CP VIOLATION IN BEAUTY DECAYS*

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Precision tests of the Kobayashi-Maskawa model of CP violation are discussed, pointing out possible signatures for other sources of CP violation and for new flavor-changing operators. The current status of the most accurate tests is summarized.

Keywords: CP violation; CKM; New Physics.

PACS Nos.: 13.25.Hw, 11.30.Er, 12.15.Ji, 14.40.Nd

1. Introduction

It took thirty-seven years from the discovery of a tiny CP violating effect of order 10^{-3} in $K_L \to \pi^+\pi^-$ [1] to a first observation of a breakdown of CP symmetry outside the strange meson system. A large CP asymmetry of order one between rates of initial B^0 and $\bar{B}^0$ decays to $J/\psi K_S$ was measured in summer 2001 by the Babar and Belle Collaborations.[2] A sizable however smaller asymmetry had been anticipated twenty years earlier[3] in the framework of the Kobayashi-Maskawa (KM) model of CP violation,[4] in the absence of crucial information on b quark couplings. The asymmetry was observed in a time-dependent measurement as suggested,[5] thanks to the long B^0 lifetime and the large B^0-$\bar{B}^0$ mixing.[6] The measured asymmetry, fixing (in the standard phase convention[7]) the sine of the phase 2β ($\equiv 2\phi_1$) $\equiv 2\arg(V_{tb}V_{td}^*)$ of the top-quark dominated B^0-$\bar{B}^0$ mixing amplitude, was found to be in good agreement with other determinations of Cabibbo-Kobayasi-Maskawa (CKM) parameters,[8,9] including a recent precise measurement of B_s-$\bar{B}_s$ mixing.[10] This showed that the CKM phase γ ($\equiv \phi_3$) $\equiv \arg(V_{ub}^*)$, which seems to be unable to account for the observed cosmological baryon asymmetry,[11] is the dominant source of CP violation in flavor-changing processes. With this confirmation the next pressing question became whether small contributions beyond the CKM framework occur in CP violating flavor-changing processes, and whether such effects can be observed.

One way of answering this question is by over-constraining the CKM unitarity triangle through precise CP conserving measurements related to the lengths of the

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*Based on review talks given at several recent conferences.

sides of the triangle. An alternative and more direct way, focusing on the origin of CP violation in the CKM framework, is to measure β and γ in a variety of B decay modes. Different values obtained from asymmetries in several processes, or values different from those imposed by other constraints, could provide clues for new sources of CP violation and for new flavor-changing interactions. Such phases and interactions occur in the low energy effective Hamiltonian of extensions of the Standard Model (SM) including models based on supersymmetry.[12]

In this presentation we will focus on the latter approach based primarily on CP asymmetries, using also complementary information on hadronic B decay rates which are expected to be related to each other in the CKM framework. In the next section we outline several of the most relevant processes and the theoretical tools applied for their studies, quoting numerous papers where these ideas have been originally proposed and where more details can be found.[13] Sections 3, 4 and 5 describe a number of methods in some detail, summarizing at the end of each section the current experimental situation. Section 6 discusses several tests for NP effects, while Section 7 concludes.

2. Processes, methods and New Physics effects

Whereas testing the KM origin of CP violation in most hadronic B decays requires separating strong and weak interaction effects, in a few "golden modes" CP asymmetries are unaffected by strong interactions. For instance, the decay $B^0 \to J/\psi K_S$ is dominated by a single tree-level quark transition $\bar{b} \to \bar{c}c\bar{s}$, up to a correction smaller than a fraction of a percent.[14,15,16,17] Thus, the asymmetries measured in this process and in other decays dominated by $\bar{b} \to \bar{c}c\bar{s}$ have already provided a rather precise measurement of $\sin 2\beta$,[18,19,20]

$$\sin 2\beta = 0.678 \pm 0.025 \ . \tag{1}$$

This value permits two solutions for β at $21.3°$ and at $68.7°$. Time-dependent angular studies of $B^0 \to J/\psi K^{*0}$,[21] and time-dependent Dalitz analyses of $B^0 \to Dh^0$ $(D \to K_S\pi^+\pi^-, h^0 = \pi^0, \eta, \omega)$[22] measuring $\cos 2\beta > 0$ have excluded the second solution at a high confidence level, implying

$$\beta = (21.3 \pm 1.0)° \ . \tag{2}$$

Since $B^0 \to J/\psi K_S$ proceeds through a CKM-favored quark transition, contributions to the decay amplitude from physics at a higher scale are expected to be very small, potentially identifiable by a tiny direct asymmetry in this process or in $B^+ \to J/\psi K^+$.[23]

Another process where the determination of a weak phase is not affected by strong interactions is $B^+ \to DK^+$, proceeding through tree-level amplitudes $\bar{b} \to \bar{c}u\bar{s}$ and $\bar{b} \to \bar{u}c\bar{s}$. The interference of these two amplitudes, from $\bar{D}^0$ and D^0 which can always decay to a common hadronic final state, leads to decay rates and a CP asymmetry which measure very cleanly the relative phase γ between these

amplitudes.[24,25] The trick here lies in recognizing the measurements which yield this fundamental CP-violating quantity. Physics beyond the SM is expected to have a negligible effect on this determination of γ which relies on the interference of two tree amplitudes.

B decays into pairs of charmless mesons, such as $B \to \pi\pi$ (or $B \to \rho\rho$) and $B \to K\pi$ (or $B \to K^*\rho$), involve contributions of both tree and penguin amplitudes which carry different weak and strong phases.[14,26,27] Contrary to the case of $B \to DK$, the determination of β and γ using CP asymmetries in charmless B decays involves two correlated aspects which must be considered: its dependence on strong interaction dynamics and its sensitivity to potential New Physics (NP) effects. This sensitivity follows from the CKM and loop suppression of penguin amplitudes, implying that new heavy particles at the TeV mass range, replacing the W boson and the top-quark in the penguin loop, may have sizable effects.[28]. In order to claim evidence for physics beyond the SM from a determination of β and γ in these processes one must handle first the question of dynamics. There are two approaches for treating the dynamics of charmless hadronic B decays:

(1) Study systematically strong interaction effects in the framework of QCD.
(2) Identify by symmetry observables which do not depend on QCD dynamics.

The first approach faces the difficulty of having to treat precisely long distance effects of QCD including final state interactions. Remarkable theoretical progress has been made recently in proving a leading-order (in $1/m_b$) factorization formula for these amplitudes in a heavy quark effective theory approach to perturbative QCD.[29,30,31] However, there remain differences between ways of treating in different approaches power counting, the scale of Wilson coefficients, end-point quark distribution functions of light mesons, and nonperturbative contributions from charm loops.[32] Also, the nonperturbative input parameters in these calculations involve non-negligible uncertainties. These parameters include heavy-to-light form factors at small momentum transfer, light-cone distribution amplitudes, and the average inverse momentum fraction of the spectator quark in the B meson. The resulting inaccuracies in calculating magnitudes and strong phases of amplitudes prohibit a precise determination of γ from measured decay rates and CP asymmetries. Also, the calculated rates and asymmetries cannot provide a clear case for physics beyond the SM in cases where the results of a calculation deviate slightly from the measurements.

In the second approach one applies isospin symmetry to obtain relations among several decay amplitudes. For instance, using the distinct behavior under isospin of tree and penguin operators contributing in $B \to \pi\pi$, a judicious choice of observables permits a determination of γ or α ($\equiv \phi_2) = \pi - \beta - \gamma$. [33] The same analysis applies in B decays to pairs of longitudinally polarized ρ mesons. In case that an observable related to the subdominant penguin amplitude is not measured with sufficient precision, it may be replaced in the analysis by a CKM-enhanced SU(3)-related observable, in which a large theoretical uncertainty is translated to a small

error in γ. The precision of this method is increased by including contributions of higher order electroweak penguin amplitudes, which are related by isospin to tree amplitudes.[34,35] With sufficient statistics one should also take into account isospin-breaking corrections of order $(m_d - m_u)/\Lambda_{\mathrm{QCD}} \sim 0.02$,[36,37] and an effect caused by the ρ meson width.[38] A similar analysis proposed for extracting γ in $B \to K\pi$ [39,40] requires using flavor SU(3) instead of isospin for relating electroweak penguin contributions and tree amplitudes.[35,41] While flavor SU(3) is usually assumed to be broken by corrections of order $(m_s - m_d)/\Lambda_{\mathrm{QCD}} \sim 0.3$, in this particular case a rather precise recipe for SU(3) breaking is provided by QCD factorization, reducing the theoretical uncertainty in γ to only a few degrees.[42]

Charmless B decays, which are sensitive to physics beyond the SM [28], provide a rich laboratory for studying various signatures of NP. A large variety of theories have been studied in this context, including supersymmetric models, models involving tree-level flavor-changing Z or Z' couplings, models with anomalous three-gauge-boson couplings and other models involving an enhanced chromomagnetic dipole operator.[43,44] The following effects have been studied and will be discussed in Section 6 in a model-independent manner:

(1) Within the SM, the three values of γ extracted from $B \to \pi\pi$, $B \to K\pi$ and $B^+ \to DK^+$ are equal. As we will explain, these three values are expected to be different in extensions of the SM involving new low energy four-fermion operators behaving as $\Delta I = 3/2$ in $B \to \pi\pi$ and as $\Delta I = 1$ in $B \to K\pi$.

(2) Other signatures of anomalously large $\Delta I = 1$ operators contributing to $B \to K\pi$ are violations of isospin sum rules, holding in the SM for both decay rates and CP asymmetries in these decays.[45,46,47]

(3) Time-dependent asymmetries in $B^0 \to \pi^0 K_S$, $B^0 \to \phi K_S$ and $B^0 \to \eta' K_S$ and in other $b \to s$ penguin-dominated decays may differ substantially from the asymmetry $\sin 2\beta \sin \Delta mt$, predicted approximately in the SM.[26,43,48] Significant deviations are expected in models involving anomalous $|\Delta S| = 1$ operators behaving as $\Delta I = 0$ or $\Delta I = 1$.

(4) An interesting question, which may provide a clue to the underlying New Physics once deviations from SM predictions are observed, is how to diagnose the value of ΔI in NP operators contributing to $|\Delta S| = 1$ charmless B decays. We will discuss an answer to this question which has been proposed recently.[49]

3. Determining γ in $B \to DK$

In this section we will discuss in some length a rather rich and very precise method for determining γ in processes of the form $B \to D^{(*)}K^{(*)}$, which uses both charged and neutral B mesons and a large variety of final states. It is based on a broad idea that any coherent admixture of a state involving a $\bar{D}^0$ from $\bar{b} \to \bar{c}u\bar{s}$ and a state with D^0 from $\bar{b} \to \bar{u}c\bar{s}$ can decay to a common final state.[24,25] The interference between the two channels, $B \to D^{(*)0}K^{(*)}$, $D^0 \to f_D$ and $B \to \bar{D}^{(*)0}K^{(*)}$, $\bar{D}^0 \to$

f_D, involves the weak phase difference γ, which may be determined with a high theoretical precision using a suitable choice of measurements. Effects of D^0-$\bar{D}^0$ mixing are negligible.[50] While some of these processes are statistically limited, combining them together is expected to reduce the experimental error in γ. In addition to (quasi) two-body B decays, the D or D^* in the final state may be accompanied by any multi-body final state with quantum numbers of a kaon.[25]

Each process in this large class of neutral and charged B decays is characterized by two pairs of parameters, describing complex ratios of amplitudes for D^0 and $\bar{D}^0$ for the two steps of the decay chain (we use a convention $r_B, r_f \geq 0, 0 \leq \delta_B, \delta_f < 2\pi$),

$$\frac{A(B \to D^{(*)0}K^{(*)})}{A(B \to \bar{D}^{(*)0}K^{(*)})} = r_B e^{i(\delta_B + \gamma)} \ , \qquad \frac{A(D^0 \to f_D)}{A(\bar{D}^0 \to f_D)} = r_f e^{i\delta_f} \ . \qquad (3)$$

In three-body decays of B and D mesons, such as $B \to DK\pi$ and $D \to K\pi\pi$, the two pairs of parameters (r_B, δ_B) and (r_f, δ_f) are actually functions of two corresponding Dalitz variables describing the kinematics of the above three-body decays. The sensitivity of determining γ depends on r_B and r_f because this determination relies on an interference of D^0 and $\bar{D}^0$ amplitudes. For D decay modes with $r_f \sim 1$ (see discussion below) the sensitivity increases with the magnitude of r_B.

For each of the eight sub-classes of processes, $B^{+,0} \to D^{(*)}K^{(*)+,0}$, one may study a variety of final states in neutral D decays. The states f_D may be divided into four families, distinguished qualitatively by their parameters (r_f, δ_f) defined in Eq. (3):

(1) f_D = CP-eigenstate[24,25,51] $(K^+K^-, K_S\pi^0, \text{etc.}); r_f = 1, \delta_f = 0, \pi$.
(2) f_D = flavorless but non-CP state[52] $(K^+K^{*-}, K^{*+}K^-, \text{etc.}); r_f = \mathcal{O}(1)$.
(3) f_D = flavor state[53] $(K^+\pi^-, K^+\pi^-\pi^0, \text{etc.}); r_f \sim \tan^2\theta_c$.
(4) f_D = 3-body self-conjugate state[54] $(K_S\pi^+\pi^-); r_f, \delta_f$ vary across the Dalitz plane.

In the first family, CP-odd states occur in Cabibbo-favored D^0 and $\bar{D}^0$ decays, while CP-even states occur in singly Cabibbo-suppressed decays. The second family of states occurs in singly Cabibbo-suppressed decays, the third family occurs in Cabibbo-favored $\bar{D}^0$ decays and in doubly Cabibbo-suppressed D^0 decays, while the last state is formally a Cabibbo-favored mode for both D^0 and $\bar{D}^0$.

The parameters r_B and δ_B in $B \to D^{(*)}K^{(*)}$ depend on whether the B meson is charged or neutral, and may differ for K vs K^*,[55] and for D vs D^*, where a neutral D^* can be observed in $D^* \to D\pi^0$ or $D^* \to D\gamma$.[56] The ratio r_B involves a CKM factor $|V_{ub}V_{cs}/V_{cb}V_{us}| \simeq 0.4$ in both B^+ and B^0 decays, and a color-suppression factor in B^+ decays, while in B^0 decays both $\bar{b} \to \bar{c}u\bar{s}$ and $\bar{b} \to \bar{u}c\bar{s}$ amplitudes are color-suppressed. A rough estimate of the color-suppression factor in these decays may be obtained from the color-suppression measured in corresponding CKM-favored decays, $B \to D\pi, D^*\pi, D\rho, D^*\rho$, where the suppression is found to be in the range $0.3 - 0.5$.[57] Thus, one expects $r_B(B^0) \sim 0.4$, $r_B(B^+) = (0.3 -$

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$0.5)r_B(B^0)$ in all the processes $B^{+,0} \to D^{(*)}K^{(*)+,0}$. We note that three-body B^+ decays, such as $B^+ \to D^0 K^+ \pi^0$, are not color-suppressed, making these processes advantageous by their potentially large value of r_B which varies in phase space.[58,59]

The above comparison of $r_B(B^+)$ and $r_B(B^0)$ may be quantified more precisely by expressing the four ratios $r_B(B^0)/r_B(B^+)$ in $B \to D^{(*)}K^{(*)}$ in terms of reciprocal ratios of known magnitudes of amplitudes:[60]

$$\frac{r_B(B^0 \to D^{(*)}K^{(*)0})}{r_B(B^+ \to D^{(*)}K^{(*)+})} \simeq \sqrt{\frac{(B^+ \to \bar{D}^{(*)0}K^{(*)+})}{(B^0 \to \bar{D}^{(*)0}K^{(*)0})}} \ . \tag{4}$$

This follows from an approximation,

$$A(B^0 \to D^{(*)0}K^{(*)0}) \simeq A(B^+ \to D^{(*)0}K^{(*)+}) \ , \tag{5}$$

where the B^0 and B^+ processes are related to each other by replacing a spectator d quark by a u quark. While formally Eq. (5) is not an isospin prediction, it may be obtained using an isospin triangle relation,[61]

$$A(B^0 \to D^{(*)0}K^{(*)0}) = A(B^+ \to D^{(*)0}K^{(*)+}) + A(B^+ \to D^{(*)+}K^{(*)0}), \tag{6}$$

and neglecting the second amplitude on the right-hand-side which is "pure annihilation".[62] This amplitude is expected to be suppressed by a factor of four or five relative to the other two amplitudes appearing in (6) which are color-suppressed. Evidence for this kind of suppression is provided by corresponding ratios of CKM-favored amplitudes,[57] $|A(B^0 \to D_s^- K^+)/\sqrt{2}A(\bar{D}^0\pi^0)| = 0.23 \pm 0.03, |A(B^0 \to D_s^{*-} K^+)/\sqrt{2}A(\bar{D}^{*0}\pi^0)| < 0.24$.

Applying Eq. (4) to measured branching ratios,[57,63] one finds

$$\frac{r_B(B^0)}{r_B(B^+)} = \left\{ \begin{array}{cccc} B \to DK & B \to DK^* & B \to D^*K & B \to D^*K^* \\ 2.9 \pm 0.4 & 3.7 \pm 0.3 & > 2.2 & > 3.0 \end{array} \right. \tag{7}$$

This agrees with values of $r_B(B^0)$ near 0.4 and $r_B(B^+)$ between 0.1 and 0.2. Note that in spite of the expected larger values of r_B in B^0 decays, from the point of view of statistics alone (without considering the question of flavor tagging and the efficiency of detecting a K_S in $B^0 \to D^{(*)}K^0$), B^+ and B^0 decays may fare comparably when studying γ. This follows from (5) because the statistical error on γ scales roughly as the inverse of the smallest of the two interfering amplitudes.

We will now discuss the actual manner by which γ can be determined using *separately* three of the above-mentioned families of final states f_D. We will mention advantages and disadvantages in each case. For illustration of the method we will consider $B^+ \to f_D K^+$. We will also summarize the current status of these measurements in all eight decay modes $B^{+,0} \to D^{(*)}K^{(*)+,0}$.

3.1. f_D = CP-eigenstates

One considers four observables consisting of two charge-averaged decay rates for even and odd CP states, normalized by the decay rate into a D^0 flavor state,

$$R_{\text{CP}\pm} \equiv \frac{\Gamma(D_{\text{CP}\pm}K^-) + \Gamma(D_{\text{CP}\pm}K^+)}{\Gamma(D^0 K^-)} \ , \tag{8}$$

and two CP asymmetries for even and odd CP states,

$$A_{\text{CP}\pm} \equiv \frac{\Gamma(D_{\text{CP}\pm}K^-) - \Gamma(D_{\text{CP}\pm}K^+)}{\Gamma(D_{\text{CP}\pm}K^-) + \Gamma(D_{\text{CP}\pm}K^+)} \ . \tag{9}$$

In order to avoid dependence of $R_{\text{CP}\pm}$ on errors in D^0 and D_{CP} branching ratio measurements one uses a definition of $R_{\text{CP}\pm}$ in terms of ratios of B decay branching ratios into DK and $D\pi$ final states.[59] The four observables $R_{\text{CP}\pm}$ and $A_{\text{CP}\pm}$ provide three independent equations for r_B, δ_B and γ,

$$R_{\text{CP}\pm} = 1 + r_B^2 \pm 2r_B \cos\delta_B \cos\gamma \ , \tag{10}$$

$$A_{\text{CP}\pm} = \pm 2r_B \sin\delta_B \sin\gamma / R_{\text{CP}\pm} \ . \tag{11}$$

While in principle this is the simplest and most precise method for extracting γ, up to a discrete ambiguity, in practice this method is sensitive to r_B^2, because $(R_{\text{CP}+} + R_{\text{CP}-})/2 = 1 + r_B^2$. This becomes very difficult for charged B decays where one expects $r_B \sim 0.1 - 0.2$, but may be feasible for neutral B decays where $r_B \sim 0.4$. An obvious signature for a non-zero value of r_B would be observing a difference between $R_{\text{CP}+}$ and $R_{\text{CP}-}$ which is linear in this quantity.

Studies of $B^+ \to D_{\text{CP}}K^+, B^+ \to D_{\text{CP}}K^{*+}$ and $B^+ \to D^*_{\text{CP}}K^+$ have been carried out recently,[64,65,66] each consisting of a few tens of events. A nonzero difference $R_{\text{CP}+} - R_{\text{CP}-}$ at 2.6 standard deviations, measured in $B^+ \to D_{\text{CP}}K^{*+}$,[64] is probably a statistical fluctuation. A larger difference is anticipated in $B^0 \to D_{CP}K^{*0}$, as the value of r_B in this process is expected to be three or four times larger than in $B^+ \to DK^{*+}$. [See Eq. (7).] Higher statistics is required for a measurement of γ using this method.

3.2. f_D = flavor state

Consider a flavor state f_D in Cabibbo-favored $\bar{D}^0$ decays, accessible also to doubly Cabibbo-suppressed D^0 decays, such that one has $r_f \sim \tan^2\theta_c$ in Eq. (3). One studies the ratio of two charge-averaged decay rates, for decays into $\bar{f}_D K$ and $f_D K$,

$$R_f \equiv \frac{\Gamma(f_D K^-) + \Gamma(\bar{f}_D K^+)}{\Gamma(\bar{f}_D K^-) + \Gamma(f_D K^+)} \ , \tag{12}$$

and the CP asymmetry,

$$A_f \equiv \frac{\Gamma(f_D K^-) - \Gamma(\bar{f}_D K^+)}{\Gamma(f_D K^-) + \Gamma(\bar{f}_D K^+)} \ . \tag{13}$$

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These observables are given by

$$R_f = r_B^2 + r_f^2 + 2r_B\, r_f\, \cos(\delta_B - \delta_f)\, \cos\gamma\ , \qquad (14)$$

$$A_f = 2r\, r_f\, \sin(\delta_B - \delta_f)\, \sin\gamma/R_f\ , \qquad (15)$$

where a multiplicative correction $1 + \mathcal{O}(r_B r_f) \sim 1.01$ has been neglected in (14).

These two observables involve three unknowns, $r_B, \delta_B - \delta_f$ and γ. One assumes r_f to be given by the measured ratio of doubly Cabibbo-suppressed and Cabibbo-favored branching ratios. Thus, one needs at least two flavor states, f_D and f'_D, for which two pairs of observables (R_f, A_f) and $(R_{f'}, A_{f'})$ provide four equations for the four unknowns, $r_B, \delta_B - \delta_f, \delta_B - \delta_{f'}, \gamma$. The strong phase differences $\delta_f, \delta_{f'}$ can actually be measured at a ψ'' charm factory,[67] thereby reducing the number of unknowns to three.

While the decay rate in the numerator of R_f is rather low, the asymmetry A_f may be large for small values of r_B around 0.1, as it involves two amplitudes with a relative magnitude r_f/r_B.

So far, only upper bounds have been measured for R_f implying upper limits on r_B in several processes, $r_B(B^+ \to DK^+) < 0.2$,[68,69,70] $r_B(B^+ \to D^*K^+) < 0.2$,[68] $r(B^+ \to DK^{*+}) < 0.4$,[71] and $r_B(B^0 \to DK^{*0}) < 0.4$.[63,72] Further constraints on r_B in the first three processes have been obtained by studying D decays into CP-eigenstates and into the state $K_S\pi^+\pi^-$. Using $r_B(B^0 \to DK^{*0})/r_B(B^+ \to DK^{*+}) = 3.7 \pm 0.3$ in (7) and assuming that $r_B(B^+ \to DK^{*+})$ is not smaller than about 0.1, one may conclude that a nonzero measurement of $r_B(B^0 \to DK^{*0})$ should be measured soon. The signature for $B^0 \to D^0K^{*0}$ events would be two kaons with opposite charges.

3.3. $f_D = K_S\pi^+\pi^-$

The amplitude for $B^+ \to (K_S\pi^+\pi^-)_D K^+$ is a function of the two invariant-mass variables, $m_\pm^2 \equiv (p_{K_S} + p_{\pi\pm})^2$, and may be written as

$$A(B^+ \to (K_S\pi^+\pi^-)_D K^+) = f(m_+^2, m_-^2) + r_B e^{i(\delta_B + \gamma)} f(m_-^2, m_+^2)\ . \qquad (16)$$

In B^- decay one replaces $m_+ \leftrightarrow m_-$, $\gamma \to -\gamma$. The function f may be written as a sum of about twenty resonant and nonresonant contributions modeled to describe the amplitude for flavor-tagged $\bar{D}^0 \to K_S\pi^+\pi^-$ which is measured separately.[73,74] This introduces a model-dependent uncertainty in the analysis. Using the measured function f as an input and fitting the rates for $B^\pm \to (K_S\pi^+\pi^-)_D K^\pm$ to the parameters, r_B, δ_B and γ, one then determines these three parameters.

The advantage of using $D \to K_S\pi^+\pi^-$ decays over CP and flavor states is being Cabibbo-favored and involving regions in phase space with a potentially large interference between D^0 and $\bar{D}^0$ decay amplitudes. The main disadvantage is the uncertainty introduced by modeling the function f.

Two recent analyses of comparable statistics by Belle and Babar, combining $B^\pm \to DK^\pm, B^\pm \to D^*K^\pm$ and $B^\pm \to DK^{*\pm}$, obtained values [73] $\gamma = [53^{+15}_{-18} \pm 3 \pm$

9(model)]° and $\gamma = [92\pm41\pm11\pm12(\text{model})]°$.[74] [This second value does not use the process $B^+ \to D(K_S\pi^+)_{K^*}$, also studied by the same group,[75].] The larger errors in the second analysis are correlated with smaller values of the extracted parameters r_B in comparison with those extracted in the first study. The model-dependent errors may be reduced by studying at CLEO-c the decays $D_{CP\pm} \to K_S\pi^+\pi^-$, providing further information on strong phases in D decays.[67]

Conclusion: The currently most precise value of γ is $\gamma = [53^{+15}_{-18} \pm 3 \pm 9(\text{model})]°$, obtained from $B^\pm \to D^{(*)}K^{(*)\pm}$ using $D \to K_S\pi^+\pi^-$. These errors may be reduced in the future by combining the study of *all D decay modes* in $B^{+,0} \to D^{(*)}K^{(*)+,0}$. The decay $B^0 \to DK^{*0}$ seems to carry a high potential because of its expected large value of r_B. Decays $B^0 \to D^{(*)}K^0$ may also turn useful, as they have been shown to provide information on γ without the need for flavor tagging of the initial B^0.[60,76]

4. The currently most precise determination of γ: $B \to \pi\pi, \rho\rho, \rho\pi$

4.1. $B \to \pi\pi$

The amplitude for $B^0 \to \pi^+\pi^-$ contains two terms, conventionally denoted "tree" (T) and "penguin" (P) amplitudes,[14,26] involving a weak CP-violating phase γ and a strong CP-conserving phase δ, respectively:

$$A(B^0 \to \pi^+\pi^-) = |T|e^{i\gamma} + |P|e^{i\delta} \ . \tag{17}$$

Time-dependent decay rates, for an initial B^0 or a $\overline{B}^0$, are given by

$$\Gamma(B^0(t)/\overline{B}^0(t) \to \pi^+\pi^-) = e^{-\Gamma t}\Gamma_{\pi^+\pi^-} [1 \pm C_{+-}\cos\Delta mt \mp S_{+-}\sin\Delta mt] \ , \tag{18}$$

where

$$S_{+-} = \frac{2\text{Im}(\lambda_{\pi\pi})}{1 + |\lambda_{\pi\pi}|^2} \ , \quad C_{+-} = \frac{1 - |\lambda_{\pi\pi}|^2}{1 + |\lambda_{\pi\pi}|^2} \ , \quad \lambda_{\pi\pi} \equiv e^{-2i\beta}\frac{A(\overline{B}^0 \to \pi^+\pi^-)}{A(B^0 \to \pi^+\pi^-)} \ . \tag{19}$$

One has[14]

$$S_{+-} = \sin 2\alpha + 2|P/T|\cos 2\alpha \sin(\beta + \alpha)\cos\delta + \mathcal{O}(|P/T|^2) \ ,$$
$$C_{+-} = 2|P/T|\sin(\beta + \alpha)\sin\delta + \mathcal{O}(|P/T|^2) \ . \tag{20}$$

This tells us two things:
(1) The deviation of S_{+-} from $\sin 2\alpha$ and the magnitude of C_{+-} increase with $|P/T|$, which can be estimated to be $|P/T| \sim 0.5$ by comparing $B \to \pi\pi$ rates with penguin-dominated $B \to K\pi$ rates.[77]
(2) $\Gamma_{\pi^+\pi^-}$, S_{+-} and C_{+-} are insufficient for determining $|T|, |P|, \delta$ and γ (or α). Further information on these quantities may be obtained by applying isospin symmetry to all $B \to \pi\pi$ decays.

In order to carry out an isospin analysis,[33] one uses the fact that the three physical $B \to \pi\pi$ decay amplitudes and the three $\bar{B} \to \pi\pi$ decay amplitudes,

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depending each on two isospin amplitudes, obey triangle relations of the form,

$$A(B^0 \to \pi^+\pi^-)/\sqrt{2} + A(B^0 \to \pi^0\pi^0) - A(B^+ \to \pi^+\pi^0) = 0 \ . \qquad (21)$$

Furthermore, the penguin amplitude is pure $\Delta I = 1/2$; hence the $\Delta I = 3/2$ amplitude carries a week phase γ, $A(B^+ \to \pi^+\pi^0) = e^{2i\gamma}A(B^- \to \pi^-\pi^0)$. Defining $\sin 2\alpha_{\text{eff}} \equiv S_{+-}/(1 - C_{+-}^2)^{1/2}$, the difference $\alpha_{\text{eff}} - \alpha$ is then determined by an angle between corresponding sides of the two isospin triangles sharing a common base, $|A(B^+ \to \pi^+\pi^0)| = |A(B^- \to \pi^-\pi^0)|$. A sign ambiguity in $\alpha_{\text{eff}} - \alpha$ is resolved by two model-independent features which are confirmed experimentally, $|P|/|T| \leq 1, |\delta| \leq \pi/2$. This implies $\alpha < \alpha_{\text{eff}}$.[78]

Table I.   Branching ratios and CP asymmetries in $B \to \pi\pi$, $B \to \rho\rho$.

Decay mode	Branching ratio (10^{-6})	$A_{CP} = -C$	S
$B^0 \to \pi^+\pi^-$	5.16 ± 0.22	0.38 ± 0.07	-0.61 ± 0.08
$B^+ \to \pi^+\pi^0$	5.7 ± 0.4	0.04 ± 0.05	
$B^0 \to \pi^0\pi^0$	1.31 ± 0.21	$0.36^{+0.33}_{-0.31}$	
$B^0 \to \rho^+\rho^-$	$23.1^{+3.2}_{-3.3}$	0.11 ± 0.13	-0.06 ± 0.18
$B^+ \to \rho^+\rho^0$	18.2 ± 3.0	-0.08 ± 0.13	
$B^0 \to \rho^0\rho^0$	1.07 ± 0.38		

Current CP-averaged branching ratios and CP asymmetries for $B \to \pi\pi$ and $B \to \rho\rho$ decays are given in Table I,[20] where $A_{CP} \equiv -C$ for decays to CP eigenstates. An impressive experimentally progress has been achieved in the past two years in extracting a precise value for α_{eff}, $\alpha_{\text{eff}} = (110.6^{+3.6}_{-3.2})°$. However, the error on $\alpha_{\text{eff}} - \alpha$ using the isospin triangles is still large. An upper bound, given by CP-averaged rates and a direct CP asymmetry in $B^0 \to \pi^+\pi^-$,[79,80]

$$\cos 2(\alpha_{\text{eff}} - \alpha) \geq \frac{\left(\frac{1}{2}\Gamma_{+-} + \Gamma_{+0} - \Gamma_{00}\right)^2 - \Gamma_{+-}\Gamma_{+0}}{\Gamma_{+-}\Gamma_{+0}\sqrt{1 - C_{+-}^2}} \ , \qquad (22)$$

leads to $0 < \alpha_{\text{eff}} - \alpha < 31°$ at 1σ. Adding in quadrature the error in α_{eff} and the uncertainty in $\alpha - \alpha_{\text{eff}}$, this implies $\alpha = (95 \pm 16)°$ or $\gamma = (64 \pm 16)°$ by . A similar central value but a smaller error, $\alpha = (97 \pm 11)°$, has been reported recently by the Belle Collaboration.[81] The possibility that a penguin amplitude in $B^0 \to \pi^+\pi^-$ may lead to a large CP asymmetry S for values of α near $90°$ where $\sin 2\alpha = 0$ was anticipated fifteen years ago.[82]

The bound on $\alpha_{\text{eff}} - \alpha$ may be improved considerably by measuring a nonzero direct CP asymmetry in $B^0 \to \pi^0\pi^0$. This asymmetry can be shown to be *large and positive* (see Eq. (46) in Sec. 5.2), implying a large rate for $\bar{B}^0$ but a small rate for B^0. Namely, the triangle (21) is expected to be squashed, while the $\bar{B}$ triangle is roughly equal sided.

An alternative way of treating the penguin amplitude in $B^0 \to \pi^+\pi^-$ is by combining within flavor SU(3) the decay rate and asymmetries in this process with

rates and asymmetries in $B^0 \to K^0 \pi^+$ or $B^0 \to K^+ \pi^-$.[77] The ratio of $\Delta S = 1$ and $\Delta S = 0$ tree amplitudes in these processes, excluding CKM factors, is taken to be given by f_K/f_π assuming factorization, while the ratio of corresponding penguin amplitudes is allowed to vary by ± 0.22 around one. A current update of this rather conservative analysis obtains [83]

$$\gamma = (73 \pm 4^{+10}_{-8})^\circ \ , \tag{23}$$

where the first error is experimental, while the second one is due to an uncertainty in SU(3) breaking. A discussion of SU(3) breaking factors relating $B^0 \to \pi^+\pi^-$ and $B^0 \to K^+\pi^-$ is included in Section 5.2.

4.2. $B \to \rho\rho$

Angular analyses of the pions in ρ decays have shown that $B^0 \to \rho^+\rho^-$ is dominated almost 100% by longitudinal polarization [20]. This simplifies the isospin analysis of CP asymmetries in these decays to becoming similar to $B^0 \to \pi^+\pi^-$. The advantage of $B \to \rho\rho$ over $B \to \pi\pi$ is the relative small value of $(\rho^0\rho^0)$ in comparison with $(\rho^+\rho^-)$ and $(\rho^+\rho^0)$ (see Table I), indicating a smaller $|P/T|$ in $B \to \rho^+\rho^-$ ($|P/T| < 0.3$ [8]) than in $B^0 \to \pi^+\pi^-$ ($|P/T| \sim 0.5$ [77]). Eq. (22) leads to an upper bound on $\alpha_{\rm eff} - \alpha$ in $B \to \rho\rho$, $0 < \alpha_{\rm eff} - \alpha < 17^\circ$ (at 1σ). The asymmetries for longitudinal ρ's given in Table I imply $\alpha_{\rm eff} = (91.7^{+5.3}_{-5.2})^\circ$. Thus, one finds $\alpha = (83 \pm 10)^\circ$ or $\gamma = (76 \pm 10)^\circ$ by adding errors in quadrature.

A stronger bound on $|P/T|$ in $B^0 \to \rho^+\rho^-$, leading to a more precise value of γ, may be obtained by relating this process to $B^+ \to K^{*0}\rho^+$ within flavor SU(3). [84] One uses the branching ratio and fraction of longitudinal rate measured for this process [20], $(K^{*0}\rho^+) = (9.2 \pm 1.5) \times 10^{-6}$, $f_L(K^{*0}\rho^+) = 0.48 \pm 0.08$, to normalize the penguin amplitude in $B^0 \to \rho^+\rho^-$. Including a conservative uncertainty from SU(3) breaking and smaller amplitudes, one finds a value

$$\gamma = (71.4^{+5.8}_{-8.8} \ {}^{+4.7}_{-1.7})^\circ \ , \tag{24}$$

where the first error is experimental and the second one theoretical.

The current small theoretical error in γ requires including isospin breaking effects in studies based on isospin symmetry. The effect of electroweak penguin amplitudes on the isospin analyses of $B \to \pi\pi$ and $B \to \rho\rho$ has been calculated and was found to move γ slightly higher by an amount $\Delta\gamma_{\rm EWP} = 1.5^\circ$.[34,35] Other corrections, relevant to methods using π^0 and ρ^0, includng π^0-η-η' mixing, ρ-ω mixing, and a small $I = 1$ $\rho\rho$ contribution allowed by the ρ-width, are each smaller than one degree.[36,37,38]

Conclusion: Taking an average of the two values of γ in (23) and (24) obtained from $B^0 \to \pi^+\pi^-$ and $B^0 \to \rho^+\rho^-$, and including the above-mentioned EWP correction, one finds

$$\gamma = (73.5 \pm 5.7)^\circ \ . \tag{25}$$

A third method of measuring γ (or α) in time-dependent Dalitz analyses of $B^0 \to (\rho\pi)^0$ involves a much larger error,[85] and has a small effect on the overall averaged value of the weak phase. We note that $\sin\gamma$ is close to one and its relative error is only 3%, the same as the relative error in $\sin 2\beta$ and slightly smaller than the relative error in $\sin\beta$.

5. Rates, asymmetries, and γ in $B \to K\pi$

5.1. *Extracting γ in $B \to K\pi$*

The four decays $B^0 \to K^+\pi^-, B^0 \to K^0\pi^0, B^+ \to K^0\pi^+, B^+ \to K^+\pi^0$ involve a potential for extracting γ, provided that one is sensitive to interference between a dominant isoscalar penguin amplitude and a small tree amplitude contributing to these processes. This idea has led to numerous suggestions for determining γ in these decays starting with a proposal made in 1994.[86,87] An interference between penguin and tree amplitudes may be identified in two ways:

(1) Two different properly normalized $B \to K\pi$ rates.
(2) Nonzero direct CP asymmetries.

Table II.   Branching ratios and asymmetries in $B \to K\pi$.

Decay mode	Branching ratio (10^{-6})	A_{CP}
$B^0 \to K^+\pi^-$	19.4 ± 0.6	-0.097 ± 0.012
$B^+ \to K^+\pi^0$	12.8 ± 0.6	0.047 ± 0.026
$B^+ \to K^0\pi^+$	23.1 ± 1.0	0.009 ± 0.025
$B^0 \to K^0\pi^0$	10.0 ± 0.6	-0.12 ± 0.11

Current branching ratios and CP asymmetries are summarized in Table II.[20] Three ratios of rates, calculated using the ratio of B^+ and B^0 lifetimes, $\tau_+/\tau_0 = 1.076 \pm 0.008$,[20] are:

$$R \equiv \frac{\Gamma(B^0 \to K^+\pi^-)}{\Gamma(B^+ \to K^0\pi^+)} = 0.90 \pm 0.05 \ ,$$
$$R_c \equiv \frac{2\Gamma(B^+ \to K^+\pi^0)}{\Gamma(B^+ \to K^0\pi^+)} = 1.11 \pm 0.07 \ ,$$
$$R_n \equiv \frac{\Gamma(B^0 \to K^+\pi^-)}{2\Gamma(B^0 \to K^0\pi^0)} = 0.97 \pm 0.07 \ . \tag{26}$$

The largest deviation from one, observed in the ratio R at 2σ, is insufficient for claiming unambiguous evidence for a non-penguin contribution. An upper limit, $R < 0.965$ at 90% confidence level, would imply $\gamma \leq 79°$ using $\sin^2\gamma \leq R$,[88] which neglects however "color-suppressed" EWP contributions.[89] As we will argue now, these contributions and "color-suppressed" tree amplitudes are actually not suppressed as naively expected.

The nonzero asymmetry measured in $B^0 \to K^+\pi^-$ provides first evidence for an interference between penguin (P') and tree (T') amplitudes with a nonzero relative strong phase. Such an interference occurs also in $B^+ \to K^+\pi^0$ where no asymmetry has been observed. An assumption that other contributions to the latter asymmetry are negligible has raised some questions about the validity of the CKM framework. In fact, a color-suppressed tree amplitude (C'), also occurring in $B^+ \to K^+\pi^0$,[86] resolves this "puzzle" if this amplitude is comparable in magnitude to T'. Indeed, several studies have shown that this is the case,[90,91,92,93,94] also implying that color-suppressed and color-favored EWP amplitudes are of comparable magnitudes.[35] For consistency between the two CP asymmetries in $B^0 \to K^+\pi^-$ and $B^+ \to K^+\pi^0$, the strong phase difference between C' and T' must be negative and cannot be very small.[95] This seems to stand in contrast to QCD calculations using a factorization theorem.[29,31,94]

The small asymmetry $A_{CP}(B^+ \to K^+\pi^0)$ implies bounds on the sine of the strong phase difference δ_c between $T' + C'$ and P'. The cosine of this phase affects $R_c - 1$ involving the decay rates for $B^+ \to K^0\pi^+$ and $B^+ \to K^0\pi^+$. A question studied recently is whether the two upper bounds on $|\sin\delta_c|$ and $|\cos\delta_c|$ are consistent with each other or, perhaps, indicate effects of NP. Consistency was shown by proving a sum rule involving $A_{CP}(B^+ \to K^+\pi^0)$ and $R_c - 1$, in which an electroweak penguin (EWP) amplitude plays an important role. We will now present a proof of the sum rule, which may provide important information on γ.[95]

The two amplitudes for $B^+ \to K^0\pi^+, K^+\pi^0$ are given in terms of topological contributions including P', T' and C',

$$A(B^+ \to K^0\pi^+) = (P' - \frac{1}{3}P'^c_{EW}) + A' \ ,$$

$$A(B^+ \to K^+\pi^0) = (P' - \frac{1}{3}P'^c_{EW}) + (T' + P'_{EW}) + (C' + P'_{EW}) + A' \ , \quad (27)$$

where P'_{EW} and P'^c_{EW} are color-favored and color-suppressed EWP contributions. The small annihilation amplitude A' and a small u quark contribution to P' involving a CKM factor $V^*_{ub}V_{us}$ will be neglected ($|V^*_{ub}V_{us}|/|V^*_{cb}V_{cs}| = 0.02$). Evidence for the smallness of these terms can be found in the small CP asymmetry measured for $B^+ \to K^0\pi^+$. Large terms would require rescattering and a sizable strong phase difference between these terms and P'.

Flavor SU(3) symmetry relates $\Delta I = 1, I(K\pi) = 3/2$ electroweak penguin and tree amplitudes through a calculable ratio δ_{EW} [35,41],

$$T' + C' + P'_{EW} + P'^c_{EW} = (T' + C')(1 - \delta_{EW}e^{-i\gamma}) \ ,$$

$$\delta_{EW} = -\frac{3}{2}\frac{c_9 + c_{10}}{c_1 + c_2}\frac{|V^*_{tb}V_{ts}|}{|V^*_{ub}V_{us}|} = 0.60 \pm 0.05 \ . \quad (28)$$

The error in δ_{EW} is dominated by the current uncertainty in $|V_{ub}|/|V_{cb}| = 0.104 \pm 0.007$ [57], including also a smaller error from SU(3) breaking estimated using QCD factorization. Eqs. (27) and (28) imply [96]

$$R_c = 1 - 2r_c\cos\delta_c(\cos\gamma - \delta_{EW}) + r_c^2(1 - 2\delta_{EW}\cos\gamma + \delta_{EW}^2) \ , \quad (29)$$

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$$A_{CP}(B^+ \to K^+\pi^0) = -2r_c \sin\delta_c \sin\gamma/R_c \quad , \tag{30}$$

where $r_c \equiv |T' + C'|/|P' - \frac{1}{3}P'^c_{EW}|$ and δ_c is the strong phase difference between $T' + C'$ and $P' - \frac{1}{3}P'^c_{EW}$.

The parameter r_c is calculable in terms of measured decay rates, using broken flavor SU(3) which relates $T' + C'$ and $T + C$ dominating $B^+ \to \pi^+\pi^0$ by a factorization factor f_K/f_π (neglecting a tiny EWP term in $B^+ \to \pi^+\pi^0$),[87]

$$|T' + C'| = \sqrt{2}\frac{V_{us}}{V_{ud}}\frac{f_K}{f_\pi}|A(B^+ \to \pi^+\pi^0)| \quad . \tag{31}$$

Using branching ratios from Tables I and II, one finds

$$r_c = \sqrt{2}\frac{V_{us}}{V_{ud}}\frac{f_K}{f_\pi}\sqrt{\frac{(B^+ \to \pi^+\pi^0)}{(B^+ \to K^0\pi^+)}} = 0.198 \pm 0.008 \quad . \tag{32}$$

The error in r_c does not include an uncertainty from assuming factorization for SU(3) breaking in $T' + C'$. While this assumption should hold well for T', it may not be a good approximation for C' which as we have mentioned is comparable in magnitude to T' and carries a strong phase relative to it. Thus one should allow a 10% theoretical error when using factorization for relating $B \to K\pi$ and $B \to \pi\pi$ $T + C$ amplitudes, so that

$$r_c = 0.20 \pm 0.01 \text{ (exp)} \pm 0.02 \text{ (th)} \quad . \tag{33}$$

Eliminating δ_c in Eqs. (29) and (30) by retaining terms which are linear in r_c, one finds

$$\left(\frac{R_c - 1}{\cos\gamma - \delta_{EW}}\right)^2 + \left(\frac{A_{CP}(B^+ \to K^+\pi^0)}{\sin\gamma}\right)^2 = (2r_c)^2 + \mathcal{O}(r_c^3) \quad . \tag{34}$$

This sum rule implies that at least one of the two terms whose squares occur on the left-hand-side must be sizable, of the order of $2r_c = 0.4$. The second term, $|A_{CP}(B^+ \to K^+\pi^0)|/\sin\gamma$, is already smaller than $\simeq 0.1$, using the current 2σ bounds on γ and $|A_{CP}(B^+ \to K^+\pi^0)|$. Thus, the first term must provide a dominant contribution. For $R_c \simeq 1$, this implies $\gamma \simeq \arccos\delta_{EW} \simeq (53.1 \pm 3.5)°$. This range is expanded by including errors in R_c and $A_{CP}(B^+ \to K^+\pi^0)$. Currently one only obtains an upper bound $\gamma \leq 88°$ at 90% confidence level.[95] This bound is consistent with the value obtained in (25) from $B \to \pi\pi$ and $B \to \rho\rho$, but is not competitive with the latter precision.

Conclusion: The current constraint obtained from R_c and $A_{CP}(B^+ \to K^+\pi^0)$ is $\gamma \leq 88°$ at 90% confidence level. Further improvement in the measurement of R_c (which may, in fact, be very close to one) is required in order to achieve a precision in γ comparable to that obtained in $B \to \pi\pi, \rho\rho$.

5.2. *Symmetry relations for $B \to K\pi$ rates and asymmetries*

The following two features imply rather precise sum rules in the CKM framework, both for $B \to K\pi$ decay rates and CP asymmetries:

(1) The dominant penguin amplitude is $\Delta I = 0$.

(2) The four decay amplitudes obey a linear isospin relation,[39]

$$A(K^+\pi^-) - A(K^0\pi^+) - \sqrt{2}A(K^+\pi^0) + \sqrt{2}A(K^0\pi^0) . \qquad (35)$$

An immediate consequence of these features are two isospin sum rules, which hold up to terms which are quadratic in small ratios of non-penguin to penguin amplitudes,[45,46,47]

$$\Gamma(K^+\pi^-) + \Gamma(K^0\pi^+) = 2\Gamma(K^+\pi^0) + 2\Gamma(K^0\pi^0) , \qquad (36)$$

$$\Delta(K^+\pi^-) + \Delta(K^0\pi^+) = 2\Delta(K^+\pi^0) + 2\Delta(K^0\pi^0) , \qquad (37)$$

where

$$\Delta(K\pi) \equiv \Gamma(\bar{B} \to \bar{K}\bar{\pi}) - \Gamma(B \to K\pi) . \qquad (38)$$

Quadratic corrections to (36) have been calculated in the SM and were found to be a few percent.[97,98,99] This is the level expected in general for isospin-breaking corrections which must therefore also be considered. The above two features imply that these $\Delta I = 1$ corrections are suppressed by a small non-penguin to penguin ratio and are negligible.[100] Indeed, this sum rule holds experimentally within a 5% error.[101] One expects the other sum rule (37) to hold at a similar precision.

The CP rate asymmetry sum rule (37), relating the four CP asymmetries, leads to a prediction for the asymmetry in $B^0 \to K^0\pi^0$ in terms of the other three asymmetries which have been measured with higher precision,

$$A_{CP}(B^0 \to K^0\pi^0) = -0.140 \pm 0.043 . \qquad (39)$$

While this value is consistent with experiment (see Table II), higher accuracy in this asymmetry measurement is required for testing this straightforward prediction.

Relations between CP asymmetries in $B \to K\pi$ and $B \to \pi\pi$ following from approximate flavor SU(3) symmetry of QCD [102] are not expected to hold as precisely as isospin relations, but may still be interesting and useful. An important question relevant to such relations is how to include SU(3)-breaking effects, which are expected to be at a level of 20-30%. Here we wish to discuss two SU(3) relations proposed twelve years ago,[103,104] one of which holds experimentally within expectation, providing some lesson about SU(3) breaking, while the other has a an interesting implication for future applications of the isospin analysis in $B \to \pi\pi$.

A most convenient proof of SU(3) relations is based on using a diagramatic approach, in which diagrams with given flavor topologies replace reduced SU(3) matrix elements.[86] In this language, the amplitudes for B^0 decays into pairs of charged or neutral pions, and pairs of charged or neutral π and K, are given by:

$$-A(B^0 \to \pi^+\pi^-) = T + (P + 2P^c_{EW}/3) + E + PA ,$$

$$
\begin{aligned}
-\sqrt{2}A(B^0 \to \pi^0\pi^0) &= C - (P - P_{EW} - P^c_{EW}/3) - E - PA \; , \\
-A(B^0 \to K^+\pi^-) &= T' + (P' + 2P'^c_{EW}/3) \; , \\
-\sqrt{2}A(B^0 \to K^0\pi^0) &= C' - (P' - P'_{EW} - P'^c_{EW}/3) \; .
\end{aligned}
\tag{40}
$$

The combination $E + PA$, representing exchange and penguin annihilation topologies, is expected to be $1/m_b$-suppressed relative to T and C,[31,62] as demonstrated by the small branching ratio measured for $B^0 \to K^+K^-$.[20] This term will be neglected.

Expressing amplitudes in terms of CKM factors and SU(3)-invariant amplitudes including strong phases, one may write

$$
\begin{aligned}
T &= V^*_{ub}V_{ud}|\mathcal{T}| \; , & P + 2P^c_{EW}/3 &= V^*_{tb}V_{td}|\mathcal{P}|e^{i\delta} \; , \\
T' &= V^*_{ub}V_{us}|\mathcal{T}| \; , & P' + 2P'^c_{EW}/3 &= V^*_{tb}V_{ts}|\mathcal{P}|e^{i\delta} \; , \\
C &= V^*_{ub}V_{ud}|\mathcal{C}| \; , & P - P_{EW} - P^c_{EW}/3 &= V^*_{tb}V_{td}|\tilde{\mathcal{P}}|e^{i\tilde{\delta}} \; , \\
C' &= V^*_{ub}V_{us}|\mathcal{C}| \; , & P' - P'_{EW} - P'^c_{EW}/3 &= V^*_{tb}V_{ts}|\tilde{\mathcal{P}}|e^{i\tilde{\delta}} \; .
\end{aligned}
\tag{41}
$$

Using the identity

$$
\mathrm{Im}\,(V^*_{ub}V_{ud}V_{tb}V^*_{td}) = -\mathrm{Im}\,(V^*_{ub}V_{us}V_{tb}V^*_{ts}) \; ,
\tag{42}
$$

one finds[103,104]

$$
\Delta(B^0 \to K^+\pi^-) = -\Delta(B^0 \to \pi^+\pi^-)
\tag{43}
$$

$$
\Delta(B^0 \to K^0\pi^0) = -\Delta(B^0 \to \pi^0\pi^0) \; ,
\tag{44}
$$

where Δ is the CP rate difference defined in (38).

Quoting products of branching ratios and asymmetries from Tables I and II, Eq. (43) reads

$$
-1.88 \pm 0.24 = -1.96 \pm 0.37 \; .
\tag{45}
$$

This SU(3) relation works well and requires no SU(3)-breaking. An SU(3) breaking factor f_K/f_π in $\mathcal{T}$ but not in $\mathcal{P}$, or in both $\mathcal{T}$ and $\mathcal{P}$, are currently excluded at a level of 1.0σ, or 1.75σ. More precise CP asymmetry measurements in $B^0 \to K^+\pi^-$ and $B^0 \to \pi^+\pi^-$ are required for determining the pattern of SU(3) breaking in tree and penguin amplitudes.

Using the prediction (39) of the $B \to K\pi$ asymmetry sum rule, Eq. (44) predicts

$$
A_{CP}(B^0 \to \pi^0\pi^0) = 1.07 \pm 0.38 \; .
\tag{46}
$$

The error is dominated by current errors in CP asymmetries for $B^+ \to K^0\pi^+$ and $B^+ \to K^+\pi^0$, and to a less extent by the error in $(\pi^0\pi^0)$. SU(3) breaking in amplitudes could modify this prediction by a factor f_π/f_K if this factor applies to $\mathcal{C}$, and less likely by $(f_\pi/f_K)^2$. A large positive CP asymmetry, favored in all three cases, will affect future applications of the isospin analysis in $B \to \pi\pi$. It implies that while the $\bar{B}$ isospin triangle is roughly equal-sided, the B triangle is squashed. A twofold ambiguity in the value of γ disappears in the limit of a flat B triangle.[24]

Conclusion: The isospin sum rule for $B \to K\pi$ decay rates holds well, while the CP asymmetry sum rule predicts $A_{CP}(B^0 \to K^0\pi^0) = -0.140 \pm 0.043$. The different asymmetries in $B^0 \to K^+\pi^-$ and $B^+ \to K^+\pi^0$ can be explained by an amplitude C' comparable to T' and involving a relative negative strong phase, and should not be considered a "puzzle". An SU(3) relation for $B^0 \to \pi\pi$ and $B^0 \to K\pi$ CP asymmetries works well for charged modes. The corresponding relation for neutral modes predicts a large positive asymmetry in $B^0 \to \pi^0\pi^0$. Improving asymmetry measurements can provide tests for SU(3) breaking factors.

6. Tests for small New Physics effects

6.1. *Values of γ*

We have described three ways for extracting a value for γ relying on interference of distinct pairs of quark amplitudes, $(b \to c\bar{u}s, b \to u\bar{c}s), (b \to c\bar{c}s, b \to u\bar{u}s)$ and $(b \to c\bar{c}d, b \to u\bar{u}d)$. The three pairs provide a specific pattern for CP violation in the CKM framework, which is expected to be violated in most extensions of the SM. The respective values of γ obtained in $B \to D^{(*)}K^{(*)}$ and $B \to K\pi$ are consistent with those extracted in $B \to \pi\pi, \rho\pi$, but are not yet sufficiently precise for testing small NP effects in charmless B decays. Further experimental improvements are required, in particular in the former two types of processes. While the value of γ in $B \to D^{(*)}K^{*)}$ is not expected to be affected by NP, the other two classes of processes involving penguin loops are susceptible to such effects. The extraction of γ in $B \to \pi\pi \ \rho\rho$ assumes that γ is the phase of a $\Delta I = 3/2$ tree amplitude, while an additional EWP contribution is included using isospin. The extracted value could be modified by a new $\Delta I = 3/2$ operator. Similarly, the value of γ extracted in $B \to K\pi$ is affected by a potential new $\Delta I = 1$ operator, because the amplitude (28), playing an essential role in this method, is pure $\Delta I = 1$.

6.2. *$B \to K\pi$ sum rule*

Charmless $|\Delta S| = 1$ B and B_s decays are particularly sensitive to NP effects, as new heavy particles at the TeV mass range may replace the the W boson and top-quark in the penguin loop dominating these amplitudes.[28] The sum rule (36) for $B \to K\pi$ decay rates provides a test for such effects. However, as we have argued from isospin considerations, it is only affected by quadratic $\Delta I = 1$ amplitudes including NP contributions. Small NP amplitudes, contributing quadratically to the sum rule, cannot be separated from SM corrections, which are by themselves at a level of a few percent. This is the level to which the sum rule has already been tested. We will argue below for evidence showing that potential NP contributions to $|\Delta S| = 1$ charmless decays must be suppressed by roughly an order of magnitude relative to the dominant $b \to s$ penguin amplitudes.

6.3. *Values of S, C in $|\Delta S| = 1$ $B^0 \to f_{CP}$ decays*

A class of $b \to s$ penguin-dominated B^0 decays to CP-eigenstates has recently attracted considerable attention. This includes final states XK_S and XK_L, where $X = \phi, \pi^0, \eta', \omega, f_0, \rho^0, K^+K^-, K_S K_S, \pi^0\pi^0$, for which measured asymmetries $-\eta_{CP}S$ and C are quoted in Table III. [The asymmetries S and $C = -A_{CP}$ were defined in (18) for $B^0 \to \pi^+\pi^-$. Observed modes with K_L in the final states obey $\eta_{CP}(XK_L) = -\eta_{CP}(XK_S)$.] In these processes, a value $S = -\eta_{CP}\sin 2\beta$ (for states

Table III.   Asymmetries S and C in $B^0 \to XK_S$.

X	ϕ	π^0	η'	ω	$f_0(980)$
$-\eta_{CP}S$	0.39 ± 0.18	0.33 ± 0.21	0.61 ± 0.07	0.48 ± 0.24	0.42 ± 0.17
C	0.01 ± 0.13	0.12 ± 0.11	-0.09 ± 0.06	-0.21 ± 0.19	-0.02 ± 0.13

X	ρ^0	K^+K^-	$K_S K_S$	$\pi^0\pi^0$
$-\eta_{CP}S$	0.20 ± 0.57	$0.58^{+0.18}_{-0.13}$	0.58 ± 0.20	-0.72 ± 0.71
C	0.64 ± 0.46	0.15 ± 0.09	-0.14 ± 0.15	0.23 ± 0.54

with CP-eigenvalue η_{CP}) is expected approximately.[26,43] These predictions involve hadronic uncertainties at a level of several percent, of order λ^2, $\lambda \sim 0.2$. It has been pointed out some time ago[105] that it is difficult to separate these hadronic uncertainties within the SM from NP contributions to decay amplitudes if the latter are small (as will be argued below). Corrections to $S = -\eta_{CP}\sin 2\beta$ and values for the asymmetries C have been calculated in the SM using methods based on QCD factorization[106,107] and flavor SU(3),[90,108,109] and were found to be between a few percent up to above ten percent within hadronic uncertainties. In the next subsection we will discuss other indirect evidence showing that NP contributions to S and C must be small.

Whereas the deviation of S from $-\eta_{CP}\sin 2\beta$ is process-dependent, a generic result has been proven a long time ago for both S and C, to first order in $|c/p|$,[14]

$$\Delta S \equiv -\eta_{CP}S - \sin 2\beta = 2\frac{|c|}{|p|}\cos 2\beta \sin\gamma \cos\Delta \ ,$$
$$C = 2\frac{|c|}{|p|}\sin\gamma \sin\Delta \ . \tag{47}$$

Here p and c are penguin and color-suppressed tree amplitudes involving a small ratio and relative weak and strong phases γ and Δ, respectively. This implies $\Delta S > 0$ for $|\Delta| < \pi/2$, which can be argued for several of the above decays using QCD arguments[106,107] or SU(3) fits.[109] (Note that while $|p|$ is measurable in certain decay rates up to first order corrections, $|c|$ and Δ involve sizable hadronic uncertainties in QCD calculations.) In contrast to this expectation, the central values measured for ΔS are negative for all decays. (See Table III.) Consequently, one finds an averaged value $\sin 2\beta_{\text{eff}} = 0.53 \pm 0.05$,[20] to be compared with $\sin 2\beta = 0.678 \pm 0.025$. Two measurements which seem particularly interesting are $-\eta_{CP}S_{\phi K_S} = 0.39 \pm 0.18$,

where a positive correction of a few percent to $\sin 2\beta$ is expected in the SM,[106,107] and $-\eta_{CP}S_{\pi^0 K_S} = 0.33 \pm 0.21$, where a rather large positive correction to $\sin 2\beta$ is expected shifting this asymmetry to a value just above 0.8.[90]

While the current averaged value of $\sin 2\beta_{\text{eff}}$ is tantalizing, experimental errors in S and C must be reduced further to make a clear case for physics beyond the SM. Assuming that the discrepancy between improved measurements and calculated values of S and C persists beyond theoretical uncertainties, can this provide a clue to the underlying New Physics? Since many models could give rise to a discrepancy,[28,43,44] one would seek signatures characterizing classes of models rather than studying the effects in specific models. One way of classifying extensions of the SM is by the isospin behavior of the new effective operators contributing to $b \to sq\bar{q}$ transitions.

6.4. *Diagnosis of ΔI for New Physics operators*

Four-quark operators in the effective Hamiltonian associated with NP in $b \to sq\bar{q}$ transitions can be either isoscalar or isovector operators. We will now discuss a study proposed recently in order to isolate $\Delta I = 0$ or $\Delta I = 1$ operators, thus determining corresponding NP amplitudes and CP violating phases.[49] We will show that since S and C in the above processes combine $\Delta I = 0$ or $\Delta I = 1$ contributions, separating these contributions requires using also information from other two asymmetries, which are provided by isospin-reflected decay processes.

Two $|\Delta S| = 1$ charmless B (or B_s) decay processes, related by isospin reflection, $R_I : u \leftrightarrow d$, $\bar{u} \leftrightarrow -\bar{d}$, can always be expressed in term of common $\Delta I = 0$ and $\Delta I = 1$ amplitudes B and A in the form:

$$A(B^+ \to f) = B + A , \quad A(B^0 \to R_I f) = \pm(B - A) . \tag{48}$$

A proof of this relation uses a sign change of Clebsch-Gordan coefficients under $m \leftrightarrow -m$.[49] The description (48) applies, in particular, to pairs of processes involving all the B^0 decay modes listed in Table III, and B^+ decay modes where final states are obtained by isospin reflection from corresponding B^0 decay modes. Decay rates for pairs of isospin-reflected B decay processes, and for $\bar{B}$ decays to corresponding charge conjugate final states are therefore given by (we omit inessential common kinematic factors),

$$\begin{aligned} \Gamma_+ = |B + A|^2 , \qquad \Gamma_0 = |B - A|^2 , \\ \Gamma_- = |\bar{B} + \bar{A}|^2 , \qquad \Gamma_{\bar{0}} = |\bar{B} - \bar{A}|^2 . \end{aligned} \tag{49}$$

The amplitudes $\bar{B}$ and $\bar{A}$ are related to B and A by a change in sign of all weak phases, whereas strong phases are left unchanged.

For each pair of processes one defines four asymmetries: an isospin-dependent CP-conserving asymmetry,

$$A_I \equiv \frac{\Gamma_+ + \Gamma_- - \Gamma_0 - \Gamma_{\bar{0}}}{\Gamma_+ + \Gamma_- + \Gamma_0 + \Gamma_{\bar{0}}} , \tag{50}$$

two CP-violating asymmetries for B^+ and B^0,

$$A_{CP}^+ \equiv \frac{\Gamma_- - \Gamma_+}{\Gamma_- + \Gamma_+} \quad , \qquad - C \equiv A_{CP}^0 \equiv \frac{\Gamma_{\bar 0} - \Gamma_0}{\Gamma_{\bar 0} + \Gamma_0} \quad , \tag{51}$$

and the time-dependent asymmetry S in B^0 decays,

$$S = \frac{2\mathrm{Im}\lambda}{1 + |\lambda|^2} \quad , \quad \lambda \equiv \eta_{CP} \frac{\bar B - \bar A}{B - A} e^{-2i\beta} \quad , \tag{52}$$

In the Standard Model, the isoscalar amplitude B contains a dominant penguin contribution, B_P, with a CKM factor $V_{cb}^* V_{cs}$. The residual isoscalar amplitude,

$$\Delta B \equiv B - B_P \quad , \tag{53}$$

and the amplitude A, consist each of contributions smaller than B_P by about an order of magnitude.[29,30,31,32,86] These include terms with a much smaller CKM factor $V_{ub}^* V_{us}$, and a higher order electroweak penguin amplitude with CKM factor $V_{tb}^* V_{ts}$. Thus, one expects

$$|\Delta B| \ll |B_P| \quad , \quad |A| \ll |B_P| \quad . \tag{54}$$

Consequently, the asymmetries A_I, $A_{CP}^{+,0}$ and ΔS are expected to be small, of order $2|A|/|B|$ and $2|\Delta B|/|B_P|$. In contrast, potentially large contributions to ΔB and A from NP, comparable to B_P, would most likely lead to large asymmetries of order one. An unlikely exception is the case when both $\Delta B/B_P$ and A/B_P are purely imaginary, or almost purely imaginary. This would require very special circumstances such as fine-tuning in specific models. Excluding cancellations between NP and SM contributions in both CP-conserving and CP violating asymmetries, tests for the hierarchy (54) become tests for the smallness of corresponding potential NP contributions to B and A.

There exists ample experimental information showing that asymmetries A_{CP}^+ are small in processes related by isospin reflection to the decay modes in Table III. Upper limits on the magnitudes of most asymmetries are at a level of ten or fifteen percent [e.g., $A_{CP}^+(K^+\phi) = 0.034 \pm 0.044$, $A_{CP}^+(K^+\eta') = 0.031 \pm 0.026$], while others may be as large as twenty or thirty percent [$A_{CP}^+(K^+\rho^0) = 0.31^{+0.11}_{-0.10}$]. Similar values have been measured for isospin asymmetries A_I [e.g., $A_I(K^+\phi) = -0.037 \pm 0.077$, $A_I(K^+\eta') = -0.001 \pm 0.033$, $A_I(K^+\rho^0) = -0.16 \pm 0.10$].[49] Since these two types of asymmetries are of order $2|\Delta B|/|B_P|$ and $2|A|/|B_P|$, this confirms the hierarchy (54), which can be assumed to hold also in the presence of NP.

We will take by convention the dominant penguin amplitude B_P to have a zero weak phase and a zero strong phase, referring all other strong phases to it. Writing

$$B = B_P + \Delta B \quad , \quad \bar B = B_P + \Delta \bar B \quad , \tag{55}$$

and expanding the four asymmetries to leading order in $\Delta B/B_P$ or A/B_P, one has

$$\Delta S = \cos 2\beta \left[\frac{\mathrm{Im}(\bar{A} - A)}{B_P} - \frac{\mathrm{Im}(\Delta \bar{B} - \Delta B)}{B_P} \right] \quad , \tag{56}$$

$$A_I = \frac{\mathrm{Re}(\bar{A} + A)}{B_P} \quad , \tag{57}$$

$$A_{CP}^+ = \frac{\mathrm{Re}(\bar{A} - A)}{B_P} + \frac{\mathrm{Re}(\Delta \bar{B} - \Delta B)}{B_P} \quad , \tag{58}$$

$$A_{CP}^0 = -\frac{\mathrm{Re}(\bar{A} - A)}{B_P} + \frac{\mathrm{Re}(\Delta \bar{B} - \Delta B)}{B_P} \quad . \tag{59}$$

The four asymmetries provide the following information:

- The $\Delta I = 0$ and $\Delta I = 1$ contributions in CP asymmetries are separated by taking sums and differences,

$$A_{CP}^{\Delta I=0} \equiv \frac{1}{2}(A_{CP}^+ + A_{CP}^0) = \frac{\mathrm{Re}(\Delta \bar{B} - \Delta B)}{B_P} \quad , \tag{60}$$

$$A_{CP}^{\Delta I=1} \equiv \frac{1}{2}(A_{CP}^+ - A_{CP}^0) = \frac{\mathrm{Re}(\bar{A} - A)}{B_P} \quad . \tag{61}$$

- $\mathrm{Re}A/B_P$ and $\mathrm{Re}\bar{A}/B_P$ may be separated by using information from $A_{CP}^{\Delta I=1}$ and A_I.
- ΔS is governed by an *imaginary* part of a combination of $\Delta I = 0$ and $\Delta I = 1$ terms which cannot be separated in B decays. Such a separation is possible in B_s decays to pairs of isospin-reflected decays, e.g. $B_s \to K^+ K^-, K_S K_S$ or $B_s \to K^{*+} K^{*-}, K^{*0} \bar{K}^{*0}$, where 2β in the definition of ΔS (47) is now replaced by the small phase of B_s-$\bar{B}_s$ mixing.

One may take one step further under the assumption that strong phases associated with NP amplitudes are small relative to those of the SM and can be neglected.[110] This assumption, which must be confronted by data, is reasonable because rescattering from a leading $b \to sc\bar{c}$ amplitude is likely the main source of strong phases, while rescattering from a smaller $b \to sq\bar{q}$ NP amplitude is then a second-order effect. In the convention (55), where the strong phase of B_P is set equal to zero, ΔB and A have the same strong phase δ, and involve weak phases ϕ_B and ϕ_A, respectively,

$$\Delta B = |\Delta B|e^{i\delta}e^{i\phi_B} \quad , \qquad A = |A|e^{i\delta}e^{i\phi_A} \quad . \tag{62}$$

Since the four asymmetries are first order in small ratios of amplitudes, one may take B_P in their expression to be given by the square root of Γ_+ or Γ_0, thereby neglecting second order terms. The four observables (56)-(59) can then be shown to determine $|A|, \phi_A$ and $|\Delta B| \sin \phi_B$.[49] The combination $|\Delta B| \cos \phi_B$ adds coherently to B_P and cannot be fixed independently.

The amplitudes ΔB and A consist of process-dependent SM and NP contributions. Assuming that the former are calculable, either using methods based on

QCD-factorization or by fitting within flavor SU(3) these and other B decay rates and asymmetries, the four asymmetries determine the magnitude and CP violating phase of a $\Delta I = 1$ NP amplitude and the imaginary part of a $\Delta I = 0$ NP amplitude. In some cases, e.g., $B \to \phi K$ or $B \to \eta' K_S$, stringent upper bounds on SM contributions may suffice. In the pair $B^+ \to K^+\pi^0, B^0 \to K^0\pi^0$, the four measured asymmetries [using the predicted value (39)] are $A_I = 0.087 \pm 0.038, A_{CP}^{\Delta I=0} = -0.047 \pm 0.025, A_{CP}^{\Delta I=1} = 0.094 \pm 0.025, \Delta S = -0.35 \pm 0.21$. Some reduction of errors is required for an implementation of this method.

Conclusion: There exists ample experimental evidence in pairs of isospin-reflected $b \to s$ penguin-dominated decays that potential NP amplitudes must be small. Assuming that these amplitudes involve negligible strong phases, one may determine the magnitude and CP violating phase of a NP $\Delta I = 1$ amplitude and the imaginary part of a $\Delta I = 0$ amplitude.

6.5. *Null or nearly-null tests*

We have not discussed null tests of the CKM framework.[111] Evidence for physics beyond the Standard Model may show-up as (small) nonzero asymmetries in processes where they are extremely small in the CKM framework. A well-known example is $B^+ \to \pi^+\pi^0$, where the CP asymmetry is expected to be a small fraction of a percent including EWP amplitudes.[34,35] We have only discussed *exclusive hadronic B* decays, where QCD calculations involve hadronic uncertainties. A more robust calculation exists for the direct CP asymmetry in *inclusive radiative* decays $B \to X_s\gamma$, found to be smaller than one percent.[112] The current upper limit on this asymmetry is at least an order of magnitude larger.[113]

Time-dependent asymmetries in radiative decays $B^0 \to K_S\pi^0\gamma$, for a $K_S\pi^0$ invariant-mass in the K^* region and for a larger invariant-mass range including this region, are interesting because they test the photon helicity, predicted to be dominantly right-handed in B^0 decays and left-handed in $\bar{B}^0$ decays.[105,114] The asymmetry, suppressed by m_s/m_b, is expected to be several percent in the SM and can be very large in extensions where spin-flip is allowed in $b \to s\gamma$. While dimensional arguments seem to indicate a possible larger asymmetry in the SM, of order $\Lambda_{\rm QCD}/m_b \sim 10\%$,[115] calculations using perturbative QCD[116] and QCD factorization[117] find asymmetries of a few percent. The current averaged values, for the K^* region and for a larger invariant-mass range including this region, are $S((K_S\pi^0)_{K^*}\gamma) = -0.28 \pm 0.26$ and $S(K_S\pi^0\gamma) = -0.09 \pm 0.24$.[20,118] These measurements must be improved to become sensitive to the level predicted in the SM, or to provide evidence for physics beyond the SM.

7. Summary

The Standard Model passed with great success numerous tests in the flavor sector, including a variety of measurements of CP asymmetries related to the CKM phases

β and γ. Small potential New Physics corrections may occur in $\Delta S = 0$ and $|\Delta S| = 1$ penguin amplitudes, affecting the extraction of γ and modifying asymmetries in $|\Delta S| = 1$ B^0 decays to CP eigenstates and isospin-related B^+ decays. Higher precision than achieved so far is required for claiming evidence for such effects and for sorting out their isospin structure.

Similar studies can be performed with B_s mesons produced at hadron colliders and at e^+e^- colliders running at the $\Upsilon(5S)$ resonance. Time-dependence in $B_s \to D_s^- K^+$ and $B_s \to J/\psi\phi$ or $B_s \to J/\psi\eta$ measures γ and the small phase of the B_s-$\bar{B}_s$ mixing amplitude.[119] Comparing time-dependence and angular analysis in $B_s \to J/\psi\phi$ with $b \to s$ penguin-dominated processes including $B_s \to \phi\phi, B_s \to K^{*+}K^{*-}, B_s \to K^{*0}\bar{K}^{*0}$ provides a methodic search for potential NP effects. Work on B_s decays has just begun at the Tevatron.[120] One is looking forward to first results from the LHC.

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