Approaching the Heisenberg limit in an atom laser

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We present experimental results showing the decreased divergence and improved transverse profile of an atom laser produced by an optical Raman transition, compared to one produced by an RF transition. We present a theoretical model to compare with experimental results and find excellent agreement. This model predicts a limit to the beam quality of a Raman atom laser. This limit is a factor of 1.3 above the fundamental (Heisenberg) limit.

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Experiments in ultracold dilute atomic gases have had an enormous impact on physics. The realization of phenomena such as Bose-Einstein condensates (BECs), degenerate Fermi gases, BEC-BCS crossover systems, and many others have resulted in many fundamental insights and a wealth of new results in both experiment and theory. One exciting system to emerge from this research is the atom laser, a highly coherent, directional beam of degenerate atoms, controllably released from a BEC [1, 2, 3, 4, 5, 6, 7]. The atom lasers demonstrated so far have produced beams many orders of magnitude brighter than is possible with thermal atomic beams [8].

Atom laser beams show great potential for use in studies of fundamental physics and in high precision measurements [9]. To fulfill this potential, it will be important to produce atom lasers that are diffraction limited, just as it was important for optical beams. The beam quality factor M^2, introduced for atom lasers by J.-F. Riou *et al.* [10], is a measure of how far the beam deviates from the Heisenberg limit, and is defined by

$$M^2 = \frac{2}{\hbar}\Delta x \Delta p_x, \tag{1}$$

where Δx is the beam width, measured at the waist, and Δp_x is the transverse momentum spread. A number of experimental works have shown that the beam quality of an atom laser is strongly affected by the interaction of the outcoupled atoms with the BEC from which it is produced [10, 11, 12, 13]. As the atoms fall through the condensate, the repulsive interaction acts as a diverging lens to the outcoupled atoms. This leads to a divergence in the atom laser beam and a poor quality transverse beam profile. Such behavior may cause problems in, for example, coherent atom interferometry, or in mode matching the atom beam to a high finesse optical cavity of small mode volume. Experiments on atom lasers in waveguides have produced beams with improved spatial profile [7]. However, to make precision measurements with atom interferometry is likely to require propagation in free space [14].

In a recent Letter [10], it was shown that the quality of a free space atom laser is improved by outcoupling from the base of the condensate. Our scheme, however, enables the production of a high quality atom laser while outcoupling from the center of the condensate. This is desirable for a number of reasons: First, outcoupling from the center allows the highest possible output flux for a given outcoupling rate and hence a lower classical noise

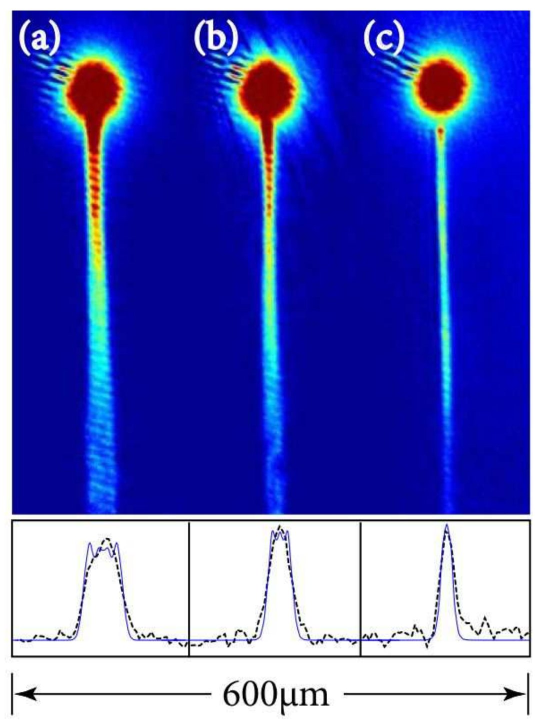

FIG. 1: (color online). Top: Sequence of atom laser beams showing the improved beam profile of a Raman atom laser. The atom laser beams were produced using RF (a) and Raman (b and c) transitions. The angle between the Raman beams (see Fig. 2) was $\theta = 30^\circ$ in (b) and $\theta = 140^\circ$ in (c), corresponding to a kick of 0.3 cm/s and 1.1 cm/s respectively. The outcoupling rate differs between each atom laser. Below: Comparison of experimental (dashed) and theoretical (solid) beam profiles 500 μm below the BEC. The height of each theoretical curve has been scaled to match experimental data.

level [15]. Second, outcoupling from the center allows the longest operating time (for a quasicontinuous atom laser) since the condensate can be drained completely. Third, outcoupling from the center minimizes the sensitivity of the output coupling to condensate excitations or external fluctuations.

In a recent Letter [8], we have demonstrated a continuously outcoupled atom laser where the output coupler is a coherent multi-photon (Raman) transition [6, 16]. In this scheme, the atoms receive a momentum kick from the absorption and emission of photons. They leave the condensate more quickly, so that adverse effects due to the mean-field repulsion from the condensate are reduced. In this Letter, we report measurements of a two-fold improvement in the beam quality M^2 using this outcoupling. In Fig. 1, we show absorption images of atom laser beams outcoupled from the center of a BEC with (a) negligible momentum kick, (b) a kick of 0.3 cm/s, and 1.1 cm/s (c). As the kick increases, the divergence is reduced and the beam profile improved.

In our experiment we create ^{87}Rb BECs of 5×10^5 atoms in the $|F = 1, m_F = -1\rangle$ state via standard runaway evaporation of laser cooled atoms. We use a highly stable, water cooled QUIC magnetic trap (axial frequency $\omega_y = 2\pi \times 15$ Hz and radial frequency $\omega_\rho = 2\pi \times 150$ Hz, with a bias field of $B_0 = 1$ G). We control drifts in the magnetic bias by using high stability power supplies and passive thermal management. This stability allows us to precisely and repeatably address the condensate.

We produce the atom laser by transferring the atoms to the untrapped $|F = 1, m_F = 0\rangle$ state and letting them fall under gravity. To outcouple atoms with negligible momentum kick we induce spin flips via an RF field of a frequency corresponding to the Zeeman shift in the center of the condensate. Alternatively, we couple to the untrapped state via an optical Raman transition. The setup is shown in Fig. 2. Two optical Raman beams, separated by an angle θ, propagate in the plane of gravity and the magnetic trap bias field. The momentum transfer to the atoms through absorption and emission of the photons is $2\hbar k \sin(\theta/2)$, with k the wave number of the laser beams. The Raman laser beams are produced from one 700 mW diode laser. We can turn the laser power on or off in less than 200 ns using a fast switching AOM in a double pass configuration. After the switching AOM, the light is split and sent through two separate AOMs, again each in a double pass configuration. The frequency difference between the AOMs corresponds to the Zeeman plus kinetic energy difference between the initial and final states of the two-photon Raman transition. We stabilize the frequency difference by running the 80 MHz function generators driving the AOMs from a single oscillator. The beams are then coupled via single mode, polarization maintaining optical fibers directly to the BEC through a collimating lens and waveplate, providing a maximum intensity of 2500 mW/cm^2 per beam at the BEC. The polarization of the beams is optimized to achieve maximum outcoupling with a downward kick and corresponds to π polarization for the upper beam and σ^+ for the lower beam.

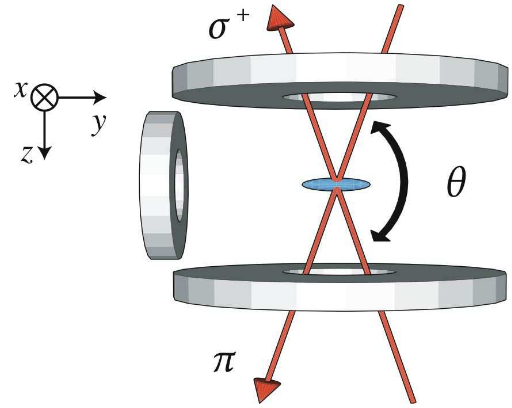

FIG. 2: (color online) Experimental schematic (not to scale) showing the BEC, Raman lasers, and trapping coils.

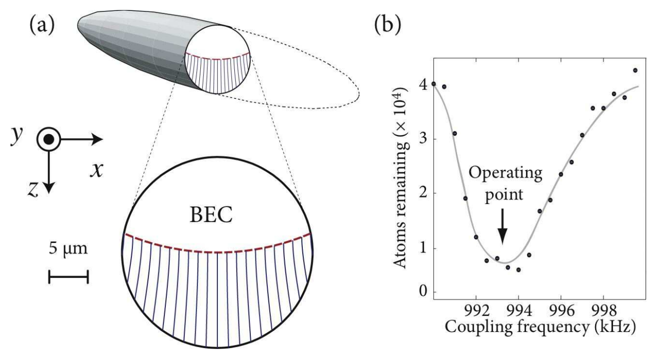

FIG. 3: (color online) (a) Cross section along the two strong axes of the magnetic trap, showing the BEC, outcoupling surface, and atom laser trajectories. Note that the field of view is rotated 90° with respect to Fig. 2.
(b) Output coupling spectroscopy showing the operating point at the center of the BEC, solid curve to guide the eye.

The outcoupling resonance is set to the center of the BEC for both RF and Raman outcoupling, as shown in Fig. 3 (a). This point is found by performing spectroscopy on the BEC using 100 ms of weak output coupling at varying detunings, and measuring the number of atoms remaining in the condensate after the output coupling time [3]. A typical calibration curve is shown in Fig. 3 (b), in this case for an $F = 2$ condensate and for RF outcoupling. The arrow indicates the point at which we operate both RF and Raman output couplers. We further check this frequency by ensuring that a continuous beam can still be produced when the initial condensate is very small, which can only happen when outcoupling from the center.

We observe the system using standard absorption

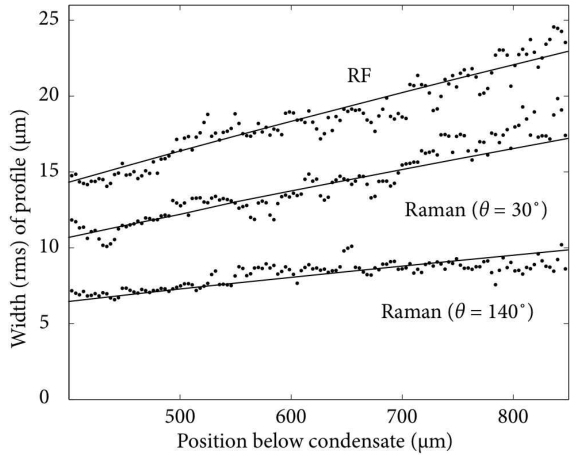

FIG. 4: The rms beam width for RF and Raman atom lasers. The dots represent experimental measurements and the solid curves our theoretical predictions.

imaging on the $F = 2 \rightarrow F' = 3$ transition, with a 200 μs pulse of repumping light ($F = 1 \rightarrow F' = 2$) along the z-direction, 1 ms prior to imaging. To calculate the beam quality factor M^2, we extract the rms width of each atom laser beam, as a function of fall distance, from both our experimental results and theoretical results, as shown in Fig. 4. Technical details, predominantly the condensate expansion during trap switch off, prevent the observation of the beam width less than 400 μm below the condensate. From this we calculate the transverse velocity spread Δv_x using $(\Delta x(t))^2 = (\Delta x_0)^2 + (\Delta v_x)^2 t^2$, where Δx_0 is the beam width at the waist (equal to the condensate width).

To model the system, we use a two-step method following [10]. Inside the condensate, we use the WKB approximation, by integrating the phase along the classical trajectories of atoms moving in the Thomas-Fermi potential of the condensate (an inverted paraboloid) [12]. After this, we propagate the atom laser wavefunction using a Kirchoff-Fresnel diffraction integral over the surface of the condensate:

$$\psi(\mathbf{r}) = \int_S d\mathbf{S}' \cdot [G\,\nabla'\,\psi - \psi\,\nabla'\,G], \tag{2}$$

where $G = G(\mathbf{r}, \mathbf{r}')$ is the Green's function for the Hamiltonian in the gravitational potential $V(\mathbf{r}) = -mgz$ [17]. Therefore, the model includes only interactions between condensate atoms and beam atoms; interactions between atoms within the beam are ignored. The integral in Eq. (2) is formally a two dimensional surface integral over the whole condensate. However for simplicity, following [10], we neglect divergence in the weak trapping axis and only consider cross sections in the plane of the strong trapping axes, and so the integral becomes one dimensional. A 3D wavefunction is built up by calculating the atom laser in a series of planes along the weak trapping axis.

The atom laser state is unaffected to first order by the magnetic field, but is weakly anti-trapped due to the second order Zeeman effect, with a trapping frequency of $\omega_{\text{2nd}} = 2\pi \times 2.6$ Hz. The transverse position of an atom in such a potential is

$$x(t) = x_0 \cosh(\omega_{\text{2nd}} t) \approx x_0(1 + \omega_{\text{2nd}}^2 t^2/2). \tag{3}$$

For the 1 mm propagation we consider here the transverse position is affected by less than 3 %, and we therefore neglect the second order Zeeman shift in our model.

We have checked the validity of this model against a solution of the full 3D Gross-Pitaesvskii (GP) equation, including beam-beam interactions. To find the atom laser wavefunction at large distances below the condensate (up to 1 mm), we transfer the GP model to a freely falling frame once the atom laser wavefunction has reached steady state. (A detailed description of this method will be the subject of a future publication.) The two models give excellent agreement.

We calculate the theoretical M^2 of the atom laser using the formalism above. We find that as the kick increases, the beam quality improves, and that the quality asymptotes to that of a beam that does not interact with the condensate, so the transverse velocity spread of the atom laser is equal to that of the condensate. We calculate this limit to be $M^2 = 1.3$ for our conditions, using the condensate wavefunction which is calculated numerically. We check this by propagating the atom laser using the formalism presented above, using a kick sufficiently large that the mean field component of the transverse velocity spread is negligible. This limit of $M^2 = 1.3$ is greater than the Heisenberg limit $M^2 = 1$ because the BEC, which is the source of the atom laser, is not a minimum uncertainty state due to the repulsive condensate-self interaction.

We also present a simple and intuitive model to calculate the (mean field induced) transverse velocity spread $(\Delta v_x)_{\text{MF}}$ of a Raman atom laser. As above, we neglect divergence along the weak trapping axis, and so we consider the atom laser in cross section, in the plane of the two strong trapping axes. This model is valid in the regime where the velocity kick is large, specifically when $v_{z0} \gg \omega R$, where R is the Thomas-Fermi radius, and in the regime where the condensate sag is large, $z_{\text{sag}} \gg R$. The second condition allows us to approximate the outcoupling surface as a horizontal plane through the center of the condensate, (that is, the initial positions are $x_0 \in [-R, +R]$ and $z_0 = 0$).

Classically, an atom outcoupled at initial position x_0 has a transverse velocity after leaving the condensate:

$$v_x(x_0) = x_0\,\omega \sinh(\omega\, t_e) \approx x_0 \omega^2 t_e, \tag{4}$$

where t_e is the time for an atom to leave the condensate (the escape time). For a large kick, atoms travel in almost straight lines along the kick direction, so the escape time

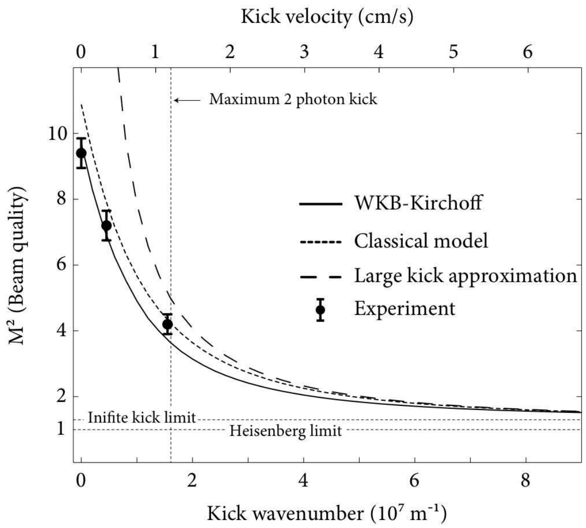

FIG. 5: Measured quality factor M^2 of the atom laser compared to theoretical models. The three theoretical curves show M^2 calculated by the WKB and Kirchoff integral technique (solid curve), and two classical models, one in which the mean field component of the transverse velocity spread $(\Delta v_x)_{\rm MF}$ was calculated numerically (dotted curve), and one from the result given in Eq. (6) (dashed curve). The number of atoms in the calculation was set to $N = 5 \times 10^5$. The uncertainty in the measured M^2 values is calculated from the statistical variation in each beam.

is the vertical distance to the condensate edge, divided by the kick magnitude, $t_e \approx \sqrt{(R^2 - x_0^2)}/v_{z0}$. The mean field transverse velocity is calculated:

$$(\Delta v_x)^2_{\rm MF} = \int_{-R}^{R} v_x(x_0)^2 n(x_0) dx_0, \tag{5}$$

where $n(x_0) = \frac{3}{4R}(1 - \frac{x_0^2}{R^2})$ is the linear number density across the out-coupling surface in the Thomas-Fermi regime. The transverse velocity spread due to the mean field is found to be:

$$(\Delta v_x)_{\rm MF} = \frac{2}{\sqrt{35}} \frac{\omega^2 R^2}{v_{z0}} \approx 0.3 \frac{\omega^2 R^2}{v_{z0}} = 0.6 \frac{\mu}{m} \frac{1}{v_{z0}}. \tag{6}$$

In Fig. 5 we show the measured M^2 of our atom laser. The theoretical curves include an M^2 calculated from the WKB and Kirchoff formalism, and two from classical trajectories: one numerical and one using the approximation given in Eq. (6). The classical trajectories calculations can only find the mean field component of the transverse velocity spread; we add it in quadrature to the calculated velocity spread of the atom laser on the outcoupling surface. We find excellent agreement between theory and experiment.

Our calculations show that the mean field component of the transverse velocity spread is 2.8 times the velocity spread of the condensate. This is for the maximum momentum transfer we can achieve using a two photon kick. It should be possible for the beam quality to reach the condensate limit, either by reducing the trapping frequency, directing the photon kick towards the BEC center using highly focused Raman beams, or using higher order Raman transitions [18].

In conclusion, we have measured the M^2 beam quality parameter of a continuously outcoupled Raman atom laser, and shown that quality is more than twice that than of an RF atom laser. We have presented two theoretical models, a WKB approximation / Kirchoff integral model, and a simple model based on classical trajectories, and shown them to be in good agreement with experimental results. These models indicate that the limit of large kick is equivalent to turning off the mean field interaction between the atom laser beam and the condensate, so that the atom laser beam divergence is only due to the initial condensate wavefunction. The M^2 value of our current atom laser beam is 4.2 ± 0.4, within a factor of three of the limit of $M^2 = 1.3$ for our parameters. We have shown that it should be possible to achieve an atom laser at this limit.

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