V^2,U*V)
13 u=sum(U)/length(U)
14 v=sum(V)/length(V)
15 covUV = (sum(U*V) - u*v) / length(Y)
16 s.dU=sqrt(1/length(Y)*(sum(U^2)-u^2))
17 s.dV=sqrt(1/length(Y)*(sum(V^2)-v^2))
18 rUV = covUV/(s.dU*s.dV)
19 \text{ rUV}
```

R code Exa 10.2 Correct Value Of Correlation Coefficient

```
1 \#Page number -10.8
2 \# \text{Example number} - -10.2
3
4 n = 25
5 \text{ CsX} = 125 - 6 - 8 + 8 + 6
                                 #Corrected sum of X
                                      #Corrected sum of Y
6 CsY = 100 - 14 - 6 + 12 + 8
7 CsX2=650-6^2-8^2+8^2+6^2
                                      #Corrected sum of X<sup>2</sup>
                                      #Corrected sum of Y^2
8 CsY2=460-14^2-6^2+12^2+8^2
9 CsXY = 508 - 6 * 14 - 8 * 6 + 8 * 12 + 6 * 8
                                      #Corrected sum of XY
10 meanX = CsX/n
11 meanY=CsY/n
12 covXY=1/n*CsXY-meanX*meanY
13 s.dX=sqrt(1/n*CsX2-meanX^2)
14 s.dY=sqrt(1/n*CsY2-meanY^2)
15 rXY = covXY/(s.dX*s.dY)
16     round(rXY,2)
```

R code Exa 10.14 Correlation Coefficient

```
1 #Page number -- 10.18
2 #Example number -- 10.14
3
4 n=100
5 v=c(-2,-1,0,1,2,3)
6 g=c(8,19,35,22,10,6)
7 a=v^2*g
8 uf=c(-19,0,31,56)
9 u2f=c(19,0,31,112)
10 suvf=c(9,0,13,30)
11 meanU=sum(uf)/n
```

```
12  meanV=sum(v*g)/n
13  covUV=1/n*sum(suvf)-meanU*meanV
14  s.dU=sqrt(1/n*sum(u2f)-meanU^2)
15  s.dU
16  s.dV=sqrt(1/n*sum(a)-meanV^2)
17  s.dV
18  rUV=covUV/(s.dU*s.dV)
19  rUV
```

R code Exa 10.15 Correlation Coefficient Between X And Y

```
1 \# \text{Page number} - -10.20
 2 \# \text{Example number} - -10.15
 3
4 X = c(-1, 1)
5 Y = c(0,1)
6 X1=c(1/8,2/8)
7 X2 = c(3/8, 2/8)
8 \text{ gy} = X1 + X2
9 px=c(3/8,5/8)
10 data.frame(Y,X1,X2,gy)
11
12 EX = sum(X*px); EX
13 EX2 = sum(X^2*px); EX2
14 \text{ varX}=EX2-EX^2
15 varX
16
17 EY = sum(Y * gy); EY
18 EY2=sum(Y^2*gy); EY2
19 varY=EY2-EY^2
20 varY
21
22 \text{ EXY} = 0 * -1 * 1 / 8 + 0 * 1 * 3 / 8 + 1 * -1 * 2 / 8 + 1 * 1 * 2 / 8
23 EXY
24 \text{ covXY} = \text{EXY} - \text{EX} \times \text{EY}
```

```
25 rXY=covXY/sqrt(varX*varY)
26 rXY
```

R code Exa 10.17 Rank Correlation Coefficient

```
1 #Page number -- 10.25
2 #Example number -- 10.17
3
4 n=16
5 r_i_m=c(seq(1,16,1)) #
Ranks in math
6 r_i_p=c(1,10,3,4,5,7,2,6,8,11,15,9,14,12,16,13)
#Ranks in physics
7 d = r_i_m-r_i_p
8 data.frame(r_i_m,r_i_p,d,d^2)
9
10 #Rank correlation coefficient
11 p=1-(6*sum(d^2))/(n*(n^2-1))
12 p
```

R code Exa 10.18 Rank Correlation Between 3 Judges

```
1 #Page number -- 10.25
2 #Example number -- 10.18
3
4 n=10
5 a=c(1,6,5,10,3,2,4,9,7,8)
6 b=c(3,5,8,4,7,10,2,1,6,9)
7 c=c(6,4,9,8,1,2,3,10,5,7)
8 d1=a-b
9 d2=a-c
10 d3=b-c
11 data.frame(a,b,c,d1,d2,d3,d1^2,d2^2,d3^2)
```

```
12
13 #Rank correlation coefficient
14 Pab=1-(6*sum(d1^2))/(n*(n^2-1))
15 Pab
16 Pac=1-(6*sum(d2^2))/(n*(n^2-1))
17 Pac
18 Pbc=1-(6*sum(d3^2))/(n*(n^2-1))
19 Pbc
```

R code Exa 10.19 Rank Correlation Coefficient

```
#Page number -- 10.26
#Example number -- 10.19

X = c (68,64,75,50,64,80,75,40,55,64)

Y = c (62,58,68,45,81,60,68,48,50,70)

RankX = c (4,6,2.5,9,6,1,2.5,10,8,6)

RankY = c (5,7,3.5,10,1,6,3.5,9,8,2)

d = RankX - RankY

data . frame (X,Y,RankX,RankY,d,d^2)

#Total correction

Correction = 2*(4-1)/12+3*(9-1)/12

p = 1 - (6*(sum(d^2)+Correction+1/2))/(n*(n^2-1))

p
```

Chapter 11

Linear And Curvilinear Regression

R code Exa 11.1 Equations Of Two Lines Of Regression

```
1 \# Page number - 11.9
2 \# \text{Example number} - -11.1
4 X = c (65,66,67,67,68,69,70,72)
5 Y=c(67,68,65,68,72,72,69,71)
6 #Equation of line regression of Y on X
7 b=lm(Y^X)
8 a=summary(b)$coefficients[1,1]
                 # Intercept
10 c=summary(b)$coefficients[2,1]
11 c
                 # Slope
12 #From the above, we can get the equation of Y on X
13 \#Equation ---->Y=c*X+a----->Y=0.66*X+23.66
14
15 #Equation of line regressiono of X on Y
16 \text{ e=lm}(X^Y)
17 f=summary(e)$coefficients[1,1]
18 f
                 # Intercept
19 g=summary(e)$coefficients[2,1]
```

```
20 g # Slope
21 #From the above, we can get the equation of X on Y
22 #Equation—>X=g*Y+f——>X=0.545*Y+30.36
23
24 #Calculating the value
25 x=0.545*70+30.36
26 x # Answer little vary due to rounding off
```

R code Exa 11.2 Record An Analysis On Correlation Data

```
1 \#Page number -11.10
2 \#Example number --11.2
4 a=matrix(c(8,40,-10,-18),nrow=2,ncol=2)
5 \text{ b=matrix}(c(-66,214),nrow=2,ncol=1)
6 r = solve(a,b)
8 x=r[1,1]
9 y=r[2,1]
10 by x = 8/10
                           #Regression coefficient of Y
      on X
11 bxy = 18/40
                           #Regression coefficient of X
      on Y
12 z=sqrt(byx*bxy)
13 z
14 s.dy=solve(1/5,4/5)
                           #Standard deviation of Y
15 \text{ s.dy}
```

R code Exa 11.3 Correlation Coefficient

```
1 #Page number - 11.11
2 #Example number - - 11.3
3
```

```
4 kolkata = c(65, 2.5)
5 \text{ mumbai} = c(67, 3.5)
6 type=c("Average Price", "Standard Deviation")
7 q=data.frame(type,kolkata,mumbai)
8 q
9
10 x = q[1,2]; x
11 s.dx=q[2,2]; s.dx
                                     #Standard deviation
      of x
12 y = q[1,3]; y
                                          #Standard
13 s.dy = q[2,3]; s.dy
      deviation of y
14
15 slope=0.8*3.5/2.5
16 intercept = 67+0.8*65*3.5/2.5
17 #From the above, we can get the equation of Y on X
18 #Equation ---->Y=slope*X+intercept
19
20 #Calculating the value
21 Y = 67 + 0.8 * 3.5 / 2.5 * (70 - 65)
22 Y
```

R code Exa 11.6 Coefficient Of Regression And Regression Equation

```
#Page number - 11.14
#Example number - - 11.6

dobs=c(seq(1,10,1))

X=c(1,1,2,2,3,3,4,5,6,7)

Y=c(2,7,7,10,8,12,10,14,11,14)

data.frame(obs,X,Y,X^2,X^3,X^4,X*Y,X^2*Y)

a=matrix(c(10,34,154,34,154,820,154,820,4774),nrow = 3,ncol = 3)

b=matrix(c(95,377,1849),nrow=3,ncol = 1)
```

```
11 t=solve(a,b)
12 t
13 #From the above, we can get the equation
14 #Equation—> Y= 1.80 + 3.48*X - 0.2689*X^2
```

R code Exa 11.7 Fit Exponential Curve

```
1 \#Page number -11.15
2 \# \text{Example number} - -11.7
3
4 obs=c(seq(1,8,1))
5 \text{ X} = \mathbf{c} (1,2,3,4,5,6,7,8)
6 Y=c(1.0,1.2,1.8,2.5,3.6,4.7,6.6,9.1)
7 U = log(Y)
8 data.frame(obs,X,Y,U,X*U,X^2)
10 a=matrix(c(8,36,36,204),nrow=2,ncol=2)
11 b=matrix(c(3.7393,22.7385),nrow=2,ncol=1)
12 t=solve(a,b)
13 t
14 #Taking antilog
15 b=0.1408
16 d=0.6821
17
18 \#Equation ----> Y= 0.6821* (1.38) \hat{X}
```

Chapter 12

Additional Topics On Correlation And Regression

R code Exa 12.7 Partial And Multiple Correlation Coefficient

```
1 #Page number -- 12.31
2 #Example number -- 12.7
3
4 r12=0.77
5 r13=0.72
6 r23=0.52
7
8 r12.3=(r12-r13*r23)/sqrt((1-r13^2)*(1-r23^2))
9 r12.3
10 R1.23=sqrt((r12^2+r13^2-2*r12*r13*r23)/(1-r23^2))
11 R1.23
```

R code Exa 12.8 Distribution

```
    #Page number - -12.32
    #Example number - -12.8
```

```
3
4 \text{ s.d1=2}
5 \text{ s.d2=3}
6 \text{ s.d3=3}
7 r12=0.7
8 \text{ r21=0.7}
9 r13=0.5
10 \text{ r31=0.5}
11 \text{ r} 23 = 0.5
12 \text{ r}32=0.5
13
14 #Part (i)
15 r23.1=(r23-r21*r31)/sqrt((1-r21^2)*(1-r31^2))
16 r23.1
17 #Part (ii)
18 R1.23=\sqrt{(r12^2+r13^2-2*r12*r13*r23)/(1-r23^2)}
19 R1.23
20 #Part (iii)
21 r12.3=0.6
22 r13.2=0.2425
23 \text{ s.d1.3} = 2 * \text{sqrt} (1 - \text{r13}^2); \text{s.d1.3}
24 \text{ s.d2.3=3*sqrt}(1-r23^2); \text{s.d2.3}
25 s.d1.2=2*sqrt(1-r12^2);s.d1.2
26 \text{ s.d3.2=3*sqrt}(1-r23^2); \text{s.d3.2}
27 b12.3=r12.3*s.d1.3/s.d2.3;b12.3
28 b13.2=r13.2*s.d1.2/s.d3.2;b13.2
29
30 #Part (iv)
31 w=matrix(c(1,r12,r13,r12,1,r23,r13,r23,1),nrow=3,
       ncol=3)
32 det(w)
33 w1=matrix(c(1,r23,r23,1),nrow=2,ncol=2)
34 det(w1)
35 \text{ s.d1.23=2*sqrt(det(w)/det(w1));s.d1.23}
```

R code Exa 12.9 Regression Equation

```
1 \#Page number -12.33
2 \# \text{Example number} - -12.9
3
4 r12=0.8
5 \text{ r21=0.8}
6 \text{ r13} = -0.4
7 \text{ r31} = -0.4
8 \text{ r} 23 = -0.56
9 \text{ r}32 = -0.56
10
11 w=matrix(c(1,r12,r13,r12,1,r23,r13,r23,1),nrow=3,
      ncol=3)
12 det(w)
13 w11=matrix(c(1,r23,r23,1),nrow=2,ncol=2)
14 det(w11)
15 w12=matrix(c(r21,r31,r23,1),nrow=2,ncol=2)
16 \det(w12)*-1
17 w13=r23*r12-r13
18 w13
19
20 #Required equation
21 \#0.686/4.42*(X1-28.02)-0.576/1.10*(X2-4.91)-0.048/85
      *(X3-594)
```

Theory Of Attributes

R code Exa 13.2 Finding Frequencies

```
1 \#Page number -13.5
2 \#Example number -13.2
4 ABi=738; AjC=225; Aji=1196
5 \text{ kBC} = 204; \text{kBi} = 1762; \text{kjC} = 171;
6 \text{ kji} = 21842
7 \text{ ABC} = 149
9 A = ABC + ABi + AjC + Aji; A
10 B=ABC+ABi+kBC+kBi; B
11 C = ABC + AjC + kBC + kjC; C
12 AB=ABC+Aji; AB
                                         # Answer is wrong in
       the example
13 AC = ABC + AjC; AC
14 BC=ABC+kBC; BC
15
16 N=ABC+ABi+AjC+Aji+kBC+kBi+kjC+kji; N
```

R code Exa 13.11 Find if A And B Are Independent Or Positively Associated Or Negatively Associated

```
1 \#Page number --13.13
2 \#Example number -13.11
4 #Part (i)
5 N = 1000
6 A=470; B=620; AB=320
7 k = AB - A * B / N
8 if (k>0) print ("A and B are positively associated")
10 #Part (ii)
11 A=490; AB=294; i=570; iB=380
12 N = A + i
13 B = AB + iB
14 k = AB - A * B / N
15 if (k<0) print ("A and B are negatively associated")
16
17 #Part (iii)
18 AB = 256; iB = 768; Aj = 48; ij = 144
19 \quad A = AB + Aj
20 B = AB + iB
21 N = AB + Aj + iB + ij
22 k = AB - A * B / N
23 if (k<0) print("A and B are independent")
```

R code Exa 13.12 Heredity In A Family

```
1 #Page number -- 13.14
2 #Example number -- 13.12
3
4 #A:: Dark eye-colour of father
5 #B:: Dark eye-colour of son
```

```
7 AB=50; Aj=79; iB=89; ij=782
9 Q = (AB*ij-Aj*iB)/(AB*ij+Aj*iB)
                                               #Yule's
      Coefficient of Association
10 Q
11
12 A = AB + Aj; A
13 B=AB+iB;B
14 i=iB+ij;i
15 j = Aj + ij; j
16 N = A + i
17 N
18 AB0=A*B/N; round (ABO)
19 Aj0=A*j/N; round(Aj0)
20 iB0=i*B/N; round(iB0)
21 ij0=i*j/N;round(ij0)
```

R code Exa 13.14 Coefficient Of Association

```
1 \#Page number -13.15
2 \# \text{Example number} - -13.14
3
4 #A:: Boys
5 #i:: Girls
6 #B:: Successful candidates
7 #j:: Failed candidates
8
9 N = 800
10 Aj = 80; iB = 260
11 AB=380; ij=80
12 Attributes=c("B","j")
13 A = c (380,80)
14 i = c(260,80)
15 Total=c(A[1]+i[1],A[2]+i[2])
16 data.frame(Attributes, A, i, Total)
```

R code Exa 13.15 Coefficient Of Association

```
1 \#Page number -13.15
2 \# \text{Example number} - -13.15
3
4 #A:: Majority
5 #i:: Minority
6 #B:: Regular players
7 #j:: Not Regular players
9 i = 250 + 200 + 150
10 sprintf("No. of minority students: %d",i)
11 \quad A = 120 + 100 + 80
12 sprintf("No. of major students: %d",A)
13 iB=200+150+90
14 sprintf("No. of minor regular players: %d",iB)
15 \quad AB = 48 + 30 + 12
16 sprintf("No. of major regular players: %d", AB)
17
18 Aj = A - AB; Aj
19 ij=i-iB;ij
20
21 Q = (AB*ij-Aj*iB)/(AB*ij+Aj*iB)
                                              #Yule's
      Coefficient of Association
22 Q
```

Large Sample Theory

R code Exa 14.1 Limits Between Which Probability 3 Or 4 Lies

```
1 #Page number --14.12
2 \# Example number - -14.1
3
4 n = 9000
5 X = 3240
7 #H0::That die is unbiased
8 #Probability of success
9 P=1/6+1/6
10 Q = 1 - P
11 #H1::That die is baised
12
13 Z=(X-n*P)/sqrt(n*P*Q)
14
15 #Probability limits
16 p = 3240/9000; q = 1-p
17 11=p-3*sqrt(p*q/n)
18 ul=p+3*sqrt(p*q/n)
19 sprintf("Probability of getting 3 or 4 certainly
      lies between %f and %f", round(11,3), round(u1,3))
```

R code Exa 14.2 Percentage Of Bad Pineapples

```
1 \#Page number -14.13
2 \# \text{Example number} - -14.2
3
4 n = 500
5 X = 65
                     #Proportion on Number of bad
6 p=X/n
      pineapples
7 q = 1 - p
8 #S.E of proportion
9 \text{ s.e=} \text{sqrt}(p*q/n)
10 #Probability limits
11 ll=p-3*sqrt(p*q/n)
12 ul=p+3*sqrt(p*q/n)
13 sprintf("Percentage of bad pineapples in the
      consignment between %f and %f", round(11,3), round(
      ul,3))
```

R code Exa 14.4 Rice And Wheat Eaters

```
10 Q=1-P
11
12 Z=(p-P)/sqrt(P*Q/n)
13
14 #At 1% level of significance
15 if(Z<2.532)
16 sprintf("Hence, H0 hypothesis accepted")
17 sprintf("Rice and wheat are equally popular in Maharashtra State")</pre>
```

R code Exa 14.6 Number Of Observations

```
1 #Page number -- 14.14
2 #Example number -- 14.6
3
4 P=0.95
5 p=40/200
6 q=1-p
7 n=p*q*3.8416/0.0025
8 round(n)
```

R code Exa 14.7 Test For Significance For Difference Of Proportions

```
1 #Page number -- 14.16
2 #Example number -- 14.7
3
4 #H1: P1!=P2 (two tailed test)
5 n1=400
6 n2=600
7 X1=200; X2=325
8 p1=X1/n1
9 p2=X2/n2
```

```
11  p=(n1*p1+n2*p2)/(n1+n2)
12  q=1-p
13
14  z=abs((p1-p2)/sqrt(p*q*(1/n1+1/n2)))
15  z
16  if(z<1.96)  print("Mean and women do not differ on flyover proposal")</pre>
```

R code Exa 14.10 Test For Significance For Difference Of Proportions

```
1 \#Page number -14.19
2 \# \text{Example number} - -14.10
4 n1=1000; p1=800/1000; q1=1-p1
5 n2=1200; p2=800/1200; q2=1-p2
6
7 #H0: P1=P2
8 #H1: P1>P2 (Right-tailed test)
9
10 p=(n1*p1+n2*p2)/(n1+n2);p
11 \quad q = 1 - p
12
13 z=abs((p1-p2)/sqrt(p*q*(1/n1+1/n2)))
14 z
15 if(z>1.96) print("HO hypothesis is rejected")
16 sprintf("There is significant decrease in consuption
       of tea after increase in excise duty")
```

R code Exa 14.15 Proportion Of Failures

```
    1 #Page number - -14.22
    2 #Example number - -14.15
    3
```

```
4    n1=400; p1=300/400; q1=1-p1
5    n2=500; p2=300/500; q2=1-p2
6
7    #H0: P1=P2
8    #H1: P1!=P2 (two tailed test)
9
10    p=(n1*p1+n2*p2)/(n1+n2); p
11    q=1-p
12    s.e.p1p2=sqrt(p*q*1/(n1+n2)*500/400)
13    s.e.p1p2
14    z=(p-p1)/s.e.p1p2
15    abs(z)  # Answer is wrong in the book
16    sprintf("H0 is rejected")
```

R code Exa 14.18 Arithmetic Mean And Standard Deviation

```
1 \#Page number -14.26
2 \# \text{Example number} - -14.18
3
4 \# H0: u=30.5
5 \#H1: u < 30.5 \text{ (Left-tailed test)}
7 AgeLastBirthday=c("16-20","21-25","26-30","31-35","
      36-40")
8 f=c(12,22,20,30,16)
9 midx=c((16+20)/2,(25+21)/2,(26+30)/2,(31+35)/
      2,(36+40)/2
10 d = (midx - 28) / 5
11 \quad a=f*d
12 b=f*d^2
13 data.frame(AgeLastBirthday,f,midx,d,a,b)
14
15 u = 30.5
16 \quad x = 28 + 5 * 16 / 100
17 \text{ s=}5*\text{sqrt}(164/100-(16/100)^2)
```

```
18

19 z=abs((x-u)/sqrt(s^2/100))

20 z

21 if(z>1.645) print("Reject H0 hypothesis")
```

R code Exa 14.26 Food Expenditure Of Two Populations

```
1 #Page number -- 14.31
2 #Example number -- 14.26
3
4 n1=400; n2=400
5 x1=250; x2=220
6 s.d1=40; s.d2=55
7
8 #H0:: u1=u2
9 #H1:: u1!=u2 (two tailed test)
10
11 z=abs((x1-x2)/sqrt(40^2/400+55^2/400))
12 z
13 if(z>3) print("H0 rejected")
14 sprintf("Expenditure of two population of shoppers in market A and B differ significantly")
```

R code Exa 14.28 Difference In Average Weights Of Items

```
1 #Page number -- 14.32
2 #Example number -- 14.28
3
4 n1=250; n2=400
5 x1=120; x2=124
6 s.d1=12; s.d2=14
7 s.e.x1x2=sqrt(s.d1^2/n1+s.d2^2/n2)
8 s.e.x1x2
```

```
9
10 \# H0 :: u1=u2
11 #H1::u1!=u2 (two tailed test)
12
13 z=(x1-x2)/s.e.x1x2
14 z
15 if(z>3) print("H0 rejected")
16 sprintf("There is significant difference between
      sample means")
17
18 \# |u1-u2| limits
19 11 = abs(x1-x2) - 2.58 * s.e.x1x2
                                         #lower limit
20 ul = abs(x1-x2) + 2.58*s.e.x1x2
                                         #upper limit
21 sprintf("|u1-u2| varies between %f and %f",11,u1)
```

R code Exa 14.30 Relating Heights Of Country A And Country B

```
1 \#Page number -14.34
2 \# \text{Example number} - -14.30
3
4 n1=1000; n2=1200
5 \times 1 = 67.42; \times 2 = 67.25
6 \text{ s.d1}=2.58; \text{s.d2}=2.5
7
8 #Part (i)
9 #H0:: u1=u2
10 #H1::u1!=u2 (two tailed test)
12 z=abs((x1-x2)/sqrt(s.d1^2/n1+s.d2^2/n2))
13 z
14 if(z<1.96) print("H0 accepted")
15 sprintf("There is no significant difference between
      sample means")
16
17 #Part (ii)
```

Exact Sampling Distributions I

R code Exa 15.9 Precision Of Instrument

```
1 #Page number -15.25
2 \# \text{Example number} - -15.9
4 X=c(2.5,2.3,2.4,2.3,2.5,2.7,2.5,2.6,2.6,2.6,2.7,2.5)
5 m=sum(X)/length(X)
                                         # Mean
6 \quad a = round(X-m, 2)
7 b=a^2
8 data.frame(X,m,a,b)
9
10 #Null Hypothesis
11 \text{ HO} = 0.16
12 #Alternative Hypothesis
13 \#H1 > 0.16
14 \quad x = sum(b)/H0
15 x
16
17 sprintf("H0 may be accepted")
18 sprintf("The data are consistent with the hypothesis
       that the precision of the instrument is 0.16")
```

R code Exa 15.12 Digits And Frequency In Telephone Directory

```
1 \#Page number -15.27
2 \#\text{Example number} - -15.12
4 digits=c(seq(0,9,1))
5 \text{ f} = c(1026, 1107, 997, 966, 1075, 933, 1107, 972, 964, 853)
      # Observed Frequency
6 m=sum(f)/length(f)
                                                          #
      Mean
  e = rep(m, 10)
                                                # Expected
      Frequency
8 a = (f - e)^2
9 b = round(a/e,3)
10 data.frame(digits,f,e,a,b)
11
12 #H0 is null hypothesis
13 x = sum(b)
14 x
15
16 sprintf("HO may be rejected")
17 sprintf("The digits are not uniformly distributed in
       the directory")
```

R code Exa 15.14 Survey Of 800 Families

```
1 #Page number--15.29
2 #Example number--15.14
3
4 #Probability of male birth
5 p=1/2
6 #Probability of female birth
```

```
7 q = 1/2
8 \text{ n_m_b=c(seq(0,4,1))}
9 f=c(32,178,290,236,64)
                                          # Number of
      families
10 e = c()
11 for (i in n_m_b){
       e[i+1]=800*choose(4,i)*p^4 # Frequency of
          male birth
13 }
14 a = (f - e)^2
15 b = round(a/e,3)
16 data.frame(n_m_b,f,e,a,b)
17
18 #H0 is null hypothesis
19 x = sum(b)
20 x
21
22 sprintf("H0 may be rejected")
23 sprintf("Male and female births are not equally
      probable")
```

R code Exa 15.15 Fit a Poisson Distribution

R code Exa 15.18 8000 Graduates

```
1 \#Page number -15.34
2 \# \text{Example number} - -15.18
4 Class=c("Male employed", "Male unemployed", "Female
      employed", "Female unemployed")
5 \text{ f=c} (1480,5720,120,680)
                                                   #
      Observed Frequency
6 = c (7200*1600/8000,7200-1440,1600-1440,6400-5760)
      # Expected Frequency
7 a = (f - e)^2
8 b=round(a/e,2)
9 data.frame(Class,f,e,b)
10
11 #H0 is null hypothesis
12 x = sum(b)
13 x
14
15 sprintf("H0 may be rejected")
16 sprintf("Appointment is based on the basis of sex")
```

R code Exa 15.20 Two Researchers

```
    #Page number - -15.36
    #Example number - -15.20
```

```
3
4 E86=126*200/300
5 E60 = 93 * 200 / 300
6 \quad E44 = 69 * 200 / 300
8 Researcher=c("X","Y")
9 bavg=c(E86,126-E86)
                                       # Below Average
10 avg=c(E60,93-E60)
                                            # Average
11 aavg=c(E44,69-E44)
                                       # Above Average
12 genius=c(200-192,12-8)
13 data.frame(Researcher, bavg, avg, aavg, genius)
14
15 f = c(86,60,44,10,40,33,25,2)
16 \text{ e=} \text{c} (84,62,46,8,42,31,23,4)
17 a = (f - e)^2
18 b = round(a/e,3)
19 data.frame(f,e,a,b)
20
21 x = sum(b[1:6])
22 x
23 sprintf("No difference in sampling techniques used
      by two researchers")
```

Exact Sampling Distributions II

R code Exa 16.6 Sales Of Soap Bars Advertising Campaign

R code Exa 16.7 Sample Of 10 Boys IQ

```
1 #Page number -16.14
2 \# \text{Example number} - -16.7
3
4 n = 10
5 u = 100
6 #H0:: Null Hypothesis ----> mean IQ of 100 in the
      population u=100
7 #H1:: Alternative Hypothesis ---->u!=100
9 x=c(70,120,110,101,88,83,95,98,107,100)
10 m = sum(x)/n
                              #Mean
11 m
12 \quad a = x - m
13 b=a^2
14 data.frame(x,a,b)
15 \ s2 = sum(b)/9
16 s2
17 t=abs((m-u)/sqrt(s2/n))
18 t
19 sprintf("HO is accepted")
```

R code Exa 16.10 Gain In Weights By Diet A And B

```
10 x1=c(25,32,30,34,24,14,32,24,30,31,35,25)
11 \quad m1 = sum(x1)/n1
                              # Mean
12 m1
13 \quad a1 = x1 - m1
14 b1=a1^2
15 data.frame(x1,a1,b1)
16
17 #Diet B
18 x2=c(44,34,22,10,47,31,40,30,32,35,18,21,35,29,22)
19 m2 = sum(x2)/n2
                              # Mean
20 \text{ m}2
21 \quad a2 = x2 - m2
22 b2=a2^2
23 data.frame(x2,a2,b2)
24
25 	ext{ s2=(sum(b2)+sum(b1))/(n1+n2-2)}
26 t = abs((m1-m2)/sqrt(s2*(1/n1+1/n2)))
27 t
28
29 sprintf("HO is accepted")
30 sprintf("Two diets do not differ significantly")
```

R code Exa 16.12 Heights Of Sailors And Soldiers

```
1 #Page number -- 16.20
2 #Example number -- 16.12
3
4 # SAILORS
5 X = c (63,65,68,69,71,72)
6 A = 68
7 d = X - A
8 a = sum (d^2)
9 data frame (X,d,d^2)
10
11 # SOLDIERS
```

```
12 Y=c(61,62,65,66,69,69,70,71,72,73)
13 B=66
14 \, D = Y - B
15 b = sum(D^2)
16 data.frame(Y,D,D^2)
17
                                     # Mean
18 \text{ mx} = A + \text{sum}(d)/6; \text{mx}
19 p=a-(sum(d))^2/6;p
20
21 my=B + sum(D)/10; my
                                     # Mean
22 q=b-(sum(D))^2/10;q
23
24 S2 = (1/(6+10-2))*(p+q)
25 S2
26
27 t = (mx - my) / sqrt (S2 * (1/6 + 1/10))
28 t
29
30 sprintf("Null hypothesis can be retained")
31 sprintf("The sailors are on the average taller than
      the soilders.")
```

 $\bf R$ code Exa $\bf 16.15$ Increase In Weights Of Animals By Food A And Food B

```
1 #Page number -- 16.22
2 #Example number -- 16.15
3
4 # Part (i)
5 # Food A
6 X=c(49,53,51,52,47,50,52,53)
7 A=50
8 d=X-A
9 a=sum(d^2)
10 data.frame(X,d,d^2)
```

```
11
12 # Food B
13 Y=c(52,55,52,53,50,54,54,53)
14 B=52
15 \, D = Y - B
16 b = sum(D^2)
17 data.frame(Y,D,D^2)
18
19 mx = A + sum(d)/8; mx
                                    # Mean
20 p=a-(sum(d))^2/8;p
21
22 my=B + sum(D)/8; my
                                    # Mean
23 q=b-(sum(D))^2/8;q
24
25 S2=(1/(8+8-2))*(p+q)
26 S2
27
28 t=(mx-my)/sqrt(S2*(1/8+1/8))
29 t
30
31 sprintf("Null hypothesis rejected")
32 sprintf("Food B is superior to Food A")
33
34 # Part (ii)
35 X
36 Y
37 d = X - Y
38 data.frame(X,Y,d,d^2)
39
40 n = length(X)
41 \text{ md} = \text{sum}(d)/n
42 \, \text{md}
43 S2=1/(n-1)*(sum(d^2)-(sum(d))^2/n)
44 S2
45
46 t= abs(md/sqrt(S2/n))
47 round(t,3)
48
```

R code Exa 16.18 Random Sample Of 625 Pairs Of Observations

```
1 \#Page number -16.25
2 \# \text{Example number} - -16.18
4 #H0:: Null Hypothesis ---->p=0
5 \#H0:: The value of r=0.2 is not significant
7 r = 0.2
8 n = 625
9 t=r*sqrt(n-2)/sqrt(1-r^2)
10 t
11
12 sprintf("Null hypothesis is rejected")
13 sprintf("The sample correlation is significant of
      correlation in the population.")
14
15 #95% confidence limits
16 11=r - 1.96*(1-r^2)/sqrt(n)
                                               # lower
      limit
17 11
                                           # upper limit
18 ul=r + 1.96*(1-r^2)/sqrt(n)
19 ul
20
21 #99% confidence limits
22 \ 11=r - 2.58*(1-r^2)/sqrt(n)
                                               # lower
      limit
23 round(11,3)
24 \text{ ul=r} + 2.58*(1-r^2)/sqrt(n)
                                           # upper limit
25 ul
```

R code Exa 16.26 F Test

```
#Page number -- 16.38
#Example number -- 16.26

n1 = 8
n2 = 10
dx = 84.4
dy = 102.6
SX2 = dx / (n1 - 1); SX2
SY2 = dy / (n2 - 1); SY2

F = SX2 / SY2

if (F < 3.29) print ("H0 may be accepted")</pre>
```

R code Exa 16.27 F Test And T Test

```
1 \# Page number - -16.38
2 \#Example number -16.27
3
4 n1=10
5 n2=12
6 \text{ mx} = 15
7 \, \text{my} = 14
8 \, dx = 90
9 \, dy = 108
10
11 \# F-Test
12 SX2=dx/(n1-1); SX2
13 SY2=dy/(n2-1); SY2
14
15 F = SX2/SY2
16 F
```

```
17
18 if(F<2.90) print("H0 of equality of populatin may be accepted")
19
20 # T-Test
21 S2=1/(n1+n2-2)*(dx+dy)
22 S2
23 t=(mx-my)/sqrt(S2*(1/n1+1/n2))
24 t
25
26 sprintf("Samples have been drawn from the same normal population")</pre>
```

Some Additional Topics

R code Exa 20.1 Fit A Straight Line

```
1 #Page number -- 20.8
2 #Example number -- 20.1
3
4 x = seq(0,4,1)
5 e = x - 2
6 y = c(1,1.8,3.3,4.5,6.3)
7 q = e
8 y0 = y * e
9 data.frame(x,e,y,q,y0)
10
11 b0 = sum(y) / length(x)
12 b0
13 b1 = sum(y0) / sum(e^2)
14 b1
15 #WE get the following equation
16 # y = 1.33 * x + 0.72
```

R code Exa 20.2 Fit A Second Degree Parabola

```
1 \#Page number -20.9
2 \# \text{Example number} --20.2
4 x = seq(0.5,3,0.5)
5 e = 4 * x - 7
6 y=c(72,110,158,214,290,380)
7 q = e
8 q2=c(5,-1,-4,-4,-1,5)
9 y0=y*q
10 y1 = y * q2
11 data.frame(x,e,y,q,q2,y0,y1)
12
13 b0=sum(y)/length(x)
14 b0
15 b1=sum(y0)/sum(e^2)
17 b2 = sum(y1) / sum(q2^2)
18 b2
19
20 #WE get the following equation
21 \# y = 106.32 * x^2 - 128.04 * x + 83.08
```