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Remarks on N_c dependence of decays of exotic baryons

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We calculate the N_c dependence of the decay widths of exotic eikosiheptaplet within the framework of Chral Quark Soliton Model. We also discuss generalizations of regular baryon representations for arbitrary N_c.

§1. Introduction

One of the most puzzling results of the chiral quark-soliton model (χQSM) for exotic baryons consists in a very small hadronic decay width,[1)] governed by the decay constant $G_{\overline{10}}$. While the small mass of exotic states is rather generic for all chiral models[1)–3)] the smallness of the decay width appears as a subtle cancelation of three different terms that contribute to $G_{\overline{10}}$. Decay width in solitonic models[4)] is calculated in terms of a matrix element $\mathcal{M}$ of the collective axial current operator corresponding to the emission of a pseudoscalar meson φ[1)] – see Ref. 5) for criticism of this approach:

$$\hat{O}^{(8)}_{\varphi} = 3\sum_{i=1}^{3}\left(G_0\, D^{(8)}_{\varphi i} - G_1\, d_{ibc}\, D^{(8)}_{\varphi b}\,\hat{S}_c - \frac{G_2}{\sqrt{3}}\, D^{(8)}_{\varphi 8}\,\hat{S}_i\right) \times p^i_{\varphi}. \tag{1·1}$$

For notation see Ref. 1). Constants $G_{0,1,2}$ are constructed from the so called *moments of inertia* that are calculable in χQSM. The decay width is given as

$$\Gamma_{B\to B'+\varphi} = \frac{1}{8\pi}\frac{p_{\varphi}}{M\,M'}\overline{\mathcal{M}^2} = \frac{1}{8\pi}\frac{p^3_{\varphi}}{M\,M'}\overline{\mathcal{A}^2}. \tag{1·2}$$

The "bar" over the amplitude squared denotes averaging over initial and summing over final spin (and, if explicitly indicated, over isospin).

For $B^{(\overline{10})} \to B'^{(8)} + \varphi$ for spin "up" and $\vec{p}_{\varphi} = (0,0,p_{\varphi})$ we have

$$\mathcal{M} = \left\langle 8_{1/2}, B' \right| \hat{O}^{(8)}_{\varphi} \left| \overline{10}_{1/2}, B \right\rangle = -\frac{3G_{\overline{10}}}{\sqrt{15}}\left(\begin{array}{cc|c} 8 & 8 & \overline{10} \\ \varphi & B' & B \end{array}\right) \times p_{\varphi} \tag{1·3}$$

and

$$G_{\overline{10}} = G_0 - G_1 - \frac{1}{2}G_2. \tag{1·4}$$

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In order to have an estimate of the width (1·2) the authors of Ref. 1) calculated $G_{\overline{10}}$ in the nonrelativistic limit[6] of χQSM and got $G_{\overline{10}} \equiv 0$. It has been shown that this cancelation between terms that scale differently with N_c ($G_0 \sim N_c^{3/2}$, $G_{1,2} \sim N_c^{1/2}$) is in fact consistent with large N_c counting,[7] since

$$G_{\overline{10}} = G_0 - \frac{N_c+1}{4}G_1 - \frac{1}{2}G_2 \tag{1·5}$$

where the N_c dependence comes from the SU(3) Clebsch-Gordan coefficients calculated for large N_c In the nonrelativistic limit (NRL):

$$G_0 = -(N_c+2)\,G, \quad G_1 = -4G, \quad G_2 = -2G, \quad G \sim N_c^{1/2}. \tag{1·6}$$

In this paper we ask whether the similar cancelation takes place for the decays of 27 of spin 1/2 and 3/2. We also discuss the possible modifications of the N_c dependence of the decay width due to the different choice of the large N_c generalizations of regular SU(3) multiplets.

§2. Baryons in large N_c limit

Soliton is usually quantized as quantum mechanical symmetric top with two moments of inertia $I_{1,2}$:

$$M_B^{(\mathcal{R})} = M_{\rm cl} + \frac{1}{2I_1}S(S+1) + \frac{1}{2I_2}\left(C_2(\mathcal{R}) - S(S+1) - \frac{N_c^2}{12}\right) + \delta_B^{(\mathcal{R})}. \tag{2·1}$$

Here S denotes baryon spin, $C_2(\mathcal{R})$ the Casimir operator for the SU(3) representation $\mathcal{R} = (p,q)$:

$$C_2(\mathcal{R}) = \frac{1}{3}\left(p^2 + q^2 + pq + 3(p+q)\right) \tag{2·2}$$

and quantities $\delta_B^{(\mathcal{R})}$ denote matrix elements of the SU(3) breaking hamiltonian:

$$\hat{H}' = \frac{N_c}{3}\sigma + \alpha D_{88}^{(8)}) + \beta Y + \frac{\gamma}{\sqrt{3}}D_{8A}^{(8)}\hat{J}_A. \tag{2·3}$$

Model parameters that can be found in Ref. 8)

$$\alpha = -\frac{N_c}{3}(\sigma+\beta), \quad \beta = -m_s\frac{K_2}{I_2},\ \gamma = 2m_s\left(\frac{K_1}{I_1} - \frac{K_2}{I_2}\right),\ \sigma = \frac{2}{N_c}\frac{m_s}{m_u+m_d}\varSigma_{\pi N}$$

scale with N_c in the following way:

$$i_{1,2} = 3I_{1,2}/N_c \quad \text{where} \quad i_{1,2} \sim \mathcal{O}(N_c^0),\ \sigma,\beta,\gamma \sim \mathcal{O}(m_s N_c^0). \tag{2·4}$$

Here $\varSigma_{\pi N}$ is pion-nucleon sigma term and m_q denote current quark masses. Numerically $\sigma > |\beta|\,, |\gamma|$.

So far we have specified *explicit* N_c dependence (2·4) that follows from the fact that model parameters) are given in terms of the quark loop. Another type of the

N_c dependence comes from the constraint[9] that selects $\mathrm{SU}(3)_{\mathrm{flavor}}$ representations $\mathcal{R} = (p, q)$ containing states with hypercharge $Y_R = N_c/3$. Therefore for arbitrary N_c ordinary baryon representations have to extended and one has to specify which states correspond to the physical ones. Usual choice[10]

$$\text{"}8\text{"} = (1, (N_c - 1)/2)\,,\ \text{"}10\text{"} = (3, (N_c - 3)/2)\,,\ \text{"}\overline{10}\text{"} = (0, (N_c + 3)/2)\,, \qquad (2{\cdot}5)$$

depicted in Fig. 1 corresponds – in the quark language – to the case when each time when N_c is increased by 2, a spin-isospin singlet (but charged) $\overline{3}$ diquark is added, as depicted in Fig. 2.

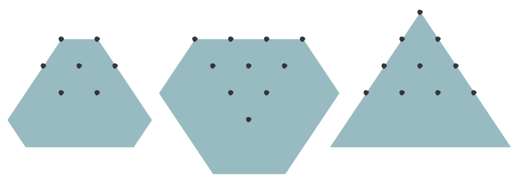

Fig. 1. Standard generalization of SU(3) flavor baryon representations for arbitrary N_c

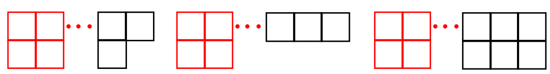

Fig. 2. Adding $\overline{3}$ diquarks to regular SU(3) baryon representations 8, 10 and $\overline{10}$ corresponds to the representation set of Fig.1.

Extension (2·5) leads to (1·5). It implies that mass differences between centers of multiplets scale differently with N_c:

$$\mathit{\Delta}_{10-8} = \frac{3}{2I_1} \sim \mathcal{O}(1/N_c), \quad \mathit{\Delta}_{\overline{10}-8} = \frac{N_c+3}{4I_2} \sim \mathcal{O}(1). \qquad (2{\cdot}6)$$

The fact that $\mathit{\Delta}_{\overline{10}-8} \neq 0$ in large N_c limit, triggered recently discussion on the validity of the semiclassical quantization for exotic states.[11] Since in the chiral limit the momentum p_φ of the outgoing meson scales according to (2·6), overall N_c dependence of the decay width is strongly affected by its third power (1·2):

$$\mathit{\Gamma}_{B\to B'+\varphi} \sim \frac{1}{N_c^2}\mathcal{O}(\overline{\mathcal{A}^2})\mathcal{O}(p_\varphi^3). \qquad (2{\cdot}7)$$

Phenomenologically, however, scaling (2·6) is not sustained. Indeed, meson momenta in $\mathit{\Delta}$ and $\mathit{\Theta}$ decays are almost identical (assuming $M_{\mathit{\Theta}}^{(\overline{10})} \simeq 1540$ MeV):

$$p_\pi \simeq 225 \text{ MeV},\ p_K \simeq 268 \text{ MeV}. \qquad (2{\cdot}8)$$

Unfortunately, going off $\mathrm{SU}(3)_{\mathrm{flavor}}$ limit does not help. Explicitly

$$\delta^{(8)} = \frac{N_c}{3}\sigma + \frac{(N_c-3)}{3}\beta + \frac{(N_c-2)\alpha + \frac{3}{2}\gamma}{N_c+7} + \left(\beta + \frac{3(N_c+2)\alpha - \frac{1}{2}(2N_c+9)\gamma}{(N_c+3)(N_c+7)}\right)Y$$

$$+\frac{(6\alpha+(N_c+6)\gamma)}{(N_c+3)(N_c+7)}\left(\frac{Y^2}{4}-I(I+1)\right)=3\sigma+2\beta-\sigma Y+\ldots \tag{2·9}$$

$$\delta^{(10)}=\frac{N_c}{3}\sigma+\frac{(N_c-3)(N_c+4)}{(N_c+1)(N_c+9)}\alpha+\frac{N_c-3}{3}\beta+\frac{5(N_c-3)}{2(N_c+1)(N_c+9)}\gamma$$
$$+\left(\beta+\frac{3(N_c-1)\alpha-\frac{5}{2}(N_c+3)\gamma}{(N_c+1)(N_c+9)}\right)Y=3\sigma+2\beta-\sigma Y+\ldots \tag{2·10}$$

$$\delta^{(\overline{10})}=\frac{N_c}{3}\sigma+\frac{N_c(N_c-3)}{(N_c+3)(N_c+9)}\alpha+\frac{N_c-3}{3}\beta-\frac{3(N_c-3)}{2(N_c+3)(N_c+9)}\gamma$$
$$+\left(\beta+\frac{6N_c\alpha-9\gamma}{2(N_c+3)(N_c+9)}\right)Y=5\sigma+4\beta-\sigma Y+\ldots \tag{2·11}$$

where $\ldots$ denote terms $\mathcal{O}(1/N_c)$, Y and I denote *physical* hypercharge and isospin.

Interestingly in all cases in the large N_c limt, m_s splittings are proportional to the hypercharge differences only. In this limit $\Sigma-\Lambda$ splitting in the octet is zero and this degeneracy is lifted in the next order at $\mathcal{O}(1/N_c)$. This explains the smallness of $\Sigma-\Lambda$ mass difference. Additionally $\delta_N^{(8)}\simeq\delta_\Delta^{(10)}$ up to higher order terms $\mathcal{O}(1/N_c^2)$, however $\delta_\Theta^{(\overline{10})}-\delta_N^{(8)}\simeq\sigma+2\beta>0$. This implies that

$$M_\Theta^{(\overline{10})}-M_N^{(8)}=\frac{3}{2I_2}-\frac{1}{20}\alpha+\beta-\frac{3}{40}\gamma\rightarrow\frac{3}{4i_2}+\sigma+2\beta+\mathcal{O}(1/N_c),$$
$$M_\Delta^{(10)}-M_N^{(8)}=\frac{3}{2I_1}-\frac{7}{40}\alpha-\frac{21}{80}\gamma\rightarrow\mathcal{O}(1/N_c). \tag{2·12}$$

The first equation shows that the $\Theta-N\neq 0$ in the large N_c limit even if m_s corrections are included. We will come back to this problem in the last section.

§3. Decay constants of twentysevenplet for large N_c

In this section we shall consider decays of eikosiheptaplet (27-plet)

$$\text{"27"}=(2,(N_c+1)/2) \tag{3·1}$$

that can have either spin 1/2 or 3/2, the latter being lighter. Mass differences read

$$\Delta_{27_{3/2}-8}=\frac{3}{2I_1}+\frac{N_c+1}{4I_2}\sim\mathcal{O}(1),\quad \Delta_{27_{1/2}-8}=\frac{N_c+7}{4I_2}\sim\mathcal{O}(1),$$
$$\Delta_{27_{3/2}-10}=\frac{N_c+1}{4I_2}\sim\mathcal{O}(1),\qquad \Delta_{27_{1/2}-10}=-\frac{3}{2I_1}+\frac{N_c+7}{4I_2}\sim\mathcal{O}(1),$$
$$\Delta_{27_{3/2}-\overline{10}}=\frac{3}{2I_1}-\frac{1}{2I_2}\sim\mathcal{O}(1/N_c),\quad \Delta_{27_{1/2}-\overline{10}}=\frac{1}{I_2}\sim\mathcal{O}(1/N_c). \tag{3·2}$$

Matrix elements for the decays of eikosiheptaplet (with $S_3=1/2$) read:

$$\mathcal{A}(B_{27_{3/2}}\rightarrow B_8'+\varphi)=3\left(\begin{array}{cc|c} 8 & \text{"8"} & \text{"27"} \\ \varphi & B' & B \end{array}\right)\sqrt{\frac{8(N_c+5)}{9(N_c+3)(N_c+9)}}\times G_{27},$$

$$\mathcal{A}(B_{27_{3/2}} \to B'_{10} + \varphi) = -3 \left(\begin{array}{cc|c} 8 & "10" & "27" \\ \varphi & B' & B \end{array} \right) \sqrt{\frac{(N_c-1)(N_c+7)}{9(N_c+1)(N_c+3)(N_c+9)}} \times F_{27},$$

$$\mathcal{A}(B_{27_{3/2}} \to B'_{\overline{10}} + \varphi) = 3 \left(\begin{array}{cc|c} 8 & "\overline{10}" & "27" \\ \varphi & B' & B \end{array} \right) \sqrt{\frac{2(N_c+1)(N_c+7)}{3(N_c+3)(N_c+9)}} \times E_{27}, \tag{3·3}$$

and

Decay	Large N_c NRL		Scaling in NRL
$27_{3/2} \to 8_{1/2}$	$G_{27} = G_0 - \frac{N_c-1}{4} G_1$	$= -3G$	$N_c^{1/2}$
$27_{3/2} \to 10_{3/2}$	$F_{27} = G_0 - \frac{N_c-1}{4} G_1 - \frac{3}{2} G_2$	$= 0$	0
$27_{3/2} \to \overline{10}_{1/2}$	$E_{27} = G_0 + G_1$	$= -(N_c+6)G$	$N_c^{3/2}$

For $S = 1/2$ and $S_3 = 1/2$ we have:

$$\mathcal{A}(B_{27_{1/2}} \to B'_{8} + \varphi) = -3 \left(\begin{array}{cc|c} 8 & "8" & "27" \\ \varphi & B' & B \end{array} \right) \sqrt{\frac{(N_c+1)(N_c+5)}{9(N_c+3)(N_c+7)(N_c+9)}} \times H_{27},$$

$$\mathcal{A}(B_{27_{1/2}} \to B'_{10} + \varphi) = -3 \left(\begin{array}{cc|c} 8 & "10" & "27" \\ \varphi & B' & B \end{array} \right) \sqrt{\frac{8(N_c-1)}{9(N_c+3)(N_c+9)}} \times G'_{27},$$

$$\mathcal{A}(B_{27_{1/2}} \to B'_{\overline{10}} + \varphi) = 3 \left(\begin{array}{cc|c} 8 & "\overline{10}" & "27" \\ \varphi & B' & B \end{array} \right) \frac{N_c+4}{\sqrt{9(N_c+3)(N_c+9)}} \times H'_{27}, \tag{3·4}$$

Decay	Large N_c NRL		Scaling in NRL
$27_{1/2} \to 8_{1/2}$	$H_{27} = G_0 - \frac{N_c+5}{4} G_1 + \frac{3}{2} G_2$	$= 0$	0
$27_{1/2} \to 10_{3/2}$	$G'_{27} = G_0 - \frac{N_c+5}{4} G_1$	$= 3G$	$N_c^{1/2}$
$27_{1/2} \to \overline{10}_{1/2}$	$H'_{27} = G_0 + \frac{2N_c+5}{2N_c+8} G_1 + \frac{3}{2N_c+8} G_2$	$= -\frac{(N_c+3)(N_c+7)}{N_c+4} G$	$N_c^{3/2}$

In order to calculate the N_c behavior of the width we have to know the N_c dependence of the flavor Clebsch-Gordan coefficients that depend on the states involved. For the decays into 8 and 10 the only possible channels are $\Theta_{27} \to N(\Delta) + K$, and the pertinent Clebsches do not depend on N_c. For the decays into $\overline{10}$ we have $\Theta_{27} \to \Theta_{\overline{10}} + \pi$ that scales like $\mathcal{O}(1)$ and $\Theta_{27} \to N_{\overline{10}} + K$ that scales like $\mathcal{O}(1/\sqrt{N_c})$. The resulting scaling of $\Gamma_{\Theta_{27} \to B' + \varphi}$ calculated from Eq.(2·7) reads as follows:

decay of $\Theta_{27_{3/2}}$	N_c scaling exact	NRL	decay of $\Theta_{27_{1/2}}$	N_c scaling exact	NRL
$\to N_8 + K$	$\mathcal{O}(1)$	$\mathcal{O}(1/N_c^2)$	$\to N_8 + K$	$\mathcal{O}(1)$	0
$\to \Delta_{10} + K$	$\mathcal{O}(1)$	0	$\to \Delta_{10} + K$	$\mathcal{O}(1)$	$\mathcal{O}(1/N_c^2)$
$\to N_{\overline{10}} + K$	$\mathcal{O}(1/N_c^3)$	$\mathcal{O}(1/N_c^3)$	$\to N_{\overline{10}} + K$	$\mathcal{O}(1/N_c^3)$	$\mathcal{O}(1/N_c^3)$
$\to \Theta_{\overline{10}} + \pi$	$\mathcal{O}(1/N_c^2)$	$\mathcal{O}(1/N_c^2)$	$\to \Theta_{\overline{10}} + \pi$	$\mathcal{O}(1/N_c^2)$	$\mathcal{O}(1/N_c^2)$

Interestingly, we see that whenever the exact scaling is $\mathcal{O}(1)$ the nonrelativistic cancelation (exact or partial) lowers the power of N_c, whereas in the case when the width has *good* behavior for large N_c, there is no NRL cancelation.

§4. Alternative choices for large N_c multiplets

So far we have only considered the "standard" generalization (2·5) of baryonic $SU(3)_{\rm flavor}$ representations for large N_c. This choice is based on the requirement that generalized baryonic states have physical spin, isospin and strangeness, however their hypercharge and charge are not physical.[10] Moreover the generalization of the octet is not selfadjoint and antidecuplet is not complex conjugate of decuplet. Some years ago it has been proposed to consider alternative schemes.[12]

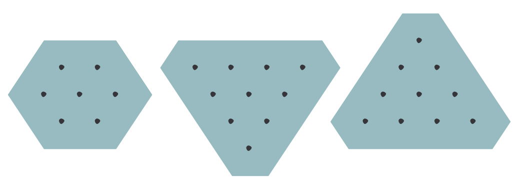

Fig. 3. Generalization of SU(3) flavor representations in which octet is selfadjoint

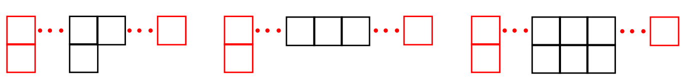

Fig. 4. Adding triquarks to regular SU(3) baryon representations 8, 10 and $\overline{10}$ corresponds to the representation set of Fig.3.

If we require the generalized octet to be self-adjoint we are led to the following set of representations

$$"8" = (N_c/3, N_c/3)\,, \quad "10" = ((N_c+6)/3, (N_c-3)/3)\,, \quad "\overline{10}" = "10"^* \qquad (4·1)$$

that are depicted in Figs. 3 and 4. This means that we enlarge N_c in steps of 3 adding each time a *uds* triquark. Generalized states have physical isospin, hypercharge (and charge), but unphysical strangeness and spin that is of the order of N_c. With this choice both Δ_{10-8}, $\Delta_{\overline{10}-8} \neq 0$ in large N_c limit:

$$\Delta_{10-8} = (N_c/6 - 1)/I_1, \quad \Delta_{\overline{10}-8} = (N_c/6 - 1)/I_2. \tag{4·2}$$

With this power counting we can calculate large N_c approximation of the meson momenta in the decays of Δ and Θ:

$$\begin{aligned} \Delta \to N \qquad & p_\pi = \sqrt{(M_\Delta - M_N)^2 - m_\pi^2} = 256 \text{ MeV}, \\ \Theta \to N \qquad & p_K = \sqrt{(M_\Theta - M_N)^2 - m_K^2} = 339 \text{ MeV} \end{aligned} \tag{4·3}$$

that are much closer to the physical values (2·8) than (2·6).

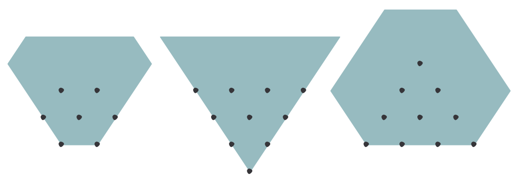

Fig. 5. Generalization of SU(3) flavor representations in which decuplet is fully symmetric $(0, q)$.

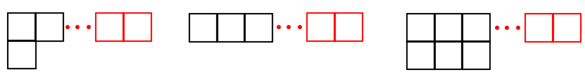

Fig. 6. Adding sextet diquarks to regular SU(3) baryon representations 8, 10 and $\overline{10}$ corresponds to the representation set of Fig.5.

Finally let us mention a third possibility in which we require generalized decuplet to be a completely symmetric $\mathrm{SU(3)_{flavor}}$ representation for arbitrary N_c. This leads to (see Figs. 5 and 6):

$$\text{"}8\text{"} = (N_c - 2, 1) \qquad \text{"}10\text{"} = (N_c, 0) \qquad \text{"}\overline{10}\text{"} = (N_c - 3, 3)\,. \tag{4·4}$$

Interestingly this choice has a smooth limit to the one flavor case. In the quark language it amounts to adding a symmetric diquark to the original $\mathrm{SU(3)_{flavor}}$ representation when increasing N_c in steps of 2. As seen from Fig. 5 physical states are situated at the bottom of infinite representations (4·4) and therefore have unphysical strangeness, charge (hypercharge) and also spin.

The mass splittings for this choice read

$$\Delta_{10-8} = N_c/\,2I_1, \quad \Delta_{\overline{10}-8} = 3/\,2I_2. \tag{4·5}$$

Here the generalized decuplet remains split from the "8", while $\Delta_{\overline{10}-8} \to 0$ for large N_c. The phase space factor for Θ decay is therefore suppressed with respect to the one of Δ.

§5. Summary

In this short note we have shown that very small width of exotic baryons – if they exist – cannot be explained by the standard N_c counting alone. Certain degree of *nonrelativisticity* is needed to ensure cancelations between different terms in the decay constants. This phenomenon observed firstly for antidecuplet, is also operative for the decays of eikosiheptaplet. We have shown that in χQSM in the nonrelativistic limit all decays are suppressed for large N_c. Exact cancelations occur for $\Theta_{27_{3/2}} \to \Delta_{10} + K$ and $\Theta_{27_{1/2}} \to N_8 + K$, leading N_c terms cancel for $\Theta_{27_{3/2}} \to N_8 + K$ and $\Theta_{27_{1/2}} \to \Delta_{10} + K$. For $27 \to \overline{10}$ there are no cancelations, but the phase space is N_c^{-3} suppressed.

We have also briefly discussed nonstandard generalizations of regular baryon representations for arbitrary N_c. For $N_c > 3$ bayons are no longer composed from 3 quarks and therefore they form large $\mathrm{SU}(3)_{\mathrm{flavor}}$ representations that reduce to octet, decuplet and antidecuplet for $N_c = 3$. The standard way to generalize regular baryon representations is to add antisymmetric antitriplet diqaurk when N_c is increased in intervals of 2. This choice fulfils many reasonable requirements; most importantly for $\mathrm{SU}(2)_{\mathrm{flavor}}$ these representations form regular isospin multiplets. However, representations (2·5) do not obey conjugation relations characteristic for regular representations. Therefore we have proposed generalization (4·1) that satisfies conjugation relations. Most important drawback of (4·1) is that spin $S \sim N_c$ that contradicts semiclassical quantization. Nevertheless as a result meson momenta emitted in 10 and $\overline{10}$ decays scale in the same way with N_c (4·3), consistently with "experimental" values (2·8), whereas for (2·5) the scaling is different (2·6).

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