Non-Equilibrium Josephson and Andreev Current through Interacting Quantum Dots

Marco G. Pala,[1] Michele Governale,[2] and Jürgen König[2]

[1]IMEP-MINATEC (UMR CNRS/INPG/UJF 5130), BP 257, 38016 Grenoble, France

[2]Institut für Theoretische Physik III, Ruhr-Universität Bochum, 44780 Bochum, Germany

(Dated: August 30, 2013)

We present a theory of transport through interacting quantum dots coupled to normal and superconducting leads in the limit of weak tunnel coupling. A Josephson current between two superconducting leads, carried by first-order tunnel processes, can be established by non-equilibrium proximity effect. Both Andreev and Josephson current is suppressed for bias voltages below a threshold set by the Coulomb charging energy. A π-transition of the supercurrent can be driven by tuning gate or bias voltages.

PACS numbers: 74.45.+c,73.23.Hk,73.63.Kv,73.21.La

Non-equilibrium transport through superconducting systems attracted much interest since the demonstration of a Superconductor-Normal-Superconductor (SNS) transistor [1]. In such a device, supercurrent suppression and its sign reversal (π-transition) are achieved by driving the quasi-particle distribution out of equilibrium by means of applied voltages [2, 3, 4, 5]. Another interesting issue in mesoscopic physics is transport through quantum dots attached to superconducting leads. For DC transport through quantum dots coupled to a normal and a superconducting lead, subgap transport is due to Andreev reflection [6, 7, 8, 9, 10, 11]. Also transport between two superconductors through a quantum dot has been studied extensively. The limit a non-interacting dot has been investigated in Ref. 12. Several authors considered the regime of weak tunnel coupling where the electrons forming a Cooper pair tunnel one by one via virtual states [13, 14, 15, 16]. Multiple Andreev reflection through localized levels was investigated in Refs. 17, 18. Numerical approaches based on the non-crossing approximation [19], the numerical renormalization group [20] and Monte Carlo [21] have also been used. The authors of Ref. 22 compare different approximation schemes, such as mean field and second-order perturbation in the Coulomb interaction. Experimentally, the supercurrent through a quantum dot has been measured through dots realized in carbon nanotubes [23] and in indium arsenide nanowires [24].

In this Letter we study the transport properties of a system composed of an interacting single-level quantum dot between two equilibrium superconductors where a third, normal lead is used to drive the dot out of equilibrium. We relate the subgap current flowing into the superconductors to the pair amplitude of the dot $\langle d_\downarrow d_\uparrow \rangle$, induced by proximity effect. The latter is calculated by means of a kinetic equation derived from a real-time diagrammatic technique. In particular, we identify nonequilibrium situations in which a Josephson current carried by first-order tunnel processes is established.

Model.—We consider a single-level quantum dot connected to two superconducting and one normal lead via tunnel junctions, see Fig. 1. The total Hamilto-

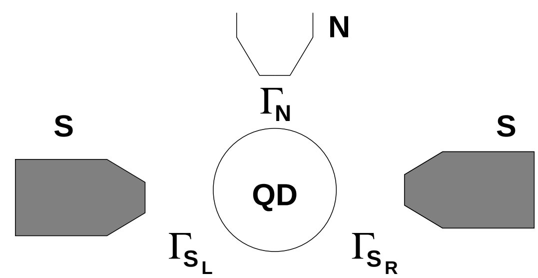


FIG. 1: Setup: a single-level quantum dot is connected by tunnel junctions to one normal and two superconducting leads with tunneling rates $\Gamma_{\rm N}$ and $\Gamma_{\rm S_{L,R}}$, respectively.

nian is given by $H = H_{\rm D} + \sum_{\eta={\rm N,S}_L,{\rm S}_R}(H_\eta + H_{{\rm tunn},\eta})$. The quantum dot is described by the Anderson model $H_{\rm D} = \sum_\sigma \epsilon d^\dagger_\sigma d_\sigma + U n_\uparrow n_\downarrow$, where $n_\sigma = d^\dagger_\sigma d_\sigma$ is the number operator for spin $\sigma =\uparrow,\downarrow$, ϵ is the energy level, and U is the charging energy for double occupation. The leads, labeled by $\eta = {\rm N, S_L, S_R}$, are modeled by $H_\eta = \sum_{k\sigma} \epsilon_k c^\dagger_{\eta k\sigma} c_{\eta k\sigma} - \sum_k \left(\Delta_\eta c^\dagger_{\eta k\uparrow} c^\dagger_{\eta -k\downarrow} + {\rm H.c.}\right)$, where Δ_η is the superconducting order parameter ($\Delta_{\rm N} = 0$). The tunneling Hamiltonians are $H_{{\rm tunn},\eta} = V_\eta \sum_{k\sigma}\left(c^\dagger_{\eta k\sigma} d_\sigma + {\rm H.c.}\right)$. Here, V_η are the spin- and wavevector-independent tunnel matrix elements, and $c_{\eta k\sigma}(c^\dagger_{\eta k\sigma})$ and $d_\sigma(d^\dagger_\sigma)$ represent the annihilation (creation) operators for the leads and dot, respectively. The tunnel-coupling strengths are characterized by $\Gamma_\eta = 2\pi|V_\eta|^2 \sum_k \delta(\omega - \epsilon_k)$.

Current formula.— We start with deriving a general formula for the charge current in lead η by generalizing to superconducting leads the approach of Ref. [25]. For this, it is convenient to use the operators $\psi_{\eta k} = (c_{\eta k\uparrow}, c^\dagger_{\eta -k\downarrow})^{\rm T}$ and $\phi = (d_\uparrow, d^\dagger_\downarrow)^{\rm T}$ in Nambu formalism. The current from lead η is expressed as $J_\eta = e\langle dN_\eta/dt\rangle = i(e/\hbar)\langle[H, N_\eta]\rangle = i(e/\hbar)\langle[H_{{\rm tunn},\eta}, N_\eta]\rangle$ [26], with $N_\eta = \sum_k \psi^\dagger_{\eta k}\tau_3\psi_{\eta k}$, where τ_1, τ_2, τ_3 indicate the Pauli matrices in Nambu space. Evaluating the commutator leads to

$$J_\eta = -\frac{2e}{\hbar}\sum_k \int \frac{d\omega}{2\pi}\mathrm{Re}\left\{\mathrm{Tr}\left[\tau_3 \mathbf{V}_\eta \mathbf{G}^<{}_{\mathrm{D},\eta k}(\omega)\right]\right\}, \quad (1)$$

with $\mathbf{V}_\eta = \mathrm{Diag}(V_\eta, -V_\eta^*)$ and the lead–dot lesser Green's functions $(\mathbf{G}^<{}_{\mathrm{D},\eta k}(\omega))_{m,n}$ that are the Fourier transforms of $i\langle\psi^\dagger_{\eta kn}(0)\phi_m(t)\rangle$. In the following, we assume the tunneling matrix elements V_η to be real (any phase of V_η can be gauged away by substituting $\Delta_\eta \to \Delta_\eta \exp(-2i\arg V_\eta)$). The Green's function $\mathbf{G}^<{}_{\mathrm{D},\eta k}$ is related to the full dot Green's functions and the lead Green's functions by a Dyson equation in Keldysh formalism [27]. Using this relation and assuming energy-independent tunnel rates Γ_η, we obtain for the current $J_\eta = J_{1\eta} + J_{2\eta}$ with

$$\begin{aligned} J_{1\eta} &= \frac{e}{\hbar}\int\frac{d\omega}{2\pi}\Gamma_\eta D_\eta(\omega)\mathrm{Im}\left\{\mathrm{Tr}\left[\tau_3\left[\mathbf{1} - \frac{\mathbf{\Delta}_\eta}{\omega-\mu_\eta}\right]\right.\right. \\ &\qquad \left.\left.\left(2\mathbf{G}^{\mathrm{R}}(\omega)f_\eta(\omega) + \mathbf{G}^<(\omega)\right)\right]\right\}, \quad (2) \\ J_{2\eta} &= \frac{e}{\hbar}\int\frac{d\omega}{2\pi}\Gamma_\eta S_\eta(\omega)\mathrm{Re}\left\{\mathrm{Tr}\left[\tau_3\frac{\mathbf{\Delta}_\eta}{|\Delta_\eta|}\mathbf{G}^<(\omega)\right]\right\}, \quad (3) \end{aligned}$$

where $\mathbf{\Delta}_\eta = \begin{pmatrix} 0 & \Delta_\eta \\ \Delta_\eta^* & 0\end{pmatrix}$, and $f_\eta(\omega) = [1+\exp(\omega - \mu_\eta)/(k_{\mathrm{B}}T))]^{-1}$ is the Fermi function, with T being the temperature and k_{B} the Boltzmann constant. The dot Green's functions $(\mathbf{G}^<{}_{\mathrm{D}}(\omega))_{m,n}$ and $\left(\mathbf{G}^{\mathrm{R}}{}_{\mathrm{D}}(\omega)\right)_{m,n}$ are defined as the Fourier transforms of $i\langle\phi_n^\dagger(0)\phi_m(t)\rangle$ and $-i\theta(t)\langle\{\phi_m(t),\phi_n^\dagger(0)\}\rangle$, respectively. The two weighting functions $D_\eta(\omega)$ and $S_\eta(\omega)$ are given by

$$\begin{aligned} D_\eta(\omega) &= \frac{|\omega-\mu_\eta|}{\sqrt{(\omega-\mu_\eta)^2 - |\Delta_\eta|^2}}\theta(|\omega-\mu_\eta| - |\Delta_\eta|) \\ S_\eta(\omega) &= \frac{|\Delta_\eta|}{\sqrt{|\Delta_\eta|^2 - (\omega-\mu_\eta)^2}}\theta(|\Delta_\eta| - |\omega-\mu_\eta|). \end{aligned}$$

The term $J_{1\eta}$ involves energies above the gap. For $\eta = \mathrm{N}$, this is the only contribution, and the current reduces to the result presented in Ref. 25. For superconducting leads, $J_{1\eta}$ describes quasiparticle transport, but also includes anomalous components of the Green's functions. In the limit of large superconducting gap, only subgap contributions to the current, $J_{2\eta}$, describing Josephson as well as Andreev tunneling remain. In the following we consider the large-superconducting-gap limit ($|\Delta_\eta| \to \infty$), where the current simplifies to

$$J_\eta = \frac{2e}{\hbar}\Gamma_\eta |\langle d_\downarrow d_\uparrow\rangle| \sin(\Psi - \Phi_\eta), \quad (4)$$

with Φ_η being the phase of Δ_η and $\langle d_\downarrow d_\uparrow\rangle = |\langle d_\downarrow d_\uparrow\rangle|\exp(i\Psi)$ the pair amplitude of the dot that has to be determined in the presence of Coulomb interaction, coupling to all (normal and superconducting) leads and in non-equilibrium due to finite bias voltage.

We now focus on a symmetric three-terminal setup with $\Gamma_{\mathrm{S_L}} = \Gamma_{\mathrm{S_R}} = \Gamma_{\mathrm{S}}$, $\Delta_{\mathrm{S_L}} = |\Delta|\exp(i\Phi/2)$ and $\Delta_{\mathrm{S_R}} = |\Delta|\exp(-i\Phi/2)$, and $\mu_{\mathrm{S_L}} = \mu_{\mathrm{S_R}} = 0$. The quantities of interest are the the current that flows between the two superconductors (Josephson current) $J_{\mathrm{jos}} = (J_{\mathrm{S_L}} - J_{\mathrm{S_R}})/2$ and the current in the normal lead (Andreev current) $J_{\mathrm{and}} = J_{\mathrm{N}} = -(J_{\mathrm{S_L}} + J_{\mathrm{S_R}})$.

To proximize the quantum dot, the dot states for empty and double occupation should be energetically almost degenerate. Furthermore, in *equilibrium* situations, temperature must be low enough, $k_{\mathrm{B}}T < \Gamma_{\mathrm{S}}$, to resolve the influence of the superconductors on the quantum-dot spectrum [28]. In this paper, however, we focus on the opposite limit, $k_{\mathrm{B}}T > \Gamma_{\mathrm{S}}$, in which proximity is due to *non-equilibrium* occupation of the dot only.

Kinetic equations for quantum-dot degrees of freedom.– The Hilbert space of the dot is four dimensional: the dot can be empty, singly occupied with spin up or down, or doubly occupied, denoted by $|\chi\rangle \in \{|0\rangle, |\uparrow\rangle, |\downarrow\rangle, |D\rangle \equiv d_\uparrow^\dagger d_\downarrow^\dagger|0\rangle)\}$, with energies E_0, $E_\uparrow = E_\downarrow$, E_D. For convenience we define the detuning as $\delta = E_D - E_0 = 2\epsilon + U$. The dot dynamics is fully described by its reduced density matrix ρ_{D}, with matrix elements $P^{\chi_1}_{\chi_2} \equiv (\rho_{\mathrm{D}})_{\chi_2\chi_1}$. The dot pair amplitude $\langle d_\downarrow d_\uparrow\rangle$ is given by the off-diagonal matrix element P^0_D. The time evolution of the reduced density matrix is described by the kinetic equations

$$\begin{aligned} &\frac{d}{dt}P^{\chi_1}_{\chi_2}(t) + \frac{i}{\hbar}(E_{\chi_1} - E_{\chi_2})P^{\chi_1}_{\chi_2}(t) \\ &= \sum_{\chi_1',\chi_2'}\int_{t_0}^t dt' W^{\chi_1\chi_1'}_{\chi_2\chi_2'}(t,t')P^{\chi_1'}_{\chi_2'}(t'). \quad (5) \end{aligned}$$

We define the generalized transition rates by $W^{\chi_1\chi_1'}_{\chi_2\chi_2'} \equiv \int_{-\infty}^t dt' W^{\chi_1\chi_1'}_{\chi_2\chi_2'}(t,t')$, which are the only quantities to be evaluated in the stationary limit. Together with the normalization condition $\sum_\chi P_\chi = 1$, Eq. (5) determines the matrix elements of ρ_{D}. Furthermore, in Eq. (5) we retain only linear terms in the tunnel strengths Γ_η and the detuning δ. Hence, we calculate the rates $W^{\chi_1\chi_1'}_{\chi_2\chi_2'}$ to the lowest (first) order in Γ_η for $\delta = 0$. This is justified in the transport regime $\Gamma_{\mathrm{S}}, \Gamma_{\mathrm{N}}, \delta < k_{\mathrm{B}}T$.

The rates are evaluated by means of a real-time diagrammatic technique [29], that we generalize to include superconducting leads. We find for the (first-order) diagonal rates $W_{\chi_1\chi_2} \equiv W^{\chi_1\chi_2}_{\chi_1\chi_2}$ the expressions $W_{\sigma 0} = \Gamma_{\mathrm{N}} f_{\mathrm{N}}(-U/2)$; $W_{0\sigma} = \Gamma_{\mathrm{N}}[1 - f_{\mathrm{N}}(-U/2)]$; $W_{D\sigma} = \Gamma_{\mathrm{N}} f_{\mathrm{N}}(U/2)$; $W_{\sigma D} = \Gamma_{\mathrm{N}}[1 - f_{\mathrm{N}}(U/2)]$. The N lead also contributes to the rates $W^{DD}_{00} = -\Gamma_{\mathrm{N}}[1 + f_{\mathrm{N}}(-U/2) - f_{\mathrm{N}}(U/2) + iB]$ and $W^{00}_{DD} = -\Gamma_{\mathrm{N}}[1 - f_{\mathrm{N}}(-U/2) + f_{\mathrm{N}}(U/2) - iB]$ where $B = \frac{1}{\pi}\mathrm{Re}\left[\psi\left(\frac{1}{2} + i\frac{U/2-\mu_{\mathrm{N}}}{2\pi T}\right) - \psi\left(\frac{1}{2} + i\frac{-U/2-\mu_{\mathrm{N}}}{2\pi T}\right)\right]$, with μ_{N} being the chemical potential of the normal lead and $\psi(z)$ the Digamma function. Notice that B vanishes when $\mu_{\mathrm{N}} = 0$ or $U = 0$. The superconducting leads do not enter here due to the gap in the quasi-particle density of states. These leads, though, contribute to the off-diagonal rates $W^{00}_{0D} = W^{DD}_{0D} = \alpha(\Phi); W^{0D}_{00} = W^{0D}_{DD} = \alpha^*(\Phi); W^{D0}_{00} = W^{D0}_{DD} = -\alpha(\Phi); W^{00}_{D0} = W^{DD}_{D0} = -\alpha^*(\Phi)$, where $\alpha(\Phi) = 2i\Gamma_S \cos(\Phi/2)$.

For an intuitive representation of the system dynamics we define, in analogy to Ref. [30], a dot isospin by

$$I_x = \frac{P^D_0 + P^0_D}{2};\; I_y = i\frac{P^D_0 - P^0_D}{2};\; I_z = \frac{P_D - P_0}{2}. \quad (6)$$

From Eq. (5), we find that in the stationary limit the isospin dynamics can be separated into three parts, $0 = d\mathbf{I}/dt = (d\mathbf{I}/dt)_{\rm acc} + (d\mathbf{I}/dt)_{\rm rel} + (d\mathbf{I}/dt)_{\rm rot}$, with

$$\hbar\left(\frac{d\mathbf{I}}{dt}\right)_{\rm acc} = -\frac{\Gamma_N}{2}[1 - f_N(-U/2) - f_N(U/2)]\hat{\mathbf{e}}_z \quad (7)$$

$$\hbar\left(\frac{d\mathbf{I}}{dt}\right)_{\rm rel} = -\Gamma_N[1 + f_N(-U/2) - f_N(U/2)]\mathbf{I} \quad (8)$$

$$\hbar\left(\frac{d\mathbf{I}}{dt}\right)_{\rm rot} = \mathbf{I}\times\mathbf{B}_{\rm eff} \quad (9)$$

where $\hat{\mathbf{e}}_z$ is the z-direction and $\mathbf{B}_{\rm eff} = \{2\Gamma_S\cos(\Phi/2), 0, -\Gamma_N B - 2\epsilon - U\}$ is an effective magnetic field in the isospin space. The accumulation term Eq. (7) builds up a finite isospin, while the relaxation term Eq. (8) decreases it. Finally, Eq. (9) describes a rotation of the isospin direction.

Non-equilibrium Josephson current.— In the isospin language the current in the superconducting leads is

$$J_{S_{L,R}} = \frac{2e}{\hbar}\Gamma_S\left[I_y\cos(\Phi/2) \pm I_x\sin(\Phi/2)\right], \quad (10)$$

where the upper(lower) sign refers to the left(right) superconducting lead. The I_y component contributes to the Andreev current, while I_x is responsible for the Josephson current. By inspection of Eqs. (7)-(9), it is apparent that for a finite Josephson current $J_{\rm jos}$, we need that the z-component, $-\Gamma_N B - 2\epsilon - U$, of the effective magnetic field acting on the isospin is non zero. The Josephson current and the Andreev current read

$$J_{\rm jos} = -\frac{e\Gamma_S}{\hbar}\frac{[2\epsilon + U + \Gamma_N B]\Gamma_S\sin(\Phi)}{|\mathbf{B}_{\rm eff}|^2 + \Gamma_N^2[1 + f_N(-U/2) - f_N(U/2)]^2} \times\frac{1 - f_N(-U/2) - f_N(U/2)}{1 + f_N(-U/2) - f_N(U/2)} \quad (11)$$

$$J_{\rm and} = -\frac{e\Gamma_S}{\hbar}\frac{2\Gamma_N\Gamma_S[1 + \cos(\Phi)]}{|\mathbf{B}_{\rm eff}|^2 + \Gamma_N^2[1 + f_N(-U/2) - f_N(U/2)]^2} \times[1 - f_N(-U/2) - f_N(U/2)]. \quad (12)$$

These results take into account only first-order tunnel processes, i.e. the rates $W^{\chi_1\chi_1'}_{\chi_2\chi_2'}$ are computed to first order in Γ_η. The factor $[1 - f_N(-U/2) - f_N(U/2)]$ ensures that no finite dot-pair amplitude can be established if the chemical potential of the normal lead, μ_N, is inside the interval $[-U/2, U/2]$ by at least $k_B T$. In this situation both the Josephson and the Andreev currents vanish. On the other hand, this factor takes the value -1 if $\mu_N > U/2$ and the value $+1$ if $\mu_N < -U/2$. Hence, the sign of the Josephson current can be reversed by the applied voltage (voltage driven π-transition). The considerations above establish the importance of the non-equilibrium voltage to induce and control proximity effect in the interacting quantum dot. In Fig. 2 we show in a density plot of (a) $J_{\rm jos}$ and (b) $J_{\rm and}$ for $\Phi = \pi/2$ as a function of the voltage μ_N and the level position ϵ. Both the control of proximity effect by the chemical potential μ_N and the voltage driven π-transition are clearly visible. If the detuning is too large, $|\delta + \Gamma_N B| > \sqrt{\Gamma_N^2 + 4\Gamma_S^2\cos^2(\Phi/2)}$, it becomes difficult to build a superposition of the states $|0\rangle$ and $|D\rangle$, which is necessary to establish proximity. As a consequence, the Josephson and the Andreev current are algebraically suppressed by δ^{-1} and δ^{-2}, respectively. Fig. 3 shows the Josephson current as a function of $\delta = 2\epsilon + U$. The fact that the Josephson current is non zero for $\delta = 0$ is due to the term $\Gamma_N B$, i.e. of the interaction induced contribution to the z-component of

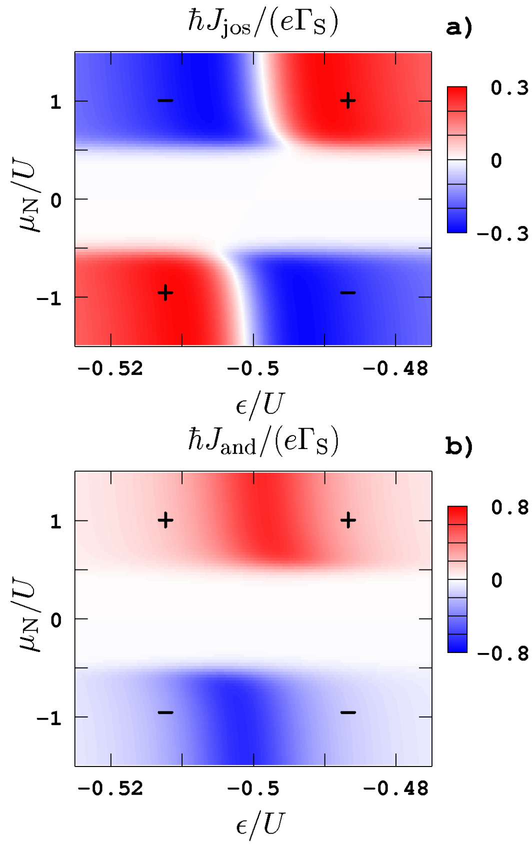


FIG. 2: (color online) Density plot of the a) Josephson and b) Andreev current, for fixed superconducting-phase difference $\Phi = \pi/2$, as a function of the dot-level position ϵ and of the chemical potential of the normal lead μ_N. The symbols $\pm$ refer to the sign of the current. The other parameters are $\Gamma_S = \Gamma_N = 0.01U$, and $k_B T = 0.05U$.

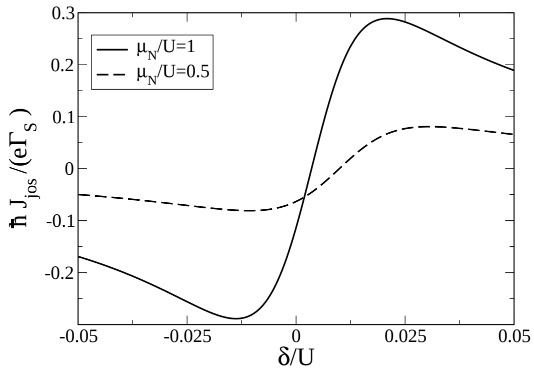


FIG. 3: Josephson current, for fixed superconducting-phase difference $\Phi = \pi/2$, as a function of the detuning $\delta = E_D - E_0 = 2\epsilon + U$ for different values of the chemical potential. The other parameters are $\Gamma_\mathrm{S} = \Gamma_\mathrm{N} = 0.01U$ and $k_\mathrm{B}T = 0.05U$.

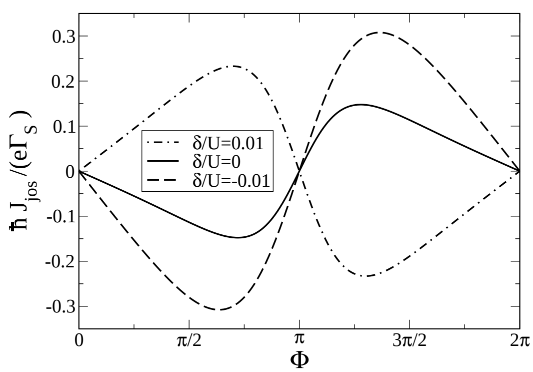


FIG. 4: Josephson current as a function of the superconducting-phase difference Φ for different values of the detuning. The other parameters are $\Gamma_\mathrm{S} = \Gamma_\mathrm{N} = 0.01U$, $\mu_\mathrm{N} = U$, and $k_\mathrm{B}T = 0.05U$.

the effective field $\mathbf{B}_\mathrm{eff}$ acting on the isospin. The term $|B|$ has a maximum at $\mu_\mathrm{N} = U/2$, which causes this effect to be more pronounced at the onset of transport. The fact that the value of the Josephson current varies on a scale smaller than temperature indicates its nonequilibrium nature.

A π-transition of the Josephson current can also be achieved by changing the sign of $\delta + \Gamma_\mathrm{N} B$, as shown in Fig. 4 where J_jos is plotted as a function of the phase difference Φ for different values of the level position. Notice that the current for $\delta = 0$ ($\epsilon = -U/2$) is different from zero only due to the presence of the term $\Gamma_\mathrm{N} B$ acting on the isospin.

Conclusion.— We have studied non-equilibrium proximity effect in an interacting single-level quantum dot weakly coupled to two superconducting and one normal leads. By applying a bias voltage between normal and superconducting leads, a Josephson current carried by first-order tunneling processes, is established. A π-transition can be driven either by bias or gate voltage. The charging energy of the quantum dot defines a threshold bias voltage below which proximity is suppressed.

We would like to thank W. Belzig, R. Fazio, A. Shnirman, and A. Volkov for useful discussions. M.G. and J.K. acknowledge the hospitality of Massey University, Palmerston North, and of the CAS Oslo, respectively.

[1] J.J.A. Baselmans, A.F. Morpurgo, B.J. van Wees, T.M. Klapwijk, Nature **397**, 43 (1999).
[2] A. F. Volkov, Phys. Rev. Lett. **74**, 4730 (1995).
[3] F.K. Wilhelm, G. Schön, and A.D. Zaikin, Phys. Rev. Lett. **81**, 1682 (1998).
[4] S.-K. Yip, Phys. Rev. B 58, 5803 (1998).
[5] F. Giazotto, T.T. Heikkilä, F. Taddei, R. Fazio, J.P. Pekola, and F. Beltram, Phys. Rev. Lett. **92**, 137001 (2004).
[6] R. Fazio and R. Raimondi, Phys. Rev. Lett. **80**, 2913 (1998); Phys. Rev. Lett. **82**, 4950 (1999).
[7] K. Kang, Phys. Rev. B **58**, 9641 (1998).
[8] P. Schwab and R. Raimondi, Phys. Rev. B **59**, 1637 (1999).
[9] A. A. Clerk, V. Ambegaokar, and S. Hershfield, Phys. Rev. B **61**, 3555 (2000).
[10] S. Shapira, E. H. Linfield, C. J. Lambert, R. Seviour, A. F. Volkov, and A. V. Zaitsev, Phys. Rev. Lett. **84**, 159 (2000).
[11] J. C. Cuevas, A. L. Yeyati, and A. Martín-Rodero, Phys. Rev. B **63**, 094515 (2001).
[12] C. W. J. Beenakker and H. van Houten, in *Single-Electron Tunneling and Mesoscopic Devices*, edited by H. Koch and H. Lübbig, Springer, Berlin, 1992, pp. 175-179.
[13] L. I. Glazman and K. A. Matveev, JETP Lett. **49**, 659 (1989).
[14] B. I. Spivak and S. A. Kivelson, Phys. Rev. B **43**, 3740 (1991).
[15] A.V. Rozhkov, D.P. Arovas, and F. Guinea, Phys. Rev. B **64**, 233301 (2001).
[16] Y. Avishai, A. Golub, and A.D. Zaikin, Phys. Rev. B **67**, 041301(R) (2003).
[17] A. L. Yeyati, J. C. Cuevas, A. López-Dávalos, and A. Martín-Rodero, Phys. Rev. B **55**, R6137 (1997).
[18] G. Johansson, E. N. Bratus, V. S. Shumeiko, and G. Wendin, Phys. Rev. B **60**, 1382 (1999).
[19] S. Ishizaka, J Sone, and T. Ando, Phys. Rev. B **52**, 8358 (1995).
[20] Mahn-Soo Choi, Minchul Lee, K. Kang, and W. Belzig, Phys. Rev. B **70**, 020502(R) (2004).
[21] F. Siano and R. Egger, Phys. Rev. Lett. **93**, 047002 (2004).
[22] E. Vecino, A. Martín-Rodero, and A. L. Yeyati, Phys. Rev B **68**, 035105 (2003).
[23] M. R. Buitelaar, T. Nussbaumer, and C. Schönenberger, Phys. Rev. Lett. **89**, 256801 (2002); J.-P. Cleuziou, W. Wernsdorfer, V. Bouchiat, T. Ondarçuhu, and M. Monthioux, Nature Nanotechnology **1**, 53 (2006); P. Jarillo-Herrero, J. A. van Dam, and L. P. Kouwenhoven, Nature **439**, 953 (2006); H. I. Jørgensen, K. Grove-Rasmussen, T. Novotný, K. Flensbert, and P. E. Lindelof, Phys. Rev. Lett. **96**, 207003 (2006).
[24] J.A. van Dam, Y.V. Nazarov, E.P.A.M. Bakkers, S. De Franceschi, and L.P. Kouwenhoven, Nature **442**,

667 (2006); T. Sand-Jespersen *et al.* unpublished; cond-mat/0703264.

[25] Y. Meir and N.S. Wingreen, Phys. Rev. Lett. **68**, 2512 (1992).

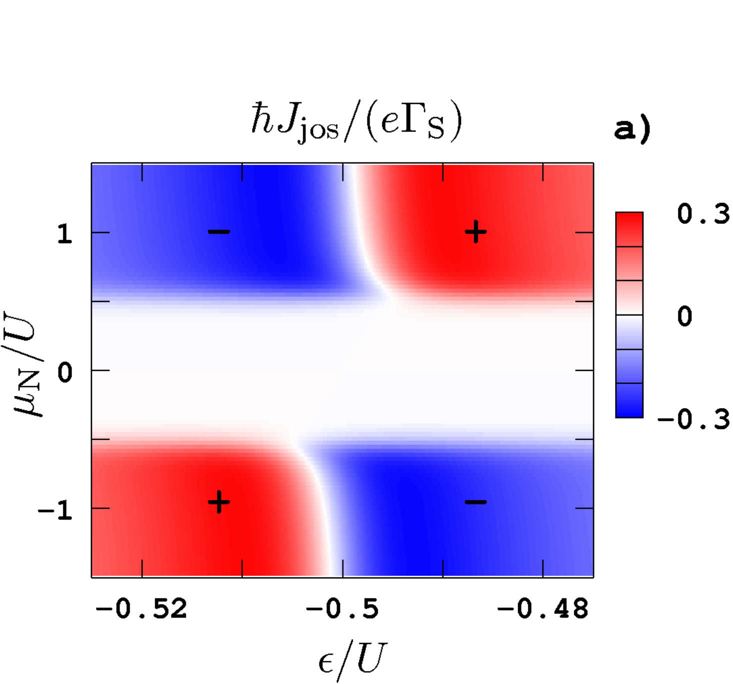
[26] Note that $[H_\eta, N_\eta] \neq 0$ but $\langle [H_\eta, N_\eta] \rangle = 0$ for $\eta = \mathrm{S}_{\mathrm{L,R}}$.

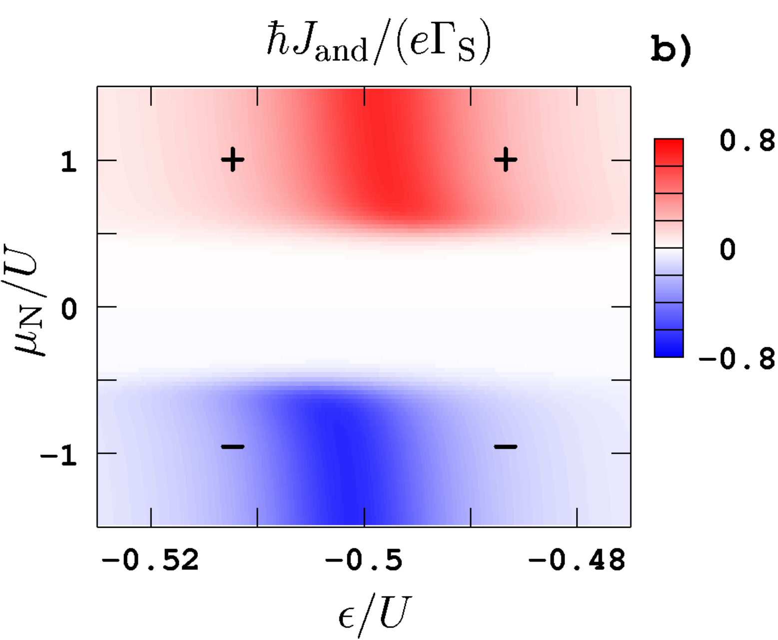
[27] The Dyson equation reads $\mathbf{G}^{<}{}_{\mathrm{D},\eta k}(\omega) = \mathbf{G}^{\mathrm{R}}(\omega)\mathbf{V}_\eta^\dagger \mathbf{g}_{\eta k}^{<}(\omega) + \mathbf{G}^{<}(\omega)\mathbf{V}_\eta^\dagger \mathbf{g}_{\eta k}^{\mathrm{A}}(\omega)$, where $\mathbf{G}^{\mathrm{R}(<)}(\omega)$ is the retarded (lesser) dot Green's function, and and $\mathbf{g}_{\eta k}^{\mathrm{A}(<)}(\omega)$ the lead advanced (lesser) Green's function.

[28] This can, e.g., be seen in the exactly-solvable limit of $U = 0$ together with $\Gamma_{\mathrm{N}} = 0$, where the Josephson current is $J_{\mathrm{jos}} = -(e/2\hbar)\Gamma_{\mathrm{S}}^2 \sin(\Phi)\left[f(-\epsilon_{\mathrm{A}}(\Phi)) - f(\epsilon_{\mathrm{A}}(\Phi))\right]/\epsilon_{\mathrm{A}}(\Phi)$ with $\epsilon_{\mathrm{A}}(\Phi) = \sqrt{\epsilon^2 + \Gamma_{\mathrm{S}}^2\cos^2(\Phi/2)}$.

[29] J. König, H. Schoeller, and G. Schon, Phys. Rev. Lett **76**, 1715 (1996); J. König,J. Schmid, H. Schoeller, and G. Schon, Phys. Rev. B **54** 16820 (1996).

[30] M. Braun, J. König, and J. Martinek, Phys. Rev. B **70**, 195345 (2004).


$\hbar J_{\rm jos}/(e\Gamma_{\rm S})$
a)
0.3
0
−0.3
1
0
−1
$\mu_{\rm N}/U$
−0.52
−0.5
−0.48
ϵ/U
−
+
+
−



$\hbar J_{\rm and}/(e\Gamma_{\rm S})$
b)
0.8
0
-0.8
+
+
−
−
1
0
-1
$\mu_{\rm N}/U$
-0.52
-0.5
-0.48
ϵ/U