
Common Envelope Evolution Redux

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Summary. Common envelopes form in dynamical time scale mass exchange, when the envelope of a donor star engulfs a much denser companion, and the core of the donor plus the dense companion star spiral inward through this dissipative envelope. As conceived by Paczynski and Ostriker, this process must be responsible for the creation of short-period binaries with degenerate components, and, indeed, it has proven capable of accounting for short-period binaries containing one white dwarf component. However, attempts to reconstruct the evolutionary histories of close double white dwarfs have proven more problematic, and point to the need for enhanced systemic mass loss, either during the close of the first, slow episode of mass transfer that produced the first white dwarf, or during the detached phase preceding the final, common envelope episode. The survival of long-period interacting binaries with massive white dwarfs, such as the recurrent novae T CrB and RS Oph, also presents interpretative difficulties for simple energetic treatments of common envelope evolution. Their existence implies that major terms are missing from usual formulations of the energy budget for common envelope evolution. The most plausible missing energy term is the energy released by recombination in the common envelope, and, indeed, a simple reformulation the energy budget explicitly including recombination resolves this issue.

1 Introduction

From the realization [26, 27, et seq.] that all cataclysmic variables (CVs) are interacting binary stars, their existence posed a dilemma for theories of binary evolution. The notion that close binary stars might evolve in ways fundamentally different from isolated stars was rooted in the famous ‘Algol paradox’ (that the cooler, lobe-filling subgiant or giant components among these well-known eclipsing binaries are less massive, but more highly evolved, than their hotter main-sequence companions). The resolution of that paradox invoked large-scale mass transfer reversing the initial mass ratios of these binaries [34]. Indeed, model calculations assuming conservation of total mass and orbital angular momentum are qualitatively consistent with the main features of Algol-type binaries. Even if quantitative consistency between models

and observational data generally requires some losses of mass and angular momentum among Algol binaries (e.g., [10, 21, 8]), the degree of those losses is typically modest, and the remnant binary is expected to adhere closely to an equilibrium core mass-radius relation for low-mass giant stars (see, e.g., the pioneering study of AS Eri by Refsdal, Roth & Weigert [46]). Those remnant binaries are typically of long orbital period (days to weeks) in comparison with CVs, and furthermore typically contain helium white dwarfs of low mass, especially in the short-period limit. In contrast, CVs evidently contain relatively massive white dwarfs, in binary systems of much shorter orbital periods (hours), that is, with much smaller total energies and orbital angular momenta.

In an influential analysis of the Hyades eclipsing red dwarf/white dwarf binary BD +16° 516 (= V471 Tau), Vauclair [57] derived a total system mass less than the turnoff mass of the Hyades, and noted that the cooling age of the white dwarf component was much smaller than the age of the cluster. He speculated that V471 Tau in its present state was the recent product of the ejection of a planetary nebula by the white dwarf. Paczynski [41] realized that, immediately prior to that event, the white dwarf progenitor must have been an asymptotic giant branch star of radius $\sim 600 R_{\odot}$, far exceeding its current binary separation $\sim 3 R_{\odot}$. He proposed that the dissipation of orbital energy provided the means both for planetary nebula ejection and for the severe orbital contraction between initial and final states, a process he labeled ‘common envelope evolution’ (not to be confused with the common envelopes of contact binary stars). Discovery soon followed of the first ‘smoking gun’, the short-period eclipsing nucleus of the planetary nebula Abell 63 [3].

Over the succeeding three decades, there have been a number of attempts to build detailed physical models of common envelope evolution (see [55] for a review). These efforts have grown significantly in sophistication, but this phenomenon presents a daunting numerical challenge, as common envelope evolution is inherently three-dimensional, and the range of spatial and temporal scales needed to represent a common envelope binary late in its inspiral can both easily exceed factors of 10^3 . Determining the efficiency with which orbital energy is utilized in envelope ejection requires such a code to conserve energy over a similarly large number of dynamical time scales.

Theoretical models of common envelope evolution are not yet capable of predicting the observable properties of objects in the process of inspiral. If envelope ejection is to be efficient, then the bulk of dissipated orbital energy must be deposited in the common envelope on a time scale short compared with the thermal time scale of the envelope, else that energy be lost to radiation. The duration of the common envelope phase thus probably does not exceed $\sim 10^3$ years. However, general considerations of the high initial orbital angular momenta of systems such as the progenitor of V471 Tau, and the fact that most of the orbital energy is released the envelope only very late in the inspiral have led to a consensus view [60, 32, 33, 65, 50] that the planetary nebulae they eject should be bipolar in structure, with dense equatorial rings

absorbing most of the initial angular momentum of the binary, and higher-velocity polar jets powered by the late release of orbital energy. Indeed, this appears to be a signature morphology of planetary nebulae with binary nuclei (e.g., [2]), although it may not be unique to binary nuclei.

2 The Energetics of Common Envelope Evolution

Notwithstanding the difficulties in modeling common envelope evolution in detail, it is possible to calculate with some confidence the initial total energy and angular momentum of a binary at the onset of mass transfer, and the corresponding orbital energy and angular momentum of any putative remnant of common envelope evolution.

Consider an initial binary of component masses M_1 and M_2 , with orbital semimajor axis A_i . Its initial total orbital energy is

$$E_{\text{orb},i} = -\frac{GM_1M_2}{2A_i} . \quad (1)$$

Let star 1 be the star that initiates interaction upon filling its Roche lobe. If M_{1c} is its core mass, and $M_{1e} = M_1 - M_{1c}$ its envelope mass, then we can write the initial total energy of that envelope as

$$E_e = -\frac{GM_1M_{1e}}{\lambda R_{1,L}} , \quad (2)$$

where $R_{1,L}$ is the Roche lobe radius of star 1 at the onset of mass transfer (the orbit presumed circularized prior to this phase), and λ is a dimensionless parameter dependent on the detailed structure of the envelope, but presumably of order unity. For very simplified models of red giants – condensed polytropes [40, 14, 16] – λ is a function only of $m_e \equiv M_e/M = 1 - M_c/M$, the ratio of envelope mass to total mass for the donor, and is well-approximated by

$$\lambda^{-1} \approx 3.000 - 3.816m_e + 1.041m_e^2 + 0.067m_e^3 + 0.136m_e^4 , \quad (3)$$

to within a relative error $< 10^{-3}$.

For the final orbital energy of the binary we have

$$E_{\text{orb},f} = -\frac{GM_{1c}M_2}{2A_f} , \quad (4)$$

where A_f is of course the final orbital separation. If a fraction α_{CE} of the difference in orbital energy is consumed in unbinding the common envelope,

$$\alpha_{\text{CE}} \equiv \frac{E_e}{(E_{\text{orb}}^{(f)} - E_{\text{orb}}^{(i)})} , \quad (5)$$

then

$$\frac{A_f}{A_i} = \frac{M_{1c}}{M_1} \left[1 + \left(\frac{2}{\alpha_{CE} \lambda r_{1,L}} \right) \left(\frac{M_1 - M_{1c}}{M_2} \right) \right]^{-1}, \quad (6)$$

where $r_{1,L} \equiv R_{1,L}/A_i$ is the dimensionless Roche lobe radius of the donor at the start of mass transfer. In the classical Roche approximation, $r_{1,L}$ is a function only of the mass ratio, $q \equiv M_1/M_2$ [7]:

$$r_{1,L} \approx \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})}. \quad (7)$$

Typically, the second term in brackets in (6) dominates the first term.

As formulated above, our treatment of the outcome of common envelope evolution neglects any sources or sinks of energy beyond gravitational terms and the thermal energy content of the initial envelope (incorporated in the parameter λ). The justification for this assumption is again that common envelope evolution must be rapid compared to the thermal time scale of the envelope. This implies that radiative losses (or nuclear energy gains – see below) are small. They, as well as terminal kinetic energy of the ejecta, are presumably reflected in ejection efficiencies $\alpha_{CE} < 1$. We neglect also the rotational energy of the common envelope (invariably small in magnitude compared to its gravitational binding energy), and treat the core of the donor star and the companion star as inert masses, which neither gain nor lose mass or energy during the course of common envelope evolution. One might imagine it possible that net accretion of mass by the companion during inspiral might compromise this picture. However, the common envelope is typically vastly less dense than the companion star, and may be heated to roughly virial temperature on infall. A huge entropy barrier arises at the interface between the initial photosphere of the companion and the common envelope in which it is now embedded, with a difference in entropy per particle of order $(\mu m_H/k)\Delta s \approx 4$ –6. The rapid rise in temperature and decrease in density through the interface effectively insulates the accreting companion thermally, and strongly limits the fraction of the very rarified common envelope it can retain upon exit from that phase [62, 15].

Common envelope evolution entails systemic angular momentum losses as well as systemic mass and energy losses. Writing the orbital angular momentum of the binary,

$$J = \left[\frac{GM_1^2 M_2^2 A(1 - e^2)}{M_1 + M_2} \right]^{1/2}, \quad (8)$$

in terms of the total orbital energy, $E = -GM_1 M_2 / 2A$, we find immediately that the ratio of final to initial orbital angular momentum is

$$\frac{J_f}{J_i} = \left(\frac{M_{1c}}{M_1} \right)^{3/2} \left(\frac{M_{1c} + M_2}{M_1 + M_2} \right)^{-1/2} \left(\frac{E_i}{E_f} \right)^{1/2} \left(\frac{1 - e_f^2}{1 - e_i^2} \right)^{1/2}. \quad (9)$$

Since $M_{1c} < M_1$ and we expect the initial orbital eccentricity to be small ($e_i \approx 0$), it follows that any final energy state lower than the initial state ($|E_f| >$

$|E_i|$) requires the loss of angular momentum. The reverse is not necessarily true, so it is the energy budget that most strongly constrains possible outcomes of common envelope evolution.

3 Does Common Envelope Evolution Work?

As an example of common envelope energetics, let us revisit the pre-CV V471 Tau, applying the simple treatment outlined above. It is a member of the Hyades, an intermediate-age metal-rich open cluster ($t = 650$ Myr, $[\text{Fe}/\text{H}] = +0.14$) with turnoff mass $M_{\text{TO}} = 2.60 \pm 0.06 M_{\odot}$ [28]. The cooling age of the white dwarf is much smaller than the age of the cluster ($t_{\text{cool,WD}} = 10^7$ yr [39] – but see the discussion there of the paradoxical fact that this most massive of Hyades white dwarfs is also the youngest). Allowing for the possibility of significant mass loss in a stellar wind prior to the common envelope phase, we may take M_{TO} for an upper limit to the initial mass M_1 of the white dwarf component. The current masses for the white dwarf and its dK2 companion, as determined by O’Brien et al. [39] are $M_{\text{WD}} = 0.84 \pm 0.05 M_{\odot}$, $M_{\text{K}} = 0.93 \pm 0.07 M_{\odot}$, with orbital separation $A = 3.30 \pm 0.08 R_{\odot}$. A $2.60 M_{\odot}$ star of Hyades metallicity with a $0.84 M_{\odot}$ core lies on the thermally-pulsing asymptotic giant branch, with radius (maximum in the thermal pulse cycle) which we estimate at $R_i = 680 R_{\odot} = R_{1,\text{L}}$, making $A_i = 1450 R_{\odot}$. With this combination of physical parameters, we derive an estimate of $\alpha_{\text{CE}}\lambda = 0.057$ for V471 Tau. Equation (3) then implies $\alpha_{\text{CE}} = 0.054$. This estimate of course ignores any mass loss prior to common envelope evolution (which would drive α_{CE} to lower values), or orbital evolution since common envelope evolution (which would drive α_{CE} to higher values). In any event, the status of V471 Tau would appear to demand only a very small efficiency of envelope ejection.¹

The fact that V471 Tau is a double-lined eclipsing member of a well-studied cluster provides an exceptionally complete set of constraints on its prior evolution. In all other cases of short-period binaries with degenerate or compact components, available data are inadequate to fix simultaneously both the initial mass of the compact component and the initial binary separation, for example. To validate the energetic arguments outlined above, one must resort to consistency tests, whether demonstrating the existence of physically-plausible initial conditions that could produce some individual system, or else

¹ The anomalously small value of α_{CE} deduced for V471 Tau may be connected to its puzzlingly high white dwarf mass and luminosity: O’Brien et al. [39] suggest that it began as a hierarchical triple star, in which a short-period inner binary evolved into contact, merged (as a blue straggler), and later engulfed its lower-mass companion in a common envelope. An overmassive donor at the onset of common envelope evolution would then have a more massive core than produced by its contemporaries among primordially single stars, and it would fill its Roche lobe with a more massive envelope at somewhat shorter orbital period, factors all consistent with a larger value of α_{CE} having led to V471 Tau as now observed.

following a plausible distribution of primordial binaries wholesale through the energetics of common envelope evolution and showing that, after application of appropriate observational selection effects, the post-common-envelope population is statistically consistent with the observed statistics of the selected binary type. In the cases of interacting binaries, such as CVs, one should allow further for post-common-envelope evolution. Nevertheless, within these limitations, binary population synthesis models show broad consistency between the outcomes of common envelope evolution and the statistical properties of CVs and pre-CVs [4, 24, 44, 17, 63], as well as with most super-soft X-ray sources [5], for assumed common envelope ejection efficiencies typically of order $\alpha_{\text{CE}} \approx 0.3\text{--}0.5$.

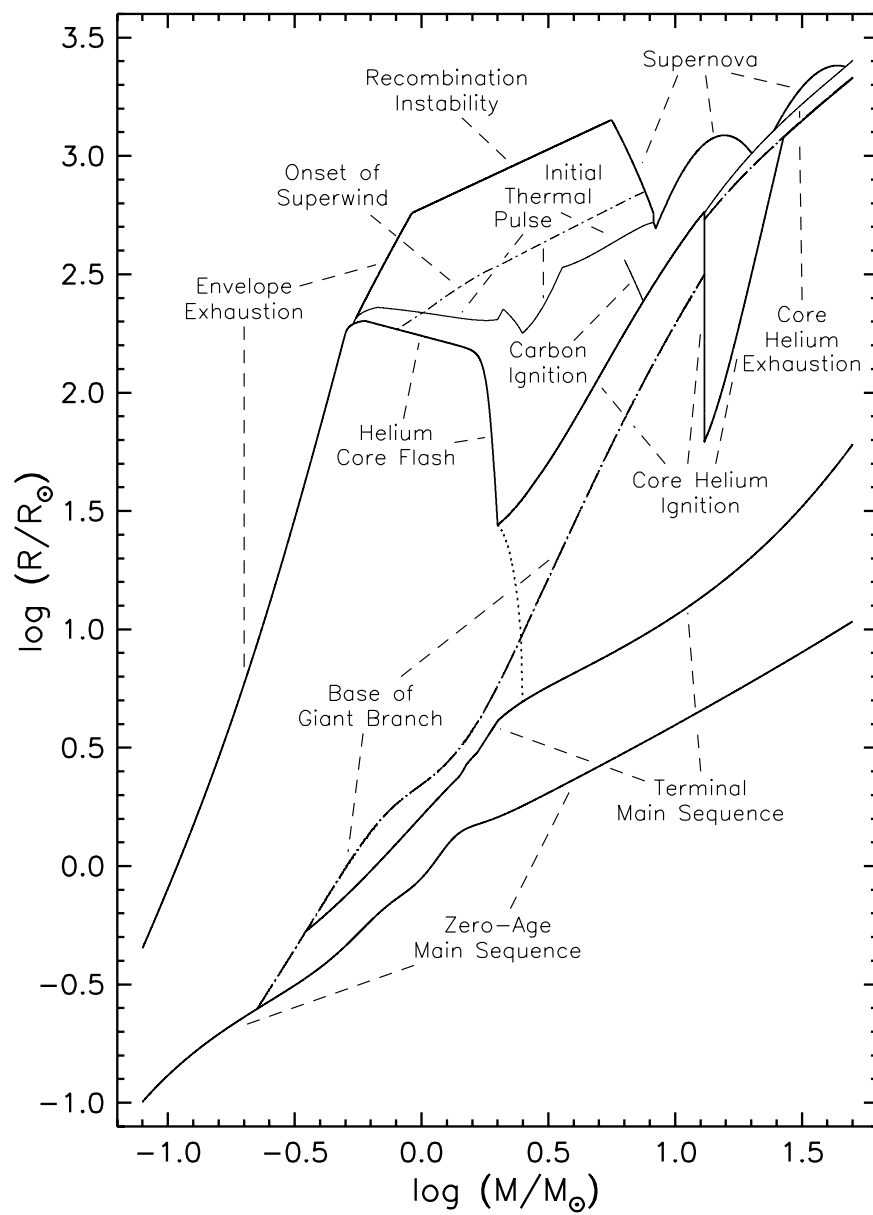
A useful tool in reconstructing the evolutionary history of a binary, used implicitly above in analyzing V471 Tau, is the mass-radius diagram spanned by single stars of the same composition as the binary. Figure 1 illustrates such a diagram for solar-composition stars from $0.08 M_{\odot}$ to $50 M_{\odot}$. In it are plotted various critical radii marking, as a function of mass, the transition from one evolutionary phase to the next.² Since the Roche lobe of a binary component represents a dynamical limit to its size, its orbital period fixes the mean density at which that star fills its Roche lobe,

$$\log P_{\text{orb}}(\text{d}) \approx \frac{3}{2} \log(R_{\text{L}}/R_{\odot}) - \frac{1}{2} \log(M/M_{\odot}) - 0.455, \quad (10)$$

to within a very weak function of the binary mass ratio. The mass and radius of any point in Fig. 1 therefore fixes the orbital period at which such a star would fill its Roche lobe, just as the orbital period of a binary fixes the evolutionary state at which such a star initiates mass transfer.

² Not all evolutionary phases are represented here. In a binary, a donor initiates mass transfer when it first fills its Roche lobe; if it would have done so at a prior stage of evolution, then its present evolutionary state is ‘shadowed’, in the sense that it only occurs by virtue of the binary *not* having filled its lobe previously. Thus, for example, low- and intermediate-mass stars cannot in general initiate mass transfer during core helium burning, because they would have filled their Roche lobes on the initial ascent of the giant branch.

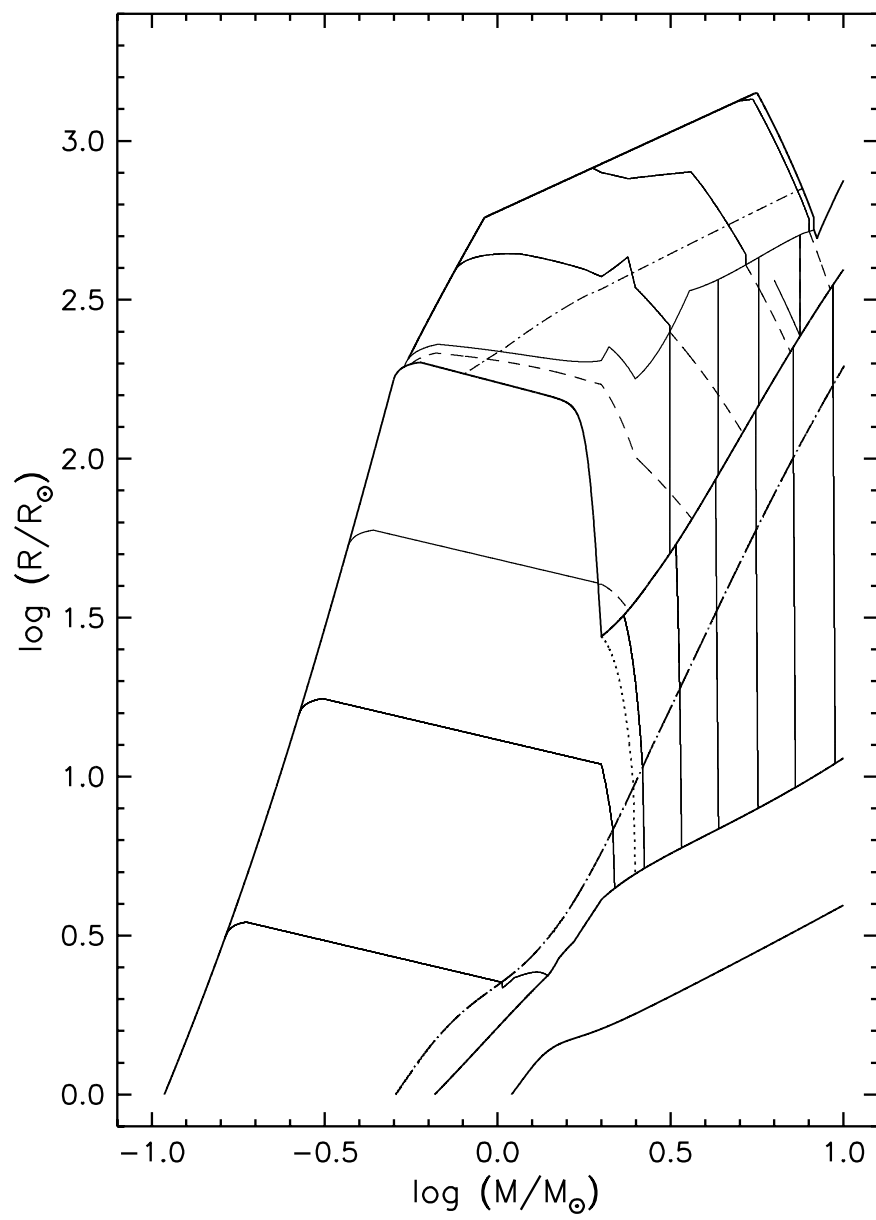
Fig. 1 (facing page). The mass-radius diagram for stars of solar metallicity, constructed from the parametric models of stellar evolution by Hurley, Pols, & Tout [19] and models of thermally-pulsing asymptotic giant branch stars by Wagenhuber & Weiss [58]. Also plotted in the locus of asymptotic giant branch stars at the onset of the superwind, after Willson [64]; beyond this radius, systemic mass loss drives orbital expansion faster than nuclear evolution drives stellar expansion, and a binary will no longer be able to initiate tidal mass transfer. The unlabeled dotted line terminating at the junction between lines labeled ‘helium core flash’ and ‘core helium ignition’ marks the division between those helium cores (at lower masses) which evolve to degeneracy if stripped of their envelope, and those (at higher masses) which ignite helium non-degenerately and become helium stars.



In Fig. 2, the corresponding core masses of low- and intermediate-mass stars are plotted in the mass-radius diagram. For a binary which is the immediate product of common envelope evolution, the mass of the most recently formed white dwarf (presumably the spectroscopic primary) equals the core mass of the progenitor donor star. That donor (presuming it to be of solar metallicity) must be located somewhere along the corresponding core mass sequence in Fig. 2, with the radius at any point along that sequence corresponding to the Roche lobe radius at the onset of the mass transfer, and the mass at that point corresponding to the initial total mass of the donor. Thus, if the mass of the most recently-formed white dwarf is known, it is possible to identify a single-parameter (e.g., initial mass or initial radius of the donor) family of possible common-envelope progenitors.

Using a mapping procedure similar to this, Nelemans & Tout [35] recently explored possible progenitors for detached close binaries with white dwarf components. Broadly speaking, they found solutions using (6) for almost all systems containing only one white dwarf component. Only three putative post-common-envelope systems failed to yield physically-plausible values of $\alpha_{\text{CE}\lambda}$: AY Cet (G5 III + DA, $P_{\text{orb}} = 56.80$ d [49]), Sanders 1040 (in M67: G4 III + DA, $P_{\text{orb}} = 42.83$ d [56]), and HD 185510 (=V1379 Aql: gK0 + sdB, $P_{\text{orb}} = 20.66$ d [22]). The first two of these systems are non-eclipsing, but photometric masses for their white dwarf components are extremely low (estimated at $\sim 0.25 M_{\odot}$ and $0.22 M_{\odot}$, respectively), with Roche lobe radii consistent with the limiting radii of very low-mass giants as they leave the giant branch (cf. Fig. 2, above). They are thus almost certainly post-Algol binaries, and not post-common-envelope binaries. HD 185510 is an eclipsing binary; a spectroscopic orbit exists only for the gK0 component [9]. The mass ($0.304 \pm 0.015 M_{\odot}$) and radius ($0.052 \pm 0.010 R_{\odot}$) of the sdB component, deduced from model atmosphere fitting of IUE spectra combined with solution of the eclipse light curve, place it on a low-mass white dwarf cooling curve, rather than among helium-burning subdwarfs [22]. Indeed, from fitting very detailed evolutionary models to this system, Nelson & Eggleton [38] found a

Fig. 2 (facing page). The mass-radius diagram for low- and intermediate-mass stars, as in Fig. 1, but with loci of constant core mass added. The solid lines added correspond to core masses interior to the hydrogen-burning shell, dashed lines to those interior to the helium-burning shell. Solid lines intersecting the base of the giant branch (dash-dotted curve) correspond to helium core masses of to 0.15, 0.25, 0.35, 0.5, 0.7, 1.0, 1.4, and $2.0 M_{\odot}$; those between helium ignition and the initial thermal pulse to 0.7, 1.0, 1.4, and $2.0 M_{\odot}$, and those beyond the initial thermal pulse to 0.7, 1.0, and $1.4 M_{\odot}$. Dashed lines between helium ignition and initial thermal pulse correspond to carbon-oxygen core masses of 0.35, 0.5, 0.7, 1.0, and $1.4 M_{\odot}$. Beyond the initial thermal pulse, helium and carbon-oxygen core masses converge, with the second dredge-up phase reducing helium core masses above $\sim 0.8 M_{\odot}$ to the carbon-oxygen core.



post-Algol solution they deemed acceptable. It thus appears that these three problematic binaries are products of quasi-conservative mass transfer, and not common envelope evolution.

The close double white dwarfs present a more difficult conundrum, however. Nelemans et al. [36, 37, 35] found it impossible using the energetic arguments (6) outlined above to account for the existence of a most known close double white dwarfs. Mass estimates can be derived for spectroscopically detectable components of these systems from their surface gravities and effective temperatures (determined from Balmer line fitting). The deduced masses are weakly dependent on the white dwarf composition, and may be of relatively modest accuracy, but they are independent of the uncertainties in orbital inclination afflicting orbital solutions. These mass estimates place the great majority of detectable components in close double white dwarf binaries below $\sim 0.46 M_{\odot}$, the upper mass limit for pure helium white dwarfs (e.g., [51]). They are therefore pure helium white dwarfs, or perhaps hybrid white dwarfs (low-mass carbon-oxygen cores with thick helium envelopes). While reconstructions of their evolutionary history yield physically-reasonable solutions for the final common envelope phase, with values for $0 < \alpha_{\text{CE}} \lambda < 1$, the preceding phase of mass transfer, which gave rise to the first white dwarf, is more problematic. If it also proceeded through common envelope evolution, the deduced values of $\alpha_{\text{CE}} \lambda \leq -4$ for that phase are unphysical. Nelemans & Tout [35] interpreted this paradox as evidence that descriptions of common envelope evolution in terms of orbital energetics, as described above, are fundamentally flawed.

4 An Alternative Approach to Common Envelope Evolution?

Nelemans et al. [36] proposed instead parameterizing common envelope evolution in terms of γ , the ratio of the fraction of angular momentum lost to the fraction of mass lost:

$$\frac{J_i - J_f}{J_i} = \gamma \frac{M_1 - M_{1,c}}{M_1 + M_2} . \quad (11)$$

Both initial and final orbits are assumed circular, so the ratio of final to initial orbital separations becomes

$$\frac{A_f}{A_i} = \left(\frac{M_1}{M_{1c}} \right)^2 \left(\frac{M_{1c} + M_2}{M_1 + M_2} \right) \left[1 - \gamma \left(\frac{M_1 - M_{1c}}{M_1 + M_2} \right) \right]^2 . \quad (12)$$

Among possible solutions leading to known close double white dwarfs, Nelemans & Tout [35] find values $1 < \gamma \lesssim 4$ required for the second (final) common envelope phase, and $0.5 \lesssim \gamma < 3$ for the first (putative) common envelope phase. They note that values in the range $1.5 < \gamma < 1.7$ can be found among possible solutions for all common envelope phases in their sample, not only

those leading to known double white dwarfs, but those leading to known pre-CV and sdB binaries as well.

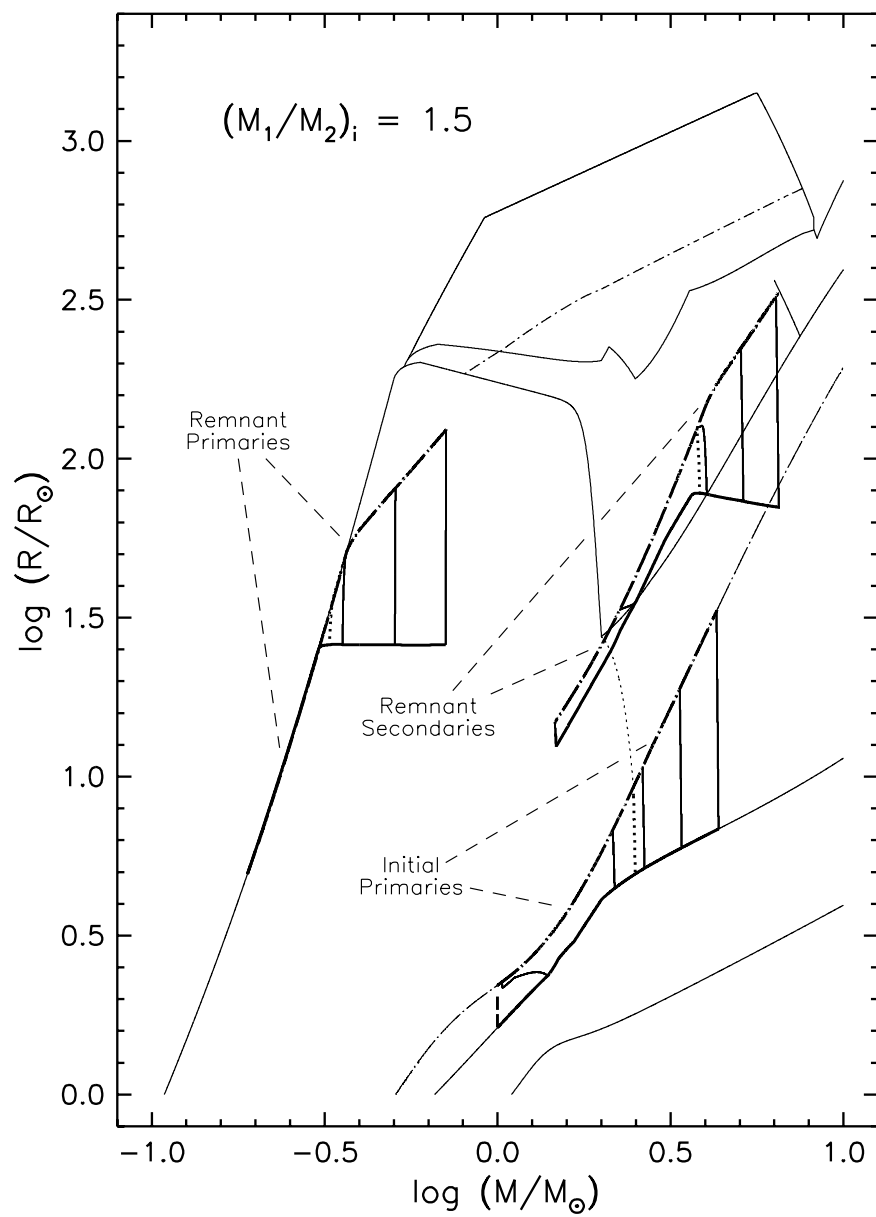
The significance of this finding is itself open to debate. At one extreme, it would seem implausible for any mechanism to remove less angular momentum per unit mass than the orbital angular momentum per unit mass of either component in its orbit (so-called Jeans-mode mass loss). At the other extreme, a firm upper limit to γ is set by vanishing final orbital angular momentum, J_f . If M_{1c} and M_2 can be regarded as fixed, the corresponding limits on γ are

$$\left(\frac{M_1 + M_2}{M_1 - M_{1c}}\right) > \gamma > \left(\frac{M_1 + M_2}{M_1 - M_{1c}}\right) \left[1 - \left(\frac{M_{1c}}{M_1}\right) \left(\frac{M_1 + M_2}{M_{1c} + M_2}\right)\right]. \quad (13)$$

In a fairly typical example, $M_{1c} = M_2 = \frac{1}{4}M_1$, γ is inevitably tightly constrained for any conceivable outcome: $\frac{5}{3} > \gamma > \frac{5}{8}$. The ratio of final to initial orbital separation, A_f/A_i , is extremely sensitive to γ near the upper limit of its range. It is therefore not surprising to find empirical estimates of γ clustering as they do – their values merely affirm the fact that A_f must typically be much smaller than A_i .

The unphysically large or, more commonly, *negative* values of $\alpha_{CE}\lambda$ noted above for the first mass transfer phase in the production of close white dwarf binaries [35] implies that the orbital energies of these binaries have *increased* through this phase (or, at any rate, decreased by significantly less than the nominal binding energies of their common envelopes). Such an increase in orbital energy is a hallmark of slow, quasi-conservative mass transfer, on a thermal or nuclear time scale. Thermal time scale mass transfer is driven by relaxation of the donor star toward thermal evolution; the re-expansion of the donor following mass ratio reversal is powered by the (nuclear) energy outflow from the core of the star. Likewise, the bulk expansion of the donor star in nuclear time scale mass transfer draws energy from nuclear sources in that star. It appears, therefore, that the first phase of mass transfer among known close double white dwarfs cannot have been a common envelope phase, but must instead have been a quasi-conservative phase, notwithstanding the difficulties that conclusion presents, as we shall now see.

The dilemma that the close double white dwarfs present is illustrated in Figs. 3 and 4. Figure 3 shows the distribution of immediate remnants of mass transfer among solar-metallicity binaries of low and intermediate mass, for a relatively moderate initial mass ratio. Conservation of total mass and orbital angular momentum have been assumed. The remnants of the initial primary include both degenerate helium white dwarfs, and nondegenerate helium stars which have lost nearly all of their hydrogen envelopes. The helium white dwarfs lie almost entirely along the left-hand boundary, the line labeled ‘envelope exhaustion’ in Fig. 1. (The extent of this sequence is more apparent in the distribution of remnant secondaries.) Their progenitors have enough angular momentum to accommodate core growth in the terminal phases of mass transfer. In the calculation shown, the least massive cores grow from

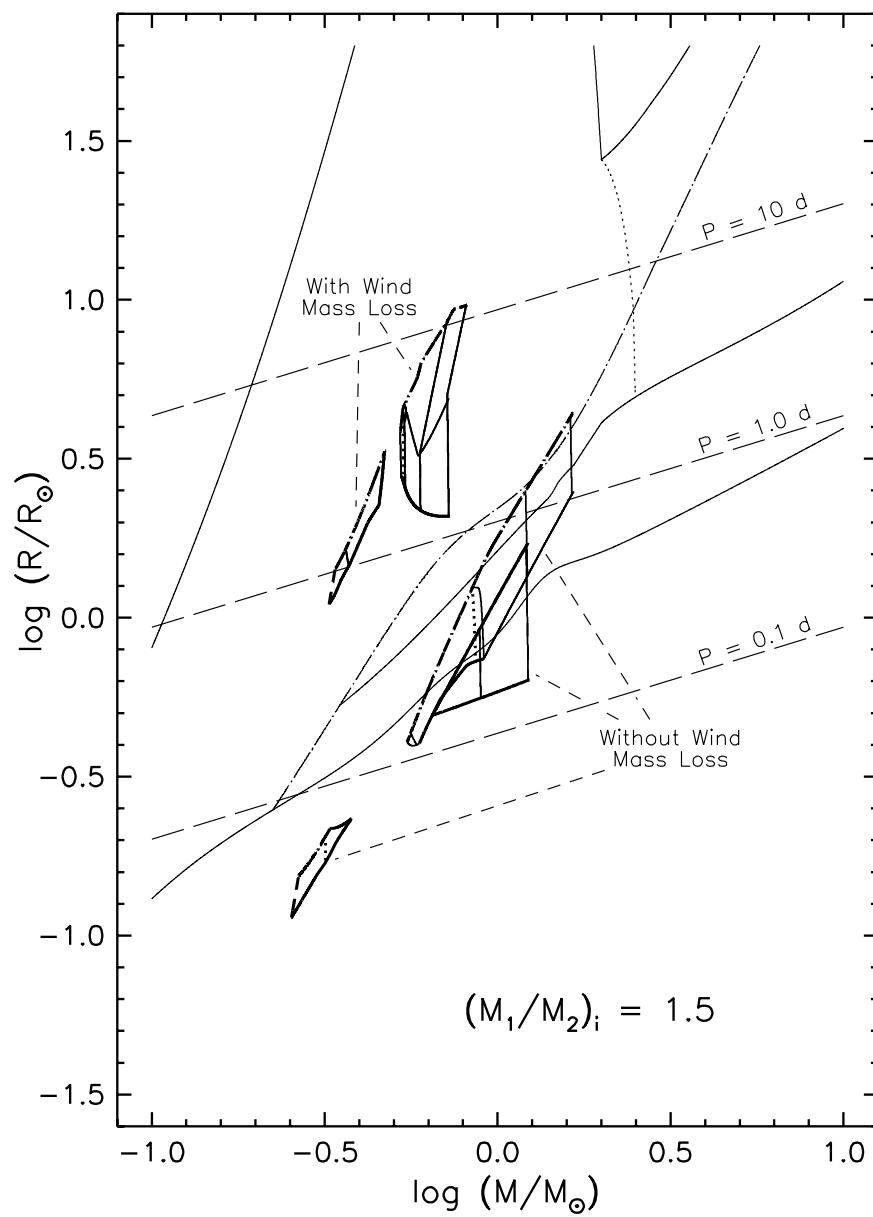


$0.11 M_{\odot}$ to $\sim 0.18 M_{\odot}$ by the completion of mass transfer. In contrast, virtually all binaries leaving nondegenerate helium star remnants have too little angular momentum to recover thermal equilibrium before they have lost their hydrogen envelopes; for them, there is no slow nuclear time scale phase of mass transfer, and core growth during mass transfer is negligible. The lowest-mass helium star remnants have nuclear burning lifetimes comparable to their hydrogen-rich binary companions, now grown through mass accretion. Those more massive than $\sim 0.8 M_{\odot}$ develop very extended envelopes during shell helium burning, and will undergo a second phase of mass transfer from primary to secondary, not reflected here; such massive white dwarfs are absent in the Nelemans & Tout [35] sample, and so are omitted here.

In Fig. 4, the remnants of the first phase of conservative mass transfer illustrated in Fig. 3 are followed through the second phase of mass transfer, using (6). Because the remnants of the first phase have second-phase donors much more massive than their companions, and nearly all have deep convective envelopes, they are unstable to dynamical time scale mass transfer, and undergo common envelope evolution. The systems labeled ‘Without Wind Mass Loss’ have been calculated assuming that no orbital evolution or mass loss occurs between the end of the first phase of mass transfer and common envelope evolution. It is assumed furthermore that $\alpha_{\text{CE}} = 1$, in principle marking the most efficient envelope ejection energetically possible. Binary orbital periods of 0.1, 1.0, and 10 days (assuming equal component masses) are indicated for reference.

Observed close double white dwarfs, as summarized by Nelemans & Tout [35], have a median orbital period of 1.4 d, and mass (spectroscopic primary) $0.39 M_{\odot}$. Among double-lines systems, nearly equal white dwarf masses are strongly favored, with the median $q = 1.00$. Clearly, most observed double white dwarfs are too long in orbital period (have too much total energy and angular momentum) to have evolved in the manner assumed here. Furthermore, the computed binary mass ratios are typically more extreme than observed, with the second-formed core typically 1.3-2.5 times as massive as the first. The problem is that, while remnant white dwarfs or low-mass he-

Fig. 3 (facing page). Products of mass- and angular momentum-conservative mass transfer for a typical initial mass ratio. The radii indicated refer to *Roche lobe radii* at the onset or termination of mass transfer, as appropriate. To avoid common envelope evolution, the donor stars (the region outlined in bold toward the lower right in the diagram) must have radiative envelopes, and so arise between the terminal main sequence and base of the giant branch. Their mass transfer remnants are outlined in bold at the center-left of the diagram, with the remnant accretors at upper right. The regions mapped are truncated in each case at a lower initial donor mass of $1.0 M_{\odot}$ and upper initial donor core mass of $0.7 M_{\odot}$. Lines of constant initial core mass (with values as in Fig. 2) are indicated for the initial and remnant primaries. Lines of constant remnant primary mass are indicated for the remnant secondaries.



lium stars with suitable masses can be produced in the first, conservative mass transfer phase, the remnant companions have envelope masses too large, and too tightly bound, to survive the second (common envelope) phase of interaction at orbital separations and periods as large as observed. Evidently, the progenitors of these double white dwarfs have lost a significant fraction of their initial mass, while gaining in orbital energy, prior to the final common envelope phase. These requirements can be fulfilled by a stellar wind, provided that the process is slow enough that energy losses in the wind can be continuously replenished from nuclear energy sources.

The requisite mass loss and energy gain are possible with stellar wind mass loss during the non-interactive phase between conservative and common envelope evolution, or with stellar winds in nuclear time-scale mass transfer or the terminal (recovery) phase of thermal time-scale mass transfer. Systemic mass loss during or following conservative mass transfer will (in the absence of angular momentum losses) shift the remnant regions to the left and upward in Fig. 3 (subject to the limit posed by envelope exhaustion), while systemic angular momentum losses shift them downwards. More extreme initial mass ratios shift them downwards to the left.

The net effect of wind mass loss is illustrated by the regions labeled ‘With Wind Mass Loss’ in Fig. 4. For simplicity, it is assumed here that half the remnant mass of the original secondary was lost in a stellar wind prior to the common envelope phase. Mass loss on this scale not only significantly reduces the mass of the second-formed white dwarf relative to the first, but the concomitant orbital expansion produces wider remnant double white dwarfs, bringing this snapshot model into good accord with the general properties of real systems. Losses of this magnitude might be unprecedented among single stars prior to their terminal superwind phase, but they have been a persistent feature of evolutionary studies of Algol-type binaries [10, and references

Fig. 4 (facing page). Remnants of the second, common envelope, phase of mass transfer of the systems shown in Fig. 3. Masses refer to the final remnants of the original secondaries, and radii to their Roche lobe radii. Two groups of remnants are shown. Those at lower right labeled ‘Without Wind Mass Loss’ follow directly from the distributions of remnant primaries and secondaries shown in Fig. 3. Because the remnant secondaries straddle the helium ignition line in Fig. 3, across which core masses are discontinuous (see Fig. 2), the distribution of their post-common-envelopes remnants is fragmented, some appearing as degenerate helium white dwarf remnants (lower left), some as helium main sequence star remnants (lower center), and the remainder as shell-burning helium star remnants (upper center). These latter two groups overlap in the mass-radius diagram. The remnant distributions labeled ‘With Wind Mass Loss’ assume that the remnant secondaries of conservative mass transfer lose half their mass in a stellar wind prior to common envelope evolution. They too are fragmented, into degenerate helium white dwarf remnants (lower left) and shell-burning helium stars (upper right). Within each group of remnants, lines of constant remnant primary mass are shown, as in Fig. 3.

therein] and, indeed, of earlier studies of close double white dwarf formation [11]. In the present context, their existence appears inescapable, if not understood.

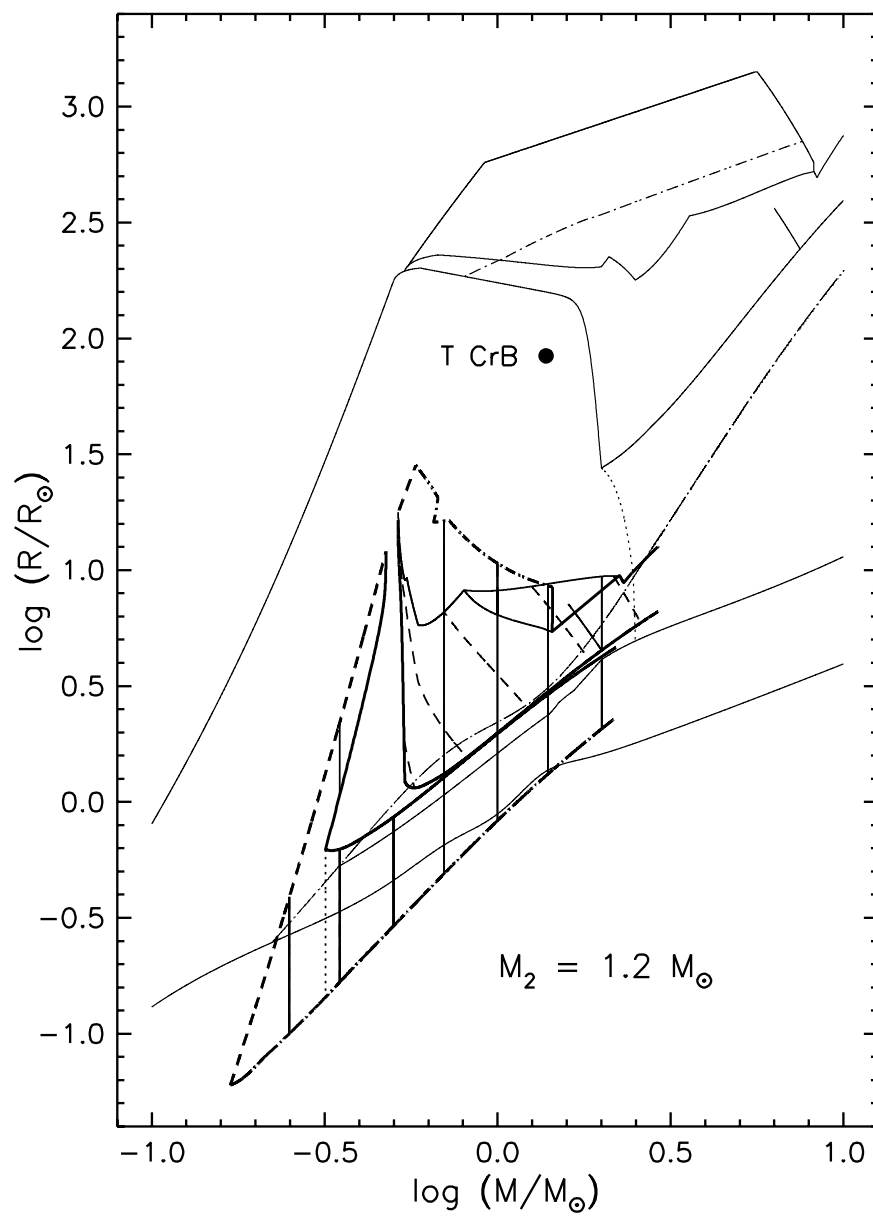
5 Long-Period Post-Common-Envelope Binaries and the Missing Energy Problem

If the properties of short-period binaries with compact components can be reconciled with the outcomes of common envelope evolution as expected from simple energetics arguments, a challenge to this picture still comes from the survival of symbiotic stars and recurrent novae at orbital separations too large to have escaped tidal mass transfer earlier in their evolution. Notwithstanding this author's earlier hypothesis that the outbursting component in the recurrent nova T CrB (and its sister system RS Oph) might be a nondegenerate star undergoing rapid accretion [59, 29, 61], it is now clear that the hot components in both of these systems must indeed be hot, degenerate dwarfs [48, 6, 1, 18]. Furthermore, the short outburst recurrence times of these two binaries demand that the degenerate dwarfs in each must have masses very close to the Chandrasekhar limit.

The complexion of the problem posed by these systems can be illustrated by a closer examination of T CrB itself. Its orbital period ($P = 227.53$ d) and spectroscopic mass function ($f(m) = 0.299 M_{\odot}$) are well-established from the orbit of the donor M3 III star [23]. The emission-line orbit for the white dwarf [25] now appears very doubtful [18], but the system shows very strong ellipsoidal variation (e.g., [1]), suggesting that the system is near a grazing eclipse. Following Hric, et al. [18], I adopt $M_{\text{WD}} = 1.38 M_{\odot}$ and $M_{\text{M3}} = 1.2 M_{\odot}$. The Roche lobe radius of the white dwarf is then $R_{\text{L,WD}} = 84 R_{\odot}$, nearly an order of magnitude larger than can be accommodated from the energetics arguments presented above, even for $\alpha_{\text{CE}} = 1$, assuming solar metallicity for the system (see Fig. 5). A similar discrepancy occurs for RS Oph.

It is evident that these long-period binaries are able to tap some energy source not reflected in the energy budget in (6). One possibility, discussed repeatedly in studies of planetary nebula ejection ([30, 42], more recently [58, 12, 13]) is that the recombination energy of the envelope comes

Fig. 5 (facing page). Post-common-envelope masses and Roche lobe radii for binaries consisting of a white dwarf or helium star plus a $1.2 M_{\odot}$ companion, computed with $\alpha_{\text{CE}} = 1$. Remnant systems inhabit the regions outlined in bold, and spanned vertically by lines of constant white dwarf/helium star mass of 0.25, 0.35, 0.5, 0.7, 1.0, 1.4, and $2.0 M_{\odot}$. Other initial sequences, encoded as in Fig. 2, have been mapped through common envelope evolution. The location in this diagram of the white dwarf in the recurrent nova T CrB is also indicated.



into play. For solar composition material (and complete ionization), that recombination energy amounts to 15.4 eV amu^{-1} , or $1.49 \times 10^{13} \text{ erg g}^{-1}$. For tightly-bound envelopes on the initial giant branch of the donor, this term is of little consequence; but near the tip of the low-mass giant branch, and on the upper asymptotic giant branch of intermediate-mass stars, it can become comparable with, or even exceed, the gravitational potential energy of the envelope. In the model calculations of thermally-pulsing asymptotic giant branch stars by Wagenhuber & Weiss [58], the threshold for spontaneous ejection by envelope recombination occurs consistently when the stellar surface gravity at the peak thermal pulse luminosity falls to

$$\log g_{\text{HRI}} = -1.118 \pm 0.042 . \quad (14)$$

This threshold marks the presumed upper limit to the radii of lower-mass asymptotic giant branch stars in Figs. 1 et seq. in the present paper. In fact, the total energies of the envelopes of these stars formally becomes decidedly positive even before the onset of the superwind phase, also shown in these figures.

Whether single stars successfully tap this ionization energy in ejecting planetary nebulae is still debated, but the circumstances of mass transfer in binary systems would seem to provide a favorable environment for doing so. In the envelopes of extended giants and asymptotic giant branch stars, photospheric electron densities and opacities are dominated by heavy elements; the middle of the hydrogen ionization zone is buried at optical depths of order $\tau \sim 10^5$. Adiabatic expansion of the envelope of the donor into the Roche lobe of its companion can therefore trigger recombination even as the recombination radiation is itself trapped and reprocessed within the flow, much as the same process occurs in rising convective cells.

Other possible energy terms exist that have been neglected in the energetics arguments above: rotational energy, tidal contributions, coulomb energy, magnetic fields, etc. But Virial arguments preclude most of these terms from amounting to more than a minor fraction of the internal energy content of the common envelope at the onset of mass transfer, when the energy budget is established. The only plausible energy source of significance is the input from nuclear reactions. In order for that input to be of consequence, it must of course occur on a time scale short compared with the thermal time scale of the common envelope. Taam [53] explored the possibility that shell burning in an asymptotic giant branch core could be stimulated by mixing induced dynamically in the common envelope (see also [54, 52]). Nothing came of this hypothesis: mixing of fresh material into a burning shell required taking low-density, high-entropy material from the common envelope and mixing it downward many pressure scale heights through a strongly stable entropy gradient to the high-density, low-entropy burning region. In the face of strong buoyancy forces, dynamical penetration is limited to scales of order a pressure scale height.

6 Common Envelope Evolution with Recombination

The notion that recombination energy may be of importance specifically to common envelope evolution is not new. It has been included, at least parametrically in earlier studies, for example, by Han et al. [13], who introduced a second α -parameter, α_{th} , characterizing the fraction of the initial thermal energy content of the common envelope available for its ejection. The initial energy kinetic/thermal content of the envelope is constrained by the Virial Theorem, however, and it is not clear that there is a compelling reason for treating it differently from, say, the orbital energy input from the inspiraling cores. We choose below to formulate common envelope evolution in terms of a single efficiency parameter, labeled here β_{CE} to avoid confusion with α_{CE} as defined above.

By combining the standard stellar structure equations for hydrostatic equilibrium and mass conservation, we can obtain an expression for the gravitational potential energy, Ω_{e} , of the common envelope:

$$\Omega_{\text{e}} \equiv - \int_{M_{\text{c}}}^{M_{*}} \frac{GM}{r} dM = 3 PV \Big|_{R_{\text{c}}}^{R_{*}} - 3 \int_{V_{\text{c}}}^{V_{*}} P dV , \quad (15)$$

where subscripts c refer to the core-envelope boundary, and $*$ to the stellar surface. This is, of course, the familiar Virial Theorem applied to a stellar interior.

It is convenient to split the pressure in this integral into non-relativistic (particle), P_{g} , and relativistic (photon), P_{r} , parts. The envelopes of giants undergoing common envelope evolution are sufficiently cool and non-degenerate to make the classical ideal gas approximation an excellent one for the particle gas. One can then write

$$P = P_{\text{g}} + P_{\text{r}} = \frac{2}{3}u_{\text{g}} + \frac{1}{3}u_{\text{r}} , \quad (16)$$

where u_{g} and u_{r} are kinetic energy densities of particle and radiation gases, respectively. The *total* internal energy density of the gas is

$$u = u_{\text{g}} + u_{\text{r}} + u_{\text{int}} , \quad (17)$$

where the term u_{int} now appearing represents non-kinetic contributions to the total energy density of the gas, principally the dissociation and ionization energies plus internal excitation energies of bound atoms and molecules. The overwhelmingly dominant terms in u_{int} are the ionization energies: $u_{\text{int}} \approx \rho\chi_{\text{eff}}$.

Integrating over the stellar envelope, we obtain for the total energy E_{e} of the envelope:

$$\begin{aligned} E_{\text{e}} &= \Omega_{\text{e}} + U_{\text{e}} \\ &= \left(3 PV \Big|_{R_{\text{c}}}^{R_{*}} - 2U_{\text{g}} - U_{\text{r}} \right) + (U_{\text{g}} + U_{\text{r}} + U_{\text{int}}) \\ &= -4\pi R_{\text{c}}^3 P_{\text{c}} - U_{\text{g}} + U_{\text{int}} , \end{aligned} \quad (18)$$

where we explicitly take $P_* \rightarrow 0$. In fact, experience shows that, for red-giant like structures, R_c is so small that the first right-hand term in the last equality can generally be neglected. In that case, we get the familiar Virial result, but with the addition of a term involving the ionization/excitation/dissociation energy available in the gas, $U_{\text{int}} \approx M_e \chi_{\text{eff}}$, which becomes important for diffuse, loosely-bound envelopes.

In the context of common envelope evolution, it is of course the dissipated orbital energy, $E_{\text{orb}}^{(i)} - E_{\text{orb}}^{(f)}$, that must unbind the envelope. However, the inclusion of U_{int} in E_e now opens the possibility that the common envelope began with *positive* total energy; that is, in the usual α_{CE} -prescription, it is possible for λ^{-1} to be zero or even negative, which has the undesirable consequence that α_{CE} need not lie in the interval $0 \leq \alpha_{\text{CE}} \leq 1$ for all physically-possible outcomes. However, the gravitational potential energy of the envelope, Ω_e , is negative-definite, and by comparing it with all available energy sources (orbital energy released plus internal energy of the envelope), we can define an ejection efficiency β_{CE} that has the desired property, $0 \leq \beta_{\text{CE}} \leq 1$:

$$\beta_{\text{CE}} \equiv \frac{\Omega_e}{(E_{\text{orb}}^{(f)} - E_{\text{orb}}^{(i)}) - U_e} = \frac{4\pi R_c^3 P_c + 2U_g + U_r}{E_{\text{orb}}^{(i)} - E_{\text{orb}}^{(f)} + U_g + U_r + U_{\text{int}}} . \quad (19)$$

By analogy to the form factor λ in the conventional α_{CE} formalism above, we can define separate form factors λ_{Ω} for the gravitational potential energy and λ_P for the gas plus radiation contributions to the (kinetic) internal energy of the envelope:

$$\Omega_e = \frac{GMM_e}{\lambda_{\Omega} R} \quad \text{and} \quad U_g + U_r = \frac{GMM_e}{\lambda_P R} , \quad (20)$$

In contrast, the recombination energy available can be written simply in terms of an average ionization energy per unit mass,

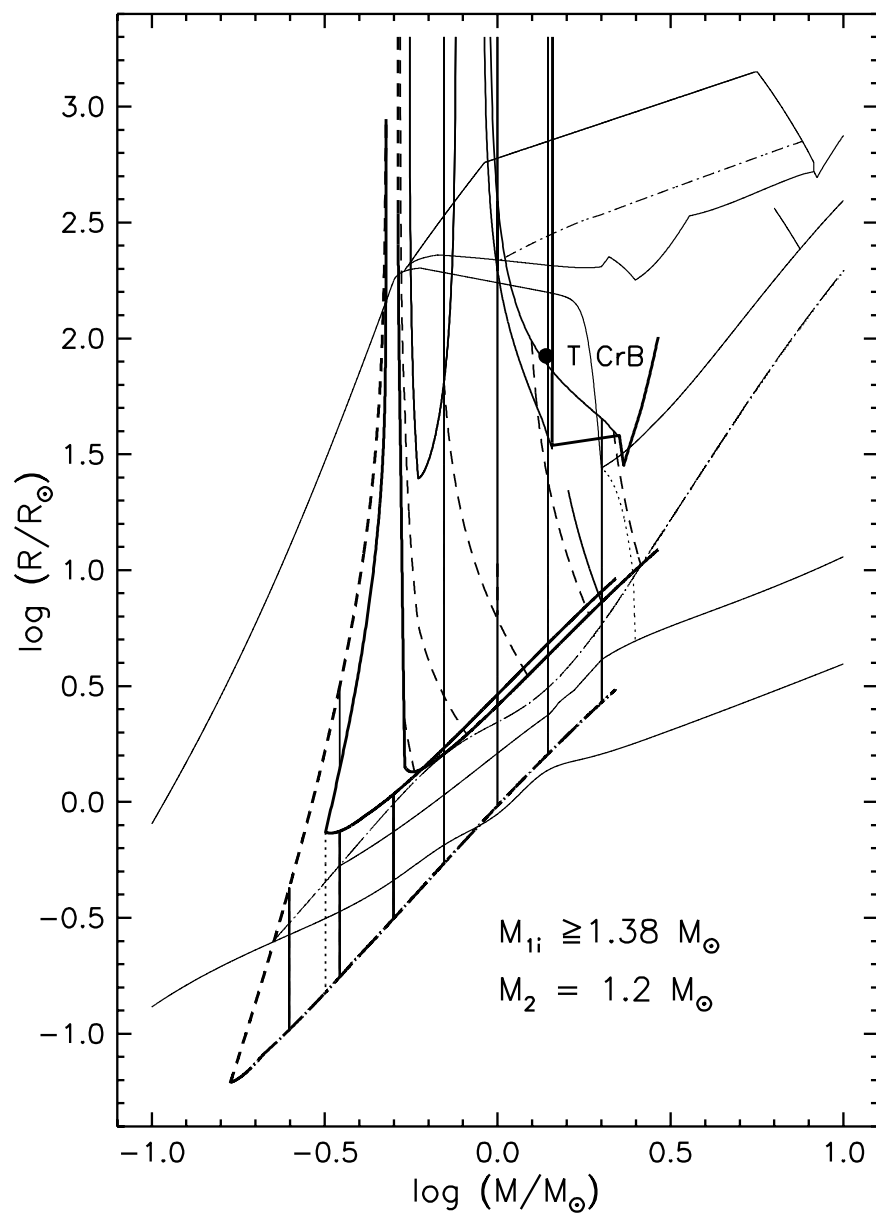
$$U_{\text{int}} = M_e \chi_{\text{eff}} . \quad (21)$$

The ratio of final to initial orbital separation then becomes

$$\frac{A_f}{A_i} = \frac{M_{1c}}{M_1} \left[1 + 2 \left(\frac{1}{\beta_{\text{CE}} \lambda_{\Omega} r_{1,L}} - \frac{1}{\lambda_P r_{1,L}} - \frac{\chi_{\text{eff}} A_i}{GM_1} \right) \left(\frac{M_1 - M_{1c}}{M_2} \right) \right]^{-1} . \quad (22)$$

In the limit that radiation pressure P_r , ionization energy (U_{int}), and the boundary term ($4\pi R_c^3 P_c$) are all negligible, then $2\lambda_{\Omega} \rightarrow \lambda_P \rightarrow \lambda$ and $\beta_{\text{CE}} \rightarrow 2\alpha_{\text{CE}}/(1 + \alpha_{\text{CE}})$.

Fig. 6 (facing page). Post-common-envelope masses and Roche lobe radii as in Fig. 5, but with recombination energy included, computed from (22) with $\beta_{\text{CE}} = 1$, with the approximation $2\lambda_{\Omega} = \lambda_P = \lambda$ from (3). At small separations, the differences are inconsequential, but substantially larger final separations are allowed when $A_f \gtrsim 10 R_{\odot}$ ($R_L \gtrsim 3 R_{\odot}$).



The ability to tap the recombination energy of the envelope has a profound effect on the the final states of the longest-period intermediate-mass binaries, those that enter common envelope evolution with relatively massive, degenerate carbon-oxygen (or oxygen-neon-magnesium) cores. As is evident in Fig. 6, possible final states span a much broader range of final orbital separations. Indeed, for the widest progenitor systems, the (positive) total energy of the common envelope can exceed the (negative) orbital energy of the binary, making arbitrarily large final semimajor axes energetically possible.³ The inclusion of recombination energy brings both T CrB and RS Oph within energetically accessible post-common-envelope states. It suffices as well to account for the exceptionally long-period close double white binary PG 1115+166, as suggested by Maxted, et al. [31].

7 Conclusions

Re-examination of global constraints on common envelope evolution leads to the following conclusions:

Both energy and angular momentum conservation pose strict limits on the outcome of common envelope evolution. Of these two constraints, however, energy conservation is much the more demanding.

The recent study of close double white dwarf formation by Nelemans & Tout [35] shows clearly that their progenitors can have lost little orbital energy through their first episodes of mass transfer. Since common envelope ejection must be rapid if it is to be efficient, its energy budget is essentially fixed at its onset by available thermal and gravitational terms. The preservation of orbital energy through that first phase of mass transfer therefore indicates that the observed close double white dwarfs escaped common envelope formation in that first mass transfer phase. They evidently evolved through quasi-conservative mass transfer. However, strictly mass- and angular momentum-conservative mass transfer leaves remnant accretors that are too massive and compact to account for any but the shortest-period close double white dwarfs. Significant mass loss and the input of orbital energy prior to the onset of the second (common envelope) phase of mass transfer are required. The requisite energy source must be of nuclear evolution, which is capable of driving orbital expansion and stellar wind losses during the slower (thermal recovery or nuclear time scale) phases of quasi-conservative mass transfer, or during the interval between first and second episodes of mass transfer. Details of this process remain obscure, however.

³ The final orbit remains constrained by the finite initial orbital angular momentum of the binary. Final semimajor axes much in excess of the initial semimajor axis may be energetically allowed, but the finite angular momentum available means that they cannot be circular – see (9) – an effect which has been neglected in Fig. 6.

Long-period cataclysmic variables such as T CrB and RS Oph pose a more extreme test of common envelope energetics. With their massive white dwarfs, the evident remnants of much more massive initial primaries, they are nevertheless too low in total systemic mass to be plausible products of quasi-conservative mass transfer, but too short in orbital period to have escaped tidal mass transfer altogether. They must be products of common envelope evolution, but to have survived at their large separations, they demand the existence of a latent energy reservoir in addition to orbital energy to assist in envelope ejection. It appears that these binaries efficiently tap ionization/recombination energy in ejecting their common envelopes. That reservoir is demonstrably adequate to account for the survival of these binaries. Its inclusion requires only a simple revision to the parameterization of common envelope ejection efficiency.

Acknowledgement. This work owes its existence to both the encouragement and the patience of Gene Milone, to whom I am most grateful. Thanks go as well to Ron Taam for a useful discussion of possible loopholes in common envelope theory, and to Jarrod Hurley for providing the source code described in Hurley, et al. (2000). This work was supported in part by grant AST 0406726 to the University of Illinois, Urbana-Champaign, from the US National Science Foundation.

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