

Strong Phase and $D^0 - \bar{D}^0$ mixing at BES-III

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Most recently, both BaBar and Belle experiments found evidences of neutral D mixing. In this paper, we discuss the constraints on the strong phase difference in $D^0 \rightarrow K\pi$ decay from the measurements of the mixing parameters, y' , y_{CP} and x at the B factories. With CP tag technique at $\psi(3770)$ peak, the extraction of the strong phase difference at BES-III are discussed. The sensitivity of the measurement of the mixing parameter y is estimated in BES-III experiment at $\psi(3770)$ peak. Finally, we also make an estimate on the measurements of the mixing rate R_M .

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Due to the smallness of $\Delta C = 0$ amplitude in the Standard Model (SM), $D^0 - \bar{D}^0$ mixing offers a unique opportunity to probe flavor-changing interactions which may be generated by new physics. The recent measurements from BaBar and Belle experiments indicate that the $D^0 - \bar{D}^0$ mixing may exist [1, 2]. At the B factories, the decay time information can be used to extract the neutral D mixing parameters. At $t = 0$ the only term in the amplitude is the direct doubly-Cabibbo-suppressed (DCS) mode $D^0 \rightarrow K^+\pi^-$, but for $t > 0$ $D^0 - \bar{D}^0$ mixing may contribute through the sequence $D^0 \rightarrow \bar{D}^0 \rightarrow K^+\pi^-$, where the second stage is Cabibbo favored (CF). The interference of this term with the DCS contribution involves the lifetime and mass differences of the neutral D mass eigenstates, as well as the final-state strong phase difference $\delta_{K\pi}$ between the CF and the DCS decay amplitudes. This interference plays a key role in the measurement of the mixing parameters at time-dependent measurements.

With the assumption of CPT invariance, the mass eigenstates of $D^0 - \bar{D}^0$ system are $|D_1\rangle = p|D^0\rangle + q|\bar{D}^0\rangle$ and $|D_2\rangle = p|D^0\rangle - q|\bar{D}^0\rangle$ with eigenvalues $\mu_1 = m_1 - \frac{i}{2}\Gamma_1$ and $\mu_2 = m_2 - \frac{i}{2}\Gamma_2$, respectively, where the m_1 and Γ_1 (m_2 and Γ_2) are the mass and width of D_1 (D_2). For the method of detecting $D^0 - \bar{D}^0$ mixing involving the $D^0 \rightarrow K\pi$ decay mentioned above, in order to separate the DCS decay from the mixing signal, one must study the time-dependent decay rate. The proper-time evolution of the particle states $|D_{\text{phys}}^0(t)\rangle$ and $|\bar{D}_{\text{phys}}^0(t)\rangle$ are given by

$$\begin{aligned} |D_{\text{phys}}^0(t)\rangle &= g_+(t)|D^0\rangle - \frac{q}{p}g_-(t)|\bar{D}^0\rangle, \\ |\bar{D}_{\text{phys}}^0(t)\rangle &= g_+(t)|\bar{D}^0\rangle - \frac{p}{q}g_-(t)|D^0\rangle, \end{aligned} \quad (1)$$

where

$$g_{\pm} = \frac{1}{2}(e^{-im_2t - \frac{1}{2}\Gamma_2t} \pm e^{-im_1t - \frac{1}{2}\Gamma_1t}), \quad (2)$$

with definitions

$$\begin{aligned} m &\equiv \frac{m_1 + m_2}{2}, \quad \Delta m \equiv m_2 - m_1, \\ \Gamma &\equiv \frac{\Gamma_1 + \Gamma_2}{2}, \quad \Delta\Gamma \equiv \Gamma_2 - \Gamma_1, \end{aligned} \quad (3)$$

Note the sign of Δm and $\Delta\Gamma$ is to be determined by experiments.

In practice, one define the following mixing parameters

$$x \equiv \frac{\Delta m}{\Gamma}, \quad y \equiv \frac{\Delta\Gamma}{2\Gamma}. \quad (4)$$

The time-dependent decay amplitudes for $D_{\text{phys}}^0(t) \rightarrow K^+\pi^-$ and $\bar{D}_{\text{phys}}^0(t) \rightarrow K^-\pi^+$ are described as

$$\begin{aligned} \langle K^+\pi^- | \mathcal{H} | D_{\text{phys}}^0(t) \rangle &= g_+(t)A_{K^+\pi^-} - \frac{q}{p}g_-(t)\bar{A}_{K^+\pi^-} \\ &= \frac{q}{p}\bar{A}_{K^+\pi^-}[\lambda g_+(t) - g_-(t)], \end{aligned} \quad (5)$$

$$\begin{aligned} \langle K^-\pi^+ | \mathcal{H} | \bar{D}_{\text{phys}}^0(t) \rangle &= g_+(t)\bar{A}_{K^-\pi^+} - \frac{p}{q}g_-(t)A_{K^-\pi^+} \\ &= \frac{p}{q}A_{K^-\pi^+}[\bar{\lambda}g_+(t) - g_-(t)], \end{aligned} \quad (6)$$

where $A_{K^+\pi^-} = \langle K^+\pi^- | \mathcal{H} | D^0 \rangle$, $\bar{A}_{K^+\pi^-} = \langle K^+\pi^- | \mathcal{H} | \bar{D}^0 \rangle$, $A_{K^-\pi^+} = \langle K^-\pi^+ | \mathcal{H} | D^0 \rangle$, and $\bar{A}_{K^-\pi^+} = \langle K^-\pi^+ | \mathcal{H} | \bar{D}^0 \rangle$. Here, λ and $\bar{\lambda}$ are defined as:

$$\lambda \equiv \frac{p}{q} \frac{A_{K^+\pi^-}}{\bar{A}_{K^+\pi^-}}, \quad (7)$$

$$\bar{\lambda} \equiv \frac{q}{p} \frac{\bar{A}_{K^-\pi^+}}{A_{K^-\pi^+}}. \quad (8)$$

From Eqs. (5) and (6), one can derive the general expression for the time-dependent decay rate, in agreement

with [3, 4]:

$$\begin{aligned} \frac{d\Gamma(D_{\text{phys}}^0(t) \rightarrow K^+\pi^-)}{dt\mathcal{N}} &= |\bar{A}_{K^+\pi^-}|^2 \left| \frac{q}{p} \right|^2 e^{-\Gamma|t|} \times \\ &[(|\lambda|^2 + 1)\cosh(y\Gamma t) + \\ &(|\lambda|^2 - 1)\cos(x\Gamma t) + \\ &2\mathcal{R}e(\lambda)\sinh(y\Gamma t) + \\ &2\mathcal{I}m(\lambda)\sin(x\Gamma t)] \end{aligned} \quad (9)$$

$$\begin{aligned} \frac{d\Gamma(\bar{D}_{\text{phys}}^0(t) \rightarrow K^-\pi^+)}{dt\mathcal{N}} &= |A_{K^-\pi^+}|^2 \left| \frac{p}{q} \right|^2 e^{-\Gamma|t|} \times \\ &[(|\bar{\lambda}|^2 + 1)\cosh(y\Gamma t) + \\ &(|\bar{\lambda}|^2 - 1)\cos(x\Gamma t) + \\ &2\mathcal{R}e(\bar{\lambda})\sinh(y\Gamma t) + \\ &2\mathcal{I}m(\bar{\lambda})\sin(x\Gamma t)] \end{aligned} \quad (10)$$

where \mathcal{N} is a common normalization factor. In order to simplify the above formula, we make the following definition:

$$\frac{q}{p} \equiv (1 + A_M)e^{-i\beta}, \quad (11)$$

where β is the weak phase in mixing and A_M is a real-valued parameter which indicates the magnitude of CP violation in the mixing. For $f = K^-\pi^+$ final state, we define

$$\frac{A_{K^+\pi^-}}{\bar{A}_{K^+\pi^-}} \equiv -\sqrt{r'}e^{-i\alpha'}; \quad \frac{\bar{A}_{K^-\pi^+}}{A_{K^-\pi^+}} \equiv -\sqrt{r}e^{-i\alpha}, \quad (12)$$

where r' and α' (r and α) are the ratio and relative phase of the DCS decay rate and the CF decay rate. Then, λ and $\bar{\lambda}$ can be parameterized as

$$\lambda = -\sqrt{r'} \frac{1}{1 + A_M} e^{-i(\alpha' - \beta)}, \quad (13)$$

$$\bar{\lambda} = -\sqrt{r}(1 + A_M)e^{-i(\alpha + \beta)}. \quad (14)$$

In order to demonstrate the CP violation in decay, we define $\sqrt{r'} \equiv \sqrt{R_D}(1 + A_D)$ and $\sqrt{r} \equiv \sqrt{R_D} \frac{1}{1 + A_D}$. Thus, Eqs. (13) and (14) can be expressed as

$$\lambda = -\sqrt{R_D} \frac{1 + A_D}{1 + A_M} e^{-i(\delta - \phi)}, \quad (15)$$

$$\bar{\lambda} = -\sqrt{R_D} \frac{1 + A_M}{1 + A_D} e^{-i(\delta + \phi)}, \quad (16)$$

where $\delta = \frac{\alpha' + \alpha}{2}$ is the averaged phase difference between DCS and CF processes, and $\phi = \frac{\alpha - \alpha'}{2} + \beta$.

We can characterize the CP violation in the mixing amplitude, the decay amplitude, and the interference between amplitudes with and without mixing, by real-valued parameters A_M , A_D , and ϕ as in Ref [5, 6]. In the limit of CP conservation, A_M , A_D and ϕ are all zero. $A_M = 0$ means no CP violation in mixing, namely, $|q/p| = 1$; $A_D = 0$ means no CP violation in decay, for this case, $r = r' = R_D = |\bar{A}_{K^-\pi^+}/A_{K^-\pi^+}|^2 = |A_{K^+\pi^-}/\bar{A}_{K^+\pi^-}|^2$; $\phi = 0$ means no CP violation in the interference between decay and mixing.

In experimental searches, one can define CF decay as right-sign (RS) and DCS decay or via mixing followed by a CF decay as wrong-sign (WS). Here, we define the ratio of WS to RS decays as for D^0 :

$$R(t) = \frac{d\Gamma(D_{\text{phys}}^0(t) \rightarrow K^+\pi^-)}{dt\mathcal{N} \times e^{-\Gamma|t|} \times 2|\bar{A}_{K^+\pi^-}|^2}, \quad (17)$$

and for \bar{D}^0 :

$$\bar{R}(t) = \frac{d\Gamma(\bar{D}_{\text{phys}}^0(t) \rightarrow K^-\pi^+)}{dt\mathcal{N} \times e^{-\Gamma|t|} \times 2|A_{K^-\pi^+}|^2}, \quad (18)$$

Taking into account that $\lambda, \bar{\lambda} \ll 1$ and $x, y \ll 1$, keeping terms up to order x^2, y^2 and R_D in the expressions, neglecting CP violation in mixing, decay and the interference between decay with and without mixing ($A_M = 0$, $A_D = 0$, and $\phi = 0$), expanding the time-dependent for $xt, yt \lesssim \Gamma^{-1}$, combining Eqs. (9) and (10), we can write Eqs. (17) and (18) as

$$R(t) = \bar{R}(t) = R_D + \sqrt{R_D}y'\Gamma t + \frac{x'^2 + y'^2}{4}(\Gamma t)^2, \quad (19)$$

where

$$x' = x\cos\delta + y\sin\delta, \quad (20)$$

$$y' = -x\sin\delta + y\cos\delta. \quad (21)$$

In the limit of SU(3) symmetry, $A_{K^+\pi^-}$ and $\bar{A}_{K^+\pi^-}$ ($A_{K^-\pi^+}$ and $\bar{A}_{K^-\pi^+}$) are simply related by CKM factors, $A_{K^+\pi^-} = (V_{cd}V_{cs}^*/V_{cs}V_{ud}^*)\bar{A}_{K^+\pi^-}$ [7]. In particular, $A_{K^+\pi^-}$ and $\bar{A}_{K^+\pi^-}$ have the same strong phase, leading to $\alpha' = \alpha = 0$ in Eq. (12). But the SU(3) symmetry is broken according to the recent precise measurements from the B factories, the ratio [5]:

$$\mathcal{R} = \frac{\mathcal{BR}(D^0 \rightarrow K^+\pi^-)}{\mathcal{BR}(\bar{D}^0 \rightarrow K^+\pi^-)} \left| \frac{V_{ud}V_{cs}^*}{V_{us}V_{cd}^*} \right|^2, \quad (22)$$

is unity in the SU(3) symmetry limit. But, the world average for this ratio is

$$\mathcal{R}_{exp} = 1.21 \pm 0.03, \quad (23)$$

computed from the individual measurements using the standard method of Ref. [4]. Since the SU(3) is broken in $D \rightarrow K\pi$ decays at the level of 20%, in which

case the strong phase δ should be non-zero. Recently, a time-dependent analysis in $D \rightarrow K\pi$ has been performed based on 384 fb^{-1} luminosity at $\Upsilon(4S)$ [1]. By assuming CP conservation, they obtained the following neutral D mixing results

$$\begin{aligned} R_D &= (3.03 \pm 0.16 \pm 0.10) \times 10^{-3}, \\ x'^2 &= (-0.22 \pm 0.30 \pm 0.21) \times 10^{-3}, \\ y' &= (9.7 \pm 4.4 \pm 3.1) \times 10^{-3}. \end{aligned} \quad (24)$$

TABLE I: Experimental results used in the paper. Only one error is quoted, we have combined in quadrature statistical and systematic contributions.

Parameter	BaBar ($\times 10^{-3}$)	Belle ($\times 10^{-3}$)	Technique
x'^2	-0.22 ± 0.37 [1]	$0.18^{+0.21}_{-0.23}$ [8]	$K\pi$
y'	9.7 ± 5.4 [1]	$0.6^{+4.0}_{-3.9}$ [8]	$K\pi$
R_D	3.03 ± 0.19 [1]	3.64 ± 0.17 [8]	$K\pi$
y_{CP}	-	13.1 ± 4.1 [2]	$K^+K^-, \pi^+\pi^-$
x	-	8.0 ± 3.4 [9]	$K_S\pi^+\pi^-$
y	-	3.3 ± 2.8 [9]	$K_S\pi^+\pi^-$

The result is inconsistent with the no-mixing hypothesis with a significance of 3.9 standard deviations. The results from BaBar and Belle are in agreement within 2 standard deviation on the exact analysis of y' measurement by using $D \rightarrow K\pi$ as listed in Table I. As indicated in Eq. (23), the strong phase δ should be non-zero due to the SU(3) violation. One has to know the strong phase difference exactly in order to extract the direct mixing parameters, x and y as defined in Eqs. (4). However, at the B factory, it is hard to do that with a model-independent way [7, 10]. In order to extract the strong phase δ we need data near the $D\bar{D}$ threshold to do a CP tag as discussed in Ref. [7]. Here, we would like to figure out the possible physics solution of the strong phase δ by using the recent results from the B factories with different decay modes, so that we can have an idea about the sensitivity to measure the strong phase at the BES-III project.

In Ref [2], Belle collaboration also reported the result of $y_{CP} = \frac{\tau(D^0 \rightarrow K^+\pi^-)}{\tau(D^0 \rightarrow f_{CP})} - 1$, where $f_{CP} = K^+K^-$ and $\pi^+\pi^-$

$$y_{CP} = (13.1 \pm 3.2 \pm 2.5) \times 10^{-3}. \quad (25)$$

The result is about 3.2σ significant deviation from zero (non-mixing). In the limit of CP symmetry, $y_{CP} = y$ [11, 12]. In the decay of $D^0 \rightarrow K_S\pi^+\pi^-$, Belle experiment has done a Dalitz plot (DP) analysis [9], they obtained the direct mixing parameters x and y as

$$x = (8.0 \pm 3.4) \times 10^{-3}, \quad y = (3.3 \pm 2.8) \times 10^{-3}, \quad (26)$$

where the error includes both statistic and systematic uncertainties. Since the parameterizations of the resonances

on the DP are model-dependent, the results suffer from large uncertainties from the DP model. In this analysis, they see a significance of 2.4 standard deviations from non-mixing. Here, we will use the value of x measured in the DP analysis for further discussion. As shown in Eq. (21), once y , y' and x are known, it is straightforward to extract the strong phase difference between DCS and CF decay in $D^0 \rightarrow K\pi$ decay. If taking the measured central values of x , $y_{CP}(\approx y)$, and y' as input parameters, we found two-fold solutions for $\tan\delta$ as below:

$$\tan\delta = 0.35 \pm 0.63, \text{ or } -7.14 \pm 29.13, \quad (27)$$

which are corresponding to $(19 \pm 32)^\circ$ and $(-82^\circ \pm 30^\circ)$, respectively.

At $\psi(3770)$ peak, to extract the mixing parameter y , one can make use of rates for exclusive $D^0\bar{D}^0$ combination, where both the D^0 final states are specified (known as double tags or DT), as well as inclusive rates, where either the D^0 or \bar{D}^0 is identified and the other D^0 decays generically (known as single tags or ST) [13]. With the DT tag technique [14, 15], one can fully consider the quantum correlation in $C = -1$ and $C = +1$ $D^0\bar{D}^0$ pairs produced in the reaction $e^+e^- \rightarrow D^0\bar{D}^0(n\pi^0)$ and $e^+e^- \rightarrow D^0\bar{D}^0\gamma(n\pi^0)$ [13, 16, 17], respectively.

For the ST, in the limit of CP conservation, the rate of D^0 decays into a CP eigenstate is given as [13]:

$$\Gamma_{f_\eta} \equiv \Gamma(D^0 \rightarrow f_\eta) = 2A_{f_\eta}^2 [1 - \eta y], \quad (28)$$

where f_η is a CP eigenstate with eigenvalue $\eta = \pm 1$, and $A_{f_\eta} = |\langle f_\eta | \mathcal{H} | D^0 \rangle|$ is the real-valued decay amplitude.

For the DT case, Gronau *et. al.* [7] and Xing [18] have considered time-integrated decays into correlated pairs of states, including the effects of non-zero final state phase difference. As discussed in Ref. [7], the rate of $(D^0\bar{D}^0)^{C=-1} \rightarrow (l^\pm X)(f_\eta)$ is described as [7]:

$$\begin{aligned} \Gamma_{l;f_\eta} \equiv \Gamma[(l^\pm X)(f_\eta)] &= A_{l^\pm X}^2 A_{f_\eta}^2 (1 + y^2) \\ &\approx A_{l^\pm X}^2 A_{f_\eta}^2, \end{aligned} \quad (29)$$

where $A_{l^\pm X} = |\langle l^\pm X | \mathcal{H} | D^0 \rangle|$, here, we neglect y^2 term since $y \ll 1$.

For $C = -1$ initial $D^0\bar{D}^0$ state, y can be expressed in term of the ratios of DT rates and the double ratios of ST rates to DT rates [13]:

$$y = \frac{1}{4} \left(\frac{\Gamma_{l;f_+} \Gamma_{f_-}}{\Gamma_{l;f_-} \Gamma_{f_+}} - \frac{\Gamma_{l;f_-} \Gamma_{f_+}}{\Gamma_{l;f_+} \Gamma_{f_-}} \right). \quad (30)$$

For a small y , its error, $\Delta(y)$, is approximately $1/\sqrt{N_{l^\pm X}}$, where $N_{l^\pm X}$ is the total number of $(l^\pm X)$ events tagged with CP -even and CP -odd eigenstates. The number $N_{l^\pm X}$ of CP tagged events is related to the total number of $D^0\bar{D}^0$ pairs $N(D^0\bar{D}^0)$ through $N_{l^\pm X} \approx N(D^0\bar{D}^0)[\mathcal{BR}(D^0 \rightarrow l^\pm + X) \times \mathcal{BR}(D^0 \rightarrow f_\pm) \times \epsilon_{tag}] \approx 1.5 \times 10^{-3} N(D^0\bar{D}^0)$, here we take the branching ratios-efficiency factor $(\mathcal{BR}(D^0 \rightarrow f_\pm) \times \epsilon_{tag})$ for tagging

CP eigenstates is about 1.1% (the total branching ratio into CP eigenstates is larger than about 5% [4]). We find

$$\Delta(y) = \frac{\pm 26}{\sqrt{N(D^0 \bar{D}^0)}} = \pm 0.003. \quad (31)$$

If we take the central value of y from the measurement of y_{CP} at Belle experiment [2], thus, at BES-III experiment [19], with $20fb^{-1}$ data at $\psi(3770)$ peak, the significance of the measurement of y could be around 4.3σ deviation from zero.

We can also take advantage of the coherence of the D^0 mesons produced at the $\psi(3770)$ peak to extract the strong phase difference δ between DCS and CF decay amplitudes that appears in the time-dependent mixing measurement in Eq. (19) [7, 13]. Because the CP properties of the final states produced in the decay of the $\psi(3770)$ are anti-correlated [16, 17], one D^0 state decaying into a final state with definite CP properties immediately identifies or tags the CP properties of the other side. As discussed in Ref. [7], the process of one D^0 decaying to $K^-\pi^+$, while the other D^0 decaying to a CP eigenstate f_η can be described as

$$\begin{aligned} \Gamma_{K\pi;f_\eta} &\equiv \Gamma[(K^-\pi^+)(f_\eta)] \approx A^2 A_{f_\eta}^2 |1 + \eta \sqrt{R_D} e^{-i\delta}|^2 \\ &\approx A^2 A_{f_\eta}^2 (1 + 2\eta \sqrt{R_D} \cos\delta), \end{aligned} \quad (32)$$

where $A = |\langle K^-\pi^+ | \mathcal{H} | D^0 \rangle|$ and $A_{f_\eta} = |\langle f_\eta | \mathcal{H} | D^0 \rangle|$ are the real-valued decay amplitudes, and we have neglected the y^2 terms in Eq. (32). In order to estimate the total sample of events needed to perform a useful measurement of δ , one defined [7, 10] an asymmetry

$$\mathcal{A} \equiv \frac{\Gamma_{K\pi;f_+} - \Gamma_{K\pi;f_-}}{\Gamma_{K\pi;f_+} + \Gamma_{K\pi;f_-}}, \quad (33)$$

where $\Gamma_{K\pi;f_\pm}$ is defined in Eq. (32), which is the rates for the $\psi(3770) \rightarrow D^0 \bar{D}^0$ configuration to decay into flavor eigenstates and a CP -eigenstates f_\pm . Eq. (32) implies a small asymmetry, $\mathcal{A} = 2\sqrt{R_D} \cos\delta$. For a small asymmetry, a general result is that its error $\Delta\mathcal{A}$ is approximately $1/\sqrt{N_{K^-\pi^+}}$, where $N_{K^-\pi^+}$ is the total number of events tagged with CP -even and CP -odd eigenstates. Thus one obtained

$$\Delta(\cos\delta) \approx \frac{1}{2\sqrt{R_D} \sqrt{N_{K^-\pi^+}}}. \quad (34)$$

The expected number $N_{K^-\pi^+}$ of CP -tagged events can be connected to the total number of $D^0 \bar{D}^0$ pairs $N(D^0 \bar{D}^0)$ through $N_{K^-\pi^+} \approx N(D^0 \bar{D}^0) \mathcal{BR}(D^0 \rightarrow K^-\pi^+) \times \mathcal{BR}(D^0 \rightarrow f_\pm) \times \epsilon_{tag} \approx 4.2 \times 10^{-4} N(D^0 \bar{D}^0)$ [7], here, as in Ref [7], we take the branching ratio-times-efficiency factor $\mathcal{BR}(D^0 \rightarrow f_\pm) \times \epsilon_{tag} = 1.1\%$. With the measured $R_D = (3.03 \pm 0.19) \times 10^{-3}$ and $\mathcal{BR}(D^0 \rightarrow$

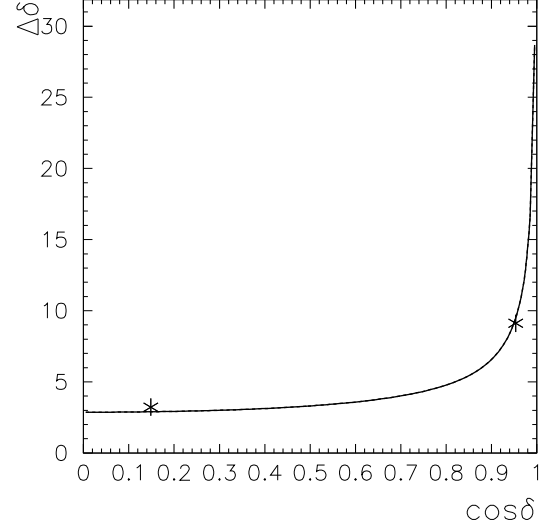


FIG. 1: Illustrative plot of the expected error ($\Delta\delta$) of the strong phase with various central values of $\cos\delta$. The expected error of $\cos\delta$ is 0.04 by ssuming $20fb^{-1}$ data at $\psi(3770)$ peak at BES-III. The two asterisks correspond to $\delta = 19^\circ$ and -82° , respectively.

$K^-\pi^+$) = 3.8% [4], one found [7]

$$\Delta(\cos\delta) \approx \frac{\pm 444}{\sqrt{N(D^0 \bar{D}^0)}}. \quad (35)$$

At BESIII, about 72×10^6 $D^0 \bar{D}^0$ pairs can be collected with 4 years' running. If considering both $K^-\pi^+$ and $K^+\pi^-$ final states, we thus estimate that one may be able to reach an accuracy of about 0.04 for $\cos\delta$. Figure 1 shows the expected error of the strong phase δ with various central values of $\cos\delta$. With the expected $\Delta(\cos\delta) = \pm 0.04$, the sensitivity of the strong phase varies with the physical value of $\cos\delta$. For $\delta = 19^\circ$ and -82° , the expected error could be $\Delta(\delta) = \pm 2.9^\circ$ and $\pm 8.7^\circ$, respectively.

By combing the measurements of x in $D^0 \rightarrow K_S \pi \pi$ and y_{CP} from Belle, one can obtain $R_M = (1.18 \pm 0.6) \times 10^{-4}$. At the $\psi(3770)$ peak, $D^0 \bar{D}^0$ pair are produced in a state that is quantum-mechanically coherent [16, 17]. This enables simple new method to measure D^0 mixing parameters in a way similar proposed in Ref. [7]. At BES-III, the measurement of R_M can be performed unambiguously with the following reactions [16]:

- (i) $e^+e^- \rightarrow \psi(3770) \rightarrow D^0 \bar{D}^0 \rightarrow (K^\pm \pi^\mp)(K^\pm \pi^\mp),$
 - (ii) $e^+e^- \rightarrow \psi(3770) \rightarrow D^0 \bar{D}^0 \rightarrow (K^- e^+ \nu)(K^- e^+ \nu),$
 - (iii) $e^+e^- \rightarrow D^- D^{*+} \rightarrow (K^+ \pi^- \pi^-)(\pi_{soft}^+[K^+ e^- \nu]).$
- (36)

Reaction (i) in Eq. (36) can be normalized to $D^0 \bar{D}^0 \rightarrow$

$(K^-\pi^+)(K^+\pi^-)$, the following time-integrated ratio is obtained by neglecting CP violation:

$$\frac{N[(K^-\pi^+)(K^-\pi^+)]}{N[(K^-\pi^+)(K^+\pi^-)]} \approx \frac{x^2 + y^2}{2} = R_M. \quad (37)$$

For the case of semileptonic decay, as (ii) in Eq. (36), we have

$$\frac{N(l^\pm l^\pm)}{N(l^\pm l^\mp)} = \frac{x^2 + y^2}{2} = R_M, \quad (38)$$

The observation of reaction (i) would be definite evidence for the existence of $D^0 - \bar{D}^0$ mixing since the final state $(K^\pm\pi^\mp)(K^\pm\pi^\mp)$ can not be produced from DCS decay due to quantum statistics [16, 17]. In particular, the initial $D^0\bar{D}^0$ pair is in an odd eigenstate of C which will preclude, in the absence of mixing between the D^0 and \bar{D}^0 over time, the formation of the symmetric state required by Bose statistics if the decays are to be the same final state. This final state is also very appealing experimentally, because it involves a two-body decay of both charm mesons, with energetic charged particles in the final state that form an overconstrained system. Particle identification is crucial in this measurement because if both the kaon and pion are misidentified in one of the two D -meson decays in the event, it becomes impossible to discern whether mixing has occurred. At BESIII, where the data sample is expected to be 20 fb^{-1} integrated luminosity at $\psi(3770)$ peak, the limit will be 10^{-4} at 95% C.L. for R_M , but only if the particle identification capabilities are adequate.

Reactions (ii) and (iii) offer unambiguous evidence for the mixing because the mixing is searched for in the semileptonic decays for which there are no DCS decays. Of course since the time-evolution is not measured, observation of Reactions (ii) and (iii) actually would indicate the violation of the selection rule relating the change in charm to the change in leptonic charge which holds true in the standard model [16].

In Table II, the sensitivity for R_M measurements in different decay modes are estimated with 4 years' run at BEPCII.

TABLE II: The sensitivity for R_M measurements at BES-III with different decay modes with 4 years' run at BESPCII

$D^0\bar{D}^0$ Mixing		
Reaction	Events RS($\times 10^4$)	Sensitivity $R_M(\times 10^{-4})$
$\psi(3770) \rightarrow (K^-\pi^+)(K^-\pi^+)$	10.4	1.0
$\psi(3770) \rightarrow (K^-e^+\nu)(K^-e^+\nu)$	8.9	3.7
$\psi(3770) \rightarrow (K^-e^+\nu)(K^-\mu^+\nu)$	8.1	
$\psi(3770) \rightarrow (K^-\mu^+\nu)(K^-\mu^+\nu)$	7.3	

In the limit of CP conservation, by combing the measurements of x in $D^0 \rightarrow K_S\pi\pi$ and y_{CP} from Belle, one can obtain $R_M = (1.18 \pm 0.6) \times 10^{-4}$. With 20fb^{-1} data

at BES-III, about 12 events for the process $D^0\bar{D}^0 \rightarrow (K^\pm\pi^\mp)(K^\pm\pi^\mp)$ can be produced. One can observe 3.0 events after considering the selection efficiency at BE-III, which could be about 25% for the four charged particles. The background contamination due to double particle misidentification is about 0.6 event with 20fb^{-1} data at BES-III [20]. Table III lists the expected mixing signal for $N_{sig} = N(K^\pm\pi^\mp)(K^\pm\pi^\mp)$, background N_{bkg} , and the Poisson probability $P(n)$, where n is the possible number of observed events in experiment. In Table III, we assume the $R_M = 1.18 \times 10^{-4}$, the expected number of mixing signal events are estimated with 10fb^{-1} and 20fb^{-1} , respectively.

TABLE III: The expected mixing signal for $N_{sig} = N(K^\pm\pi^\mp)(K^\pm\pi^\mp)$, background N_{bkg} , and the Poisson probability $P(n)$ in 10 fb^{-1} and 20 fb^{-1} at BES-III at $\psi(3770)$ peak, respectively. Here, we take the mixing rate $R_M = 1.18 \times 10^{-4}$.

	$10 \text{ fb}^{-1} (\psi(3770))$ 36 million $D^0\bar{D}^0$	$20 \text{ fb}^{-1} (\psi(3770))$ 72 million $D^0\bar{D}^0$
N_{sig}	1.5	3.0
N_{bkg}	0.3	0.6
$P(n=0)$	15.7%	2.5%
$P(n=1)$	29.1%	9.1%
$P(n=2)$	26.9%	16.9%
$P(n=3)$	16.6%	20.9%
$P(n=4)$	7.7%	19.3%
$P(n=5)$	2.8%	14.3%
$P(n=6)$	0.9%	8.8%
$P(n=7)$	0.2%	4.7%
$P(n=8)$	0.1%	2.2%
$P(n=9)$	0.01%	0.9%

In conclusion, we discuss the constraints on the strong phase difference in $D^0 \rightarrow K\pi$ decay according to the most recent measurements of y' , y_{CP} and x from B factories. We estimate the sensitivity of the measurement of mixing parameter y at $\psi(3770)$ peak in BES-III experiment. With 20 fb^{-1} data, the uncertainty $\Delta(y)$ could be 0.003. Thus, assuming y at a percent level, we can make a measurement of y at a significance of 4.3σ deviation from zero. The sensitivity of the strong phase difference at BES-III are obtained by using data near the $D\bar{D}$ threshold with CP tag technique at BES-III experiment. Finally, we estimated the sensitivity of the measurements of the mixing rate R_M , and find that BES-III experiment may not be able to make a significant measurement of R_M with current luminosity by using coherent $D\bar{D}$ state at $\psi(3770)$ peak.

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