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Dark energy and neutrino model in SUSY
– Remarks on active and sterile neutrinos mixing –

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We consider a Mass Varying Neutrinos (MaVaNs) model in supersymmetric theory. The model includes effects of supersymmetry breaking transmitted by the gravitational interaction from the hidden sector, in which supersymmetry was broken, to the dark energy sector. Then evolutions of the neutrino mass and the equation of state parameter of the dark energy are presented in the model. It is remarked that only the mass of a sterile neutrino is variable in the case of the vanishing mixing between the left-handed and a sterile neutrino on cosmological time scale. The finite mixing makes the mass of the left-handed neutrino variable.

1. Introduction

Cosmological observations have provided the strong evidence that the Universe is flat and its energy density is dominated by the dark energy component whose negative pressure causes the cosmic expansion to accelerate.[1] In order to clarify the origin of the dark energy, one has tried to understand the connection of the dark energy with particle physics.

In the Mass Varying Neutrinos (MaVaNs) scenario proposed by Fardon, Nelson and Weiner, relic neutrinos could form a negative pressure fluid and cause the present cosmic acceleration.[2] In the model, an unknown scalar field, which is called "acceleron", is introduced and neutrinos are assumed to interact through a new scalar force. The acceleron sits at the instantaneous minimum of its potential, and the cosmic expansion only modulates this minimum through changes in the number density of neutrinos. Therefore, the neutrino mass is given by the acceleron, in other words, it depends on the number density of neutrinos and changes with the expansion of the Universe. The equation of state parameter w and the energy density of the dark energy also evolve with the neutrino mass. Those evolutions depend

*talked at the International Workshop on Neutrino Masses and Mixings, University of Shizuoka, Shizuoka, Japan, December 17-19, 2006 on the form of a scalar potential and the relation between the acceleron and the neutrino mass strongly. Some examples of the potential have been considered.[3]

The idea of the variable neutrino mass was considered at first in a model of neutrino dark matter and was discussed for neutrino clouds.[4] Interacting dark energy scalar with neutrinos was considered in the model of a sterile neutrino.[5] The coupling to the left-handed neutrino and its implication on the neutrino mass limit from baryogenesis was discussed.[6] In the context of the MaVaNs scenario, there have been a lot of works. [7,8,9,10,11,12]

In this talk, we present a MaVaNs model including the supersymmetry breaking effect mediated by the gravity. Then we show evolutions of the neutrino mass and the equation of state parameter in the model.

2. MaVaNs Model in Supersymmetric Theory

We discuss the Mass Varying Neutrinos scenario in supersymmetric theory and present a model.

We assume a chiral superfield A in dark sector. A is assumed to be a singlet under the gauge group of the standard model. It is difficult to construct a viable MaVaNs model without fine-tunings in some parameters when one assumes one chiral superfield in dark sector, which couples to only the left-handed lepton doublet superfield. [8] Therefore, we assume that the superfield A couples to both the left-handed lepton doublet superfield L and the right-handed neutrino superfield R. For simplicity, we consider the MaVaNs scenario in one generation of neutrinos.[a]

In such framework, we suppose the following superpotential,

$$W = \frac{\lambda}{6}A^3 + \frac{M_A}{2}AA + m_D LA + M_D LR + \frac{M_R}{2}RR, \tag{1}$$

where λ is a coupling constant of $\mathcal{O}(1)$ and M_A, M_D, M_R and m_D are mass parameters.[b] The scalar and the spinor component of A are represented by ϕ and ψ, respectively. The scalar component corresponds to the acceleron which cause the present cosmic acceleration. The spinor component is a sterile neutrino. The third term of the right-hand side in Eq. (1) is derived from the Yukawa coupling such as $yLAH$ with $y < H >= m_D$, where H is the Higgs doublet.

In the MaVaNs scenario, the dark energy is assumed to be composed of the neutrinos and the scalar potential for the acceleron. Therefore, the energy density of the dark energy is given as

$$\rho_{\rm DE} = \rho_\nu + V(\phi). \tag{2}$$

Since only the acceleron potential contributes to the dark energy, we assume the vanishing vacuum expectation values of sleptons, and thus we find the following

[a]Three generations model of this scenario has presented in non supersymmetric theory.[9]
[b]Other supersymmetric model so called "hybrid model" has been proposed.[10]

effective scalar potential,

$$V(\phi) = \frac{\lambda^2}{4}|\phi|^4 + M_A^2|\phi|^2 + m_D^2|\phi|^2. \tag{3}$$

We can write down a lagrangian density from Eq. (1),

$$\mathcal{L} = \lambda\phi\psi\psi + M_A\psi\psi + m_D\nu_L\psi + M_D\nu_L\nu_R + M_R\nu_R\nu_R + h.c.. \tag{4}$$

It is noticed that the lepton number conservation in the dark sector is violated because this lagrangian includes both $M_A\psi\psi$ and $m_D\nu_L\psi$. After integrating out the right-handed neutrino, the effective neutrino mass matrix is given by

$$\mathcal{M} \simeq \begin{pmatrix} c & m_D \\ m_D & M_A + \lambda\phi \end{pmatrix}, \tag{5}$$

in the basis of (ν_L, ψ), where $c \equiv -M_D^2/M_R$ and we assume $\lambda\phi \ll M_D \ll M_R$. The first term of the $(1,1)$ element of this matrix corresponds to the usual term given by the seesaw mechanism in the absence of the acceleron. We obtain masses of the left-handed and a sterile neutrino as follows,

$$m_{\nu_L} = \frac{c + M_A + \lambda < \phi >}{2} + \frac{\sqrt{[c - (M_A + \lambda < \phi >)]^2 + 4m_D^2}}{2}, \tag{6}$$

$$m_\psi = \frac{c + M_A + \lambda < \phi >}{2} - \frac{\sqrt{[c - (M_A + \lambda < \phi >)]^2 + 4m_D^2}}{2}. \tag{7}$$

It is remarked that only the mass of a sterile neutrino is variable in the case of the vanishing mixing ($m_D = 0$) between the left-handed and a sterile neutrino on cosmological time scale. The finite mixing ($m_D \neq 0$) makes the mass of the left-handed neutrino variable. We will consider these two cases of $m_D = 0$ and $m_D \neq 0$ later.

In the MaVaNs scenario, there are two constraints on the scalar potential. The first one comes from cosmological observations. It is that the magnitude of the present dark energy density is about $0.74\rho_c$. ρ_c is the critical density. Thus, the first constraint turns to

$$V(\phi^0) = 0.74\rho_c - \rho_\nu^0, \tag{8}$$

where "0" means the present value.

The second one is the stationary condition:

$$\frac{\partial \rho_{\mathrm{DE}}}{\partial \phi} = \frac{\partial \rho_\nu}{\partial \phi} + \frac{\partial V(\phi)}{\partial \phi} = 0. \tag{9}$$

In this scenario, the neutrino mass is represented by a function of the acceleron; $m_\nu = f(\phi)$. Since the energy density of the neutrino varies on cosmological times scale, the vacuum expectation value of the acceleron also varies. This property makes the neutrino mass variable. If $\partial m_\nu/\partial\phi \neq 0$, Eq. (9) is equivalent to

$$\frac{\partial \rho_{\mathrm{DE}}}{\partial m_\nu} = \frac{\partial \rho_\nu}{\partial m_\nu} + \frac{\partial V(\phi(m_\nu))}{\partial m_\nu} = 0. \tag{10}$$

Eq. (10) is rewritten by using the cosmic temperature T:

$$\frac{\partial V(\phi)}{\partial m_\nu} = -T^3 \frac{\partial F(\xi)}{\partial \xi}, \tag{11}$$

where $\xi \equiv m_\nu/T$, $\rho_\nu = T^4 F(\xi)$ and

$$F(\xi) \equiv \frac{1}{\pi^2} \int_0^\infty \frac{dy y^2 \sqrt{y^2+\xi^2}}{e^y+1}. \tag{12}$$

We can get the time evolution of the neutrino mass from Eq. (11). Since the stationary condition should be always satisfied in the evolution of the Universe, this one at the present epoch is the second constraint on the scalar potential:

$$\left.\frac{\partial V(\phi)}{\partial m_\nu}\right|_{m_\nu = m_\nu^0} = -T^3 \left.\frac{\partial F(\xi)}{\partial \xi}\right|_{m_\nu = m_\nu^0, T=T_0}. \tag{13}$$

In addition to two constraints for the potential, we also have two relations between the vacuum expectation value of the acceleron and the neutrino masses at the present epoch:

$$m_{\nu_L}^0 = \frac{c + M_A + \lambda < \phi >^0}{2} + \frac{\sqrt{[c - (M_A + \lambda < \phi >^0)]^2 + 4m_D^2}}{2}, \tag{14}$$

$$m_\psi^0 = \frac{c + M_A + \lambda < \phi >^0}{2} - \frac{\sqrt{[c - (M_A + \lambda < \phi >^0)]^2 + 4m_D^2}}{2}. \tag{15}$$

Next, let us consider the dynamics of the acceleron field. In order that the acceleron does not vary significantly on distance of inter-neutrino spacing, the acceleron mass at the present epoch must be less than $\mathcal{O}(10^{-4}\text{eV})$ [2]. Here and below, we fix the present acceleron mass as

$$m_\phi^0 = 10^{-4}\ \text{eV}. \tag{16}$$

Once we adjust parameters which satisfy five equations (8) and (13)$\sim$(16), we can have evolutions of the neutrino masses by using the Eq. (11).

The dark energy is characterized by the evolution of the equation of state parameter w. The equation of state is derived from the energy conservation law and the stationary condition Eq. (11):

$$w + 1 = \frac{[4 - h(\xi)]\rho_\nu}{3\rho_{\text{DE}}}, \tag{17}$$

where

$$h(\xi) \equiv \frac{\xi \frac{\partial F(\xi)}{\partial \xi}}{F(\xi)}. \tag{18}$$

It seems that w in this scenario depend on the neutrino mass and the cosmic temperature. This means that w varies with the evolution of the Universe unlike the cosmological constant.

In the last of this section, we comment on the hydrodynamical stability of the dark energy in the MaVaNs scenario. The speed of sound squared in the neutrino-acceleron fluid is given by

$$c_s^2 = \frac{\dot{p}_{\rm DE}}{\dot{\rho}_{\rm DE}} = \frac{\dot{w}\rho_{\rm DE} + w\dot{\rho}_{\rm DE}}{\dot{\rho}_{\rm DE}}, \tag{19}$$

where $p_{\rm DE}$ is the pressure of the dark energy. Recently, it was argued that when neutrinos are non-relativistic, this speed of sound squared becomes negative in this scenario.[11] The emergence of an imaginary speed of sound means that the MaVaNs scenario with non-relativistic neutrinos is unstable, and thus the fluid in this scenario cannot acts as the dark energy. However, finite temperature effects provide a positive contribution to the speed of sound squared and avoid this instability. [12] Then, a model should satisfy the following condition,

$$\frac{\partial m_\nu}{\partial z}\left(1 - \frac{5aT^2}{3m_\nu^2}\right) + \frac{25aT_0^2(z+1)}{3m_\nu} > 0, \tag{20}$$

where z is the redshift parameter, $z \equiv (T/T_0) - 1$, and

$$a \equiv \frac{\int_0^\infty \frac{dyy^4}{e^y+1}}{2\int_0^\infty \frac{dyy^2}{e^y+1}} \simeq 6.47. \tag{21}$$

The first and the second term of left hand side in Eq. (20) are negative and positive contributions to the speed of sound squared, respectively. We find that a model which leads to small $\partial m_\nu/\partial z$ is favored. A model with a small power-law scalar potential; $V(\phi) = \Lambda^4(\phi/\phi^0)^k$, $k \ll 1$, and a constant dominant neutrino mass; $m_\nu = C + f(\phi)$, $f(\phi) \ll C$, leads to small $\partial m_\nu/\partial z$.[c] Actually, some models have been presented.[9]

3. Effect of supersymmetry breaking

Let us consider effect of supersymmetry breaking in the dark sector. We assume a superfield X, which breaks supersymmetry, in the hidden sector, and the chiral superfield A in the dark sector is assumed to interact with the hidden sector only through the gravity. This framework is shown graphically in Fig. 1. Once supersymmetry is broken at TeV scale, its effect is transmitted to the dark sector through the following operators:

$$\int d^4\theta \frac{X^\dagger X}{M_{p\ell}^2} A^\dagger A, \quad \int d^4\theta \frac{X^\dagger + X}{M_{p\ell}} A^\dagger A, \tag{22}$$

where $M_{p\ell}$ is the Planck mass. Then, the scale of soft terms $F_X(\mathrm{TeV}^2)/M_{p\ell} \sim \mathcal{O}(10^{-3}\text{-}10^{-2}\mathrm{eV})$ is expected. In the "acceleressence" scenario, this scale is identified with the dark energy scale.[14] We consider only one superfield which breaks

[c] A model with the masses of the left-handed neutrinos given by the see-saw mechanism is unstable even if it has a small power-law scalar potential.[13]

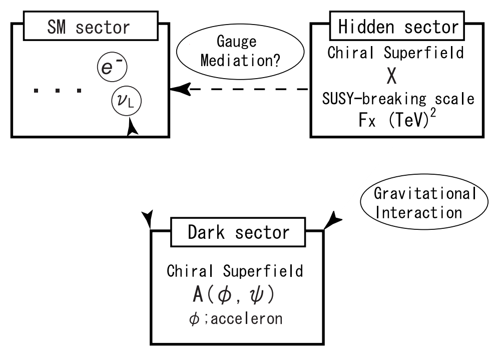

Fig. 1. The illustration of interactions among three sectors. The dark sector couples to the left-handed neutrino through a new scalar force in the MaVaNs scenario. The dark sector is also assumed to be related with the hidden sector only through the gravity.

supersymmetry for simplicity. If one extends the hidden sector, one can consider a different mediation mechanism between the standard model and the hidden sector from one between the dark and the hidden sector.

In this framework, taking supersymmetry breaking effect into account, the scalar potential is given by

$$V(\phi) = \frac{\lambda^2}{4}|\phi|^4 - \frac{\kappa}{3}(\phi^3 + h.c.) + M_A^2|\phi|^2 + m_D^2|\phi|^2 - m^2|\phi|^2 + V_0, \tag{23}$$

where κ and m are supersymmetry breaking parameters, and V_0 is a constant determined by the condition that the cosmological constant is vanishing at the true minimum of the acceleron potential.

We consider two types of the neutrino mass matrix in this scalar potential. They are the cases of the vanishing and the finite mixing between the left-handed and a sterile neutrino.

3.1. *Case of the Vanishing Mixing*

When the mixing between the left-handed and a sterile neutrino is vanishing, $m_D = 0$ in the neutrino mass matrix (5). Then we have the masses of the left-handed and a sterile neutrino as

$$m_{\nu_L} = c, \tag{24}$$

$$m_\psi = M_A + \lambda < \phi > . \tag{25}$$

In this case, we find that only the mass of a sterile neutrino is variable on cosmological time scale due to the second term of the right hand side in Eq. (25).

Let us adjust parameters which satisfy Eqs. (8) and (13)$\sim$(16). In Eq. (8), the scalar potential Eq. (23) is used. Putting typical values for four parameters by hand as follows:

$$\lambda = 1, \quad m_D = 0, \quad m_{\nu_L}^0 = 2 \times 10^{-2} \text{ eV}, \quad m_\psi^0 = 10^{-2} \text{ eV}, \tag{26}$$

we have

$$
\begin{aligned}
&< \phi >^0 \simeq -1.31 \times 10^{-5} \text{ eV}, \quad c = 2 \times 10^{-2} \text{ eV}, \quad M_A \simeq 10^{-2} \text{ eV}, \\
&m \simeq 10^{-2} \text{ eV}, \quad \kappa \simeq 4.34 \times 10^{-3} \text{ eV}.
\end{aligned} \tag{27}
$$

We need fine-tuning between M_A and m in order to satisfy the constraint on the present accerelon mass of Eq. (16).

We show evolutions of the mass of a sterile neutrino and the equation of state parameter in Figs. 2, 3 and 4. The behavior of the mass of a neutrino near the present epoch is shown in Fig. 3. We find that the mass of a sterile neutrino have varied slowly in this epoch. This means that the first term of the left hand side in Eq. (20), which is a negative contribution to the speed of sound squared, is tiny. We can also check the positive speed of sound squared in a numerical calculation. Therefore, the neutrino-acceleron fluid is hydrodynamically stable and acts as the dark energy.

3.2. *Case of the Finite Mixing*

Next, we consider the case of the finite mixing between the left-handed and a sterile neutrino ($m_D \neq 0$). In this case, the left-handed and a sterile neutrino mass are given by

$$
m_{\nu_L} = \frac{c + M_A + \lambda < \phi >}{2} + \frac{\sqrt{[c - (M_A + \lambda < \phi >)]^2 + 4m_D^2}}{2}, \tag{28}
$$

$$
m_\psi = \frac{c + M_A + \lambda < \phi >}{2} - \frac{\sqrt{[c - (M_A + \lambda < \phi >)]^2 + 4m_D^2}}{2}. \tag{29}
$$

We find that both masses of the left-handed and a sterile neutrino are variable on cosmological time scale due to the term of the acceleron dependence.

Taking typical values for four parameters as

$$
\lambda = 1, \quad m_D = 10^{-3} \text{ eV}, \quad m_{\nu_L}^0 = 2 \times 10^{-2} \text{ eV}, \quad m_\psi^0 = 10^{-2} \text{ eV}, \tag{30}
$$

we have

$$
\begin{aligned}
&< \phi >^0 \simeq -1.31 \times 10^{-5} \text{ eV}, \quad c \simeq 1.99 \times 10^{-2} \text{ eV}, \quad M_A \simeq 1.01 \times 10^{-2} \text{ eV}, \\
&m \simeq 1.02 \times 10^{-2} \text{ eV}, \quad \kappa \simeq 4.34 \times 10^{-3} \text{ eV}.
\end{aligned} \tag{31}
$$

where we required that the mixing between the active and a sterile neutrino is tiny. In our model, the small present value of the acceleron is needed to satisfy the constraints on the scalar potential in Eqs. (8) and (13).

Values of parameters in (31) are almost same as the case of the vanishing mixing (27). However, the mass of the left-handed neutrino is variable unlike the vanishing mixing case. The time evolution of the left-handed neutrino mass is shown in Fig. 5. The mixing does not affect the evolution of a sterile neutrino mass and the equation of state parameter, which are shown in Figs. 6, 7. Since the variation in the mass of the left-handed neutrino is not vanishing but extremely small, the model can also avoid the instability of speed of sound.

Finally, we comment on the smallness of the evolution of the neutrino mass at the present epoch. In our model, the mass of the left-handed and a sterile neutrino include the constant part. A variable part is a function of the acceleron. In the present epoch, the constant part dominates the neutrino mass because the present value of the acceleron should be small. This smallness of the value of the acceleron is required from the cosmological observation and the stationary condition in Eqs. (8) and (13).

4. Summary

We presented a supersymmetric MaVaNs model including effects of the supersymmetry breaking mediated by the gravity. Evolutions of the neutrino mass and the equation of state parameter have been calculated in the model. Our model has a chiral superfield in the dark sector, whose scalar component causes the present cosmic acceleration, and the right-handed neutrino superfield. In our framework, supersymmetry is broken in the hidden sector at TeV scale and the effect is assumed to be transmitted to the dark sector only through the gravity. Then, the scale of soft parameters of $\mathcal{O}(10^{-3}\text{-}10^{-2})$(eV) is expected.

We considered two types of model. One is the case of the vanishing mixing between the left-handed and a sterile neutrino. Another one is the finite mixing case. In the case of the vanishing mixing, only the mass of a sterile neutrino had varied on cosmological time scale. In the epoch of $0 \leq z \leq 20$, the sterile neutrino mass had varied slowly. This means that the speed of sound squared in the neutrino acceleron fluid is positive, and thus this fluid can act as the dark energy. In the finite mixing case, the mass of the left-handed neutrino had also varied. However, the variation is extremely small and the effect of the mixing does not almost affect the evolution of the sterile neutrino mass and the equation of state parameter. Therefore, this model can also avoid the instability.

References

1. A. G. Riess *et al.*, Astron. J. **116**, 1009 (1998); S. Perlmutter *et al.*, Astrophys. J. **517**, 565 (1999); P. de Bernardis *et al.*, Nature **404**, 955 (2000); A. Baldi *et al.*, Astrophys. J. **545**, L1 (2000), [Erratum-ibid. **558**, L145 (2001)]; A. T. Lee *et al.*, Astrophys. J. **561**, L1 (2001); R. Stompor *et al.*, Astrophys. J. **561**, L7 (2001); N. W. Halverson *et al.*, Astrophys. J. **568**, 38 (2002); C. L. Bennet *et al.*, Astrophys. J. Suppl. **148**, 1 (2003); D. N. Spergel *et al.*, Astrophys. J. Suppl. **148**, 175 (2003), [astro-ph/0603449]; J. A. Peacock *et al.*, Nature **410**, 169 (2001); W. J. Percival *et al.*, Mon. Not. Roy. Astron. Soc. **327**, 1297 (2001); M. Tegmark *et al.*, Phys. Rev. D **69**, 103501 (2004); K. Abazajian *et al.*, Astron. J. **128**, 502 (2004); U. Seljak *et al.*, Phys. Rev. D **71**, 103515 (2005); P. McDonald, U. Seljak, R. Cen, P. Bode, and J. P. Ostriker, Mon. Not. Roy. Astron. Soc. **360**, 1471 (2005).
2. R. Fardon, A. E. Nelson and N. Weiner, JCAP. **0410**, 005 (2004).
3. R. D. Peccei, Phys. Rev. D **71**, 023527 (2005).
4. M. Kawasaki, H. Murayama and T. Yanagida, Mod. Phys. Lett. A **7**, 563 (1992); G. J.

Stephenson, T. Goldman and B. H. J. McKellar, Int. J. Mod. Phys. A **13**, 2765 (1998), Mod. Phys. Lett. A **12**, 2391 (1997).

5. P. Q. Hung, hep-ph/0010126.
6. P. Gu, X-L. Wang and X-Min. Zhang, Phys. Rev. D **68**, 087301 (2003).
7. D. B. Kaplan, A. E. Nelson, N. Weiner, Phy. Rev. Lett. **93**, 091801 (2004); V. Barger, D. Marfatia and K. Whisnant, Phys. Rev. D**73**, 013005 (2006); P-H. Gu, X-J. Bi, B. Feng, B-L. Young and X. Zhang, hep-ph/0512076; X-J. Bi, P. Gu, X-L. Wang and X-Min. Zhang, Phys. Rev. D **69**, 113007 (2004); P. Gu and X-J. Bi, Phys. Rev. D **70**, 063511 (2004); P. Q. Hung and H. Päs, Mod. Phys. Lett. A **20**, 1209 (2005); V. Barger, P. Huber and D. Marfatia, Phys. Rev. Lett. **95**, 211802 (2005); M. Cirelli and M. C. Gonzalez-Garcia and C. Peña-Garay, Nucl. Phys. B **719**, 219 (2005); X-J. Bi, B. Feng, H. Li and X-Min. Zhang, Phys. Rev. D**72** 123523, (2005); A. W. Brookfield, C. van de Bruck, D. F. Mota and D. Tocchini-Valentini, Phys. Rev. Lett. **96**, 061301 (2006); R. Horvat, JCAP **0601**, 015 (2006); R. Barbieri, L. J. Hall, S. J. Oliver and A. Strumia, Phys. Lett. B **625**, 189 (2005); N. Weiner and K. Zurek, Phys. Rev. D **74**, 023517 (2006); H. Li, B. Feng, J-Q. Xia and X-Min. Zhang, Phys. Rev. D **73**, 103503 (2006); A. W. Brookfield, C. van de Bruck, D. F. Mota and D. Tocchini-Valentini, Phys. Rev. D **73**, 083515 (2006); P-H. Gu, X-J. Bi and X. Zhang, hep-ph/0511027; E. Ma and U. Sarkar, Phys. Lett. B**638**, 356 (2006); A. Zanzi, Phys. Rev. D **73**, 124010 (2006); R. Takahashi and M. Tanimoto, Phys. Rev. D **74**, 055002 (2006); A. Ringwald and L. Schrempp, JCAP. **0610**, 012 (2006); R. Takahashi and M. Tanimoto, hep-ph/0610347; S. Das and N. Weiner, astro-ph/0611353; C.T. Hill, I. Mocioiu, E.A. Paschos and U. Sarkar, hep-ph/0611284; L. Schrempp, astro-ph/0611912.
8. R. Takahashi and M. Tanimoto, Phys. Lett. B**633**, 675 (2006).
9. M. Honda, R. Takahashi and M. Tanimoto, JHEP **0601**, 042 (2006).
10. R. Fardon, A. E. Nelson and N. Weiner, JHEP **0603**, 042 (2006).
11. N. Afshordi, M. Zaldarriaga and K. Kohri, Phys. Rev. D **72**, 065024 (2005).
12. R. Takahashi and M. Tanimoto, JHEP **0605**, 021 (2006).
13. C. Spitzer, astro-ph/0606034.
14. Z. Chacko, L. J. Hall and Y. Nomura, JCAP. **0410**, 011 (2004).

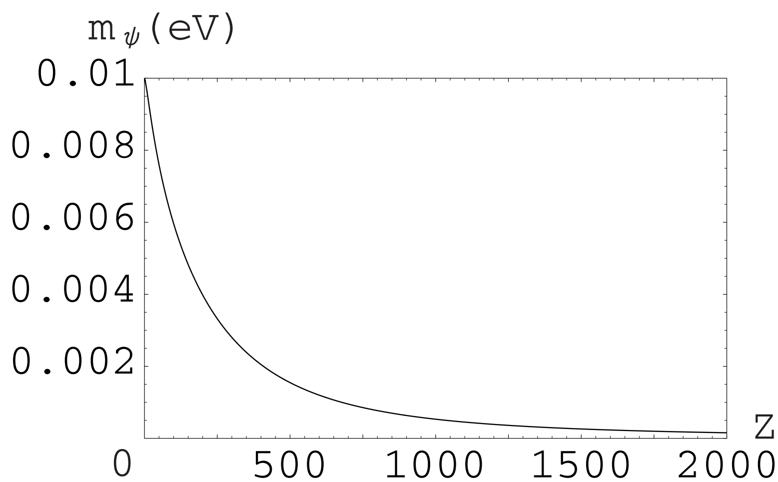

Fig. 2. Evolution of the mass of a sterile neutrino ($0 \leq z \leq 2000$)

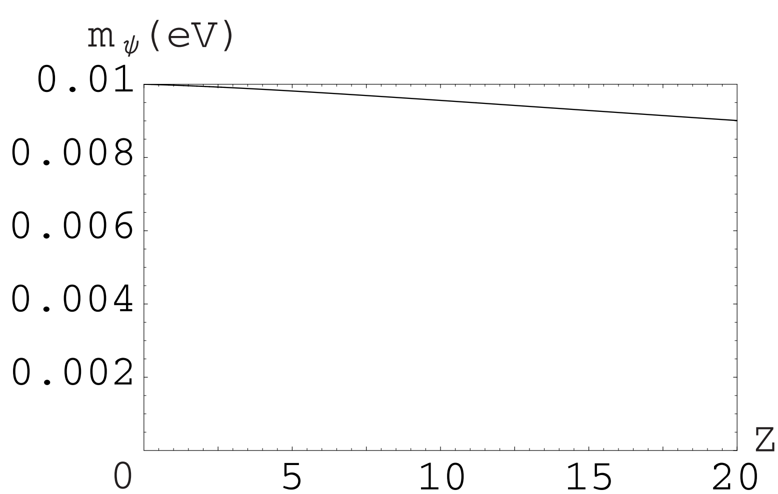

Fig. 3. Evolution of the mass of a sterile neutrino ($0 \leq z \leq 20$)

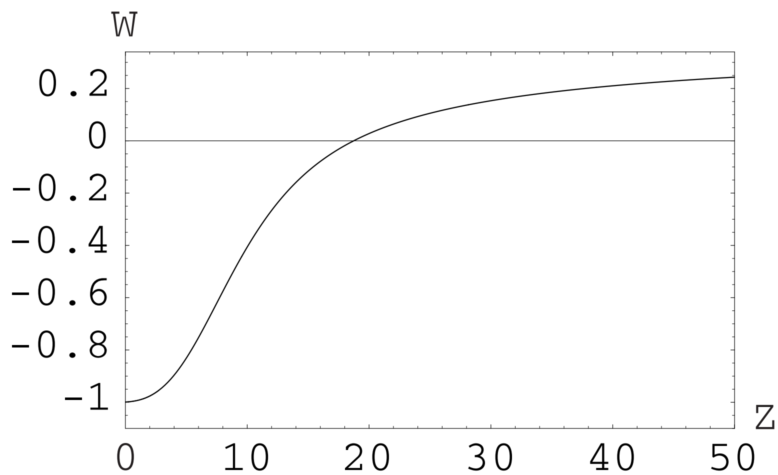

Fig. 4. Evolution of w $(0 \leq z \leq 50)$

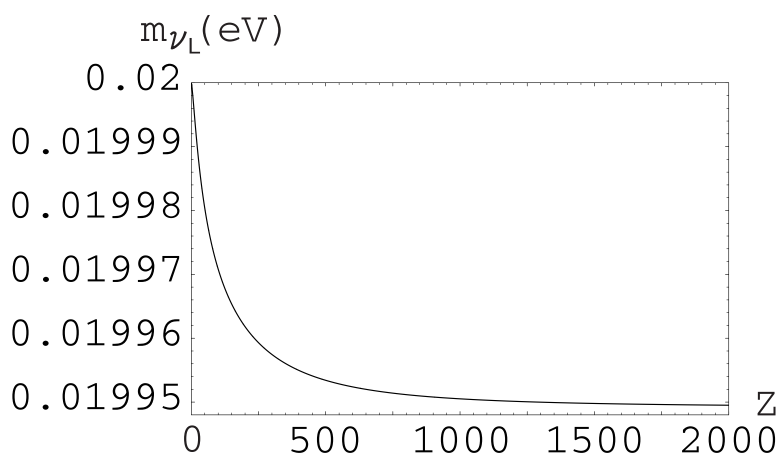

Fig. 5. Evolution of the mass of the left-handed neutrino $(0 \leq z \leq 2000)$

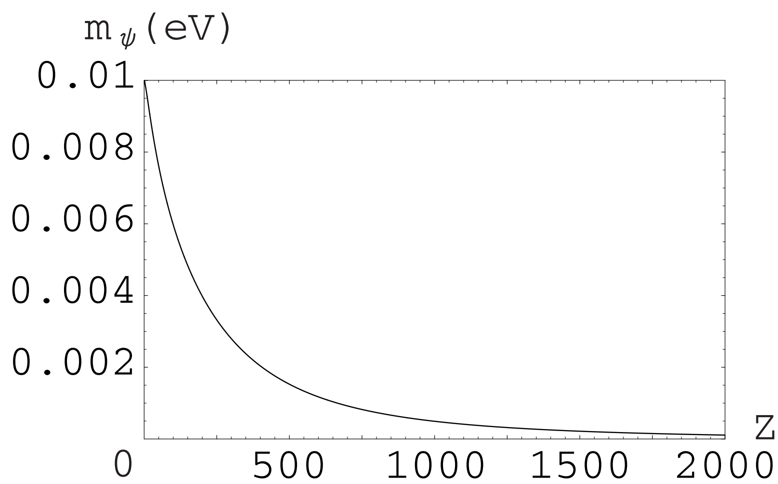

Fig. 6. Evolution of the mass of a sterile neutrino ($0 \leq z \leq 2000$)

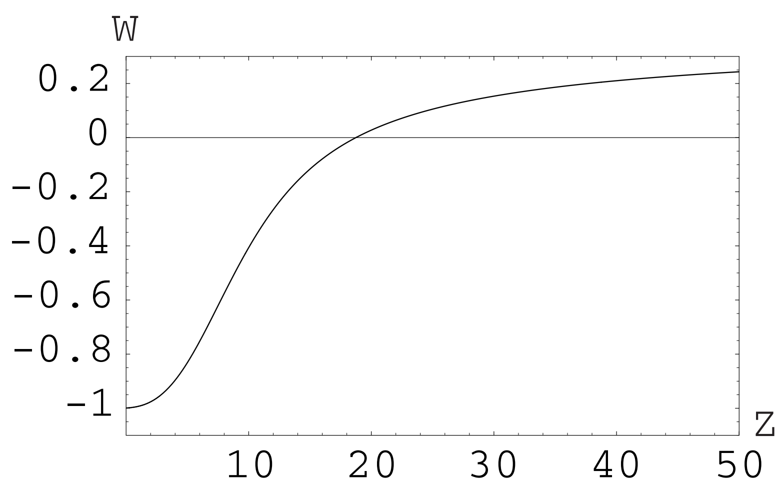

Fig. 7. Evolution of w ($0 \leq z \leq 50$)