QED⊗QCD Resummation and Shower/ME Matching for LHC Physics*

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Abstract

We present the theory of QED⊗QCD resummation and its interplay with shower/matrix element matching in precision LHC physics scenarios. We illustrate the theory using single heavy gauge boson production at hadron colliders.

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1 Introduction

In the imminent LHC environment, where one expects to have an experimental luminosity precision tag at the level of 2%, [1] the requirement for the theoretical precision tag on the corresponding luminosity processes, such as single W, Z production with the subsequent decay into light lepton pairs, should be at the 0.67% level in order not to compromise, unnecessarily, the over-all precision of the respective LHC luminosity determinations. This dictates that multiple gluon and photon radiative effects must be controlled at the stated precision. The theory of $QED \otimes QCD$ exponentiation [2] allows for the simultaneous resummation of multiple gluon and multiple photon radiative effects in LHC physics processes, to be realized ultimately by MC methods on an event-by-event basis in the presence of parton showers, in a framework which allows us to systematically improve the accuracy of the calculations without double-counting of effects, in principle to all orders in both α_s and α. Such a theoretical framework opens the way to the desired theoretical precision tag on the LHC luminosity processes.

Our starting point for the new QED⊗QCD resummation theory [2] is the QCD resummation theory presented in Ref. [3]. This resummation is an exact rearrangement of the QCD perturbative series based on the $N = 1$ term in the exponent in the formal proof of exponentiation in non-Abelian gauge theories *in the eikonal approximation*, as given in Ref. [4]. This exponential is augmented with a sum of residuals which take into account the remaining contributions to the perturbative series exactly to all orders in α_s.[1] We therefore have an exact result whereas the resummation theory in Ref. [4] and those in Refs. [5–7] are approximate. Recently, an alternative resummation theory, the soft-collinear effective theory(SCET) [8], has been developed to treat double resummation of soft and collinear effects. Since we have an exact re-arrangement of the perturbative series, we could introduce the results from Refs. [5–8] into our representation as well. Such introductions will appear elsewhere.

The need for the extension of the QCD resummation theory to QED⊗ QCD resummation was already suggested by the results in Refs. [9–14], where

[1]If desired, our overall expoential factor can be made to include all of the terms in the exponent in Ref. [4], in principle.

it was shown that in the evolution of the structure functions the inclusion of the QED contributions leads to effects at the level of $\sim 0.3\%$, already almost half of the error budget discussed above. We will find similar size effects from the threshold region of heavy gauge boson production. All of these must be taken into account if one wants $\sim 1.0\%$ for the theoretical precision tag.

The discussion is organized as follows. In Section 2, we review the extension of the YFS theory to an exact resummation theory for QCD. Section 3 presents the further extension to QED⊗QCD. Section 4 contains the application to heavy gauge boson production with the attendant discussion of shower/ME matching. Section 5 contains some concluding remarks.

2 Extension of YFS Theory to QCD

We consider a parton-level single heavy boson production process such as $q + \bar{q}' \rightarrow V + n(g) + X \rightarrow \bar{\ell}\ell' + n(g) + X$, where $V = W^{\pm}, Z$, and $\ell = e, \mu,\ \ell' = \nu_e, \nu_\mu(e, \mu)$ respectively for $V = W^+(Z)$, and $\ell = \nu_e, \nu_\mu,\ \ell' = e, \mu$ respectively for $V = W^-$. It has been established [3] that the cross section may be expressed as

$$\begin{aligned} d\hat{\sigma}_{\rm exp} = \sum_n d\hat{\sigma}^n = e^{\rm SUM_{IR}(QCD)} \sum_{n=0}^{\infty} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j} \\ \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(p_1+p_2-q_1-q_2-\sum k_j)+D_{\rm QCD}} \\ \times \tilde{\bar{\beta}}_n(k_1, \ldots, k_n) \frac{d^3 p_2}{p_2^0} \frac{d^3 q_2}{q_2^0} \end{aligned} \tag{1}$$

where gluon residuals $\tilde{\bar{\beta}}_n(k_1, \ldots, k_n)$, defined by Ref. [3], are free of all infrared divergences to all orders in $\alpha_s(Q)$. The functions $\mathrm{SUM}_{IR}(\mathrm{QCD})$ and $D_{\rm QCD}$, together with the basic infrared functions $B^{nls}_{\rm QCD}$, $\tilde{B}^{nls}_{\rm QCD}$, and $\tilde{S}^{nls}_{\rm QCD}$ are specified in Ref. [3]. We call attention to the essential compensation between the left over genuine non-Abelian IR virtual and real singularities between the phase space integrals $\int d\mathrm{Ph}\ \bar{\beta}_n$ and $\int d\mathrm{Ph}\ \bar{\beta}_{n+1}$ that really allows us to isolate $\tilde{\bar{\beta}}_j$ and distinguishes QCD from QED, where no such compensation occurs. The result in (1) has been realized by Monte Carlo methods [3]. See also Refs. [15–17] for exact $\mathcal{O}(\alpha_s^2)$ and Refs. [18–20] for exact $\mathcal{O}(\alpha)$ results on the heavy gauge boson production processes which we discuss here.

Apparently, we can not emphasize too much the exactness of (1). Some confusion seems to exist because it does not show explicitly an ordered exponential operator for an appropriate ordering prescription, path-ordered, time-ordered, etc. The essential point is that, in (1), we have evaluated the matrix elements of these operators and written the result in terms of the over-all exponent shown therein and the residuals $\tilde{\bar{\beta}}_j$. This allows us to maintain exactness to all orders in α_s.

3 QED⊗QCD Resummation Theory

The new $QED \otimes QCD$ theory is obtained by simultaneously exponentiating the large IR terms in QCD and the exact IR divergent terms in QED, so that we arrive at the new result

$$
\begin{aligned}
d\hat{\sigma}_{\rm exp} = e^{\rm SUM_{IR}(QCED)} \sum_{n,m=0}^{\infty} \int \prod_{j_1=1}^{n} \frac{d^3k_{j_1}}{k_{j_1}} \prod_{j_2=1}^{m} \frac{d^3{k'}_{j_2}}{{k'}_{j_2}} \\
\int \frac{d^4y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum {k'}_{j_2})+D_{\rm QCED}} \\
\times \tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)\frac{d^3p_2}{p_2^{\,0}}\frac{d^3q_2}{q_2^{\,0}},
\end{aligned}
\tag{2}
$$

where the new YFS [21,22] residuals, $\tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)$, with n hard gluons and m hard photons, defined in Ref. [2], represent the successive application of the YFS expansion first for QCD and subsequently for QED.

The functions $\rm SUM_{IR}(QCED)$, $D_{\rm QCED}$ are determined from their QCD analogs $\rm SUM_{IR}(QCD)$, $D_{\rm QCD}$ via the substitutions

$$
\begin{aligned}
B^{nls}_{\rm QCD} &\rightarrow B^{nls}_{\rm QCD} + B^{nls}_{\rm QED} &&\equiv B^{nls}_{\rm QCED},\\
\tilde{B}^{nls}_{\rm QCD} &\rightarrow \tilde{B}^{nls}_{\rm QCD} + \tilde{B}^{nls}_{\rm QED} &&\equiv \tilde{B}^{nls}_{\rm QCED},\\
\tilde{S}^{nls}_{\rm QCD} &\rightarrow \tilde{S}^{nls}_{\rm QCD} + \tilde{S}^{nls}_{\rm QED} &&\equiv \tilde{S}^{nls}_{\rm QCED}
\end{aligned}
\tag{3}
$$

everywhere in expressions for the latter functions given in Ref. [3]. We stress that if desired the exponent corresponding the $N^{\rm th}$ Gatherall exponent for $N>1$ can be systematically included in the QCD exponents $\mathrm{SUM}_{\mathrm{IR}}(\mathrm{QCD})$, D_{QCD} if desired, with a corresponding change in the respective residuals $\tilde{\bar\beta}_{n,m}(k_1,\ldots,k_n;\ k'_1,\ldots,k'_m)$. The residuals $\tilde{\bar\beta}_{n,m}(k_1,\ldots,k_n;\ k'_1,\ldots,k'_m)$ are free of all infrared singularities, and the result in (2) is a representation that is exact and that can therefore be used to make contact with parton shower MC's without double counting or the unnecessary averaging of effects such as the gluon azimuthal angular distribution relative to its parent's momentum direction.

In the respective infrared algebra (QCED) in (2), the average Bjorken x values

$$\begin{aligned} x_{avg}(\mathrm{QED}) &\cong \gamma(\mathrm{QED})/(1+\gamma(\mathrm{QED})), \\ x_{avg}(\mathrm{QCD}) &\cong \gamma(\mathrm{QCD})/(1+\gamma(\mathrm{QCD})), \end{aligned}$$

where $\gamma(A)=\frac{2\alpha_A\mathcal{C}_A}{\pi}(L_s-1)$, $A=$ QED, QCD, with $\mathcal{C}_A=Q_f^2, C_F$, respectively, for $A=$ QED, QCD and the big log L_s, imply that QCD dominant corrections happen an order of magnitude earlier than those for QED. This means that the leading $\tilde{\bar\beta}_{0,0}$-level gives already a good estimate of the size of the interplay between the higher order QED and QCD effects which we will use to illustrate (2) here.

4 QED$\otimes$ QCD Threshold Corrections and Shower/ME Matching at the LHC

The cross section for the processes $pp\rightarrow V+n(\gamma)+m(g)+X\rightarrow \bar\ell\ell'+n'(\gamma)+m(g)+X$, where V,ℓ,ℓ' are the vector-boson / lepton combinations defined in Section 3, may be constructed from the parton-level cross section via the usual formula (we use the standard notation here [2])

$$d\sigma_{\mathrm{exp}}=\sum_{i,j}\int dx_i dx_j F_i(x_i)F_j(x_j)d\hat\sigma_{\mathrm{exp}}(x_ix_js), \tag{4}$$

In this section, we will use the result in (2) here with semi-analytical methods and structure functions from Ref. [23] to examine the size of QED$\otimes$QCD threshold corrections. A Monte Carlo realization will appear elsewhere [24].

First, we wish to make contact with the existing literature and standard practice for QCD parton showers as realized by HERWIG [25] and/or PYTHIA [26]. Eventually, we will also make contact with the new parton distribution function evolution MC algorithm in Ref. [27]. We intend to combine our exact YFS-style resummation calculus with HERWIG and/or PYTHIA by using the latter to generate a parton shower starting from the initial (x_1, x_2) point at factorization scale μ, after this point is provided by the $\{F_i\}$. This combination of theoretical constructs can be systematically improved with exact fully exclusive results order-by-order in α_s, where currently the state of the art in such a calculation is the work in Ref. [28] which accomplishes the combination of an exact $\mathcal{O}(\alpha_s)$ correction with HERWIG, where the gluon azimuthal angle is averaged in the combination.

The issue of this being an exact rearrangement of the QCD and QED perturbative series requires some comment. Unlike the threshold resummation techniques in Refs. [5–7], we have a resummation which is valid over the entire phase space. Thus, it is readily applicable to an exact treatment of the respective phase space in its implementation via MC methods.

We may illustrate how the combination with PYTHIA/HERWIG may proceed as follows. We note that, for example, if we use a quark mass m_q as our collinear limit regulator, DGLAP [29] evolution of the structure functions allows us to factorize all the terms that involve powers of the big log $L_c = \ln \mu^2/m_q^2 - 1$ in such a way that the evolved structure function contains the effects of summing the leading big logs $L = \ln \mu^2/\mu_0^2$ where the evolution involves initial data at the scale μ_0. This gives us a result independent of m_q for $m_q \downarrow 0$. In the DGLAP theory, the factorization scale μ represents the largest p_{T} of the gluon emission included in the structure function.

In practice, when we use these structure functions with an exact result for the residuals in (2), it means that we must in the residuals omit the contributions from gluon radiation at scales below μ. This can be shown to amount in most cases to replacing $L_s = \ln \hat{s}/m_q^2 - 1 \rightarrow L_{nls} = \ln \hat{s}/\mu^2$ but in any case it is immediate how to limit the p_T in the gluon emission[2] so that we do not double count effects. In other words, we apply the standard QCD factorization of mass singularities to the cross section in (2) in the standard way. We may do it with either the mass regulator for the collinear singularities or with dimensional regularization of such singularities. The final result should be independent of this regulator and this is something that we may use as a cross-check on the results.

This would in practice mean the following: We first make an event with the formula in (4) which would produce an initial beam state at (x_1, x_2) for the two hard interacting partons at the factorization scale μ from the structure functions $\{F_j\}$ and a corresponding final state X from the exponentiated cross section in $d\hat{\sigma}_{\rm exp}(x_i x_j s)$, where we stress that the latter has had all collinear singularities factorized so that it is much more convergent then its analog in LEP physics for the electroweak theory for example. The standard Les Houches procedure [30] of showering this event (x_1, x_2, X) would then be used, employing backward evolution of the initial partons. If we restrict the p_T as we have indicated above, there would be no double counting of effects. Let us call this p_T matching of the shower from the backward evolution and the matrix elements in the QCED exponentiated cross section.

It is possible, however, to be more accurate in the use of the exact result in (2). Just as the residuals $\bar{\tilde{\beta}}_{n,m}(k_1, \ldots, k_n;\, k'_1, \ldots, k'_m)$ are computed order by order in perturbation theory from the corresponding exact perturbative results by expanding the exponents in (2) and comparing the appropriate corresponding coefficients of the respective powers of $\alpha^n \alpha_s^m$, so too can the shower formula which is used to generate the backward evolution be expanded so that the product of the shower formula's perturbative expansion, the perturbative expansion of the exponents in (2), and the perturbative expansions of the residuals can be written as an over-all expansion in powers of $\alpha^n \alpha_s^m$ and required to match the respective calculated exact result for given order. In this way, new shower subtracted residuals, $\{\hat{\bar{\tilde{\beta}}}_{n,m}(k_1, \ldots, k_n;\, k'_1, \ldots, k'_m)\}$, are calculated that can be used for the entire gluon $p_{\rm T}$ phase space with an accuracy of the cross section that should in principle be improved compared with the first procedure for shower matching presented above. Both approaches are under investigation, where we note that the shower subtracted

[2] Here, we refer to both on-shell and off-shell emitted gluons.

residuals have been realized for the exact $\mathcal{O}(\alpha)$ luminosity Bhabha process at DAPHNE energies by the authors in Ref. [31].

Returning to the general discussion, we compute, with and without QED, the ratio $r_{\rm exp} = \sigma_{\rm exp}/\sigma_{\rm Born}$, where we do not use the narrow resonance approximation, for we wish to set a paradigm for precision heavy vector boson studies. The formula which we use for $\sigma_{\rm Born}$ is obtained from that in (4) by substituting $d\hat\sigma_{\rm Born}$ for $d\hat\sigma_{\rm exp}$ therein, where $d\hat\sigma_{\rm Born}$ is the respective parton-level Born cross section. Specifically, we have from (1) the $\bar{\bar{\tilde\beta}}_{0,0}$-level result

$$\begin{aligned}\hat\sigma_{\rm exp}(x_1x_2s) = \int_0^{v_{\rm max}} & dv\ \gamma_{\rm QCED}\ v^{\gamma_{\rm QCED}-1}F_{\rm YFS}(\gamma_{\rm QCED}) \\ & e^{\delta_{\rm YFS}}\hat\sigma_{\rm Born}((1-v)x_1x_2s)\end{aligned} \tag{5}$$

where we intend the well-known results for the respective parton-level Born cross sections and the value of $v_{\rm max}$ implied by the experimental cuts under study.

What is new here is the value for the QED$\otimes$QCD exponent

$$\gamma_{\rm QCED} = \left(2Q_f^2\frac{\alpha}{\pi} + 2C_F\frac{\alpha_s}{\pi}\right)L_{nls} \tag{6}$$

where $L_{nls} = \ln x_1x_2s/\mu^2$ when μ is the factorization scale. The functions $F_{\rm YFS}(\gamma_{\rm QCED})$ and $\delta_{\rm YFS}(\gamma_{\rm QCED})$ are well-known [22] as well:

$$\begin{aligned}F_{YFS}(\gamma_{\rm QCED}) &= \frac{e^{-\gamma_{\rm QCED}\gamma_E}}{\Gamma(1+\gamma_{\rm QCED})}, \\ \delta_{YFS}(\gamma_{\rm QCED}) &= \frac{1}{4}\gamma_{\rm QCED} + \left(Q_f^2\frac{\alpha}{\pi} + C_F\frac{\alpha_s}{\pi}\right)\left(2\zeta(2)-\frac{1}{2}\right),\end{aligned} \tag{7}$$

where $\zeta(2)$ is Riemann's zeta function of argument 2, i.e., $\pi^2/6$, and γ_E is Euler's constant, i.e., $0.5772\ldots$.

Using these formulas in (4) allows us to get the results

$$r_{\rm exp} = \begin{cases} 1.1901\ , & \text{QCED} \equiv \text{QCD+QED},\ \text{LHC} \\ 1.1872\ , & \text{QCD},\ \text{LHC} \\ 1.1911\ , & \text{QCED} \equiv \text{QCD+QED},\ \text{Tevatron} \\ 1.1879\ , & \text{QCD},\ \text{Tevatron}. \end{cases} \tag{8}$$

We see that QED is at the level of .3% at both LHC and FNAL. This is stable under scale variations [2]. We agree with the results in Refs. [15,16,18–20] on both of the respective sizes of the QED and QCD effects. Furthermore, the QED effect is similar in size to structure function results found in Refs. [9–13].

5 Conclusions

We have shown that YFS theory (EEX and CEEX), when extended to non-Abelian gauge theory, allows simultaneous exponentiation of QED and QCD, QED$\otimes$QCD exponentiation. For QED$\otimes$QCD we find that full MC event generator realization is possible in a way that combines our calculus with HERWIG and PYTHIA in principle. Semi-analytical results for QED (and QCD) threshold effects agree with literature on Z production. As QED is at the .3% level, it is needed for LHC theory predictions at $\lesssim 1\%$. The corresponding analysis of the W production is in progress. We have illustrated a firm theoretical basis for the realization of the complete $\mathcal{O}(\alpha_s^2, \alpha\alpha_s, \alpha^2)$ results needed for the FNAL/LHC/RHIC/ILC physics and all of the latter are in progress.

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