Meta-Stable Brane Configuration of Product Gauge Groups

Changhyun Ahn

Department of Physics, Kyungpook National University, Taegu 702-701, Korea

`ahn@knu.ac.kr`

Abstract

Starting from the $\mathcal{N} = 1$ $SU(N_c) \times SU(N_c')$ gauge theory with fundamental and bifundamental flavors, we apply the Seiberg dual to the first gauge group and obtain the $\mathcal{N} = 1$ dual gauge theory with dual matters including the gauge singlets. By analyzing the F-term equations of the superpotential, we describe the intersecting type IIA brane configuration for the meta-stable nonsupersymmetric vacua of this gauge theory. By introducing an orientifold 6-plane, we generalize to the case for $\mathcal{N} = 1$ $SU(N_c) \times SO(N_c')$ gauge theory with fundamental and bifundamental flavors. Finally, the $\mathcal{N} = 1$ $SU(N_c) \times Sp(N_c')$ gauge theory with matters is also described very briefly.

1 Introduction

It is well-known that the $\mathcal{N} = 1$ $SU(N_c)$ QCD with fundamental flavors has a vanishing superpotential before we deform this theory by mass term for quarks. The vanishing superpotential in the electric theory makes it easier to analyze its nonvanishing dual magnetic superpotential. Sometimes by tuning the various rotation angles between NS5-branes and D6-branes appropriately, even if the electric theory has nonvanishing superpotential, one can make the nonzero superpotential to vanish in the electric theory. Two procedures, deforming the electric gauge theory by adding the mass for the quarks and taking the Seiberg dual magnetic theory from the electric theory, are crucial to find out meta-stable supersymmetry breaking vacua in the context of dynamical supersymmetry breaking [1, 2]. Some models of dynamical supersymmetry breaking can be studied by gauging the subgroup of the flavor symmetry group by either field theory analysis or using the brane configuration [1].

In this paper, starting from the known $\mathcal{N} = 1$ supersymmetric electric gauge theories, we construct the $\mathcal{N} = 1$ supersymmetric magnetic gauge theories by brane motion and linking number counting. The dual gauge group appears only on the first gauge group. Based on their particular limits of corresponding magnetic brane configurations in the sense that their electric theories do not have any superpotentials except the mass deformations for the quarks, we describe the intersecting brane configurations of type IIA string theory for the meta-stable nonsupersymmetric vacua of these gauge theories.

We focus on the cases where the whole gauge group is given by a product of two gauge groups. One example can be realized by three NS5-branes with D4- and D6-branes, and the other by four NS5-branes with D4- and D6-branes. For the latter, the appropriate orientifold 6-plane should be located at the center of this brane configuration in order to have two gauge groups. Of course, it is also possible, without changing the number of gauge groups, to have the brane configuration consisting of five NS5-branes and orientifold 6-plane, at which the extra NS5-brane is located, with D4- and D6-branes, but we'll not do this particular case in this paper.

In section 2, we review the type IIA brane configuration that contains three NS5-branes, corresponding to the electric theory based on the $\mathcal{N} = 1$ $SU(N_c) \times SU(N'_c)$ gauge theory [4, 5, 6] with matter contents and deform this theory by adding the mass term for the quarks. Then we construct the Seiberg dual magnetic theory which is $\mathcal{N} = 1$ $SU(\widetilde{N}_c) \times SU(N'_c)$ gauge theory with corresponding dual matters as well as various gauge singlets, by brane motion and linking number counting. We do not touch the part of second gauge group $SU(N'_c)$ in

---

[1]For the type IIA brane configuration description of $\mathcal{N} = 1$ supersymmetric gauge theory, see the review paper [3].

this dual process.

In section 3, we consider the nonsupersymmetric meta-stable minimum by looking at the magnetic brane configuration we obtained in section 2 and present the corresponding intersecting brane configuration of type IIA string theory, along the line of [7, 8, 9, 10, 11](see also [12, 13, 14]) and we describe M-theory lift of this supersymmetry breaking type IIA brane configuration.

In section 4, we describe the type IIA brane configuration that contains four NS5-branes, corresponding to the electric theory based on the $\mathcal{N} = 1$ $SU(N_c) \times SO(N'_c)$ gauge theory [15] with matter contents and deform this theory by adding the mass term for the quarks. Then we take the Seiberg dual magnetic theory which is $\mathcal{N} = 1$ $SU(\widetilde{N}_c) \times SO(N'_c)$ gauge theory with corresponding dual matters as well as various gauge singlets, by brane motion and linking number counting. The part of second gauge group $SO(N'_c)$ in this dual process is not changed under this process.

In section 5, the nonsupersymmetric meta-stable minimum by looking at the magnetic brane configuration we obtained in section 4 is constructed and we present the corresponding intersecting brane configuration of type IIA string theory and describe M-theory lift of this supersymmetry breaking type IIA brane configuration, as we did in section 3.

In section 6, we describe the similar application to the $\mathcal{N} = 1$ $SU(N_c) \times Sp(N'_c)$ gauge theory [15] briefly and make some comments for the future directions.

2 The $\mathcal{N} = 1$ supersymmetric brane configuration of $SU(N_c) \times SU(N'_c)$ gauge theory

After reviewing the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N} = 1$ $SU(N_c) \times SU(N'_c)$ gauge theory [4, 5, 6], we construct the Seiberg dual magnetic theory which is $\mathcal{N} = 1$ $SU(\widetilde{N}_c) \times SU(N'_c)$ gauge theory.

2.1 Electric theory with $SU(N_c) \times SU(N'_c)$ gauge group

The gauge group is given by $SU(N_c) \times SU(N'_c)$ and the matter contents [4, 5, 6] are given by

• N_f chiral multiplets Q are in the fundamental representation under the $SU(N_c)$, N_f chiral multiplets $\widetilde{Q}$ are in the antifundamental representation under the $SU(N_c)$ and then Q are in the representation $(\mathbf{N_c}, \mathbf{1})$ while $\widetilde{Q}$ are in the representation $(\overline{\mathbf{N_c}}, \mathbf{1})$ under the gauge group

• N'_f chiral multiplets Q' are in the fundamental representation under the $SU(N'_c)$, N'_f chiral multiplets $\widetilde{Q'}$ are in the antifundamental representation under the $SU(N'_c)$ and then Q'

are in the representation $(\mathbf{1}, \mathbf{N_c'})$ while $\widetilde{Q'}$ are in the representation $(\mathbf{1}, \overline{\mathbf{N_c'}})$ under the gauge group

• The flavor singlet field X is in the bifundamental representation $(\mathbf{N_c}, \overline{\mathbf{N_c'}})$ under the gauge group and its complex conjugate field $\widetilde{X}$ is in the bifundamental representation $(\overline{\mathbf{N_c}}, \mathbf{N_c'})$ under the gauge group

In the electric theory, since there exist N_f quarks Q, N_f quarks $\widetilde{Q}$, one bifundamental field X which will give rise to the contribution of N_c' and its complex conjugate $\widetilde{X}$ which will give rise to the contribution of N_c', the coefficient of the beta function of the first gauge group factor is

$$b_{SU(N_c)} = 3N_c - N_f - N_c'$$

and similarly since there exist N_f' quarks Q', N_f' quarks $\widetilde{Q'}$, one bifundamental field X which will give rise to the contribution of N_c and its complex conjugate $\widetilde{X}$ which will give rise to the contribution of N_c, the coefficient of the beta function of the second gauge group factor is

$$b_{SU(N_c')} = 3N_c' - N_f' - N_c.$$

The anomaly free global symmetry is given by $[SU(N_f) \times SU(N_f')]^2 \times U(1)^3 \times U(1)_R$ [4, 5, 6] and let us denote the strong coupling scales for $SU(N_c)$ as Λ_1 and for $SU(N_c')$ as Λ_2. The theory is asymptotically free when $b_{SU(N_c)} = 3N_c - N_f - N_c' > 0$ for the $SU(N_c)$ gauge theory and when $b_{SU(N_c')} = 3N_c' - N_f' - N_c > 0$ for the $SU(N_c')$ gauge theory.

The type IIA brane configuration for this theory can be described by N_c color D4-branes (01236) suspended between a middle NS5-brane (012345) and the right NS5'-brane (012389) (denoted by $NS5_R'$-brane) along x^6 direction, together with N_f D6-branes (0123789) which are parallel to $NS5_R'$-brane and have nonzero (45) directions. Moreover, an extra left NS5'-brane (denoted by $NS5_L'$-brane) is located at the left hand side of a middle NS5-brane along the x^6 direction and there exist N_c' color D4-branes suspended between them, with N_f' D6-branes which have zero (45) directions. These are shown in Figure 1 explicitly. See also [3] for the brane configuration.

By realizing that the two outer $NS5_{L,R}'$-branes are perpendicular to a middle NS5-brane and the fact that N_f D6-branes are parallel to $NS5_R'$-brane and N_f' D6-branes are parallel to $NS5_L'$-brane, the classical superpotential vanishes. However, one can deform this theory. Then the classical superpotential by deforming this theory by adding the mass term for the quarks Q and $\widetilde{Q}$, along the lines of [1, 11, 10, 9, 8, 7], is given by

$$W = mQ\widetilde{Q} \tag{2.1}$$

and this type IIA brane configuration can be summarized as follows [2]:

- One middle NS5-brane with worldvolume (012345).
- Two NS5'-branes with worldvolume (012389).
- N_f D6-branes with worldvolume (0123789) located at the positive region in v direction.
- N_c color D4-branes with worldvolume (01236). These are suspended between a middle NS5-brane and $NS5'_R$-brane.
- N'_c color D4-branes with worldvolume (01236). These are suspended between $NS5'_L$-brane and a middle NS5-brane.

Now we draw this electric brane configuration in Figure 1 and we put the coincident N_f D6-branes in the nonzero v direction. If we ignore the left $NS5'_L$-brane, N'_c D4-branes and N'_f D6-branes, then this brane configuration corresponds to the standard $\mathcal{N} = 1$ SQCD with the gauge group $SU(N_c)$ with N_f massive flavors. The electric quarks Q and $\widetilde{Q}$ correspond to strings stretching between the N_c color D4-branes with N_f D6-branes, the electric quarks Q' and $\widetilde{Q'}$ correspond to strings between the N'_c color D4-branes with N'_f D6-branes and the bifundamentals X and $\widetilde{X}$ correspond to strings stretching between the N_c color D4-branes with N'_c color D4-branes.
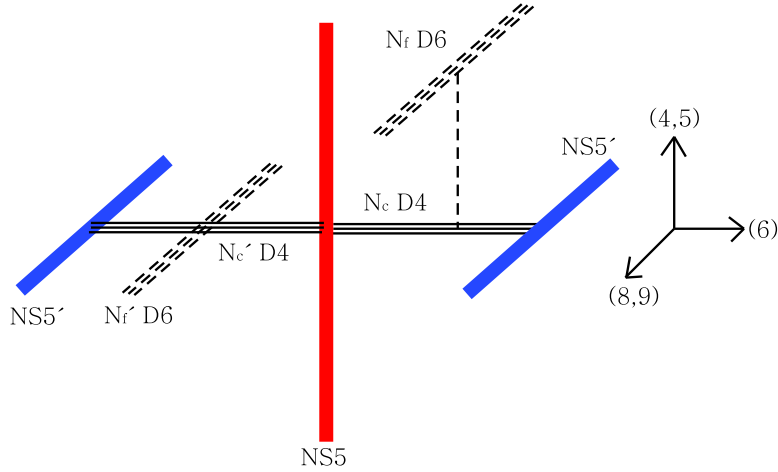

Figure 1: The $\mathcal{N} = 1$ supersymmetric electric brane configuration of $SU(N_c) \times SU(N'_c)$ with N_f chiral multiplets Q, N_f chiral multiplets $\widetilde{Q}$, N'_f chiral multiplets Q', N'_f chiral multiplets $\widetilde{Q'}$, the flavor singlet bifundamental field X and its complex conjugate bifundamental field $\widetilde{X}$. The N_f D6-branes have nonzero v coordinates where $v = m$ for equal massive case of quarks $Q, \widetilde{Q}$ while Q' and $\widetilde{Q'}$ are massless.

---

[2]We introduce two complex coordinates $v \equiv x^4 + ix^5$ and $w \equiv x^8 + ix^9$ for simplicity.

4

2.2 Magnetic theory with $SU(\widetilde{N}_c) \times SU(N'_c)$ gauge group

One can consider dualizing one of the gauge groups regarding as the other gauge group as a spectator. One takes the Seiberg dual for the first gauge group factor $SU(N_c)$ while remaining the second gauge group factor $SU(N'_c)$ unchanged. Also we consider the case where $\Lambda_1 >> \Lambda_2$, in other words, the dualized group's dynamical scale is far above that of the other spectator group.

Let us move a middle NS5-brane to the right all the way past the right $NS5'_R$-brane. For example, see [12, 13, 14, 11, 10, 9, 8, 7]. After this brane motion, one arrives at the Figure 2. Note that there exists a creation of N_f D4-branes connecting N_f D6-branes and $NS5'_R$-brane. Recall that the N_f D6-branes are perpendicular to a middle NS5-brane in Figure 1. The linking number [16] of NS5-brane from Figure 2 is $L_5 = \frac{N_f}{2} - \widetilde{N}_c$. On the other hand, the linking number of NS5-brane from Figure 1 is $L_5 = -\frac{N_f}{2} + N_c - N'_c$. Due to the connection of N'_c D4-branes with $NS5'_R$-brane, the presence of N'_c in the linking number arises. From these two relations, one obtains the number of colors of dual magnetic theory

$$\widetilde{N}_c = N_f + N'_c - N_c. \tag{2.2}$$

The linking number counting looks similar to the one in [7] where there exists a contribution from O4-plane.

Let us draw this magnetic brane configuration in Figure 2 and recall that we put the coincident N_f D6-branes in the nonzero v directions in the electric theory, along the lines of [12, 13, 14, 11, 10, 9, 8, 7]. The N_f created D4-branes connecting between D6-branes and $NS5'_R$-brane can move freely in the w direction. Moreover since N'_c D4-branes are suspending between two equal $NS5'_{L,R}$-branes located at different x^6 coordinate, these D4-branes can slide along the w direction also. If we ignore the left $NS5'_L$-brane, N'_c D4-branes and N'_f D6-branes(detaching these from Figure 2), then this brane configuration corresponds to the standard $\mathcal{N} = 1$ SQCD with the magnetic gauge group $SU(\widetilde{N}_c = N_f - N_c)$ with N_f massive flavors [12, 13, 14].

The dual magnetic gauge group is given by $SU(\widetilde{N}_c) \times SU(N'_c)$ and the matter contents are given by

• N_f chiral multiplets q are in the fundamental representation under the $SU(\widetilde{N}_c)$, N_f chiral multiplets $\widetilde{q}$ are in the antifundamental representation under the $SU(\widetilde{N}_c)$ and then q are in the representation $(\mathbf{\widetilde{N}_c}, \mathbf{1})$ while $\widetilde{q}$ are in the representation $(\overline{\mathbf{\widetilde{N}_c}}, \mathbf{1})$ under the gauge group

• N'_f chiral multiplets Q' are in the fundamental representation under the $SU(N'_c)$, N'_f chiral multiplets $\widetilde{Q'}$ are in the antifundamental representation under the $SU(N'_c)$ and then Q'
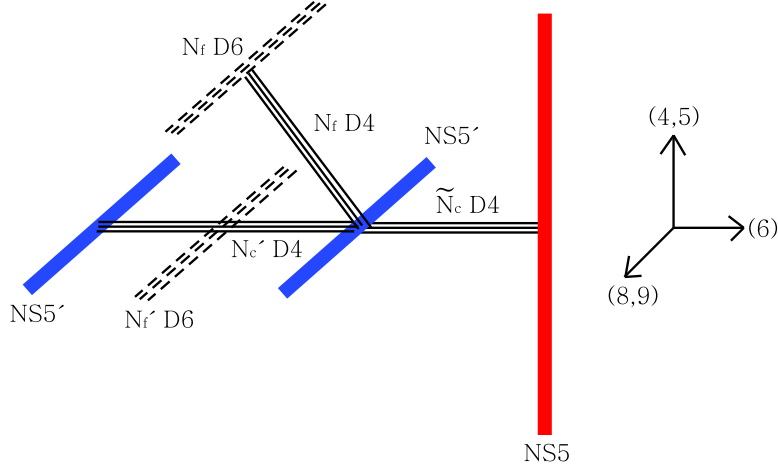
5

Figure 2: The $\mathcal{N} = 1$ supersymmetric magnetic brane configuration of $SU(\widetilde{N}_c = N_f + N'_c - N_c) \times SU(N'_c)$ with N_f chiral multiplets q, N_f chiral multiplets $\widetilde{q}$, N'_f chiral multiplets Q', N'_f chiral multiplets $\widetilde{Q'}$, the flavor singlet bifundamental field Y and its complex conjugate bifundamental field $\widetilde{Y}$ as well as N_f fields F', its complex conjugate N_f fields $\widetilde{F'}$, N_f^2 fields M and the gauge singlet Φ. There exist N_f flavor D4-branes connecting D6-branes and $NS5'_R$-brane.

are in the representation $(\mathbf{1}, \mathbf{N'_c})$ while $\widetilde{Q'}$ are in the representation $(\mathbf{1}, \overline{\mathbf{N'_c}})$ under the gauge group

• The flavor singlet field Y is in the bifundamental representation $(\widetilde{\mathbf{N}_c}, \overline{\mathbf{N'_c}})$ under the gauge group and its complex conjugate field $\widetilde{Y}$ is in the bifundamental representation $(\overline{\widetilde{\mathbf{N}_c}}, \mathbf{N'_c})$ under the gauge group

There are $(N_f + N'_c)^2$ gauge singlets in the first dual gauge group factor as follows:

• N_f-fields F' are in the fundamental representation under the $SU(N'_c)$, its complex conjugate N_f-fields $\widetilde{F'}$ are in the antifundamental representation under the $SU(N'_c)$ and then F' are in the representation $(\mathbf{1}, \mathbf{N'_c})$ under the gauge group while $\widetilde{F'}$ are in the representation $(\mathbf{1}, \overline{\mathbf{N'_c}})$ under the gauge group

These additional N_f $SU(N'_c)$ fundamentals and N_f $SU(N'_c)$ antifundamentals are originating from the $SU(N_c)$ chiral mesons $\widetilde{X}Q$ and $X\widetilde{Q}$ respectively. It is clear to see that from the Figure 2, since the N_f D6-branes are parallel to the $NS5'_R$-brane, the newly created N_f D4-branes can slide along the plane consisting of these $D6$-branes and $NS5'_R$-brane arbitrarily. Then strings stretching between the N_f D6-branes and N'_c D4-branes will give rise to these additional N_f $SU(N'_c)$ fundamentals and N_f $SU(N'_c)$ antifundamentals.

• N_f^2-fields M are in the representation $(\mathbf{1}, \mathbf{1})$ under the gauge group

This corresponds to the $SU(N_c)$ chiral meson $Q\widetilde{Q}$ and the fluctuations of the singlet M

correspond to the motion of N_f flavor D4-branes along (789) directions in Figure 2.

• The $N_c'^2$-fields Φ is in the representation $(\mathbf{1}, \mathbf{N_c'^2 - 1}) \oplus (\mathbf{1}, \mathbf{1})$ under the gauge group

This corresponds to the $SU(N_c)$ chiral meson $X\widetilde{X}$ and note that X has a representation $\overline{\mathbf{N_c'}}$ of $SU(N_c')$ while $\widetilde{X}$ has a representation $\mathbf{N_c'}$ of $SU(N_c')$. The fluctuations of the singlet Φ correspond to the motion of N_c' D4-branes suspended two $NS5'_{L,R}$-branes along the (789) directions in Figure 2.

In the dual theory, since there exist N_f quarks q, N_f quarks $\widetilde{q}$, one bifundamental field Y which will give rise to the contribution of N_c' and its complex conjugate $\widetilde{Y}$ which will give rise to the contribution of N_c', the coefficient of the beta function for the first gauge group factor [6] is

$$b^{mag}_{SU(\widetilde{N}_c)} = 3\widetilde{N}_c - N_f - N_c' = 2N_f + 2N_c' - 3N_c$$

where we inserted the number of colors given in (2.2) in the second equality and since there exist N_f' quarks Q', N_f' quarks $\widetilde{Q}'$, one bifundamental field Y which will give rise to the contribution of $\widetilde{N}_c$, its complex conjugate $\widetilde{Y}$ which will give rise to the contribution of $\widetilde{N}_c$, N_f fields F', its complex conjugate N_f fields $\widetilde{F}'$ and the singlet Φ which will give rise to N_c', the coefficient of the beta function of second gauge group factor [6] is

$$b^{mag}_{SU(N_c')} = 3N_c' - N_f' - \widetilde{N}_c - N_f - N_c' = N_c' + N_c - 2N_f - N_f'.$$

Therefore, both $SU(\widetilde{N}_c)$ and $SU(N_c')$ gauge couplings are IR free by requiring the negativeness of the coefficients of beta function. One can rely on the perturbative calculations at low energy for this magnetic IR free region $b^{mag}_{SU(\widetilde{N}_c)} < 0$ and $b^{mag}_{SU(N_c')} < 0$. Note that the $SU(N_c')$ fields in the magnetic theory are different from those of the electric theory. Since $b_{SU(N_c')} - b^{mag}_{SU(N_c')} > 0$, $SU(N_c')$ is more asymptotically free than $SU(N_c')^{mag}$ [6]. Neglecting the $SU(N_c')$ dynamics, the magnetic $SU(\widetilde{N}_c)$ is IR free when $N_f + N_c' < \frac{3}{2}N_c$ [6].

The dual magnetic superpotential, by adding the mass term (2.1) for Q and $\widetilde{Q}$ in the electric theory which is equal to put a linear term in M in the dual magnetic theory, is given by

$$W_{dual} = \left(Mq\widetilde{q} + YF'\widetilde{q} + \widetilde{Y}q\widetilde{F'} + \Phi Y\widetilde{Y} \right) + mM \tag{2.3}$$

where the mesons in terms of the fields defined in the electric theory are

$$M \equiv Q\widetilde{Q}, \qquad \Phi \equiv X\widetilde{X}, \qquad F' \equiv \widetilde{X}Q, \qquad \widetilde{F'} \equiv X\widetilde{Q}.$$

By orientifolding procedure(O4-plane) into this brane configuration, the $q(Q)$ and $\widetilde{q}(\widetilde{Q})$ are equivalent to each other, the $Y(X)$ and $\widetilde{Y}(\widetilde{X})$ become identical and F' and $\widetilde{F'}$ become the

7

same. Then the reduced superpotential is identical with the one in [7]. Here q and $\widetilde{q}$ are fundamental and antifundamental for the gauge group index respectively and antifundamentals for the flavor index. Then, $q\widetilde{q}$ has rank $\widetilde{N}_c$ while m has a rank N_f. Therefore, the F-term condition, the derivative the superpotential W_{dual} with respect to M, cannot be satisfied if the rank N_f exceeds $\widetilde{N}_c$. This is so-called rank condition and the supersymmetry is broken. Other F-term equations are satisfied by taking the vacuum expectation values of $Y, \widetilde{Y}, F'$ and $\widetilde{F'}$ to vanish.

The classical moduli space of vacua can be obtained from F-term equations

$$
\begin{aligned}
q\widetilde{q} + m &= 0, & \widetilde{q}M + \widetilde{F'}\widetilde{Y} = 0, \\
Mq + YF' &= 0, & F'\widetilde{q} + \widetilde{Y}\Phi = 0, \\
\widetilde{q}Y &= 0, & q\widetilde{F'} + \Phi Y = 0, \\
\widetilde{Y}q &= 0, & Y\widetilde{Y} = 0.
\end{aligned}
$$

Then, it is easy to see that there exist three reduced equations

$$
\widetilde{q}M = 0 = Mq, \qquad q\widetilde{q} + m = 0
$$

and other F-term equations are satisfied if one takes the zero vacuum expectation values for the fields $Y, \widetilde{Y}, F'$ and $\widetilde{F'}$. Then the solutions can be written as follows:

$$
\begin{aligned}
< q > &= \begin{pmatrix} \sqrt{m}e^{\phi}\mathbf{1}_{\widetilde{N}_c} \\ 0 \end{pmatrix}, < \widetilde{q} >= \begin{pmatrix} \sqrt{m}e^{-\phi}\mathbf{1}_{\widetilde{N}_c} & 0 \end{pmatrix}, < M >= \begin{pmatrix} 0 & 0 \\ 0 & \Phi_0 \mathbf{1}_{N_f-\widetilde{N}_c} \end{pmatrix} \\
< Y > &= < \widetilde{Y} >=< F' >=< \widetilde{F'} >= 0.
\end{aligned}
$$

(2.4)

Let us expand around a point on (2.4), as done in [1]. Then the remaining relevant terms of superpotential are given by

$$
W_{dual}^{rel} = \Phi_0 \left(\delta\varphi\, \delta\widetilde{\varphi} + m \right) + \delta Z\, \delta\varphi\, \widetilde{q}_0 + \delta\widetilde{Z}\, q_0 \delta\widetilde{\varphi}
$$

by following the same fluctuations for the various fields as in [9]:

$$
\begin{aligned}
q &= \begin{pmatrix} q_0 \mathbf{1}_{\widetilde{N}_c} + \frac{1}{\sqrt{2}}(\delta\chi_+ + \delta\chi_-)\mathbf{1}_{\widetilde{N}_c} \\ \delta\varphi \end{pmatrix}, & \widetilde{q} &= \begin{pmatrix} \widetilde{q}_0 \mathbf{1}_{\widetilde{N}_c} + \frac{1}{\sqrt{2}}(\delta\chi_+ - \delta\chi_-)\mathbf{1}_{\widetilde{N}_c} & \delta\widetilde{\varphi} \end{pmatrix}, \\
M &= \begin{pmatrix} \delta Y & \delta Z \\ \delta\widetilde{Z} & \Phi_0 \mathbf{1}_{N_f-\widetilde{N}_c} \end{pmatrix}
\end{aligned}
$$

as well as the fluctuations of $Y, \widetilde{Y}, F'$ and $\widetilde{F'}$. Note that there exist also three kinds of terms, the vacuum $< q >$ multiplied by $\delta\widetilde{Y}\delta\widetilde{F'}$, the vacuum $< \widetilde{q} >$ multiplied by $\delta F'\delta Y$, and the vacuum $< \Phi >$ multiplied by $\delta Y\delta\widetilde{Y}$. However, by redefining these, they do not enter the contributions for the one loop result, up to quadratic order. As done in [17], one gets that $m_{\Phi_0}^2$ will contain $(\log 4 - 1) > 0$ implying that these are stable.

8

3 Nonsupersymmetric meta-stable brane configuration of $SU(N_c) \times SU(N_c')$ gauge theory

Now we recombine $\widetilde{N}_c$ D4-branes among N_f flavor D4-branes connecting between D6-branes and $NS5'_R$-brane with those connecting between $NS5'_R$-brane and NS5-brane and push them in $+v$ direction from Figure 2. After this procedure, there are no color D4-branes between $NS5'_R$-brane and NS5-brane. For the flavor D4-branes, we are left with only $(N_f - \widetilde{N}_c)$ flavor D4-branes.

Then the minimal energy supersymmetry breaking brane configuration is shown in Figure 3, along the lines of [12, 13, 14, 11, 10, 9, 8, 7]. If we ignore the left $NS5'_L$-brane, N_c' D4-branes and N_f' D6-branes(detaching these from Figure 3), as observed already, then this brane configuration corresponds to the minimal energy supersymmetry breaking brane configuration for the $\mathcal{N} = 1$ SQCD with the magnetic gauge group $SU(\widetilde{N}_c = N_f - N_c)$ with N_f massive flavors [12, 13, 14].
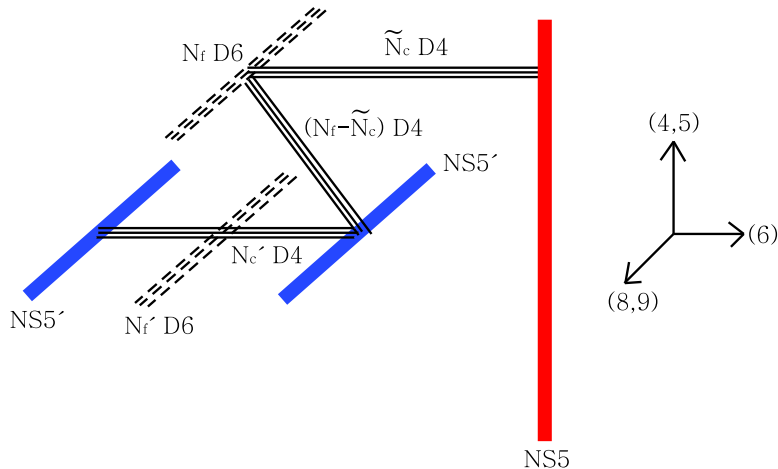

Figure 3: The nonsupersymmetric minimal energy brane configuration of $SU(\widetilde{N}_c = N_f + N_c' - N_c) \times SU(N_c')$ with N_f chiral multiplets q, N_f chiral multiplets $\widetilde{q}$, N_f' chiral multiplets Q', N_f' chiral multiplets $\widetilde{Q'}$, the flavor singlet bifundamental field Y and its complex conjugate bifundamental field $\widetilde{Y}$ and various gauge singlets.

The type IIA/M-theory brane construction for the $\mathcal{N} = 2$ gauge theory was described by [18] and after lifting the type IIA description to M-theory, the corresponding magnetic M5-brane configuration [3] with equal mass for the quarks where the gauge group is given by

---

[3]The M5-brane lives in (0123) directions and is wrapping on a Riemann surface inside (4568910) directions. The Taub-NUT space in (45610) directions is parametrized by two complex variables v and y and the flat two dimensions in (89) directions by a complex variable w. See [14] for the relevant discussions.

$SU(\widetilde{N}_c) \times SU(N'_c)$, in a background space of $xt = v^{N'_f} \prod_{k=1}^{N_f}(v - e_k)$ where this four dimensional space replaces (45610) directions, is described by

$$t^3 + (v^{\widetilde{N}_c} + \cdots)t^2 + (v^{N'_c} + \cdots)t + v^{N'_f}\prod_{k=1}^{N_f}(v - e_k) = 0 \tag{3.1}$$

where e_k is the position of the D6-branes in the v direction(for equal massive case, we can write $e_k = m$) and we have ignored the lower power terms in v in t^2 and t denoted by $\cdots$ and the scales for the gauge groups in front of the first term and the last term, for simplicity. For fixed x, the coordinate t corresponds to y.

From this curve (3.1) of cubic equation for t above, the asymptotic regions for three NS5-branes can be classified by looking at the first two terms providing NS5-brane asymptotic region, next two terms providing $NS5'_R$-brane asymptotic region and the final two terms giving $NS5'_L$-brane asymptotic region as follows

1. $v \to \infty$ limit implies

$$w \to 0, \quad y \sim v^{\widetilde{N}_c} + \cdots \quad \text{NS asymptotic region.}$$

2. $w \to \infty$ limit implies

$$v \ \to \ m, \quad y \sim w^{N_f + N'_f - N'_c} + \cdots \quad NS'_L \text{ asymptotic region,}$$
$$v \ \to \ m, \quad y \sim w^{N'_c - \widetilde{N}_c} + \cdots \quad NS'_R \text{ asymptotic region.}$$

Here the two $NS5'_{L,R}$-branes are moving in the $+v$ direction holding everything else fixed instead of moving D6-branes in the $+v$ direction, in the spirit of [14]. The harmonic function sourced by the D6-branes can be written explicitly by summing over two contributions from the N_f and N'_f D6-branes and similar analysis to both solve the differential equation and find out the nonholomorphic curve can be done [14, 10, 9, 8, 7]. An instability from a new M5-brane mode arises.

4 The $\mathcal{N} = 1$ supersymmetric brane configuration of $SU(N_c) \times SO(N'_c)$ gauge theory

After reviewing the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N} = 1$ $SU(N_c) \times SO(N'_c)$ gauge theory [15], we describe the Seiberg dual magnetic theory which is $\mathcal{N} = 1$ $SU(\widetilde{N}_c) \times SO(N'_c)$ gauge theory.

4.1 Electric theory with $SU(N_c) \times SO(N_c')$ gauge group

The gauge group is given by $SU(N_c) \times SO(N_c')$ and the matter contents [15](similar matter contents are found in [4]) are given by

- N_f chiral multiplets Q are in the fundamental representation under the $SU(N_c)$, N_f chiral multiplets $\widetilde{Q}$ are in the antifundamental representation under the $SU(N_c)$ and then Q are in the representation $(\mathbf{N_c}, \mathbf{1})$ while $\widetilde{Q}$ are in the representation $(\overline{\mathbf{N_c}}, \mathbf{1})$ under the gauge group

- $2N_f'$ chiral multiplets Q' are in the fundamental representation under the $SO(N_c')$ and then Q' are in the representation $(\mathbf{1}, \mathbf{N_c'})$ under the gauge group

- The flavor singlet field X is in the bifundamental representation $(\mathbf{N_c}, \mathbf{N_c'})$ under the gauge group and the flavor singlet $\widetilde{X}$ is in the bifundamental representation $(\overline{\mathbf{N_c}}, \mathbf{N_c'})$ under the gauge group

In the electric theory, since there exist N_f quarks Q, N_f quarks $\widetilde{Q}$, one bifundamental field X which will give rise to the contribution of N_c' and its complex conjugate $\widetilde{X}$ which will give rise to the contribution of N_c', the coefficient of the beta function of the first gauge group factor is

$$b_{SU(N_c)} = 3N_c - N_f - N_c'$$

and similarly, since there exist $2N_f'$ quarks Q', one bifundamental field X which will give rise to the contribution of N_c and its complex conjugate $\widetilde{X}$ which will give rise to the contribution of N_c, the coefficient of the beta function of the second gauge group factor is

$$b_{SO(N_c')} = 3(N_c' - 2) - 2N_f' - 2N_c.$$

The anomaly free global symmetry is given by $SU(N_f)^2 \times SU(2N_f') \times U(1)^2 \times U(1)_R$ and let us denote the strong coupling scales for $SU(N_c)$ as Λ_1 and for $SO(N_c')$ as Λ_2, as in previous section. The theory is asymptotically free when $b_{SU(N_c)} > 0$ for the $SU(N_c)$ gauge theory and when $b_{SO(N_c')} > 0$ for the $SO(N_c')$ gauge theory.

The type IIA brane configuration of $\mathcal{N} = 2$ gauge theory [19] consists of four NS5-branes (012345) which have different x^6 values, N_c and N_c' D4-branes (01236) suspended between them, $2N_f$ and $2N_f'$ D6-branes (0123789) and an orientifold 6 plane (0123789) of positive Ramond charge [4]. According to $\mathbf{Z}_2$ symmetry of orientifold 6-plane(O6-plane) sitting at $v = 0$ and $x^6 = 0$, the coordinates (x^4, x^5, x^6) transform as $-(x^4, x^5, x^6)$, as usual. See also [3] for the discussion of O6-plane.

---

[4]There are many different brane configurations with O6-plane in the literature and some of them are present in [20, 21, 22, 23, 24].

By rotating the third and fourth NS5-branes which are located at the right hand side of O6-plane, from v direction toward $-w$ and $+w$ directions respectively, one obtains $\mathcal{N} = 1$ theory. Their mirrors, the first and second NS5-branes which are located at the left hand side of O6-plane, can be rotated in a $\mathbf{Z}_2$ symmetric manner due to the presence of O6-plane simultaneously. That is, if the first NS5-brane rotates by an angle $-\omega$ in (v, w) plane, denoted by $NS5_{-\omega}$-brane [3], then the mirror image of this NS5-brane, the fourth NS5-brane, is rotated by an angle ω in the same plane, denoted by $NS5_{\omega}$-brane. If the second NS5-brane rotates by an angle θ in (v, w) plane, denoted by $NS5_{\theta}$-brane [3], then the mirror image of this NS5-brane, the third NS5-brane, is rotated by an angle $-\theta$ in the same plane, denoted by $NS5_{-\theta}$-brane. For more details, see the Figure 4 [5].

We also rotate the N'_f D6-branes which are located between the second NS5-brane and an O6-plane and make them be parallel to $NS5_{\theta}$-brane and denote them as $D6_{\theta}$-brane with zero v coordinate(the angle between the unrotated D6-branes and $D6_{\theta}$-branes is equal to $\frac{\pi}{2} - \theta$) and its mirrors N'_f D6-branes appear as $D6_{-\theta}$-branes between the O6-plane and third NS5-brane. There is no coupling between the adjoint field and the quarks since the rotated $D6_{\theta}$-branes are parallel to the rotated $NS5_{\theta}$-brane [5, 3]. Similarly, the N_f D6-branes which are located between the third NS5-brane and the fourth NS5-brane can be rotated and we can make them be parallel to $NS5_{\omega}$-brane and denote them as $D6_{\omega}$-branes with nonzero v coordinate(the angle between the unrotated D6-branes and $D6_{\omega}$-branes is equal to $\frac{\pi}{2} - \omega$) and its mirrors N_f D6-branes appear as $D6_{-\omega}$-branes between the first NS5-brane and the second NS5-brane.

Moreover the N_c D4-branes are suspended between the first NS5-brane and the second NS5-brane(and its mirrors) and the N'_c D4-branes are suspended between the second NS5-brane and the third NS5-brane.

For this brane setup [6], the classical superpotential is given by [15]

$$W = -\frac{1}{4}\left[\frac{1}{4\tan(\omega - \theta)} + \frac{1}{\tan 2\theta}\right]\text{tr}(X\widetilde{X})^2 + \frac{\text{tr}\,X\widetilde{X}\widetilde{X}X}{4\sin 2\theta} + \frac{(\text{tr}\,X\widetilde{X})^2}{4N_c\tan(\omega - \theta)}. \qquad (4.1)$$

It is easy to see that when θ approaches 0 and ω approaches $\frac{\pi}{2}$, then this superpotential vanishes.

---

[5]The angles of θ_1 and θ_2 in [15] are related to the angles θ and ω as follows: $\theta = \theta_1$ and $\omega = \theta_2$.

[6]For arbitrary angles θ and ω, the superpotential for the $SU(N_c)$ sector is given by $W = X\phi\widetilde{X} + \tan(\omega - \theta)\,\text{tr}\,\phi^2$ where ϕ ia an adjoint field for $SU(N_c)$. There is no coupling between ϕ and N_f quarks because $D6_{\pm\omega}$-branes are parallel to $NS5_{\pm\omega}$-branes. The superpotential for the $SO(N'_c)$ sector is given by $W = X\phi_A\widetilde{X} + X\phi_S\widetilde{X} + \tan\theta\,\text{tr}\,\phi_A^2 - \frac{1}{\tan\theta}\,\text{tr}\,\phi_S^2$ where ϕ_A and ϕ_S are an adjoint field and a symmetric tensor for $SO(N'_c)$ [25]. After integrating out ϕ, ϕ_A and ϕ_S, the whole superpotential can be written as in (4.1).

Now one summarizes the supersymmetric electric brane configuration with their worldvolumes in type IIA string theory as follows.

- $NS5_{-\omega}$-brane with worldvolume by both (0123) and two spatial dimensions in (v, w) plane and with negative x^6.

- $NS5_\theta$-brane with worldvolume by both (0123) and two spatial dimensions in (v, w) plane and with negative x^6.

- $NS5_{-\theta}$-brane with worldvolume by both (0123) and two spatial dimensions in (v, w) plane and with positive x^6.

- $NS5_\omega$-brane with worldvolume by both (0123) and two spatial dimensions in (v, w) plane and with positive x^6.

- N'_f $D6_\theta$-branes with worldvolume by both (01237) and two spatial dimensions in (v, w) plane and with negative x^6 and $v = 0$.

- N'_f $D6_{-\theta}$-branes with worldvolume by both (01237) and two space dimensions in (v, w) plane and with positive x^6 and $v = 0$.

- N_f $D6_\omega$-branes with worldvolume by both (01237) and two spatial dimensions in (v, w) plane and with positive x^6. Before the rotation, the distance from N_c color D4-branes in the $+v$ direction is nonzero.

- N_f $D6_{-\omega}$-branes with worldvolume by both (01237) and two space dimensions in (v, w) plane and with negative x^6. Before the rotation, the distance from N_c color D4-branes in the $-v$ direction is nonzero.

- O6-plane with worldvolume (0123789) with $v = 0 = x^6$.

- N_c D4-branes connecting $NS5_{-\omega}$-brane and $NS5_\theta$-brane, with worldvolume (01236) with $v = 0 = w$(and its mirrors).

- N'_c D4-branes connecting $NS5_\theta$-brane and $NS5_{-\theta}$-brane, with worldvolume (01236) with $v = 0 = w$.

We draw the type IIA electric brane configuration in Figure 4 which was basically given in [15] already but the only difference is to put N_f D6-branes in the nonzero v direction in order to obtain nonzero masses for the quarks which are necessary to obtain the meta-stable vacua.

4.2 Magnetic theory with $SU(\widetilde{N}_c) \times SO(N'_c)$ gauge group

One takes the Seiberg dual for the first gauge group factor $SU(N_c)$ while remaining the second gauge group factor $SO(N'_c)$, as in previous case. Also we consider the case where $\Lambda_1 >> \Lambda_2$, in other words, the dualized group's dynamical scale is far above that of the other spectator group.
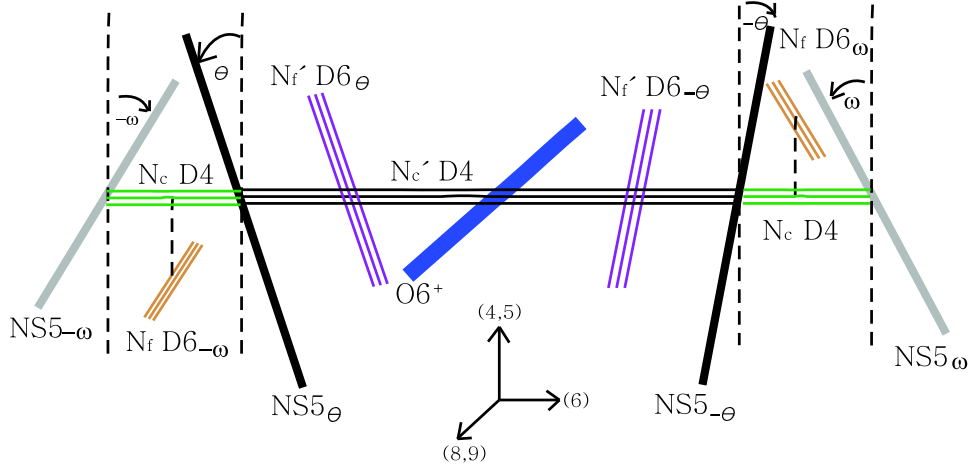
Figure 4: The $\mathcal{N} = 1$ supersymmetric electric brane configuration of $SU(N_c) \times SO(N'_c)$ with N_f chiral multiplets Q, N_f chiral multiplets $\widetilde{Q}$, $2N'_f$ chiral multiplets Q', the flavor singlet bifundamental field X and its complex conjugate bifundamental field $\widetilde{X}$. The N_f $D6_\omega$-branes have nonzero v coordinates where $v = m$(and its mirrors) for equal massive case of quarks $Q, \widetilde{Q}$ while Q' is massless.

Let us move the $NS5_{-\theta}$-brane to the right all the way past the right $NS5_\omega$-brane(and its mirrors to the left). After this brane motion, one arrives at the Figure 5. Note that there exists a creation of N_f D4-branes connecting N_f $D6_\omega$-branes and $NS5_\omega$-brane(and its mirrors). Recall that the N_f $D6_\omega$-branes are not parallel to the $NS5_{-\theta}$-brane in Figure 4(and its mirrors). The linking number of $NS5_{-\theta}$-brane from Figure 5 is $L_5 = \frac{N_f}{2} - \widetilde{N}_c$. On the other hand, the linking number of $NS5_{-\theta}$-brane from Figure 4 is $L_5 = -\frac{N_f}{2} + N_c - N'_c$. From these, one gets the number of colors in dual magnetic theory

$$\widetilde{N}_c = N_f + N'_c - N_c. \tag{4.2}$$

Let us draw this magnetic brane configuration in Figure 5 and remember that we put the coincident N_f $D6_\omega$-branes in the nonzero v directions(and its mirrors). The N_f created D4-branes connecting between $D6_\omega$-branes and $NS5_\omega$-brane can move freely in the w direction, as in previous case. Moreover, since N'_c D4-branes are suspending between two unequal $NS5_{\pm\omega}$-branes located at different x^6 coordinate, these D4-branes cannot slide along the w direction, for arbitrary rotation angles. If we are detaching all the branes except $NS5_\omega$-brane, $NS5_{-\theta}$-brane, $D6_\omega$-branes, N_f D4-branes and $\widetilde{N}_c$ D4-branes from Figure 5, then this brane configuration corresponds to $\mathcal{N} = 1$ SQCD with the magnetic gauge group $SU(\widetilde{N}_c = N_f - N_c)$ with N_f massive flavors with tilted NS5-branes.

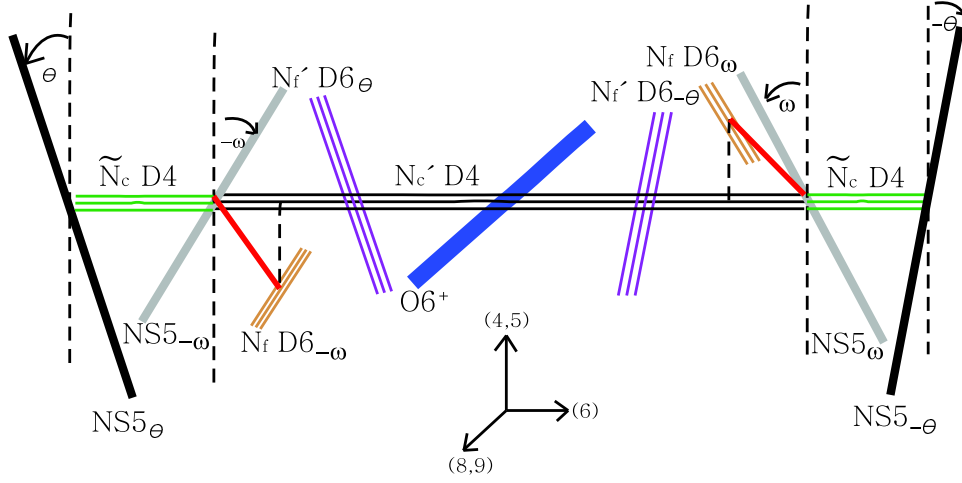The dual magnetic gauge group is given by $SU(\widetilde{N}_c) \times SO(N'_c)$ and the matter contents are given by

14

Figure 5: The $\mathcal{N} = 1$ supersymmetric magnetic brane configuration of $SU(\widetilde{N}_c = N_f + N_c' - N_c) \times SO(N_c')$ with N_f chiral multiplets q, N_f chiral multiplets $\widetilde{q}$, $2N_f'$ chiral multiplets Q', the flavor singlet bifundamental field Y and its complex conjugate bifundamental field $\widetilde{Y}$ as well as N_f fields F', its complex conjugate N_f fields $\widetilde{F'}$, N_f^2 fields M and the gauge singlet Φ. There exist N_f flavor D4-branes connecting $D6_\omega$-branes and $NS5_\omega$-brane(and its mirrors).

- N_f chiral multiplets q are in the fundamental representation under the $SU(\widetilde{N}_c)$, N_f chiral multiplets $\widetilde{q}$ are in the antifundamental representation under the $SU(\widetilde{N}_c)$ and then q are in the representation $(\widetilde{\mathbf{N}}_{\mathbf{c}}, \mathbf{1})$ while $\widetilde{q}$ are in the representation $(\overline{\widetilde{\mathbf{N}}_{\mathbf{c}}}, \mathbf{1})$ under the gauge group

- $2N_f'$ chiral multiplets Q' are in the fundamental representation under the $SO(N_c')$ and then Q' are in the representation $(\mathbf{1}, \mathbf{N}_{\mathbf{c}}')$ under the gauge group

- The flavor singlet field Y is in the bifundamental representation $(\widetilde{\mathbf{N}}_{\mathbf{c}}, \mathbf{N}_{\mathbf{c}}')$ under the gauge group and its complex conjugate field $\widetilde{Y}$ is in the bifundamental representation $(\overline{\widetilde{\mathbf{N}}_{\mathbf{c}}}, \mathbf{N}_{\mathbf{c}}')$ under the gauge group

There are $(N_f + N_c')^2$ gauge singlets in the first dual gauge group factor as follows:

- N_f-fields F' are in the fundamental representation under the $SO(N_c')$, N_f-fields $\widetilde{F'}$ are in the fundamental representation under the $SO(N_c')$ and then F' are in the representation $(\mathbf{1}, \mathbf{N}_{\mathbf{c}}')$ under the gauge group while $\widetilde{F'}$ are in the representation $(\mathbf{1}, \mathbf{N}_{\mathbf{c}}')$ under the gauge group

These additional $2N_f$ $SO(N_c')$ vectors are originating from the $SU(N_c)$ chiral mesons $\widetilde{X}Q$ and $X\widetilde{Q}$ respectively. It is easy to see that from the Figure 5, since the $D6_{-\omega}$-branes are parallel to the $NS5_{-\omega}$-brane, the newly created N_f D4-branes can slide along the plane consisting of $D6_{-\omega}$-branes and $NS5_{-\omega}$-brane arbitrarily(and its mirrors). Then strings connecting the N_f $D6_{-\omega}$-branes and N_c' D4-branes will give rise to these additional $2N_f$ $SO(N_c')$ vectors.

- N_f^2-fields M are in the representation $(\mathbf{1}, \mathbf{1})$ under the gauge group

This corresponds to the $SU(N_c)$ chiral meson $Q\widetilde{Q}$ and the fluctuations of the singlet M correspond to the motion of N_f flavor D4-branes along (789) directions in Figure 5.

- The $N_c'^2$ singlet Φ is in the representation $(\mathbf{1}, \mathbf{adj}) \oplus (\mathbf{1}, \mathbf{symm})$ under the gauge group

This corresponds to the $SU(N_c)$ chiral meson $X\widetilde{X}$ and note that both X and $\widetilde{X}$ have representation $\mathbf{N'_c}$ of $SO(N_c')$. In general, the fluctuations of the singlet Φ correspond to the motion of N_c' D4-branes suspended two $NS5_{\pm\omega}$-branes along the (789) directions in Figure 5.

In the dual theory, since there exist N_f quarks q, N_f quarks $\widetilde{q}$, one bifundamental field Y which will give rise to the contribution of N_c' and its complex conjugate $\widetilde{Y}$ which will give rise to the contribution of N_c', the coefficient of the beta function of the first gauge group factor with (4.2) is

$$b_{SU(\widetilde{N}_c)}^{mag} = 3\widetilde{N}_c - N_f - N_c' = 2N_f + 2N_c' - 3N_c$$

and since there exist $2N_f'$ quarks Q', one bifundamental field Y which will give rise to the contribution of $\widetilde{N}_c$, its complex conjugate $\widetilde{Y}$ which will give rise to the contribution of $\widetilde{N}_c$, N_f fields F', its complex conjugate N_f fields $\widetilde{F'}$ and the singlet Φ which will give rise to N_c', the coefficient of the beta function is

$$b_{SO(N_c')}^{mag} = 3(N_c' - 2) - 2N_f' - 2\widetilde{N}_c - 2N_f - 2N_c' = -N_c' + 2N_c - 4N_f - 2N_f' - 6.$$

Therefore, both $SU(\widetilde{N}_c)$ and $SO(N_c')$ gauge couplings are IR free by requiring the negativeness of the coefficients of beta function. One can rely on the perturbative calculations at low energy for this magnetic IR free region $b_{SU(\widetilde{N}_c)}^{mag} < 0$ and $b_{SO(N_c')}^{mag} < 0$. Note that the $SO(N_c')$ fields in the magnetic theory are different from those of the electric theory. Since $b_{SO(N_c')} - b_{SO(N_c')}^{mag} > 0$, $SO(N_c')$ is more asymptotically free than $SO(N_c')^{mag}$. Neglecting the $SO(N_c')$ dynamics, the magnetic $SU(\widetilde{N}_c)$ is IR free when $N_f + N_c' < \frac{3}{2}N_c$, as in previous case.

The dual magnetic superpotential, by adding the mass term for Q and $\widetilde{Q}$ in the electric theory which is equal to put a linear term in M in the dual magnetic theory, is given by [7]

$$W_{dual} = \left[(\Phi^2 + \cdots) + Q'\Phi Q' + Mq\widetilde{q} + Y\widetilde{F'}\widetilde{q} + \widetilde{Y}qF' + \Phi Y\widetilde{Y} \right] + mM \qquad (4.3)$$

where the mesons in terms of the fields defined in the electric theory are

$$M \equiv Q\widetilde{Q}, \qquad \Phi \equiv X\widetilde{X}, \qquad F' \equiv \widetilde{X}Q, \qquad \widetilde{F'} \equiv X\widetilde{Q}.$$

---

[7]There appears a mismatch between the number of colors from field theory analysis and those from brane motion when we take the full dual process on the two gauge group factors simultaneously [15]. By adding $4N_f'$ D4-branes to the dual brane configuration without affecting the linking number counting, this mismatch can be removed. Similar phenomena occurred in [5, 26]. Then this turned out that there exists a deformation ΔW generated by the meson $Q'X\widetilde{X}Q'$. This is exactly the second term, $Q'\Phi Q'$, in (4.3). In previous example, there is no such deformation term in (2.3).

We abbreviated all the relevant terms and coefficients appearing in the quartic superpotential for the bifundamentals in electric theory (4.1) and denote them here by $\Phi^2 + \cdots$. Here q and $\widetilde{q}$ are fundamental and antifundamental for the gauge group index respectively and antifundamentals for the flavor index. Then, $q\widetilde{q}$ has rank $\widetilde{N}_c$ and m has a rank N_f. Therefore, the F-term condition, the derivative the superpotential W_{dual} with respect to M, cannot be satisfied if the rank N_f exceeds $\widetilde{N}_c$ and the supersymmetry is broken. Other F-term equations are satisfied by taking the vacuum expectation values of $Y, \widetilde{Y}, F', \widetilde{F'}$ and Q' to vanish.

The classical moduli space of vacua can be obtained from F-term equations and one gets

$$
\begin{aligned}
q\widetilde{q} + m &= 0, & \widetilde{q}M + F'\widetilde{Y} &= 0, \\
Mq + Y\widetilde{F'} &= 0, & \widetilde{F'}\widetilde{q} + \widetilde{Y}\Phi &= 0, \\
\widetilde{q}Y &= 0, & qF' + \Phi Y &= 0, \\
\widetilde{Y}q &= 0, & Q'Q' + Y\widetilde{Y} &= 0, \\
\Phi Q' &= 0.
\end{aligned}
$$

Then, it is easy to see that there exists a solution

$$
\widetilde{q}M = 0 = Mq, \qquad q\widetilde{q} + m = 0.
$$

Other F-term equations are satisfied if one takes the zero vacuum expectation values for the fields $Y, \widetilde{Y}, F', Q'$ and $\widetilde{F'}$. Then the solutions can be written as

$$
\begin{aligned}
<q> &= \begin{pmatrix} \sqrt{m}e^{\phi}\mathbf{1}_{\widetilde{N}_c} \\ 0 \end{pmatrix}, <\widetilde{q}> = \begin{pmatrix} \sqrt{m}e^{-\phi}\mathbf{1}_{\widetilde{N}_c} & 0 \end{pmatrix}, <M> = \begin{pmatrix} 0 & 0 \\ 0 & \Phi_0\mathbf{1}_{N_f - \widetilde{N}_c} \end{pmatrix} \\
<Y> &= <\widetilde{Y}> = <F'> = <\widetilde{F'}> = <Q'> = 0.
\end{aligned} \tag{4.4}
$$

Let us expand around a point on (4.4), as done in [1]. Then the remaining relevant terms of superpotential are given by

$$
W_{dual}^{rel} = \Phi_0\left(\delta\varphi\,\delta\widetilde{\varphi} + m\right) + \delta Z\,\delta\varphi\,\widetilde{q}_0 + \delta\widetilde{Z}\,q_0\delta\widetilde{\varphi}
$$

by following the similar fluctuations for the various fields as in [9]. Note that there exist also four kinds of terms, the vacuum $<q>$ multiplied by $\delta\widetilde{Y}\delta F'$, the vacuum $<\widetilde{q}>$ multiplied by $\delta\widetilde{F'}\delta Y$, the vacuum $<\Phi>$ multiplied by $\delta Y\delta\widetilde{Y}$, and the vacuum $<\Phi>$ multiplied by $\delta Q'\delta Q'$. However, by redefining these, they do not enter the contributions for the one loop result, up to quadratic order. As done in [17], one gets that $m_{\Phi_0}^2$ will contain $(\log 4 - 1) > 0$ implying that these are stable.

5 Nonsupersymmetric meta-stable brane configuration of $SU(N_c) \times SO(N_c')$ gauge theory

Since the electric superpotential (4.1) vanishes for $\theta = 0$ and $\omega = \frac{\pi}{2}$, the corresponding magnetic superpotential in (4.3) does not contain the terms $\Phi^2 + \cdots$ and it becomes

$$W_{dual} = \left(Q'\Phi Q' + Mq\widetilde{q} + Y\widetilde{F'}\widetilde{q} + \widetilde{Y}qF' + \Phi Y\widetilde{Y} \right) + mM.$$

Now we recombine $\widetilde{N}_c$ D4-branes among N_f flavor D4-branes connecting between $D6_{\omega=\frac{\pi}{2}} = D6$-branes and $NS5_{\omega=\frac{\pi}{2}} = NS5_R'$-brane with those connecting between $NS5_R'$-brane and $NS5_{-\theta=0} = NS5_R$-brane(and its mirrors) and push them in $+v$ direction from Figure 5. Of course their mirrors will move to $-v$ direction in a $\mathbf{Z}_2$ symmetric manner due to the $O6^+$-plane. After this procedure, there are no color D4-branes between $NS5_R'$-brane and $NS5_R$-brane. For the flavor D4-branes, we are left with only $(N_f - \widetilde{N}_c)$ D4-branes(and its mirrors).

Then the minimal energy supersymmetry breaking brane configuration is shown in Figure 6. If we ignore all the branes except $NS5_R'$-brane, $NS5_R$-brane, D6-branes, $(N_f - \widetilde{N}_c)$ D4-branes and $\widetilde{N}_c$ D4-branes, as observed already, then this brane configuration corresponds to the minimal energy supersymmetry breaking brane configuration for the $\mathcal{N} = 1$ SQCD with the magnetic gauge group $SU(\widetilde{N}_c)$ with N_f massive flavors [12, 13, 14]. Note that N_c' D4-branes can slide w direction for this brane configuration.

The type IIA/M-theory brane construction for the $\mathcal{N} = 2$ gauge theory was described by [19] and after lifting the type IIA description we explained so far to M-theory, the corresponding magnetic M5-brane configuration with equal mass for the quarks where the gauge group is given by $SU(\widetilde{N}_c) \times SO(N_c')$, in a background space of $xt = (-1)^{N_f+N_f'}v^{2N_f'+4}\prod_{k=1}^{N_f}(v^2 - e_k^2)$ where this four dimensional space replaces (45610) directions, is characterized by

$$t^4 + (v^{\widetilde{N}_c} + \cdots)t^3 + (v^{N_c'} + \cdots)t^2 + (v^{\widetilde{N}_c} + \cdots)t + v^{2N_f'+4}\prod_{k=1}^{N_f}(v^2 - e_k^2) = 0.$$

From this curve of quartic equation for t above, the asymptotic regions can be classified by looking at the first two terms providing $NS5_R$-brane asymptotic region, next two terms providing $NS5_R'$-brane asymptotic region, next two terms providing $NS5_L'$-brane asymptotic region, and the final two terms giving $NS5_L$-brane asymptotic region as follows:

1. $v \to \infty$ limit implies

$$w \to 0, \quad y \sim v^{\widetilde{N}_c} + \cdots \quad NS5_R \text{ asymptotic region},$$
$$w \to 0, \quad y \sim v^{2N_f+2N_f'-\widetilde{N}_c+4} + \cdots \quad NS5_L \text{ asymptotic region}.$$

Figure 6: The nonsupersymmetric minimal energy brane configuration of $SU(\widetilde{N}_c = N_f + N_c' - N_c) \times SO(N_c')$ with N_f chiral multiplets q, N_f chiral multiplets $\widetilde{q}$, $2N_f'$ chiral multiplets Q', the flavor singlet bifundamental field Y and its complex conjugate bifundamental field $\widetilde{Y}$ and gauge singlets. The N_c' D4-branes and $2(N_f - \widetilde{N}_c)$ D4-branes can slide w direction freely in a $\mathbf{Z}_2$ symmetric way.

2. $w \to \infty$ limit implies

$$
\begin{aligned}
v &\to -m, & y &\sim w^{\widetilde{N}_c - N_c'} + \cdots & NS5_L' \text{ asymptotic region,} \\
v &\to +m, & y &\sim w^{N_c' - \widetilde{N}_c} + \cdots & NS5_R' \text{ asymptotic region.}
\end{aligned}
$$

Now the two $NS5_{L,R}'$-branes are moving in the $\pm v$ direction holding everything else fixed instead of moving D6-branes in the $\pm v$ direction. Then the mirrors of D4-branes are moved appropriately. The harmonic function sourced by the D6-branes can be written explicitly by summing of three contributions from the N_f and N_f' D6-branes(and its mirrors) plus an O6-plane, and similar analysis to solve the differential equation and find out the nonholomorphic curve can be done [14, 10, 9, 8, 7]. In this case also, we expect an instability from a new M5-brane mode.

6   Discussions

So far, we have dualized only the first gauge group factor in the gauge group $SU(N_c) \times SO(N_c')$. What happens if we dualize the second gauge group factor $SO(N_c')$?(For the case $SU(N_c) \times SU(N_c')$, the behavior of dual for the second gauge group will be the same as when we take the dual for the first gauge group factor.) This can be done by moving the $NS5_\theta$-brane and N_f' $D6_\theta$-branes that can be located at the nonzero v coordinate for massive quarks Q', to

the right passing through O6-plane(and their mirrors to the left). According to the linking number counting, one obtains the dual gauge group $SU(N_c) \times SO(\widetilde{N}'_c = 2N_c + 2N'_f - N'_c + 4)$. One can easily see that there is a creation of N'_f D4-branes connecting $NS5_\theta$-brane and $D6_\theta$-branes(and its mirrors). Then from the brane configuration, there exist the additional $2N'_f$ $SU(N_c)$ quarks originating from the $SO(N'_c)$ chiral mesons $Q'X \equiv \widetilde{F'}$ and $Q'\widetilde{X} \equiv F'$. The deformed superpotential $\Delta W = Q'X\widetilde{X}Q'$ can be interpreted as the mass term of $F'\widetilde{F'}$. Then one can write dual magnetic superpotential in this case. However, it is not clear how the recombination of color and flavor D4-branes and splitting procedure between them in the construction of meta-stable vacua arises since there is no extra NS5-brane between two $NS5_{\pm\theta}$-branes. If there exists an extra NS5-brane at the origin of our brane configuration(then the gauge group and matter contents will change), it would be possible to construct the corresponding meta-stable brane configuration. It would be interesting to study these more in the future.

As already mentioned in [8] and section 4, the matter contents in [4] are different from the ones in section 4 with the same gauge group. In other words, the theory of $SU(N_c) \times SO(N'_c)$ with X, which transform as fundamental in $SU(N_c)$ and vector in $SO(N'_c)$, a antisymmetric tensor A in $SU(N_c)$, as well as fundamentals for $SU(N_c)$ and vectors for $SO(N'_c)$ can confine either $SU(N_c)$ factor or $SO(N'_c)$ factor. This theory can be described by the web of branes in the presence of $O4^-$-plane and orbifold fixed points. With two NS5-branes and $O4^-$-plane, by modding out $\mathbf{Z}_3$ symmetry acting on (v, w) as $(v, w) \rightarrow (v\exp(\frac{2\pi i}{3}), w\exp(\frac{2\pi i}{3}))$, the resulting gauge group will be $SU(N_c) \times SO(N_c+4)$ with above matter contents [27]. Similar analysis for $SU(N_c) \times Sp(\frac{N_c}{2}-2)$ gauge group with opposite $O4^+$-plane can be done. Then in this case, the matter in $SU(N_c)$ will be a symmetric tensor S and other matter contents are present also. It would be interesting to see whether this gauge theory and corresponding brane configuration will provide a meta-stable vacuum.

Let us comment on other possibility where the gauge group is given by $SU(N_c) \times Sp(N'_c)$ and the matter contents are given by

• N_f chiral multiplets Q are in the fundamental representation under the $SU(N_c)$, N_f chiral multiplets $\widetilde{Q}$ are in the antifundamental representation under the $SU(N_c)$ and then Q are in the representation $(\mathbf{N_c}, \mathbf{1})$ while $\widetilde{Q}$ are in the representation $(\overline{\mathbf{N_c}}, \mathbf{1})$ under the gauge group

• $2N'_f$ chiral multiplets Q' are in the fundamental representation under the $Sp(N'_c)$ and then Q' are in the representation $(\mathbf{1}, \mathbf{2N'_c})$ under the gauge group

• The flavor singlet field X is in the bifundamental representation $(\mathbf{N_c}, \mathbf{2N'_c})$ under the gauge group and the flavor singlet $\widetilde{X}$ is in the bifundamental representation $(\overline{\mathbf{N_c}}, \mathbf{2N'_c})$ under

the gauge group

One can compute the coefficients of beta functions of the each gauge group factor, as we did for previous examples.

The type IIA brane configuration of an electric theory is exactly the same as the Figure 4 except the RR charge O6-plane with negative sign. The classical superpotential [8] is given by [15]

$$W = -\frac{1}{4}\left[\frac{1}{4\tan(\omega-\theta)} + \frac{1}{\tan 2\theta}\right]\text{tr}(X\widetilde{X})^2 - \frac{\text{tr}\,X\widetilde{X}\widetilde{X}X}{4\sin 2\theta} + \frac{(\text{tr}\,X\widetilde{X})^2}{4N_c\tan(\omega-\theta)}. \tag{6.1}$$

In this case, when θ approaches $\frac{\pi}{2}$ and ω approaches 0, then this superpotential vanishes.

The dual magnetic gauge group is given by $SU(\widetilde{N}_c = N_f + 2N'_c - N_c) \times Sp(N'_c)$ with the same number of colors of dual theory as those in previous cases and the matter contents are given by

• N_f chiral multiplets q are in the fundamental representation under the $SU(\widetilde{N}_c)$, N_f chiral multiplets $\widetilde{q}$ are in the antifundamental representation under the $SU(\widetilde{N}_c)$ and then q are in the representation $(\widetilde{\mathbf{N}}_\mathbf{c}, \mathbf{1})$ while $\widetilde{q}$ are in the representation $(\overline{\widetilde{\mathbf{N}}}_\mathbf{c}, \mathbf{1})$ under the gauge group

• $2N'_f$ chiral multiplets Q' are in the fundamental representation under the $Sp(N'_c)$ and then Q' are in the representation $(\mathbf{1}, \mathbf{2N'_c})$ under the gauge group

• The flavor singlet field Y is in the bifundamental representation $(\widetilde{\mathbf{N}}_\mathbf{c}, \mathbf{2N'_c})$ under the gauge group and its complex conjugate field $\widetilde{Y}$ is in the bifundamental representation $(\overline{\widetilde{\mathbf{N}}}_\mathbf{c}, \mathbf{2N'_c})$ under the gauge group

There are $(N_f + 2N'_c)^2$ gauge singlets in the first dual gauge group factor

• N_f-fields F' are in the fundamental representation under the $Sp(N'_c)$, N_f-fields $\widetilde{F'}$ are in the fundamental representation under the $Sp(N'_c)$ and then F' are in the representation $(\mathbf{1}, \mathbf{2N'_c})$ under the gauge group while $\widetilde{F'}$ are in the representation $(\mathbf{1}, \mathbf{2N'_c})$ under the gauge group

• N_f^2-fields M are in the representation $(\mathbf{1}, \mathbf{1})$ under the gauge group

• The $4N'^2_c$ singlet Φ is in the representation $(\mathbf{1}, \mathbf{adj}) \oplus (\mathbf{1}, \mathbf{antisymm})$ under the gauge group

The dual magnetic superpotential for arbitrary angles is given by (4.3) with appropriate $Sp(N'_c)$ invariant metric J. The stability analysis can be done similarly.

---

[8]The superpotential for the $Sp(N'_c)$ sector is given by $W = X\phi_A\widetilde{X} + X\phi_S\widetilde{X} + \tan\theta\,\text{tr}\,\phi_S^2 - \frac{1}{\tan\theta}\,\text{tr}\,\phi_A^2$ where ϕ_S and ϕ_A are an adjoint field(symmetric tensor) and an antisymmetric tensor for $Sp(N'_c)$ [25]. Note that there is a sign change in the second trace term of the superpotential in (6.1), compared to (4.1).

After following the procedure from Figure 4 to Figure 5 with opposite RR charge for O6-plane and by taking the limit where $\theta \to \frac{\pi}{2}$ and $\omega \to 0$, the minimal energy supersymmetry breaking brane configuration is shown in Figure 7.
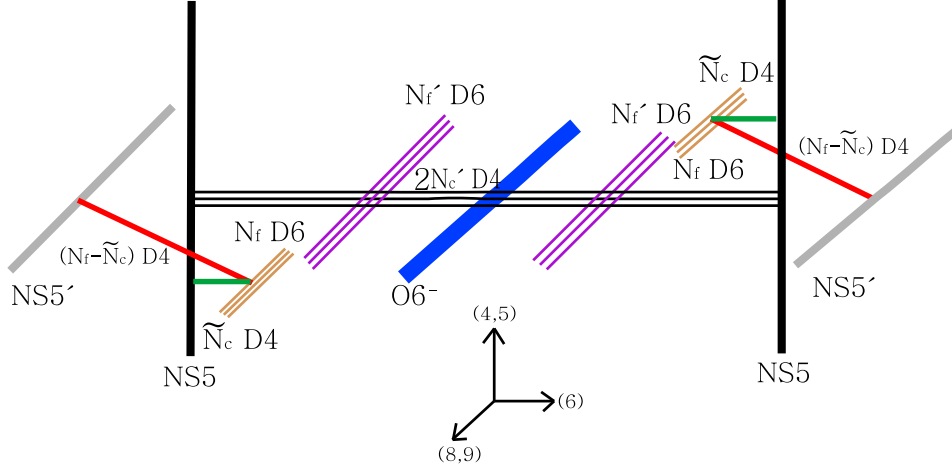

Figure 7: The nonsupersymmetric minimal energy brane configuration of $SU(\widetilde{N}_c = N_f + 2N'_c - N_c) \times Sp(N'_c)$ with N_f chiral multiplets q, N_f chiral multiplets $\widetilde{q}$, $2N'_f$ chiral multiplets Q', the flavor singlet bifundamental field Y and its complex conjugate bifundamental field $\widetilde{Y}$ and gauge singlets. Note the RR charge of O6-plane is negative and its charge is equivalent to -4 D6-branes. The $2N'_c$ D4-branes and $2(N_f - \widetilde{N}_c)$ D4-branes can slide w direction freely in a $\mathbf{Z}_2$ symmetric way.

Compared to the previous nonsupersymmetric brane configuration in Figure 6, the role of NS5-brane and NS5'-brane is interchanged to each other: undoing the Seiberg dual in the context of [13]. This kind of feature of recombination and splitting between color D4-branes and flavor D4-branes occurs in [8]. At the electric brane configuration, N_f D6-branes are perpendicular to NS5-brane and this leads to the coupling between the quarks and adjoint in the superpotential. However, the overall coefficient function including this extra terms vanishes and eventually the whole electric superpotential will vanish according to the above limit we take.

From the quartic equation with the presence of opposite RR charge for O6-plane, in a background space of $xt = (-1)^{N_f + N'_f} v^{2N'_f - 4} \prod_{k=1}^{N_f} (v^2 - e_k^2)$,

$$t^4 + (v^{\widetilde{N}_c} + \cdots)t^3 + (v^{N'_c} + \cdots)t^2 + (v^{\widetilde{N}_c} + \cdots)t + v^{2N'_f - 4}\prod_{k=1}^{N_f}(v^2 - e_k^2) = 0,$$

the asymptotic regions can be classified as follows:

1. $v \to \infty$ limit implies

$$
\begin{aligned}
w &\to 0, \quad y \sim v^{N_c' - \widetilde{N}_c} \cdots \quad NS5_R \text{ asymptotic region,} \\
w &\to 0, \quad y \sim v^{\widetilde{N}_c - N_c'} + \cdots \quad NS5_L \text{ asymptotic region.}
\end{aligned}
$$

2. $w \to \infty$ limit implies

$$
\begin{aligned}
v &\to -m, \quad y \sim w^{2N_f + 2N_f' - \widetilde{N}_c - 4} + \cdots \quad NS5_L' \text{ asymptotic region,} \\
v &\to +m, \quad y \sim w^{\widetilde{N}_c} + \cdots \quad NS5_R' \text{ asymptotic region.}
\end{aligned}
$$

In [28], the $SU(7) \times \widetilde{Sp}(1)$ model and $SU(9) \times \widetilde{Sp}(2)$ model can be obtained by dualizing the $SU(7) \times SU(2)$ model with a bifundamental and two antifundamentals for $SU(7)$ and a fundamental for $SU(2)$ and the $SU(9) \times SU(2)$ with a bifundamental and two antifundamentals for $SU(9)$ and a fundamental for $Sp(1)$ respectively(Note that $Sp(1) \sim SU(2)$). The matter contents in an electric theory are different from those in previous paragraph. The matter contents in the magnetic description are given by an antisymmetric tensor and a fundamental in the first gauge group as well as a bifundamental, a fundamental in the second gauge group and two antifundamentals in the first gauge group. There exists a nonzero dual magnetic superpotential. Also the dual description the $SU(7) \times \widetilde{Sp}(1)$ model and $SU(9) \times \widetilde{Sp}(2)$ model can be constructed from the antisymmetric models of Affleck-Dine-Seiberg by gauging a maximal flavor symmetry and adding the extra matter to cancel all anomalies and extra flavor.

On the other hand, the models $SU(2N_c + 1) \times SU(2)$ have its brane box model description in [29] where the above examples correspond to $N_c = 3$ and $N_c = 4$ respectively. In particular, the case where $N_c = 1$(the gauge group is $SU(3) \times SU(2)$, i.e., $(3, 2)$ model [30]) was described by brane box model with superpotential or without superpotential. Then it would be interesting to obtain the Seiberg dual for these models using brane box model and look for the possibility of having meta-stable vacua for these models. Moreover, this gauge theory was generalized to $SU(2N_c + 1) \times Sp(N_c')$ model with a bifundamental and $2N_c'$ antifundamentals for $SU(2N_c + 1)$ and a fundamental for $Sp(N_c')$ and its dual description $SU(2N_c + 1) \times Sp(\widetilde{N}_c' = N_c - N_c' - 1)$ with a bifundamental and $2N_c'$ antifundamentals for $SU(2N_c + 1)$ and a fundamental for $Sp(N_c')$ as well as two gauge singlets [28]. For the particular range of N_c, the dual theory is IR free, not asymptotically free.

According to [31], $SU(2N_c)$ with antisymmetric tensor and antifundamentals can be described by two gauge groups $Sp(2N_c - 4) \times SU(2N_c)$ with bifundamental and antifundamentals for $SU(2N_c)$. Some of the brane realization with zero superpotential was given in the brane box model in [29]. Similarly from the result of [32] by following the method of [31], the dual

description for $SU(2N_c + 1)$ with antisymmetric tensor and fundamentals can be represented by two gauge group factors. This dual theory breaks the supersymmetry at the tree level. Similar discussions are present in [33]. Then it would be interesting to construct the corresponding Seigerg dual and see how the electric theory and its magnetic theory can be mapped into each other in the brane box model.

Ther are also different directions concerning on the meta-stable vacua in different contexts and some of the relevant works are present in [34]-[43] where some of them use anti D-branes and some of them describe the type IIB theory and it would be interesting to find out how similarities if any appear and what are the differences in what sense between the present work and those works.

References

[1] K. Intriligator, N. Seiberg and D. Shih, "Dynamical SUSY breaking in meta-stable vacua," JHEP **0604**, 021 (2006) [arXiv:hep-th/0602239].

[2] K. Intriligator and N. Seiberg, "Lectures on supersymmetry breaking," [arXiv:hep-ph/0702069].

[3] A. Giveon and D. Kutasov, "Brane dynamics and gauge theory," Rev. Mod. Phys. **71**, 983 (1999) [arXiv:hep-th/9802067].

[4] K. A. Intriligator, R. G. Leigh and M. J. Strassler, "New examples of duality in chiral and nonchiral supersymmetric gauge theories," Nucl. Phys. B **456**, 567 (1995) [arXiv:hep-th/9506148].

[5] J. H. Brodie and A. Hanany, "Type IIA superstrings, chiral symmetry, and N = 1 4D gauge theory dualities," Nucl. Phys. B **506**, 157 (1997) [arXiv:hep-th/9704043].

[6] E. Barnes, K. Intriligator, B. Wecht and J. Wright, "N = 1 RG flows, product groups, and a-maximization," Nucl. Phys. B **716**, 33 (2005) [arXiv:hep-th/0502049].

[7] C. Ahn, "Meta-Stable Brane Configuration and Gauged Flavor Symmetry," [arXiv:hep-th/0703015].

[8] C. Ahn, "More on meta-stable brane configuration," [arXiv:hep-th/0702038].

[9] C. Ahn, "Meta-stable brane configuration with orientifold 6 plane," [arXiv:hep-th/0701145], to appear in JHEP.

[10] C. Ahn, "M-theory lift of meta-stable brane configuration in symplectic and orthogonal gauge groups," Phys. Lett. B **647**, 493 (2007) [arXiv:hep-th/0610025].

[11] C. Ahn, "Brane configurations for nonsupersymmetric meta-stable vacua in SQCD with adjoint matter," Class. Quant. Grav. **24**, 1359 (2007) [arXiv:hep-th/0608160].

[12] H. Ooguri and Y. Ookouchi, "Meta-stable supersymmetry breaking vacua on intersecting branes," Phys. Lett. B **641**, 323 (2006) [arXiv:hep-th/0607183].

[13] S. Franco, I. Garcia-Etxebarria and A. M. Uranga, "Non-supersymmetric meta-stable vacua from brane configurations," JHEP **0701**, 085 (2007) [arXiv:hep-th/0607218].

[14] I. Bena, E. Gorbatov, S. Hellerman, N. Seiberg and D. Shih, "A note on (meta)stable brane configurations in MQCD," JHEP **0611**, 088 (2006) [arXiv:hep-th/0608157].

[15] E. Lopez and B. Ormsby, "Duality for SU x SO and SU x Sp via branes," JHEP **9811**, 020 (1998) [arXiv:hep-th/9808125].

[16] A. Hanany and E. Witten, "Type IIB superstrings, BPS monopoles, and three-dimensional gauge dynamics," Nucl. Phys. B **492**, 152 (1997) [arXiv:hep-th/9611230].

[17] D. Shih, "Spontaneous R-Symmetry Breaking in O'Raifeartaigh Models," [arXiv:hep-th/0703196].

[18] E. Witten, "Solutions of four-dimensional field theories via M-theory," Nucl. Phys. B **500**, 3 (1997) [arXiv:hep-th/9703166].

[19] K. Landsteiner and E. Lopez, "New curves from branes," Nucl. Phys. B **516**, 273 (1998) [arXiv:hep-th/9708118].

[20] K. Landsteiner, E. Lopez and D. A. Lowe, "Supersymmetric gauge theories from branes and orientifold six-planes," JHEP **9807**, 011 (1998) [arXiv:hep-th/9805158].

[21] C. Ahn, K. Oh and R. Tatar, "Comments on SO/Sp gauge theories from brane configurations with an O(6) plane," Phys. Rev. D **59**, 046001 (1999) [arXiv:hep-th/9803197].

[22] K. Landsteiner, E. Lopez and D. A. Lowe, "Duality of chiral N = 1 supersymmetric gauge theories via branes," JHEP **9802**, 007 (1998) [arXiv:hep-th/9801002].

[23] I. Brunner, A. Hanany, A. Karch and D. Lust, "Brane dynamics and chiral non-chiral transitions," Nucl. Phys. B **528**, 197 (1998) [arXiv:hep-th/9801017].

[24] S. Elitzur, A. Giveon, D. Kutasov and D. Tsabar, "Branes, orientifolds and chiral gauge theories," Nucl. Phys. B **524**, 251 (1998) [arXiv:hep-th/9801020].

[25] C. Csaki, M. Schmaltz, W. Skiba and J. Terning, "Gauge theories with tensors from branes and orientifolds," Phys. Rev. D **57**, 7546 (1998) [arXiv:hep-th/9801207].

[26] C. Ahn, K. Oh and R. Tatar, "Branes, geometry and N = 1 duality with product gauge groups of SO and Sp," J. Geom. Phys. **31**, 301 (1999) [arXiv:hep-th/9707027].

[27] J. D. Lykken, E. Poppitz and S. P. Trivedi, "M(ore) on chiral gauge theories from D-branes," Nucl. Phys. B **520**, 51 (1998) [arXiv:hep-th/9712193].

[28] K. A. Intriligator and S. D. Thomas, "Dual descriptions of supersymmetry breaking," [arXiv:hep-th/9608046].

[29] A. Hanany and A. Zaffaroni, "On the realization of chiral four-dimensional gauge theories using branes," JHEP **9805**, 001 (1998) [arXiv:hep-th/9801134].

[30] I. Affleck, M. Dine and N. Seiberg, "Dynamical Supersymmetry Breaking In Four-Dimensions And Its Phenomenological Implications," Nucl. Phys. B **256**, 557 (1985).

[31] M. Berkooz, "The Dual of supersymmetric SU(2k) with an antisymmetric tensor and composite dualities," Nucl. Phys. B **452**, 513 (1995) [arXiv:hep-th/9505067].

[32] P. Pouliot, "Duality in SUSY $SU(N)$ with an Antisymmetric Tensor," Phys. Lett. B **367**, 151 (1996) [arXiv:hep-th/9510148].

[33] P. Pouliot and M. J. Strassler, "Duality and Dynamical Supersymmetry Breaking in $Spin(10)$ with a Spinor," Phys. Lett. B **375**, 175 (1996) [arXiv:hep-th/9602031].

[34] S. Murthy, "On supersymmetry breaking in string theory from gauge theory in a throat," [arXiv:hep-th/0703237].

[35] R. Argurio, M. Bertolini, S. Franco and S. Kachru, "Metastable vacua and D-branes at the conifold," [arXiv:hep-th/0703236].

[36] A. Giveon and D. Kutasov, "Gauge symmetry and supersymmetry breaking from intersecting branes," [arXiv:hep-th/0703135].

[37] Y. E. Antebi and T. Volansky, "Dynamical supersymmetry breaking from simple quivers," [arXiv:hep-th/0703112].

[38] M. Wijnholt, "Geometry of particle physics," [arXiv:hep-th/0703047].

[39] J. J. Heckman, J. Seo and C. Vafa, "Phase structure of a brane/anti-brane system at large N," [arXiv:hep-th/0702077].

[40] R. Tatar and B. Wetenhall, "Metastable vacua, geometrical engineering and MQCD transitions," JHEP **0702**, 020 (2007) [arXiv:hep-th/0611303].

[41] H. Verlinde, "On metastable branes and a new type of magnetic monopole," [arXiv:hep-th/0611069].

[42] M. Aganagic, C. Beem, J. Seo and C. Vafa, "Geometrically induced metastability and holography," [arXiv:hep-th/0610249].

[43] R. Argurio, M. Bertolini, S. Franco and S. Kachru, "Gauge / gravity duality and meta-stable dynamical supersymmetry breaking," JHEP **0701**, 083 (2007) [arXiv:hep-th/0610212].