THE SMALL DEVIATIONS OF MANY-DIMENSIONAL DIFFUSION PROCESSES AND RAREFACTION BY BOUNDARIES

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ABSTRACT. We lead the algorithm of expansion of sojourn probability of many-dimensional diffusion processes in small domain. The principal member of this expansion defines normalizing coefficient for special limit theorems.

Introduction.

Let $\xi(t)$ be a random process with measurable phase space $(X, \Sigma(X))$. Consider the measurable connected domain $D \in \Sigma(X)$ and small parameter ϵ. The investigations of asymptotics of sojourn probability (small deviations)

$$P(\xi(t) \in \epsilon D, \quad t \in [0, T]) \tag{1}$$

is jointed with many practice and theoretical problems [1-4]. In the literature, it was researched both rough asymptotics of principal member of (1)(log from it)[5] and exact asymptotics of diffusion processes of (1)[6-8]. In the works [9,10] was proved of algorithms of expansions of exact asymptotics of small deviation for diffusion and piecewise deterministic random processes for one-dimensional case.

The purpose this article is to present the algorithm of expansion of small deviation for many-dimensional diffusion processes and to define all constants of principal member.

In Section 1 our main result is stated and proved. In section 2 we consider the limits theorems about numbers of unabsorbed diffusion particles by boundaries of small domain.

I. The expansion.

We shall investigate of asymptote of following probability

$$P(\epsilon, x) = P(\xi(t) \in \epsilon D, \quad 0 \le t \le T), \quad \epsilon \to 0,$$

where $\xi(t) \in R^d$ is solution of the following stochastic differential equation

$$d\xi(t) = a(t, \xi(t))dt + \sum_{i=1}^{d} b_i(\xi(t))dw_i(t), \quad \xi(0) = x \in \epsilon D. \tag{2}$$

where functions

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$$b_i(x), \ a(t,x) : R^d \to R^d \quad \text{and} \quad R_+ \times R^d \to R^d.$$

are differentiable.

Set $\sigma_{ij}(x) = \sum_k b_k^i(x) b_k^j(x).$

It is known that $P(\epsilon, x) = u_0^\epsilon(T, x)$. Here $u_0^\epsilon(t, x)$ is solution of the following parabolic boundary problem at $0 \leq t \leq T$

$$\frac{\partial u_0^\epsilon(t,x)}{\partial t} = \frac{1}{2} \sum_{i,j=1}^d \sigma_{ij}(x) \frac{\partial^2 u_0^\epsilon(t,x)}{\partial x_i \partial x_j} + \sum_{i=1}^d a_i(T-t,x) \frac{\partial u_0^\epsilon(t,x)}{\partial x_i}, \quad x \in D_\epsilon;$$

$$u(t,x)|_{t=0} = 1; \quad x \in D_\epsilon; \quad u(t,x) = 0 \quad x \in \partial D_\epsilon, \quad 0 \leq t \leq T. \tag{3}$$

where $D_\epsilon = \epsilon D$. It is assumed that D is a connected bounded domain from R^m; the boundary ∂Q is the Lyapunov surface $C^{(1,\lambda)}$ and $0 \in D$. We interest of the asymptotic expansion $\epsilon \to 0$ of solution this problem $u_0^\epsilon(t,x)$ at $\epsilon \to 0$.

We define the differential operator $A : \frac{1}{2} \sum_{1 \leq i,j \leq d} \sigma_{ij}(0) \frac{\partial^2}{\partial x_i \partial x_j}$. Let σ be a matrix with the following property

$$\sum_{1 \leq i,j \leq d} \sigma_{ij}(0) z_i z_j \geq \mu |\vec{z}|^2.$$

Here μ, there is a fixed positive number, and $\vec{z} = (z_1, \cdots, z_d)$ is an arbitrary real vector.

This operator acts in the following space

$$H_A = \{u : u \in L_2(D) \cap Au \in L_2(D) \cap u(\partial D) = 0\}$$

with inner product $(u,v)_A = (Au, v)$. Here $(,)$ is inner product in $L_2(Q)$. The operator A is a positive operator[11]. It is known that the following eigenvalue problem

$$Au = -\lambda u, \quad u(\partial D) = 0$$

has infinite set of real eigenvalues $\lambda_i \to \infty$ and

$$0 < \lambda_1 < \lambda_2 < \cdots < \lambda_s < \cdots.$$

The corresponding eigenfunctions

$$f_{11}, \ldots, f_{1n_1}, \cdots, f_{s1}, \ldots, f_{sn_s}, \cdots$$

form the complete system of functions both in H_A and $L_2^0(Q) := \{u : u \in L_2(Q) \cap u(\partial Q) = 0\}$. Here the number n_k is equal to multiplicity of eigenvalue λ_k.

It is often convenient to present the system of eigenfunctions by one index: $\{f_n(z)\}$. The corresponding system of eigenvalues $\{\lambda_n\}$ will be with recurrences. We shall use it too.

We introduce the spectral function

$$e(x, y, \lambda) = \sum_{\lambda_j \leq \lambda} f_j(x) f_j(y).$$

We shall need in the following theorem from the monograph [12].

Theorem 1 ([12].Th.17.5.3). . *There exists such constant C_α that*

$$\sup_{x,y\in D} \sqrt{|D^\alpha_{x,y}e(x,y,\lambda)|} \leq C_\alpha \lambda^{(n+|\alpha|)/2}$$

Here α is multi-index.

Theorem 2. . *If the surface ∂D is Lyapunov surface and*

$$\sup_{(t,z)\in[0,T]\times D, 1\leq i,j\leq d} \max\left\{|\frac{\partial a_i(t,z)}{\partial z_j}|, |\frac{\partial b_i(z)}{\partial z_j}|, |\frac{\partial a_i(T-t,z)}{\partial t}|\right\} < \infty$$

then the following relation takes place at $\epsilon \to 0$

$$P(\epsilon, z\epsilon) = \exp\left\{-\lambda_1 \frac{T}{\epsilon^2} + \int_0^T \mu(t)dt\right\} \sum_{m=1}^{n_1} c_{1m} f_{1m}(z)(1+O(\epsilon)), \quad at \quad z \in D,$$

where

$$\mu(t) = \sum_{i,j}\left(\frac{1}{2}\sigma_{ij}(0)a_i(t,0)a_j(t,0) - \delta_{ij}a_i(t,0)a_j(t,0)\right)$$

and $c_{1m} = \int\limits_D f_{1m}(z)dz$.

Proof. Make the change of variables and function

$$x_i = z_i\epsilon, \quad u_1^\epsilon = u_0^\epsilon \exp\left\{\epsilon \sum_{k=1}^d a_k(T-t,0)z_k\right\},$$

Now we obtain the following parabolic problem for function u_1^ϵ

$$\frac{\partial u_1^\epsilon(t,z)}{\partial t} = \frac{1}{2\epsilon^2}\sum_{i,j=1}^d \sigma_{ij}(\epsilon z)\frac{\partial^2 u_1^\epsilon(t,z)}{\partial z_i \partial z_j} + \frac{1}{\epsilon}\sum_{i,j}\left(a_i(T-t,\epsilon z) - \frac{1}{2}\sigma_{ij}(\epsilon z)a_j(T-t,0)\right)\frac{\partial u_1^\epsilon(t,z)}{\partial z_i}$$

$$+ \sum_{i,j}\left(\frac{1}{2}\sigma_{ij}(\epsilon z)a_i(T-t,0)a_j(T-t,0) - \delta_{ij}a_i(T-t,0)a_j(T-t,\epsilon z) - \epsilon\frac{\partial a_i(T-t,0)}{\partial t}z_i\right)u_1^\epsilon, \quad z \in D;$$

$$u_1^\epsilon(t,z)|_{t=0} = \exp\left\{\epsilon\sum_{k=1}^d a_k(T,0)z_k\right\}; \quad z \in D; \quad u_1^\epsilon(t,z) = 0 \quad z \in \partial D, \quad 0 \leq t \leq T.$$

$$(4)$$

We will construct the asymptotic expansion of solution for this initial - boundary problem in the following form

$$u_1^\epsilon(t,z) = \sum_{k\geq 0} v_k(t,z)\epsilon^k. \qquad (5)$$

Note that the famous expansion

$$\exp\left\{\epsilon\sum_{k=1}^{d}a_k(T,0)z_k\right\} = 1 + \epsilon\sum_{k=1}^{d}a_k(T,0)z_k + \frac{1}{2!}\left(\epsilon\sum_{k=1}^{d}a_k(T,0)z_k\right)^2 + \cdots,$$

defines the initial conditions for v_k, $\quad k \geq 0$:

$$v_0(0,z) = 1, \quad v_1(0,z) = \sum_{k=1}^{d}a_k(T,0)z_k, \quad v_2(0,z) = \frac{1}{2}\left(\sum_{k=1}^{d}a_k(T,0)z_k\right)^2\cdots.$$

Using the first fragment of Taylor series in zero point under conditions of theorem we can obtain the following representations

$$\sigma_{ij}(\epsilon z) = \sigma_{ij}(0) + \epsilon\sigma_{ij}^{\epsilon}(z), \quad a_i(T-t,\epsilon z) = a_i(T-t,0) + \epsilon a_i^{\epsilon}(T-t,z), 1 \leq i,j \leq d \quad (6)$$

where

$$\sup_{z\in D,\epsilon\in[0,1],1\leq i,j\leq d}|\sigma_{ij}^{\epsilon}(z)| < \infty, \qquad \sup_{z\in D,t\in[0,T],\epsilon\in[0,1],1\leq i\leq d}|a_i^{\epsilon}(T-t,z)| < \infty$$

Now, after substitution of (5),(6) to (4) we conclude that the v_0 satisfies the problem

$$\frac{\partial v_0}{\partial t} = \frac{1}{2\epsilon^2}\left(\sum_{i,j=1}^{d}\sigma_{ij}(0)\frac{\partial^2}{\partial z_i\partial z_j}\right)v_0 + \mu(t)v_0 \tag{7}$$

$$v_0|_{\partial D} = 0; \quad v_0(0,z) = 1, \quad z \in D.$$

Here

$$\mu(t) = \sum_{i,j}\left(\frac{1}{2}\sigma_{ij}(0)a_i(T-t,0)a_j(T-t,0) - \delta_{ij}a_i(T-t,0)a_j(T-t,0)\right).$$

Further, let us denote by $B_{\epsilon}(t,z)$ the operator $C^2(D) \to C(D)$, for $f \in C^2(D)$ it's defined as follows:

$$B^{\epsilon}(t,z)f =$$

$$\frac{1}{2\epsilon}\sum_{i,j=1}^{d}\sigma_{ij}^{\epsilon}(z)\frac{\partial^2 f}{\partial z_i\partial z_j} + \frac{1}{\epsilon}\sum_{i,j}\left(a_i(T-t,\epsilon z) - \frac{1}{2}\sigma_{ij}(\epsilon z)a_j(T-t,0)\right)\frac{\partial f}{\partial z_i} +$$

$$+\epsilon\sum_{i,j=1}^{d}\left(\frac{1}{2}\sigma_{ij}^{\epsilon}(z)a_i(T-t,0)a_j(T-t,0) - \delta_{ij}a_i(T-t,0)a_j^{\epsilon}(T-t,z) - \frac{\partial a_i(T-t,0)}{\partial t}z_i\right)f =$$

$$=: \frac{1}{2\epsilon}\sum_{i,j=1}^{d}\sigma_{ij}^{\epsilon}(z)\frac{\partial^2 f}{\partial z_i\partial z_j} + \frac{1}{\epsilon}A_1^{\epsilon}(t,z)f + \epsilon A_2^{\epsilon}(t,z).$$

Now, formally the functions $v_k, k \geq 1$ are defined by the following recurrence system problems

$$\frac{\partial v_k}{\partial t} = \frac{1}{2\epsilon^2} \left(\sum_{i,j=1}^{d} \sigma_{ij}(0) \frac{\partial^2}{\partial z_i \partial z_j} \right) v_k + B_\epsilon(t,z) v_{k-1} \tag{8}$$

$$v_0|_{\partial D} = 0; \quad v_k(0,z) = \frac{1}{k!} \left(\sum_{k=1}^{d} a_k(T-t,0) z_k \right)^k, z \in D.$$

We shall solve the problems of $(7),(8)$ by method of separation of variables. According to this method the solutions are defined in the form

$$v_k(t,z) = \sum_{n \geq 1} q_{k,n}(t) f_n(z). \tag{9}$$

For definition of principal number it suffices to construct of the v_0. If we substitute (9) at $k = 0$ to (7) then we obtain

$$\sum_{n \geq 1} \left\{ -\dot{q}_{0,n}(t) - \frac{\lambda_n}{\epsilon^2} q_{0,n}(t) + \mu(t) q_{0,n}(t) \right\} f_n(z) = 0.$$

Set $c_{0,n} = \int_D f_n(z) dz$ (coefficients of expansion of indicator of set D). The initial condition of v_0 has the following stating

$$v_0(0,z) = \sum_{n \geq 1} q_{0,n}(0) f_n(z) = \sum_{n \geq 1} c_{0,n} f_n(z) = \sum_{l \geq 1} \sum_{m=1}^{n_l} c_{0,lm} f_{lm}(z), \quad z \in D.$$

By definition of system of functions $\{f_n(z)\}$, now we have the system of ordinary differential equations

$$\dot{q}_{0,n}(t) + \left(\frac{\lambda_n}{\epsilon^2} - \mu(t) \right) q_{0,n}(t) = 0, \quad q_{0,n}(0) = c_{0,n}.$$

From the latter one we have

$$q_{0,n}(t) = c_{0,n} \exp \left\{ -\frac{\lambda_n}{\epsilon^2} t + \int_0^t \mu(s) ds \right\}.$$

Set

$$A_0 = \sup_{\epsilon \leq 1, z \in D; i,j} |\sigma_{ij}^\epsilon(z)|, \quad L_0 = \sum_{l \geq 1, 1 \leq m \leq n_l} (c_{0,ml})^2.$$

$$A_1 = \sup_{0 \leq \epsilon \leq 1, z \in D, t \in [0,T]; i,j} \left| a_i(T-t, \epsilon z) - \frac{1}{2} \sigma_{ij}(\epsilon z) a_j(T-t, 0) \right|.$$

$$A_2 =$$

$$= \sup_{0 \leq \epsilon \leq 1, z \in D, t \in [0,T]; i,j} \left| \frac{1}{2} \sigma_{ij}^\epsilon(z) a_i(T-t,0) a_j(T-t,0) - \delta_{ij} a_i(T-t,0) a_j^\epsilon(T-t,z) - \frac{\partial a_i(T-t,0)}{\partial t} z_i \right\|$$

We have the following relations for eigenvalues λ_l

$$k_1 l^{2/d} \leq \lambda_l \leq k_2 l^{2/d}, \quad \max(k_1, k_2) < \infty$$

,

Applying Cauchy-Bunyakovskii inequality, Theorem 1 and the latter one, we get

$$\left| \sum_{i,j} a_{i,j}^{\epsilon}(z) \frac{\partial^2 v_0}{\partial z_i \partial z_j} \right| = \left| \sum_l \exp\left(-\lambda_l t \epsilon^{-2} + \int_0^t \mu(s)ds \right) \sum_{m=1}^{n_l} c_{0,ml} \sum_{i,j} a_{i,j}^{\epsilon}(z) \frac{\partial^2 f_{ml}(z)}{\partial z_i \partial z_j} \right| \leq$$

$$\leq A_0 d \sum_l \exp\left(-\lambda_l t \epsilon^{-2} + \int_0^t \mu(s)ds \right) \left(\sum_{m=1}^{n_l} (c_{0,ml})^2 \right)^{\frac{1}{2}} \left(\sum_{m=1}^{n_l} \sum_{i,j} \left(\frac{\partial^2 f_{ml}(z)}{\partial z_i \partial z_j} \right)^2 \right)^{\frac{1}{2}} \leq$$

$$\leq A_0 d C_{2,2} L_0 \sum_{l \geq 1} \exp\left(-\frac{\lambda_l t}{\epsilon^2} + \int_0^t \mu(s)ds \right) \lambda_l^{\frac{d}{2}+2} \leq \exp\left(-\frac{\lambda_1 t}{\epsilon^2} \right) K_0. \qquad (10)$$

Here $K_0 < \infty$.

Reasoning similarly we convince ourselves that for other parts of $B^{\epsilon}(t,z)v_0$ the following estimations take place

$$|A_1^{\epsilon}(t,z)v_0| \leq A_1 d C_{1,1} L_0 \sum_{l \geq 1} \exp\left(-\frac{\lambda_l t}{\epsilon^2} + \int_0^t \mu(s)ds \right) \lambda_l^{\frac{d}{2}+1} \leq \exp\left(-\frac{\lambda_1 t}{\epsilon^2} \right) K_{0,1}; \quad (11)$$

$$|A_2^{\epsilon}(t,z)v_0| \leq A_2 d C_{0,0} L_0 \sum_{l \geq 1} \exp\left(-\frac{\lambda_l t}{\epsilon^2} + \int_0^t \mu(s)ds \right) \lambda_l^{\frac{d}{2}} \leq \exp\left(-\frac{\lambda_1 t}{\epsilon^2} \right) K_{0,2}, \quad (12)$$

where $\max\{K_{0,1}, K_{0,2}\} < \infty$.

Now let us estimate the coefficients $\beta_n^{\epsilon}(t)$ of expansion of $B^{\epsilon}(t,z)v_0$ by system $\{f_n\}_{n \geq 1}$. Applying (10)-(12) and Cauchy-Bunyakovskii inequality, we get

$$|\beta_n^{\epsilon}(t)| = \left| \int_D B^{\epsilon}(t,z)v_0(t,z)f_n(z)dz \right| \leq$$

$$\leq \left(\int_D \left(B^{\epsilon}(t,z)v_0 \right)^2 dz \right)^{\frac{1}{2}} \left(\int_D f_n^2(z)dz \right)^{\frac{1}{2}} \leq$$

$$\leq \exp(-\lambda_1 t \epsilon^{-2}) \left(\frac{K_0 + K_{0,1}}{\epsilon} + \epsilon K_{0,2} \right) |D|.$$

The latter one now gives

$$\left| \int_0^t \beta_n^{\epsilon}(s)ds \right| \leq \epsilon \gamma_{\epsilon}(t), \qquad (13)$$

where
$$\sup_{0\leq\epsilon\leq1,t\in[0,T]}\gamma_\epsilon(t)<\infty.$$

Finally, let us estimate the difference $r^\epsilon(t,z)=u_1^\epsilon(t,z)-v_0(t,z)$. By definition, $r^\epsilon(t,z)$ is solution of the following problem

$$\frac{\partial r^\epsilon}{\partial t}=\frac{1}{2\epsilon^2}\left(\sum_{i,j=1}^d\sigma_{ij}(0)\frac{\partial^2}{\partial z_i\partial z_j}\right)r^\epsilon+B_\epsilon(T-t,z)v_0\quad z\in D; \tag{14}$$

$$r^\epsilon(t,z)|_{t=0}=\exp\left\{\epsilon\sum_{k=1}^d a_k(T,0)z_k\right\}-1;\quad z\in D;\quad r^\epsilon(t,z)=0\quad z\in\partial D,\quad 0\leq t\leq T.$$

It is clear that $r^\epsilon(0,z)$ we can present as $\epsilon r_1^\epsilon(0,z)$, where $r_1^\epsilon(0,z)$ is uniform bounded function of variables $\epsilon\in[0,1]$ and $z\in D$. So, the coefficients of expansion this function by system $\{f_n(z)\}$ have the following forms

$$\int_D r^\epsilon(0,z)f_n(z)dz=\epsilon\mu_n^\epsilon,\quad\text{where}\quad\sup_{0\leq\epsilon\leq1}\sum_{n\geq1}(\mu_n^\epsilon)^2=M<\infty. \tag{13}$$

Now we have the solution of (14) in the following form

$$r^\epsilon(t,z)=\epsilon\sum_{n\geq1}\mu_n^\epsilon\exp\{-\lambda_n t\epsilon^{-2}+\int\beta_n^\epsilon(s)ds\}f_n(z)$$

Applying latter one ,(13),(15), Theorem 1 and Cauchy-Bunyakovskii inequality we get at $t>0$

$$|r^\epsilon(t,z)\epsilon^{-1}|\leq\left(\sum_{n\geq1}(\mu_n^\epsilon)^2\right)^{\frac{1}{2}}C_{0,0}\sum_{n\geq1}\exp\{-\lambda_n t\epsilon^{-2}+\int\beta_n^\epsilon(s)ds\}\lambda_n^{\frac{d}{2}}\}\leq$$

$$\leq MC_{0,0}\exp\{-\lambda_1 t\epsilon^{-2}\}K_{0,3},\quad\text{where}\quad K_{0,3}<\infty.$$

The proof of theorem is completed.

Remark 1. *According to the above system of problems for definition of the functions $v_k, k\geq1$, we outline the construction of coefficients $q_{k,n})(t)$ for the series (8):*

$$\dot{q}_{k,n}(t)=+\left(\frac{\lambda_n}{\epsilon^2}+\mu_{k-1,n}^\epsilon(t)\right)q_{k,n}(t),$$

$$q_{k,n}(0)=\int_D v_k(0,z)f_n(z)dz=\frac{1}{k!}\int_D\left(\sum_{m=1}^d a_m(T,0)z_m\right)^m f_n(z)dz$$

Here $\mu_{k-1,n}^\epsilon(t)=\int_D f_n(z)B^\epsilon(t,z)v_{k-1}(t,z)dz$.

Remark 2. *Theorem 2 is coordinated with results of works [6-8] where the principal member of small deviations in ball are investigated for more simple SDE.*

II. The rarefaction of set of diffusion processes by boundaries of small domains.

The following problem was investigated in works[13,14]. Let a set identical diffusion random processes start at the initial time from the different points of domain D. These processes are diffusion processes with absorbtion on the boundary ∂D. We are interested in distribution of the number yet absorbed at the moment T. The initial number and initial position of diffusion processes are defined either a random Poisson measure[14] or deterministic measure [13]. The proved limits theorems described the situation when $T \to \infty$ and initial number of diffusion processes depended on T and it increased at the rise of T. The role of normalizing function played principal member of asymptote of solution of according parabolic problem at $T \to \infty$.

Henceforth we shall assume that considered diffusion processes satisfy of the SDE (2) with different initial points.

Now we consider the situation when initial number of absorbing diffusion processes in small domain ϵD depends on $\epsilon \to 0$ and it increase under the condition of decrease of ϵ. It is not hard to show, that now normalizing function is the principal member of parabolic problem (3) at $\epsilon \to 0$.

The proofs of stated below theorems repeat the proofs of according theorems from [13,14] almost word for word.

We will denote by $\eta(\epsilon, T)$ the number of remaining processes in the region ϵD at the moment T.

We will also assume that σ-additive measure ν is given on the Σ_ν- algebra sets from D, $\quad \nu(D) < \infty$. All eigenfunctions $f_{ij} : D \to R^1$ are (Σ_ν, Σ_Y) measurable. Here Σ_Y is system of Borel sets from R^1. Let $\Rightarrow$ denote the weak convergence of random values or measures.

At the beginning we assume that initial number and position of diffusion processes are defined by deterministic measure $N(\epsilon B, \epsilon), B \in D$. Thus, $N(\epsilon B, \epsilon)$ is equal to number of starting points in the set ϵB.

Let us denote by $\nu_\epsilon(\cdot)$ the measure

$$\nu_\epsilon(\epsilon B) = \exp\left(-\frac{T\lambda_1}{2\epsilon^2}\right) N(\epsilon B, \epsilon).$$

where $B \in \Sigma_\nu$.

By definition of measure $\nu_\epsilon(\cdot)$, we have

$$d\nu_\epsilon(x) = \begin{cases} \exp\left(-\frac{T\lambda_1}{2\epsilon^2}\right), & \text{if} \quad x = x_k, \quad k = 1, \cdots, N(\epsilon D, \epsilon) \\ 0, & \text{otherwise.} \end{cases}$$

Theorem 3. *Under the assumptions of the Theorem 2 let the $N(\epsilon\cdot, \epsilon)$ satisfies the condition*

$$\nu_\epsilon(\epsilon \; \cdot) \underset{\epsilon \to 0}{\Rightarrow} \nu(\cdot).$$

Then $\eta(\epsilon, T) \Rightarrow \eta(T)$ if $\epsilon \to 0$ where $\eta(T)$ has Poisson distribution function with parameter

$$a(T) = \exp\left(\int_0^T \mu(s)ds\right)\int_D F(z)d\nu(z),$$

where $F(z) = \sum_{i=1}^{n_1} f_{1i}(z)c_{1i}, \quad c_{1i} = \int_Q f_{1i}(z)dz$
and $\mu(t)$ is the function from Theorem 2.

Now we consider the case when the initial number and positions of processes are defined by the random Poisson measure $\mu(\cdot, \epsilon)$ in ϵD:

$$P(\mu(\epsilon A, \epsilon) = k) = \frac{m^k(\epsilon A, \epsilon)}{k!}e^{-m(\epsilon A, \epsilon)},$$

where $m(\epsilon \cdot, \epsilon)$ is finitely additive positive measure on ϵD for fixed ϵ.
    We assign

$$g(\epsilon) = \exp\left(-\frac{T\lambda_1}{2\epsilon^2}\right).$$

Theorem 4. *Under the assumptions of the Theorem 2 we suppose that $m(\epsilon\cdot, \epsilon)$ holds the condition*

$$\lim_{\epsilon \to 0} m(\epsilon B, \epsilon)g(\epsilon) = \nu(B), \quad B \in \Sigma_\nu.$$

Then $\eta(\epsilon, T) \Rightarrow \eta(T)$ if $\epsilon \to 0$ where $\eta(T)$ has the Poisson distribution function with the parameter $a(T)$ from Theorem 3.

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