Understanding the Flavor Symmetry Breaking and Nucleon Flavor-Spin Structure within Chiral Quark Model

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Abstract

In χQM, a quark can emit Goldstone bosons. The flavor symmetry breaking in the Goldstone boson emission process is used to intepret the nucleon flavor-spin structure. In this paper, we study the inner structure of constituent quarks implied in χQM caused by the Goldstone boson emission process in nucleon. From a simplified model Hamiltonian derived from χQM, the intrinsic wave functions of constituent quarks are determined. Then the obtained transition probabilities of the emission of Goldstone boson from a quark can give a reasonable interpretation to the flavor symmetry breaking in nucleon flavor-spin structure.

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I. INTRODUCTION

The measurements of the polarized structure functions of the nucleon in deep inelastic scattering(DIS) experiments[1, 2, 3, 4] show the complication in proton spin structure. Only a portion of the proton spin is carried by valence quarks. Moreover, several experiments[5, 6, 7] clearly indicate the $\bar{u}$-$\bar{d}$ asymmetry as well as the existence of the strange quark content $\bar{s}$ in the proton sea. Also the distribution of strange quark in the proton sea is polarized negative. The DIS results deviate significantly from the naïve quark model (NQM) expectation.

NQM gives many fairly good description of hadron properties. Why does NQM work? It is a puzzle that the quarks inside a hadron could be treated as non-relativistic particles in NQM. The chiral quark model (χQM) tries to bridge between QCD and NQM. It was originated by Weinberg[8] and formulated by Manohar and Georgi[9]. Between the QCD confinement scale ($\Lambda_{\rm QCD} \simeq$200MeV) and a chiral symmetry breaking scale ($\Lambda_{\chi\rm SB} \simeq$1GeV), the strong interaction is described by an effective Lagrangian of quarks q, gluons g and Numbu-Goldstone bosons Π. An important feature of the χQM is that, betweetn $\Lambda_{\rm QCD}$ and $\Lambda_{\chi\rm SB}$, the internal gluon effects in a hadron can be small compared to the internal Goldstone bosons Π and quarks q, so the effective degrees of freedom in this region can be q and Π.

It is interesting that χQM can also be used to explain why NQM does not work in the above DIS experiments. By the emission of Goldstone boson, χQM allows the fluctuation of a quark q into a recoiling quark plus a Goldstone boson $q \to q'\Pi$. The $q'\Pi$ system then further splits to generate quark sea through

- the helicity-flipping process

$$q_\uparrow \longrightarrow \Pi + q'_\downarrow \longrightarrow (q\bar{q}') + q'_\downarrow \tag{1}$$

- and the helicity-non-flipping process

$$q_\uparrow \longrightarrow \Pi + q'_\uparrow \longrightarrow (q\bar{q}') + q'_\uparrow \tag{2}$$

where the subscript indicates the helicity of quark. In both the process, $q'\Pi$ is in a relative P-wave state. In the helicity-flipping process (1), the orbital angular momentum along helicity direction must be $\langle l_z \rangle = +1$. In the helicity-non-flipping process (2), $\langle l_z \rangle = 0$. The process cause a modification of the spin content of the nucleon because a quark changes its helicity in (1). Also it causes a modification of the flavor content because the generated quark sea from Π is flavor dependent[10, 11].

χQM was first used to explain the nucleon sea flavor asymmetry and the smallness of the quark spin fraction by Eichten, Hinchliffe and Quigg[10]. The flavor asymmetry of sea quark distribution arises from the mass differences in different quark flavors and in different Goldstone bosons. Only the lightest Goldstone Boson π was considered since its contribution dominates. From a perturbation calculation, the probability for an up quark to emit a π^+ was estimated to be $a = 0.083$. This would induce a flavor asymmetry in parton distributions of nucleon and other hadrons.

However, the estimated transition probability is not enough to full account the flavor asymmetry in DIS experiments. Contribution from other Π's and even η' was considered by Cheng and Li[11]. Explicit $SU_f(3)$ breaking in the transition probabilities was later introduced in refs. 12, 13 and further used by several authors[14, 15, 16, 17, 18, 19]. Nevertheless, in all these calculation, the transition probabilities were put into model by hand. To fit the experimenal data, the probability of an up quark emitting π^+ needs to be set to $a \gtrsim 0.1$, which is about 20% larger than the purturbation calculation.

We should not be surprised by this discrepancy since the χQM works in a region right above the QCD confinement scale Λ_{QCD}. There one may expect the confinement effect is important and the purturbative calculation of QCD may contain large error. However, there is another essential difference between above model calculations and the purturbation calcultion. In the purturbation calculation, the emitted Goldstone bosons are virtual particles. In the above model calculations which are closely related to NQM, however, the Goldstone bosons are close to mass shell under the non-relativistic approximation.

Since χQM can be a bridge between NQM and QCD, it is interesting to explore χQM from NQM side where we use the wave function method. This will give the above model calculation a concrete foundation in NQM and help us further understand the flavor symmetry breaking mechanism.

In this paper, we will use wave function method to investigate the flavor symmetry breaking in χQM. In a conventional quark model[20], a hadron consists of confined constituent quarks and its wave function is constructed in the configuration space of the constituent quarks. To incorporate the transition process of emitting Goldstone boson of χQM into the quark model, the constituent quarks will have intrinsic wave functions within the configuration $q + q'\Pi$.

In Sec. II, we first present the composite wave function of constituent quarks including components of $q'\Pi$. The wave functions and the transition probabilities of $q \to q'\Pi$ are determined from a simplified χQM Hamiltionian. In Sec. III and Sec. IV, the obtained transition probabilities are used to calculate nucleon flavor-spin structure and baryon octet magnetic moments respectively. The numerical results and a brief summary are presented in Sec. V.

II. THE WAVE FUNCTION OF A CONSTITUENT QUARK

In χQM, the effective Lagrangian below the chiral symmetry breaking scale $\Lambda_{\chi\mathrm{QM}}$ involves quarks, gluons, and Goldstone bosons. The first few terms in this Lagrangian are[9]:

$$\begin{aligned}\mathcal{L}_{\chi\mathrm{QM}} &= \bar{\psi}(iD_\mu + V_\mu)\gamma^\mu\psi + ig_A\bar{\psi}A_\mu\gamma^\mu\gamma^5\psi \\ &- m\bar{\psi}\psi + \frac{1}{4}f_\pi^2 \mathrm{tr}\partial^\mu\Sigma^\dagger\partial_\mu\Sigma + ...\end{aligned} \tag{3}$$

where $D_\mu = \partial_\mu + igG_\mu$ is the gauge-covariant derivative of QCD, G_μ the gluon field and g the strong coupling constant. The dimensionless axial-vector coupling $g_A = 0.7524$ is determined from the axial charge of the nucleon. m represents the constituent quark masses due to chiral symmetry breaking. The pseudoscalar decay constant is $f_\pi \approx 93$MeV. The Σ field, vector currents V_μ and axial-vector currents A_μ are given in terms of the Goldstone boson fields Φ

$$\Phi = \begin{bmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K^0} & -\frac{2}{\sqrt{6}}\eta \end{bmatrix}, \tag{4}$$

$$\Sigma = \exp(i\frac{\sqrt{2}\Phi}{f_\pi}), \tag{5}$$

$$\begin{pmatrix} V_\mu \\ A_\mu \end{pmatrix} = \frac{1}{2}(\xi^\dagger\partial_\mu\xi \pm \xi\partial_\mu\xi^\dagger), \tag{6}$$

$$\xi = \exp(i\frac{\Phi}{\sqrt{2}f_\pi}). \tag{7}$$

An expansion of the currents in powers of Φ/f_π yields the effective interaction between Π and q[10]

$$\mathcal{L}_I = -\frac{g_A}{\sqrt{2}f_\pi}\bar{\psi}\partial_\mu \Phi \gamma^\mu \gamma_5 \psi. \tag{8}$$

This allows the fluctuation of a quark into a recoil quark plus a Goldstone boson $q \to q'\Pi$.

In quark model, a hadron is built with constituent quarks. In accordance with χQM, we should treat a constituent quark as a composite particle including such components $q'\Pi$. Here we denote the wave function of a composite constituent quark as $|q\rangle\rangle$. At rest,

$$|q\rangle\rangle = z^q|q\rangle + \sum_{q'\Pi} x^q_{q'\Pi}|q'\Pi\rangle. \tag{9}$$

In our paper, the state normalization relation is always taken as

$$\langle p|p'\rangle = \delta^3(\mathbf{p} - \mathbf{p}'). \tag{10}$$

The above wave function is of essential importance in our work. The square of the modulus of the coefficient of each $q'\Pi$ configuration is just the probability for the corresponding Π emission process

$$P_{q\to q'\Pi} = |x^q_{q'\Pi}|^2, \tag{11}$$

and

$$|z^q|^2 = (1 - \sum_{q'\Pi} P_{q\to q'\Pi})$$

is the probability of no Π emission.

To determine the wave function (9), we first construct a simplified Hamiltonian in the degrees of freedom q and Π,

$$H = H_0 + H_B + H_I. \tag{12}$$

H_0 represents the kinetic energies of q and Π. It reads

$$H_0 = \int d^3x \left\{ \bar{\psi}(i\alpha\cdot\nabla + m)\psi + \frac{1}{2}\mathrm{Tr}[\dot{\Phi}^2 + (\nabla\Phi)^2] + \frac{1}{2}\sum_{\Pi} m^2_\Pi (\Phi^\Pi)^2 \right\}, \tag{13}$$

where m_Π is the physical mass of Π which is nonzero and nondegenerate.

$$H_I = -\int d^3x \mathcal{L}_I, \tag{14}$$

is the χQM interaction. H_B is an accessary interaction which is needed to bind the $q'\Pi$ together. In our simplified Hamiltonian, we will not disscuss the explicit formalism of H_B.

Instead, we will put some physical restriction condition on it later in this section, which is sufficient to our calculation.

From H_0, we can expand free fields ψ and Π in terms of creation and annihilation operators

$$\psi^q(x) = \int \frac{d^3p}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E^q_{\mathbf{p}}}} \sum_s \left[a^q_{\mathbf{p}s} u^q(\mathbf{p}, s) e^{-ip\cdot x} + b^{q\dagger}_{\mathbf{p}s}(t) v^q(\mathbf{p}, s) e^{ip\cdot x}\right]_{p^0=E^q_{\mathbf{p}}}, \tag{15}$$

$$\Phi^\Pi(x) = \int \frac{d^3p}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E^\Pi_{\mathbf{p}}}} \left[c^\Pi_{\mathbf{p}} e^{-ip\cdot x} + c^{\Pi\dagger}_{\mathbf{p}} e^{ip\cdot x}\right]_{p^0=E^\Pi_{\mathbf{p}}}, \tag{16}$$

where

$$E^q_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m_q^2}$$

is the quark energy of flavor q,

$$E^\Pi_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m_\Pi^2}$$

is the energy of Goldstone boson Π. $a^{q\dagger}_{\mathbf{p}s}$ and $b^{q\dagger}_{\mathbf{p}r}$ are the creation operators of quark q and anti-quark $\bar{q}$

$$\{a^q_{\mathbf{p}r}, a^{q\dagger}_{\mathbf{p}'s}\} = \{b^q_{\mathbf{p}r}, b^{q\dagger}_{\mathbf{p}'s}\} = \delta^{(3)}(\mathbf{p} - \mathbf{p}')\delta_{rs}. \tag{17}$$

$c^{\Pi\dagger}_{\mathbf{p}}$ is the creation operator of Π

$$[c^\Pi_{\mathbf{p}}, c^{\Pi\dagger}_{\mathbf{p}'}] = \delta^{(3)}(\mathbf{p} - \mathbf{p}'). \tag{18}$$

Next, we will replace the field ψ and Φ in the Hamiltonian (12) with the free field of (15) and (16). Then we can express the Hamiltonian in creation and annihilation operators, for example

$$H_0 = \sum_q \sum_s \int d^3p\; E^q_{\mathbf{p}}[a^{q\dagger}_{\mathbf{p}s} a^q_{\mathbf{p}s} + b^{q\dagger}_{\mathbf{p}s} b^q_{\mathbf{p}s}] + \sum_\Pi \int d^3p\; E^\Pi_{\mathbf{p}} c^{\Pi\dagger}_{\mathbf{p}} c^\Pi_{\mathbf{p}}. \tag{19}$$

In all the model calculations [11, 12, 13, 14, 15, 16, 17, 18, 19], the emitted Π is assumed bound to the quark source. To represent that $q'\Pi$ are bound, we use the well known SHO function as their spatial wave function

$$|q\Pi\rangle = \frac{1}{\sqrt{N}}(-i) \int d^3p |\mathbf{p}| e^{-\frac{p^2}{2\lambda^2}}\; [Y_1(\theta, \phi)\; c^{\Pi\dagger}_{-\mathbf{p}}\; a^{q\dagger}_{\mathbf{p}}]_{1/2}\; |0\rangle, \tag{20}$$

$$\begin{aligned} |q\Pi \uparrow\rangle = {} & \frac{1}{\sqrt{N}}\sqrt{\frac{2}{3}}(-i) \int d^3p |\mathbf{p}| e^{-\frac{p^2}{2\lambda^2}}\; Y_{11}(\theta, \phi)\; c^{\Pi\dagger}_{-\mathbf{p}}\; a^{q\dagger}_{\mathbf{p}\downarrow}\; |0\rangle \\ & - \frac{1}{\sqrt{N}}\sqrt{\frac{1}{3}}(-i) \int d^3p |\mathbf{p}| e^{-\frac{p^2}{2\lambda^2}}\; Y_{10}(\theta, \phi)\; c^{\Pi\dagger}_{-\mathbf{p}}\; a^{q\dagger}_{\mathbf{p}\uparrow}\; |0\rangle, \end{aligned} \tag{21}$$

where λ is the "characteristic radius" parameter in Gaussian function. $1/\sqrt{N}$ is the normalization factor,

$$N = \int dp\, p^4\, e^{-\frac{p^2}{\lambda^2}} = \frac{3}{8}\sqrt{\pi}\lambda^5. \tag{22}$$

However, we need a binding interaction H_B in the Hamiltonian. Yet we do not know how to write out the explicit form of H_B. However, H_B should provide enough binding energy. That is, for the $q'\Pi$ system, we must have

$$\langle q\Pi|H_0 + H_B|q\Pi\rangle \leq m_q + m_\Pi. \tag{23}$$

That is

$$E_B = \langle q\Pi|H_B|q\Pi\rangle \leq m_q + m_\Pi - \langle q\Pi|H_0|q\Pi\rangle = m_q - E^q + m_\Pi - E^\Pi. \tag{24}$$

As a roughly estimation, we will take the mininum value of E_B

$$E_B = -\max_{q,\Pi}\{E^q - m_q + E^\Pi - m_\Pi\} = -(E^u - m_u + E^\pi - m_\pi). \tag{25}$$

Then the wave function of a composite constituen quark is determined from Schrödinger equation

$$H|q\rangle\rangle = M_q|q\rangle\rangle. \tag{26}$$

After taking the above simplification, we need only solve a matrix eigen-value problem

$$\begin{pmatrix} a & B \\ B^T & C \end{pmatrix} \begin{pmatrix} z^q \\ X^q \end{pmatrix} = M_q \begin{pmatrix} z^q \\ X^q \end{pmatrix}, \tag{27}$$

where

$$\begin{aligned} a\delta^3(0) &= \langle q|H|q\rangle, \\ B_{q'\Pi}\delta^3(0) &= \langle q|H|q'\Pi\rangle, \\ C_{q'\Pi;q''\Pi'}\delta^3(0) &= \langle q'\Pi|H|q''\Pi'\rangle, \\ X^q_{q'\Pi} &= x^q_{q'\Pi}. \end{aligned}$$

For example, let us consider the process u emitting Π. There are four possible $|q'\Pi\rangle$ states generated by the fluctuations of a u quark: $|u\pi^0\rangle$, $|u\eta\rangle$, $|d\pi^+\rangle$ and $|sK^+\rangle$. Thus

$$|u\rangle\rangle = z^u|u\rangle + x^u_{u\pi^0}|u\pi^0\rangle + x^u_{u\eta}|u\eta\rangle + x^u_{d\pi^+}|d\pi^+\rangle + x^u_{sK^+}|sK^+\rangle. \tag{28}$$

Taking these wave functions as basis, we can calculate the matrix of the Hamiltonian in (27).

$$a = m_u. \tag{29}$$

C is diagonalized. Its diagonal matrix elements are calculated from H_0

$$C_{u\pi^0;u\pi^0} = \frac{1}{N}\int dp\, p^4\, e^{-\frac{p^2}{\lambda^2}}(\sqrt{\mathbf{p}^2+m_u^2}+\sqrt{\mathbf{p}+m_{\pi^0}^2})+E_B, \tag{30}$$

$$C_{u\eta;u\eta} = \frac{1}{N}\int dp\, p^4\, e^{-\frac{p^2}{\lambda^2}}(\sqrt{\mathbf{p}^2+m_u^2}+\sqrt{\mathbf{p}+m_{\eta}^2})+E_B, \tag{31}$$

$$C_{d\pi^+;d\pi^+} = \frac{1}{N}\int dp\, p^4\, e^{-\frac{p^2}{\lambda^2}}(\sqrt{\mathbf{p}^2+m_d^2}+\sqrt{\mathbf{p}+m_{\pi^+}^2})+E_B, \tag{32}$$

$$C_{sK^+;sK^+} = \frac{1}{N}\int dp\, p^4\, e^{-\frac{p^2}{\lambda^2}}(\sqrt{\mathbf{p}^2+m_s^2}+\sqrt{\mathbf{p}+m_{K^+}^2})+E_B. \tag{33}$$

B is calculated from H_I

$$B_{u\pi^0} = -\frac{g_A}{2\sqrt{2}\pi f_\pi}\frac{1}{\sqrt{N}}\int dp\, p^4 e^{-\frac{p^2}{2\lambda^2}}\frac{1}{\sqrt{4E^u_{\mathbf{p}}E^{\pi^0}_{-\mathbf{p}}}}\cdot\sqrt{E^u_{\mathbf{p}}+m_u}\left(1+\frac{E^{\pi^0}_{-\mathbf{p}}}{E^u_{\mathbf{p}}+m_u}\right), \tag{34}$$

$$B_{u\eta} = -\frac{g_A}{2\sqrt{6}\pi f_\pi}\frac{1}{\sqrt{N}}\int dp\, p^4 e^{-\frac{p^2}{2\lambda^2}}\frac{1}{\sqrt{4E^u_{\mathbf{p}}E^{\eta}_{-\mathbf{p}}}}\cdot\sqrt{E^u_{\mathbf{p}}+m_u}\left(1+\frac{E^{\eta}_{-\mathbf{p}}}{E^u_{\mathbf{p}}+m_u}\right), \tag{35}$$

$$B_{d\pi^+} = -\frac{g_A}{2\pi f_\pi}\frac{1}{\sqrt{N}}\int dp\, p^4 e^{-\frac{p^2}{2\lambda^2}}\frac{1}{\sqrt{4E^d_{\mathbf{p}}E^{\pi^+}_{-\mathbf{p}}}}\cdot\sqrt{E^d_{\mathbf{p}}+m_d}\left(1+\frac{E^{\pi^+}_{-\mathbf{p}}}{E^d_{\mathbf{p}}+m_d}\right), \tag{36}$$

$$B_{sK^+} = -\frac{g_A}{2\pi f_\pi}\frac{1}{\sqrt{N}}\int dp\, p^4 e^{-\frac{p^2}{2\lambda^2}}\frac{1}{\sqrt{4E^s_{\mathbf{p}}E^{K^+}_{-\mathbf{p}}}}\cdot\sqrt{E^s_{\mathbf{p}}+m_s}\left(1+\frac{E^{K^+}_{-\mathbf{p}}}{E^s_{\mathbf{p}}+m_s}\right). \tag{37}$$

By diagonalizing this Hamiltonian matrix, we will obtain a new mass of the constituent u quark M_u and its composite wave function. The constituent masses and wave functions of d and s quarks can be obtained similarly. We have

$$|d\rangle\rangle = z^d|d\rangle + x^d_{d\pi^0}|d\pi^0\rangle + x^d_{d\eta}|d\eta\rangle + x^d_{u\pi^-}|u\pi^-\rangle + x^d_{sK^0}|sK^0\rangle, \tag{38}$$

$$|s\rangle\rangle = z^s|s\rangle + x^s_{s\eta}|s\eta\rangle + x^s_{d\bar{K}^0}|d\bar{K}^0\rangle + x^s_{uK^-}|uK^-\rangle. \tag{39}$$

From isospin symmetry, $m_u = m_d$, we have

$$z^d = z^u; \quad x^d_{d\pi^0} = -x^u_{u\pi^0}; \quad x^d_{u\pi^-} = x^u_{d\pi^+}; \quad \ldots \tag{40}$$

However, since $m_u \neq m_s$, one should notice that

$$z^s \neq z^u; \quad x^s_{d\bar{K}^0} \neq x^d_{sK^0}; \quad x^s_{uK^-} \neq x^u_{sK^+}. \tag{41}$$

After the diagonalization, the Goldstone bosons Π are separated from quarks q approximately. With only degrees of freedom q one can re-build the quark model and so M_u, M_d, M_s should be regarded as the constituent quark masses in quark model.

III. FLAVOR AND SPIN STRUCTURE OF PROTON

Having known the wave functions of constituent quark q and the transition amplitudes of q emitting each Goldstone Bosons Π, we are able to calculate the quark distribution in a constituent quark following refs. 11, 12, 13. In χQM, Π will further split into a quark-antiquark pair. By substituting the quark contents of Π into wave functions (28), (38) and (39), we can rewrite the wave functions of constituent quark q as

$$\begin{aligned}
|u\rangle\rangle &= z^u|u\rangle + \left(\frac{x^u_{u\eta}}{\sqrt{6}} + \frac{x^u_{u\pi^0}}{\sqrt{2}}\right)|u(u\bar{u})\rangle + \left(\frac{x^u_{u\eta}}{\sqrt{6}} - \frac{x^u_{u\pi^0}}{\sqrt{2}}\right)|u(d\bar{d})\rangle \\
&\quad - \frac{2x^u_{u\eta}}{\sqrt{6}}|u(s\bar{s})\rangle + x^u_{d\pi^+}|d(u\bar{d})\rangle + x^u_{sK^+}|s(u\bar{s})\rangle, \qquad (42)\\
|d\rangle\rangle &= z^u|d\rangle + \left(\frac{x^u_{u\eta}}{\sqrt{6}} - \frac{x^u_{u\pi^0}}{\sqrt{2}}\right)|d(u\bar{u})\rangle + \left(\frac{x^u_{u\eta}}{\sqrt{6}} + \frac{x^u_{u\pi^0}}{\sqrt{2}}\right)|d(d\bar{d})\rangle \\
&\quad - \frac{2x^u_{u\eta}}{\sqrt{6}}|d(s\bar{s})\rangle + x^u_{d\pi^+}|u(d\bar{u})\rangle + x^u_{sK^+}|s(d\bar{s})\rangle, \qquad (43)\\
|s\rangle\rangle &= z^s|s\rangle + \frac{x^s_{s\eta}}{\sqrt{6}}|s(u\bar{u})\rangle + \frac{x^s_{s\eta}}{\sqrt{6}}|s(d\bar{d})\rangle - \frac{2x^s_{s\eta}}{\sqrt{6}}|s(s\bar{s})\rangle \\
&\quad + x^s_{d\bar{K}^0}|d(s\bar{d})\rangle + x^s_{uK^-}|u(s\bar{u})\rangle. \qquad (44)
\end{aligned}$$

Then the antiquark and quark flavor contents of the proton (uud) are

$$\bar{u} = 2\left|\frac{x^u_{u\eta}}{\sqrt{6}} + \frac{x^u_{u\pi^0}}{\sqrt{2}}\right|^2 + \left|\frac{x^u_{u\eta}}{\sqrt{6}} - \frac{x^u_{u\pi^0}}{\sqrt{2}}\right|^2 + |x^u_{d\pi^+}|^2, \qquad u = \bar{u} + 2, \qquad (45)$$

$$\bar{d} = \left|\frac{x^u_{u\eta}}{\sqrt{6}} + \frac{x^u_{u\pi^0}}{\sqrt{2}}\right|^2 + 2\left|\frac{x^u_{u\eta}}{\sqrt{6}} - \frac{x^u_{u\pi^0}}{\sqrt{2}}\right|^2 + 2|x^u_{d\pi^+}|^2, \qquad d = \bar{d} + 1, \qquad (46)$$

$$\bar{s} = 2|x^u_{u\eta}|^2 + 3|x^u_{sK^+}|^2, \qquad s = \bar{s}. \qquad (47)$$

Some important quantities depending on the above quark distribution are: the Gottfried sum rule $I_G = \frac{1}{3} + \frac{2}{3}(\bar{u} - \bar{d})$ whose deviation indicates the $\bar{u}$-$\bar{d}$ asymmetry in proton sea; $\bar{u}/\bar{d}$ measured through the ratio of muon pair production cross sections; and the fractions of quark flavors in proton $f_q = \frac{q+\bar{q}}{\Sigma(q+\bar{q})}$, $f_3 = f_u - f_d$ and $f_8 = f_u + f_d - 2f_s$.

We can further calculate the spin structure of proton. Here one should consider the effects of configuration mixing generated by spin-spin forces[21]. We take the baryon wave functions from the quark model calculation[22, 23, 24]. The proton wave function for example, is expressed as

$$\left|P, \frac{1}{2}^+\right\rangle = 0.90|P_8^2 S_S\rangle - 0.34|P_8^2 S_S'\rangle - 0.27|P_8^2 S_M\rangle \tag{48}$$

where the baryon $SU(6)\otimes O(3)$ wave functions are denoted as $|B_N^{2S+1}L_\sigma\rangle$, N is $SU(3)$ multiplicity. S, L are the total spin and total orbital angular momentum while $\sigma = S, M, A$ denotes the permutation symmetry of $SU(6)$. The spin polarization functions will be remarkably affected by configuration mixing. Following refs. 15, 17, we define the number operator by

$$\hat{N} = n_{u\uparrow}u_\uparrow + n_{u\downarrow}u_\downarrow + n_{d\uparrow}d_\uparrow + n_{d\downarrow}d_\downarrow + n_{s\uparrow}s_\uparrow + n_{s\downarrow}s_\downarrow,$$

where $n_{q\uparrow}$, $n_{q\downarrow}$ are the number of $q_\uparrow$, $q_\downarrow$ quarks. The spin structure of the "mixed" proton is given by

$$\begin{aligned}\hat{P} &\equiv \left\langle P, \frac{1}{2}^+\right| N \left|P, \frac{1}{2}^+\right\rangle \\ &= (0.90^2 + 0.34^2)\left(\frac{5}{3}u_\uparrow + \frac{1}{3}u_\downarrow + \frac{1}{3}d_\uparrow + \frac{2}{3}d_\downarrow\right) + 0.27^2\left(\frac{4}{3}u_\uparrow + \frac{2}{3}u_\downarrow + \frac{2}{3}d_\uparrow + \frac{1}{3}d_\downarrow\right).\end{aligned} \tag{49}$$

The spin structure after considering Π-emission is obtained by replacing for every quark in eq. (49) by

$$q_{\uparrow,\downarrow} \longrightarrow (1 - \Sigma P_i)q_{\uparrow,\downarrow} + P_{flipping}(q_{\uparrow,\downarrow}) + P_{non-flipping}(q_{\uparrow,\downarrow}), \tag{50}$$

where $P_{flipping}(q_{\uparrow,\downarrow})$ and $P_{non-flipping}(q_{\uparrow,\downarrow})|$ are the probabilities of quark helicity flipping and non-flipping for $q_{\uparrow,\downarrow}$ respectively. For example, in the case of $u_\uparrow$ quark we have,

$$P_{flipping}(u_\uparrow) = \frac{2}{3}\left[(|x^u_{u\pi^0}|^2 + |x^u_{u\eta}|^2)u_\downarrow + |x^u_{d\pi^+}|^2 d_\downarrow + |x^u_{sK^+}|^2 s_\downarrow\right],$$

and

$$P_{non-flipping}(u_\uparrow) = \frac{1}{3}\left[(|x^u_{u\pi^0}|^2 + |x^u_{u\eta}|^2)u_\uparrow + |x^u_{d\pi^+}|^2 d_\uparrow + |x^u_{sK^+}|^2 s_\uparrow\right].$$

Finally the spin polarization functions defined as $\Delta q = q_\uparrow - q_\downarrow$ are

$$\begin{aligned}
\Delta u &= (0.90^2+0.34^2)\left[\frac{4}{3}-\left(\frac{114|x^u_{u\pi^0}|^2+48|x^u_{u\eta}|^2+36|x^u_{sK^+}|^2}{27}\right)\right] \\
&\quad + 0.27^2\left[\frac{2}{3}-\left(\frac{66|x^u_{u\pi^0}|^2+24|x^u_{u\eta}|^2+18|x^u_{sK^+}|^2}{27}\right)\right], \qquad (51)\\
\Delta d &= (0.90^2+0.34^2)\left[-\frac{1}{3}+\left(\frac{6|x^u_{u\pi^0}|^2+12|x^u_{u\eta}|^2+9|x^u_{sK^+}|^2}{27}\right)\right] \\
&\quad + 0.27^2\left[\frac{1}{3}-\left(\frac{42|x^u_{u\pi^0}|^2+12|x^u_{u\eta}|^2+9|x^u_{sK^+}|^2}{27}\right)\right], \qquad (52)\\
\Delta s &= -\frac{|x^u_{sK^+}|^2}{3}. \qquad (53)
\end{aligned}$$

There are several measured quantities which can be expressed in terms of the above spin polarization functions. The quantities usually calculated are $\Delta_3 = \Delta u - \Delta d$ and $\Delta_8 = \Delta u + \Delta d - 2\Delta s$, obtained from the neutron β-decay and the weak decays of hyperons respectively. Another important quantity is the flavor singlet component of the total quark spin content defined as $2\Delta\Sigma = \Delta u + \Delta d + \Delta s$. We also calculate some weak axial-vector form factors which are also related to the spin polarization functions, $(G_A/G_V)_{\Lambda\to p} = \frac{1}{3}(2\Delta u - \Delta d - \Delta s)$, $(G_A/G_V)_{\Sigma^-\to n} = \Delta d - \Delta s$, and $(G_A/G_V)_{\Xi^-\to\Lambda} = \frac{1}{3}(\Delta u + \Delta d - 2\Delta s)$.

IV. BARYON OCTET MAGNETIC MOMENTS

Consider the relative angular momentum between quark and Goldstone boson Π, the magnetic moment operator of a $q\Pi$ system is

$$\begin{aligned}
\hat{\mu}_{q\Pi} &= \frac{e_q}{m_q}\,\hat{\mathbf{s}} + \frac{e_q}{2\sqrt{\mathbf{p}_q^2+m_q^2}}\frac{\sqrt{\mathbf{p}_\Pi^2+m_\Pi^2}}{\sqrt{\mathbf{p}_q^2+m_q^2}+\sqrt{\mathbf{p}_\Pi^2+m_\Pi^2}}\,\hat{\mathbf{l}} \\
&\quad + \frac{e_\Pi}{2\sqrt{\mathbf{p}_\Pi^2+m_\Pi^2}}\frac{\sqrt{\mathbf{p}_q^2+m_q^2}}{\sqrt{\mathbf{p}_q^2+m_q^2}+\sqrt{\mathbf{p}_\Pi^2+m_\Pi^2}}\,\hat{\mathbf{l}} \qquad (54)
\end{aligned}$$

where e_q and e_Π are the electric charges carried by q and Π respectively, $\hat{\mathbf{s}}$ the quark spin operator and $\hat{\mathbf{l}}$ the relative angular momentum bewteen q and Π. The first term in Eq(54) is the intrinsic magnetic moment of quark and the other two terms are the contribution of the orbital angular momentum. Here we have to consider the relativistic effect since the relative momentum of q or Π are comparable to their masses in the $q\Pi$ system

$$\mathbf{p}_{q,\Pi} \sim \Lambda \sim m_{q,\Pi}.$$

With the SHO wave functions of (20), the magnetic moment of $q\Pi$ system (54) can be readily calculated. Then we can re-calculate the magnetic moments of constituent quarks taking into account of the relativistic effect. For example, the magnetic moments of the u quark is

$$\mu_u = |z^u|^2 \langle u_\uparrow|\hat{\mu}|u_\uparrow\rangle + P_{u\to u\pi^0}\langle u\pi^0|\hat{\mu}|u\pi^0\rangle + P_{u\to u\eta}\langle u\eta|\hat{\mu}|u\eta\rangle$$
$$+ P_{u\to d\pi^+}\langle d\pi^+|\hat{\mu}|d\pi^+\rangle + P_{u\to sK^+}\langle sK^+|\hat{\mu}|sK^+\rangle, \qquad (55)$$

where

$$\langle u_\uparrow|\hat{\mu}|u_\uparrow\rangle = \frac{e_u}{2m_u}, \qquad (56)$$

and the contribution from $q\Pi$ systems are

$$\langle u\pi^0|\hat{\mu}|u\pi^0\rangle = -\frac{e_u}{6m_u} + \frac{e_u}{3N}\int dp\, \frac{\sqrt{p^2+m_\pi^2}}{\sqrt{p^2+m_u^2}+\sqrt{p^2+m_\pi^2}}\frac{1}{\sqrt{p^2+m_u^2}}p^4\, e^{-\frac{p^2}{\lambda^2}}, \qquad (57)$$

$$\langle u\eta|\hat{\mu}|u\eta\rangle = -\frac{e_u}{6m_u} + \frac{e_u}{3N}\int dp\, \frac{\sqrt{p^2+m_\eta^2}}{\sqrt{p^2+m_u^2}+\sqrt{p^2+m_\eta^2}}\frac{1}{\sqrt{p^2+m_u^2}}p^4\, e^{-\frac{p^2}{\lambda^2}}, \qquad (58)$$

$$\langle d\pi^+|\hat{\mu}|d\pi^+\rangle = -\frac{e_d}{6m_d} + \frac{e_d}{3N}\int dp\, \frac{\sqrt{p^2+m_\pi^2}}{\sqrt{p^2+m_d^2}+\sqrt{p^2+m_\pi^2}}\frac{1}{\sqrt{p^2+m_d^2}}p^4\, e^{-\frac{p^2}{\lambda^2}}$$
$$+ \frac{e_{\pi^+}}{3N}\int dp\, \frac{\sqrt{p^2+m_d^2}}{\sqrt{p^2+m_d^2}+\sqrt{p^2+m_\pi^2}}\frac{1}{\sqrt{p^2+m_\pi^2}}p^4\, e^{-\frac{p^2}{\lambda^2}}, \qquad (59)$$

$$\langle sK^+|\hat{\mu}|sK^+\rangle = -\frac{e_s}{6m_s} + \frac{e_s}{3N}\int dp\, \frac{\sqrt{p^2+m_K^2}}{\sqrt{p^2+m_s^2}+\sqrt{p^2+m_K^2}}\frac{1}{\sqrt{p^2+m_s^2}}p^4\, e^{-\frac{p^2}{\lambda^2}}$$
$$+ \frac{e_{K^+}}{3N}\int dp\, \frac{\sqrt{p^2+m_s^2}}{\sqrt{p^2+m_s^2}+\sqrt{p^2+m_K^2}}\frac{1}{\sqrt{p^2+m_K^2}}p^4\, e^{-\frac{p^2}{\lambda^2}}. \qquad (60)$$

The magnetic moments of d and s quarks can be calculated similarly.

One can easily obtain the octet baryon magnetic moments by replacing the valence quarks inside the baryons with the corresponding constituent quarks. Again we take proton as an example,

$$\mu_p = (0.90^2 + 0.34^2)\left(\frac{4}{3}\mu_u - \frac{1}{3}\mu_d\right) + 0.27^2\left(\frac{2}{3}\mu_u + \frac{1}{3}\mu_d\right). \qquad (61)$$

The magnetic moments for other octet baryons can be calculated similarly.

V. NUMERICAL RESULTS AND CONCLUSIONS

In the numberical calculation, most of the parameters can be taken from the experimental data or the chiral quark model. We collect these fixed input parameters of our calculation in Table I. Here we have used the the physical masses of Goldstone bosons[25].

TABLE I: The fixed input parameters from chiral quark model and experimental data.

g_A	f_π(MeV)	m_π(MeV)	m_K(MeV)	m_η(MeV)
0.7524	93	135	494	548

For the quark masses, since our work focus on the inner context of the constituent quarks in quark model, naturally we will refer to the quark masses from quark model, instead of the chiral quark model values. Here we will use the quark masse values from the widely accepted Isgur's quark model[20] as shown in Table II. However, one should be cautious that, in our model, it is the quark with the Goldstone boson mixing which corresponds to the constituent quark in quark model. That is mass values M_q after the diagonalization process should be set to the quark masses in Isgur's model. Our strategy is to adjust the quark masses m_q in the model hamiltonian to fit the M_q values.

Finally we are left only with one free parameter λ which describe the confinement of the emitted Goldstone boson in our model. An overall fit to the experimental data of nucleon flavor-spin structure and octet baryon magnetic moments shows that the best value should be λ=152MeV. With this value of λ and a minimun binding energe $E_B = -218$MeV, The "bare" values of quark masses m_q without Goldstone boson mixing are shown also in Table II.

TABLE II: The quark masses with vs. without Goldstone boson mixing.

λ	E_B(MeV)	$m_{u,d}$(MeV)	m_s(MeV)	$M_{u,d}$(MeV)	M_s(MeV)
152	-218	288	474	220	419

Transition probabilities of the light and strange quarks to various $q'\Pi$ systems are given in Table III and IV respectively. The probability of a u quark emitting a π^+ $P(u \to d + \pi^+)$=0.145 is significantly larger than the purturbation calculation a=0.083. Also, we notice that the asymmetry between the probabilities of $u(d) \to s + K$ and $s \to u(d) + \bar{K}$. Whether this asymmetry leads to any observable consequence in hadron structure needs further investigation.

Next, we will compare our calculate results with the experimental data. Since our em-

TABLE III: Transition probabilities of a u quark to various $q'\Pi$ systems and the mass of constituent u quark.

$u \rightarrow u + \pi^0$	$u \rightarrow u + \eta$	$u \rightarrow d + \pi^+$	$u \rightarrow s + K^+$	no GB-emission	M_u
0.072	0.003	0.145	0.010	0.770	220MeV

TABLE IV: Transition probabilities of a s quark to various $q'\Pi$ systems and the mass of constituent s quark.

$s \rightarrow s + \eta$	$s \rightarrow u + K^-$	$s \rightarrow d + \bar{K}^0$	no GB-emission	M_s
0.012	0.071	0.071	0.846	419MeV

phasis is on the substructure of a constituent quark in NQM, here we also quote the results from NQM. In Table V, the calculated flavor and spin structures of the proton are shown. It should be mentioned that the quark spin polarization functions can be further corrected by the gluon anomaly[13, 15, 17, 26, 27, 28] through

$$\Delta q(Q^2) = \Delta q - \frac{\alpha_s(Q^2)}{2\pi}\Delta g(Q^2), \tag{62}$$

and the flavor singlet component of the total helicity is modified accordingly as

$$\Delta\Sigma(Q^2) = \Delta\Sigma - \frac{3\alpha_s(Q^2)}{4\pi}\Delta g(Q^2), \tag{63}$$

where $\Delta q(Q^2)$ and $\Delta\Sigma(Q^2)$ are the experimentally measured quantities, Δq and $\Delta\Sigma$ correspond to the calculated quantities without gluon correction. Using the experimental data $\Sigma(Q^2 = 5\text{GeV}^2) = 0.19 \pm 0.02$[2], $\alpha_s(Q^2 = 5\text{GeV}^2) = 0.285 \pm 0.013$[25], and our result $\Delta\Sigma$=0.346, the gluon polarization $\Delta g(Q^2)$ is estimated to be 2.293. Both the results with and without gluon polarization corrections are presented in Table V. The inclusion of gluon polarization leads to a better agreement with experimental data for the spin structure.

The calculated magnetic moments of octet baryons are given in Table VI. Although the deviation is somewhat around 30% in the case of Ξ^-, our overall fit to octet baryon magnetic moments is in good agreement with experiments. Also it should be mentioned that even in the case of Ξ^- the fit can perhaps be improved if corrections due to pion loops are taken into account[30, 31].

In the model calculations [11, 12, 13, 14, 15, 16, 17, 18, 19], the Goldstone boson sector in χQM is usually extended to include the η' meson with $U(3)$ symmetry. According to Cheng

TABLE V: The calculated values for the quark flavor distribution functions and spin polarization functions in proton, as compared with experimental data and NQM results.

	Data	NQM	Our Model	
			With Δg	Without Δg
Δu	0.85 ± 0.05[2]	1.33	0.864	0.968
Δd	-0.41 ± 0.05[2]	-0.33	-0.377	-0.274
Δs	-0.07 ± 0.05[2]	0	-0.107	-0.003
$\Delta_3 = (G_A/G_V)_{n\to p}$	1.270 ± 0.003[25]	1.67	1.242	1.242
$(G_A/G_V)_{\Lambda\to p}$	0.718 ± 0.015[25]	1	0.737	0.737
$(G_A/G_V)_{\Sigma\to n}$	-0.340 ± 0.017[25]	-0.33	-0.270	-0.270
$(G_A/G_V)_{\Xi\to\Lambda}$	0.25 ± 0.05[25]	0.33	0.234	0.234
Δ_8	0.58 ± 0.025[2]	1	0.701	0.701
$\Delta\Sigma$	0.19 ± 0.02[2]	0.5	0.190	0.346
$\bar{u}$	–		0.264	
$\bar{d}$	–		0.392	
$\bar{s}$	–		0.036	
$\bar{u} - \bar{d}$	-0.118 ± 0.015[6]	0	-0.128	
$\bar{u}/\bar{d}$	0.67 ± 0.06[6]	1	0.674	
I_G	0.254 ± 0.005[6]	0.33	0.248	
f_u	–		0.577	
f_d	–		0.407	
f_s	0.10 ± 0.06[29]	0	0.017	
f_3	–		0.170	
f_8	–		0.950	
f_3/f_8	0.21 ± 0.05[14]	0.33	0.179	

and Li[11], in the large N_c limit of QCD, there are nine Goldstone bosons including the usual octet and the singlet η'. Thus an constituent quark can also transit to a quark-η' system. We have also made an $U(3)$ calculation. With the inclusion of η', we find that the probabilities for η'-emission from light and strange quarks $P(u \to u + \eta')$=$P(d \to d + \eta')$=0.0021 and

TABLE VI: The caculated octet baryon magnetic moments in nuclear magneton, as compared with experiments and the results of NQM.

Octet baryons	Data[25]	NQM[32]	Our model
p	2.79 ± 0.00	2.72	2.73
n	-1.91 ± 0.00	-1.81	-1.91
Σ^-	-1.16 ± 0.025	-1.01	-1.23
Σ^+	2.46 ± 0.01	2.61	2.67
Ξ^0	-1.25 ± 0.0014	-1.41	-1.36
Ξ^-	-0.65 ± 0.002	-0.50	-0.44
Λ	-0.61 ± 0.004	-0.59	-0.56
$\Sigma\Lambda$	1.61 ± 0.08	1.51	1.63

$P(s \to s + \eta')$=0.0018 which are negligibly small as compared to those of octet Goldstone boson emissions. We therefore conclude that the contribution of η' is not important, due to the obvious axial $U(1)$ symmetry breaking in meson mass spectra $m_{\eta'} > m_{K,\eta}$.

To summarize, the χQM builds a bridge between the QCD and low-energy quark model. This allow us to understand the mechanism of flavor symmetry breaking and nucleon flavor-spin structure in NQM through the consideration of the sea quark and Goldstone bosons in the substructure of constituent quarks. Using the simple SHO wave function, we have modeled the wave functions of the composite constituent quarks and thus estimated the transition probabilities for Goldstone boson emissions. These transition probabilities indeed reflect the flavor SU(3) symmetry breaking in χQM from the differences in quark masses $m_s > m_{u,d}$ and differences in Goldstone bosons masses $m_{K,\eta} > m_\pi$ and roughly in agreement with the parametriation of other model calculations [11, 12, 13, 14, 15, 16, 17, 18, 19]. The fit to both the flavor-spin structure of nucleon and octet baryon magnetic moments are in good agreement with experiments.

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