Fermi-liquid effects in the transresistivity in quantum Hall double layers near $\nu = 1/2$

Natalya A. Zimbovskaya

Department of Physics and Astronomy, St. Cloud State University,
720 Fourth Avenue South, St. Cloud, MN 56301, USA;
Urals State Mining University, Kuibysheva Str. 30, Yekaterinburg, Russia, 620000
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Here, we present theoretical studies of the temperature and magnetic field dependences of the Coulomb drag transresistivity between two parallel layers of two dimensional electron gases in quantum Hall regime near half filling of the lowest Landau level. It is shown that Fermi-liquid interactions between the relevant quasiparticles could give a significant effect on the transresistivity, providing its independence of the interlayer spacing for spacings taking on values reported in the experiments. Obtained results agree with the experimental evidence.

During the last decade double-layer two-dimensional electron gas (2DEG) systems were of significant interest due to many remarkable phenomena they exhibit, including so called Coulomb drag. In Coulomb drag experiments two 2DEGs are arranged close to each other, so that they can interact via Coulomb forces. A current I is applied to one layer of the system, and the voltage V_D in the other nearby layer is measured, with no current allowed to flow in that layer. The ratio $-V_D/I$ gives a transresistivity ρ_D which characterizes the strength of the effect. The physical interpretation of the Coulomb drag is that momentum is tranferred from the current carrying layer to the nearby one due to interlayer interactions [1, 2, 3].

It was shown theoretically [4, 5] and confirmed with experiments [5] that the transresistivity between two 2DEGs in quantum Hall regime at one half filling of the lowest Landau level for both layers is proportional to $T^{4/3}$ (T is the temperature of the system) which is quite different from the temperature dependence of ρ_D in the absence of the external magnetic field applied to 2DEGs. This temperature dependence of the drag at $\nu = 1/2$ originates from the ballistic contribution to the transresistivity. The latter reflects the response of the two-layer system to the driving disturbance of finite wave vector $\mathbf{q}$ and finite frequency ω when considering scales are smaller than the mean free path l of electrons $(ql \gg 1)$, and times are shorter than their scattering time τ $(\omega\tau \gg 1)$ [6].

In further experiments [7] the Coulomb drag was measured between 2DEGs where the layer filling factor was varied around $\nu = 1/2$. The transresistivity was reported to be enhanced quadratically with $\Delta\nu = \nu - 1/2$. It was also reported that the curvature of the enhancement depended on temperature but it was insensitive to both sign of $\Delta\nu$ and distance d between the layers. The present work is motivated with these experiments of [7]. We calculate the transresistivity between two layers of 2DEGs subject to a strong magnetic field which provides ν close to $1/2$ for both layers.

We start from the well-known expression [1, 3] which relates the Coulomb drag transresistivity to density-density components of the polarization in the layers $\Pi_{(1)}(\mathbf{q},\omega)$ and $\Pi_{(2)}(\mathbf{q},\omega)$:

$$\rho_D = \frac{1}{2(2\pi)^2}\frac{h}{e^2}\frac{1}{Tn^2}\int\frac{q^2 d\mathbf{q}}{(2\pi)^2}\int\frac{\hbar d\omega}{\sinh^2(\hbar\omega/2T)}$$
$$\times|U(\mathbf{q},\omega)|^2 \mathrm{Im}\Pi_{(1)}(\mathbf{q},\omega)\mathrm{Im}\Pi_{(2)}(\mathbf{q},\omega). \qquad (1)$$

Here, $U(\mathbf{q},\omega)$ is the screened interlayer Coulomb interaction, and electron densities in the layers are supposed to be equal $(n_1 = n_2 = n)$.

Within the usual Composite Fermion (CF) approach [8] a single layer polarizability describes that part of the density-current electromagnetic response which is irreducible with respect to the Coulomb interaction. Adopting for simplicity the RPA, we obtain the following expression for the 2×2 polarizability matrix:

$$\Pi^{-1} = (K^0)^{-1} + C^{-1}. \qquad (2)$$

Here, the matrix K^0 gives the response of noninteracting CFs and C is the Chern-Simons interaction matrix. Assuming for certainty the wave vector $\mathbf{q}$ to lie in the "x" direction we have:

$$C = \begin{pmatrix} 0 & \dfrac{iq}{4\pi\hbar} \\ -\dfrac{iq}{4\pi\hbar} & 0 \end{pmatrix}. \qquad (3)$$

Starting from the expression (2) we arrive at the following results for the density-density response function $\Pi_{00(i)}(\mathbf{q},\omega)$:

$$\Pi_{00(i)}(\mathbf{q},\omega) = \Pi_{(i)}(\mathbf{q},\omega)$$

$$= \frac{K^0_{00(i)}(\mathbf{q},\omega)}{1 - \dfrac{8i\pi\hbar}{q}K^0_{01(i)}(\mathbf{q},\omega) - \left(\dfrac{4\pi\hbar}{q}\right)^2\Delta_{(i)}(\mathbf{q},\omega)}. \qquad (4)$$

Here

$$\Delta_{(i)}(\mathbf{q},\omega) = K^0_{00(i)}(\mathbf{q},\omega)K^0_{11(i)}(\mathbf{q},\omega) + \left(K^0_{01(i)}(\mathbf{q},\omega)\right)^2. \tag{5}$$

Within the RPA response functions included in Eqs. (4), (5) are simply related to components of the CF conductivity tensor $\tilde{\sigma}$ [8]:

$$\frac{1}{\tilde{\sigma}^{(i)}_{xx}(\mathbf{q},\omega)} = \frac{iq^2}{\omega e^2}\left[\frac{1}{K^0_{00(i)}(\mathbf{q},\omega)} - \frac{1}{K^0_{00(i)}(\mathbf{q},0)}\right];$$

$$\tilde{\sigma}^{(i)}_{yy}(\mathbf{q},\omega) = -\frac{ie^2}{\omega}\left[K^0_{11(i)}(\mathbf{q},\omega) - K^0_{11(i)}(\mathbf{q},0)\right];$$

$$\tilde{\sigma}^{(i)}_{xy} = -\tilde{\sigma}^{(i)}_{yx} = \frac{ie^2}{q}K^0_{01(i)}(\mathbf{q},\omega). \tag{6}$$

To proceed we calculate the components of the CF conductivity at ν slightly away from $1/2$. In this case CFs experience a nonzero effective magnetic field $B_{eff} = B - B_{1/2}$. We concentrate on the ballistic contribution to the transresistivity, so we need asymptotics for the relevant conductivity components applicable in a nonlocal ($ql \gg 1$) and high frequency ($\omega\tau \gg 1$) regime. Corresponding expressions for $\tilde{\sigma}_{ij}$ were obtained in earlier works [8]. However, these results are not appropriate for our analysis for they do not provide a smooth passage to the $B_{eff} \to 0$ limit at finite $\mathbf{q}$. Due to this reason we do not use them in further calculations. To get a suitable approximation for the CF conductivity we start from the standard solution of the Boltzmann transport equation for the CF distribution function. This gives us the following results for the CF conductivity components for a single layer [9]:

$$\tilde{\sigma}_{\alpha\beta} = \frac{m^*e^2}{(2\pi\hbar)^2}\frac{1}{\Omega}\int_0^{2\pi}d\psi v_\alpha(\psi)\exp\left[-\frac{iq}{\Omega}\int_0^\psi v_x(\psi'')d\psi''\right]$$

$$\times \int_{-\infty}^\psi v_\beta(\psi')\exp\left[\frac{iq}{\Omega}\int_0^{\psi'}v_x(\psi'')d\psi''\right.$$

$$\left.+\frac{1}{\Omega\tau}(\psi'-\psi)(1-i\omega\tau)\right]d\psi'. \tag{7}$$

Here, m^*, Ω are the CF effective mass and the cyclotron frequency at the effective magnetic field B_{eff}; ψ is the angular coordinate of the CF cyclotron orbit. Now we carry out some formal transformations of this expression (7) following the way proposed before [9, 10]. First, we expand the CF velocity components $v_\beta(\psi')$ in Fourier series:

$$v_\beta(\psi') = \sum_k v_{k\beta}\exp(ik\psi'). \tag{8}$$

Substituting this expansion (8) into (7) we obtain:

$$\tilde{\sigma} = \frac{m^*e^2}{(2\pi\hbar)^2}\sum_k v_{k\beta}\int_0^{2\pi}d\psi v_\alpha(\psi)\exp(ik\psi)$$

$$\times \int_{-\infty}^0 \exp\left[\left(ik\Omega - i\omega + \frac{1}{\tau} + iqv_x(\psi)\right)\theta\right.$$

$$\left.+iq\int_0^\theta\left(v_x(\psi+\Omega\theta') - v_x(\psi)\right)d\theta'\right]d\theta \tag{9}$$

where $\theta = (\psi' - \psi)/\Omega$.

Then we introduce a new variable η which is related to the variable θ as follows:

$$\eta = \left(ik\Omega - i\omega + \frac{1}{\tau} + iqv_x(\psi)\right)\theta$$

$$+iq\int_0^\theta\left[v_x(\psi+\Omega\theta') - v_x(\psi)\right]d\theta', \tag{10}$$

and we arrive at the result:

$$\tilde{\sigma}_{\alpha\beta} = \frac{im^*e^2}{(2\pi\hbar)^2}\sum_k v_{k\beta}\int_{-\infty}^0 e^\eta d\eta$$

$$\times\int_0^{2\pi}\frac{v_\alpha(\psi)\exp(ik\psi)}{\omega + i/\tau - k\Omega - qv_x(\psi+\Omega\theta)}d\psi. \tag{11}$$

Under the conditions of interest $\omega\tau \gg 1$, $ql \gg 1$, and also assuming that the filling factor is close to $\nu = 1/2$, so that $qv_F \gg \Omega$ (v_F is the CFs Fermi velocity), the variable θ is approximately equal to $\eta\tau(1 + iql\cos\psi + ik\Omega\tau - i\omega\tau)^{-1}$. Taking this into account and expanding the last term in the denominator of (11) in powers of $\Omega\theta$ we obtain:

$$qv_x(\psi + \Omega\theta)$$

$$\approx qv_x(\psi) + \eta\Omega q\tau(1 + iql\cos\psi + ik\Omega\tau - i\omega\tau)^{-1}\frac{dv_x}{d\psi}$$

$$+ q\frac{\eta^2}{2}(\Omega\tau)^2(1 + iql\cos\psi + ik\Omega\tau - i\omega\tau)^{-2}\frac{d^2v_x}{d\psi^2}. \tag{12}$$

Substituting this asymptotic expression into (9) we can calculate first terms of the expansions of relevant components of the CF conductivity in powers of the small parameter $(qR)^{-1}$ where $R = v_F/\Omega$ is the CF cyclotron radius. Within the "collisionless" limit $1/\tau \to 0$ we have:

$$\tilde{\sigma}_{xx} = -N\frac{i\omega}{q^2}e^2\left\{1 + \frac{i\delta}{\sqrt{1-\delta^2}} + \frac{i\delta}{\sqrt{(1-\delta^2)^5}}\right.$$

$$\left.\times\frac{1}{2(qR)^2}\left(1 - \frac{5}{4}\frac{1}{1-\delta^2}\right)\right\}; \tag{13}$$

$$\tilde{\sigma}_{yy} = N\frac{v_Fe^2}{q}\left\{\sqrt{1-\delta^2} + i\delta + \frac{1}{2(qR)^2}\right.$$

$$\left.\times\left[\frac{7}{4}\frac{1}{\sqrt{(1-\delta^2)^5}} - \frac{1}{\sqrt{(1-\delta^2)^3}}\right]\right\}; \tag{14}$$

$$\tilde{\sigma}_{xy} = iN\frac{v_Fe^2}{q}\frac{\delta}{2qR}\left[\frac{1}{\sqrt{1-\delta^2}} + \frac{\delta^2}{\sqrt{(1-\delta^2)^3}}\right]. \tag{15}$$

Here, $N = m^*/2\pi\hbar^2$ is the density of states at the CF Fermi surface, and $\delta = \omega/qv_F$. Using these results we can easily get approximations for the functions $K^0_{\alpha\beta(i)}(\mathbf{q},\omega)$ $(\alpha,\beta = 0.1)$ and, subsequently, the desired density-density response function given by (4). It was shown [3] that the integral over ω in the expression for ρ_D (1) is dominated by $\omega \sim T$, and the major contribution to the integral over q in this expression comes from $q \sim k_F(T/T_0)^{1/3}$, where k_F is the Fermi wave vector and the scaling temperature T_0 is defined below. Therefore we get an estimate for δ, namely $\delta \sim (T/\mu)(T_0/T)^{1/3}$, where μ is the chemical potential of a single 2DEG included in the bilayer. For the parameter T_0 taking on values of the order of room temperature, δ is small compared to unity at low temperatures $(T \sim 1K)$.

Here, we limit ourselves to the case of two identical layers $(\Pi_{(1)} = \Pi_{(2)} \equiv \Pi)$. For $\delta \ll 1$ we obtain the approximation:

$$\Pi_{00}(\mathbf{q},\omega)$$

$$= \frac{q^3}{q^3\left(\frac{dn}{d\mu}\right)^{-1} - 8\pi i\hbar\omega k_F\left(1 + 2(k_F R)^{-1} + \frac{3}{8}(qR)^{-2}\right)}.$$

$$(16)$$

Here, $dn/d\mu$ is the compressibility of the $\nu = 1/2$ state which is defined as [3]:

$$\frac{dn}{d\mu} \equiv \Pi_{00}(\mathbf{q} \to 0;\ \omega \to 0) = \frac{3m^*}{8\pi\hbar^2}. \qquad (17)$$

This differs from the compressibility of the noninteracting 2DEG in the absence of an external magnetic field (the latter equals N). The difference in the compressibility values is a manifestation of the Chern-Simons interaction in strong magnetic fields.

In following calculations we adopt the expression used in the work [3] for the screened interlayer potential $U(\mathbf{q},\omega)$, namely:

$$U(\mathbf{q},\omega) = \frac{1}{2}\frac{V_b + U_b}{1 + \Pi(\mathbf{q},\omega)(V_b + U_b)}$$

$$-\frac{1}{2}\frac{V_b - U_b}{1 + \Pi(\mathbf{q},\omega)(V_b - U_b)} \qquad (18)$$

where $V_b(\mathbf{q}) = 2\pi e^2/\epsilon q$ and $U_b(\mathbf{q}) = (2\pi e^2/\epsilon q)e^{-qd}$ are Fourier components of the bare Coulomb potentials for intralayer and interlayer interactions, respectively, and ϵ is the dielectric constant. Substituting (18) into (1) and using our result (16) for $\Pi(\mathbf{q},\omega)$ we can present the transresistvity in the "ballistic" regime as:

$$\rho_D = \rho_{D0} + \delta\rho_D. \qquad (19)$$

Here, the first term ρ_{D0} is the transresistivity at $\nu = 1/2$ when the effective magnetic field is zero, and the second term gives a correction arising in a nonzero effective magnetic field (away from $\nu = 1/2$). As it was to be expected, our expression for ρ_{D0} coincides with the already known result [3]:

$$\rho_{D0} = \frac{h}{e^2}\frac{\Gamma(7/3)\zeta(4/3)}{3\sqrt{3}}\left(\frac{T}{T_0}\right)^{4/3} \qquad (20)$$

with $T_0 = (\pi e^2 nd/\epsilon)(1 + \alpha)$, and

$$\frac{1}{\alpha} = \frac{2\pi e^2 d}{\epsilon}\frac{dn}{d\mu}. \qquad (21)$$

The leading term of the correction $\delta\rho_D$ at low temperatures $(T/T_0) \ll 1$ can be written as follows:

$$\delta\rho_D = \frac{2}{3}\rho_{D0}\frac{1}{k_F R}\left(1 + \frac{3}{8}\frac{1}{k_F R}\right) + a^2\frac{h}{e^2}\left(\frac{2T}{T_0}\right)^{2/3}\frac{1}{(k_F R)^2}$$

$$\approx \frac{4}{3}\rho_{D0}\Delta\nu\left(1 + \frac{3}{4}\Delta\nu\right) + 4a^2\frac{h}{e^2}\left(\frac{2T}{T_0}\right)^{2/3}(\Delta\nu)^2.$$

$$(22)$$

Here, the dimensionless positive constant a^2 can be approximated as:

$$a^2 = \frac{7}{24\sqrt{3}}\int_0^\infty\left(\frac{y^{2/3}}{\sinh^2 y} - \frac{1}{y^{4/3}\cosh^2 y}\right)dy. \qquad (23)$$

We have to remark that our result (23) cannot be used in the limit $T \to 0$. Actually, this expression provides a good asymptotic form for the coefficient a^2 when $(Tk_F l/\mu)^{1/3} \geq 1.5$. Assuming that the mean free path is of the order of $1.0\mu m$ as in the experiments [11] on dc magnetotransport in a single modulated 2DEG at ν close to $1/2$, and using the estimate of [7] for the electron density $n = 1.4 \times 10^{15}m^{-2}$, we obtain that the expression (23) gives good approximation for a^2 when T/μ is no less than 10^{-2}.

It follows from our results (19), (22) that transresistivity ρ_D enhances nearly quadratically with $\Delta\nu$ when the filling factor deviates from $\nu = 1/2$. The linear in $\Delta\nu$ term is also present in the expression for $\delta\rho_D$. This causes an asymmetric shape of the plot of Eq. (22) relative to $\Delta\nu = 0$. However, this asymmetry is not very significant for the linear term is smaller than the last term on the right hand side of (22). This difference in magnitudes is due to different temperature dependences of the considered terms. The first term including the linear in $(k_F R)^{-1}$ correction is proportional to $(T/T_0)^{4/3}$, whereas the second one is proportional to $(T/T_0)^{2/3}$ and predominates at low temperatures. So, the magnetic field dependence of the transresistivity near $\nu = 1/2$ matchs that observed in the experiments (See Fig. 1).

Keeping only the greatest term in (22), the ratio ρ_D/ρ_{D0} can be presented in the form:

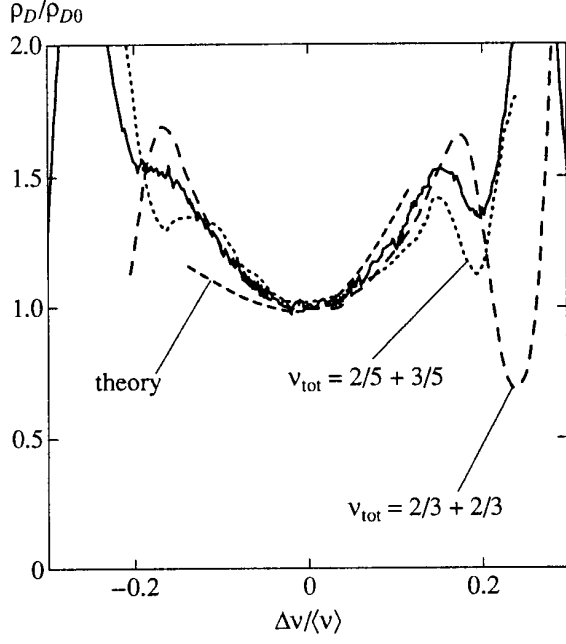$$\frac{\rho_D}{\rho_{D0}} = 4\beta(\Delta\nu)^2 + 1. \qquad (24)$$

FIG. 1: Scaled drag resistivity versus $\Delta\nu$ at T = 0.6; lowest dashed curve is the plot of Eq. (22) at $m^* = 4m_b$; $A_0 = 15$, and remaining curves present experimental data of [7];
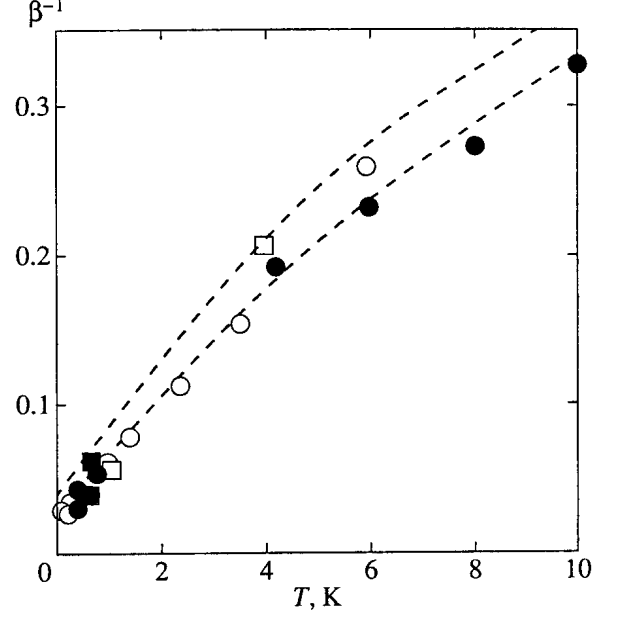

FIG. 2: Temperature dependence of the coefficient β^{-1} for interlayer distances $d = 10nm$ (upper curve) and $d = 22.5nm$ (lower curve) compared to the summary of experimental curvature at both spacings [7]

Here, the coefficient β equals:

$$\beta = \frac{3\sqrt{3}a^2}{\Gamma(7/3)\zeta(4/3)}\left(\frac{2T_0}{T}\right)^{2/3}. \qquad (25)$$

This coefficient is proportional to the curvature of the plot of Eq. (22) assuming that the first term is neglected. The curvature reveals a strong dependence on temperature whose character also agrees with experiments of [7] as it is shown in Fig. 2.

A striking feature in the experimental results is that they appear to be nonsensitive to the distance between the 2DEGs. Sets of data corresponding to samples with different interlayer spacings $d_A = 10nm$ and $d_B = 22.5nm$ fall on the same curve. This concerns both magnetic field dependence of the transresistivity and temperature dependence of the parameter β. Results of the present analysis provide a possible explanation for this feature. It follows from (20)–(25) that the dependence of ρ_D of the interlayer spacing is completely included in the characteristic temperature T_0 which is defined with Eq. (21). The above quantity is nearly independent of the interlayer separation d when the parameter α takes on values larger that unity. Estimating the parameter α as it is given by Eq. (21), we obtain that the condition $\alpha > 1$ could be satisfied for small values of the compressibility of the $\nu = 1/2$ state. However, within the RPA the effective mass of CFs coincides with the single electron band mass m_b which takes on the value $m_b \approx 0,07m_e$ for GaAs wells (m_e is the mass of a free

electron). Using this value to estimate the compressibility as it is introduced by Eq. (17) we get $\alpha \approx 0.44$. This is too small to provide insensitivity of the coefficient β determined by Eq. (25) to the interlayer distance for interlayer spacings reported in the experiments [3]. The above discrepancy could be removed taking into account Fermi liquid interactions among quasiparticles (CFs). To include Fermi liquid effects into consideration we write the renormalized polarizability Π^* in the form [8]:

$$\Pi^{*-1} = \Pi^{-1} + F_{(0)} + F_{(1)}. \qquad (26)$$

Here, Π is the polarizability of noninteracting CFs defined with Eq. (2), and the remaining terms present contributions arising due to Fermi liquid interaction in the CF system. Only contributions from the first and greatest two terms in the expansion of the Fermi liquid interaction function in Legendre polynomials (f_0 and f_1, respectively) are kept in Eq. (26) to avoid too lengthy calculations. Matrix elements of the 2×2 matrices $F_{(0)}$ and $F_{(1)}$ equal:

$$F_{(0)} = \begin{pmatrix} f_0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\qquad (27)$$

$$F_{(1)} = \begin{pmatrix} \dfrac{m^* - m_b}{ne^2}\dfrac{\omega^2}{q^2} & 0 \\ 0 & -\dfrac{m^* - m_b}{ne^2} \end{pmatrix}.$$

Within the Fermi liquid theory the effective mass m^* is related to the "bare" mass m_b as follows:

$$\frac{1}{m_b} = \frac{1}{m^*} + \frac{f_1}{2\pi\hbar^2} \equiv \frac{1+A_1}{m^*}. \quad (28)$$

Using these expressions (26)–(28) and carrying out calculations within the relevant limit $\delta \ll 1$, we obtain that the expression for the density-density response function for a single layer keeps the form given by Eq. (16) where the compressibility $dn/d\mu$ is replaced with the quantity $dn^*/d\mu$ renormalized due to the Fermi liquid interaction:

$$\frac{dn^*}{d\mu} = \frac{3m^*}{8\pi\hbar^2}\left(1+\frac{3m^*}{8\pi\hbar^2}f_0\right)^{-1} \equiv \frac{dn}{d\mu}\left(1+\frac{dn}{d\mu}f_0\right)^{-1}. \quad (29)$$

For strongly correlated quasiparticles this renormalization may significantly reduce the compressibility of the CF liquid, and, consequently, increase the value of the parameter α. It is usually assumed [3, 8] that the Fermi liquid renormalization of the effective mass significantly changes its value: $m^* \sim 5-10\ m_b$. This gives for the Fermi liquid coefficient A_1 values of the order of 10. Using this estimate, and substituting our renormalized compressibility (29) into the expression (21) we arrive at the conclusion that $dn^*/d\mu$ is low enough for the condition $\alpha > 1$, to be satisfied when the Fermi liquid parameter $A_0 \equiv f_0/2\pi\hbar^2$ takes on values of the order of $10-100$. This conclusion does not seem an unrealistic one for it is reasonable to expect A_0 to be of the order or greater than the next Fermi liquid parameter A_1. We obtain a reasonably good agreement between the plot of our Eq. (22) and the experimental results, using $A_0 = 15$ and $A_1 = 3$ ($m^* = 4m_b$). (Fig. 1).

Our results for temperature dependence of β^{-1} also agree with the results of experiments [7]. The upper curve in Fig.2 corresponds to the double-layer system with with smaller interlayer spacing $d_A = 10nm$ which gives $T_0 = 487K$, and the lower curve exhibits the temperature dependence of β^{-1} for greater spacing $d_B = 22.5nm$ ($T_0 = 587K$). The curves do not coincide but they are arranged rather close to each other.

Finally, the results of the present analysis enable us to qualitatively describe all important features observed in experiments of [7] on the Coulomb drag slightly away from one half filling of lowest Landau levels of both interacting 2DEG. They also give us grounds to treat these experimental results as one more evidence of strong Fermi liquid interaction in the CF system near one half filling of the lowest Landau level. The above interaction provides a significant reduction of the compressibility of the CF liquid and a consequent enhancement in the screening length in single layers. Essentially, the parameter α characterizes the ratio of the Thomas–Fermi screening length in a single 2DEG at $\nu = 1/2$ and the separation between the layers [3]. When $\alpha > 1$, intralayer interactions predominate those between the layers which could be the reason for low sensitivity of the bilayer to changes in the interlayer spacing. It is likely that here is an explanation for the reported nearly independence of the drag on the interlayer separation [7]. We believe that at larger distances between the layers the dependence of the transresistivity of d could be revealed in the experiments. At the same time the results of [7] give us a valuable opportunity to estimate a strength of Fermi liquid interactions between quasiparticles at $\nu = 1/2$ state which is important for further studies of such systems.

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[1] L. Zheng and A.H. MacDonald, Phys. Rev. B **48**, 8203 (1993).

[2] A. Kamenev and Y. Oreg, Phys. Rev. B **52**, 7516 (1995).

[3] I. Ussishkin and A. Stern, Phys. Rev. B **56**, 4013 (1997).

[4] S. Sakhi, Phys. Rev. B **56**, 4098 (1997);

[5] M.P. Lilly, J.P. Eisenstein, L.N. Pfeiffer and K.W. West, Phys. Rev. Lett. **80**, 1714 (1998).

[6] When the external driving disturbance applied to one of the layers is of small **q**,ω ($ql \ll 1, \omega\tau \ll 1$) the transresisitivity is dominated with the diffusion contribution, and new effects could emerge (See e.g. F. von Oppen, S.H. Simon and A. Stern, Phys. Rev. Lett. **87**, 106803 (2001) and references therein).

[7] M.P. Lilly, J.P. Eisenstein, L.N. Pfeiffer and K.W. West, cond-mat/9909231.

[8] B.I. Halperin, P.A. Lee and N. Read, Phys. Rev. B **47**, 7312 (1993); S.H. Simon and B.I. Halperin, Phys. Rev. B **48**, 17368 (1993).

[9] N.A. Zimbovskaya and J.L. Birman, Phys. Rev. B, **60** 16762 (1999).

[10] N.A. Zimbovskaya, "Local Geometry of the Fermi Surface and High-Frequency Phenomena in Metals", Springer-Verlag, New York, 2001.

[11] R.L. Willett, K.W. West, and L.N. Pfeiffer Phys. Rev. Lett. **83**, 2624 (1999).