Low Energy Aspects of Heavy Meson Decays .*

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I discuss low energy aspects of heavy meson decays, where there is at least one heavy meson in the final state. Examples are $B-\overline{B}$ mixing, $B \to D\overline{D}$, $B \to D\eta'$, and $B \to D\gamma$. The analysis is performed in the heavy quark limit within heavy-light chiral perturbation theory. Coefficients of $1/N_c$ suppressed chiral Lagrangian terms (beyond factorization) have been estimated by means of a heavy-light chiral quark model.

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1. Introduction

In this paper we consider non-leptonic "heavy meson to heavy meson(s)" transitions, for instance $B-\overline{B}$-mixing [1], $B \to D\bar{D}$ [2] and with only one D-meson in the final state, like $B \to D\eta'$ [3] and $B \to \gamma\, D^*$ [4, 5, 6].

The methods [7] used to describe heavy to light tansitions like $B \to \pi\pi$ and $B \to K\pi$ are not suited for the decays we consider. We use heavy-light chiral perturbation theory (HLχPT). Lagrangian terms corresponding to factorization are then determined to zeroth order in $1/m_Q$, where m_Q is the mass of the heavy quark (b or c). For $B-\overline{B}$-mixing we have also calculated $1/m_b$ corrections [1].

Colour suppressed $1/N_c$ terms beyond factorization can be written down, but their coefficients are unknown. However, these coefficients can be calculated within a heavy-light chiral quark model (HLχQM) [8] based on the heavy quark effective theory (HQEFT) [9] and HLχPT [10]. The $1/N_c$ suppressed non-factorizable terms calculated in this way will typically be proportional to a model dependent gluon condensate [1, 2, 3, 6, 8, 11].

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2. Quark Lagrangians for non-leptonic decays

The effective non-leptonic Lagrangian at quark level has the form [12]:

$$\mathcal{L}_W = \sum_i C_i(\mu)\, \hat{Q}_i(\mu) \; , \tag{1}$$

where the Wilson coefficients C_i contain G_F and KM factors. Typically, the operators are four quark operators being the product of two currents:

$$\hat{Q}_i \;=\; j_W^\mu(q_1 \to q_2)\, j^W_\mu(q_3 \to q_4) \; , \tag{2}$$

where $j_W^\mu(q_i \to q_j) = \overline{(q_j)_L}\, \gamma^\mu\, (q_i)_L$, and some of the quarks $q_{i,j}$ are heavy. To leading order in $1/N_c$, matrix elements of $\hat{Q}_i$ factorize in products of matrix elements of currents. Non-factorizable $1/N_c$ suppressed terms are obtained from "coloured quark operators". Using Fierz transformations and

$$\delta_{ij}\delta_{ln} = \frac{1}{N_c}\delta_{in}\delta_{lj} \;+\; 2\; t^a_{in}\; t^a_{lj} \; , \tag{3}$$

where t^a are colour matrices, we may rewrite the operator $\hat{Q}_i$ as

$$\hat{Q}_i^F = \frac{1}{N_c}\, j_W^\mu(q_1 \to q_4)\; j^W_\mu(q_3 \to q_2) + 2\, j_W^\mu(q_1 \to q_4)^a\; j^W_\mu(q_3 \to q_2)^a \; , \tag{4}$$

where $j_W^\mu(q_i \to q_j)^a = \overline{(q_j)_L}\, \gamma^\mu\, t^a\, (q_i)_L$ is a left-handed coloured current. The quark operators in $\hat{Q}_i^F$ give $1/N_c$ suppressed terms.

3. Heavy-light chiral perturbation theory

The QCD Lagrangian involving light and heavy quarks is:

$$\mathcal{L}_{Quark} = \pm\overline{Q_v^{(\pm)}} iv \cdot DQ_v^{(\pm)} + \mathcal{O}(m_Q^{-1}) \;+\; \bar{q}i\gamma \cdot Dq \;+ ... \tag{5}$$

where $Q_v^{(\pm)}$ are the quark fields for a heavy quark and a heavy anti-quark with velocity v, q is the light quark triplet, and $iD_\mu = i\partial_\mu - e_q A_\mu - g_s t^a A^a_\mu$. The bosonized Lagrangian have the following form, consistent with the underlying symmetry [10]:

$$\mathcal{L}_\chi(Bos) = \mp Tr\left[\overline{H_a^{(\pm)}}(iv \cdot \mathcal{D}_{fa})H_f^{(\pm)}\right] - g_\mathcal{A} Tr\left[\overline{H_a^{(\pm)}}H_f^{(\pm)}\gamma_\mu\gamma_5\mathcal{A}^\mu_{fa}\right] + ... \tag{6}$$

where the covariant derivative is $i\mathcal{D}^\mu_{fa} \equiv \delta_{af}(i\partial^\mu - e_H A^\mu) - \mathcal{V}^\mu_{fa}$; a, f being SU(3) flavour indices. The axial coupling is $g_\mathcal{A} \simeq 0.6$. Furthermore,

$$\mathcal{V}_\mu(\text{or}\, \mathcal{A}_\mu) \;=\; \pm\frac{i}{2}(\xi^\dagger\partial_\mu\xi \pm \xi\partial_\mu\xi^\dagger) \; , \tag{7}$$

where $\xi = exp(i\Pi/f)$, and Π is a 3 by 3 matrix containing the light mesons $(\pi, K\eta)$, and the heavy $(1^-, 0^-)$ doublet field (P_μ, P_5) is

$$H^{(\pm)} = P_\pm(P_\mu^{(\pm)}\gamma^\mu - iP_5^{(\pm)}\gamma_5) \ , \ P_\pm = (1 \pm \gamma \cdot v)/2 \ , \tag{8}$$

where superscripts $(\pm)$ means meson and anti-meson respectively. To bosonize the non-leptonic quark Lagrangian, we need to bosonize the currents. Then the b, c, and $\overline{c}$ quarks are treated within HQEFT, which means the replacements $b \rightarrow Q_{v_b}^{(+)}$, $c \rightarrow Q_{v_c}^{(+)}$, and $\overline{c} \rightarrow Q_{\bar{v}}^{(-)}$. Then the bosonization of currents within HQEFT for decay of a heavy B-meson will be:

$$\overline{q_L}\,\gamma^\mu\, Q_{v_b}^{(+)} \ \longrightarrow \ \frac{\alpha_H}{2} Tr\left[\xi^\dagger\gamma^\mu L\, H_b^{(+)}\right] \ \equiv \ J_b^\mu \ , \tag{9}$$

where L is the left-handed projector in Dirac space, and $\alpha_H = f_H\sqrt{M_H}$ for $H = B, D$ before pQCD and chiral corrections are added. Here, $H_b^{(+)}$ represents the heavy meson (doublet) containing a b-quark. For creation of a heavy anti-meson $\overline{B}$ or $\overline{D}$, the corresponding currents $J_{\bar{b}}^\mu$ and $J_{\bar{c}}^\mu$ are given by (9) with $H_b^{(+)}$ replaced by $H_b^{(-)}$ and $H_c^{(-)}$, repectively. For the $B \rightarrow D$ transition we have

$$\overline{Q_{v_b}^{(+)}}\,\gamma^\mu\, LQ_{v_c}^{(+)} \ \longrightarrow \ -\zeta(\omega) Tr\left[\overline{H_c^{(+)}}\gamma^\mu L H_b^{(+)}\right] \ \equiv \ J_{b\rightarrow c}^\mu \ , \tag{10}$$

where $\zeta(\omega)$ is the Isgur-Wise function, and $\omega = v_b \cdot v_c$. For creation of $D\overline{D}$ pair we have the same expression for the current $J_{c\bar{c}}^\mu$ with $H_b^{(+)}$ replaced by $H_c^{(-)}$, and $\zeta(\omega)$ replaced by $\zeta(-\lambda)$, where $\lambda = \bar{v} \cdot v_c$. In addition there are $1/m_Q$ corrections for $Q = b, c$. The low velocity limit is $\omega \rightarrow 1$. For $B \rightarrow D\overline{D}$ and $B \rightarrow D^*\gamma$ one has $\omega \simeq 1.3$, and $\omega \simeq 1.6$, respectively.

3.1. Factorized lagrangians for non-leptonic processes

For $B - \overline{B}$ mixing, the factorized bosonized Lagrangian is

$$\mathcal{L}_B = C_B\ J_b^\mu\ (J_{\bar{b}})^\mu \ , \tag{11}$$

where C_B is a short distance Wilson coefficient (containing $(G_F)^2$), which is taken at $\mu = \Lambda_\chi \simeq 1$ GeV, and the currents are given by (9).

For processes obtained from two different four quark operators for $b \rightarrow c\bar{c}q$ $(q = d, s)$, we find the factorized Lagrangian corresponding to Fig. 1:

$$\mathcal{L}_{Fact}^{Spec} = (C_2 + \frac{C_1}{N_c}) J_{b\rightarrow c}^\mu\ (J_{\bar{c}})_\mu \ , \tag{12}$$

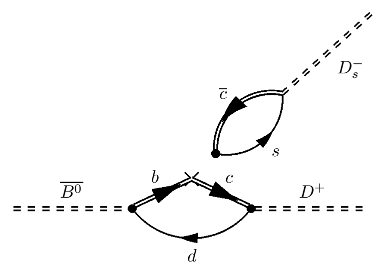

Fig. 1. Factorized contribution for $\overline{B_d^0} \to D^+ D_s^-$ through the spectator mechanism, which does not exist for decay mode $\overline{B_d^0} \to D_s^+ D_s^-$.

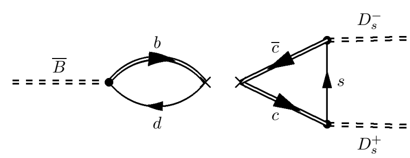

Fig. 2. Factorized contribution for $\overline{B_d^0} \to D_s^+ D_s^-$ through the annihilation mechanism, which give zero contributions if both D_s^+ and D_s^- are pseudoscalars.

where $C_i = \frac{4}{\sqrt{2}} G_F V_{cb} V_{cq}^* a_i$, and [13] $a_1 \simeq -0.35 - 0.07i$, $a_2 \simeq 1.29 + 0.08i$. We have considered the process $\overline{B_d^0} \to D_s^+ D_s^-$. Note that there is no factorized contribution to this process if both D-mesons in the final state are pseudoscalars! But the factorized contribution to $\overline{B_d^0} \to D^+ D_s^-$ will be the starting point for chiral loop contributions to the process $\overline{B_d^0} \to D_s^+ D_s^-$.

The factorizable term from annihilation is shown in Fig. 2, and is:

$$\mathcal{L}_{Fact}^{Ann} = (C_1 + \frac{C_2}{N_c}) \, J_{c\bar{c}}^{\mu} \, (J_b)_\mu \; . \tag{13}$$

Because $(C_1 + C_2/N_c)$ is a non-favourable combination of the Wilson coefficients, this term will give a small non-zero contribution if at least one of the mesons in the final state is a vector.

3.2. Possible $1/N_c$ suppressed tree level terms

For $B - \bar{B}$ mixing, we have for instance the $1/N_c$ suppressed term

$$Tr\left[\xi^\dagger \sigma^{\mu\alpha} L \, H_b^{(+)}\right] \cdot Tr\left[\xi^\dagger \sigma_{\mu\alpha} R H_{\bar{b}}^{(-)}\right] \; . \tag{14}$$

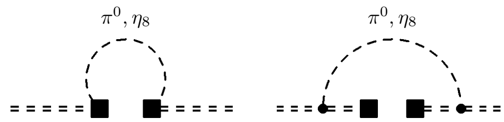

Fig. 3. Chiral corrections to $B - \overline{B}$ mixing, i.e the bag parameter B_{B_q} for $q = d, s$. The black boxes are weak vertices.

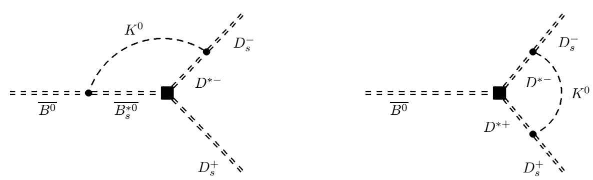

Fig. 4. Two classes of non-factorizable chiral loops for $\overline{B_d^0} \to D_s^+ D_s^-$ based on the factorizable amplitude proportional to the IW function $\sim \zeta(\omega)$.

For $B \to D\bar{D}$, we have for instance the terms

$$Tr\left[\xi^\dagger \sigma^{\mu\alpha} L\, H_b^{(+)}\right] \cdot Tr\left[\overline{H_c^{(+)}} \gamma_\alpha L H_{\bar{c}}^{(-)} \gamma_\mu\right] \;, \tag{15}$$

$$Tr\left[\xi^\dagger \sigma^{\mu\alpha} L\, H_b^{(+)}\right] \cdot Tr\left[\overline{H_c^{(+)}} \gamma_\alpha L H_{\bar{c}}^{(-)}\right] (\bar{v} - v_c)_\mu \;\; . \tag{16}$$

One needs a framework to estimate the coefficients of such terms. We use the HLχQM, which will pick a certain linear combination of $1/N_c$ terms.

3.3. Chiral loops for non-leptonic processes

Within HLχPT, the leading chiral corrections are proportional to

$$\chi(M) \equiv (\frac{g_{\mathcal{A}} m_M}{4\pi f})^2 \ln(\frac{\Lambda_\chi^2}{m_M^2}) \;, \tag{17}$$

where m_M is the appropriate light meson mass and Λ_χ is the chiral symmetry breaking scale, which is also the matching scale within our framework.

For $B - \overline{B}$ mixing there are chiral loops obtained from (6) and (11) shown in Fig. 3. These have to be added to the factorized contribution.

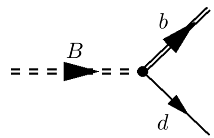

Fig. 5. The $HL\chi$QM ansatz: Vertex for quark meson interaction

For the process $\overline{B^0_d} \to D_s^+ D_s^-$ we obtain a chiral loop amplitude corresponding to Fig. 4. This amplitude is complex and depend on ω and λ defined previously. It has been recently shown [5] that $(0^+, 1^+)$ states in loops should also be added to the result.

4. The heavy-light chiral quark model

The Lagrangian for HLχQM [8] contains the Lagrangian (5):

$$\mathcal{L}_{HL\chi QM} = \mathcal{L}_{HQET} + \mathcal{L}_{\chi QM} + \mathcal{L}_{Int} \,, \tag{18}$$

where $\mathcal{L}_{HQET}$ is the heavy quark part of (5), and the light quark part is

$$\mathcal{L}_{\chi QM} = \overline{\chi}\,[\gamma^\mu (iD_\mu + \mathcal{V}_\mu + \gamma_5 \mathcal{A}_\mu) - m]\,\chi \,. \tag{19}$$

Here $\chi_L = \xi^\dagger q_L$ and $\chi_R = \xi q_R$ are flavour rotated light quark fields, and m is the light constituent mass. The bosonization of the (heavy-light) quark sector is performed via the ansatz:

$$\mathcal{L}_{Int} = -G_H \left[\overline{\chi}_f \, \overline{H_v^f} \, Q_v \, + \overline{Q_v} \, H_v^f \, \chi_f \right] \,. \tag{20}$$

The coupling G_H is determined by bosonization through the loop diagrams in Fig 6. The bosonization lead to relations between the model dependent parameters G_H, m, and $\langle \frac{\alpha_s}{\pi} G^2 \rangle$, and the quadratic-, linear, and logarithmic- divergent integrals $I_1, I_{3/2}, I_1$, and the physical quantities f_π, $\langle \overline{q} q \rangle$, $g_{\mathcal{A}}$ and f_H $(H = B, D)$. For example, the relation obtained for identifying the kinetic term is:

$$- iG_H^2 N_c \,(I_{3/2} + 2mI_2 + \frac{i(8-3\pi)}{384 N_c m^3} \langle \frac{\alpha_s}{\pi} G^2 \rangle) = 1 \,, \tag{21}$$

where we have used the prescription:

$$g_s^2 G^a_{\mu\nu} G^a_{\alpha\beta} \; \to 4\pi^2 \langle \frac{\alpha_s}{\pi} G^2 \rangle \frac{1}{12} (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}) \,. \tag{22}$$

The parameters are fitted in strong sector, with $\langle \frac{\alpha_s}{\pi} G^2 \rangle = [(0.315 \pm 0.020)$ GeV$]^4$, and $G_H{}^2 = \frac{2m}{f^2} \rho$, where $\rho \simeq 1$. For details , see [8].

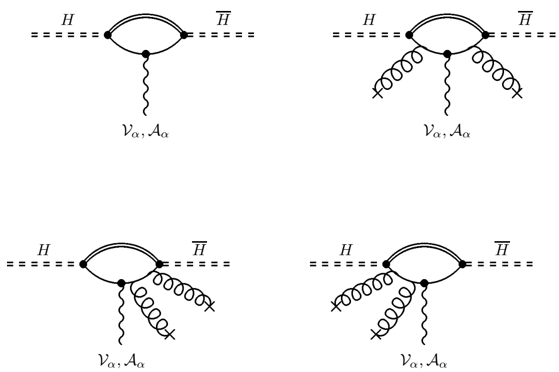

Fig. 6. Diagrams generating the strong chiral lagrangian at mesonic level. The kinetic term and and the axial vector term $\sim g_{\mathcal{A}}$.

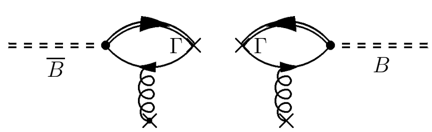

Fig. 7. Non-factorizable contribution to $B - \overline{B}$ mixing; $\Gamma \equiv t^a \, \gamma^\mu \, L$

5. $1/N_c$ terms from HLχQM

To obtain the $1/N_c$ terms for $B - \overline{B}$ mixing in Fig. 7 , we need the bosonization of colored current in the quark operators of eq. (4):

$$\left(\overline{q_L} \, t^a \, \gamma^\alpha \, Q_{v_b}^{(+)}\right)_{1G} \longrightarrow -\frac{G_H \, g_s}{64\pi} \, G^a_{\mu\nu} Tr \left[\xi^\dagger \gamma^\alpha L \, H_b^{(+)} \Sigma_{\mu\nu}\right] \; , \tag{23}$$

$$\Sigma^{\mu\nu} = \sigma^{\mu\nu} \; - \frac{2\pi f^2}{m^2 \, N_c} [\sigma^{\mu\nu}, \gamma \cdot v_b]_+ \; . \tag{24}$$

This coloured current is also used for $B \to D\overline{D}$ in Fig. 8, for $B \to D \, \eta'$ in Fig. 9, and for $B \to \gamma D^*$ in Fig. 10 In addition there are more complicated bosonizations of coloured currents as indicated in Fig. 8.

For $B \to D \, \eta'$ and $B \to \gamma D^*$ decays there are two different four quark operators, both for $b \to c\bar{u}q$ and $b \to \bar{c}uq$, respectively. At $\mu = 1$ GeV they have Wilson coefficients $a_2 \simeq 1.17$, $a_1 \simeq -0.37$ (up to prefactors G_F and

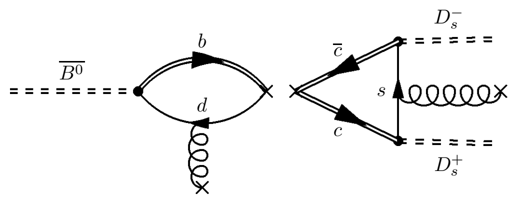

Fig. 8. Non-factorizable $1/N_c$ contribution for $\overline{B^0} \to D_s^+ D_s^-$ through the annihilation mechanism with additional soft gluon emision.

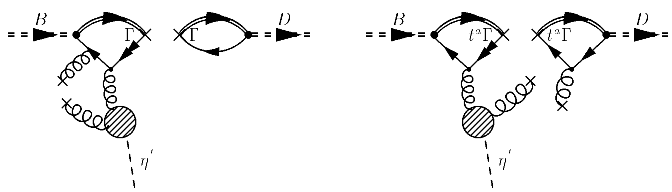

Fig. 9. Diagram for $B \to D\eta'$ within $HL\chi QM$. $\Gamma = \gamma^\mu(1-\gamma_5)$

KM-factors). For $B \to D\,\eta'$, we must also attach a propagating gluon to the $\eta' gg^*$-vertex. Note that for $\overline{B^0_{s,d}} \to \gamma D^{0*}$, the $1/N_c$ suppressed mechanism in Fig. 10 dominates, unlike $\overline{B^0_{s,d}} \to \overline{\gamma D^{0*}}$. Factorized contributions are proportional to either the favourable contribution $a_f \;=\; a_2 + a_1/N_c \simeq 1.05$ or the non-favourable contribution $a_{nf} \;=\; a_1 + a_2/N_c \simeq 0.02$.

5.1. $1/m_c$ *correction terms*

For the $B \to D$ transition we have the $1/m_c$ suppressed terms:

$$\frac{1}{m_c}\, Tr\left[\left(Z_0 \overline{H_c^{(+)}} + Z_1 \gamma^\alpha \overline{H_c^{(+)}} \gamma_\alpha + Z_2 \overline{H_c^{(+)}} \gamma \cdot v_b\right) \gamma^\alpha L H_b^{(+)}\right] \;, \qquad (25)$$

where the Z_i's are calculable within HLχQM. The relative size of $1/m_c$ corrections are typically of order $20-30\%$.

6. Results

6.1. $B - \overline{B}$ *mixing*

The result for the B(ag) parameter in $B - \overline{B}$-mixing has the form [1]

$$\hat{B}_{B_q} = \frac{3}{4}\,\widetilde{b}\left[1 + \frac{1}{N_c}\left(1-\delta_G^B\right) + \frac{\tau_b}{m_b} + \frac{\tau_\chi}{32\pi^2 f^2}\right] \;, \qquad (26)$$

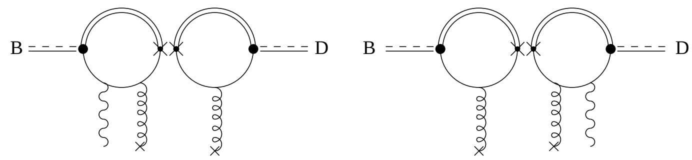

Fig. 10. Non-factorizable contributions to $B \to \gamma D^*$ from the coloured operators

similar to the $K - \overline{K}$-mixing case [11]. From perturbative QCD we have $\widetilde{b} \simeq 1.56$ at $\mu = \Lambda_\chi = 1$ GeV. From calculations within the HLχQM we obtain, $\delta_G^B = 0.5 \pm 0.1$ and $\tau_b = (0.26 \pm 0.04)$GeV, and from chiral corrections $\tau_{\chi,s} = (-0.10 \pm 0.04)\mathrm{GeV}^2$, and $\tau_{\chi,d} = (-0.02 \pm 0.01)\mathrm{GeV}^2$. We obtained

$$\hat{B}_{B_d} = 1.51 \pm 0.09 \qquad \hat{B}_{B_s} = 1.40 \pm 0.16 \; , \tag{27}$$

in agreement with lattice results.

6.2. $B \to D\,\overline{D}$ decays

Keeping the chiral logs and the $1/N_c$ terms from the gluon condensate, we find the branching ratios in the "leading approximation". For decays of $\bar{B}_d^0$ ($\sim V_{cb}V_{cd}^*$) and $\bar{B}_s^0$ ($\sim V_{cb}V_{cs}^*$) we obtain branching ratios of order few $\times 10^{-4}$ and $\times 10^{-3}$, respectively Then we have to add counterterms $\sim m_s$ for chiral loops. These may be estimated in HLχQM.

6.3. $B \to D\,\eta'$ and $B \to \gamma D^*$ decays

The result corresponding to Fig. 9 is:

$$Br(B \to D\eta^{'}) \, \simeq \, 2 \times 10^{-4} \; . \tag{28}$$

The partial branching ratios from the mechanism in Fig. 10 are [6]

$$Br(\overline{B_d^0} \to \gamma \, D^{*0})_G \, \simeq 1 \times 10^{-5} \quad ; \quad Br(\overline{B_s^0} \to \gamma \, D^{*0})_G \, \simeq 6 \times 10^{-7} \; . \tag{29}$$

The corresponding factorizable contribibutions are roughly two orders of magnitude smaller. Note that the process $\overline{B_d^0} \to \gamma \, \overline{D^{*0}}$ has substantial meson exchanges (would be chiral loops for $\omega \to 1$), and is different.

7. Conclusions

Our low energy framework is well suited to $B - \overline{B}$ mixing, and to some extent to $B \to D\overline{D}$. Work continues to include $(0^+, 1^+)$, states, counterterms, and $1/m_c$ terms. Note that the amplitude for $\overline{B_d^0} \to D_s^+ D_s^-$ is zero in the factorized limit. For processes like $B \to D\eta'$ and $B \to D\gamma$ we can give order of magnitude estimates when factorization give zero or small amplitudes.

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REFERENCES

[1] A. Hiorth and J. O. Eeg, Eur. Phys. J. direct **C30**, 006 (2003) (see also references therein).

[2] J.O. Eeg, S. Fajfer , and A. Hiorth, Phys.Lett. **B570**, 46-52 (2003);
J. O. Eeg, S. Fajfer, and A. Prapotnik Eur. Phys. J. **C42**, 29-36 (2005).
See also: J.O. Eeg, S. Fajfer, J. Zupan, Phys. Rev. **D 64**, 034010 (2001).

[3] J. O. Eeg, A. Hiorth, A. D. Polosa, Phys. Rev. **D 65**, 054030 (2002).

[4] B.Grinstein and R.F. Lebed, Phys.Rev. **D60**, 031302(R) (1999).

[5] O. Antipin and G. Valencia, Phys.Rev. **D74**, 054015 (2006), hep-ph/0606065.

[6] J.A. Macdonald Sørensen and J.O. Eeg, hep-ph/0605078.

[7] M. Beneke *et. al*, Phys. Rev. Lett. **83**, 1914 (1999); C. W. Bauer *et al.* Phys. Rev. D **70**, 054015 (2004)

[8] A. Hiorth and J. O. Eeg, Phys. Rev. **D 66**, 074001 (2002), and references therein.

[9] For a review, see M. Neubert, Phys. Rep. **245**, 259 (1994).

[10] For a review, see: R. Casalbuoni *et al.* Phys. Rep. **281**, 145 (1997).

[11] S. Bertolini, J.O. Eeg and M. Fabbrichesi, Nucl. Phys. **B449**, 197 (1995);
V. Antonelli *et al.* Nucl. Phys. **B469**, 143 (1996); S. Bertolini *et al.* Nucl. Phys. **B514**, 63 (1998); *ibid* **B514**, 93 (1998).

[12] See for example: G. Buchalla, A. J. Buras, M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996), and references therein.

[13] B. Grinstein *et al.*, Nucl. Phys. **B363** , 19 (1991). R. Fleischer, Nucl. Phys. **B 412**, 201 (1994).