Effects of Dirac sea on pion propagation in asymmetric nuclear matter

Subhrajyoti Biswas and Abhee K. Dutt-Mazumder
Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata-700 064, INDIA

We study pion propagation in asymmetric nuclear matter (ANM) with particular focus on the excitation of the Dirac sea. One of the interesting consequences of pion propagation in ANM is the mode splitting for the different charged states of pions. It is seen that the effect of Dirac vacuum is to enhance such splitting. Results for both the pseudovector (PV) and pseudoscalar (PS) couplings are presented.

PACS numbers: 13.75Gx, 13.75 Cs, 25.70Gh
Keywords: Dirac sea, isospin, symmetry, collective modes

I. INTRODUCTION

Pions in nuclear physics assume a special status. It is responsible for the spin-isospin dependent long range part of the nuclear force. In addition, there are variety of physical phenomena related to the pion propagation in nuclear matter. One of the fascinating ideas in relation to the pion-nucleon dynamics in nuclear matter is the pion condensation [1]. This might happen if there exists space like zero energy excitation of pionic modes. The short-range correlation, on the other hand, removes such a possibility at least at densities near the saturation densities. In the context of relativistic heavy ion collisions (RHIC), the importance of medium modified pion spectrum was discussed by Mishustin, where it was shown that due to lowering of pion energy pion in nuclear matter might carry a bulk amount of entropy [2]. Subsequently, Gyulassy and Greiner studied pionic instability in great detail in the context of RHIC. The production of pionic modes in nuclear collisions was also discussed in [3].

In experiments medium dependent pion dispersion relation can also be probed via the measurements of dilepton invariant mass spectrum. The lepton pairs produced with invariant mass near the ρ pole are sensitive to the slope of the pion dispersion relation in matter [4]. Particularly the softening of momentum dependence of the pion dispersion relation in matter leads to higher yield of the dileptons. Gale and Kapusta were first to realize that the in-medium pion dynamics can be studied by measuring lepton pair productions [5]. Most of the earlier studies of in-medium pion properties were performed in the non-relativistic frame work [6–8]. A quasi-relativistic approach was taken in [9–11] where the calculations were extended to finite temperature. In particular, [11] discusses various non-collective modes with the possibility of pion condensation. In [4], on the other hand, the dilepton production rates were calculated using non-relativistic pion dispersion relations. Ref.[12] treated the problem relativistically but free fermi gas model was used, while in [13] pion propagation was studied by extending the Walecka model including delta baryon. In recent years, there has been significant progress to calculate dilepton production rates involving pionic properties in a more realistic framework [4, 5, 11, 14–16].

In the present paper we study pion dispersion relations in ANM in a relativistic model. This is important as most of the calculations, as mentioned above, are either restricted to symmetric nuclear matter (SNM) or performed in the non-relativistic framework. Here we focus on the propagating modes of various charged states of pions which are non degenerate in ANM. The importance of relativistic corrections and density dependent pion mass splitting in ANM in the context of deriving pion-nucleus optical potential was discussed in [17]. The formalism adopted in [17] was that of chiral perturbation theory. Recently, in the context of astrophysics, pionic properties in ANM has also been studied by involving Nambu-Jona-Lasinio model [18, 19]. Motivated by [17] present authors revisited the problem in ref.[20] where not only the static self-energy responsible for the mass splitting but the full dispersion relations for the various charged states of pions were calculated after performing relevant density expansion in terms of the Fermi momentum. However, in our previous work the effect of the Dirac sea was completely ignored which, we address here. Moreover, unlike previous work, here we report results both for the pseudoscalar (PS) and pseudovector (PV) interactions. In [20] we have discussed another interesting possibility of the density driven π-η mixing in ANM. However, quantitatively, the mixing was found to be a higher order effect and does not affect the pion dispersion relations at the leading order in density. Hence in the present paper we neglect π-η mixing.

The plan of the paper is as follows. In section II we present the formalism followed by sub sections A and B where the results for self-energy and dispersion relations are presented for PV and PS interaction respectively. In section III we present our results and in section IV we conclude. Detailed derivations for the Dirac part of the pion self-energy for PV and PS couplings have been relegated to appendix A and B respectively.

II. FORMALISM

To study pion dispersion relations in ANM, we adopt the techniques of quantum hadrodynamics (QHD) [21]. The pion, while propagating through nuclear matter, receives self-energy corrections both due to particle-antiparticle ($N\bar{N}$) and particle-hole (Nh) excitations. The pion-self energy in matter can be evaluated in the same way as vacuum where only the free nucleon propagator has to be replaced by the medium modified propagator. This change is due to Pauli blocking which forbids on mass shell propagation of the nucleons below the Fermi sea. Explicitly,

$$G_i(k) = G_i^F(k) + G_i^D(k). \tag{1}$$

where,

$$G_i^F(k) = \frac{\not{k} + M_i^*}{k^2 - M_i^{*2} + i\zeta} \quad , \quad G_i^D(k) = \frac{i\pi(\not{k} + M_i^*)}{E_i^*}\delta(k_0 - E_i^*)\theta(k_i^F - |\mathbf{k}|) \tag{2}$$

Here, $G_i^F(k)$ and $G_i^D(k)$ represent the free and the density dependent part of the propagator. It should, however, be noted that in the free part the nucleon mass is replaced by its effective mass in matter. In Eq.(2) k is the nucleon momentum; k_i^F $(i = p, n)$ denotes the Fermi momentum and M_i^* is the in-medium nucleon mass modified due to scalar mean field while the modification of the propagator due to the vector mean field is ignored [22] in the present calculation. In the mean-field model, scalar and vector fields are replaced by their vacuum expectation values. The nucleon energy is $E_i^* = \sqrt{M_i^{*2} + \mathbf{k}^2}$. We, from now onwards, use k_p and k_n to denote the proton and neutron Fermi momentum respectively. In QHD the effective nucleon mass is determined from the following self-consistent condition [22].

$$M_i^* = M_i - \frac{g_s^2}{m_s^2}(\rho_p^s + \rho_n^s) \tag{3}$$

where M_i and m_s are the masses of free nucleon and sigma (σ) meson respectively and g_s is the sigma-nucleon coupling constant. The last term of Eq.(3) appears as the nucleon mass changes due to scalar mean field interaction [22] where ρ_i^s represents scalar density given by

$$\rho_i^s = \frac{M_i^*}{2\pi^2}\left[E_i^* k_i - M_i^{*2}\ln\left(\frac{E_i^* + k_i}{M_i^*}\right)\right] \tag{4}$$

It is clear from Eq.(3) that $\Delta M = M_n - M_p = \Delta M^*$ as the nucleon masses are modified by scalar mean field [22] which does not distinguish between n and p. Here, for the moment we neglect explicit symmetry breaking (n-p mass difference) *i.e.* $M_p^* = M_n^* = M^*$. By solving the Eq.(3) and Eq.(4) self-consistently M^* is calculated.

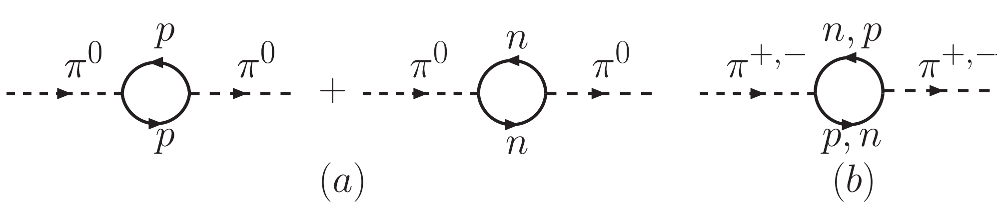

FIG. 1: (a) represents the one-loop self-energy diagram for π^0, and (b) represents the same for $\pi^{\pm}$.

To study how the pion masses are modified in the ANM, the in-medium self-energy of pion-nucleon interaction is calculated using the one-loop diagram shown in the Fig.(1). The self-energy is given by,

$$\Sigma^*(q) = -i\int \frac{d^4k}{(2\pi)^4} Tr[\{i\Gamma(q)\}iG_{p(n)}(k+q)\{i\Gamma(-q)\}iG_{p(n)}(k)] \tag{5}$$

where $G_{p(n)}$ denotes the in-medium proton (neutron) propagator. $\Gamma(q)$ is the vertex factor. It is $-i\gamma_5$ for PS coupling and $-i\frac{f_\pi}{m_\pi}\gamma_5\gamma_\mu q^\mu$ for PV coupling respectively. Using Eq.(1) and Eq.(2), the expression for self-energy given in Eq.(5) reduces to the following form :

$$\Sigma^*(q) = -ig^2 \int \frac{d^4k}{(2\pi)^4} \mathbf{T} \tag{6}$$

Here g is g_π (f_π/m_π) for PS (PV) coupling. For $\pi^\pm$ the coupling constant g_π (or f_π) has to be replaced by $\sqrt{2}g_\pi$ (or $\sqrt{2}f_\pi$ where $\sqrt{2}$ is the isospin factor for $\pi^\pm$). $\mathbf{T}$ is the trace factor and it consists of four parts :

$$\mathbf{T} = \mathbf{T}^{FF} + \mathbf{T}^{FD} + \mathbf{T}^{DF} + \mathbf{T}^{DD} \tag{7}$$

Detailed expressions for $\mathbf{T}^{FF}$, $\mathbf{T}^{FD}$ and $\mathbf{T}^{DF}$ will be discussed later. Here the term $\mathbf{T}^{DD}$ contains the product of two delta functions ($G^D(k)G^D(k+q)$) which put both the loop-nucleons on shell implying the cut in the loop (Fig.2a). This means that pion can decay into nucleon-antinucleon (Fig.2b) pair which happens only in the high momentum limit $i.e$ $q > 2k_{p,n}$ and also $q_0 > 2E^F_{p,n}$ where $E^F_{p,n}$ is the Fermi energy for proton (or neutron). Under this conditions only $\mathbf{T}^{DD}$ contributes to the self-energy. But in the present calculation, we investigate low momentum (of pion) collective excitations only [21]. Therefore $\mathbf{T}^{DD}$ is neglected.

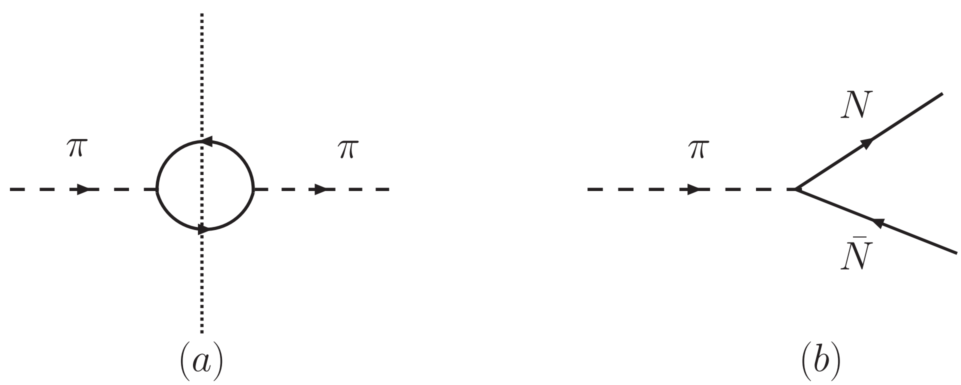

FIG. 2: (a) represents cutting of loop implied by the product of two delta functions and (b) represents the decay of pion into nucleon-antinucleon.

From Eq.(6) the pion self-energy can be written as

$$\begin{aligned} \Sigma^*(q) &= \Sigma^{*FF}(q) + \Sigma^{*(FD+DF)}(q) \\ &= -ig^2 \int \frac{d^4k}{(2\pi)^4} \mathbf{T}^{FF} - ig^2 \int \frac{d^4k}{(2\pi)^4} \left(\mathbf{T}^{FD} + \mathbf{T}^{DF}\right) \end{aligned} \tag{8}$$

The self-energies for different charged states of pion are calculated using the one-loop diagram shown in the Fig.(1). The loop diagram gives us the trace factor $\mathbf{T}$. The first term of Eq.(8) is same as the pion self-energy in vacuum with $M_i \to M_i^*$. This part is divergent.

A. PV coupling

To describe pion nucleon dynamics we first consider the pseudovector interaction which naturally gives rise to strong p-wave and weak s-wave interaction [23]. The disadvantage of the PV coupling, as is well known, is that it is not renormalizable. The PV interaction Lagrangian is given by,

$$\mathcal{L}^{PV} = -\frac{f_\pi}{m_\pi} \bar{\Psi}_N \gamma_5 \gamma^\mu \partial_\mu \left(\vec{\tau} \cdot \vec{\Phi}_\pi\right) \Psi_N \tag{9}$$

Here $f_\pi = 92.4$ [17] is called the weak pion decay constant. Ψ_N and Φ_π are the nucleon and pion field respectively and $\vec{\tau}$ is the Pauli isospin matrix.

First we discuss the FF part of pion self-energy for PV coupling. The trace factor of FF part is

$$\begin{aligned}\mathbf{T}^{FF}_{PV} &= -2\ Tr[\gamma_5\gamma^\mu q_\mu iG^F(k)\gamma_5\gamma^\nu q_\nu iG^F(k+q)] \\ &= -8\ \left[\frac{M^{*2}q^2 + k\cdot(k+q)q^2 - 2(k\cdot q)(k+q)\cdot q}{(k^2-M^{*2})((k+q)^2-M^{*2})}\right]\end{aligned} \tag{10}$$

Here factor 2 appears in Eq.(10) due to the fact that for π^0, proton loop and neutron loop are added (see Fig. 1a) and for $\pi^\pm$, each vertex of the loop diagram (see Fig. 1b) brings in a factor of $\sqrt{2}$. Now the FF part of the self-energy for PV coupling is denoted by $\Sigma^{*FF}_{PV}(q)$. From Eq.(8) and Eq.(10) we get,

$$\Sigma^{*FF}_{PV}(q) = 8i\left(\frac{f_\pi}{m_\pi}\right)^2\int\frac{d^4k}{(2\pi)^4}\left[\frac{M^{*2}q^2 + k\cdot(k+q)q^2 - 2(k\cdot q)(k+q)\cdot q}{(k^2-M^{*2})((k+q)^2-M^{*2})}\right] \tag{11}$$

The term $\Sigma^{*FF}_{PV}(q)$ in Eq.(11) is divergent. To remove the divergence we need to regularize $\Sigma^{*FF}_{PV}(q)$. But problem arises from the extra factor of q^2 in the numerator of the expression given in Eq.(11) for which all the divergences cannot be easily eliminated by simply adding the counterterms to the original Lagrangian and higher order diagrams require more counterterms, hence π-N pseudovector interaction is non-renormalizable. We use the dimensional regularization [24–26] technique to regularize the FF part of the pion self-energy which takes the following form after subtraction as shown in appendix A,

$$\Sigma^{*R}_{PV}(q) \;=\; \frac{q^2}{2\pi^2}\left(\frac{f_\pi}{m_\pi}\right)^2\left[2M^{*2}\int_0^1 dx\ \ln\left(\frac{M^{*2}-q^2x(1-x)}{M^{*2}-m_\pi^2x(1-x)}\right)\right] \tag{12}$$

The imaginary part of $\Sigma^{*FF}_{PV}(q)$ is

$$\mathrm{Im}\ \Sigma^{*FF}_{PV}(q) = -\left(\frac{f_\pi}{m_\pi}\right)^2\left[\frac{q}{\pi}2M^{*2}\sqrt{q^2-4M^{*2}}\right]\ \theta\left(q^2-4M^{*2}\right) \tag{13}$$

It is observed from Eq.(13) that Im $\Sigma^{*FF}_{PV}(q)$ is non-vanishing only if $q^2 > 4M^{*2}$. Now $\Sigma^{*R}_{PV}(q)$ is approximated to

$$\Sigma^{*R}_{PV}(q) \simeq \mathcal{C}q^2 - \mathcal{D}q^4 \tag{14}$$

where,

$$\left.\begin{aligned}\mathcal{C} &= \left(\tfrac{f_\pi M^*}{m_\pi \pi^2}\right)^2\left[\tfrac{m_\pi^2}{6M^{*2}}\right] \\ \mathcal{D} &= \left(\tfrac{f_\pi M^*}{m_\pi \pi^2}\right)^2\left[\tfrac{1}{6M^{*2}}\right]\end{aligned}\right\} \tag{15}$$

Now we discuss the (FD+DF) part of PV coupling. The trace of the (FD+DF) part for π^0,

$$T^{FD}_{PV} + T^{DF}_{PV} = Tr[\gamma_5\not{q}G^F_p(k+q)\gamma_5\not{q}G^D_p(k) + \gamma_5\not{q}G^D_p(k+q)\gamma_5\not{q}G^F_p(k)] + [p\rightarrow n] \tag{16}$$

and for $\pi^{+(-)}$,

$$T^{FD}_{PV} + T^{DF}_{PV} = Tr[\gamma_5\not{q}G^F_{p(n)}(k+q)\gamma_5\not{q}G^D_{n(p)}(k) + \gamma_5\not{q}G^D_{p(n)}(k+q)\gamma_5\not{q}G^F_{n(p)}(k)] \tag{17}$$

In pure neutron (or proton) matter one of the terms of Eq.(17) *viz* $G^D_{p(n)} = 0$ for the charged pion states. The same argument holds true for the neutral pion where only two terms would contribute which can be observed from Eq.(16). In case of pure neutron (or proton) matter $p(n)$ appears as the intermediate state. Now the $(FD + DF)$ part of the self-energy for π^0 and $\pi^\pm$ can be written as

$$\Sigma_{PV}^{*0(FD+DF)}(q) = -8\left(\frac{f_\pi}{m_\pi}\right)^2 \int \frac{d^3k}{(2\pi)^3 E^*} \mathbf{A}_{PV} \tag{18}$$

$$\begin{aligned}\Sigma_{PV}^{*\pm(FD+DF)}(q) &= -8\left(\frac{f_\pi}{m_\pi}\right)^2 \int \frac{d^3k}{(2\pi)^3 E^*} [\mathbf{A}_{PV} \mp \mathbf{B}_{PV}] \\ &= \Sigma_{PV}^{*0(FD+DF)}(q) \mp \delta\Sigma_{PV}^{*(FD+DF)}(q)\end{aligned} \tag{19}$$

Where

$$\mathbf{A}_{PV} = \left[\frac{M^{*2}q^4}{q^4 - 4(k\cdot q)^2}\right](\theta_p + \theta_n) \tag{20}$$

$$\mathbf{B}_{PV} = \frac{1}{2}\left[1 + \frac{4M^{*2}q^2}{q^4 - 4(k\cdot q)^2}\right](k\cdot q)(\theta_p - \theta_n) \tag{21}$$

with $\theta_{p,n} = \theta(k_{p,n} - |\mathbf{k}|)$. We restrict ourselves in the long wavelength limit *i.e.* when the pion momentum ($\mathbf{q}$) is small compared to the Fermi momentum ($k_{p,n}$) of the system where the many body effects manifests strongly. In this case particle propagation can be understood in terms of collective excitation [21] of the system which permits analytic solutions of the dispersion relations [21, 27]. But in the short wavelength limit *i.e.* when the pion momentum ($\mathbf{q}$) is much larger than the Fermi momentum ($k_{p,n}$), particle dispersion approaches to that of the free propagation.

In the long wavelength limit *i.e.* the collective excitations near the Fermi surface, permit us to calculate the $(FD + DF)$ part of the pion-self energy analytically. In this case we can neglect the term q^4 compared to the term $4(k\cdot q)^2$ from the denominator of $\mathbf{A}_{PV}$ and $\mathbf{B}_{PV}$ in Eqs. (20) and (21). This is called hard nucleon loop (HNL) approximation [27]. Explicitly, after a straight forward calculation, we get,

$$\Sigma_{PV}^{*0(FD+DF)}(q) = \frac{1}{2}\left(\frac{f_\pi M^*}{m_\pi \pi}\right)^2 \left[\left(\ln\left|\frac{1+v_p}{1-v_p}\right| - c_0 \ln\left|\frac{c_0+v_p}{c_0-v_p}\right|\right) + \left(\ln\left|\frac{1+v_n}{1-v_n}\right| - c_0 \ln\left|\frac{c_0+v_n}{c_0-v_n}\right|\right)\right] \tag{22}$$

and

$$\begin{aligned}\delta\Sigma_{PV}^{*(FD+DF)}(q) &= \left(\frac{f_\pi}{m_\pi \pi}\right)^2 \left[\frac{2}{3}k_p^3 q_0 - \frac{M^{*2}q^2}{|\mathbf{q}|}\left(E_p^* \ln\left|\frac{c_0+v_p}{c_0-v_p}\right| - \frac{2M^*}{\sqrt{c_0^2-1}} \tan^{-1}\frac{k_p\sqrt{c_0^2-1}}{c_0 M^*}\right)\right] \\ &- \left(\frac{f_\pi}{m_\pi \pi}\right)^2 \left[\frac{2}{3}k_n^3 q_0 - \frac{M^{*2}q^2}{|\mathbf{q}|}\left(E_n^* \ln\left|\frac{c_0+v_n}{c_0-v_n}\right| - \frac{2M^*}{\sqrt{c_0^2-1}} \tan^{-1}\frac{k_n\sqrt{c_0^2-1}}{c_0 M^*}\right)\right]\end{aligned} \tag{23}$$

where $v_{p,n} = k_{p,n}/E^*_{p,n}$, $E^*_{p,n} = \sqrt{M^{*2} + k^2_{p,n}}$ and $c_0 = q_0/|\mathbf{q}|$. The approximate results of Eqs.(22) and (23) are

$$\Sigma_{PV}^{*0(FD+DF)}(q) \simeq \mathcal{A}\frac{q^4}{q_0^2} + \mathcal{B}q^2 \tag{24}$$

$$\delta\Sigma_{PV}^{*(FD+DF)}(q) \simeq \mathcal{E}q_0 \tag{25}$$

where,

$$\left.\begin{aligned}\mathcal{A} &= \left(\tfrac{f_\pi M^*}{m_\pi \pi}\right)^2 \left[\tfrac{1}{3}\left(\tfrac{k_P^3}{E_p^{*3}} + \tfrac{k_n^3}{E_n^{*3}}\right)\right] \\ \mathcal{B} &= \left(\tfrac{f_\pi M^*}{m_\pi \pi}\right)^2 \left[\tfrac{1}{5}\left(\tfrac{k_P^5}{E_p^{*5}} + \tfrac{k_n^5}{E_n^{*5}}\right)\right] \\ \mathcal{E} &= \left(\tfrac{f_\pi M^*}{m_\pi \pi}\right)^2 \left[\tfrac{2}{5}\left(\tfrac{k_p^5}{M^{*4}} - \tfrac{k_n^5}{M^{*4}}\right)\right]\end{aligned}\right\} \tag{26}$$

The total pion self-energy for PV coupling is

$$\Sigma_{PV}^{*0,\pm}(q) = \Sigma_{PV}^{*R}(q) + \Sigma_{PV}^{*0,\pm(FD+DF)}(q) \tag{27}$$

Once the self-energy is calculated it can be used to solve the Dyson-Schwinger equation to get the dispersion relation.

$$q^2 - m^2_{\pi^{0,\pm}} - \Sigma^{*0,\pm}(q) = 0 \tag{28}$$

Here $m_{\pi^{0,\pm}}$ are the masses of π^0 and $\pi^\pm$. The approximate dispersion relations and the effective pion masses of different charged states in ANM with and without Dirac sea effect are presented below.

Using Eq.(28), the dispersion relations for $\pi^{0,\pm}$ including the effect of Dirac sea are given by,

$$q_0^2 \simeq m^{*2}_{\pi^{0,\pm}} + \left[\gamma_{\pi\pi} + 2m^{*2}_{\pi^{0,\pm}}\delta_{\pi\pi}\right]\mathbf{q}^2 + \left[\frac{\gamma^2_{\pi\pi}}{4} + \alpha_{\pi\pi} - \delta_{\pi\pi}\left(m^{*2}_{\pi^{0,\pm}} - 2\gamma_{\pi\pi}\right)\right]\frac{\mathbf{q}^4}{m^{*2}_{\pi^{0,\pm}}} \tag{29}$$

The effective masses (m^*_π) of different charged states of pion are found from Eq.(29) in the limit $|\mathbf{q}| = 0$.

$$m^{*2}_{\pi^0} \simeq \frac{m^2_{\pi^0}}{1-\Lambda_{PV}} \qquad \text{and} \qquad m^{*2}_{\pi^\pm} \simeq \frac{m^2_{\pi^\pm}}{1-(\Lambda_{PV} \pm \delta\Lambda_{PV})} \tag{30}$$

where,

$$\left.\begin{array}{ll}
\Lambda_{PV} & = \mathcal{A} + \mathcal{B} + \mathcal{C} \\
\delta\Lambda_{PV} & = \frac{\mathcal{E}}{m_{\pi^\pm}} \\
\gamma_{\pi\pi} & = 1 - \frac{\Lambda_{PV}}{1-\Lambda_{PV}} + \frac{\mathcal{B}}{1-\Lambda_{PV}} + \frac{\mathcal{C}}{1-\Lambda_{PV}} \\
\alpha_{\pi\pi} & = \frac{\mathcal{A}}{1-\Lambda_{PV}} \\
\delta_{\pi\pi} & = \frac{\mathcal{D}}{1-\Lambda_{PV}}
\end{array}\right\} \tag{31}$$

Clearly from Eq.(26), $\mathcal{E}$ indicates that the asymmetry driven mass splitting is of $\mathcal{O}(k^5_{p(n)}/M^{*4})$ for PV coupling. The dispersion relations for $\pi^{0,\pm}$ without the effect of Dirac sea are as follows,

$$q_0^2 \simeq m^{*2}_{\pi^{0,\pm}} + \gamma_{\pi\pi}\mathbf{q}^2 + \left[\frac{\gamma^2_{\pi\pi}}{4} + \alpha_{\pi\pi}\right]\frac{\mathbf{q}^4}{m^{*2}_{\pi^{0,\pm}}} \tag{32}$$

The effective pion masses without Dirac sea effect:

$$m^{*2}_{\pi^0} \simeq \frac{m^2_{\pi^0}}{1-\Omega_{PV}} \qquad \text{and} \qquad m^{*2}_{\pi^\pm} \simeq \frac{m^2_{\pi^\pm}}{1-(\Omega_{PV} \pm \delta\Omega_{PV})} \tag{33}$$

where,

$$\left.\begin{array}{ll}
\Omega_{PV} & = \mathcal{A} + \mathcal{B} \\
\delta\Omega_{PV} & = \frac{\mathcal{E}}{m^{\pi^\pm}} \\
\gamma_{\pi\pi} & = 1 - \frac{\Omega_{PV}}{1-\Omega_{PV}} + \frac{\mathcal{B}}{1-\Omega_{PV}} \\
\alpha_{\pi\pi} & = \frac{\mathcal{A}}{1-\Omega_{PV}}
\end{array}\right\} \tag{34}$$

These results are the same as that of [20] with some notational difference such as $\Omega_{PV} \to \Omega^2_{\pi\pi}$, $\delta\Omega_{PV} \to \delta\Omega^2_{\pi\pi}$, $\Omega_{PV}/(1-\Omega_{PV}) \to \chi_{\pi\pi}$ and $\mathcal{B}/(1-\Omega_{PV}) \to \beta_{\pi\pi}$. In (30) and (33), $\delta\Lambda_{PV}$ and $\delta\Omega_{PV}$ are responsible for the asymmetry parameter ($\alpha = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$) dependent mass splitting, where ρ_n and ρ_p are the neutron and proton density respectively. Clearly for SNM $\delta\Lambda_{PV}$ and $\delta\Omega_{PV}$ vanish.

B. PS coupling

In this section we consider the pseudoscalar (PS) coupling of pion to nucleon. This is renormalizable and with the same numerical value of g_π also reproduces the long range part of the nucleon-nucleon force. However, it has the great defect of producing an s-wave πN interaction which is much too strong [23]. It is to be noted here that in the present context the coupling parameters are related, *viz.* $f_\pi/m_\pi = g_\pi/2M$. With this the pseudoscalar interaction Lagrangian is given by,

$$\mathcal{L}^{PS} = -ig_\pi \bar{\Psi}_N \gamma_5 \left(\vec{\tau}\cdot\vec{\Phi}_\pi\right)\Psi_N \tag{35}$$

Here g_π is the pion-nucleon coupling constant with $\frac{g_\pi^2}{4\pi} = 12.6$ [28]. For PS coupling the trace of FF part:

$$\begin{aligned}
\mathbf{T}^{FF}_{PS} &= 2\; Tr\left[\gamma_5 iG^F(k)\gamma_5 iG^F(k+q)\right] \\
&= -8\left[\frac{M^{*2} - k\cdot(k+q)}{(k^2 - M^{*2})\left((k+q)^2 - M^{*2}\right)}\right]
\end{aligned} \tag{36}$$

We discussed about the factor 2 appears in Eq.(36) in the context of PV coupling. So the FF part of self-energy for PS coupling is calculated from Eq.(8) by substituting $\mathbf{T}^{FF}_{PS}$ and it is denoted by $\Sigma^{*FF}_{PS}(q)$.

$$\Sigma^{*FF}_{PS}(q) = 8ig_\pi^2 \int \frac{d^4k}{(2\pi)^4}\left[\frac{M^{*2} - k\cdot(k+q)}{(k^2 - M^{*2})\left((k+q)^2 - M^{*2}\right)}\right] \tag{37}$$

From Eq.(37) it is observed that $\Sigma^{*FF}_{PS}(q)$ is quadratically divergent. To eliminate these divergences we need to renormalize $\Sigma^{*FF}_{PS}(q)$. Here we again adopt the dimensional regularization [24–26] technique to regularize $\Sigma^{*FF}_{PS}(q)$ with the following results (details are discussed in appendix B).

$$\begin{aligned}
\Sigma^{*R}_{PS}(q,m_\pi) &= \frac{g_\pi^2}{2\pi^2}\Bigg[-3(M^2 - M^{*2}) + (q^2 - m_\pi^2)\left(\frac{1}{6} + \frac{M^2}{m_\pi^2}\right) - 2M^{*2}\ln\left(\frac{M^*}{M}\right) + \frac{8M^2(M-M^*)^2}{(4M^2 - m_\pi^2)} \\
&- \frac{2M^{*2}\sqrt{4M^{*2} - q^2}}{q}\tan^{-1}\left(\frac{q}{\sqrt{4M^{*2} - q^2}}\right) + \frac{2M^2\sqrt{4M^2 - m_\pi^2}}{m_\pi}\tan^{-1}\left(\frac{m_\pi}{\sqrt{4M^2 - m_\pi^2}}\right) \\
&+ \left((M^2 - M^{*2}) + \frac{m_\pi^2(M-M^*)^2}{(4M^2 - m_\pi^2)} + \frac{M^2}{m_\pi^2}(q^2 - m_\pi^2)\right)\frac{8M^2}{m_\pi\sqrt{4M^2 - m_\pi^2}}\tan^{-1}\left(\frac{m_\pi}{\sqrt{4M^2 - m_\pi^2}}\right) \\
&+ \int_0^1 dx\; 3x(1-x)q^2\;\ln\left(\frac{M^{*2} - q^2x(1-x)}{M^2 - m_\pi^2 x(1-x)}\right)\Bigg]
\end{aligned} \tag{38}$$

It is found that the result given in Eq.(38) is finite and no divergences appear further. In the appropriate kinematic regime it might generate imaginary part:

$$\begin{aligned}
\text{Im}\;\Sigma^{*FF}_{PS}(q) &= -\frac{g_\pi^2}{2\pi^2}\int_0^1 dx\;\left(M^{*2} - 3q^2x(1-x)\right)\text{Im}\left[\ln\left(M^{*2} - q^2x(1-x) - i\eta\right)\right] \\
&= -\frac{g_\pi^2}{4\pi}\left[q\sqrt{q^2 - 4M^{*2}}\right]\;\theta\left(q^2 - 4M^{*2}\right)
\end{aligned} \tag{39}$$

If we consider that $(M^* - M)$ is small enough then the term $\ln[(M^{*2} - q^2x(1-x))/(M^2 - m_\pi^2 x(1-x))]$ of Eq.(38) can be approximated to $2\ln(M^*/M)$ and the last term of Eq.(38) can be easily evaluated and the approximate result is

$$\Sigma^{*R}_{PS}(q,m_\pi) \simeq -\tilde{\mathcal{C}} + \tilde{\mathcal{D}}q^2 \tag{40}$$

where,

$$\left.\begin{aligned}\tilde{\mathcal{C}} &= \frac{g_\pi^2}{2\pi^2}\left[3(2M^2 - M^{*2}) + 2M^{*2}\ln\left(\frac{M^*}{M}\right)\right] \\ \tilde{\mathcal{D}} &= \frac{g_\pi^2}{2\pi^2}\left[3\left(\frac{M}{m_\pi}\right)^2\right]\end{aligned}\right\} \tag{41}$$

The trace of (FD+DF) part for π^0,

$$T_{PS}^{FD} + T_{PS}^{DF} = Tr[\gamma_5 G_p^F(k+q)\gamma_5 G_p^D(k) + \gamma_5 G_p^D(k+q)\gamma_5 G_p^F(k)] + [p \to n] \tag{42}$$

and for $\pi^{+(-)}$,

$$T_{PS}^{FD} + T_{PS}^{DF} = Tr[\gamma_5 G_{p(n)}^F(k+q)\gamma_5 G_{n(p)}^D(k) + \gamma_5 G_{p(n)}^D(k+q)\gamma_5 G_{n(p)}^F(k)] \tag{43}$$

The $(FD + DF)$ part of the self-energy for π^0 and $\pi^\pm$ can be written as

$$\Sigma_{PS}^{*0(FD+DF)}(q) = -8g_\pi^2 \int \frac{d^3k}{(2\pi)^3 E^*}\mathbf{A}_{PS} \tag{44}$$

$$\begin{aligned}\Sigma_{PS}^{*\pm(FD+DF)}(q) &= -8g_\pi^2 \int \frac{d^3k}{(2\pi)^3 E^*}[\mathbf{A}_{PS} \mp \mathbf{B}_{PS}] \\ &= \Sigma_{PS}^{*0(FD+DF)}(q) \mp \delta\Sigma_{PS}^{*(FD+DF)}(q)\end{aligned} \tag{45}$$

where,

$$\mathbf{A}_{PS} = \left[\frac{(k\cdot q)^2}{q^4 - 4(k\cdot q)^2}\right](\theta_p + \theta_n) \tag{46}$$

$$\mathbf{B}_{PS} = \frac{1}{2}\left[\frac{q^2(k\cdot q)}{q^4 - 4(k\cdot q)^2}\right](\theta_p - \theta_n) \tag{47}$$

As before in the long wavelength limit we neglect the term q^4 compared to the term $4(k\cdot q)^2$ from the denominator of both $\mathbf{A}_{PS}$ and $\mathbf{B}_{PS}$ in Eqs.(46) and (47). Explicitly, after a straight forward calculation we get,

$$\Sigma_{PS}^{*0(FD+DF)}(q) = \frac{g_\pi^2}{2\pi^2}\left[\left(k_p E_p^* - \frac{1}{2}M^{*2}\ln\left|\frac{1+v_p}{1-v_p}\right|\right) + \left(k_n E_n^* - \frac{1}{2}M^{*2}\ln\left|\frac{1+v_n}{1-v_n}\right|\right)\right] \tag{48}$$

and

$$\begin{aligned}\delta\Sigma_{PS}^{*(FD+DF)}(q) &= \frac{g_\pi^2}{2\pi^2}\left[\frac{1}{2}E_p^*\ln\left|\frac{c_0+v_p}{c_0-v_p}\right| - \frac{M^*}{\sqrt{c_0^2-1}}\tan^{-1}\left(\frac{k_p\sqrt{c_0^2-1}}{c_0 M^*}\right)\right]\frac{q^2}{|\mathbf{q}|} \\ &- \frac{g_\pi^2}{2\pi^2}\left[\frac{1}{2}E_n^*\ln\left|\frac{c_0+v_n}{c_0-v_n}\right| - \frac{M^*}{\sqrt{c_0^2-1}}\tan^{-1}\left(\frac{k_n\sqrt{c_0^2-1}}{c_0 M^*}\right)\right]\frac{q^2}{|\mathbf{q}|}\end{aligned} \tag{49}$$

The approximate results of Eqs.(48) and(49) are given below.

$$\Sigma_{PS}^{*0(FD+DF)}(q) \simeq \tilde{\mathcal{A}} + \tilde{\mathcal{B}} + \tilde{\mathcal{F}} - \tilde{\mathcal{G}} \tag{50}$$

$$\delta\Sigma_{PS}^{*(FD+DF)}(q) \simeq \tilde{\mathcal{E}}\frac{q^2}{q_0} \tag{51}$$

where,

$$\left.\begin{array}{ll}\tilde{\mathcal{A}} & = \frac{g_\pi^2}{2\pi^2}\left[\frac{1}{3}\left(\frac{k_p^3}{E_p^{*3}}+\frac{k_n^3}{E_n^{*3}}\right)\right]M^{*2} \\ \tilde{\mathcal{B}} & = \frac{g_\pi^2}{2\pi^2}\left[\frac{1}{5}\left(\frac{k_p^5}{E_p^{*5}}+\frac{k_n^5}{E_n^{*5}}\right)\right]M^{*2} \\ \tilde{\mathcal{F}} & = \frac{g_\pi^2}{2\pi^2}\left[\left(\frac{k_p}{E_p}+\frac{k_n}{E_n}\right)\right]M^{*2} \\ \tilde{\mathcal{G}} & = \frac{g_\pi^2}{2\pi^2}\left[k_pE_p+k_nE_n\right] \\ \tilde{\mathcal{E}} & = \frac{g_\pi^2}{2\pi^2}\left[\frac{1}{3}\left(\frac{k_p^3}{M^{*2}}-\frac{k_n^3}{M^{*2}}\right)\right]\end{array}\right\} \tag{52}$$

The total self-energy for PS coupling:

$$\Sigma_{PS}^{*0,\pm}(q) = \Sigma_{PS}^{*R}(q,m_\pi) + \Sigma_{PS}^{*0,\pm(FD+DF)}(q) \tag{53}$$

The dispersion relations can be found by solving Eq.(28). With the effect of Dirac sea $\pi^{0,\pm}$ dispersion relations are presented below.

$$q_0^2 \simeq m_{\pi^{0,\pm}}^{*2} + \mathbf{q}^2 \tag{54}$$

The effective masses ($m_{\pi^{0,\pm}}^{*2}$) with Dirac sea for different charged states of pion are given by.

$$m_{\pi^0}^{*2} \simeq \left[(\Lambda_{PS}-m_{\pi^0}^2)/\tilde{\mathcal{D}}\right] \quad \text{and} \quad m_{\pi^\pm}^{*2} \simeq \left[\frac{(\Lambda_{PS}-m_{\pi^\pm}^2)}{(1\mp\delta\Lambda_{PS})\tilde{\mathcal{D}}}\right] \tag{55}$$

where,

$$\left.\begin{array}{ll}\Lambda_{PS} & = \tilde{\mathcal{A}}+\tilde{\mathcal{B}}+\tilde{\mathcal{C}}+\tilde{\mathcal{F}}-\tilde{\mathcal{G}} \\ \delta\Lambda_{PS} & = \frac{\tilde{\mathcal{E}}}{\sqrt{(\Lambda_{PS}-m_{\pi^\pm}^2)\tilde{\mathcal{D}}}}\end{array}\right\} \tag{56}$$

For PS coupling the asymmetry driven mass splitting is of $\mathcal{O}(k_{p(n)}^3/M^{*2})$ which is different from that of PV coupling. The dispersion relations without the effect of Dirac sea for $\pi^{0,\pm}$:

$$q_0^2 \simeq m_{\pi^{0,\pm}}^{*2} + \mathbf{q}^2 \tag{57}$$

The effective masses without Dirac sea are

$$m_{\pi^0}^{*2} \simeq \left[\Omega_{PS}+m_{\pi^0}^2\right] \quad \text{and} \quad m_{\pi^\pm}^{*2} \simeq \left[\frac{\Omega_{PS}+m_{\pi^\pm}^2}{1\mp\delta\Omega_{PS}}\right] \tag{58}$$

where,

$$\left.\begin{array}{ll}\Omega_{PS} & = \tilde{\mathcal{G}}-\tilde{\mathcal{A}}-\tilde{\mathcal{B}}-\tilde{\mathcal{F}} \\ \delta\Omega_{PS} & = \frac{\tilde{\mathcal{E}}}{\sqrt{\Omega_{PS}+m_{\pi^\pm}^2}}\end{array}\right\} \tag{59}$$

This can be noted that $\delta\Lambda_{PS}$ and $\delta\Omega_{PS}$ are non-vanishing in ANM and responsible for the pion mass splitting. In SNM they vanish.

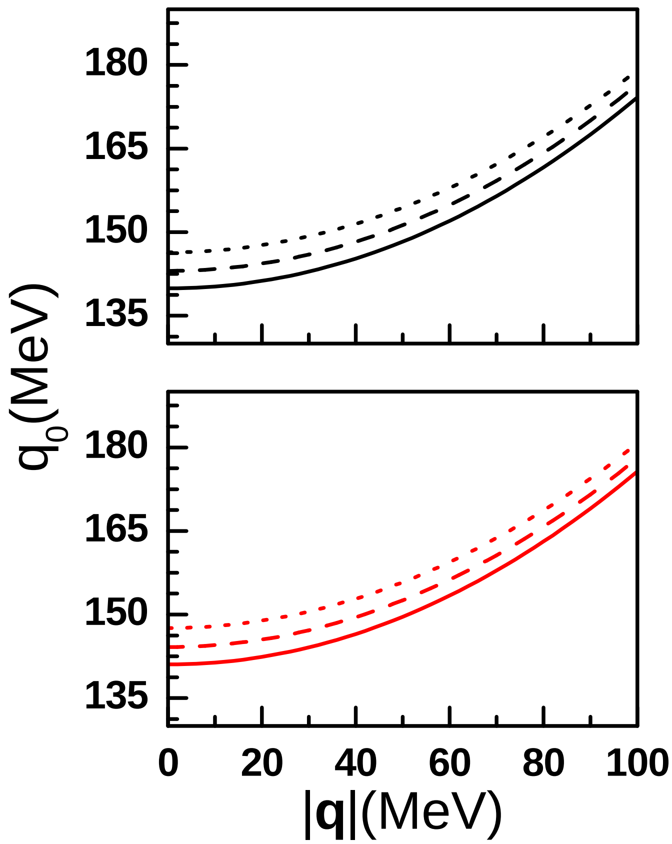

FIG. 3: Pion dispersion relation without (upper panel) and with (lower panel) the effect of Dirac sea for PV coupling. The solid, dashed and dotted curves respectively indicate the dispersion curves of π^0, π^+ and π^- at $\rho = 0.17 fm^{-1}$ and $\alpha = 0.2$.

III. RESULTS

Now we present our numerical results. Fig. 3 shows the pion dispersion relations for PV coupling for asymmetric parameter $\alpha = 0.2$ and nuclear density $\rho = 0.17 fm^{-1}$. The typical values of the pion mass shifts at normal nuclear density for Pb-like nuclei with Dirac sea effects are $\Delta m_{\pi^0} = 6.07\ MeV$, $\Delta m_{\pi^+} = 4.6\ MeV$ and $\Delta m_{\pi^-} = 8.02\ MeV$ and the corresponding values without Dirac sea contributions are 4.95 MeV, 3.47 MeV and 6.82 MeV .

The upper and lower panel of Fig. 3 shows the pion dispersion relations without and with the effect of Dirac sea respectively. It should be noted that the solid, dashed and dotted curves correspond to π^0, π^+ and π^- respectively. Though, π^+ and π^- - masses are degenerate in vacuum but their masses become non-degenerate in ANM. It is clear from Fig. 3 that the inclusion of Dirac sea, slightly increases the overall pion effective masses.

In Fig. 4 and Fig. 5, the variation of in medium pion masses with respect to asymmetry parameter (α) and nuclear density (ρ) for PV coupling are represented respectively. It is observed from Fig. 4 that effective pion masses for π^0 (solid line) and π^+ (dashed line) are same at $\alpha \approx 0.6$. For π^0 and π^- repulsive mass-shifts are observed while attractive mass shift is found for π^+.

From Fig. 5, it is clear that at $\rho \approx 1.5 fm^{-1}$; the effective π^0 and π^+ masses are indistinguishable without (upper panel) and with (lower panel) the Dirac sea. The effective mass of π^- (dotted line)increases sharply beyond the nuclear density $\rho \approx 2.0 fm^{-1}$; while that of π^+ (dotted curves) sharply decreases beyond the same nuclear density. For π^0, the effective mass (solid curve) increases slightly with the increase of nuclear density.

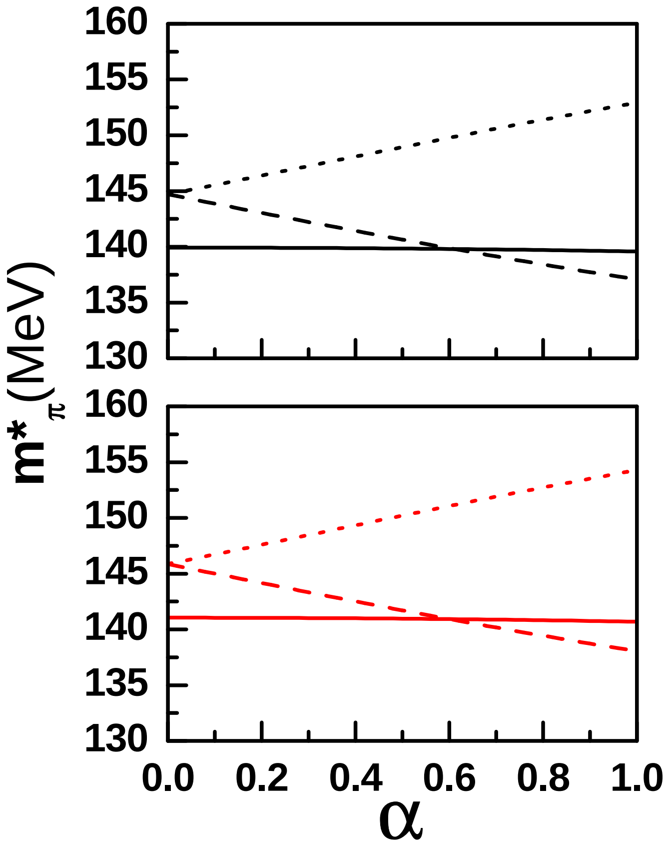

FIG. 4: Effective masses (for PV coupling) of π^0(solid line), π^+(dashed line) and π^-(dotted line) at $\rho = 0.17fm^{-1}$ for different values of asymmetry parameter (α). The upper and lower panel represent the effective masses without and with the effect of Dirac sea respectively.

The pion dispersion relations for PS coupling are shown in Fig. 6. It is observed that the dispersion curves have different momentum dependence without the effect of Dirac sea than that for the PV (Fig. 3) coupling. For PS coupling, with the effect of Dirac sea (lower panel) the dispersion curves for π^- (dotted curve) and π^+ (dashed curves) are indistinguishable.

The Fig. 7 and Fig. 8 represents the variation of the effective pion masses for PS coupling with respect to asymmetry parameter (α) and nuclear density (ρ) respectively. The effective π^- and π^+ masses are almost degenerate upto $\alpha \approx 0.4$ with the inclusion of Dirac sea contributions(Fig. 7). This is also different from the PV coupling. It is to be noted that Fermi sea and Dirac vacuum contribute with same sign in the self-energy for PS and PV couplings.

IV. CONCLUSION

In the present paper pion propagation in asymmetric nuclear matter has been studied within the framework of relativistic hadrodynamics in presence of the scalar mean field. The analytical form of the dispersion relations of the various charged states of pion in ANM have been presented after making suitable density expansions. In particular the effect of the Dirac vacuum on the pion mass splitting and dispersion relations is clearly revealed. It is shown that the corrections to the self-energy due to particle-antiparticle excitations, *viz.* the Dirac sea contributions, are

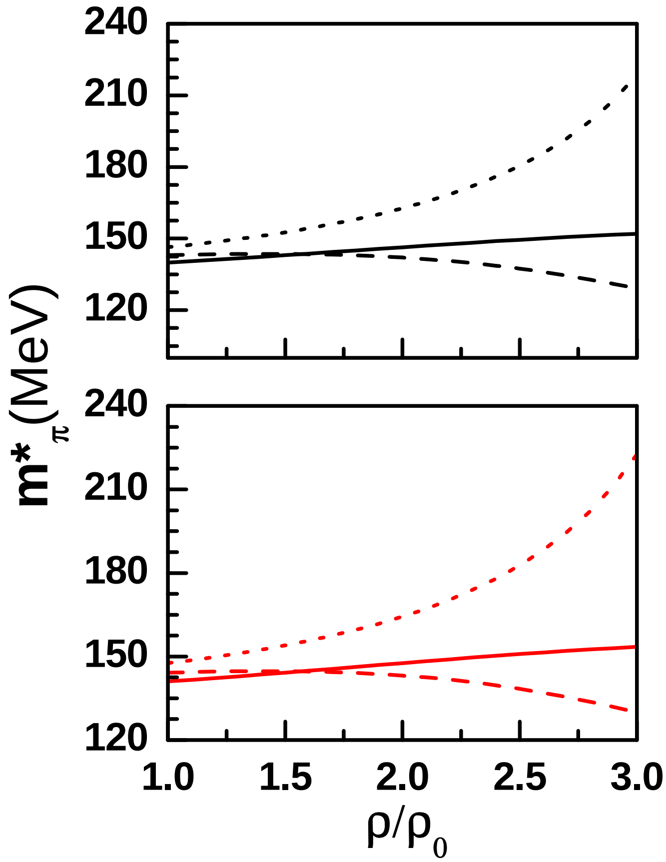

FIG. 5: Effective masses (for PV coupling) of π^0, π^+ and π^- are represented by the solid, dashed and dotted curves respectively. The curves in the upper (without Dirac sea) and lower (with Dirac sea) panel represent the variation of effective pion masses with nuclear density (ρ) at $\alpha = 0.2$.

comparable to the part related to the scattering from the Fermi sea considered in [20]. The magnitude of the effective masses of various charged states of pions are sensitive to the nature of the couplings. It should be noted that for PV and PS couplings both the Dirac vacuum and the Fermi sea corrections contribute to the self-energy with the same sign. The dispersion relations also show different density and momentum dependencies for these two cases.

Acknowledgment : The authors gratefully thank Pradip Roy and Kausik Pal for valuable comments and suggestions.

APPENDIX A

After Feynman parametrization Eq.(11) reduces to

$$
\begin{aligned}
\Sigma^{*FF}_{PV}(q) &= 8i\left(\frac{f_\pi}{m_\pi}\right)^2 \mu^{2\epsilon} \int \frac{d^N k}{(2\pi)^N} \int_0^1 dx \left[\frac{\left(M^{*2} + q^2 x(1-x) + k^2\right) q^2 - 2(k\cdot q)^2}{((k+qx)^2 + q^2 x(1-x) - M^{*2})^2}\right] \\
&= -\frac{q^2}{2\pi^2}\left(\frac{f_\pi}{m_\pi}\right)^2 \int_0^1 dx \left(4\pi\mu^2\right)^\epsilon \Gamma(\epsilon) \left[\frac{2M^{*2}}{\left(M^{*2} - q^2 x(1-x)\right)^\epsilon}\right]
\end{aligned}
$$

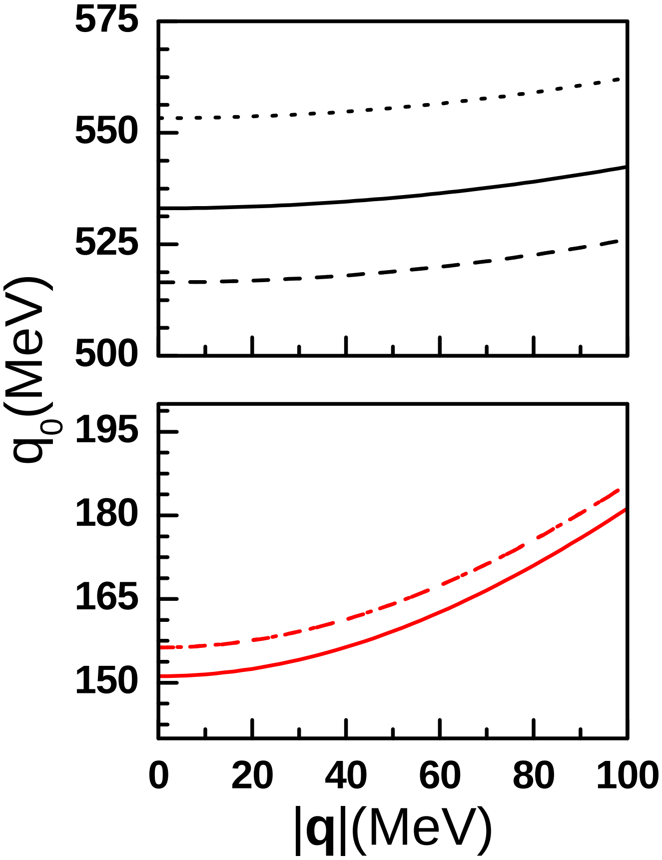

FIG. 6: The dispersion relations of pion for PS coupling at $\rho = 0.17 fm^{-1}$ and $\alpha = 0.2$. The solid, dashed and dotted curves in the upper and lower panel corresponds, respectively, to π^0, π^+ and π^- states without and with the effect of Dirac sea.

$$
\begin{aligned}
&= \frac{q^2}{2\pi^2}\left(\frac{f_\pi}{m_\pi}\right)^2 \left[2M^{*2}\left(\gamma_E - \ln\left(4\pi\mu^2\right)\right)\right] \\
&+ \frac{q^2}{2\pi^2}\left(\frac{f_\pi}{m_\pi}\right)^2 \left[2M^{*2}\int_0^1 dx\ \ln\left(M^{*2} - q^2 x(1-x)\right)\right] \\
&- \frac{q^2}{2\pi^2}\left(\frac{f_\pi}{m_\pi}\right)^2 \left[\frac{2M^{*2}}{\epsilon}\right]
\end{aligned}
\tag{A1}
$$

Here $\epsilon = 2 - \frac{N}{2}$ and μ is an arbitrary scaling parameter. We denote γ_E is the Euler-Mascheroni constant. It is observed that $\Sigma^{*FF}_{PS}(q)$ is real. The imaginary part of $\Sigma^{*FF}_{PV}(q)$ can be easily found by simply replacing $\ln\left(M^{*2} - q^2x(1-x)\right)$ with $\ln\left(M^{*2} - q^2x(1-x) - i\eta\right)$ where η is an arbitrarily small parameter and $i\eta$ comes from the denominator of G_i^F when Feynman parametrization is performed considering $i\zeta$ in the denominator of the propagator.

Here the term $\ln\left(M^{*2} - q^2x(1-x)\right)$ has branch cut only for $M^{*2} - q^2x(1-x) < 0$ and it begins at $q^2 = 4M^{*2}$ *i.e.* the threshold condition for nucleon-antinucleon pair production. So the limit of x-integration changes from $(0, 1)$ to $(\frac{1}{2} - \frac{1}{2}\alpha, \frac{1}{2} + \frac{1}{2}\alpha)$ where $\alpha = \sqrt{1 - \frac{4M^{*2}}{q^2}}$ and we used $\mathrm{Im}\ln(Z - i\eta) = -\pi$. Now,

$$
\int_{\frac{1}{2}-\frac{1}{2}\alpha}^{\frac{1}{2}+\frac{1}{2}\alpha} dx\ \theta\left(q^2 - 4M^{*2}\right) = \sqrt{1 - \frac{4M^{*2}}{q^2}}\ \theta\left(q^2 - 4M^{*2}\right) \tag{A2}
$$

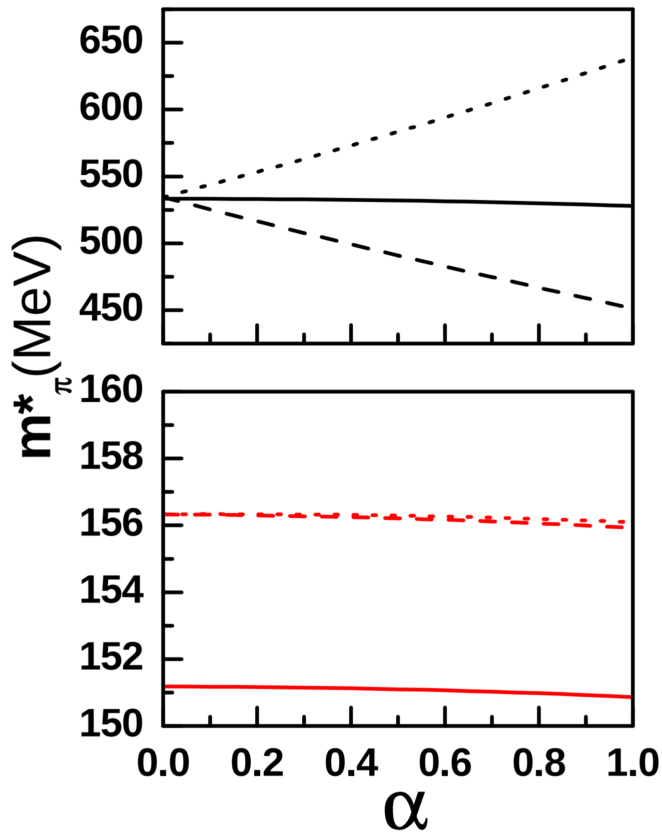

FIG. 7: Asymmetry parameter (α) dependent effective pion masses for PS coupling same as Fig. 4.

Therefore the imaginary part of $\Sigma^{*FF}_{PV}(q)$ can be found as

$$\mathrm{Im}\ \Sigma^{*FF}_{PV}(q) = -\left(\frac{f_\pi}{m_\pi}\right)^2 \left[\frac{q}{\pi} 2M^{*2}\sqrt{q^2-4M^{*2}}\right]\ \theta\left(q^2-4M^{*2}\right) \tag{A3}$$

It is clear from Eq.(A3) that $\mathrm{Im}\Sigma^{*FF}_{PV}(q)$ vanishes for $q^2 < 4M^{*2}$. With the same argument as stated for PS coupling, we excluded the imaginary part. The diverging part of $\Sigma^{*FF}_{PV}(q)$ is

$$\mathcal{D}_{PV} = -\frac{q^2}{2\pi^2}\left(\frac{f_\pi}{m_\pi}\right)^2\left[\frac{2M^{*2}}{\epsilon}\right] \tag{A4}$$

Here we use simple subtraction method to remove the divergence. So, the finite FF part of the self-energy is

$$\begin{aligned}\Sigma^{*R}_{PV}(q) &= \Sigma^{*FF}_{PV}(q) - \Sigma^{*FF}_{PV}(m_\pi)\\ &= \frac{q^2}{2\pi^2}\left(\frac{f_\pi}{m_\pi}\right)^2\left[2M^{*2}\int_0^1 dx\ \ln\left(\frac{M^{*2}-q^2x(1-x)}{M^{*2}-m_\pi^2x(1-x)}\right)\right]\end{aligned} \tag{A5}$$

APPENDIX B

After using Feynman parametrization, the term $\Sigma^{*FF}_{PS}(q)$ in Eq.(37) can be written as

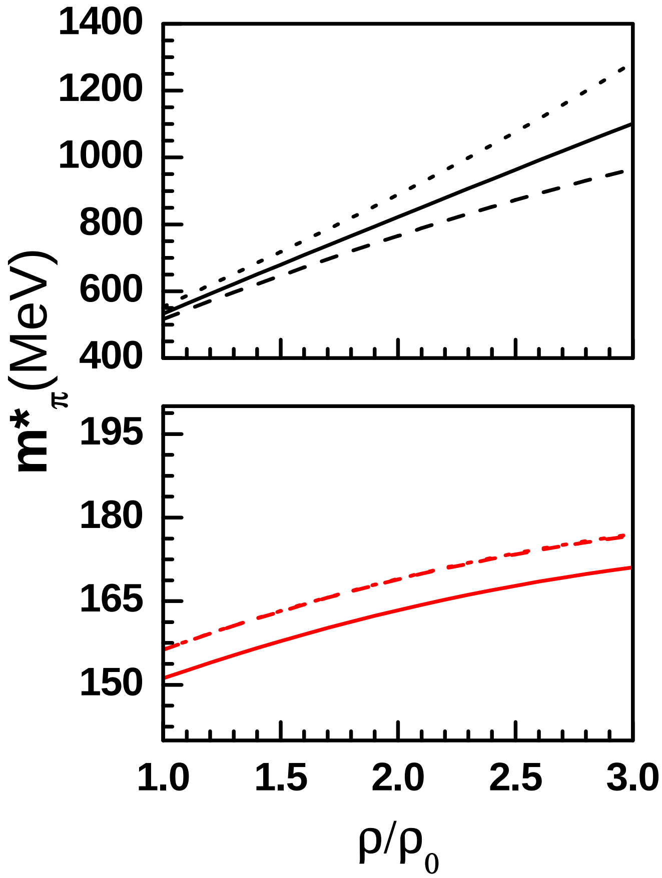

FIG. 8: Nuclear density (ρ) dependent effective pion masses (for PS coupling) at $\alpha = 0.2\ fm^{-1}$ same as Fig. 5.

$$
\begin{aligned}
\Sigma^{*FF}_{PS}(q) &= 8ig_\pi^2\mu^{2\epsilon}\int\frac{d^Nk}{(2\pi)^N}\int_0^1 dx\left[\frac{M^{*2}-k\cdot(k+q)}{((k+qx)^2+q^2x(1-x)-M^{*2})^2}\right] \\
&= \frac{g_\pi^2}{2\pi^2}\int_0^1 dx\,\left(4\pi\mu^2\right)^\epsilon\ \frac{\Gamma(\epsilon)}{1-\epsilon}\ \left[\frac{M^{*2}-3q^2x(1-x)+2\epsilon q^2x(1-x)}{\left(M^{*2}-q^2x(1-x)\right)^\epsilon}\right] \\
&= \frac{g_\pi^2}{2\pi^2}\ \frac{q^2}{3}+\frac{g_\pi^2}{2\pi^2}\ \frac{1}{\epsilon}\ \left(M^{*2}-\frac{q^2}{2}\right) \\
&- \frac{g_\pi^2}{2\pi^2}\ \left(M^{*2}-\frac{q^2}{2}\right)\left(\gamma'_E-\ln\left(4\pi\mu^2\right)\right) \\
&- \frac{g_\pi^2}{2\pi^2}\ \int_0^1 dx\,\left(M^{*2}-3q^2x(1-x)\right)\ln\left(M^{*2}-q^2x(1-x)\right)
\end{aligned}
\qquad \text{(B1)}
$$

Here $\gamma'_E = (\gamma_E - 1)$. Now the imaginary part of $\Sigma^{*FF}_{PS}(q)$ is,

$$
\begin{aligned}
\mathrm{Im}\ \Sigma^{*FF}_{PS}(q) &= -\frac{g_\pi^2}{2\pi^2}\int_0^1\ dx\ \left(M^{*2}-3q^2x(1-x)\right)\mathrm{Im}\left[\ln\left(M^{*2}-q^2x(1-x)-i\eta\right)\right] \\
&= -\frac{g_\pi^2}{4\pi}\left[q\sqrt{q^2-4M^{*2}}\right]\ \theta\left(q^2-4M^{*2}\right)
\end{aligned}
\qquad \text{(B2)}
$$

It is clear from the expression of Eq.(B1) that the second term is divergent in the limit $\epsilon \to 0$ (as $N \to 4$). To remove the divergences we need to add the counterterms [29] in the original Lagrangian interaction . The diverging part of Eq.(B1) is

$$\begin{aligned} \mathcal{D}_{PS} &= \frac{g_\pi^2}{2\pi^2}\,\frac{1}{\epsilon}\,\left(M^{*2} - \frac{q^2}{2}\right) \\ &= \frac{g_\pi^2}{2\pi^2}\left[\frac{M^2}{\epsilon} - \frac{2}{\epsilon} M g_s \phi_0 + \frac{1}{\epsilon} g_s^2 \phi_0^2 - \frac{q^2}{2\epsilon}\right] \end{aligned} \tag{B3}$$

In Eq.(B3) we substitute the effective nucleon mass $M^* = (M - g_s\phi_0)$ where M is the nucleon mass and ϕ_0 is the vacuum expectation value of the scalar field ϕ_s. The expression given in Eq.(B3) tells us that we need to be added four counter terms [29] with the original interaction Lagrangian to remove the divergences from Σ_{PS}^{*FF}. Therefore the counter term Lagrangian [29] is denoted as

$$\mathcal{L}_{CT} = -\frac{1}{2!}\beta_1 \Phi_\pi \cdot \left(\partial^2 + m_\pi^2\right) \cdot \Phi_\pi + \frac{1}{2!}\beta_2 \Phi^2 + \frac{1}{2!}\beta_3 \phi_s \Phi_\pi^2 + \frac{1}{2!2!}\beta_4 \phi^2 \Phi_\pi^2 \tag{B4}$$

The value of the counterterms β_1, β_2, β_3 and β_4 are determined by imposing the appropriate renormalization conditions.

$$\beta_1 = \left(\frac{\partial \Sigma_{PS}^{FF}(q)}{\partial q^2}\right)_{q^2=m_\pi^2} \tag{B5}$$

$$\beta_2 = \left(\Sigma_{PS}^{FF}\right)_{q^2=m_\pi^2} \tag{B6}$$

$$\beta_3 = -g_s\left(\frac{\partial \Sigma_{PS}^{FF}(q)}{\partial M}\right)_{q^2=m_\pi^2} \tag{B7}$$

$$\beta_4 = -\delta\lambda + g_s^2\left(\frac{\partial^2 \Sigma_{PS}^{FF}(q)}{\partial M^2}\right)_{q^2=m_\pi^2} \tag{B8}$$

Here β_1 and β_2 are the wave function and pion mass renormalization counterterms respectively while β_3 and β_4 are the vertex renormalizaton counterterms for the $\phi_s\Phi_\pi^2$ vertex and $\phi_s^2\Phi_\pi^2$ vertex respectively. The conditions of Eq.(B5)-(B6) implies that the pion propagator $G_\pi = [q^2 - m_\pi^2 - \Sigma_{PS}^{*R}(q)]^{-1}$ reproduces the physical mass of pions in free space. The counterterm β_4 determines the strength of coupling of the $\phi_s^2\Phi_\pi^2$ vertex. In fact $\Sigma_{PS}^{FF}(q)$ is found by simply replacing M^* with M in Eq.(B1). We can set $\delta\lambda = 0$ to minimize the effects of many-body forces in the nuclear medium [29] which is consistent with the renormalization scheme for scalar meson. Using the conditions given in Eqs.(B5)-(B8) the following results are found :

$$\begin{aligned} \beta_1 &= \frac{g_\pi^2}{2\pi^2}\left[\frac{1}{3} - \frac{1}{2}\left(\frac{1}{\epsilon} - \gamma_E' + \ln(4\pi\mu^2)\right)\right] \\ &+ \frac{g_\pi^2}{2\pi^2}\left[\int_0^1 dx\; 3x(1-x)\ln\left(M^2 - m_\pi^2 x(1-x)\right)\right] \\ &+ \frac{g_\pi^2}{2\pi^2}\left[\int_0^1 dx\;\frac{M^2 x(1-x) - 3m_\pi^2 x^2(1-x)^2}{M^2 - m_\pi^2 x(1-x)}\right] \end{aligned} \tag{B9}$$

$$\begin{aligned} \beta_2 &= \frac{g_\pi^2}{2\pi^2}\left[\frac{m_\pi^2}{2} + \left(M^2 - \frac{m_\pi^2}{3}\right)\left(\frac{1}{\epsilon} - \gamma_E' + \ln(4\pi\mu^2)\right)\right] \\ &- \frac{g_\pi^2}{2\pi^2}\left[\int_0^1 dx\;\left(M^2 - 3m_\pi^2 x(1-x)\right)\ln(M^2 - m_\pi^2 x(1-x))\right] \end{aligned} \tag{B10}$$

$$\beta_3 = \frac{g_\pi^2}{2\pi^2}\left[-g_s(2M)\left(\frac{1}{\epsilon} - \gamma_E' + \ln(4\pi\mu^2)\right)\right]$$

$$
\begin{aligned}
&+ \frac{g_\pi^2}{2\pi^2}\left[g_s(2M)\int_0^1 dx \ln(M^2 - m_\pi^2 x(1-x))\right] \\
&+ \frac{g_\pi^2}{2\pi^2}\left[g_s(2M)\int_0^1 dx \left(\frac{M^2 - 3m_\pi^2 x(1-x)}{M^2 - m_\pi^2 x(1-x)}\right)\right]
\end{aligned}
\tag{B11}
$$

$$
\begin{aligned}
\beta_4 &= -\frac{g_\pi^2}{2\pi^2}6g_s^2 + \frac{g_\pi^2}{2\pi^2}\left[2g_s\left(\frac{1}{\epsilon} - \gamma_E' + \ln(4\pi\mu^2)\right)\right] \\
&- \frac{g_\pi^2}{2\pi^2}\left[2g_s^2\int_0^1 dx \ln(M^2 - m_\pi^2 x(1-x))\right] \\
&- \frac{g_\pi^2}{2\pi^2}\left[2g_s^2\int_0^1 dx \ \frac{4M^2 m_\pi^2 x(1-x)}{(M^2 - m_\pi^2 x(1-x))^2}\right]
\end{aligned}
\tag{B12}
$$

Now the renormalized $\Sigma_{PS}^{*FF}(q)$ is

$$
\Sigma_{PS}^{*R}(q, m_\pi) = \Sigma_{PS}^{*FF}(q) - \beta_1(q^2 - m_\pi^2) - \beta_2 - \beta_3\phi_0 - \frac{1}{2}\beta_4\phi_0^2 \tag{B13}
$$

Substituting $\Sigma_{PS}^{*FF}(q)$ from Eq.(B1) and β_1, β_2, β_3, β_4 from Eqs.(B9)-(B12) in Eq.(B13) it is found that divergences in $\Sigma_{PS}^{*FF}(q)$ are completely eliminated by the counterterms. After simplification $\Sigma_{PS}^{*R}(q, m_\pi)$ reduces to

$$
\begin{aligned}
\Sigma_{PS}^{*R}(q, m_\pi) &= \frac{g_\pi^2}{2\pi^2}\Big[-3(M^2 - M^{*2}) + (q^2 - m_\pi^2)\left(\frac{1}{6} + \frac{M^2}{m_\pi^2}\right) - 2M^{*2}\ln\left(\frac{M^*}{M}\right) + \frac{8M^2(M - M^*)^2}{(4M^2 - m_\pi^2)} \\
&- \frac{2M^{*2}\sqrt{4M^{*2} - q^2}}{q}\tan^{-1}\left(\frac{q}{\sqrt{4M^{*2} - q^2}}\right) + \frac{2M^2\sqrt{4M^2 - m_\pi^2}}{m_\pi}\tan^{-1}\left(\frac{m_\pi}{\sqrt{4M^2 - m_\pi^2}}\right) \\
&+ \left((M^2 - M^{*2}) + \frac{m_\pi^2(M - M^*)^2}{(4M^2 - m_\pi^2)} + \frac{M^2}{m_\pi^2}(q^2 - m_\pi^2)\right)\frac{8M^2}{m_\pi\sqrt{4M^2 - m_\pi^2}}\tan^{-1}\left(\frac{m_\pi}{\sqrt{4M^2 - m_\pi^2}}\right) \\
&+ \int_0^1 dx\ 3x(1-x)q^2\ \ln\left(\frac{M^{*2} - q^2 x(1-x)}{M^2 - m_\pi^2 x(1-x)}\right)\Big]
\end{aligned}
\tag{B14}
$$

[1] A. B. Migdal, Rev. Mod. Phys. **50** (1978) 107.
[2] I. M. Mishustin, F. Myhrer and P. J. Siemens, Phys. Lett. **B 95** (1980) 361.
[3] G. E. Brown, E. Oset, M. Vicente Vacas and W. Weise, Nucl. Phys. **A505** (1989) 823.
[4] L.H. Xia, C.M. Ko, L. Xiong and J.Q. Wu, Nucl. Phys. **A 485** (1988) 721.
[5] C. Gale and J. Kapusta, Phys. Rev. **C 35** (1987) 2107.
[6] E. Oset, H. Toki and W. Weise, Phys. rep. **83** (1982) 281.
[7] A.B. Migdal, E.E. Saperstein, M.A. Troitsky and D.N. Voskresensky, Phys. Rep. **192** (1990) 179.
[8] V.F. Dmitriev, T. Suzuki, Nucl. Phys. **A 438**(1985)697.
[9] P. A. Henning, H. Umezawa, Nucl. Phys. bf A571 (1994) 617.
[10] C. L. Korpa and R. Malfliet, Phys. Rev. **C 52** (1995) 2757.
[11] J. Helgesson and J. Randrup, Phys. Rev. **C 52** (1995) 427.
[12] L. Liu and M. Nakano, Nucl. Phys. **A 618** (1997) 337.
[13] T. Herbert, K. Wehrberger and F. Beck, Nucl. Phys. bf A 541 (1992) 699.
[14] R. Rapp and J. Wambach, Adv. Nucl. Phys. **25** (2000) 1.
[15] C. Y. Wong, *Introduction to High-Energy Heavy-Ion Collisions*, World Scientific, Singapore, 1994.
[16] G. Chanfray and P. Schuck, Nucl. Phys. **A 555** (1993) 329.
[17] N. Kaiser and W. Weise, Phys. Lett. **B 512** (2001) 283.
[18] P. Costa, M. C. Ruivo and Yu. L. Kalinovsky, Phys. Lett. **B 560** (2003) 171.
[19] P. Costa, M. C. Ruivo, C. A. de Sousa and Yu. L. Kalinovsky, Phys. Rev. **C 70** (2004) 025204.
[20] S. Biswas and A. K. Dutt-Mazumder, Phys. Rev. **C 74** (2006) 065205.
[21] S.A. Chin, Ann. of Phys. **108** (1977) 301.

[22] B.D. Serot and J.D. Walecka, Adv. Nucl. Phys. **16** (1986) 1.
[23] J. I. Kapusta, *Finite-temperature field theory*, Cambridge University Press, 1993.
[24] G. 't Hooft and M.T. Veltmann, Nucl. Phys. **B 44** (1973) 189.
[25] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory*, Addison-Wesley Publishing Company, 1995.
[26] T. P. Cheng and L. F. Li, *Gauge theory of elementary particles*, Clarendon Press, Oxford, New York, 2006.
[27] A.K. Dutt-Mazumder, Nucl. Phys. **A 713** (2003) 119.
[28] M. Schäfer, H.C. Döngs, A. Engel and U. Mosel, Nucl. Phys. **A 575** (1994) 429.
[29] T. Matsui and B. Serot, Ann. of Phys. **144** (1982) 107.