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Gluon Radiation of an Expanding Color Skyrmion in the Quark-Gluon Plasma

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Abstract

The density of states and energy spectrum of the gluon radiation are calculated for the color current of an expanding hydrodynamic skyrmion in the quark gluon plasma with a semiclassical method. Results are compared with those in literatures.

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1 Introduction

In this letter, we address the issue of gluon radiation during the hydrodynamic stage in the evolution of the deconfined hot QCD matter or quark gluon plasma (QGP) [1] (for review see for example [2]).

The medium induced gluon radiation has been thoroughly explored in the context of final state partonic energy loss or "jet quenching" [3]. The spatially extended nuclear matter affects the processes of fragmentation and hadronization of the hard partons produced in the relativistic heavy ion collisions. Essentially all high $p_\perp$ hadronic observables are affected at collider energies and the degree of the medium modification can give a characterization of the hot QCD matter in the deconfined phase. In principle, the medium induced radiation effect emerges from thermal QCD *per se*. However, in practice, different approximation schemes are applied giving consistent results [4, 5]. On the other hand, gluon radiation has also been considered in the context of gluon density saturation in the initial stage, where a strongly interacting gluonic atmosphere is crucial for the rapid local thermalization for the deconfined QCD matter [6].

The time evolution of the RHIC "fireball" can influence the observable particle production spectra. Given a strong initial interaction, the resulting state of matter is usually modeled as a relativistic fluid undergoing a hydrodynamic flow. Generalized fluid mechanics that characterizes the long-distance physics of the transport of color charges has been developed for this purpose [7] (for review see [8]). Recently, we discovered a type of single skyrmion solutions in color fluid [9]. Moreover, we found an interesting case in which the time-dependent skyrmion expands in time, which is in accordance with the expanding nature of the fireball generated in RHIC experiments [10]. The pattern of gluon radiation pertaining to the color current of these non-static configurations is an important character of this color skyrmion. So in this letter we calculate this radiation spectrum in a semiclassical approach. The main results from our calculation are the following. There is a fast fall-off in the UV side of the spectrum but a smooth peak dominates the intermediate energy. And in IR, a long tail is the characteristic feature.

The organization of this paper is the following. In Sect. 2, after a brief review of the nonabelian fluid mechanics, we calculate the nonabelian current corresponding to the soliton solution. In Sect. 3, semiclassical gluon radiation is calculated. In Sect. 4, comparison of the radiation spectrum in our hydrodynamic approach and in other approaches is carried out.

2 Color current of an expanding soliton

Given the thermalization of hot QCD matter above the deconfinement transition temperature, the transport of the color charges in the volume of the nuclear size can be modeled by a nonlinear sigma model in a first-order formalism

$$\mathcal{L} = j^\mu \omega_\mu - F(n) - g_{eff} J^{a\mu} A^a_\mu. \tag{1}$$

This nonlinear sigma model describes an ideal fluid system. The configuration of this fluid is described by a group element field U, which shows up in the velocity field ω_μ

$$\omega_\mu = -\frac{i}{2} Tr(\sigma_3 U^\dagger \partial_\mu U). \tag{2}$$

Conjugate to the velocity is the abelian charge current j^μ. It is easy to see that the first term in the lagrangian density (1) gives rise to the canonical structure of the fluid system. The fact that we will consider only one abelian charge current means that U takes value in an $SU(2)$ group. The information about the equation of state (EOS) of the fluid is contained in the second term, which is essentially the free energy density of the fluid. In fact, energy and pressure densities are given by the ideal fluid formula

$$\epsilon = F, \quad p = nF' - F. \tag{3}$$

Here n is the invariant length of j^μ, $n^2 = j^\mu j_\mu$. The third term is the gauge coupling of the fluid with an external gluon field A^a_μ with an effective coupling g_{eff}. $J^{a\mu}$ is the nonabelian charge current which is related to the abelian current by the Eckart factorization $J^{a\mu} = Q^a j^\mu$ where Q^a is the nonabelian charge density of the fluid configuration

$$Q^a = \frac{1}{2} Tr(\sigma_3 U^\dagger \sigma^a U). \tag{4}$$

For $SU(2)$ group, $a = 1, 2, 3$.

When the temperature is relatively high, we approximate the EOS by

$$\epsilon = 3p \tag{5}$$

which is known in relativistic fluid mechanics to describe radiation. As a result, the free energy density can be obtained by integrating Eq. (3),

$$F = \frac{\beta}{4/3} n^{4/3} \tag{6}$$

where β is a dimensionless constant of integration. In this case, and without an external gluon field, the fluid system in (1) possesses a class of expanding soliton solutions which can be studied via variational and collective coordinate methods [10].

$$U = U\Big(\frac{\mathbf{x}}{R(t)}\Big), \quad R(t) \approx R_0(\frac{t}{\tau} + 1)^{4/3}\theta(t) \tag{7}$$

where R_0 and τ are the spacial and temporal characterizations of the variational soliton and $\theta(t)$ the usual step function in time direction. Physically, it is certainly very interesting to understand the origin of these two scales from a fundamental level. The approximation in (7) is valid provided $\tau \ll R_0$. This condition enables us to define a small parameter

$$\lambda = \frac{\tau}{R_0}. \tag{8}$$

For our purpose, we calculate the nonabelian current in (1) corresponding to the soliton solution in Eq. (7). To do so, the *hedgehog ansatz* is specified for the solution (7)

$$U = \cos\phi + i\sigma \cdot \hat{x} \sin\phi \tag{9}$$

where $\hat{x}$ is the unit vector and ϕ is given by the stereographic map

$$\sin\phi = \frac{2s}{1+s^2}, \quad \cos\phi = \pm\frac{1-s^2}{1+s^2}. \tag{10}$$

We write s as the dimensionless coordinate $x/R(t)$. The sign in the expression of $\cos\phi$ signifies a topological charge which is the *skyrmion number*. The negative sign gives the skyrmion number $+1$ or a skyrmion and the positive sign the skyrmion number is -1 or an anti-skyrmion. We will take the positive sign in the following. By expressing the abelian current j^μ in terms of the velocity ω_μ through the equation of motion, we derive the following expression for the nonabelian current

$$\begin{aligned} d^3x J^{a\mu} &= \left(\frac{2}{\beta}\right)^3 \cdot \frac{d^3s}{(1+s^2)^6} \cdot (\hat{s}_3^2 s^2 \dot{R}^2 - 1) \cdot \\ &\left(\delta_3^a(1-6s^2+s^4) + 4\epsilon^{a3b}\hat{s}_b s(1-s^2) + 8\hat{s}_3\hat{s}_a s^2\right) \cdot \begin{pmatrix} -\hat{s}_3 s(1+s^2)\dot{R} \\ 2\hat{s}_1\hat{s}_3 s^2 - 2\hat{s}_2 s \\ 2\hat{s}_2\hat{s}_3 s^2 + 2\hat{s}_1 s \\ 2\hat{s}_3^2 s^2 - s^2 + 1 \end{pmatrix}. \end{aligned} \tag{11}$$

The current in (11) has a natural form of a multipole expansion due to the skyrmion orientation in the color space. In this letter we only consider the effect of the lowest mode and the effects of higher polarization will be considered elsewhere. The spherically symmetric part in the current is contained only in the third component

$$\left(d^3x J^{a3}\right)_0 = -\delta_3^a \left(\frac{2}{\beta}\right)^3 \frac{d^3s}{(1+s^2)^6} P_6(s) \tag{12}$$

where $P_6(s) = 1 - 7s^2 + 7s^4 - s^6$.

3 Semiclassical gluon radiation

Now we consider the interaction between the expanding color skyrmion and the hard partons. Since the transfer momentum between hard partons is in high order to that between hard parton and soliton, we expect a hierarchy between the partonic coupling g_{YM} and the effective coupling g_{eff}. Accordingly, gluon self-interaction in terms like $F^a_{\mu\nu}F^{a\mu\nu}$ can be omitted so we can work with a free parton picture. Then the gauge coupling in (1) becomes the coupling between a classical current and a free quantum field for gluon. In this approximation, the lowest order semiclassical amplitude is given by

$$i\mathcal{M} = g_{eff}\langle 1| \int d^4x J^{a\mu} \hat{A}^a_\mu |0\rangle. \tag{13}$$

$|0\rangle$ and $|1\rangle$ are gluonic Fock vacuum and one-gluon state. The gluon factor in (13) is given by the wave function

$$\langle 1|\hat{A}^a_\mu(x)|0\rangle = \varphi^a \varepsilon_\mu \frac{e^{ik\cdot x}}{\sqrt{2\omega}} \tag{14}$$

where the color and helicity parts φ, ε will be summed over eventually. Putting the current in, we have

$$i\mathcal{M} = A(k)\int dt e^{i\omega t}\int \frac{d^3s}{(1+s^2)^6} e^{-iR(t)\mathbf{k}\cdot\mathbf{s}} P_6(s) \tag{15}$$

where $A(k) = -(2/\beta)^3 g_{eff}\varphi^3\varepsilon_3/\sqrt{2\omega}$. The spatial Fourier transformation can be completed analytically

$$i\mathcal{M} = B(k)\int dt e^{i\omega t - R(t)k} Q_4(R(t)k) \tag{16}$$

where $B(k) = \pi^2 A(k)/120$ and $Q_4(x) = 5x^2 - 5x^3 + x^4$. To go further, we need to specify $R(t)$ in this equation to the form given in (7). This gives

$$i\mathcal{M} = B(k)e^{-i\omega\tau}\frac{\eta}{\omega}\int_{\frac{\omega\tau}{\eta}}^{\infty} dt e^{i\eta t - t^{4/3}} Q_4(t^{4/3}) \tag{17}$$

where $\eta = \omega\tau/(kR_0)^{3/4}$. With onshell condition $\omega = k$, $\eta = \lambda\kappa^{1/4}$ where κ is defined to be $R_0 k$. Accordingly,

$$i\mathcal{M} = \Big(-\frac{\pi^2}{15\sqrt{2}}\Big)\Big(\frac{g_{eff}}{\beta^3}\lambda R_0^{3/2}\Big)\Big(\varphi^3\varepsilon_3 e^{-i\omega\tau}\Big)\Big(\frac{i\widetilde{\mathcal{M}}_\lambda(\kappa)}{\kappa^{5/4}}\Big) \tag{18}$$

where

$$i\widetilde{\mathcal{M}}_\lambda(\kappa) = \int_{\kappa^{3/4}}^{\infty} dt e^{i\lambda\kappa^{1/4}t - t^{4/3}} Q_4(t^{4/3}) \tag{19}$$

The radiation spectrum is given by $dE = kd\mathcal{N}$. $E(k)$ is the total energy radiated over the entire time of expansion as a function of k. The number distribution is

$$d\mathcal{N} = \sum_{c,h} |\mathcal{M}|^2 d^3k \tag{20}$$

where the summation is over colors and helicities of the gluon. In a spherically symmetric setting, $d\mathcal{N} = ndk$ where n is the density of states

$$n = 4\pi k^2 \sum_{c,h} |\mathcal{M}|^2. \tag{21}$$

By straightforward calculation,

$$\begin{aligned} n &= \alpha R_0 \lambda^2 \kappa^{-1/2} |\widetilde{\mathcal{M}}_\lambda(\kappa)|^2, & (22) \\ \frac{dE}{dk} &= \alpha \lambda^2 \kappa^{1/2} |\widetilde{\mathcal{M}}_\lambda(\kappa)|^2. & (23) \end{aligned}$$

where $\alpha \equiv (2\pi^5/225)(g_{eff}^2/\beta^6)$. The numerical results for $\lambda = 1/15, 2/15, 1/5$ are given in Fig. 1.

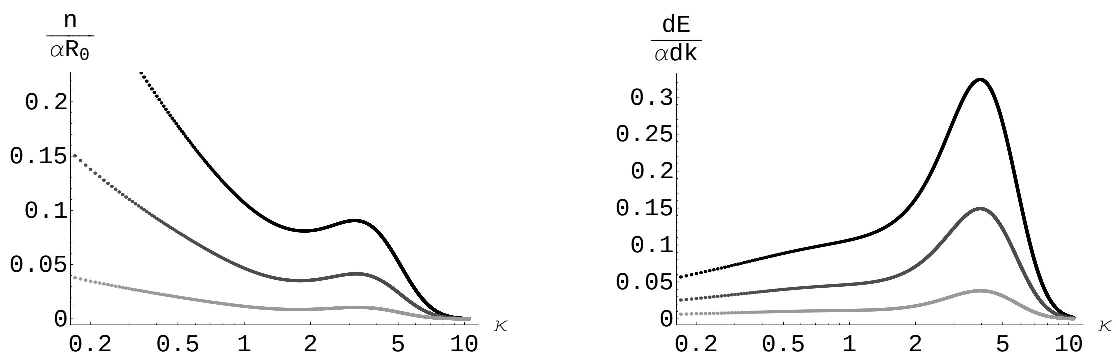

Figure 1: Density of states and energy spectrum for $\lambda = 1/5$ (Black), $2/15$ (Deep Gray) and $1/15$ (Light Gray).

4 Comparison and discussion

Understanding the pattern of gluon radiation in relativistic heavy ion collision processes is important for making an accurate determination of the physical mechanisms from the measurement of its decay products.

In [6], the authors extracted the asymptotic behavior of the number density in small k is of the $1/k$ form. In our case, the asymptotic of the number density in small k is $\sim 1/\sqrt{k}$. (See Fig. 2.) The difference comes from the fact that the medium size is taken to be infinitely large in [6] while in our case the medium size is characterized by the soliton size R_0. So the IR behavior in our case is softer.

For the case of jet quenching, the radiation energy lost is due to scattering off the hard quarks. A popular approach is to model the medium as a collection of colored static scattering

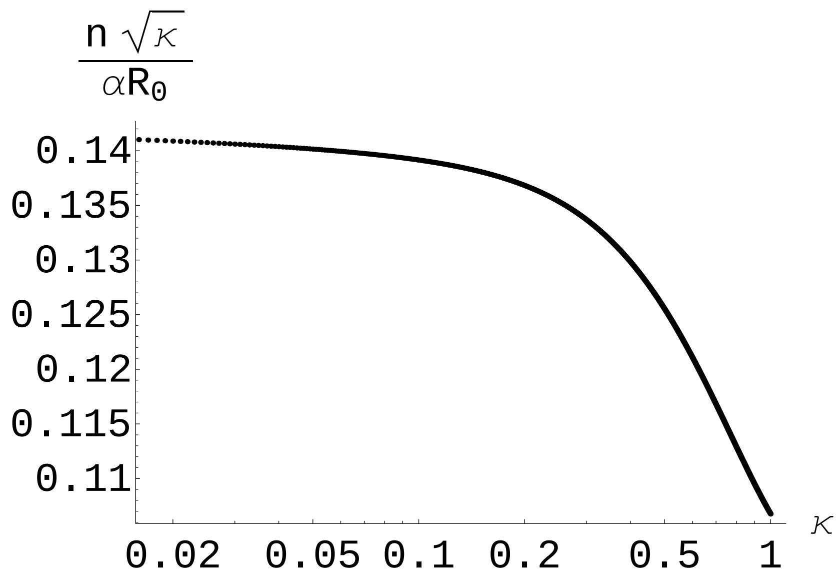

Figure 2: $n/(1/\sqrt{\kappa})$ in small k for $\lambda = .2$

centers [11]. This approach can be extended to the expanding medium [4] though the gluon radiation by the expanding medium itself is not included. In fact, the medium induced gluon radiation is characterized by the frequency

$$\omega_C = \frac{1}{2}\hat{q}L^2 \tag{24}$$

where $\hat{q}$ is the quenching parameter, estimated to be $.04 \sim .16 GeV^2/fm$, and L is the in-medium path length of a hard parton [12]. In general ω_C is significantly larger than the characteristic momentum in our case $1/R_0$. So there is a hierarchy between the medium induced gluon radiation spectrum and the gluon radiation spectrum by the medium.

Our hydrodynamical approach opens up another interesting possibility to address the eccentricity of the elliptic flow either intrinsically by considering the nonabelian color current or exogenously by considering the gluon radiation patterns. This will be the topic of the follow-up to this work.

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