

# Causal dissipative hydrodynamics for QGP fluid in 2+1 dimensions

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Effect of dissipation due to shear viscosity on the space-time evolution of QGP fluid is studied. Relaxation equations for shear stress tensors, derived from the second-order Israel-Stewart approach which ensures causal evolution, are solved simultaneously with the energy-momentum conservation equations. Comparison of evolution of ideal and viscous QGP fluid, initialized under the same conditions, e.g. same equilibration time, energy density and velocity profile, indicate that in a viscous dynamics, energy density or temperature of the fluid evolve slowly than in an ideal fluid. Cooling gets slower as viscosity increases. Transverse expansion also increases in a viscous dynamics. Enhanced life time of the QGP fluid in a viscous dynamics can have considerable effect on observables produced early in the collisions, e.g. direct photon,  $J/\psi$  suppression.

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## I. INTRODUCTION

Recent Au+Au collisions at the Relativistic Heavy Ion Collider (RHIC), at cm energy of  $\sqrt{s_{NN}}=200$  GeV, give a strong indication that in central collisions, a hot dense matter is formed [1, 2, 3, 4]. Whether the formed matter can be identified as the much sought after Quark-Gluon Plasma (QGP) as predicted in Lattice QCD simulations [5] is presently debatable. However, experimentally observed elliptic flow in non-central collisions gave a strong indication that in the collisions, thermalized collective QCD matter is formed. Additionally, success of *ideal* fluid dynamics in explaining several experimental data e.g. transverse momentum spectra of identified particles, elliptic flow etc [6], together with the string theory motivated lower limit of shear viscosity  $\eta/s \geq 1/4\pi$  [7, 8] leading to a paradigm that in Au+Au collisions, a nearly perfect fluid is created.

However, the paradigm of "perfect fluid" produced in Au+Au collisions at RHIC need to be clarified. It so happens that the ideal fluid dynamic models do have their shortcomings [9]. For example, experimentally, elliptic flow tends to saturate at large transverse momentum. The ideal fluid dynamics on the other hand predicts a continually increasing elliptic flow. The transverse momentum spectra of identified particles also starts to deviate from ideal fluid dynamics prediction beyond  $p_T \approx 1.5$  GeV. Experimentally determined HBT radii are not reproduced in the ideal fluid dynamic models, the famous "HBT puzzle" [10]. Ideal fluid dynamics also works best in central collisions and gets poorer in more peripheral collisions.

The shortcomings of ideal fluid dynamics possibly indicate greater importance of dissipative effects in the  $p_T$  ranges greater than 1.5 GeV or in more peripheral collisions. Indeed, ideal fluid is a concept, never realized in nature. As suggested in string theory motivated models,

QGP viscosity could be small,  $\eta/s \geq 1/4\pi$ , nevertheless it is non-zero. It is important to study the effect of viscosity, even if small, on space-time evolution of QGP fluid and quantify its effect. This requires a numerical implementation of relativistic dissipative fluid dynamics. Furthermore, if QGP fluid is formed in heavy ion collisions, it has to be characterized by measuring its transport coefficients, e.g. heat conductivity, bulk and shear viscosity. Theoretically, it is possible to obtain those transport coefficients in a kinetic theory model. However, in the present status of theory, the goal can not be achieved immediately, even more so for a strongly interacting QGP (sQGP). Alternatively, one can use the experimental data to obtain a "phenomenological" limit of transport coefficients of sQGP. It will also require a numerical implementation of relativistic dissipative fluid dynamics. There is another incentive to study dissipative hydrodynamics. Ideal hydrodynamics depends on the assumption of local equilibrium. Dissipative hydrodynamics on the other hand do not depend on the assumption of local equilibrium. As will be discussed below, the fluid should not be far from a equilibrium state. The range of validity of dissipative hydrodynamics thus increases. Indeed, we can explore early times of fluid evolution better in a dissipative hydrodynamics.

Theory of dissipative relativistic fluid has been formulated quite early. The original dissipative relativistic fluid equations were given by Eckart [12] and Landau and Lifshitz [13]. They are called 1st order theories. Formally, relativistic dissipative hydrodynamics are obtained from an expansion of entropy 4-current, in terms of dissipative fluxes. In 1st order theories, entropy 4-current contains terms linear in dissipative quantities. 1st order theory of dissipative hydrodynamics suffer from the problem of causality violation. Signal can travel faster than light. Causality violation is unwarranted in any theory, even more in a relativistic theory. The problem of causality violation is removed in the Israel-Stewart's 2nd order theory of dissipative fluid [14]. In 2nd order theory, expansion of entropy 4-current contains terms 2nd order in dissipative fluxes. However, these leads to complications

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that dissipative fluxes are no longer function of the state variables only. They become dynamic. The space of thermodynamic variables has to be extended to include the dissipative fluxes (e.g. heat conductivity, bulk and shear viscosity).

Even though 2nd order theory is formulated some 30 years back, significant progress towards its numerical implementation has only been made very recently [15, 16, 17, 18, 19, 20, 21, 22, 23, 24]. At the Cyclotron Centre, Kolkata, we have developed a code "AZHYDRO-KOLKATA" to simulate the hydrodynamic evolution of QGP fluid including the effect of dissipation due to shear viscosity only. The code can simulate both the 1st order and the 2nd order dissipative hydrodynamics. Some results of AZHYDRO-KOLKATA for first order dissipative hydrodynamics have been published earlier [20, 21, 22]. In the present paper, for the first time, we will present some results from AZHYDRO-KOLKATA, for 2nd order dissipative hydrodynamics in 2+1 dimensions. In the present paper, we will consider effect of dissipation in the QGP phase only. Effect of phase transition will be studied in a later publication.

The paper is organized as follows: In section II we shortly review relativistic dissipative fluid dynamics. In section III we derive the relevant equations in 2+1 dimension (assuming boost-invariance). In section IV, we discuss the equation of state, viscosity coefficient and initial conditions used in the present simulations. In V numerical results from the code AZHYDRO-KOLKATA, depicting the evolution of ideal and viscous QGP fluid will be shown. The concluding section VI summarizes our results.

## II. DISSIPATIVE FLUID DYNAMICS

In this section, I briefly discuss the phenomenological theory of dissipative hydrodynamics. More detailed exposition can be found in [14].

A simple fluid, in an arbitrary state, is fully specified by primary variables: particle current ( $N^\mu$ ), energy-momentum tensor ( $T^{\mu\nu}$ ) and entropy current ( $S^\mu$ ) and a number of additional (unknown) variables. Primary variables satisfies the conservation laws;

$$\partial_\mu N^\mu = 0, \quad (2.1)$$

$$\partial_\mu T^{\mu\nu} = 0, \quad (2.2)$$

and the 2nd law of thermodynamics,

$$\partial_\mu S^\mu \geq 0. \quad (2.3)$$

In relativistic fluid dynamics, one defines a time-like hydrodynamic 4-velocity,  $u^\mu$  (normalized as  $u^2 = 1$ ). One also define a projector,  $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ , orthogonal to the 4-velocity ( $\Delta^{\mu\nu} u_\nu = 0$ ). In equilibrium, an

unique 4-velocity ( $u^\mu$ ) exists such that the particle density ( $n$ ), energy density ( $\varepsilon$ ) and the entropy density ( $s$ ) can be obtained from,

$$N_{eq}^\mu = n u_\mu \quad (2.4)$$

$$T_{eq}^{\mu\nu} = \varepsilon u^\mu u^\nu - p \Delta^{\mu\nu} \quad (2.5)$$

$$S_{eq}^\mu = s u_\mu \quad (2.6)$$

An equilibrium state is assumed to be fully specified by 5-parameters,  $(n, \varepsilon, u^\mu)$  or equivalently by the thermal potential,  $\alpha = \mu/T$  ( $\mu$  being the chemical potential) and inverse 4-temperature,  $\beta^\mu = u^\mu/T$ . Given a equation of state,  $s = s(\varepsilon, n)$ , pressure  $p$  can be obtained from the generalized thermodynamic relation,

$$S_{eq}^\mu = p \beta^\mu - \alpha N_{eq}^\mu + \beta_\lambda T_{eq}^{\lambda\mu} \quad (2.7)$$

Using the Gibbs-Duhem relation,  $d(p\beta^\mu) = N_{eq}^\mu d\alpha - T_{eq}^{\lambda\mu} d\beta_\lambda$ , following relations can be established on the equilibrium hyper-surface  $\Sigma_{eq}(\alpha, \beta^\mu)$ ,

$$dS_{eq}^\mu = -\alpha dN_{eq}^\mu + \beta_\lambda dT_{eq}^{\lambda\mu} \quad (2.8)$$

In a non-equilibrium system, no 4-velocity can be found such that Eqs.2.4,2.5,2.6 remain valid. Tensor decomposition leads to additional terms,

$$N^\mu = N_{eq}^\mu + \delta N^\mu = n u^\mu + V^\mu \quad (2.9)$$

$$\begin{aligned} T^{\mu\nu} &= T_{eq}^{\mu\nu} + \delta T^{\mu\nu} \\ &= [\varepsilon u^\mu u^\nu - p \Delta^{\mu\nu}] + \Pi \Delta^{\mu\nu} + \pi^{\mu\nu} \\ &\quad + (W^\mu u^\nu + W^\nu u^\mu) \end{aligned} \quad (2.10)$$

$$S^\mu = S_{eq}^\mu + \delta S^\mu = s u^\mu + \Phi^\mu \quad (2.11)$$

The new terms describe a net flow of charge  $V^\mu = \Delta^{\mu\nu} N_\nu$ , heat flow,  $W^\mu = (\varepsilon + p)/n V^\mu + q^\mu$  (where  $q^\mu$  is the heat flow vector), and entropy flow  $\Phi^\mu$ .  $\Pi = -\frac{1}{3} \Delta_{\mu\nu} T^{\mu\nu} - p$  is the bulk viscous pressure and  $\pi^{\mu\nu} = [\frac{1}{2}(\Delta^{\mu\sigma} \Delta^{\nu\tau} + \Delta^{\nu\sigma} \Delta^{\mu\tau} - \frac{1}{3} \Delta^{\mu\nu} \Delta^{\sigma\tau}) T_{\sigma\tau}]$  is the shear stress tensor. Hydrodynamic 4-velocity can be chosen to eliminate either  $V^\mu$  (the Eckart frame,  $u^\mu$  is parallel to particle flow) or the heat flow  $q^\mu$  (the Landau frame,  $u^\mu$  is parallel to energy flow). In relativistic heavy ion collisions, central rapidity region is nearly baryon free and Landau's frame is more appropriate than the Eckart's frame. Dissipative flows are transverse to  $u^\mu$  and additionally, shear stress tensor is traceless. Thus a non-equilibrium state require 1+3+5=9 additional quantities, the dissipative flows  $\Pi$ ,  $q^\mu$  (or  $V^\mu$ ) and  $\pi^{\mu\nu}$ . In kinetic theory,  $N^\mu$  and  $T^{\mu\nu}$  are the 1st and 2nd moment of the distribution function. Unless the function is known a-priori, two moments do not furnish enough information to enumerate the microscopic states required to determine  $S^\mu$ , and in an arbitrary non-equilibrium state, no relation exists between,  $N^\nu$ ,  $T^{\mu\nu}$  and  $S^\mu$ . *Only in a state, close*

to a equilibrium one, such a relation can be established. Assuming that the equilibrium relation Eq.2.8 remains valid in a "near equilibrium state" also, the entropy current can be generalized as,

$$S^\mu = S_{eq}^\mu + dS^\mu = p\beta^\mu - \alpha N^\mu + \beta_\lambda T^{\lambda\mu} + Q^\mu \quad (2.12)$$

where  $Q^\mu$  is an undetermined quantity in 2nd order in deviations,  $\delta N^\mu = N^\mu - N_{eq}^\mu$  and  $\delta T^{\mu\nu} = T^{\mu\nu} - T_{eq}^{\mu\nu}$ . Detail form of  $Q^\mu$  is constrained by the 2nd law  $\partial_\mu S^\mu \geq 0$ . With the help of conservation laws and Gibbs-Duhem relation, entropy production rate can be written as,

$$\partial_\mu S^\mu = -\delta N^\mu \partial_\mu \alpha + \delta T^{\mu\nu} \partial_\mu \beta_\nu + \partial_\mu Q^\mu \quad (2.13)$$

Choice of  $Q^\mu$  leads to 1st order or 2nd order theories of dissipative hydrodynamics. In 1st order theories the simplest choice is made,  $Q^\mu = 0$ , entropy current contains terms up to 1st order in deviations,  $\delta N^\mu$  and  $\delta T^{\mu\nu}$ . Entropy production rate can be written as,

$$T\partial_\mu S^\mu = \Pi X - q^\mu X_\mu + \pi^{\mu\nu} X_{\mu\nu} \quad (2.14)$$

where,  $X = -\nabla \cdot u$ ;  $X^\mu = \frac{\nabla^\mu}{T} - u^\nu \partial_\nu u^\mu$  and  $X^{\mu\nu} = \nabla^{<\mu} u^{\nu>}$ .

The 2nd law,  $\partial_\mu S^\mu \geq 0$  can be satisfied by postulating a linear relation between the dissipative flows and thermodynamic forces,

$$\Pi = -\zeta\theta, \quad (2.15)$$

$$q^\mu = -\lambda \frac{nT^2}{\varepsilon + p} \nabla^\mu (\mu/T), \quad (2.16)$$

$$\pi^{\mu\nu} = 2\eta \nabla^{<\mu} u^{\nu>} \quad (2.17)$$

where  $\zeta$ ,  $\lambda$  and  $\eta$  are the positive transport coefficients, bulk viscosity, heat conductivity and shear viscosity respectively.

In 1st order theories, causality is violated. If, in a given fluid cell, at a certain time, thermodynamic forces vanish, corresponding dissipative fluxes also vanish instantly. Violation of causality is unwanted in any theory, even more so in relativistic theory. Causality violation of dissipative hydrodynamics is corrected in 2nd order theories [14]. In 2nd order theories, entropy current contain terms up to 2nd order in the deviations,  $Q^\mu \neq 0$ . The most general  $Q^\mu$  containing terms up to 2nd order in deviations can be written as,

$$Q^\mu = -(\beta_0 \Pi^2 - \beta_1 q^\nu q_\nu + \beta_2 \pi_{\nu\lambda} \pi^{\nu\lambda}) \frac{u^\mu}{2T} - \frac{\alpha_0 \Pi q^\mu}{T} + \frac{\alpha_1 \pi^{\mu\nu} q_\nu}{T} \quad (2.18)$$

As before, one can cast the entropy production rate ( $T\partial_\mu S^\mu$ ) in the form of Eq.2.14. Neglecting the terms involving dissipative flows with gradients of equilibrium thermodynamic quantities (both are assumed to be

small) and demanding that a linear relation exists between the dissipative flows and thermodynamic forces, following *relaxation* equations for the dissipative flows can be obtained,

$$\Pi = -\zeta(\theta + \beta_0 D\Pi) \quad (2.19)$$

$$q^\mu = -\lambda \left[ \frac{nT^2}{\varepsilon + p} \nabla^\mu \left( \frac{\mu}{T} \right) - \beta_1 Dq^\mu \right] \quad (2.20)$$

$$\pi^{\mu\nu} = 2\eta [\nabla^{<\mu} u^{\nu>} - \beta_2 D\pi_{\mu\nu}], \quad (2.21)$$

where  $D = u^\mu \partial_\mu$  is the convective time derivative. Unlike in the 1st order theories, in 2nd order theories, dynamical equations control the dissipative flows. Even if thermodynamic forces vanish, dissipative flows do not vanish instantly.

Before we proceed further, it may be mentioned that the parameters,  $\alpha$  and  $\beta_\lambda$  are not connected to the actual state ( $N^\mu, T^{\mu\nu}$ ). The pressure  $p$  in Eq.2.12 is also not the "actual" thermodynamics pressure, i.e. not the work done in an isentropic expansion. Chemical potential  $\alpha$  and 4-inverse temperature  $\beta_\lambda$  has meaning only for the equilibrium state. Their meaning need not be extended to non-equilibrium states also. However, it is possible to fit a fictitious "local equilibrium" state, point by point, such that pressure  $p$  in Eq.2.12 can be identified with the thermodynamic pressure, at least up to 1st order. The conditions of fit fixes the underlying non-equilibrium phase-space distribution.

### III. (2+1)-DIMENSIONAL VISCOUS HYDRODYNAMICS WITH LONGITUDINAL BOOST INVARIANCE

Complete dissipative hydrodynamics is a numerically challenging problem. It requires simultaneous solution of 14 partial differential equations (5 conservation equations and 9 relaxation equations for dissipative flows). We reduce the problem to solution of 6 partial differential equations (3 conservation equations and 3 relaxation equations). In the following, we will study boost-invariant evolution of baryon free QGP fluid, including the dissipative effect due to shear viscosity only. Shear viscosity is the most important dissipative effect. For example, in a baryon free QGP, heat conduction is zero and we can disregard Eq.2.20. Bulk viscosity is also zero for the QGP fluid (point particles) and Eq.2.19 can also be neglected. Shear pressure tensor has 5 independent components but the assumption of boost invariance reduces the number of independent components to three. For a baryon free fluid, we can also disregard the conservation equation Eq.2.1. With the assumption of boost-invariance, energy-momentum conservation equation  $\partial_\mu T^{\mu\eta} = 0$  become redundant and only three energy-momentum conservation equations are required to be solved.

Heavy ion collisions are best described in  $(\tau, x, y, \eta)$  coordinates, where  $\tau = \sqrt{t^2 - z^2}$  is the longitudinal

proper time and  $\eta = \frac{1}{2} \ln \frac{t+z}{t-z}$  is the space-time rapidity.  $r_\perp = (x, y)$  are the usual cartesian coordinate in the plane, transverse to the beam direction. Relevant equations concerning this coordinate transformations are given in the appendix A.

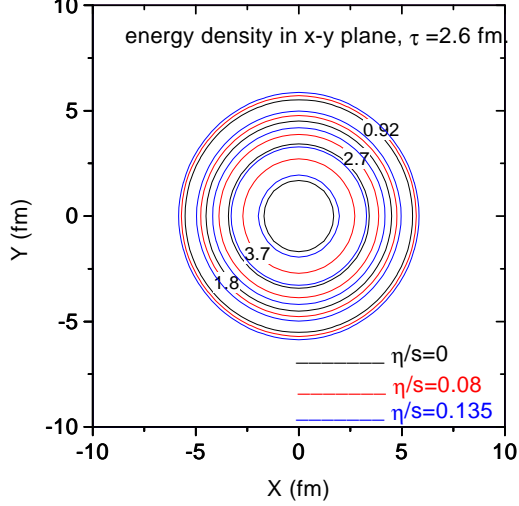


FIG. 1: (color online). Constant energy density contours in x-y plane at  $\tau=2.6$  fm. The black lines are for ideal fluid ( $\eta/s=0$ ). The red and blue lines are for viscous fluid with ADS/CFT and perturbative estimate of viscosity,  $\eta/s=0.08$  and  $0.135$ .

Explicit equations for energy-momentum conservation in  $(\tau, x, y, \eta)$  coordinates are given in the appendix B. We note that unlike in ideal fluid, in viscous fluid dynamics, conservation equations (see Eqs.B1-B3) contain additional pressure gradients due to shear viscosity. Both  $T^{\tau x}$  and  $T^{\tau y}$  components of energy-momentum tensor now evolve under additional pressure gradients. The right-most term of Eq.B3 also indicate that in viscous dynamics, longitudinal pressure is effectively reduced (note that the  $\pi^{\eta\eta}$  component is negative). Since pressure can not be negative, shear viscosity is limited by the condition,  $p + \tau^2 \pi^{\eta\eta} \geq 0$ .

As evident from the Eqs.B1-B3, in boost-invariant dissipative hydrodynamics, with shear viscosity taken into account, fluid evolution depends only on seven components of the shear stress tensors. They are  $\pi^{\tau\tau}$ ,  $\pi^{\tau x}$ ,  $\pi^{\tau y}$ ,  $\pi^{xx}$ ,  $\pi^{yy}$ ,  $\pi^{xy}$  and  $\pi^{\eta\eta}$ . However, all the seven components are not independent. Tracelessness, transversality to  $u^\mu$  and the assumption of boost-invariance reduces the independent components to three. Presently, we choose  $\pi^{xx}$ ,  $\pi^{yy}$  and  $\pi^{xy}$  as the independent components. Relaxation equations for the independent components are given in the appendix C (see Eqs.C4-C6). They are solved simultaneously with the three energy momentum conservation equations Eqs.B1-B3, with inputs as discussed below.

## IV. EQUATION OF STATE, VISCOSITY COEFFICIENT AND INITIAL CONDITIONS

### A. Equation of state

One of the most important inputs of a hydrodynamic model is the equation of state. Through this input, the macroscopic hydrodynamic models make contact with the microscopic world. In the present demonstrative calculation we will show results for the QGP phase only. In the QGP phase, we use the simple equation of state,  $p = \frac{1}{3}\epsilon$ , with energy density given as,

$$\epsilon = \frac{\pi^2}{30} g_q T^4 \quad (4.1)$$

the degeneracy factor for QGP is taken as  $g_q = 42.25$ .

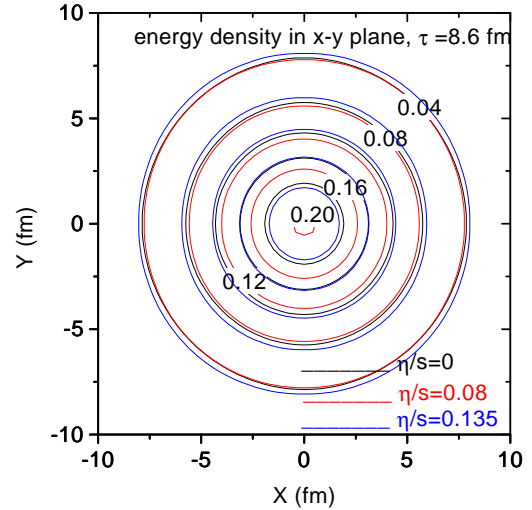


FIG. 2: (color online). same as Fig.1 but at time  $\tau=8.6$  fm.

### B. Shear viscosity coefficient

Shear viscosity coefficient ( $\eta$ ) of QGP or sQGP is quite uncertain. In perturbative regime, shear viscosity of a QGP is estimated [25, 26],

$$\eta = 86.473 \frac{1}{g^4} \frac{T^3}{\log g^{-1}}, \quad (4.2)$$

With entropy of QGP,  $s = 37 \frac{\pi^2}{15} T^3$  and  $\alpha_s \approx 0.5$ , the ratio of viscosity over the entropy, in the perturbative regime is estimated as,

$$\left(\frac{\eta}{s}\right)_{pert} \approx 0.135, \quad (4.3)$$

However, QGP produced in nuclear collisions is non-perturbative. It is strongly interacting QGP. Recently, using the ADS/CFT correspondence [7, 8] shear viscosity of a strongly coupled gauge theory,  $N=4$  SUSY YM, has been evaluated,  $\eta = \frac{\pi}{8} N_c^2 T^3$  and the entropy is given by  $s = \frac{\pi^2}{2} N_c^2 T^3$ . Thus in the strongly coupled field theory,

$$\left(\frac{\eta}{s}\right)_{\text{ADS/CFT}} = \frac{1}{4\pi} \approx 0.08, \quad (4.4)$$

which is approximately 2 times smaller than the perturbative estimate. In the present paper, we treat the shear viscosity as a parameter of the model. To demonstrate the effect of viscosity on flow and subsequent particle production, we use both the perturbative and ADS/CFT estimate of viscosity.

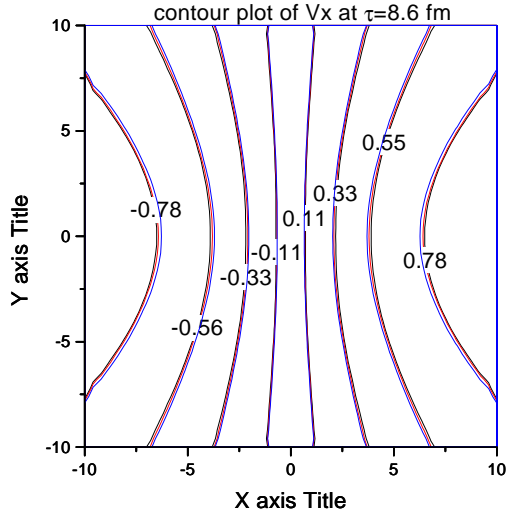


FIG. 3: (color online). contours of constant  $v_x$  in x-y plane at  $\tau=8.6$  fm. The black lines are for ideal fluid ( $\eta/s=0$ ). The red and blue lines are for viscous fluid with ADS/CFT and perturbative estimate of viscosity,  $\eta/s=0.08$  and  $0.135$ .

Shear viscosity can also be expressed in terms of sound attenuation length,  $\Gamma_s$ , defined as,

$$\Gamma_s = \frac{4\eta}{3sT} \quad (4.5)$$

$\Gamma_s$  is equivalent to mean free path and for a valid hydrodynamic description  $\Gamma_s/\tau \ll 1$ , i.e. mean free path is much less than the system size. Initial conditions of the fluid must be chosen carefully such that the validity condition  $\Gamma_s/\tau \ll 1$  remains valid initially as well as at later time also.

### C. Initial conditions

Solution of Eqs.B1-B3 require initial conditions, the initial time  $\tau_i$ , the transverse distribution of energy den-

sity  $\varepsilon(x, y)$  and the velocities  $v_x(x, y)$  and  $v_y(x, y)$ . Following [6], initial transverse energy density is parameterized geometrically. At an impact parameter  $\vec{b}$ , transverse distribution of wounded nucleons  $N_{WN}(x, y, \vec{b})$  and of binary NN collisions  $N_{BC}(x, y, \vec{b})$  to are calculated in a Glauber model. A collision at impact parameter  $\vec{b}$  is assumed to contain 25% hard scattering (proportional to number of binary collisions) and 75% soft scattering (proportional to number of wounded nucleons). Transverse energy density profile at impact parameter  $\vec{b}$  is then obtained as,

$$\varepsilon(x, y, \vec{b}) = \varepsilon_0(0.75 \times N_{WN}(x, y, \vec{b}) + 0.25 \times N_{BC}(x, y, \vec{b})) \quad (4.6)$$

with central energy density  $\varepsilon_0=30 \text{ GeV}/\text{fm}^{-3}$ . The equilibration time is chosen as  $\tau_i=0.6 \text{ fm}$  [6]. The initial velocities  $v_x$  and  $v_y$  are assumed to be zero initially.

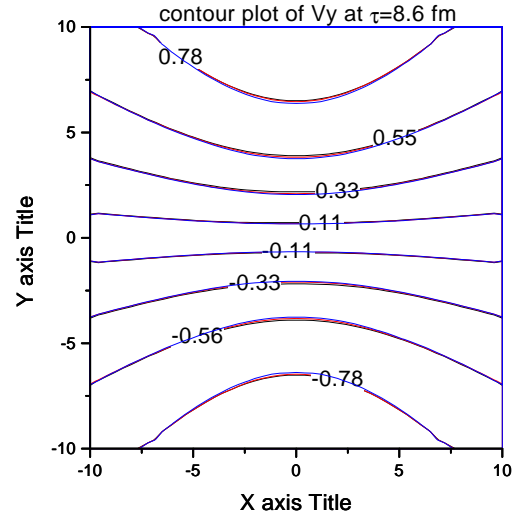


FIG. 4: (color online). contours of constant  $v_y$  in x-y plane at  $\tau=8.6$  fm. The black lines are for ideal fluid ( $\eta/s=0$ ). The red and blue lines are for viscous fluid with ADS/CFT and perturbative estimate of viscosity,  $\eta/s=0.08$  and  $0.135$ .

In dissipative hydrodynamics, one requires initial conditions for the viscous pressures also. Due to longitudinal boost invariance of the problem, we assume that viscous pressures have attained their boost-invariant values at the time of equilibration. Boost invariant values of the three independent shear stress-tensors can be easily obtained from Eqs.C4-C6,  $\sigma^{xx} = \sigma^{yy} = \theta = \frac{1}{\tau}$  and  $\sigma^{xy} = 0$  ( at the initial time  $\tau_i$ ,  $u^\mu = (1, 0, 0, 0)$ ,  $\bar{D}u^\mu = 0$ . The initial distribution of shear pressure tensors are then obtained as,

$$\pi^{xx}(x, y, \vec{b}) = 2\eta\sigma^{xx} = 2\eta/\tau_i \quad (4.7)$$

$$\pi^{yy}(x, y, \vec{b}) = 2\eta\sigma^{yy} = 2\eta/\tau_i \quad (4.8)$$

$$\pi^{xy}(x, y, \vec{b}) = 2\eta\sigma^{xy} = 0 \quad (4.9)$$

The viscous coefficient  $\eta$  is obtained using the relation,  $\eta/s = \text{const}$ ,  $\text{const}=0.08, 0.135$  for ADS/CFT and perturbative estimate respectively. For both the values of shear viscosity, the validity condition  $\Gamma_s/\tau \ll 1$  is satisfied initially. The validity condition is better satisfied at later time.

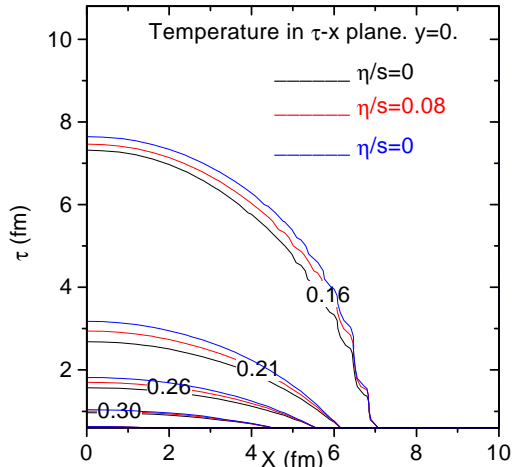


FIG. 5: (color online). Constant temperature contour in  $x - \tau$  plane, for fixed  $y=0$ . The black, red and blue lines are for ideal, viscous fluid with  $\eta/s=0.08$  and viscous fluid with  $\eta/s=0.135$ .

## V. RESULTS

### A. Evolution of the viscous QGP fluid

The energy-momentum conservation equations B1-B3, and the relaxation equations C2-C4 are solved simultaneously using the code, AZHYDRO-KOLKATA, developed at the Cyclotron Centre, Kolkata. As mentioned earlier, we have solved the equations in the QGP phase only and did not consider any phase transition. In the following we will show the results for central Au+Au collisions (impact parameter  $b = 0$  fm). To understand the effect of shear viscosity, with the same initial conditions, we have solved the energy-momentum conservation equations for ideal fluid and viscous fluid. As mentioned earlier, we have considered two values of viscosity, the ADS/CFT motivated value  $\eta/s=0.08$  and the perturbative estimate,  $\eta/s=0.135$ .

In Fig.1, we have shown the contours of constant energy density in  $x$ - $y$  plane, after an evolution of 2.6 fm. The black lines are for ideal fluid evolution. The red and blue lines are for viscous fluid with ADS/CFT ( $\eta/s=0.08$ ) and perturbative ( $\eta/s=0.135$ ) estimate of viscosity. Constant energy density contours, as depicted in Fig.1, indicate that with viscosity fluid cools slowly. Cooling gets

slower as viscosity increases. Thus at any point in the  $x$ - $y$  plane, energy density of viscous fluid is higher than that of an ideal fluid. At later time also, compared to an ideal fluid, viscous fluid evolve slowly. In Fig.2, contours of constant energy density at time  $\tau=8.6$  fm is shown. Here also we find that at any point energy density of viscous fluid is higher than its ideal counter part. The result is in accordance with our expectation. For dissipative fluid, equation of motion can be written as,

$$D\varepsilon = -(\varepsilon + p)\nabla_\mu u^\mu + \pi^{\mu\nu}\nabla_{<\mu}u_{>\nu} \quad (5.1)$$

Due to viscosity, evolution of energy density (or temperature) is slowed down.

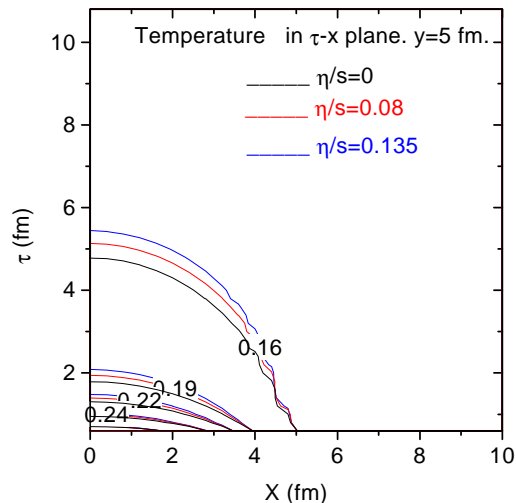


FIG. 6: (color online). same as fig.5 but at  $y=5.0$

In Fig.3 and 4, we have shown the contour plot of the fluid velocity,  $v_x$  and  $v_y$ , after evolution of 8.6 fm. As before the black lines are for the ideal fluid evolution. The red and blue lines are for viscous fluid with  $\eta/s=0.08$  and 0.135 respectively. Fluid velocities in viscous and ideal fluid differ very little. Even at late time, as shown in Fig.3 and 3, we find that for  $\eta/s=0.08-0.135$ ,  $x$  and  $y$  component of the fluid velocity show marginal difference. However, there is an indication that in a viscous fluid, velocity grow faster than in ideal fluid. But as mentioned earlier, the difference is marginal.

As seen in Fig.1-2, in viscous dynamics, QGP fluid evolves slowly. Thus life-time of the QGP phase is enhanced in viscous dynamics. To obtain an idea about the enhanced life-time, in Fig.5, we have shown the constant temperature contours in  $\tau - x$  plane, at a fixed value of  $y=0$  fm. As seen in Fig.5, temperature evolves slowly in a viscous fluid and life-time of the QGP phase is extended. For small viscosity  $\eta/s=0.08-0.135$ , the increase is not large. At the center of the fluid, for  $\eta/s=0.135$ , QGP life-time is increased approximately by 5% only. It

is even less for the ADS/CFT estimate of viscosity. However, enhancement of QGP life-time depends on the fluid cell position. In Fig.6, constant temperature contours at  $y=5$  fm is shown. For  $\eta/s=0.135$ , at  $x=0, y=5$  fm, the QGP life-time is enhanced by  $\sim 10\%$ . We conclude that in a viscous dynamics, with moderate viscosity  $\eta/s=0.08-0.135$ , QGP life-time could be enhanced by 5-10%. Enhanced lifetime of QGP in a viscous fluid can have significant effect on observables produced early in the collisions e.g. direct photon production or in  $J/\psi$  suppression.

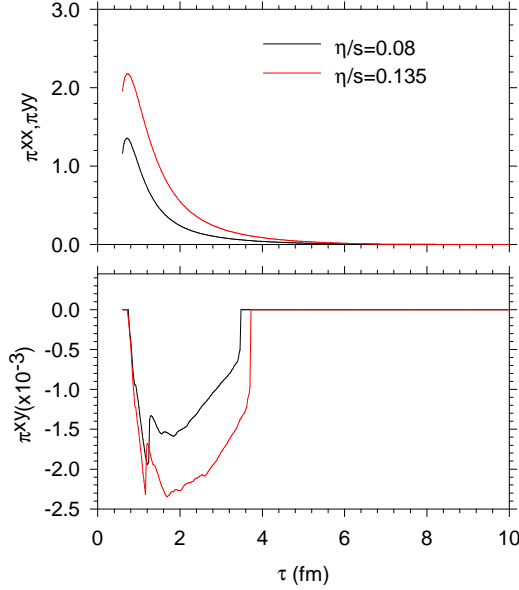


FIG. 7: (color online). In the upper panel, temporal evolution of the shear pressure tensor  $\pi^{xx}$  at the fluid cell  $x=y=0$  is shown. In the lower panel, evolution of  $\pi^{xy}$  at the fluid cell  $x=y=5$  fm is shown. The black and red lines are for ADS/CFT motivated viscosity  $\eta/s=0.08$  and perturbative estimate  $\eta/s=0.135$  respectively.

### B. Evolution of shear pressure tensors

We have assumed that initially the shear pressure tensors  $\pi^{xx}$ ,  $\pi^{yy}$  and  $\pi^{xy}$  attained their longitudinal boost-invariant values. As the fluid evolve, pressure tensors also evolve. Here we investigate the evolution of shear pressure tensors with time. In the top panel of Fig.7 evolution of shear pressure tensor  $\pi^{xx}$  at the fluid cell position  $x=y=0$  is shown. The black line is for the ADS/CFT motivated viscosity,  $\eta/s=0.08$  and the red line is for the perturbative estimate of viscosity  $\eta/s=0.135$ . Just after the start of the evolution the shear pressure tensor  $\pi^{xx}$  increases, but for a short duration and then steadily decreases with time. By 4 fm of evolution,  $\pi^{xx}$  at the center of the fluid reduces to negligibly small values. Identical

behavior is seen for the shear pressure tensor  $\pi^{yy}$ . In the bottom panel of Fig.7 we have shown the evolution of the third independent shear pressure tensor  $\pi^{xy}$ . Initially  $\pi^{xy}$  is zero. As the fluid evolve, it grows in the negative direction. We find that at the centre of the fluid ( $x=y=0$ ), it never grows. In Fig.7, temporal evolution of  $\pi^{xy}$  at the fluid cell position  $x=y=5$  fm is shown. From the initial zero value,  $\pi^{xy}$  rapidly increases in the negative direction. It reaches its maximum around  $\tau \approx 1$  fm and then decreases again. We also note that  $\pi^{xy}$  never grows to large values. Compared to  $\pi^{xx}$  or  $\pi^{yy}$  stress tensor  $\pi^{xy}$  is negligible. The results indicate that in a QGP fluid, viscous effect persist for a short duration (3-4 fm) only. At late time the fluid evolve essentially as an ideal fluid. The result is understandable. Shear viscosity depend strongly on temperature ( $\eta \propto T^3$ ). As the fluid cools, effect of viscosity decreases rapidly.

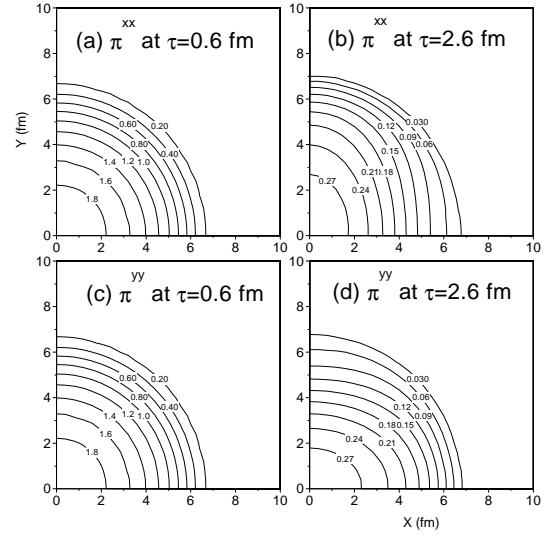


FIG. 8: (color online). In panel (a) and (b), contours of constant pressure tensor  $\pi^{xx}$  at initial time  $\tau_i=0.6$  fm and at time  $\tau=2.6$  fm is shown. In panel (c) and (d) same results for shear pressure tensor  $\pi^{yy}$  is shown.

To show the spatial distribution of the stress tensors, in Fig.8,  $\pi^{xx}$  and  $\pi^{yy}$  at initial time  $\tau_i=0.6$  fm and after an evolution of  $\tau = 2.6$  fm are shown. As shown earlier,  $\pi^{xx}$  and also  $\pi^{yy}$  rapidly decreases with time. By 2 fm of evolution they are reduced by approximately by a factor 6. It is also interesting to note that the initial x-y symmetric distribution of  $\pi^{xx}$  and  $\pi^{yy}$  quickly evolves to asymmetric distribution. With time  $\pi^{xx}$  evolves faster in the x-direction than in y-direction. Similarly,  $\pi^{yy}$  evolve faster in the y-direction than in the x-direction. For central collisions the asymmetric evolution of  $\pi^{xx}$  and  $\pi^{yy}$  counter balance each other. As shown in Fig.1 and 2, the contour plots of energy density do not show any indication of asymmetry even at late time. However, the



asymmetric pressure tensors can have important effects on elliptic flow of observables produced early in the collisions, say in elliptic flow of direct photons.

### C. Entropy generation

In a viscous fluid dynamics, entropy is generated. We can easily calculate the entropy generated during the evolution,

$$\begin{aligned}\partial_\mu S^\mu &= \frac{\pi^{\mu\nu}\pi_{\mu\nu}}{2\eta T} \\ &= \frac{1}{2\eta T} [(\pi^{\tau\tau})^2 + (\pi^{xx})^2 + (\pi^{yy})^2 + (\tau^2\pi^{\eta\eta})^2 \\ &\quad - 2(\pi^{\tau x})^2 - 2(\pi^{\tau y})^2 + 2(\pi^{xy})^2] \quad (5.2)\end{aligned}$$

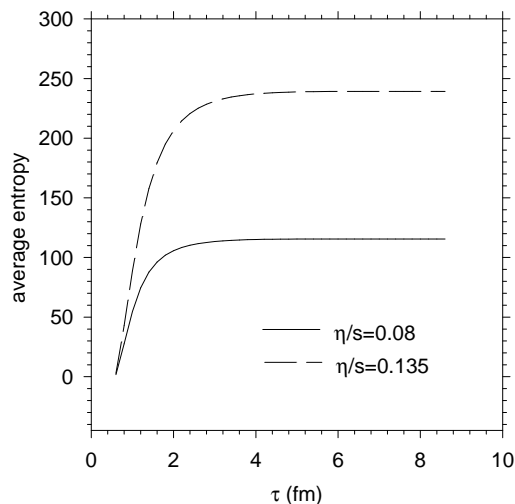


FIG. 9: Evolution of average entropy with time, for two values of viscosity, the ADS/CFT motivated viscosity  $\eta/s=0.08$  and perturbative estimate  $\eta/s=0.135$  are shown.

Evolution of spatially averaged entropy is shown in Fig.9, for the two values of viscosity coefficients  $\eta/s=0.08$  and  $0.135$ . As expected, entropy generation is more if viscosity is more. For both the values of viscosity, we find that entropy generation saturates after  $\approx 3$  fm of evolution. It is expected also. As shown previously, viscous fluxes reduces to very small values after  $\tau=3$  fm. Naturally, entropy generation is negligible thereafter.

### D. 1st order theory vs. 2nd order theory

As mentioned earlier, 1st order theory of dissipative hydrodynamics is acausal, signal can travel faster than light. This is corrected in 2nd order theory, but we

have to pay the price, relaxation equations for dissipative fluxes are required to be solved. It is interesting to compare the difference we can expect in a first order theory and in a 2nd order theory of dissipation. In Fig. 10, we have shown the contours of constant temperature in  $x-\tau$ , for a fixed  $y = 5$  fm. The black lines are for an ideal fluid. The red lines are for a viscous fluid treated in the 1st order theory. The blue lines are for viscous fluid in 2nd order theory. In 2nd order theory fluid evolve more slowly than in a first order theory. Entropy generation is also more in a 2nd order theory. In Fig.11, average entropy evolution with proper time is shown, both for the 1st order theory (the solid line) and the 2nd order theory. In 2nd order theory, approximately 80% more entropy is generated.

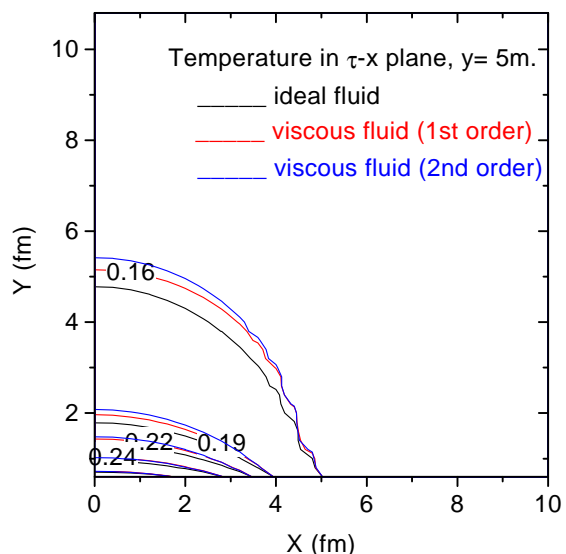


FIG. 10: (color online) constant temperature contours in  $x-\tau$  plane at  $y=5$  fm. The black lines are for ideal fluid. The red and blue lines are for viscous fluid in 1st order and in 2nd order theory respectively.  $\eta/s=0.135$ .

## VI. SUMMARY AND CONCLUSIONS

We have studied the boost-invariant hydrodynamic evolution of QGP fluid with dissipation due to shear viscosity only. In this study we have employed the Israel-Stewart's 2nd order theory of dissipative relativistic fluid. In 2nd order theory, in addition to usual thermodynamic quantities e.g. energy density, pressure, hydrodynamic velocities, dissipative flows are treated as extended thermodynamic variables. Relaxation equations for dissipative flows are solved, simultaneously with the energy-momentum conservation equations. This greatly enhances the complexity of the problem. Altogether 14 partial differential equations are required to be solved.



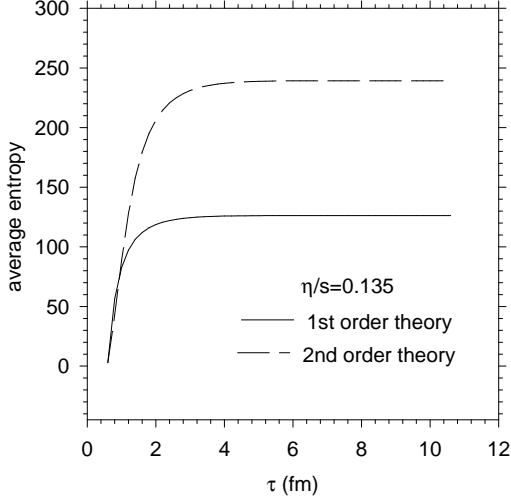


FIG. 11: Evolution of average entropy production in a 1st order (solid line) and 2nd order (dashed line) theory. 2nd order theory generate more entropy.

We simplify the problem to solution of six partial differential equations by considering the evolution of baryon free QGP fluid with longitudinal boost-invariance. We also consider dissipation due to shear viscosity only, disregarding the bulk viscosity and the heat conduction (for a baryon free QGP fluid they do not contribute). The six partial differential equations are solved using the code AZHYDRO-KOLKATA, developed at the Cyclotron Centre, Kolkata.

To bring out the effect of viscosity, we have considered the evolution of ideal as well as viscous QGP fluid. Both ideal and viscous fluid are initialized similarly, at initial time  $\tau_i = 0.6$  fm, the central entropy density is  $110 \text{ fm}^{-3}$ . Viscous dynamics require initial conditions for the shear-stress tensor components. It is assumed that at the equilibration time, the shear stress tensors components have attained their boost-invariant values. In the present paper, we have shown results for two values of viscosity coefficients, the ADS/CFT motivated value,  $\eta/s = 0.08$ , and the perturbative estimate of viscosity,  $\eta/s = 0.135$ . For both the values of viscosity ( $\eta/s = 0.08$  and  $0.135$ ), the condition of validity of viscous hydrodynamics,  $\Gamma_s/\tau \ll 1$  is satisfied all through the evolution.

Explicit simulation of ideal and viscous fluids confirms that energy density of a viscous fluid, evolve slowly than its ideal counterpart. Thus in a viscous fluid, lifetime of the QGP phase will be enhanced. Transverse expansion is also more in viscous dynamics. For a similar freeze-out condition freeze-out surface is extended in viscous fluid.

As the fluid evolve, shear pressure tensors also evolve. Explicit simulations indicate that shear pressure tensors  $\pi^{xx}$  and  $\pi^{yy}$  which are initially non zero, rapidly decreases as the fluid evolve. By 3-4 fm of evolution they reduced to very small values. The other independent shear tensor  $\pi^{zz}$  is zero initially. At later time it grow

in the negative direction but never grow to large value and is always order of magnitude smaller than the stress tensors ( $\pi^{xx}$  and  $\pi^{yy}$ ). For  $\eta/s = 0.08-0.135$ , in a QGP fluid, viscous effect persists only for a short duration  $\sim 3-4$  fm. Spatial distribution of shear pressure tensors  $\pi^{xx}$  and  $\pi^{yy}$  reveal an interesting feature of viscous dynamics. Initially  $\pi^{xx}$  and  $\pi^{yy}$  have symmetric distribution. As the fluid evolve, pressure tensors quickly become asymmetric, e.g.  $\pi^{xx}$  evolve faster in the x-direction than in the y-direction,  $\pi^{yy}$  evolve faster in y direction than in x-direction. However, in a central collision, we did not see any effect of asymmetry in the energy density distribution. In a central  $b=0$  collision, the two opposite asymmetry cancels each other.

We have not studied effects of viscous dynamics on any observable. They will be studied in a later publication. However, present results do indicate that shear viscosity, even if small  $\sim 0.08-0.135$ , can considerably affect the space-time evolution of QGP fluid. Life time of the QGP phase can increased by 5-10%. This may have important effect on observables produced early in the collisions, e.g.  $J/\psi$  suppression, direct photon production. Viscous dynamics can have important effect on elliptic flow also. As shown here, initially symmetric pressure tensors  $\pi^{xx}$  and  $\pi^{yy}$  quickly become asymmetric. In a non-central collision, the asymmetry generated due to viscous pressure can considerably affect elliptic flow.

## APPENDIX A: COORDINATE TRANSFORMATIONS

Instead of Cartesian coordinates  $x^\mu = (t, x, y, z)$  we use curvilinear coordinates in longitudinal proper time and rapidity,  $\bar{x}^m = (\tau, x, y, \eta)$ :

$$t = \tau \cosh \eta; \quad \tau = \sqrt{t^2 - z^2} \quad (\text{A1})$$

$$z = \tau \sinh \eta; \quad \eta = \frac{1}{2} \ln \frac{t+z}{t-z}. \quad (\text{A2})$$

The differentials

$$dt = d\tau \cosh \eta + d\eta \tau \sinh \eta, \quad (\text{A3})$$

$$dz = d\tau \sinh \eta + d\eta \tau \cosh \eta, \quad (\text{A4})$$

and the metric tensor is easily read off from

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu = dt^2 - dx^2 - dy^2 - dz^2 \\ &= \bar{g}_{mn} d\bar{x}^m d\bar{x}^n = d\tau^2 - dx^2 - dy^2 - \tau^2 d\eta^2, \end{aligned} \quad (\text{A5})$$

namely

$$\bar{g}_{mn} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -\tau^2 \end{pmatrix}, \quad \bar{g}^{mn} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1/\tau^2 \end{pmatrix} \quad (\text{A6})$$

In curvilinear coordinates we must replace the partial derivatives with respect to  $x^\mu$  by covariant derivatives

(denoted by a semicolon) with respect to  $\bar{x}^m$ :

$$\bar{T}_{;p}^{ik} = \frac{\partial \bar{T}^{ik}}{\partial \bar{x}^p} + \Gamma_{pm}^i \bar{T}^{mk} + \bar{T}^{im} \Gamma_{mp}^k.$$

The only non-vanishing Christoffel symbols are

$$\Gamma_{\eta\eta}^\tau = \tau; \quad \Gamma_{\tau\eta}^\eta = \Gamma_{\eta\tau}^\eta = 1/\tau. \quad (\text{A7})$$

The hydrodynamic 4-velocity  $u^\mu = \gamma(1, v_x, v_y, v_z)$  is transformed to  $\bar{u}^m = \gamma(1, v_x, v_y, 0)$ , with  $\gamma_\perp = 1/\sqrt{1-v_r^2}$ . From here on, we drop the bars over tensor components in  $\bar{x}$ -coordinates for simplicity.

The projector can be easily calculated,

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu = \begin{pmatrix} 1 - \gamma_\perp^2 & -\gamma_\perp^2 v_x & \gamma_\perp^2 v_y & 0 \\ -\gamma_\perp^2 v_x & -1 - \gamma_\perp^2 v_x^2 & -\gamma_\perp^2 v_x v_y & 0 \\ -\gamma_\perp^2 v_y & -\gamma_\perp^2 v_x v_y & -1 - \gamma_\perp^2 v_y^2 & 0 \\ 0 & 0 & 0 & \frac{1}{\tau^2} \end{pmatrix} \quad (\text{A8})$$

In  $(\tau, x, y, \eta)$  coordinate system, the convective time derivative can be obtained as,

$$D = u \cdot \partial = \gamma(\partial_\tau + v_x \partial_x + v_y \partial_y). \quad (\text{A9})$$

For future reference, we also write down the scalar expansion rate

$$\theta = \partial \cdot u = \partial_\tau u^\tau + \partial_x u^x + \partial_y u^y + \frac{u^\tau}{\tau} + \frac{u^r}{r} \quad (\text{A10})$$

## APPENDIX B: ENERGY-MOMENTUM CONSERVATION

With longitudinal boost-invariance the energy-momentum conservation equations  $T^{mn}_{;n} = 0$  yield

$$\partial_\tau \tilde{T}^{\tau\tau} + \partial_x (\tilde{T}^{\tau\tau} \bar{v}_x) + \partial_y (\tilde{T}^{\tau\tau} \bar{v}_y) = -(p + \tau^2 \pi^{\eta\eta}) \quad (\text{B1})$$

$$\partial_\tau \tilde{T}^{\tau x} + \partial_x (\tilde{T}^{\tau x} v_x) + \partial_y (\tilde{T}^{\tau x} v_y) = -\partial_x (\tilde{p} + \tilde{\pi}^{xx} - \tilde{\pi}^{\tau x} v_x) - \partial_y (\tilde{\pi}^{xy} - \tilde{\pi}^{\tau x} v_y) \quad (\text{B2})$$

$$\partial_\tau \tilde{T}^{\tau y} + \partial_x (\tilde{T}^{\tau y} v_x) + \partial_y (\tilde{T}^{\tau y} v_y) = -\partial_x (\tilde{\pi}^{xy} - \tilde{\pi}^{\tau y} v_x) - \partial_y (\tilde{p} + \tilde{\pi}^{yy} - \tilde{\pi}^{\tau y} v_y) \quad (\text{B3})$$

where  $\tilde{A}^{mn} \equiv \tau A^{mn}$ ,  $\tilde{p} \equiv \tau p$ , and  $\bar{v}_x \equiv T^{\tau x}/T^{\tau\tau}$ ,  $\bar{v}_y \equiv T^{\tau y}/T^{\tau\tau}$ .

The components of the energy momentum tensors, including the shear pressure tensor are,

$$T^{\tau\tau} = (\varepsilon + p) \gamma_\perp^2 - p + \pi^{\tau\tau} \quad (\text{B4})$$

$$T^{\tau x} = (\varepsilon + p) \gamma_\perp^2 v_x + \pi^{\tau x} \quad (\text{B5})$$

$$T^{\tau y} = (\varepsilon + p) \gamma_\perp^2 v_y + \pi^{\tau y} \quad (\text{B6})$$

In causal dissipative hydrodynamics, energy momentum conservation equations are solved simultaneously with the relaxation equations. Given an equation of state, if energy density ( $\varepsilon$ ) and fluid velocity ( $v_x$  and  $v_y$ ) distributions, at any time  $\tau_i$  are known, Eqs.B1,B2 and B3 can be integrated to obtain  $\varepsilon$ ,  $v_x$  and  $v_y$  at the next time step  $\tau_{i+1}$ . While for ideal hydrodynamics, this procedure works perfectly, viscous hydrodynamics poses a problem that shear stress-tensor components contains time derivatives,  $\partial_\tau \gamma_\perp$ ,  $\partial_\tau u^x$ ,  $\partial_\tau u^y$  etc. Thus at time step  $\tau_i$  one needs the still unknown time derivatives. Numerically, time derivatives at step  $\tau_i$  could be obtained if velocities at time step  $\tau_i$  and  $\tau_{i+1}$  are known. One possible way to circumvent the problem, is to use time derivatives of the previous step, i.e. use velocities at time step  $\tau_{i-1}$  and  $\tau_i$  to calculate the derivatives at time step  $\tau_i$  [19]. The underlying assumption that fluid velocity changes slowly with time. In 1st order theories, this problem is circumvented by calculating the time derivatives from the ideal equation of motion ,

$$Du^\mu = \frac{\nabla^\mu p}{\varepsilon + p}, \quad (\text{B7})$$

$$D\varepsilon = -(\varepsilon + p) \nabla_\mu u^\mu. \quad (\text{B8})$$

With the help of these two equations all the time derivatives can be expressed entirely in terms of spatial gradients [16, 27]. 1st order theories are restricted to contain terms at most linear in dissipative quantities. Neglect of viscous terms can contribute only in 2nd order corrections, which are neglected in 1st order theories. While the procedure is not correct in 2nd order theory, we still use it in the present calculations. The alternative procedure of using the derivative of earlier time step is not correct either.

## APPENDIX C: RELAXATION EQUATIONS FOR THE VISCOUS PRESSURE TENSOR

Being symmetric and traceless, the viscous pressure tensor  $\pi^{\mu\nu}$  has 9 independent components. The assumption of boost invariance reduces this number by 3 ( $\nabla^{(m} u^{\eta)} = 0$ ,  $m \neq \eta$ ). The transversality condition  $u_m \pi^{mn} = 0$  eliminates another three components ( $u_\eta$  vanish and thus yield no constraint). Thus, with boost-invariance the viscous pressure tensor has only three independent components. As seen in Eqs.B1,B2 and B3 in a boost-invariant evolution only seven pressure tensors

$\pi^{\tau\tau}$ ,  $\pi^{xx}$ ,  $\pi^{yy}$ ,  $\pi^{\eta\eta}$ ,  $\pi^{\tau x}$ ,  $\pi^{\tau y}$  and  $\pi^{xy}$  are of importance. Only three of these seven are independent. In an earlier publication [18], we have debated about the choice of the independent components and suggested use of either  $(\pi^{\tau\tau}, \pi^{\eta\eta}, \Delta = \pi^{xx} - \pi^{yy})$  or  $(\pi^{\tau\tau}, \pi^{\eta\eta}, \pi^{\tau x}, \pi^{\tau y})$  (which will require solution of an additional relaxation equation) as choice of independent components. However, while computing we find that the three pressure tensors  $\pi^{xx}$  and  $\pi^{yy}$  and  $\pi^{xy}$  as independent components are computationally more convenient. The choice has the advantage that the dependent shear stress tensors can be obtained from the 3 independent stress tensors by multiplying them by fluid velocity,  $v_x$  and  $v_y$  (see Eqs.

C7-C10). In any other choice of independent components (e.g.  $\pi^{\tau\tau}, \pi^{\eta\eta}, \Delta = \pi^{xx} - \pi^{yy}$ ), the evaluation of dependent stress tensors requires division by fluid velocities. Since initially, fluid velocities are assumed to be zero and they grow slowly, these choices will involve division by very small numbers. Unless proper care is not taken, division by small numbers can lead to unrealistically large values for the dependent stress tensors and ruin the computation.

The relaxation equations for the independent shear stress tensors  $\pi^{xx}$ ,  $\pi^{yy}$  and  $\pi^{xy}$ , in  $(\tau, x, y, \eta)$  co-ordinate can be written as,

$$\partial_\tau \pi^{xx} + v_x \partial_x \pi^{xx} + v_y \partial_y \pi^{xx} = -\frac{1}{\tau_\pi \gamma} (\pi^{xx} - 2\eta \sigma^{xx}) \quad (C1)$$

$$\partial_\tau \pi^{yy} + v_x \partial_x \pi^{yy} + v_y \partial_y \pi^{yy} = -\frac{1}{\tau_\pi \gamma} (\pi^{yy} - 2\eta \sigma^{yy}) \quad (C2)$$

$$\partial_\tau \pi^{xy} + v_x \partial_x \pi^{xy} + v_y \partial_y \pi^{xy} = -\frac{1}{\tau_\pi \gamma} (\pi^{xy} - 2\eta \sigma^{xy}) \quad (C3)$$

where  $\tau_\pi$  is the relaxation time,  $\tau_\pi = 2\eta/\beta_2$  (see Eq.2.21). In ultra-relativistic limit, for a Boltzman gas,  $\beta_2$  can be evaluated,  $\beta_2 \approx \frac{3}{4p}$  where  $p$  is the pressure [14]. In the present paper, we use this limit to obtain the relaxation time  $\tau_\pi$ .

The viscous pressure tensor relaxes on a time scale  $\tau_\pi$  to  $2\eta$  times the shear tensor  $\sigma^{\mu\nu} = \nabla^{(\mu} u^{\nu)}$ . The  $xx$ ,  $yy$  and  $xy$  components of the shear tensor  $\sigma^{\mu\nu}$  can be written as

$$\sigma^{xx} = -\partial_x u^x - u^x D u^x - \frac{1}{3} \Delta^{xx} \theta \quad (C4)$$

$$\sigma^{yy} = -\partial_y u^y - u^y D u^y - \frac{1}{3} \Delta^{yy} \theta \quad (C5)$$

$$\sigma^{xy} = -\frac{1}{2} [\partial_x u^y - \partial_y u^x - u^x D u^y - u^y D u^x] - \frac{1}{3} \Delta^{xy} \theta \quad (C6)$$

The dependent shear stress tensors can easily be obtained from the independent ones as,

$$\pi^{\tau x} = v_x \pi^{xx} + v_y \pi^{xy} \quad (C7)$$

$$\pi^{\tau y} = v_x \pi^{xy} + v_y \pi^{yy} \quad (C8)$$

$$\pi^{\tau\tau} = v_x^2 \pi^{xx} + v_y^2 \pi^{yy} + 2v_x v_y \pi^{xy} \quad (C9)$$

$$\tau^2 \pi^{\eta\eta} = -(1 - v_x^2) \pi^{xx} - (1 - v_y^2) \pi^{yy} + 2v_x v_y \pi^{xy} \quad (C10)$$

The expressions for the convective time derivative  $D$  and expansion scalar  $\theta = \partial u$ , in  $(\tau, x, y, \eta)$  are given in Eqs. A9 and A10.

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