Quantum mechanical approach to decoherence and relaxation generated by fluctuating environment

S.A. Gurvitz*
Department of Particle Physics, Weizmann Institute of Science, Rehovot 76100, Israel and Theoretical Division and CNLS, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

D. Mozyrsky
Theoretical Division and CNLS, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

(Dated: March 22, 2014)

We consider an electrostatic qubit, interacting with a fluctuating charge of single electron transistor (SET) in the framework of exactly solvable model. The SET plays a role of the fluctuating environment affecting the qubit's parameters in a controllable way. We derive the rate equations describing dynamics of the entire system for both weak and strong qubit-SET coupling. Solving these equation we obtain decoherence and relaxation rates of the qubit, as well as the spectral density of the fluctuating qubit's parameters. We found that in the weak coupling regime the decoherence and relaxation rates are directly related to the spectral density taken at Rabi or at zero frequency, depending on what a particular qubit's parameters is fluctuating. This relation holds also in the presence of weak back-action of the qubit on the fluctuating environment. In the case of strong back-action, such simple relationship no longer holds, even if the qubit-SET coupling is small. It does not hold either in the strong-coupling regime, even in the absence of the back-action. In addition, we found that our model predicts localization of the qubit in the strong-coupling regime, resembling that of the spin-boson model.

PACS numbers: 03.65.Yz, 05.60.Gg, 73.23.-b, 73.23.Hk

I. INTRODUCTION

The influence of environment on a single quantum system is the issue of crucial importance in quantum information science. It is mainly associated with decoherence, or dephasing, which transforms any pure state of a quantum system into a statistical mixture. Despite a large body of theoretical work devoted to decoherence, its mechanism has not been clarified enough. For instance, how decoherence is related to environmental noise, in particular in the presence of back-action of the system on the environment (quantum measurements). Moreover, decoherence is often intermixed with relaxation. Although each of them represents an irreversible process, decoherence and relaxation affect quantum systems in quite different ways.

In this paper we concentrate on an electrostatic qubit, which is considered as a generic example of two-state systems. It is realized by an electron trapped in coupled quantum dots, Fig. 1. It is reasonable to assume that the decoherence of a qubit is associated with fluctuations of the qubit parameters, $E_{1,2}$ and Ω_0, generated by the environment. Indeed, it has been demonstrated explicitly by applying a phenomenological stochastic averaging of the Schrödinger equation for the qubit (see for instance[1,2]), that the qubit state turns into a statistical mixture. It has also been shown[3] that fluctuations of the energy level of a single quantum dot generated by an external current produce dephasing of the corresponding state of the dot. The related decoherence rate was evaluated in the weak-coupling limit and found to be proportional to the spectral density of level fluctuations at zero frequency[3].

In general one can expect that the fluctuating environment should result in the relaxation of the qubit as well. The question is how the corresponding relaxation and decoherence rates are related to the environment's fluctuation spectrum, in particular when the latter is frequency dependent. Also, it is important to go beyond the weak-coupling limit usually assumed in many publications. To investigate all these problems it is very desirable to use a model that describes the effects of decoherence and relaxation in a consistent quantum mechanical way. An obvious candidate is the spin-boson model[4]. It is not easy, however, to cast it in terms of a fluctuating field affecting the qubit parameters. Also it is hard to manipulate the fluctuation spectrum in the framework of this model.

In this paper we present a different quantum mechanical model, which contains the generic features of a fluctuating field acting on a qubit. This field is generated by an electric current flowing through a quantum dot – a single electron transistor (SET). The discreteness of the electron charge creates fluctuations in the electrostatic field near the SET. If the electrostatic qubit is placed near the SET, this fluctuating field should affect the qubit behavior as shown in Fig. 2. It can produce fluctuations of the tunneling coupling between the dots (off-diagonal coupling) by narrowing the electrostatic opening connecting these dots, as in Fig. 2a, or make the energy levels of the dots fluctuate, as shown schematically in Fig. 2b. In

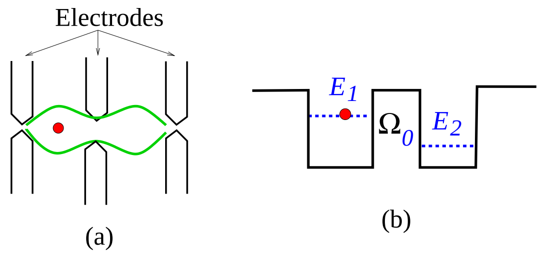

FIG. 1: Electrostatic qubit, realized by an electron trapped in a coupled-dot system (a), and its schematic representation by a double-well (b). Ω_0 denotes the coupling between the two dots.

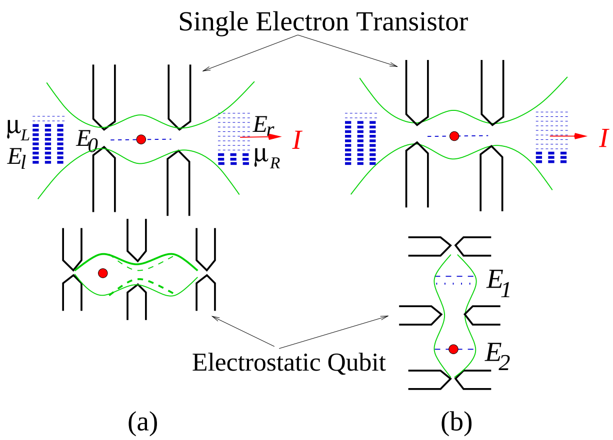

FIG. 2: Qubit near Single Electron Transistor. Here $E_{l,r}$ and E_0 denote the energy levels in the left (right) reservoirs and in the quantum dot, respectively, and $\mu_{L,R}$ are the corresponding chemical potentials. The electric current I generates fluctuations of the electrostatic opening between two dots (a), or it fluctuates the energy level of the nearest dot (b).

fact, a similar setup have been considered in many publications, where SET has been used as a qubit readout device, for instance in[5,6]. Here, however, we consider the SET as a source of noise only. Moreover, if the energy level E_0, Fig. 2, is deeply inside the voltage bias – the case we consider in the beginning, the SET would not operate as a readout device of the qubit state. Indeed, the SET current is not modulated by the qubit electron. The SET in this case represents only the fluctuating environment affecting the qubit behavior ("pure environment"[7]).

The main advantage of our model is that it can be solved exactly in the limit of high bias voltage applied on the SET, for both weak and strong coupling with the qubit. Then decoherence and relaxation rates can be extracted from the exact solution, as well as the time-correlator of the electric charge inside the SET. This will allow us to establish a relation, if any, between the frequency-dependent fluctuation spectrum of the environment and the decoherence and relaxation rates of the qubit.

By moving the energy level E_0 of the SET, which carries the current, close to the chemical potential μ_L one can study the case when the SET current is modulated by the qubit electron. The same analysis of a possible relation between the environmental noise and decoherence and relaxation of the qubit can be performed in this case too.

The plan of this paper is as follows: Sect. II presents a phenomenological description of decoherence and relaxation in the framework of Bloch equations, applied to the electrostatic qubit. Sect. III contains description of the model and the quantum rate-equation formalism, used for its solution. Detailed quantum-mechanical derivation of these equations for a specific example is presented in Appendix A. Sect. IV deals with a configuration where the SET can generate only decoherence of the qubit. We consider separately the situations when SET produces fluctuations of the tunneling coupling (Rabi frequency) or of the energy levels. The results are compared with the SET fluctuation spectrum, evaluated in Appendix B. Sect. V deals with a configuration where the SET generates both decoherence and relaxation of the qubit. Sect. VI is a summary.

II. DECOHERENCE AND RELAXATION OF A QUBIT

In this section we describe in a general phenomenological way the effect of decoherence and relaxation on the qubit behavior. Although the results are known, there still exists some confusion in the literature in this issue. We therefore need to define precisely these quantities and demonstrate how the corresponding decoherenece and relaxation rates can be extracted from the qubit density matrix.

Let us consider an electrostatic qubit, realized by an electron trapped in coupled quantum dots, Fig. 1. This system is described by the following tunneling Hamiltonian

$$H_q = E_1 a_1^\dagger a_1 + E_2 a_2^\dagger a_2 - \Omega_0(a_2^\dagger a_1 + a_1^\dagger a_2) \qquad (1)$$

where $a_{1,2}^\dagger, a_{1,2}$ are the creation and annihilation operators of the electron in the first or in the second dot. The electron wave function can be written as

$$|\Psi(t)\rangle = \left[b^{(1)}(t)a_1^\dagger + b^{(2)}(t)a_2^\dagger\right]|0\rangle \qquad (2)$$

where $b^{(1,2)}(t)$ are the probability amplitudes for finding the electron in the first or second well, obtained from the Schrödinger equation $i\partial_t|\Psi(t)\rangle = H_q|\Psi(t)\rangle$ (we adopt the units where $\hbar = 1$ and the electron charge $e = 1$). The corresponding density matrix, $\sigma_{jj'}(t) = b^{(j)}(t)b^{(j')*}(t)$, with $j, j' = \{1, 2\}$, is obtained from the equation $i\partial_t\,\sigma = [H, \sigma]$. This can be written explicitly as

$$\dot\sigma_{11} = i\Omega_0(\sigma_{21} - \sigma_{12}) \qquad (3a)$$

$$\dot\sigma_{12} = -i\epsilon\sigma_{12} + i\Omega_0(1 - 2\sigma_{11})\,, \qquad (3b)$$

where $\sigma_{22}(t) = 1 - \sigma_{11}(t)$, $\sigma_{21}(t) = \sigma_{12}^*(t)$ and $\epsilon = E_1 - E_2$. Solving these equations one easily finds that the electron oscillates between the two dots (Rabi oscillations) with frequency $\omega_R = \sqrt{4\Omega_0^2 + \epsilon^2}$. For instance, for the initial conditions $\sigma_{11}(0) = 1$ and $\sigma_{12}(0) = 1$, the probability of finding the electron in the second dot is $\sigma_{22}(t) = 2(\Omega_0/\omega_R)^2(1 - \cos\omega_R t)$. This result shows that for $\epsilon \gg \Omega_0$ the amplitude of the Rabi oscillations is small, so the electron remains localized in its initial state.

The situation is different when the qubit interacts with the environment. In this case the (reduced) density matrix of the qubit $\sigma(t)$ is obtained by tracing out the environment variables from the total density matrix. The question is how to modify Eqs. (3), written for an isolated qubit, in order to obtain the reduced density matrix $\sigma(t)$ in the environment. In general one expects that the environment could affect the qubit in two different ways. First, it can destroy the off-diagonal elements of the qubit density matrix. This process is usually referred to as decoherence (or dephasing). It can be accounted for in a phenomenological way by introducing an additional (damping) term in Eq. (3b),

$$\dot{\sigma}_{12} = -i\epsilon\sigma_{12} + i\Omega_0(1 - 2\sigma_{11}) - \frac{\Gamma_d}{2}\sigma_{12} \qquad (4)$$

where Γ_d is the decoherence rate. As a result the qubit density-matrix $\sigma(t)$ always becomes a statistical mixture in the stationary limit,

$$\sigma(t) \overset{t\to\infty}{\longrightarrow} \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}. \qquad (5)$$

This happens for any initial conditions and even for large level displacement, $\epsilon \gg \Omega_0, \Gamma_d$.

Second, the environment can put the qubit in its ground state, for instance via photon or phonon emission. This process is usually referred to as relaxation. For a symmetric qubit we would have

$$\sigma(t) \overset{t\to\infty}{\longrightarrow} \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}. \qquad (6)$$

In contrast with decoherence, Eq. (5), the relaxation process puts the qubit into a pure state. This is in fact the essential difference between decoherence and relaxation.

It is often claimed that decoherence is associated with an absence of energy transfer between the system and the environment, in contrast with relaxation. This distinction is not generally valid. For instance, if the initial qubit state corresponds to the electron in the state $|E_2\rangle$, Fig. 1, the final state after decoherence corresponds to an equal distribution between the two dots, $\langle E\rangle = (E_1 + E_2)/2$. In the case of $E_1 \gg E_2$, this process would require a large energy transfer between the qubit and the environment. In addition we note that relaxation would eliminate the off-diagonal density matrix element, but only in the basis of the qubit eigenstates. In contrast, decoherence would eliminate the off-diagonal density matrix element in any basis.

The relaxation process can be described most simply by diagonalizing the qubit Hamiltonian, Eqs. (1), to obtain $H_q = E_+ a_+^\dagger a_+ + E_- a_-^\dagger a_-$, where the operators $a_\pm$ are obtained by the corresponding rotation of the operators $a_{1,2}$[7]. Here E_+ and E_- are the ground (symmetric) and excited (antisymmetric) state energies. Then the relaxation process can be described phenomenologically in the new qubit basis $|\pm\rangle = a_\pm^\dagger|0\rangle$ as

$$\dot{\sigma}_{--}(t) = -\Gamma_r\sigma_{--}(t) \qquad (7a)$$

$$\dot{\sigma}_{+-}(t) = i(E_- - E_+)\sigma_{+-}(t) - \frac{\Gamma_r}{2}\sigma_{+-}(t)\,, \qquad (7b)$$

where $\sigma_{++}(t) = 1 - \sigma_{--}(t)$, $\sigma_{-+}(t) = \sigma_{+-}^*(t)$ and Γ_r is the relaxation rate.

In order to add decoherence, we return to the original qubit basis $|1,2\rangle = a_{1,2}^\dagger|0\rangle$ and add the damping term to the equation for the off-diagonal matrix elements, Eq. (4). We arrive at the quantum rate equation describing the qubit's behavior in the presence of both decoherence and relaxation[7,8],

$$\dot{\sigma}_{11} = i\Omega_0(\sigma_{21} - \sigma_{12}) - \Gamma_r\frac{\kappa\epsilon}{2\tilde{\epsilon}}(\sigma_{12} + \sigma_{21}) - \frac{\Gamma_r}{4}\left[1 + \left(\frac{\epsilon}{\tilde{\epsilon}}\right)^2\right](2\sigma_{11} - 1) + \Gamma_r\frac{\epsilon}{2\tilde{\epsilon}} \qquad (8a)$$

$$\dot{\sigma}_{12} = -i\epsilon\sigma_{12} + \left[i\Omega_0 + \Gamma_r\frac{\kappa\epsilon}{2\tilde{\epsilon}}\right](1 - 2\sigma_{11}) + \Gamma_r\left[\kappa - \frac{1}{2}\sigma_{12} - \kappa^2(\sigma_{12} + \sigma_{21})\right] - \frac{\Gamma_d}{2}\sigma_{12}\,, \qquad (8b)$$

where $\tilde{\epsilon} = (\epsilon^2 + 4\Omega_0^2)^{1/2}$ and $\kappa = \Omega_0/\tilde{\epsilon}$. In fact, these equations can be derived in the framework of a particular model, representing an electrostatic qubit interacting with the point-contact detector and the environment, described by the Lee model Hamiltonian[8].

Equations (8) can be rewritten in a simpler form by mapping the qubit density matrix $\sigma = \{\sigma_{11}, \sigma_{12}, \sigma_{21}\}$ to a "polarization" vector $\boldsymbol{S}(t)$ via $\sigma(t) = [1 + \boldsymbol{\tau}\cdot\boldsymbol{S}(t)]/2$, where $\tau_{x,y,z}$ are the Pauli matrices. One obtains for the symmetric case, $\epsilon = 0$,

$$\dot{S}_z = -\frac{\Gamma_r}{2} S_z - 2\Omega_0\, S_y \tag{9a}$$

$$\dot{S}_y = 2\Omega_0\, S_z - \frac{\Gamma_d + \Gamma_r}{2} S_y \tag{9b}$$

$$\dot{S}_x = -\frac{\Gamma_d + 2\Gamma_r}{2} (S_x - \bar{S}_x) \tag{9c}$$

where $\bar{S}_x = S_x(t \to \infty) = 2\Gamma_r/(\Gamma_d + 2\Gamma_r)$. Equations (9) have the form of the phenomenological Bloch equations for nuclear magnetic resonance with longitudinal and transfer relaxation times related to $\Gamma_{d,r}$ by $T_1^{-1} = (\Gamma_d + 2\Gamma_r)/2$ and $T_2^{-1} = (\Gamma_d + \Gamma_r)/2$.

Let us evaluate the probability of finding the electron in the first dot, $\sigma_{11}(t)$. Solving Eqs. (9) for the initial conditions $\sigma_{11}(0) = 1$, $\sigma_{12}(0) = 0$, we find[8]:

$$\sigma_{11}(t) = \frac{1}{2} + \frac{e^{-\Gamma_r t/2}}{4} \left(C_1 e^{-e_- t} + C_2 e^{-e_+ t} \right) \tag{10}$$

where $e_\pm = \frac{1}{4}(\Gamma_d \pm \tilde{\Omega})$, $\tilde{\Omega} = \sqrt{\Gamma_d^2 - 64\Omega_0^2}$ and $C_{1,2} = 1 \pm (\Gamma_d/\tilde{\Omega})$. Solving the same equations in the limit of $t \to \infty$, we find that the steady-state qubit density matrix is

$$\sigma(t) \stackrel{t\to\infty}{\longrightarrow} \begin{pmatrix} 1/2 & \Gamma_r/(\Gamma_d + 2\Gamma_r) \\ \Gamma_r/(\Gamma_d + 2\Gamma_r) & 1/2 \end{pmatrix} . \tag{11}$$

Thus the off-diagonal elements of the density matrix can provide us with a ratio of relaxation to decoherence rates[8].

III. DESCRIPTION OF THE MODEL

Consider the setup shown in Fig. 2. The entire system can be described by the following tunneling Hamiltonian, represented by a sum of the qubit and SET Hamiltonians and the interaction term, $H = H_q + H_{set} + H_{int}$. Here H_q is given by Eq. (1) and describes the qubit. The second term, H_{set}, describes the single-electron transistor. It can be written as

$$H_{set} = \sum_l E_l c_l^\dagger c_l + \sum_r E_r c_r^\dagger c_r + E_0 c_0^\dagger c_0 + \sum_{l,r} (\Omega_l c_l^\dagger c_0 + \Omega_r c_r^\dagger c_0 + H.c.)\,, \tag{12}$$

where $c_{l,r}^\dagger$ and $c_{l,r}$ are the creation and annihilation electron operators in the state $E_{l,r}$ of the right or left reservoir; $c_0^\dagger$ and c_0 are those for the level E_0 inside the quantum dot; and $\Omega_{l,r}$ are the couplings between the level E_0 and the level $E_{l,r}$ in the left (right) reservoir. For simplicity we assume that the quantum dot of the SET contains only one level (E_0). We also assumed a weak energy dependence of the couplings $\Omega_{l,r} \simeq \Omega_{L,R}$.

The interaction between the qubit and the SET, H_{int}, depends on a position of the SET with respect to the qubit. If the SET is placed near the middle of the qubit, Fig. 2a, then the tunneling coupling between two dots of the qubit in Eq. (1) decreases, $\Omega_0 \to \Omega_0 - \Delta\Omega_0$, whenever the quantum dot of the SET is occupied by an electron. This is due to the electron's repulsive field. In this case the interaction term can be written as

$$H_{int} = \Delta\Omega\, c_0^\dagger c_0 (a_1^\dagger a_2 + a_2^\dagger a_1)\,. \tag{13}$$

On the other hand, in the configuration shown in Fig. 2b where the SET is placed near one of the dots of the qubit, the electron repulsive field displaces the qubit energy levels by $\Delta E = U$. The interaction terms in this case can be written as

$$H_{int} = U\, a_1^\dagger a_1 c_0^\dagger c_0\,. \tag{14}$$

Consider the initial state where all the levels in the left and the right reservoirs are filled with electrons up to the Fermi levels $\mu_{L,R}$ respectively. This state will be called the "vacuum" state $|0\rangle$. The wave function for the entire system can be written as

$$|\Psi(t)\rangle = \Bigg[b_0^{(1)}(t) a_1^\dagger + \sum_l b_{1l}^{(1)}(t) a_1^\dagger c_1^\dagger c_l + \sum_{l,r} b_{rl}^{(1)}(t) a_1^\dagger c_r^\dagger c_l + \sum_{l<l',r} b_{1rll'}^{(1)}(t) a_1^\dagger c_1^\dagger c_r^\dagger c_l c_{l'} + \cdots$$
$$+ b_0^{(2)}(t) a_2^\dagger + \sum_l b_{1l}^{(2)}(t) a_2^\dagger c_1^\dagger c_l + \sum_{l,r} b_{rl}^{(2)}(t) a_2^\dagger c_r^\dagger c_l + \sum_{l<l',r} b_{1rll'}^{(2)}(t) a_2^\dagger c_1^\dagger c_r^\dagger c_l c_{l'} + \ldots \Bigg] |0\rangle, \tag{15}$$

where $b^{(j)}_\alpha(t)$ are the probability amplitudes to find the entire system in the state described by the corresponding creation and annihilation operators. These amplitudes are obtained from the Schrödinger equation $i|\dot\Psi(t)\rangle = H|\Psi(t)\rangle$, supplemented with the initial condition $b_0^{(1)}(0)=p_1$, $b_0^{(2)}(0)=p_2$, and $b^{(j)}_{\alpha\neq 0}(0)=0$, where $p_{1,2}$ are the amplitudes of the initial qubit state.

The behavior of the qubit and the SET is given by the reduced density matrix, $\sigma_{ss'}(t)$. It is obtained from the entire system's density matrix $|\Psi(t)\rangle\langle\Psi(t)|$ by tracing out the (continuum) reservoir states. The space of such a reduced density matrix consists of four discrete states $s,s'=a,b,c,d$, shown schematically in Fig. 3 for the setup of Fig. 2a. The corresponding density-matrix elements are directly related to the amplitudes $b(t)$, for instance,

$$\begin{aligned}
\sigma_{aa}(t) &= |b_0^{(1)}(t)|^2+\sum_{l,r}|b_{lr}^{(1)}(t)|^2+\sum_{l<l',r<r'}|b^{(1)}_{rr'll'}(t)|^2+\cdots && \text{(16a)}\\
\sigma_{dd}(t) &= \sum_{l}|b_{1l}^{(2)}(t)|^2+\sum_{l<l',r}|b_{1rll'}^{(2)}(t)|^2+\sum_{l<l'<l'',r<r'}|b^{(2)}_{1rr'll'}(t)|^2+\cdots && \text{(16b)}\\
\sigma_{bd}(t) &= \sum_{l}b_{1l}^{(1)}(t)b_{1l}^{(2)*}(t)+\sum_{l<l',r}b_{1rll'}^{(1)}(t)b_{1rll'}^{(2)*}(t)+\sum_{l<l'<l'',r<r'}b^{(1)}_{1rr'll'}(t)b^{(2)*}_{1rr'll'}(t)+\cdots\,. && \text{(16c)}
\end{aligned}$$

In was shown in[9,10] that the trace over the reservoir states in the system's density matrix can be performed in the strong non-equilibrium limit, $\mu_L-\mu_R\to\infty$, assuming only weak energy dependence of the transition amplitudes $\Omega_{l,r}\equiv\Omega_{L,R}$ and the density of the reservoir states, $\rho(E_{l,r})=\rho_{L,R}$. As a result we arrive at Bloch-type rate equations for the reduced density matrix without any additional assumptions. The general form of these equations is[10,11]

$$\begin{aligned}
\dot\sigma_{jj'} &= i(E_{j'}-E_j)\sigma_{jj'}+i\left(\sum_k\sigma_{jk}\tilde\Omega_{k\to j'}-\sum_k\tilde\Omega_{j\to k}\sigma_{kj'}\right)\\
&-\sum_{k,k'}\mathcal{P}_2\pi\rho(\sigma_{jk}\Omega_{k\to k'}\Omega_{k'\to j'}+\sigma_{kj'}\Omega_{k\to k'}\Omega_{k'\to j})+\sum_{k,k'}\mathcal{P}_2\pi\rho\,(\Omega_{k\to j}\Omega_{k'\to j'}+\Omega_{k\to j'}\Omega_{k'\to j})\sigma_{kk'} \qquad (17)
\end{aligned}$$

Here $\Omega_{k\to k'}$ denotes the single-electron hopping amplitude that generates the $k\to k'$ transition. We distinguish between the amplitudes $\tilde\Omega$ describing single-electron hopping between isolated states and Ω describing transitions between isolated and continuum states. The latter can generate transitions between the isolated states of the system, but only indirectly, via two consecutive jumps of an electron, into and out of the *continuum* reservoir states (with the density of states ρ). These transitions are represented by the third and the fourth terms of Eq. (17). The third term describes the transitions $(k\to k'\to j)$ or $(k\to k'\to j')$, which cannot change the number of electrons in the collector. The fourth term describes the transitions ($k\to j$ and $k'\to j'$) or ($k\to j'$ and $k'\to j$) which increase the number of electrons in the collector by one. These two terms of Eq. (17) are analogues of the "loss" (negative) and the "gain" (positive) terms in the classical rate equations, respectively. The factor $\mathcal{P}_2=\pm1$ in front of these terms is due anti-commutation of the fermions, so that $\mathcal{P}_2=-1$ whenever the loss or the gain terms in Eq. (17) proceed through a two-fermion state of the dot. Otherwise $\mathcal{P}_2=1$.

Note that the reduction of the time-dependent Schrödinger equation, $i|\dot\Psi(t)\rangle=H|\Psi(t)\rangle$, to Eqs. (17) is performed in the limit of large bias without explicit use of any Markov-type or weak coupling approximations. A detailed example of this derivation is presented in Appendix A for the case of resonant tunneling through a single level.

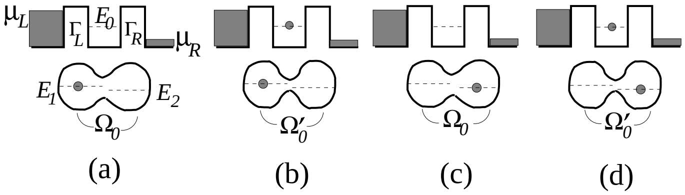

FIG. 3: The available discrete states of the entire system corresponding to the setup of Fig. 2a. $\Gamma_{L,R}$ denote the tunneling rates to the corresponding reservoirs and $\Omega_0' = \Omega_0 - \Delta\Omega$.

IV. NO BACK-ACTION ON THE ENVIRONMENT

A. Fluctuation of the Rabi frequency

Now we apply Eqs. (17) to investigate the qubit's behavior in the configurations shown in Fig. 2. First we consider the SET placed near the middle of the qubit, Figs. 2a,3. In this case the electron current through the SET will influence the coupling between two dots of the qubit, making it fluctuate between the values Ω_0 and $\Omega_0' = \Omega_0 - \delta\Omega$. The corresponding rate equations can be written straightforwardly from Eqs. (17). One finds,

$$\begin{aligned}
\dot{\sigma}_{aa} &= -\Gamma_L\sigma_{aa} + \Gamma_R\sigma_{bb} - i\Omega_0(\sigma_{ac} - \sigma_{ca}), && \text{(18a)}\\
\dot{\sigma}_{bb} &= -\Gamma_R\sigma_{bb} + \Gamma_L\sigma_{aa} - i\Omega_0'(\sigma_{bd} - \sigma_{db}), && \text{(18b)}\\
\dot{\sigma}_{cc} &= -\Gamma_L\sigma_{cc} + \Gamma_R\sigma_{dd} - i\Omega_0(\sigma_{ca} - \sigma_{ac}), && \text{(18c)}\\
\dot{\sigma}_{dd} &= -\Gamma_R\sigma_{dd} + \Gamma_L\sigma_{cc} - i\Omega_0'(\sigma_{db} - \sigma_{bd}), && \text{(18d)}\\
\dot{\sigma}_{ac} &= -i\epsilon_0\sigma_{ac} - i\Omega_0(\sigma_{aa} - \sigma_{cc}) - \Gamma_L\sigma_{ac} \\
&\qquad + \Gamma_R\sigma_{bd}, && \text{(18e)}\\
\dot{\sigma}_{bd} &= -i\epsilon_0\sigma_{bd} - i\Omega_0'(\sigma_{bb} - \sigma_{dd}) - \Gamma_R\sigma_{bd} \\
&\qquad + \Gamma_L\sigma_{ac}, && \text{(18f)}
\end{aligned}$$

where $\Gamma_{L,R} = 2\pi|\Omega_{L,R}|^2\rho_{L,R}$ are the tunneling rates from the reservoirs and $\epsilon_0 = E_1 - E_2$.

Equations (18) determine the time evolution of the qubit, as well as the SET. Indeed, the qubit (reduced) density matrix is given by:

$$\begin{aligned}
\sigma_{11}(t) &= \sigma_{aa}(t) + \sigma_{bb}(t)\,, && \text{(19a)}\\
\sigma_{12}(t) &= \sigma_{ac}(t) + \sigma_{bd}(t)\,, && \text{(19b)}
\end{aligned}$$

and $\sigma_{22}(t) = 1 - \sigma_{11}(t)$. The probability of finding the SET occupied is

$$P_1(t) \;=\; \sigma_{bb}(t) + \sigma_{dd}(t)\,. \tag{20}$$

It is given by the equation

$$\dot{P}_1(t) = \Gamma_L - \Gamma P_1(t)\,, \tag{21}$$

obtained straightforwardly from Eqs. (18). Here $\Gamma = \Gamma_L + \Gamma_R$ is the total width. The same equation for $P_1(t)$ can be obtained if the qubit is decoupled from the SET ($\delta\Omega = 0$). Thus there is no back-action of the qubit on the charge fluctuations inside the SET in the limit of large bias voltage.

Consider first the stationary limit, $t \to \infty$, where $\dot{P}_1(t) \to 0$ and $\dot{\sigma}(t) \to 0$. It follows from Eq. (21) that the probability of finding the SET occupied in this limit is $\bar{P}_1 = \Gamma_L/\Gamma$. This implies that the fluctuations of the coupling between the two qubit states, induced by the SET, would take place around the average value $\Omega = \Omega_0 - \bar{P}_1\,\delta\Omega$.

With respect to the qubit in the stationary limit, one easily obtains from Eqs. (18) that the qubit density matrix always becomes the statistical mixture (5), when $t \to \infty$. This takes place for any initial conditions and any values of the qubit and the SET parameters. Therefore the effect of the fluctuating charge inside the SET does not lead to relaxation of the qubit, but rather to its decoherence.

In order to determine the decoherence rate analytically, we perform a Laplace transform on the density matrix, $\tilde{\sigma}(E) = \int_0^\infty \sigma(t)\exp(-iEt)dE$. Then solving Eq. (18) we can determine the decoherence rate from the locations of the poles of $\tilde{\sigma}(E)$ in the complex E-plane. Consider for instance the case of $\epsilon_0 = 0$ and the symmetric SET, $\Gamma_L = \Gamma_R = \Gamma/2$. One finds from Eqs. (18) and (19a) that

$$\tilde{\sigma}_{11}(E) = \frac{i}{2E} + \frac{i}{4E - \dfrac{(2\,\delta\Omega)^2}{E + i\Gamma - 2\,\Omega} - 8\Omega} + \frac{i}{4E - \dfrac{(2\,\delta\Omega)^2}{E + i\Gamma + 2\,\Omega} + 8\Omega}\,. \tag{22}$$

Upon performing the inverse Laplace transform,

$$\sigma_{11}(t) = \int\limits_{-\infty+i0}^{\infty+i0} \tilde{\sigma}_{11}(E)\, e^{-iEt}\,\frac{dE}{2\pi i}, \tag{23}$$

and closing the integration contour around the poles of the integrand, we obtain for $\Gamma > 2\delta\Omega$ and $t \gg 1/\Gamma$

$$\sigma_{11}(t) - (1/2) \propto e^{-(\Gamma - \sqrt{\Gamma^2 - 4\delta\Omega^2})t/2} \sin(2\Omega\, t). \qquad (24)$$

Comparing this result with Eq. (10) we find that the decoherence rate is

$$\Gamma_d = 2\left(\Gamma - \sqrt{\Gamma^2 - 4\delta\Omega^2}\right) \xrightarrow{\Gamma \gg \delta\Omega} 4\delta\Omega^2/\Gamma\,. \qquad (25)$$

In a general case, $\Gamma_L \neq \Gamma_R$, we obtain in the same limit ($\Gamma_{L,R} \gg \delta\Omega$) that the decoherence rate for $\epsilon_0 \ll \Omega$ is given by:

$$\Gamma_d = \frac{16\,\delta\Omega^2}{1 + \left(\frac{\epsilon_0}{2\Omega}\right)^2} \frac{\Gamma_L \Gamma_d}{(\Gamma_L + \Gamma_d)^3} \qquad (26)$$

It is interesting to compare this result with the fluctuation spectrum of the charge inside the SET, Eq. (B8), Appendix B. We find

$$\Gamma_d = 2\,(\delta\omega_R)^2\, S_Q(0)\,, \qquad (27)$$

where $\omega_R = \sqrt{4\Omega^2 + \epsilon_0^2}$ is the Rabi frequency. The latter represents the energy splitting in the diagonalized qubit Hamiltonian. Thus $\delta\omega_R$ corresponds to the amplitude of energy level fluctuations in a single dot. The same expression for the decoherence rate of a single dot state, generated by a fluctuating energy level, has been obtained by Levinson[3] in a weak coupling limit. Note that in both cases the decoherence rate is determined by the fluctuation spectrum at zero frequency.

Although Eq. (27) has been obtained for small fluctuations $\delta\omega_R$, it might be approximately correct even if $\delta\omega_R$ is of the order of Γ. It is demonstrated in Fig. 4, where we compare $\sigma_{11}(t)$ and $\sigma_{12}(t)$, obtained from Eqs. (18) and (19) (solid line) with those from Eqs. (3a) and (4) (dashed line) for the decoherence rate Γ_d given by Eq. (27). The initial conditions correspond to $\sigma_{11}(0) = 1$ and $\sigma_{12}(0) = 0$ (respectively, $\sigma_{aa}(0) = \Gamma_R/\Gamma$ and $\sigma_{bb}(0) = \Gamma_R/\Gamma$).

In the large coupling regime ($\delta\Omega \gg \Gamma$), however, the phenomenological Bloch equations, Eqs. (3a) and (4), cannot be used. Consider for simplicity the case of $\epsilon = 0$ and $\Gamma_{L,R} = \Gamma/2$. Then one finds from Eq. (22) that the damping oscillations between the two dots take place at two different frequencies, $2\Omega \pm \sqrt{(\delta\Omega)^2 - (\Gamma/2)^2}$, instead of the one frequency, $\omega_R = 2\Omega$, given the Bloch equations. Moreover, Eq. (27) does not reproduce the decoherence (damping) rate in this limit. Indeed, one obtains from Eq. (23) that the decoherence rate $\Gamma_d = 2\Gamma$ for $\delta\Omega > \Gamma/2$, so Γ_d does not depend on the coupling ($\delta\Omega$) at all.

B. Fluctuation of the energy level

Consider the SET placed near one of the qubit dots, as shown in Fig. 2b. In this case the qubit-SET interaction term is given by Eq. (14). As a result the energy level E_1 will fluctuate under the influence of the fluctuations of the electron charge inside the SET. The available discrete states of the entire system are shown in Fig. 5. Using Eqs. (17) we can write the rate equations, similar to Eqs. (18),

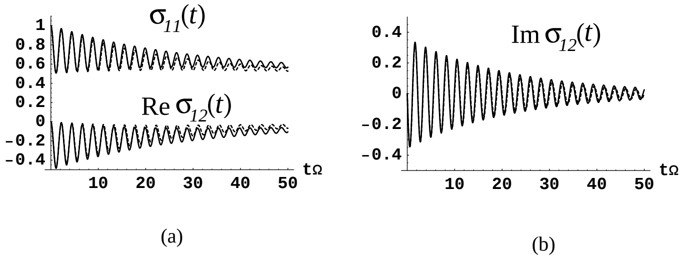

FIG. 4: The probability of finding the electron in the first dot if the qubit for $\epsilon = 2\Omega$, $\Gamma_L = \Omega$, $\Gamma_R = 2\Omega$ and $\delta\Omega = 0.5\Omega$. The solid line is the exact result, whereas the dashed line is obtained from the Bloch-type rate equations with the decoherence rate given by Eq. (27).

$$\begin{aligned}
\dot\sigma_{aa} &= -\Gamma'_L\sigma_{aa} + \Gamma'_R\sigma_{bb} - i\Omega_0(\sigma_{ac} - \sigma_{ca}), && (28a)\\
\dot\sigma_{bb} &= -\Gamma'_R\sigma_{bb} + \Gamma'_L\sigma_{aa} - i\Omega_0(\sigma_{bd} - \sigma_{db}), && (28b)\\
\dot\sigma_{cc} &= -\Gamma_L\sigma_{cc} + \Gamma_R\sigma_{dd} - i\Omega_0(\sigma_{ca} - \sigma_{ac}), && (28c)\\
\dot\sigma_{dd} &= -\Gamma_R\sigma_{dd} + \Gamma_L\sigma_{cc} - i\Omega_0(\sigma_{db} - \sigma_{bd}), && (28d)\\
\dot\sigma_{ac} &= -i\epsilon_0\sigma_{ac} - i\Omega_0(\sigma_{aa} - \sigma_{cc}) - \frac{\Gamma_L + \Gamma'_L}{2}\sigma_{ac} + \sqrt{\Gamma_R\Gamma'_R}\sigma_{bd}, && (28e)\\
\dot\sigma_{bd} &= -i(\epsilon_0 + U)\sigma_{bd} - i\Omega_0(\sigma_{bb} - \sigma_{dd}) - \frac{\Gamma_R + \Gamma'_R}{2}\sigma_{bd} + \sqrt{\Gamma_L\Gamma'_L}\sigma_{ac}\,, && (28f)
\end{aligned}$$

where $\Gamma'_{L,R}$ are the tunneling rate at the energy $E_0 + U^{12}$.

Let us assume that $\Gamma'_{L,R} = \Gamma_{L,R}$. Then it follows from Eqs. (28) that the behavior of the charge inside the SET

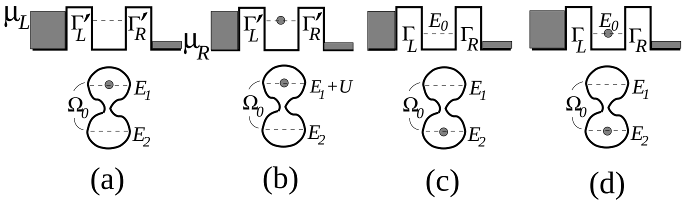

FIG. 5: The available discrete states of the entire system for the configuration shown in Fig. 2b. Here U is the repulsion energy between the electrons.

is not affected by the qubit, the same as in the previous case of the Rabi frequency fluctuations. Also the qubit density matrix becomes the mixture (5) in the stationary state for any values of the qubit and SET parameters. Hence, there is no qubit relaxation in this case either.

Since, according to Eq. (21), there is a finite probability of finding an electron inside the SET in the stationary state, $\bar{P}_1 = \Gamma_L/\Gamma$, the energy level E_1 of the qubit is shifted by $\bar{P}_1 U$. Therefore it is useful to define the "renormalized" level displacement, $\epsilon = \epsilon_0 - \bar{P}_1 U$.

As in the previous case we use the Laplace transform, $\sigma(t) \to \tilde{\sigma}(E)$, in order to determine the decoherence rate analytically. In the case of $\Gamma_L = \Gamma_R = \Gamma/2$ and $\epsilon = 0$ we obtain from Eqs. (28)

$$\tilde{\sigma}_{11}(E) = \frac{i}{2E} + \frac{i}{2E + \dfrac{32(E+i\Gamma)\Omega_0^2}{U^2 - 4E(E+i\Gamma)}}. \tag{29}$$

The position of the pole in the second term of this expression determines the decoherence rate. In contrast with Eq. (22), however, the exact analytical expression for the decoherence rate (Γ_d) is complicated, since it is given by a cubic equation. We therefore evaluate Γ_d in a different way, by substituting $E = \pm 2\Omega_0 - i\gamma$ in the second term of Eq. (29) and then expanding the latter in powers of γ. Assuming that γ is small, we keep only the first two terms of this expansion. From Eq. (10) we obtain for the decoherence rate $\Gamma_d = 4\gamma$, and thus

$$\Gamma_d = \begin{cases} \dfrac{U^2\Gamma}{2(\Gamma^2 + 4\Omega_0^2)} & \text{for} \quad U \ll (\Omega_0^2 + \Gamma\Omega_0)^{1/2} \\ \dfrac{64\Gamma\Omega_0^2}{U^2 + 16\Omega_0^2} & \text{for} \quad U \gg (\Omega_0^2 + \Gamma\Omega_0)^{1/2} \end{cases} \tag{30}$$

In general, if $\Gamma_L \neq \Gamma_R$, one finds from Eqs. (28) that $\Gamma_d = 2U^2\Gamma_L\Gamma_R/[\Gamma(\Gamma^2 + 4\Omega_0^2)]$ for $U \ll (\Omega_0^2 + \Gamma\Omega_0)^{1/2}$. The same as in the case of the Rabi-frequency fluctuations, Eq. (27), the decoherence rate in a weak coupling limit is related to the fluctuation spectrum of the SET, $S_Q(\omega)$, Eq. (B8), but now taken at $\omega = 2\Omega_0$. The latter corresponds to the level splitting of the diagonalized qubit's Hamiltonian, ω_R. Thus,

$$\Gamma_d = U^2\, S_Q(\omega_R)\,, \tag{31}$$

which can be applied also for $\epsilon \neq 0$. This is illustrated by Fig. 6 which shows $\sigma_{11}(t)$ obtained from Eqs. (28)

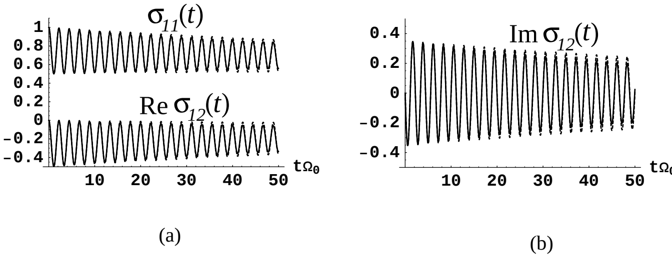

FIG. 6: The probability of finding the electron in the first dot of the qubit for $\epsilon = 2\Omega_0$, $\Gamma_L = \Omega_0$, $\Gamma_R = 2\Omega_0$ and $U = 0.5\Omega_0$. The solid line is the exact result, whereas the dashed line is obtained from the Bloch-type rate equations with the decoherence rate given by Eq. (31).

and (19) (solid line) with Eqs. (3a) and (4) (dashed line) for the decoherence rate Γ_d given by Eq. (31). As in the previous case, shown in Fig. 4, the initial conditions correspond to $\sigma_{11}(0) = 1$ and $\sigma_{12}(0) = 0$ (respectively, $\sigma_{aa}(0) = \Gamma_R/\Gamma$ and $\sigma_{bb}(0) = \Gamma_R/\Gamma$). One finds from Fig. 6 that Eq. (31) can be used for an estimation of Γ_d even for $U \sim \Gamma, \Omega_0$.

In contrast with the previous case of the Rabi-frequency fluctuations, where the decoherence rate is determined by the fluctuation spectrum at zero frequency, Eq. (27), the fluctuations of the qubit's energy level generate the decoherence rate, dependent on the fluctuation spectrum at the Rabi frequency, Eq. (31)[13]. The latter can be controlled by the qubit's levels displacement. Thus the qubit can be used for measurements of the shot-noise spectrum of the environment[14,15]. For instance, it can be done by attaching the qubit to reservoirs at different chemical potentials. In this case the resonant current would flow through the qubit. This current is given by a simple analytical expression[16] that includes explicitly the qubit's decoherence rate, Eq. (31). Thus by measuring such a current for different level displacement of the qubit (ϵ_0), one can extract the spectral density of the fluctuating environment acting on the qubit.

C. Strong-coupling limit and localization

Let us consider the limit of $U \gg (\Omega_0^2 + \Gamma\Omega_0)^{1/2}$. Our rate equation (28) are perfectly valid in this region, providing only that $E_0 + U$ is deeply inside of the potential bias. Note that in the previous case of the off-diagonal coupling fluctuations, the fluctuations amplitude has been restricted by $\delta\Omega \leq \Omega$ due to a nature of the model (Fig. 2a). We find from Eq. (30) that the decoherence rate is not directly related to the spectrum of fluctuations in strong coupling limit. In addition, the effective frequency of the qubit's Rabi oscillations (ω_R^{eff}) decreases in this limit. Indeed, by using Eqs. (29), (23), one finds that the main contribution to $\sigma_{11}(t)$, is coming from a pole of $\tilde{\sigma}_{11}(E)$, which lies on the imaginary axis. This implies that the effective frequency of Rabi oscil-

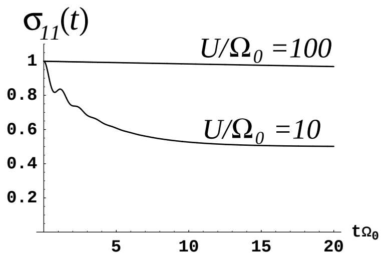

FIG. 7: The probability of finding the electron in the first dot of the qubit as given by Eqs. (28) for $\epsilon = 0$, $\Gamma_L = \Gamma_R = \Omega_0$ and two values of U.

lations vanishes in the limit $U \to \infty$. In addition, the decoherence rate $\Gamma_d \to 0$ in the same limit, Eq. (30). As a result, the electron would localize in the initial qubit state, as shown in Fig. 7.

The results displayed in this figure show that the solution of the Bloch-type rate equations, representing the damped oscillations, is very far from the exact result (solid line), obtained from Eqs. (28) and representing an electron localization in the first dot. The latter is a result of effective decrease of the Rabi frequency for large U, which slows the electron transitions between the dots, despite of the decrease of the relaxation rate. Therefore, such an environment-induced localization is different from the Zeno-type effect, generated by decoherence and perfectly described by Bloch-type equations[8,16]. The localization shown in Fig. 7 is rather similar to that in the spin-boson model[4]. It shows that in spite of their defferences, both models trace the same physics of the back-action of the environment (SET) on the qubit.

V. BACK-ACTION ON THE ENVIRONMENT

A. Weak back-action effect

Now we investigate a weak dependence of the width's $\Gamma_{L,R}$ on the energy U, Fig. 5. We keep only the linear term, $\Gamma'_{L,R} = \Gamma_{L,R} + \alpha_{L,R}U$, by assuming that U is small. In contrast with the previous examples, where the widths were energy independent, the qubit now would influence the SET current and its charge correlator. A more interesting case corresponds to asymmetric linear term, $\alpha_L \neq \alpha_R$. We take for simplicity $\alpha_L = 0$ and $\alpha_R = \alpha \neq 0$.

Similarly to the previous case we introduce the "renormalized" level displacement, $\epsilon = \epsilon_0 - (\Gamma_L/\Gamma)U$, where $\epsilon = 0$ corresponds to the aligned qubit. Solving Eqs. (28) in the steady-state limit, $\bar{\sigma} = \sigma(t \to \infty)$, and keeping only the first term in expansion in powers of U, we find for the qubit density matrix, Eqs. (19) in this limit:

$$\bar{\sigma} = \begin{pmatrix} 1/2 - \alpha\,\epsilon/(4\Gamma_R) & \alpha\Omega_0(1 + c\,\alpha\,U)/(2\Gamma_R) \\ \alpha\Omega_0(1 + c\,\alpha\,U)/(2\Gamma_R) & 1/2 + \alpha\,\epsilon/(4\Gamma_R) \end{pmatrix}, \tag{32}$$

where $c = (\alpha\epsilon - 2\Gamma)/(4\Gamma_R\Gamma)$. It follows from Eqs. (32) that the qubit's density matrix is no longer a mixture (5) in the steady-state. Indeed, the probability to occupy the lowest level is always larger than 1/2 and $\bar{\sigma}_{12} \neq 0$. This implies that relaxation takes place together with decoherence. For $\epsilon = 0$ the ratio of the relaxation and decoherence rates is given by the off-diagonal terms of the qubit density matrix, $\Gamma_d/\Gamma_r = \bar{\sigma}_{12}^{-1} - 2$, as follows from Eq. (11).

In order to find a relation between the decoherence and relaxation rates, $\Gamma_{d,r}$, and the fluctuation spectrum of the qubit energy level, $S_Q(\omega)$, we first evaluate the damping rate (γ) of the qubit's density matrix. Using Eq. (10) we find that this quantity is related to the decoherence and relaxation rates by $\gamma = (\Gamma_d + 2\Gamma_r)/4$. The same as in the previous case the rate γ is determined by poles of Laplace transformed density matrix $\sigma(t) \to \tilde{\sigma}(E)$ in the complex E-plane. Consider for simplicity the case of $\epsilon = 0$ and $\Gamma_L = \Gamma_R = \Gamma/2$. Performing the Laplace transform of Eqs. (28) we look for the poles of $\sigma_{11}(E)$ at $E = \pm 2\Omega_0 - i\gamma$ for small U. Finally we obtain

$$\Gamma_d + 2\Gamma_r = \frac{U^2}{2(\Gamma^2 + 4\Omega_0^2)}\left[\Gamma - \alpha\,U\frac{\Gamma^2 - 4\Omega_0^2}{2\,(\Gamma^2 + 4\Omega_0^2)}\right]. \tag{33}$$

for $U \ll \Omega_0$.

Now we evaluate the correlator of the charge inside the SET, $S_Q(\omega)$ which induces the energy-level fluctuations of the qubit. Using Eqs. (B6) and (28), we find,

$$S_Q(\omega) = \frac{\Gamma}{2\,(\Gamma^2 + \omega^2)} - \alpha\,U\frac{\Gamma^2 - \omega^2}{4\,(\Gamma^2 + \omega^2)^2} \tag{34}$$

for $\alpha U \ll \Gamma$. Therefore in the limit of $U \ll \Omega_0$ and $\alpha\,U \ll \Gamma$ the total damping rate of the qubit's oscillations is directly related to the spectral density of the fluctuations spectrum taken at the Rabi frequency,

$$\Gamma_d + 2\Gamma_r = U^2 S_Q(2\Omega_0). \tag{35}$$

This results represent a generalization of Eq. (31) for the case when the spectral density of the environment fluctuations is affected by the qubit. As a result, the qubit displays relaxation together with decoherence. It is remarkable that the total qubit's damping rate is still given by the fluctuation spectrum of the SET (environment) modulated by the qubit. Note that Eq. (35) can be applied only if the modulation of the tunneling rate through the SET (tunneling current) is small $\alpha\,U \ll \Gamma$, in addition to a weak distortion of the qubit ($U \ll \Omega_0$). If however the SET current is not modulated by the qubit, then the condition of a weak distortion of the qubit by the SET ($U \ll \Omega_0$) would sufficient for an evaluation of

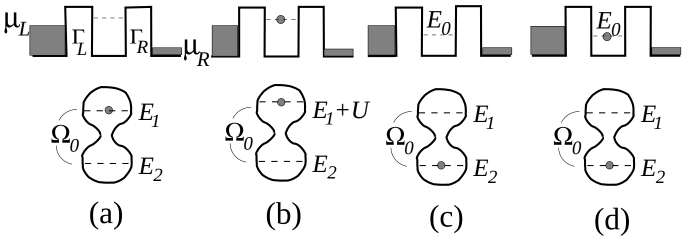

FIG. 8: The available discrete states of the entire system when the electron-electron repulsive interaction U breaks off the current through the SET.

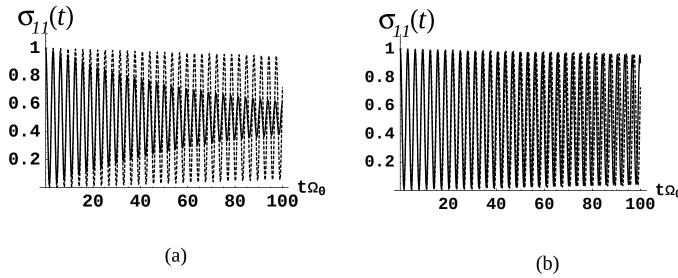

FIG. 9: (a) The probability of finding the electron in the first dot of the qubit for $\epsilon = 0$, $\Gamma_L = \Gamma_R = 0.05\Omega_0$ and $U = 0.5\Omega_0$. The solid line is obtained from Eqs. (36), whereas the dashed line corresponds to the Eq. (10) with Γ_d given by Eq. (31); (b) the same for the case, shown in Fig. 5, where the solid line corresponds to Eqs. (28).

the decoherence rate via the fluctuation spectrum of the qubit parameters.

In the case of strong modulation of the environment by the qubit, the decorerence and relaxation rates of the qubit are not directly related to the fluctuation spectrum of the environment, even if the distortion of the qubit is small. This point is illustrated by the following example.

B. Strong back-action

Until now we considered the case where $E_0 + U \ll \mu_L$, so that the interacting electron of the SET remains deeply inside the voltage bias. As a result, the charge fluctuations inside the SET were not affected by the qubit. If however, the interaction U between the qubit and the SET is such that $E_0 + U \gg \mu_L$, the qubit's oscillation would strongly affect the fluctuation of charge inside the SET. Indeed, the SET is blocked whenever the level E_1 of the qubit is occupied, Fig. 8. This case can be treated with small modification of the rate equations (28), if only $\mu_L - E_0 \gg \Gamma$ and $E_0 + U - \mu_L \gg \Gamma$, where E_0 is a level of the SET carrying the current.

The corresponding quantum rate equations describing the system are obtained directly from Eqs. (18). Assuming that the widths $\Gamma_{L,R}$ are energy independent we finds[6]

$$\dot{\sigma}_{aa} = (\Gamma_L + \Gamma_R)\sigma_{bb} - i\Omega_0(\sigma_{ac} - \sigma_{ca}), \tag{36a}$$

$$\dot{\sigma}_{bb} = -(\Gamma_R + \Gamma_L)\sigma_{bb} - i\Omega_0(\sigma_{bd} - \sigma_{db}), \tag{36b}$$

$$\dot{\sigma}_{cc} = -\Gamma_L\sigma_{cc} + \Gamma_R\sigma_{dd} - i\Omega_0(\sigma_{ca} - \sigma_{ac}), \tag{36c}$$

$$\dot{\sigma}_{dd} = -\Gamma_R\sigma_{dd} + \Gamma_L\sigma_{cc} - i\Omega_0(\sigma_{db} - \sigma_{bd}), \tag{36d}$$

$$\dot{\sigma}_{ac} = -i\epsilon_0\sigma_{ac} - i\Omega_0(\sigma_{aa} - \sigma_{cc}) - \frac{\Gamma_L}{2}\sigma_{ac} + \Gamma_R\sigma_{bd}, \tag{36e}$$

$$\dot{\sigma}_{bd} = -i(\epsilon_0 + U)\sigma_{bd} - i\Omega_0(\sigma_{bb} - \sigma_{dd}) - \left(\Gamma_R + \frac{\Gamma_L}{2}\right)\sigma_{bd}\,. \tag{36f}$$

Solving Eqs. (36) in the stationary limit, $\bar{\sigma} = \sigma(t \to \infty)$ and introducing the "renormalized" level displacement, $\epsilon = \epsilon_0 - U\Gamma_L/(2\Gamma)$, we obtain for the qubit's density matrix, Eqs. (19) in the steady state:

$$\bar{\sigma}_{11} = \frac{1}{2} - \frac{8\epsilon U}{16\epsilon^2 + 8U\epsilon + 48\Omega_0^2 + 9(U^2 + \Gamma^2)}\,, \tag{37a}$$

$$\bar{\sigma}_{12} = \frac{12U\Omega_0}{16\epsilon^2 + 8U\epsilon + 48\Omega_0^2 + 9\,(U^2 + \Gamma^2)}\,, \tag{37b}$$

where for simplicity we considered the symmetric case, $\Gamma_L = \Gamma_R = \Gamma/2$. It follows from Eqs. (37) that similarly to the previous example, the qubit's density matrix is no longer a mixture (5). The relaxation takes place together with decoherence in this case too.

Let us consider weak distortion of the qubit by the SET, $U < \Omega_0$. Although the values of U are restricted from below ($U \gg \Gamma + \mu_L - E_0$), this limit can be achieved if the level E_0 is close to the Fermi energy, providing only that $\mu_L - E_0 \gg \Gamma$, and $\Gamma \ll U$. Now we evaluate $\sigma_{11}(t)$ with the rate equations (36) and then compare it with the same quantity obtained from the Bloch equations, Eq. (10), where $\Gamma_{d,r}$ are given by Eqs. (31)and (11). The corresponding charge-correlator, $S_Q(\omega_R)$, is evaluated by Eqs. (B6) and (36). As an example, we take symmetric qubit with aligned levels, $\epsilon = 0$, $\Gamma_L = \Gamma_R = 0.05\Omega_0$ and $U = 0.5\Omega_0$. The decoherence and relaxation rates, corresponding to these parameters are respectively: $\Gamma_d/\Omega_0 = 0.0038$ and $\Gamma_r/\Omega_0 = 0.00059$.

The results are presented in Fig. 9a. The solid line shows $\sigma_{11}(t)$, obtained from the rate equations (36), where the dashed line is the same quantity obtained from Eq. (10). We find that Eq. (31) (or (35)) underestimates the actual damping rate of $\sigma_{11}(t)$ by an order of magnitude). This lies in a sharp contrast with the previous case, where the energy level of the SET is not distorted by the qubit, $\Gamma'_{L,R} = \Gamma_{L,R}$, Fig. 5. Indeed, in this case $\sigma_{11}(t)$ obtained Eq. (10) with Γ_d given by Eq. (31) and $\Gamma_r = 0$, agrees very well with that obtained from the rate equations (28), as shown in Fig. 9b.

This example clearly illustrates that the decoherence is not related to the fluctuation spectrum of the environment, even if the qubit's distortion is small, whenever the environment is strongly affected by the qubit. This is a typical case of measurement, corresponding to a noticeable response of the environment to the qubit's state (the "signal").

VI. SUMMARY

In this paper we propose a simple model describing a qubit interacting with fluctuating environment. The latter is represented by a single electron transistor (SET) in close proximity of the qubit. Then the fluctuations of the charge inside the SET generate fluctuating field acting on the qubit. In the limit of large bias voltage, the Schrödinger equation for the entire system is reduced to the Bloch-type rate equations. The resulting equations are very simple, so that one can easily analyze the limits of weak and strong coupling of the qubit with the SET.

We considered separately two different cases: (a) there is no back-action of the qubit on the SET behavior, so that the latter represents a "pure environment"; and (b) the SET behavior depends on the qubit's state. In the latter case the SET can "measure" the qubit. The setup corresponding to the "pure environment" is realized when the energy level of the SET carrying the current lies deeply inside the potential bias. The second (measurement) regime of the SET is realized when the tunneling widths of the SET are energy dependent, or when the energy level of the SET carrying the current is close enough to the Fermi level of the corresponding reservoir. Then the electron-electron interaction between the qubit and the SET modulates the electron current through the SET.

In the case of the "pure environment" ("no-measurement" regime) we investigate separately two different configurations of the qubit with respect to the SET. In the first one the SET produces fluctuations of the off-diagonal coupling (Rabi frequency) between two qubit's states. In the second configuration the SET produces fluctuations of the qubit's energy levels. In the both cases we find no relaxation of the qubit, despite the energy transfer between the qubit and the SET can take place. As a result the qubit always turns asymptotically to the statistical mixture. We also found that in both cases the decoherence rate of the qubit in the weak coupling limit is given by the spectral density of the corresponding fluctuating parameter. The difference is that in the case of the off-diagonal coupling fluctuations the spectral density is taken at zero frequency, whereas in the case of the energy level fluctuations it is taken at the Rabi-frequency.

In the case of the strong coupling limit, however, the decoherence rate is not related to the fluctuation spectrum. Moreover we found that the electron in the qubit is localized in this limit due to an effective decrease of the off-diagonal coupling. This phenomenon may resemble the localization in the spin-boson model in the strong coupling limit.

If the charge correlator and the total SET current are affected by the qubit (back-action effect), we found that the off-diagonal density-matrix elements of the qubit survive in the steady-state limit and therefore the relaxation rate is not zero. We concentrated on the case of weak coupling, when the Coulomb repulsion between the qubit and the SET is smaller then the Rabi frequency. The back-action of the qubit on the SET, however, can be weak or strong. In the first case we found that the total damping rate of the qubit due to decoherence and relaxation is again given by the spectral density of the SET charge fluctuations, *modulated by the qubit*. This relation, however, is not working if the back-action is strong. Indeed, we found that the damping rate of the qubit in this case is larger by an order of magnitude than that given by the spectral density of the corresponding fluctuating parameter.

This looks like that in the strong back-action of the qubit on the SET the major component of decoherence is not coming from the fluctuation spectrum of the qubit's parameters only, but also from the measurement "signal" of the SET. On the first sight it could agree with an analysis of Ref.[7], suggesting that the decoherence rate contains two components, generated by a measurement and by a "pure environment" (environmental fluctuations). The latter therefore represents an unavoidable decoherence, generated by any environment. Yet, in a weak coupling regime such a separation seems not working. In this case the dumping (decoherence) rate is totally determined by the environment fluctuations, even so modulated by the qubit.

Although our model deals with a particular setup, it bears the main physics of a fluctuating environment, acting on a qubit. Indeed, the Bloch-type rate equations, which we used in our analysis have a pronounced physical meaning: they relate the variation of qubit parameters with a nearby fluctuating field described by rate equations. Thus our model can be considered as a generic one. Its main advantage is that it can be easily extended to multiple coupled qubits. Such an analysis would allow to determine how decoherence scales with number of qubits, which is extremely important for a realization of quantum computations.

In addition, our model can be extended to a more complicated fluctuating environments, such as containing characteristic frequencies in its spectrum. It would formally correspond to a replacement of the SET in Fig. 2 by a double-dot (DD) coupled to the reservoirs[17]. All these situations, however, must be a subject of a separate investigation.

APPENDIX A: QUANTUM-MECHANICAL DERIVATION OF RATE EQUATIONS FOR QUANTUM TRANSPORT

Consider the resonant tunneling through the SET, shown schematically in Fig. 10. The entire system is de-

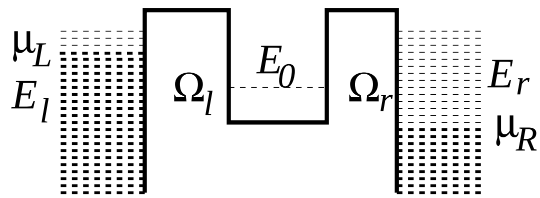

FIG. 10: Resonant tunneling through a single dot. $\mu_{L,R}$ are the Fermi energies in the collector and emitter, respectively.

scribed by the Hamiltonian H_{set}, given by Eq. (12). The wave function can be written in the same way as Eq. (15), where the variables related to the qubit are omitted,

$$|\Psi(t)\rangle = \left[b_0(t) + \sum_l b_{1l}(t) c_1^\dagger c_l + \sum_{l,r} b_{rl}(t) a_1^\dagger c_r^\dagger c_l + \sum_{l<l',r} b_{1rll'}(t) c_1^\dagger c_r^\dagger c_l c_{l'} + \cdots \right] |0\rangle. \tag{A1}$$

Substituting $|\Psi(t)\rangle$ into the time-dependent Schrödinger equation, $i\partial_t|\Psi(t)\rangle = H_{set}|\Psi(t)\rangle$, and performing the Laplace transform, $\tilde{b}(E) = \int_0^\infty \exp(iEt)\, b(t) dt$, we obtain the following infinite set of algebraic equations for the amplitudes $\tilde{b}(E)$:

$$E\tilde{b}_0(E) - \sum_l \Omega_l \tilde{b}_{1l}(E) = i \tag{A2a}$$

$$(\ E + E_l - E_1)\tilde{b}_{1l}(E) - \Omega_l \tilde{b}_0(E) - \sum_r \Omega_r \tilde{b}_{lr}(E) = 0 \tag{A2b}$$

$$(\ E + E_l - E_r)\tilde{b}_{lr}(E) - \Omega_r \tilde{b}_{1l}(E) - \sum_{l'} \Omega_{l'} \tilde{b}_{1ll'r}(E) = 0 \tag{A2c}$$

$$(\ E + E_l + E_{l'} - E_1 - E_r)\tilde{b}_{1ll'r}(E) - \Omega_{l'} \tilde{b}_{lr}(E) + \Omega_l \tilde{b}_{l'r}(E) - \sum_{r'} \Omega_{r'} \tilde{b}_{ll'rr'}(E) = 0 \tag{A2d}$$

$$\cdots\cdots\cdots$$

(The r.h.s of Eq. (A2a) reflects the initial condition.)

Let us replace the amplitude $\tilde{b}$ in the term $\sum \Omega \tilde{b}$ of each of the equations (A2) by its expression obtained from the subsequent equation. For example, substituting $\tilde{b}_{1l}(E)$ from Eq. (A2b) into Eq. (A2a) we obtain

$$\left[E - \sum_l \frac{\Omega_l^2}{E + E_l - E_1} \right] \tilde{b}_0(E) - \sum_{l,r} \frac{\Omega_l \Omega_r}{E + E_l - E_1} \tilde{b}_{lr}(E) = i. \tag{A3}$$

Since the states in the reservoirs are very dense (continuum), one can replace the sums over l and r by integrals, for instance $\sum_l \ \to \ \int \rho_L(E_l)\, dE_l$, where $\rho_L(E_l)$ is the density of states in the emitter, and $\Omega_{l,r} \to \Omega_{L,R}(E_{l,r})$. Consider the first term

$$S_1 = \int_{-\Lambda}^{\mu_L} \frac{\Omega_L^2(E_l)}{E + E_l - E_1} \rho_L(E_l) dE_l \tag{A4}$$

where Λ is the cut-off parameter. Assuming weak energy dependence of the couplings $\Omega_{L,R}$ and the density of states $\rho_{L,R}$, we find in the limit of high bias, $\mu_L = \Lambda \to \infty$

$$S_1 = -i\pi \Omega_L^2(E_i - E)\rho_L(E_1 - E) = -i\frac{\Gamma_L}{2}\, . \tag{A5}$$

Consider now the second sum in Eq. (A3).

$$S_2 = \int_{-\Lambda}^{\Lambda} \rho_R(E_r) dE_r \int_{-\Lambda}^{\Lambda} \frac{\Omega_L(E_l)\Omega_R(E_r)\tilde{b}(E, E_l, E_r)}{E + E_l - E_1} \rho_L(E_l) dE_l\, , \tag{A6}$$

where we replaced $\tilde{b}_{lr}(E)$ by $\tilde{b}(E, E_l, E_r)$ and took $\mu_L = \Lambda$, $\mu_R = -\Lambda$. In contrast with the first term of Eq. (A3), the amplitude $\tilde{b}$ is not factorized out the integral (A6). We refer to this type of terms as "cross-terms". Fortunately, all "cross-terms" vanish in the limit of large bias, $\Lambda \to \infty$. This greatly simplifies the problem and is very crucial for a transformation of the Schrödinger to the rate equations. The reason is that the poles of the integrand in the $E_l(E_r)$-variable in the "cross-terms" are on the same side of the integration contour. One can find it by using a perturbation series the amplitudes $\tilde{b}$ in powers of Ω. For instance, from iterations of Eqs. (A2) one finds

$$\tilde{b}(E, E_l, E_r) = \frac{i\Omega_L \Omega_R}{E(E + E_l - E_r)(E + E_l - E_1)} + \cdots \tag{A7}$$

The higher order powers of Ω have the same structure. Since $E \to E + i\epsilon$ in the Laplace transform, all poles of the amplitude $\tilde{b}(E, E_l, E_r)$ in the E_l-variable are below the real axis. In this case, substituting Eq. (A7) into Eq. (A6) we find

$$\lim_{\Lambda\to\infty} \int_{-\Lambda}^{\Lambda} \left[\frac{\Omega_L \Omega_R}{(E + i\epsilon)(E + E_l - E_1 + i\epsilon)^2 (E + E_l - E_r + i\epsilon)} + \cdots \right] dE_l = 0\,, \tag{A8}$$

Thus, $S_2 \to 0$ in the limit of $\mu_L \to \infty$, $\mu_R \to -\infty$.

Applying analogous considerations to the other equations of the system (A2), we finally arrive at the following set of equations:

$$(E + i\Gamma_L/2)\tilde{b}_0(E) = i \tag{A9a}$$

$$(E + E_l - E_1 + i\Gamma_R/2)\tilde{b}_{1l}(E) - \Omega_l \tilde{b}_0(E) = 0 \tag{A9b}$$

$$(E + E_l - E_r + i\Gamma_L/2)\tilde{b}_{lr}(E) - \Omega_r \tilde{b}_{1l}(E) = 0 \tag{A9c}$$

$$(E + E_l + E_{l'} - E_1 - E_r + i\Gamma_R/2)\tilde{b}_{1ll'r}(E) - \Omega_{l'} \tilde{b}_{lr}(E) + \Omega_l \tilde{b}_{l'r}(E) = 0 \tag{A9d}$$

$$\cdots\cdots\cdots$$

Eqs. (A9) can be transformed directly to the reduced density matrix $\sigma_{jj'}^{(n,n')}(t)$, where $j = 0, 1$ denote the state of the SET with an unoccupied or occupied dot and n denotes the number of electrons which have arrived at the collector by time t. In fact, as follows from our derivation, the diagonal density-matrix elements, $j = j'$and $n = n'$, form a closed system in the case of resonant tunneling through one level, Fig. 10. The off-diagonal elements, $j \neq j'$, appear in the equation of motion whenever more than one discrete level of the system carry the transport (see Eq. (17). Therefore we concentrate below on the diagonal density-matrix elements only, $\sigma_{00}^{(n)}(t) \equiv \sigma_{00}^{(n,n)}(t)$ and $\sigma_{11}^{(n)}(t) \equiv \sigma_{11}^{(n,n)}(t)$. Applying the inverse Laplace transform on finds

$$\sigma_{00}^{(n)}(t) = \sum_{l\ldots,r\ldots} \int \frac{dEdE'}{4\pi^2} \tilde{b}_{\underbrace{l\cdots}_{n}\underbrace{r\cdots}_{n}}(E) \tilde{b}^*_{\underbrace{l\cdots}_{n}\underbrace{r\cdots}_{n}}(E') e^{i(E'-E)t} \tag{A10a}$$

$$\sigma_{11}^{(n)}(t) = \sum_{l\ldots,r\ldots} \int \frac{dEdE'}{4\pi^2} \tilde{b}_{1\underbrace{l\cdots}_{n+1}\underbrace{r\cdots}_{n}}(E) \tilde{b}^*_{1\underbrace{l\cdots}_{n+1}\underbrace{r\cdots}_{n}}(E') e^{i(E'-E)t} \tag{A10b}$$

Consider, for instance, the term $\sigma_{11}^{(0)}(t) = \sum_l |b_{1l}(t)|^2$. Multiplying Eq. (A9b) by $\tilde{b}^*_{1l}(E')$ and then subtracting the complex conjugated equation with the interchange $E \leftrightarrow E'$ we obtain

$$\begin{aligned} \int \frac{dEdE'}{4\pi^2}(E' - E - i\Gamma_R) \sum_l \quad & \tilde{b}_{1l}(E)\tilde{b}^*_{1l}(E') e^{i(E'-E)t} \\ - \quad & \int \frac{dEdE'}{4\pi^2} 2\mathrm{Im} \sum_l \Omega_l \tilde{b}_{1l}(E) \tilde{b}_0^*(E') e^{i(E'-E)t} = 0 \end{aligned} \tag{A11}$$

Using Eq. (A10b) one easily finds that the first integral in Eq. (A11) equals to $-i[\dot{\sigma}_{11}^{(0)}(t) + \Gamma_R \sigma_{11}^{(0)}(t)]$. Next, substituting

$$\tilde{b}_{1l}(E) = \frac{\Omega_l \tilde{b}_0(E)}{E + E_l - E_1 + i\Gamma_R/2} \tag{A12}$$

from Eq. (A9b) into the second term of Eq. (A11), and replacing a sum by an integral, one can perform the E_l-integration in the large bias limit, $\mu_L \to \infty$, $\mu_R \to -\infty$. Then using again Eq. (A10b) one reduces the second term of Eq. (A11) to $i\Gamma_L \sigma_{00}^{(0)}(t)$. Finally, Eq. (A11) reads $\dot{\sigma}_{11}^{(0)}(t) = \Gamma_L \sigma_{00}^{(0)}(t) - \Gamma_R \sigma_{11}^{(0)}(t)$.

The same algebra can be applied for all other amplitudes $\tilde{b}(t)$. For instance, by using Eq. (A10a) one easily finds that Eq. (A9c) is converted to the following rate equation $\dot{\sigma}_{00}^{(1)}(t) = -\Gamma_L\sigma_{00}^{(1)}(t)+\Gamma_R\sigma_{11}^{(0)}(t)$. With respect to the states involving more than one electron (hole) in the reservoirs (the amplitudes like $\tilde{b}_{1ll'r}(E)$ and so on), the corresponding equations contain the Pauli exchange terms. By converting these equations into those for the density matrix using our procedure, one finds the "cross terms", like $\sum \Omega_l\tilde{b}_{l'r}(E)\Omega_{l'}\tilde{b}_{lr}^*(E')$, generated by Eq. (A9d). Yet, these terms vanish after an integration over $E_{l(r)}$ in the large bias limit, as the second term in Eq. (A3). The rest of the algebra remains the same, as described above. Finally we arrive at the following infinite system of the chain equations for the diagonal elements, $\sigma_{00}^{(n)}$ and $\sigma_{11}^{(n)}$, of the density matrix,

$$\dot{\sigma}_{00}^{(0)}(t) = -\Gamma_L\sigma_{00}^{(0)}(t)\,, \tag{A13a}$$
$$\dot{\sigma}_{11}^{(0)}(t) = \Gamma_L\sigma_{00}^{(0)}(t) - \Gamma_R\sigma_{11}^{(0)}(t)\,, \tag{A13b}$$
$$\dot{\sigma}_{00}^{(1)}(t) = -\Gamma_L\sigma_{00}^{(1)}(t) + \Gamma_R\sigma_{11}^{(0)}(t)\,, \tag{A13c}$$
$$\dot{\sigma}_{11}^{(1)}(t) = \Gamma_L\sigma_{00}^{(1)}(t) - \Gamma_R\sigma_{11}^{(1)}(t)\,, \tag{A13d}$$
$$\cdots\cdots\cdots$$

Summing over n in Eqs. (A13) we find for the reduced density matrix of the SET, $\sigma(t) = \sum_n \sigma^{(n)}(t)$, the following "classical" rate equations,

$$\dot{\sigma}_{00}(t) \;=\; -\Gamma_L\sigma_{00}(t) + \Gamma_R\sigma_{11}(t) \tag{A14a}$$
$$\dot{\sigma}_{11}(t) \;=\; \Gamma_L\sigma_{00}(t) - \Gamma_R\sigma_{11}(t) \tag{A14b}$$

These equations represent a particular case of our general quantum rate equations (17), which are derived using the above described technique[9,10].

APPENDIX B: CORRELATOR OF ELECTRIC CHARGE INSIDE THE SET.

The charge correlator inside the SET is given by $S_Q(\omega) = \bar{S}_Q(\omega) + \bar{S}_Q(-\omega)$, where

$$\bar{S}_Q(\omega) = \int_0^\infty \langle\delta\hat{Q}(0)\delta\hat{Q}(t)\rangle e^{i\omega t}dt\,. \tag{B1}$$

Here $\delta\hat{Q}(t) = c_0^\dagger(t)c_0(t) - \bar{q}$ and $\bar{q} = \bar{P}_1 = P_1(t\to\infty)$ is the average charge inside the dot. Since the initial state, $t=0$ in Eq. (B1) corresponds to the steady state, one can represent the time-correlator in the integrand (B1) as

$$\langle\delta\hat{Q}(0)\delta\hat{Q}(t)\rangle = \sum_{q=0,1} P_q(0)(q-\bar{q})(\langle Q_q(t)\rangle - \bar{q})\,, \tag{B2}$$

where $P_q(0)$ is the probability of finding the charge $q = 0,1$ inside the quantum dot in the steady state, such that $P_1(0) = \bar{q}$ and $P_0(0) = 1-\bar{q}$, and $\langle Q_q(t)\rangle = P_1^{(q)}(t)$ is the average charge in the dot at time t, starting with the initial condition $P_1^{(q)}(0) = q$. Substituting Eq. (B2) into Eq. (B1) we finally obtain

$$\bar{S}_Q(\omega) = \bar{q}(1-\bar{q})[\tilde{P}_1^{(1)}(\omega) - \tilde{P}_1^{(0)}(\omega)]\,, \tag{B3}$$

where $\tilde{P}_1^{(q)}(\omega)$ is a Laplace transform of $P_1^{(q)}(t)$. These quantities are obtained directly from the rate equations, such that $\bar{q} = \bar{\sigma}_{bb} + \bar{\sigma}_{dd}$ and $\tilde{P}_1^{(q)}(\omega) = \tilde{\sigma}_{bb}^{(q)}(\omega) + \tilde{\sigma}_{dd}^{(q)}(\omega)$, where $\bar{\sigma} = \sigma(t\to\infty)$ and $\tilde{\sigma}^{(q)}(\omega)$ is the Laplace transform $\sigma^{(q)}(t)$ with the initial conditions corresponding to the occupied ($q=1$) or unoccupied ($q=0$) SET. In order to find these quantities it is useful to rewrite the rate equations in the matrix form, $\dot{\sigma}(t) = M\sigma(t)$, representing $\sigma(t)$ as the eight-vector, $\sigma = \{\sigma_{aa},\sigma_{bb},\sigma_{cc},\sigma_{dd},\sigma_{ac},\sigma_{ca},\sigma_{bd},\sigma_{db}\}$ and M as the corresponding 8×8-matrix. Applying the Laplace transform we find the following matrix equation,

$$(i\,\omega\, I + M)\tilde{\sigma}^{(q)}(\omega) = -\sigma^{(q)}(0)\,, \tag{B4}$$

where I is the unit matrix and $\sigma^{(q)}(0)$ is the initial condition for the density-matrix obtained by projecting the total wave function (15) on occupied ($q=1$) and unoccupied ($q=0$) states of the SET in the limit of $t\to\infty$,

$$\sigma^{(1)}(0) \;=\; \mathcal{N}_1\{0,\bar{\sigma}_{bb},0,\bar{\sigma}_{dd},0,0,\bar{\sigma}_{bd},\bar{\sigma}_{db}\}\,, \tag{B5a}$$
$$\sigma^{(0)}(0) \;=\; \mathcal{N}_0\{\bar{\sigma}_{aa},0,\bar{\sigma}_{cc},0,\bar{\sigma}_{ac},\bar{\sigma}_{ca},0,0\}\,, \tag{B5b}$$

and $\mathcal{N}_1 = 1/\bar{q}$ and $\mathcal{N}_0 = 1/(1-\bar{q})$ are the corresponding normalization factors. Finally one obtains:

$$S_Q(\omega) = 2\bar{q}(1-\bar{q})\mathrm{Re}\,[\tilde{\sigma}_{bb}^{(1)}(\omega) + \tilde{\sigma}_{dd}^{(1)}(\omega) - \tilde{\sigma}_{bb}^{(0)}(\omega) - \tilde{\sigma}_{dd}^{(0)}(\omega)]. \tag{B6}$$

In the case shown in Fig. 2 one finds from Eqs. (18) or Eqs. (28) for $\Gamma'_{L,R}=\Gamma_{L,R}$ that $\bar\sigma_{ac}=\sigma_{bd}=0$, $\bar q=\Gamma_L/\Gamma$ and $\tilde\sigma^{(q)}_{bb}(\omega)+\tilde\sigma^{(q)}_{dd}(\omega)=\tilde P^{(q)}_1(\omega)$. The latter equation is given by

$$(i\omega-\Gamma)\tilde P^{(q)}_1(\omega)=-q+\frac{i\Gamma_L}{\omega}\,. \tag{B7}$$

Substituting Eq. (B7) into Eq. (B3) one obtains:

$$S_Q(\omega)=\frac{2\Gamma_L\Gamma_R}{\Gamma(\omega^2+\Gamma^2)}\,. \tag{B8}$$

Obviously, for a more general case when $\Gamma'_{L,R}\neq\Gamma_{L,R}$, or when the electron-electron interaction excites the electron inside the SET above the Fermi level, Fig. 8, the expressions for $S_Q(\omega)$, obtained from Eq. (B6) have a more complicated than Eq. (B8).

---

* Electronic address: shmuel.gurvitz@weizmann.ac.il

1 J. Shao, C. Zerbe, and P. Hanggi, Chem. Phys. **235**, 81 (1998).
2 X.R. Wang, Y.S. Zheng, and S. Yin, Phys. Rev. B**72**, 121303(R) (2005).
3 Y. Levinson, Phys. Rev. B**61**, 4748 (2000).
4 A.J. Leggett, S. Chakravarty, A.T. Dorsey, M.P.A. Fisher, A. Garg, and W. Zwerger, Rev. Mod. Phys. **59**, 1 (1987).
5 A. Käck, G. Wendin, and G. Johansson, Phys. Rev. B **67**, 035301 (2003).
6 S.A. Gurvitz and G.P. Berman, Phys. Rev. B**72**, 073303 (2005).
7 A.N. Korotkov, Phys. Rev. B**63**, 085312 (2001); *ibid*, Phys. Rev. B**63**, 115403 (2001).
8 S.A. Gurvitz, L. Fedichkin, D. Mozyrsky and G.P. Berman, Phys. Rev. Lett., **91**, 066801 (2003).
9 S.A. Gurvitz and Ya.S. Prager, Phys. Rev. B**53**, 15932 (1996).
10 S.A. Gurvitz, Phys. Rev. B**57**, 6602, (1998).
11 S.A. Gurvitz, D. Mozyrsky, and G.P. Berman, Phys.Rev. B**72**, 205341 (2005).
12 In a strict sense the quantum rate equations (17) were derived by assuming constant widths Γ. Yet these equation are also valid when the widths are weakly energy dependent, as follows from their derivations (see[9,10] and Appendix A).
13 A similar distinction between relaxation and decoherence (T_1 and T_2), generated by different components of the fluctuating field, exists in a phenomenological description of magnetic resonance, C.P. Slichter, *Principles of Magnetic Resonance*, (Springer-Verlag, 1980).
14 R.J. Schoelkopf, A.A. Clerk, S.M. Girvin, K.W. Lehnert and M.H. Devoret, *Quantum Noise in Mesoscopic Physics*, edited by Yu.V. Nazarov, (Springer, 2003).
15 E. Onac, F. Balestro, L.H. Willems van Beveren, U. Hartmann, Y.V. Nazarov, and L.P. Kouwenhoven, Phys. Rev. Lett. **96**, 176601 (2006).
16 S.A. Gurvitz, Phys. Rev. B**56**, 15215 (1997).
17 T. Gilad and S.A. Gurvitz, Phys. Rev. Lett. **97**, 116806 (2006).