Why there is something rather than nothing (out of everything)?

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The path integral over Euclidean geometries for the recently suggested density matrix of the Universe is shown to describe a microcanonical ensemble in quantum cosmology. This ensemble corresponds to a uniform (weight one) distribution in phase space of true physical variables, but in terms of the observable spacetime geometry it is peaked about complex saddle-points of the *Lorentzian* path integral. They are represented by the recently obtained cosmological instantons limited to a bounded range of the cosmological constant. Inflationary cosmologies generated by these instantons at late stages of expansion undergo acceleration whose low-energy scale can be attained within the concept of dynamically evolving extra dimensions. Thus, together with the bounded range of the early cosmological constant, this cosmological ensemble suggests the mechanism of constraining the landscape of string vacua and, simultaneously, a possible solution to the dark energy problem in the form of the quasi-equilibrium decay of the microcanonical state of the Universe.

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Euclidean quantum gravity (EQG) is a lame duck in modern particle physics and cosmology. After its summit in early and late eighties (in the form of the cosmological wavefunction proposals [1, 2] and baby universes boom [3]) the interest in this theory gradually declined, especially, in cosmological context, where the problem of quantum initial conditions was superseded by the concept of stochastic inflation [4]. EQG could not stand the burden of indefiniteness of the Euclidean gravitational action [5] and the cosmology debate of the tunneling vs no-boundary proposals [6].

Thus, a recently suggested EQG density matrix of the Universe [7] is hardly believed to be a viable candidate for the initial state of the Universe, even though it avoids the infrared catastrophe of small cosmological constant Λ, generates an ensemble of universes in the limited range of Λ, and suggests a strong selection mechanism for the landscape of string vacua [7, 8]. Here we want to justify this result by deriving it from first principles of *Lorentzian* quantum gravity applied to a microcanonical ensemble of closed cosmological models.

The statistical sum of [7, 8] is given by the Euclidean path integral over a minisuperspace of the lapse function $N(\tau)$ and scale factor $a(\tau)$ of spatially closed FRW metric $ds^2 = N^2(\tau)\,d\tau^2 + a^2(\tau)\,d^2\Omega^{(3)}$,

$$e^{-\Gamma} = \int\limits_{\rm periodic} D[\,a,N\,]\; e^{-\Gamma_E[\,a,\,N\,]}, \qquad (1)$$

$$e^{-\Gamma_E[\,a,\,N]} = \int\limits_{\rm periodic} D\phi(x)\, e^{-S_E[\,a,\,N;\,\phi(x)\,]}. \qquad (2)$$

Here $\Gamma_E[\,a,\,N]$ is the effective action of all inhomogeneous "matter" fields on minisuperspace background, which include also metric perturbations, $\Phi(x) = (\phi(x),\psi(x),A_\mu(x),h_{\mu\nu}(x),...)$. $S_E[a,N;\phi(x)]$ is the classical Eucidean action, and the integration runs over periodic fields on the Euclidean spacetime with a compactified time τ (of $S^1\times S^3$ topology).

For free matter fields $\phi(x)$ conformally coupled to gravity (which are assumed to be dominating in the system) the effective action has the form [7] $\Gamma_E[\,a,N\,] = \int d\tau\, N\mathcal{L}(a,a') + F(\eta)$, $a' \equiv da/Nd\tau$. Here $N\mathcal{L}(a,a')$ is the effective Lagrangian of its local part including the classical Einstein term (with the cosmological constant $\Lambda = 3H^2$) and the contribution of the conformal anomaly of quantum fields and their vacuum (Casimir) energy,

$$\mathcal{L}(a,a') = -aa'^2 - a + H^2a^3 + B\left(\frac{a'^2}{a} - \frac{a'^4}{6a} + \frac{1}{2a}\right). \qquad (3)$$

$F(\eta)$ is the free energy of their quasi-equilibrium excitations with the temperature given by the inverse of the conformal time $\eta = \int d\tau\, N/a$. This is a typical boson or fermion sum $F(\eta) = \pm\sum_\omega \ln\left(1\mp e^{-\omega\eta}\right)$ over field oscillators with energies ω on a unit 3-sphere. We work in units of $m_P = (3\pi/4G)^{1/2}$, and B is the constant determined by the coefficient of the Gauss-Bonnet term in the overall conformal anomaly of all fields $\phi(x)$.

Semiclassically the integral (1) is dominated by the saddle points — solutions of the Friedmann equation

$$\frac{a'^2}{a^2} + B\left(\frac{1}{2}\,\frac{a'^4}{a^4} - \frac{a'^2}{a^4}\right) = \frac{1}{a^2} - H^2 - \frac{C}{a^4}, \qquad (4)$$

modified by the quantum B-term and the radiation term C/a^4 with the constant C satisfying the bootstrap equation $C = B/2 + dF(\eta)/d\eta$. Such solutions represent garland-type instantons which exist only in the limited range $0 < \Lambda_{\rm min} < \Lambda < 3m_P^2/B$ [7, 8] and eliminate the infrared catastrophe of $\Lambda = 0$. The period of these quasi-thermal instantons is not a freely specifiable parameter, but as a function of Λ follows from this bootstrap. Therefore this is not a canonical ensemble.

Contrary to the construction above, the density matrix that we advocate here is given by the canonical path integral of *Lorentzian* quantum gravity. Its kernel in the representation of 3-metrics and matter fields denoted below as q reads

$$\rho(q_+,q_-) = e^{\Gamma}\int\limits_{q(t_\pm)=\,q_\pm} D[\,q,p,N\,]\; e^{\,i\int_{t_-}^{t_+} dt\,(p\,\dot q - N^\mu H_\mu)}, \qquad (5)$$

where the integration runs over histories of phase-space variables $(q(t), p(t))$ and the Lagrange multipliers of the gravitational constraints H_μ — lapse and shift functions $N(t) = N^\mu(t)$, interpolating between $q_\pm$ at $t_\pm$. The integration measure $D[\,q,p,N\,]$ includes the gauge-fixing factor containing the delta function $\delta(\chi) = \prod_\mu \delta(\chi^\mu)$ of gauge conditions χ^μ and the ghost factor [9] (condensed index μ includes also continuous spatial labels) [16]. It is important that the integration range of N^μ

$$-\infty < N < +\infty, \tag{6}$$

is such that it generates in the integrand the delta-functions of the constraints $\delta(H) = \prod_\mu \delta(H_\mu)$. As a consequence the kernel (5) satisfies the set of quantum Dirac constraints — Wheeler-DeWitt equations

$$\hat{H}_\mu\big(q, \partial/i\partial q\big)\,\rho(\,q, q_-\,) = 0, \tag{7}$$

and the density matrix (5) can be regarded as an operator delta-function of these constraints

$$\hat{\rho} \sim \text{“}\Big(\prod_\mu \delta(\hat{H}_\mu)\Big)\text{”}. \tag{8}$$

This notation should not be understood literally because this multiple delta-function is not well defined, for the operators $\hat{H}_\mu$ do not commute and form the quasi-algebra $[\hat{H}_\mu, \hat{H}_\nu] = i\hbar\,\hat{U}^\lambda_{\mu\nu}\hat{H}_\lambda$ with nonvanishing structure functions $\hat{U}^\lambda_{\mu\nu}$. Moreover, exact operator realization $\hat{H}_\mu$ is not known except the first two orders of semiclassical $\hbar$-expansion [10] in the models with a finite-dimensional phase space. Fortunately, we do not need a precise form of these constraints, because we will proceed with their path-integral solutions very well adjusted to the semiclassical perturbation theory. This strategy does not extend beyond typical field-theoretical considerations when the path integral is regarded more fundamental than the Schrodinger equation marred with the problems of divergent equal-time commutators, operator ordering, etc.

The very essence of our proposal is the interpretation of (5) and (8) as the density matrix of a *microcanonical* ensemble in spatially closed quantum cosmology. A simplest analogy is the density matrix of an unconstrained system having a conserved Hamiltonian $\hat{H}$ in the microcanonical state with fixed energy E, $\hat{\rho} \sim \delta(\hat{H} - E)$. Major distinction of (8) from this case is that spatially closed cosmology does not have freely specifiable constants of motion like energy or other global charges. Rather it has as constants of motion the Hamiltonian and momentum constraints H_μ, all having a particular value — zero. Therefore, the expression (8) can be considered as a most general and natural candidate for the quantum state of the *closed* Universe. Below we confirm this fact by showing that in the physical sector the corresponding statistical sum is just a uniformly distributed (with a unit weight) integral over entire phase space of true physical degrees of freedom. Thus, this is a sum over Everything. However, in terms of the observable quantities, like spacetime geometry, this distribution turns out to be nontrivially peaked around a particular set of universes. Semiclassically this distribution is given by the density matrix of Euclidean quantum gravity and the saddle-point instantons of the above type [7].

From the normalization of the density matrix in the physical Hilbert space the statistical sum follows as the path integral

$$\begin{aligned} 1 = \mathrm{Tr}_{\rm phys}\,\hat{\rho} &= \int dq\,\mu\big(q, \partial/i\partial q\big)\,\rho(q, q')\Big|_{\,q'=q} \\ &= e^{\varGamma} \int\limits_{\rm periodic} D[\,q,p,N\,]\;e^{\,i\int dt(p\,\dot{q} - N^\mu H_\mu)}, \end{aligned} \tag{9}$$

where the integration runs over periodic in time histories of $q = q(t)$. Here $\mu\big(q, \partial/i\partial q\big) = \hat{\mu}$ is the measure which distinguishes the physical inner product in the space of solutions of the Wheeler-DeWitt equations $(\psi_1|\psi_2) = \langle\psi_1|\hat{\mu}|\psi_2\rangle$ from that of the space of square-integrable functions, $\langle\psi_1|\psi_2\rangle = \int dq\,\psi_1^*\psi_2$. This measure includes the delta-function of unitary gauge conditions and an operator factor built with the aid of the relevant ghost determinant [17].

On the other hand, in terms of the physical phase space variables the Faddeev-Popov path integral equals [9]

$$\begin{aligned} &\int D[\,q,p,N\,]\;e^{\,i\int dt\,(p\,\dot{q} - N^\mu H_\mu)} \\ &\quad= \int Dq_{\rm phys}\,Dp_{\rm phys}\,e^{\,i\int dt\,(p_{\rm phys}\,\dot{q}_{\rm phys} - H_{\rm phys}(t))} \\ &\quad= \mathrm{Tr}_{\rm phys}\,\Big(\mathrm{T}\,e^{-i\int dt\,\hat{H}_{\rm phys}(t)}\Big), \end{aligned} \tag{10}$$

where T denotes the chronological ordering. Here the physical Hamiltonian $H_{\rm phys}(t)$ and its operator realization $\hat{H}_{\rm phys}(t)$ are nonvanishing only in unitary gauges explicitly depending on time [10], $\partial_t\chi^\mu(q,p,t) \neq 0$. In static gauges, $\partial_t\chi^\mu = 0$, they identically vanish, because in spatially closed cosmology the full Hamiltonian reduces to the combination of constraints.

The path integral (10) is gauge-independent on-shell [9] and coincides with that in the static gauge. Therefore, from Eqs.(9)-(10) with $\hat{H}_{\rm phys} = 0$, the statistical sum of our microcanonical ensemble equals

$$\begin{aligned} e^{-\varGamma} = \mathrm{Tr}_{\rm phys}\,\mathbf{I}_{\rm phys} &= \int dq_{\rm phys}\,dp_{\rm phys} \\ &= \text{sum over Everything.} \end{aligned} \tag{11}$$

This ultimate equipartition, not modulated by any nontrivial density of states, is a result of general covariance and closed nature of the Universe lacking any freely specifiable constants of motion. The volume integral of entire *physical* phase space, whose structure and topology is not known, is very nontrivial. However, below we show that semiclassically it is determined by EQG methods and supported by instantons of [7] spanning a bounded range of the cosmological constant.

Integration over momenta in (9) yields a Lagrangian path integral with a relevant measure and action

$$e^{-\varGamma}=\int D[\,q,N\,]\;e^{iS_L[\,q,\,N\,]}. \tag{12}$$

Integration runs over periodic fields (not indicated explicitly but assumed everywhere below) even despite the Lorentzian signature of the underlying spacetime. Similarly to the procedure of [7, 8] leading to (1)-(2) in the Euclidean case, we decompose $[\,q,N\,]$ into a minisuperspace $[\,a_L(t),N_L(t)\,]$ and the "matter" $\phi_L(x)$ variables, the subscript L indicating their Lorentzian nature. With a relevant decomposition of the measure $D[\,q,N\,]=D[\,a_L,N_L\,]\times D\phi_L(x)$, the microcanonical sum takes the form

$$e^{-\varGamma}=\int D[\,a_L,N_L\,]\;e^{i\varGamma_L[\,a_L,\,N_L\,]}, \tag{13}$$

$$e^{i\varGamma_L[\,a_L,\,N_L\,]}=\int D\phi_L(x)\;e^{iS_L[\,a_L,\,N_L;\,\phi_L(x)]}, \tag{14}$$

where $\varGamma_L[\,a_L,\,N_L\,]$ is a Lorentzian effective action. The stationary point of (13) is a solution of the effective equation $\delta\varGamma_L/\delta N_L(t)=0$. In the gauge $N_L=1$ it reads as a Lorentzian version of the Euclidean equation (4) and the bootstrap equation for the radiation constant C with the Wick rotated $\tau=it$, $a(\tau)=a_L(t)$, $\eta=i\int dt/a_L(t)$. However, with these identifications C turns out to be purely imaginary (in view of the complex nature of the free energy $F(i\int dt/a_L)$). Therefore, no periodic solutions exist in spacetime with a *real* Lorentzian metric.

On the contrary, such solutions exist in the Euclidean spacetime. Alternatively, the latter can be obtained with the time variable unchanged $t=\tau$, $a_L(t)=a(\tau)$, but with the Wick rotated lapse function

$$N_L=-iN,\quad iS_L[\,a_L,N_L;\phi_L]=-S_E[\,a,N;\phi\,]. \tag{15}$$

In the gauge $N=1$ ($N_L=-i$) these solutions exactly coincide with the instantons of [7]. The corresponding saddle points of (13) can be attained by deforming the integration contour (6) of N_L, into the complex plane to pass through the point $N_L=-i$ and relabeling the real Lorentzian t with the Euclidean τ. In terms of the Euclidean $N(\tau)$, $a(\tau)$ and $\phi(x)=\phi_L(x)$ the integrals (13) and (14) take the form of the path integrals (1) and (2) in Euclidean quantum gravity,

$$i\varGamma_L[\,a_L,\,N_L]=-\varGamma_E[\,a,\,N\,]. \tag{16}$$

However, the integration contour for the Euclidean $N(\tau)$ runs from $-i\infty$ to $+i\infty$ through the saddle point $N=1$. This is the source of the conformal rotation in Euclidean quantum gravity, which is called to resolve the problem of unboundedness of the gravitational action and effectively renders the instantons a thermal nature, even though they originate from the microcanonical ensemble. This mechanism implements the attempts of justification of EQG from canonical quantization of gravity [12] (see also [13] in black hole context).

To show this we calculate (1) in the one-loop approximation with the measure inherited from the canonical path integral (5) $D[\,a,N\,]=Da\,DN\,\mu[\,a,N\,]\,\delta[\,\chi\,]\,\mathrm{Det}\,Q$. Here $\mu[\,a,N\,]$ is a local measure determined by the Lagrangian $N\mathcal{L}(a,a')$, (3), in the local part of $\varGamma_E[\,a,N\,]$,

$$\mu_{1-\rm loop}=\prod_\tau\left(\frac{\partial^2(N\mathcal{L})}{\partial\dot a\,\partial\dot a}\right)^{1/2}=\prod_\tau\left(\frac{D}{N\,a^2a'^2}\right)^{1/2},$$
$$D=a\,a'^2(a^2-B+B\,a'^2). \tag{17}$$

The Faddeev-Popov factor $\delta[\,\chi\,]\,\mathrm{Det}\,Q$ contains a gauge condition $\chi=\chi(a,N)$ fixing the one-dimensional diffeomorphism, $\tau\to\bar\tau=\tau-f/N$, which for infinitesimal $f=f(\tau)$ has the form $\Delta^f N\equiv\bar N(\tau)-N(\tau)=\dot f$, $\Delta^f a\equiv\bar a(\tau)-a(\tau)=\dot a\,f/N$, and $Q=Q(d/d\tau)$ is a ghost operator determined by the gauge transformation of $\chi(a,N)$, $\Delta^f\chi=Q(d/d\tau)\,f(\tau)$.

The conformal mode σ of the perturbations $\delta a=\sigma a_0$ and $\delta N=\sigma N_0$ on the saddle-point background (labeled below by zero, $a=a_0+\delta a$, $N=N_0+\delta N$) originates from imposing the background gauge $\chi(a,N)=\delta N-(N_0/a_0)\,\delta a$. In this gauge $Q=a(d/d\tau)a^{-1}$, and the quadratic part of $\varGamma_E$ takes the form [11]

$$\delta^2_\sigma\varGamma_E=-\frac{3\pi m_P^2}{2}\int d\tau ND\left[\Big(\frac{\sigma}{a'}\Big)'\right]^2, \tag{18}$$

where D is given by (17). As is known from [7] for the background instantons $a_0^2(\tau)\geq a_-^2>B$ (a_- is the turning point with the smallest value of $a_0(\tau)$), so that $D>0$ everywhere except the turning points where D degenerates to zero. Therefore $\delta^2_\sigma\varGamma_E<0$ for real σ, but the Gaussian integration runs along the imaginary axes and yields the functional determinant of the positive operator — the kernel of the quadratic form (18)

$$e^{-\varGamma_{1-\rm loop}}=e^{-\varGamma_0}\,\mathrm{Det}\,Q_0\int D\sigma\,\Big(\prod_\tau D/a'^2\Big)^{1/2}e^{-\frac12\delta^2_\sigma\varGamma_E}$$
$$=e^{-\varGamma_0}\times\mathrm{Det}\Big(\frac{d}{d\tau}\Big)\left[\mathrm{Det}\Big(-\frac{1}{\sqrt D}\frac{d}{d\tau}\,D\frac{d}{d\tau}\frac{1}{\sqrt D}\Big)\right]^{-1/2}.$$

Both determinants here are subject to periodic boundary conditions and cancel each other (see [11])[18]. Therefore, the contribution of the conformal mode reduces to the selection of instantons with a fixed Euclidean time period, thus effectively endowing them with a thermal nature.

As suggested in [7, 8, 14] these instantons serve as initial conditions for inflationary universes which evolve according to the Lorentzian version of Eq.(4) and, at late stages, have two branches of a cosmological acceleration with Hubble scales $H^2_\pm=(m_P^2/B)(1\pm(1-2BH^2)^{1/2})$. If the initial $\Lambda=3H^2$ is a composite inflaton field decaying at the end of inflation, then one of the branches undergoes acceleration with $H^2_+=2m_P^2/B$. This is determined by the new quantum gravity scale suggested in [8] – the upper bound of the instanton Λ-range, $\Lambda_{\max}=3m_P^2/B$. To match the model with inflation and the dark energy phenomenon, one needs B of the order of the inflation scale in the very early Universe and $B \sim 10^{120}$ now, so that this parameter should effectively be a growing function of time.

This picture seems to fit into string theory at its low-energy field-theoretic level. Then, with a bounded range of Λ, it might constrain the landscape of string vacua [7, 8]. Moreover, string theory has a qualitative mechanism to promote the constant B to the level of a moduli variable indefinitely growing with the evolving size $R(t)$ of extra dimensions. Indeed B as a coefficient in the overall conformal anomaly of 4-dimensional quantum fields basically counts their number N, $B \sim N$. In the Kaluza-Klein (KK) theory and string theory the effective 4-dimensional fields arise as KK and winding modes with the masses [15]

$$m_{n,w}^2 = \frac{n^2}{R^2} + \frac{w^2}{\alpha'^2}\, R^2 \tag{19}$$

(enumerated by the KK and winding numbers), which break their conformal symmetry. These modes remain approximately conformally invariant as long as their masses are much smaller than the spacetime curvature, $m_{n,w}^2 \ll H_+^2 \sim m_P^2/N$. Therefore the number of *conformally invariant* modes changes with R. Simple estimates show that for pure KK modes ($w = 0$, $n \leq N$) their number grows with R as $N \sim (m_P R)^{2/3}$, whereas for pure winding modes ($n = 0$, $w \leq N$) their number grows with the decreasing R as $N \sim (m_P\alpha'/R)^{2/3}$. Thus, the effect of indefinitely growing B is possible for both scenarios with expanding or contracting extra dimensions. In the first case this is the growing tower of superhorizon KK modes which make the horizon scale $H_+ = m_P\sqrt{2/B} \sim m_P/(m_P R)^{1/3}$ indefinitely decreasing with $R \to \infty$. In the second case this is the tower of superhorizon winding modes which make this acceleration scale decrease with the decreasing R as $H_+ \sim m_P(R/m_P\alpha')^{1/3}$. This effect is flexible enough to accommodate the present day acceleration scale $H_0 \sim 10^{-60} m_P$ (though, by the price of fine-tuning an enormous coefficient of expansion or contraction of R). This gives a new dark energy mechanism driven by the conformal anomaly and transcending the inflationary and matter-domination stages starting with the state of the microcanonical distribution.

To summarize, within a minimum set of assumptions (the equipartition in the physical phase space (11)), we seem to have the mechanism of constraining the landscape of string vacua and get the full evolution of the Universe as a quasi-equilibrium decay of its initial microcanonical state. Thus, contrary to anticipations of Sidney Coleman, “there is Nothing rather than Something” [3], one can say that Something (rather than Nothing) comes from Everything.

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[16] This measure reads $D[\,q,p,N\,] = \prod_t dN\,dq\,dp\,\delta(\chi)\,J$ for unitary gauges $\chi = \chi(q)$. Here J is the ghost factor $J = \det J^\mu_\nu$, determined by the Poisson bracket of gauge conditions and constraints $J^\mu_\nu = \{\,\chi^\mu, H_\nu\}$
[17] Exact expression for $\mu\big(q, \partial/i\partial q\big)$ is not known, but its effect in the path integral (9) reduces to the insertion of one extra factor $\delta(\chi)\,J(q,p)$ into the time product of $D[\,q,p,N\,]$, so that in the leading order in $\hbar$ it reads $\mu\big(q, \partial/i\partial q\big) = \delta(\chi)\,J\big(q, \partial/i\partial q\big)$, see [10].
[18] Both operators have a zero mode to be eliminated from their determinants, because it corresponds to the conformal Killing symmetry of FRW instantons.