Solar System Constraints on Gauss-Bonnet Mediated Dark Energy

Luca Amendola[1], **Christos Charmousis**[2] **and Stephen C Davis**[3]

[1] INAF/Osservatorio Astronomico di Roma, Viale Frascati 33, 00040 Monte Porzio Catone (Roma), Italy

[2] LPT, Université Paris–Sud, Bât. 210, 91405 Orsay CEDEX, France

[3] Lorentz Institute, Postbus 9506, NL-2300 RA Leiden, The Netherlands

E-mail: amendola@mporzio.astro.it, Christos.Charmousis@th.u-psud.fr and sdavis@lorentz.leidenuniv.nl

Abstract. Although the Gauss-Bonnet term is a topological invariant for general relativity, it couples naturally to a quintessence scalar field, modifying gravity at solar system scales. We determine the solar system constraints due to this term by evaluating the post-Newtonian metric for a distributional source. We find a mass dependent, $1/r^7$ correction to the Newtonian potential, and also deviations from the Einstein gravity prediction for light-bending. We constrain the parameters of the theory using planetary orbits, the Cassini spacecraft data, and a laboratory test of Newton's law, always finding extremely tight bounds on the energy associated to the Gauss-Bonnet term. We discuss the relevance of these constraints to late-time cosmological acceleration.

Keywords: dark energy theory, gravity, string theory and cosmology

1. Introduction

Supernovae measurements [1] indicate that our Universe has entered a phase of late-time acceleration. One can question the magnitude of the acceleration and its equation of state, although given the concordance of different cosmological data, acceleration seems a robust observation (although see [2] for criticisms). Commonly, in order to explain this phenomenon one postulates the existence of a minute cosmological constant $\Lambda \sim 10^{-12}\,\mathrm{eV}^4$. This fits the data well and is the most economic explanation in terms of parameter(s). However such a tiny value is extremely unnatural from a particle physics point of view [3]. Given the theoretical problems of a cosmological constant, one hopes that the intriguing phenomenon of acceleration is a window to new observable physics. This could be in the matter sector, in the form of dark energy [4, 5], or in the gravity sector, perhaps in the form of a large distance modification of Einstein gravity [6, 7, 8, 9]. Scalar field driven dark energy, or quintessence [4] is one of the most popular of the former possibilities. However these models have important drawbacks, such as the fine tuning of the mass of the quintessence field (which has to be smaller than the actual Hubble parameter, $H_0 \sim 10^{-33}\,\mathrm{eV}$), and stability of radiative corrections from the matter sector [10] (see however [11]). Modified gravity models have the potential to avoid these problems, and can give a more profound explanation of the acceleration. However, these are far more difficult to obtain since Einstein's theory is experimentally well established [12], and the required modifications happen at very low (classical) energy scales which are (supposed to be) theoretically well understood. Furthermore, many apparently successful modified gravity models suffer from instabilities or are incompatible with gravity experiments. For example the self-accelerating solutions of DGP [8] suffer from perturbative ghosts [13], and $f(R)$ gravity theories [9] can conflict with solar system measurements and present instabilities [14].

In this paper we will consider observational constraints on a class of gravity theories which feature both dark energy and modified gravity. Specifically, we will examine solar system and laboratory constraints resulting from the response of gravity to a quintessence-like scalar field, which couples to quadratic order curvature terms such as the Gauss-Bonnet term. Such couplings arise naturally [15], and modify gravity at local and cosmological scales [15, 16]. Although the Gauss-Bonnet invariant shares many of the properties of the Einstein-Hilbert term, the resulting theory can have substantially different features, see for example [17]. It is a promising candidate for a consistent explanation of cosmological acceleration, but as we will show, can also produce undesirable effects at solar system scales.

In particular, we will determine constraints from deviations in planetary orbits around the sun, the frequency shift of signals from the Cassini probe, and table top experiments. In contrast to some previous efforts in the field [18], we will not suppose a-priori the order of the Gauss-Bonnet correction or the scalar field potential. Instead we will calculate leading order gravity corrections for each of them, and obtain constraints on the relevant coupling constants (checking they fall within the validity of our perturbative expansion). Hence our analysis will apply for large couplings, which as we will see, are in accord with Gauss-Bonnet driven effective dark energy models. In this way we will show such models generally produce significant deviations from general relativity at local scales. We also include higher order scalar field kinetic terms, although for the solutions we consider, they turn out to be subdominant.

In the next section we will present the theory in question and calculate the corrections to a post-Newtonian metric for a distributional point mass source. In section 3, we derive constraints from planetary motion, the Cassini probe, and a tabletop experiment. For the Cassini constraint, we have to explicitly derive the predicted frequency shift for our theory, as it does not fall within the usual Parametrised Post Newtonian (PPN) analysis. We discuss the implications of our results in section 4.

2. Quadratic Curvature Gravity

We will consider a theory with the second order gravitational Lagrangian

$$\mathcal{L} = \sqrt{-g}\Big\{R - (\nabla\phi)^2 - 2V(\phi)$$

$$+ \alpha \left[\xi_1(\phi)\mathcal{L}_{GB} + \xi_2(\phi)G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + \xi_3(\phi)(\nabla\phi)^2\nabla^2\phi + \xi_4(\phi)(\nabla\phi)^4\right]\Big\}\,, \quad (1)$$

which includes the Gauss-Bonnet term $\mathcal{L}_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$. Note for example that such a Lagrangian with given ξ's arises naturally from higher dimensional compactification of a pure gravitational theory [15]. On its own, in four dimensions, the Gauss-Bonnet term does not contribute to the gravitational field equations. However we emphasize that when coupled to a scalar field (as above), it has a non-trivial effect.

Throughout this paper we take the dimensionless couplings ξ_i, and their derivatives to be O(1). The parameter α then has dimensions of length squared. Similarly we assume that all derivatives of the potential V are of O(V), which in our conventions has dimensions of inverse length squared. These simplifying assumptions are very reasonable, although it is perfectly conceivable that they do not apply for our universe.

Using the post-Newtonian limit, the metric for the solar system can be written [12]

$$ds^2 = -(1 - h_{00})(c\,dt)^2 + (\delta_{ij} + h_{ij})dx^i dx^j + \mathrm{O}(\epsilon^{3/2})\,. \quad (2)$$

with $h_{00}, h_{ij} = \mathrm{O}(\epsilon)$. Here ϵ is the typical gravitational strength dimensionless parameter given by $\epsilon = Gm/(rc^2)$ where m is the typical mass scale and r the typical length scale (see below). For the solar system ϵ is at most 10^{-5}, while for cosmology, or close to the event horizon of a black hole, it is of order unity. In particular, the scale of planetary velocities v, is of order $\epsilon^{1/2}$, and so the h_{0i} components of the metric are $\mathrm{O}(\epsilon^{3/2})$. In what follows, we will take $\phi = \phi_0 + \mathrm{O}(\epsilon)$. For the linearised approximation we are using, we can adopt a post-Newtonian gauge in which the off-diagonal components of h_{ij} are zero. We can then write

$$h_{ij} = -2\Psi\delta_{ij}\,, \qquad h_{00} = -2\Phi\,, \quad (3)$$

and so $c^2\Phi$ is the Newtonian potential.

In this paper we will consider the leading order corrections in ϵ without assumptions on the magnitude of V and α. To leading order in ϵ, the Einstein equations take the nice compact form,

$$\Delta\Phi = \frac{4\pi G_0}{c^2}\rho_m - V - 2\alpha\xi_1'\mathcal{D}(\Phi + \Psi, \phi) + \mathrm{O}(\epsilon^2, \alpha\epsilon^3/r^2, V\epsilon r^2) \quad (4)$$

$$\Delta\Psi = \frac{4\pi G_0}{c^2}\rho_m + \frac{V}{2} - \alpha\left[2\xi_1'\mathcal{D}(\Psi, \phi) + \frac{\xi_2}{4}\mathcal{D}(\phi, \phi)\right] + \mathrm{O}(\epsilon^2, \alpha\epsilon^3/r^2, V\epsilon r^2) \quad (5)$$

where primes denote $\partial/\partial\phi$, and V, ξ_1', etc. are evaluated at $\phi = \phi_0$. The scalar equation is

$$\Delta\phi = V' - \alpha\left[4\xi_1'\mathcal{D}(\Phi, \Psi) + \xi_2\mathcal{D}(\Phi - \Psi, \phi) + \xi_3\mathcal{D}(\phi, \phi)\right] + \mathrm{O}(\epsilon^2, \alpha\epsilon^3/r^2, V\epsilon r^2) \quad (6)$$

We have defined the operators

$$\Delta X = \sum_i X_{,ii}\,, \qquad \mathcal{D}(X, Y) = \sum_{i,j} X_{,ij}Y_{,ij} - \Delta X \Delta Y\,. \quad (7)$$

with $i, j = 1, 2, 3$ where to leading order, the Gauss-Bonnet term is then $\mathcal{L}_{GB} = 8\mathcal{D}(\Phi, \Psi)$. For standard Einstein gravity ($V = \alpha = 0$), the solution of the above equations is

$$\Phi = \Psi = -U_m\,, \qquad \phi = \phi_0 \quad (8)$$

where

$$U_m = \frac{4\pi G_0}{c^2} \int d^3x' \frac{\rho_m(\vec{x}', t)}{|\vec{x} - \vec{x}'|} \tag{9}$$

We will now study solutions which are close to the post-Newtonian limit of general relativity, and take

$$\Phi = -U_m + \delta\Phi\,, \qquad \Psi = -U_m + \delta\Psi\,, \qquad \phi = \phi_0 + \delta\phi\,, \tag{10}$$

where $\delta\phi$, etc. are the leading order α and V dependent corrections.

Note that the Laplacian carries a distribution and therefore we have to be careful with the implementation of the $\mathcal{D}$ operator. We see that $\delta\phi$ is $\mathrm{O}(V, \alpha\epsilon^2)$, and so to leading order, we have

$$\Delta\,\delta\phi = V' - 4\alpha\xi_1'\mathcal{D}(U_m, U_m)\,. \tag{11}$$

Having calculated $\delta\phi$, we obtain

$$\Delta\,\delta\Phi = -V + 4\alpha\xi_1'\mathcal{D}(U_m, \delta\phi) \tag{12}$$

$$\Delta\,\delta\Psi = \frac{V}{2} + 2\alpha\xi_1'\mathcal{D}(U_m, \delta\phi)\,. \tag{13}$$

In the case of a spherical distributional source $\rho_m = m\delta^{(3)}(x)$,

$$U_m = \frac{G_0 m}{c^2 r}\,. \tag{14}$$

In accordance to our estimations for ϵ the solar system Newtonian potentials are $U_m \lesssim 10^{-5}$, and the velocities satisfy $v^2 \lesssim U_m$. For planets we have $U_m \lesssim 10^{-7}$ (with the maximum attained by Mercury). With the aid of the relation

$$\mathcal{D}(r^{-n}, r^{-m}) = \frac{2nm}{n+m+2}\Delta r^{-(n+m+2)} \tag{15}$$

the above expressions evaluate to

$$\phi = \phi_0 + \frac{r^2V'}{6} - 2\xi_1'\frac{\alpha(G_0m)^2}{c^4r^4} \tag{16}$$

$$\Phi = -\frac{G_0m}{c^2r}\left[1 + \frac{8\xi_1'}{3}\alpha V'\right] - \frac{r^2V}{6} - \frac{64(\xi_1')^2}{7}\frac{\alpha^2(G_0m)^3}{c^6r^7} \tag{17}$$

$$\Psi = -\frac{G_0m}{c^2r}\left[1 + \frac{4\xi_1'}{3}\alpha V'\right] + \frac{r^2V}{12} - \frac{32(\xi_1')^2}{7}\frac{\alpha^2(G_0m)^3}{c^6r^7}\,. \tag{18}$$

We find that there are now non-standard corrections to the Newtonian potential which do not follow the usual parametrised expansion, in agreement with [19], but not [18] (which uses different assumptions on the form of the theory). First of all note that the Gauss-Bonnet coupling α couples to the running of the potential of dark energy V' giving an effective γ PPN parameter. The r^2V term is typical of a theory with cosmological constant whereas the last term is the leading pure Gauss-Bonnet correction which is enhanced at small distances. If we take the usual expression for the PPN parameter $\gamma = \Psi/\Phi$, we see that it is r dependent. In using the Cassini constraint on γ we will then be careful to calculate the frequency shift from scratch using the above modified

Newtonian potential (17), (18). The $V'\alpha$ cross-terms in particular contribute to the gravitational coupling,

$$G = G_0 \left[1 + \frac{8\xi_1'}{3}\alpha V' \right] . \tag{19}$$

For the above derivation we have assumed $\delta\phi \ll U_m$, which implies $V \ll U_m/r^2$ and $\alpha \ll r^2/U_m$. This will hold in the solar system if

$$V \ll 10^{-36}\,\mathrm{m}^{-2} \qquad \text{and} \qquad \alpha \ll \begin{cases} 10^{23}\,\mathrm{m}^2 & \text{(everywhere)} \\ 10^{29}\,\mathrm{m}^2 & \text{(planets only)} \end{cases} \tag{20}$$

in geometrised units. Note that strictly speaking there is also a lower bound on our coupling constants, if the above analysis is to be valid. Indeed, if we were to find corrections of order $\epsilon^2 \sim 10^{-14}$, then it would imply that higher order corrections from general relativity were just as important as the ones appearing in (17), (18).

3. Constraints

3.1. Planetary motion

Deviations from the usual Newtonian potential will affect planetary motions, which provides a way of bounding them. This idea has been used to bound dark matter in the solar system [20], and also the value of the cosmological constant [21]. We will apply the same arguments to our theory. From the above gravitational potential (17), we obtain the Newtonian acceleration

$$g_{\mathrm{acc}}(r) = -c^2\frac{d\Phi}{dr} = -\frac{Gm}{r^2}\left[1 - \frac{Vr^3}{3r_g} - \frac{64(\alpha\xi_1')^2 r_g^2}{r^6}\right] \equiv -\frac{Gm_{\mathrm{eff}}}{r^2} \tag{21}$$

where $r_g \equiv Gm/c^2$ is gravitational radius of the mass m. The above expression gives the effective mass m_{eff} felt by a body at distance r. If the test body is a planet with semi-major axis a, we can use this formula at $r \approx a$. Its mean motion $n \equiv \sqrt{Gm/a^3}$ will then be changed by $\delta n = (n/2)(\delta m_{\mathrm{eff}}/m)$. By evaluating the statistical errors of the mean motions of the planets, $\delta n = -(3n/2)\delta a/a$, we can derive a bound on δm_{eff} and hence our deviations from general relativity

$$\frac{1}{3}\frac{\delta m_{\mathrm{eff}}}{m} = -\frac{Va^3}{9r_g} - \frac{64(\alpha\xi_1')^2 r_g^2}{3a^6} < \frac{\delta a}{a} . \tag{22}$$

The values of a for the planets are determined using Kepler's third law, with a constant Sun's mass $m_\odot$. Constraints on $\delta\Phi$ then follow from the errors δa, in the measure of a. These can be found in [22], and are also listed in the appendix for convenience. Given their different r dependence, the two corrections to δm_{eff} are unlikely to cancel. We will therefore bound them separately, giving constraints on α and V.

The strongest bound on the combination $\xi_1'\alpha$ comes from Mercury, with

$$\left(\frac{\delta a}{a}\right)_{☿} \lesssim 1.8 \times 10^{-12} \tag{23}$$

Neglecting the cosmological constant term, and using $a \approx 57 \times 10^6\,\mathrm{km}$ and $r_g \approx 1.5\,\mathrm{km}$, we find

$$|\xi_1'\alpha| \lesssim \frac{(3a^5\delta a)^{1/2}}{8r_g} \approx 3.8 \times 10^{22}\,\mathrm{m}^2\,. \tag{24}$$

We see that this is within range of validity (20) for our perturbative treatment of gravity.

In cosmology, the density fraction corresponding to the Gauss-Bonnet term is [15]

$$\Omega_{GB} = 4\xi_1'\alpha H\frac{d\phi}{dt}\,. \tag{25}$$

If this is to play the role of dark energy in our universe, it needs to take value around 0.7 at cosmological length scales (and for redshift $z \sim 1$).

Although ϕ is a dynamical field one is tempted to make the assumptions that the cosmological value of ϕ is also ϕ_0, and that $d\phi/dt \approx H$. Then we obtain a very stringent constraint on Ω_{GB}

$$|\Omega_{GB}| = 4|\xi_1'\alpha|H_0^2 \lesssim 8.8 \times 10^{-30}\,. \tag{26}$$

Given the hierarchy between cosmological and solar system scales one can always question this assumption but we will make it here, and discuss it in the concluding section. Clearly if Ω_{GB} is to explain the accelerated expansion of the universe, then one of the assumptions used to obtain the above result must be violated.

The corresponding bound for potential, which comes from the motion of Mars [21], is

$$|V| \lesssim 1.2 \times 10^{-40}\,\mathrm{m}^{-2}\,. \tag{27}$$

This implies $\Omega_V = V/(3H_0)^2 \lesssim 7.3 \times 10^{11}$, which is vastly weaker than the corresponding cosmological constraint ($\Omega_V \lesssim 1$).

3.2. Cassini spacecraft

The most stringent constraint on the PPN parameter γ was obtained from the Cassini spacecraft in 2002 while on its way to Saturn. The signals between the spacecraft and the earth pass close to the sun, whose gravitational field produces a time delay. The smallest value of r on the light ray's path defines the impact parameter b. A small impact parameter maximises the light delay. During that year's superior solar conjunction the spacecraft was $r_e = 8.43\,\mathrm{AU} = 1.26 \times 10^{12}\,\mathrm{m}$ away from the sun, and the impact parameter dropped as low as $b_{\min} = 1.6R_\odot$. A PPN analysis of the system produced the strong constraint

$$\delta\gamma \equiv \gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}\,. \tag{28}$$

Given that our theory is not PPN we have to undertake the calculation from scratch.

The above constraint comes from considering a round trip, in which the light travels from earth, grazes the sun's 'surface', reaches the spacecraft, and then returns by the same route. We take the path of the photon to be the straight line between the earth and the spacecraft, $\vec{x} = (x, b, 0)$ with x varying from $-x_e$ to $x_\oplus$. For a round trip (there and back), the additional time delay for a light ray due to the gravitational field of the sun is then

$$c\Delta t = 2\int_{-x_e}^{x_\oplus} \left[\frac{h_{00}(r)+h_{xx}(r)}{2}\right] dx = -2\int_{-x_e}^{x_\oplus} \left(\Phi+\Psi\right)\big|_{r=\sqrt{x^2+b^2}}\, dx\,. \quad (29)$$

For the solution (17) and (18), this evaluates to

$$c\Delta t = 4r_g\left[1-\frac{2\alpha\xi_1' V'}{3}\right]\ln\frac{a_\oplus r_e}{4b^2} + \left[\frac{a_\oplus^3+r_e^3}{3} + b^2(a_\oplus + r_e)\right]\frac{V}{6} + \frac{1024(\alpha\xi_1')^2 r_g^3}{b^6} \quad (30)$$

where we have assumed $x_\oplus \approx a_\oplus \gg b$, and similarly for the spacecraft.

Rather than directly measure Δt, the Cassini experiment actually found the frequency shift in the signal [23]

$$y_{\rm gr} = \frac{d\Delta t}{dt} \approx \frac{d\Delta t}{db}\frac{db}{dt}\,. \quad (31)$$

The results obtained were

$$y_{\rm gr} = -\frac{10^{-5}\,\mathrm{s}}{b}\frac{db}{dt}\left(2+\delta\gamma\right)\,. \quad (32)$$

If gravitation were to be described by the standard PPN formalism, then $\delta\gamma$ would be the possible deviation of the PPN parameter γ from the general relativity value of 1.

From (30) we obtain

$$y_{\rm gr} = -\left(2 - \frac{b^2 V(a_\oplus + r_e)}{12 r_g} + \frac{1536(\alpha\xi_1')^2 r_g^2}{b^6} - \frac{4\alpha\xi_1' V'}{3}\right)\frac{4r_g}{cb}\frac{db}{dt}\,. \quad (33)$$

Requiring that the corrections are within the errors (28) of (32), implies

$$|\xi_1'\alpha| \lesssim \frac{\sqrt{6\,\delta\gamma}}{96}\frac{b^3}{r_g} \lesssim 1.6\times 10^{20}\,\mathrm{m}^2\,. \quad (34)$$

This suggests the dark energy bound

$$|\Omega_{GB}| \lesssim 3.6\times 10^{-32}\,, \quad (35)$$

although obtaining this bound from solar system data requires major assumptions about the cosmological behaviour of ϕ as we will point out in the discussion section.

The data obtained by the spacecraft was actually for a range of impact parameters b, but we have just used the most conservative value $b = b_{\rm min} = 1.6R_\odot$. The above constraint is even stronger than (24), which was obtained for planetary motion. This is because the experiment involved smaller r, and so the possible Gauss-Bonnet effects were larger.

Taking the above expression for $y_{\rm gr}$ (33) at face value, we can also constrain the potential to be $|V| \lesssim 10^{-22}\,\mathrm{m}^2$ and the cross-term $|\alpha\xi_1' V'| \lesssim 10^{-5}$. However these are of little interest as they are much weaker than the planetary motion constraints (24),(27), and also the former is far outside the range of validity (20) of our analysis.

3.3. A table-top experiment

Laboratory experiments can also be used to obtain bounds on deviations from Newton's law. For illustration we will consider the table-top experiment described in [24]. It consists of a 60 cm copper bar, suspended at its midpoint by a tungsten wire. Two 7.3 kg masses are placed on carts far (105 cm) from the bar, and another mass of $m \approx 43\,\mathrm{g}$ is placed near (5 cm) to the side of bar. Moving the masses to the opposite sides of the bar changes in the torque felt by it. The experiment measures the torques N_{105} and $-N_5$ produced respectively by the far and near masses. The masses and distances are chosen so that the two torques roughly cancel. The ratio $R = N_{105}/N_5$ is then determined, and compared with the theoretical value. The deviation from the Newtonian result is

$$\delta_R = \frac{R_{\mathrm{expt}}}{R_{\mathrm{Newton}}} - 1 = (1.2 \pm 7) \times 10^{-4}\ . \tag{36}$$

In fact, to help reduce errors, additional measurements were taken. To account for the gravitational field of the carts that the far masses sit on, the experiment was repeated with only the carts and a $m' \approx 3\,\mathrm{g}$ near mass. The measured torque was then subtracted from the result for the loaded carts.

The Gauss-Bonnet corrections to the Newton potential (17) will alter the torques produced by all four masses, as well as the carts. Furthermore, since $\delta\Phi$ is non-linear in mass, there will be further corrections coming from cross terms. The expressions derived in section 2 are just for the gravitational field of a single mass, and so will not fully describe the above table-top experiment. However, we find that the contribution from the mass m will dominate the other corrections, and so we can get a good estimate of the Gauss-Bonnet contribution to the ratio R by just considering m.

The torque experienced by the copper bar, due to a point mass at $\vec{X} = (X, Y, Z)$ is

$$N = \int_{\mathrm{bar}} d^3x\, (\vec{x} \wedge \vec{F})_z = \rho_{\mathrm{Cu}} \int_{\mathrm{bar}} d^3x\, \frac{yX - xY}{r} c^2\, \frac{d\Phi}{dr}\bigg|_{r=|\vec{X}-\vec{x}|} \tag{37}$$

where ρ_{Cu} is the bar's density. A full list of parameters for the experiment is given in table I of [24]. The bar's dimensions are $60\,\mathrm{cm} \times 1.5\,\mathrm{cm} \times 0.65\,\mathrm{cm}$. Working in coordinates with the origin at the centre of the bar, the mass $m = 43.58\,\mathrm{g}$ is at $\vec{X} = (24.42, -4.77, -0.03)\,\mathrm{cm}$. Treating m as a point mass, Newtonian gravity, implies a torque of $N_5 \approx (8.2\,\mathrm{cm}^2)\, Gm\rho_{\mathrm{Cu}}$ is produced. The Gauss-Bonnet correction is

$$\delta N_5 = \rho_{\mathrm{Cu}} \frac{64 G^3 m^3 (\alpha\xi_1')^2}{c^4} \int_{\mathrm{bar}} d^3x\, \frac{yX - xY}{|\vec{X} - \vec{x}|^9} \approx -(0.025\,\mathrm{cm}^{-4}) \frac{(Gm)^3 (\alpha\xi_1')^2 \rho_{\mathrm{Cu}}}{c^4}\ . \tag{38}$$

To be consistent with the bound (36), we require $\delta N_5/N_5$ to be within the range of δ_R (36). This implies

$$|\alpha\xi_1'| \lesssim (18\,\mathrm{cm}^3)\, \frac{c^2 \delta_R^{1/2}}{Gm} \lesssim 1.3 \times 10^{22}\,\mathrm{m}^2 \tag{39}$$

which is comparable to the planetary constraint (24). Extrapolating it to cosmological scales gives

$$|\Omega_{GB}| \lesssim 3.1 \times 10^{-30}\,. \tag{40}$$

There are of course many more recent laboratory tests of gravity, and we expect that stronger constraints can be obtained from them. Table-top experiments frequently involve multiple gravitational sources, or gravitational fields which cannot reasonably be treated as point masses. A more detailed calculation than the one presented in section 2 will then be required. For example, the gravitational field inside a sphere or cylinder will not receive corrections of the form (17), and so any experiment involving a test mass moving in such a field requires a different analysis.

4. Discussion

We have shown that significant constraints on Gauss-Bonnet gravity can be derived from both solar system measurements and table-top laboratory experiments. The fact that the corrections to Einstein gravity are second order in curvature suggests they will automatically be small. However this does not take into account the fact that the dimensionfull coupling of the Gauss-Bonnet term must be large if it is to have any hope of producing effective dark energy. Additional constraints will come from the perihelion precession of Mercury, although the linearised analysis we have used is inadequate to determine this, and higher order (in ϵ) effects will need to be calculated.

Performing an extrapolation of our results to cosmological scales suggests that the density fraction Ω_{GB} will be far too small to explain the accelerated expansion of our universe. This agrees with the conclusions of [19]. Hence if Gauss-Bonnet gravity is to be a viable dark energy candidate, one needs to find a loophole in the above argument. This is not too difficult, and we will now turn to this question.

In particular, we have assumed no spatial or temporal evolution of the field ϕ between cosmological and solar system scales, even though the supernova measurements correspond to a higher redshift and a far different typical distance scale. A varying ϕ would of course imply that different values of ξ_i, and their derivatives, would be perceived by supernovas and the planets. It is interesting to note that the size of the bound we have found (26) is of order the square of the ratio of the solar system and the cosmological horizon scales, $s = (1\,\mathrm{AU}\,H_0)^2 \sim 10^{-30}$. Therefore one could reasonably argue that the small number appearing in (26) could in fact be due to the hierarchy scale, s, rather than a very stringent constraint on Ω_{GB}. This could perhaps be concretely realised with something similar to the chameleon effect [25] giving some constraint on the running of the quintessence theory. One other possibility is that the baryons (which make up the solar system) and dark matter (which is dominant at cosmological scales) have different couplings to ϕ [26]. Again, this would alter the relation between local and cosmological constraints.

Alternatively, it may be that our assumptions on the form of the theory should be changed. The scalar field could be coupled directly to the Einstein-Hilbert term, as in Brans-Dicke gravity. Additionally, the couplings ξ_i and their derivatives could be of different orders. The same could be true of the potential. In particular, if ϕ were to have a significant mass, this would suppress the quadratic curvature effects, as they operate via the scalar field. This would be similar to the situation in Brans-Dicke gravity, where the strong constraints on the theory can be avoided by giving the scalar a large mass (which however would inhibit acceleration).

Finally, the behaviour of the scalar field could be radically different. We took it to be $O(\epsilon)$, like the metric perturbations. However since our constraints are on the metric, and not ϕ, this need not be true. Furthermore, since the theory is quadratic, there may well be alternative solutions of the field equations, and not just the one we studied.

Hence to obtain a viable Gauss-Bonnet dark energy model, which is compatible with solar system constraints, at least one of the above assumptions must be broken. For many of the above ideas the higher order scalar kinetic terms will play a significant role. This then opens up the possibility that the higher gravity corrections will cancel each other, further weakening the constraints. We hope to address some of these issues in the near future.

Acknowledgements

CC thanks Gilles Esposito-Farese and Lorenzo Sorbo for discussions.

Appendix

For the benefit of readers without an astronomical background, we list relevant solar system parameters. The values for δa come from Table 4 of [22]. We take the Hubble constant to be $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

$R_\odot = 6.96\times10^8\,\mathrm{m}$
$r_g^\odot \equiv Gm_\odot/c^2 = 1477\,\mathrm{m}$
$H_0/c = 7.566\times10^{-27}\,\mathrm{m^{-1}}$
$1\,\mathrm{AU} \equiv a_\oplus = 149597870691\,\mathrm{m}$

$G = 6.6742\times10^{-11}\,\mathrm{m^3\,s^{-2}\,kg^{-1}}$
$c = 299792458\,\mathrm{m\,s^{-1}}$
$m_\odot = 1.989\times10^{30}\,\mathrm{kg}$

name	a (10^9 m)	δa (m)
Mercury	57.9	0.105
Venus	108	0.329
Earth	149	0.146
Mars	228	0.657
Jupiter	778	639
Saturn	1433	4.22×10^3
Uranus	2872	3.85×10^4
Neptune	4495	4.79×10^5
Pluto	5870	3.46×10^6

References

[1] A. G. Riess *et al.* [Supernova Search Team Collaboration], *Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant,* Astron. J. **116**, 1009 (1998) [astro-ph/9805201]
S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], *Measurements of Omega and Lambda from 42 High-Redshift Supernovae,* Astrophys. J. **517**, 565 (1999) [astro-ph/9812133]
A. G. Riess *et al.* [Supernova Search Team Collaboration], *Type Ia Supernova Discoveries at $z > 1$ From the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution,* Astrophys. J. **607**, 665 (2004) [astro-ph/0402512]

[2] A. Blanchard, M. Douspis, M. Rowan-Robinson and S. Sarkar, *An alternative to the cosmological concordance model,* Astron. Astrophys. **412**, 35 (2003) [astro-ph/0304237]

[3] S. Weinberg, *The cosmological constant problem,* Rev. Mod. Phys. **61**, 1 (1989)

[4] C. Wetterich, *Cosmology and the Fate of Dilatation Symmetry,* Nucl. Phys. B **302**, 668 (1988)

B. Ratra and P. J. E. Peebles, *Cosmological Consequences of a Rolling Homogeneous Scalar Field,* Phys. Rev. D **37**, 3406 (1988)

R. R. Caldwell, R. Dave and P. J. Steinhardt, *Cosmological Imprint of an Energy Component with General Equation-of-State,* Phys. Rev. Lett. **80**, 1582 (1998) [astro-ph/9708069]

C. Armendariz-Picon, V. F. Mukhanov and P. J. Steinhardt, *A dynamical solution to the problem of a small cosmological constant and late-time cosmic acceleration,* Phys. Rev. Lett. **85**, 4438 (2000) [astro-ph/0004134]

C. Armendariz-Picon, V. F. Mukhanov and P. J. Steinhardt, *Essentials of k-essence,* Phys. Rev. D **63**, 103510 (2001) [astro-ph/0006373]

T. Chiba, T. Okabe and M. Yamaguchi, *Kinetically driven quintessence,* Phys. Rev. D **62**, 023511 (2000) [astro-ph/9912463]

N. Arkani-Hamed, L. J. Hall, C. F. Kolda and H. Murayama, *A New Perspective on Cosmic Coincidence Problems,* Phys. Rev. Lett. **85**, 4434 (2000) [astro-ph/0005111]

[5] V. K. Onemli and R. P. Woodard, *Quantum effects can render* $w < -1$ *on cosmological scales,* Phys. Rev. D **70**, 107301 (2004) [gr-qc/0406098]

V. K. Onemli and R. P. Woodard, *Super-acceleration from massless, minimally coupled phi**4,* Class. Quant. Grav. **19**, 4607 (2002) [gr-qc/0204065]

[6] A. Padilla, *Cosmic acceleration from asymmetric branes,* Class. Quant. Grav. **22**, 681 (2005) [hep-th/0406157]

A. Padilla, *Infra-red modification of gravity from asymmetric branes,* Class. Quant. Grav. **22**, 1087 (2005) [hep-th/0410033]

N. Kaloper and D. Kiley, *Charting the Landscape of Modified Gravity,* hep-th/0703190

N. Kaloper, *A new dimension hidden in the shadow of a wall,* hep-th/0702206

[7] G. R. Dvali, G. Gabadadze and M. Porrati, *4D gravity on a brane in 5D Minkowski space,* Phys. Lett. B **485**, 208 (2000) [hep-th/0005016]

G. R. Dvali and G. Gabadadze, *Gravity on a brane in infinite-volume extra space,* Phys. Rev. D **63**, 065007 (2001) [hep-th/0008054]

C. Deffayet, G. R. Dvali and G. Gabadadze, *Accelerated universe from gravity leaking to extra dimensions,* Phys. Rev. D **65**, 044023 (2002) [astro-ph/0105068]

A. Lue, *The phenomenology of Dvali-Gabadadze-Porrati cosmologies,* [astro-ph/0510068]

[8] C. Deffayet, *Cosmology on a brane in Minkowski bulk,* Phys. Lett. B **502**, 199 (2001) [hep-th/0010186]

[9] S. Capozziello, S. Carloni and A. Troisi, *Quintessence without scalar fields,* astro-ph/0303041

S. Capozziello, V. F. Cardone, S. Carloni and A. Troisi, *Curvature quintessence matched with observational data,* Int. J. Mod. Phys. D **12**, 1969 (2003) [astro-ph/0307018]

S. M. Carroll, V. Duvvuri, M. Trodden and M. S. Turner, *Is cosmic speed-up due to new gravitational physics?,* Phys. Rev. D **70**, 043528 (2004) [astro-ph/0306438]

[10] S. M. Carroll, *Quintessence and the rest of the world,* Phys. Rev. Lett. **81**, 3067 (1998) [astro-ph/9806099]

C. F. Kolda and D. H. Lyth, *Quintessential difficulties,* Phys. Lett. B **458**, 197 (1999) [hep-ph/9811375]

T. Chiba, *Quintessence, the gravitational constant, and gravity,* Phys. Rev. D **60**, 083508 (1999) [gr-qc/9903094]

[11] J. A. Frieman, C. T. Hill, A. Stebbins and I. Waga, *Cosmology with ultralight pseudo Nambu-Goldstone bosons,* Phys. Rev. Lett. **75**, 2077 (1995) [astro-ph/9505060]

Y. Nomura, T. Watari and T. Yanagida, *Quintessence axion potential induced by electroweak instanton effects,* Phys. Lett. B **484**, 103 (2000) [hep-ph/0004182]

J. E. Kim and H. P. Nilles, *A quintessential axion,* Phys. Lett. B **553**, 1 (2003) [hep-ph/0210402]

K. Choi, *String or M theory axion as a quintessence,* Phys. Rev. D **62**, 043509 (2000) [hep-ph/9902292]

N. Kaloper and L. Sorbo, *Of pNGB QuiNtessence,* JCAP **0604**, 007 (2006) [astro-ph/0511543]

[12] C. M. Will, *The confrontation between general relativity and experiment,* gr-qc/0510072

C. M. Will, *Theory and experiment in gravitational physics*, Cambridge University Press

G. Esposito-Farese, *Tests of Alternative Theories of Gravity,* Proceedings of 33rd SLAC Summer Institute on Particle Physics (SSI 2005): Gravity in the Quantum World and the Cosmos, Menlo Park, California, 25 Jul – 5 Aug 2005, pp T025

[13] D. Gorbunov, K. Koyama and S. Sibiryakov, *More on ghosts in DGP model,* Phys. Rev. D **73**, 044016 (2006) [hep-th/0512097]

C. Charmousis, R. Gregory, N. Kaloper and A. Padilla, *DGP specteroscopy,* JHEP **0610**, 066 (2006) [hep-th/0604086]

K. Koyama, *Are there ghosts in the self-accelerating brane universe?,* Phys. Rev. D **72**, 123511 (2005) [hep-th/0503191]

[14] T. Chiba, *1/R gravity and scalar-tensor gravity,* Phys. Lett. B **575**, 1 (2003) [astro-ph/0307338]

A. D. Dolgov and M. Kawasaki, *Can modified gravity explain accelerated cosmic expansion?,* Phys. Lett. B **573**, 1 (2003) [astro-ph/0307285]

L. Amendola, D. Polarski, S. Tsujikawa, *Are $f(R)$ dark energy models cosmologically viable?* Phys. Rev. Lett. (in press) [astro-ph/0603703]

[15] L. Amendola, C. Charmousis and S. C. Davis, *Constraints on Gauss-Bonnet gravity in dark energy cosmologies,* JCAP **0612**, 020 (2006) [hep-th/0506137]

[16] T. Koivisto and D. F. Mota, *Cosmology and Astrophysical Constraints of Gauss-Bonnet Dark Energy,* Phys. Lett. B **644**, 104 (2007) [astro-ph/0606078]

T. Koivisto and D. F. Mota, *Gauss-Bonnet quintessence: Background evolution, large scale structure and cosmological constraints,* Phys. Rev. D **75**, 023518 (2007) [hep-th/0609155]

B. M. Leith and I. P. Neupane, *Gauss-Bonnet cosmologies: Crossing the phantom divide and the transition from matter dominance to dark energy,* hep-th/0702002

[17] P. Binetruy, C. Charmousis, S. C. Davis and J. F. Dufaux, *Avoidance of naked singularities in dilatonic brane world scenarios with a Gauss-Bonnet term,* Phys. Lett. B **544**, 183 (2002) [hep-th/0206089]

C. Charmousis, S. C. Davis and J. F. Dufaux, *Scalar brane backgrounds in higher order curvature gravity,* JHEP **0312**, 029 (2003) [hep-th/0309083]

[18] T. P. Sotiriou and E. Barausse, *Post-Newtonian expansion for Gauss-Bonnet gravity,* gr-qc/0612065

[19] G. Esposito-Farese, *Scalar-tensor theories and cosmology and tests of a quintessence-Gauss-Bonnet coupling,* gr-qc/0306018

G. Esposito-Farese, *Tests of scalar-tensor gravity,* AIP Conf. Proc. **736**, 35 (2004) [gr-qc/0409081]

[20] J. D. Anderson, E. L. Lau, A. H. Taylor, D. A. Dicus D. C. Teplitz and V. L. Teplitz *Bounds on Dark Matter in Solar Orbit,* Astrophys. J. **342**, (1989) 539

[21] M. Sereno and P. Jetzer, *Solar and stellar system tests of the cosmological constant,* Phys. Rev. D **73**, 063004 (2006) [astro-ph/0602438]

[22] E. V. Pitjeva, *High-Precision Ephemerides of Planets–EPM and Determination of Some Astronomical Constants,* Solar System Research **39**, 176 (2005)

[23] B. Bertotti, L. Iess and P. Tortora, *A test of general relativity using radio links with the Cassini spacecraft,* Nature **425**, 374 (2003)

[24] J. K. Hoskins, R. D. Newman, R. Spero and J. Schultz, *Experimental tests of the gravitational inverse square law for mass separations from 2-cm to 105-cm,* Phys. Rev. D **32**, 3084 (1985)

[25] P. Brax, C. van de Bruck, A. C. Davis, J. Khoury and A. Weltman, *Detecting dark energy in orbit: The cosmological chameleon,* Phys. Rev. D **70**, 123518 (2004) [astro-ph/0408415]

[26] L. Amendola *Coupled Quintessence*, Phys. Rev. D **62**, 043511 (2000) [astro-ph/9908023]