Resonant activation in bistable semiconductor lasers

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We theoretically investigate the possibility of observing resonant activation in the hopping dynamics of two-mode semiconductor lasers. We present a series of simulations of a rate-equations model under random and periodic modulation of the bias current. In both cases, for an optimal choice of the modulation time-scale, the hopping times between the stable lasing modes attain a minimum. The simulation data are understood by means of an effective one-dimensional Langevin equation with multiplicative fluctuations. Our conclusions apply to both Edge Emitting and Vertical Cavity Lasers, thus opening the way to several experimental tests in such optical systems.

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I. INTRODUCTION

It is currently established that stochastic fluctuations may have a constructive role in enhancing the response of nonlinear systems to an external coherent stimulus. Relevant examples are the enhancement of the decay time from a metastable state (noise–enhanced stability) [1, 2], the synchronization with a weak periodic input signal (stochastic resonance) [3] or the regularizaton of the response at an optimal noise intensity (coherence resonance) [4].

Another instance is the phenomenon of resonant activation that was discovered by Doering and Gadoua [5]. They showed that the escape of an overdamped Brownian particle over a fluctuating barrier can be enhanced by suitably choosing the correlation time of barrier fluctuations themselves. In other words, the escape time from the potential well attains a minimum for an optimal choice of such correlation time. Since its discovery, the phenomenon received a considerable attention from theorists (see e.g. Refs. [6, 7, 8, 9, 10]). Detailed studies by means of analog simulations have also been reported for both Gaussian and dichotomous fluctuations [11]. More recently, the phenomenon has been shown to occur also for the case in which the barrier oscillates periodically [12, 13].

To our knowledge, experimental evidences of resonant activation were only given for a bistable electronic circuit [14] and, very recently, for a colloidal particle subject to a periodically–modulated optical potential [15]. It is therefore important to look for other setups where the effect could be studied in detail. As a matter of fact, multimode laser systems are good candidates to investigate noise–activated dynamics like the switching among modes induced by quantum fluctuations (spontaneous emission) [16]. In particular, semiconductor lasers proved to be particularly versatile for detailed experi-

mental investigations of modulation and noise-induced phenomena like stochastic resonance [17, 18] and noise–induced phase synchronization [19]. In those previous studies, the resonance regimes are attained by a suitable random modulation of the bias current which can be tuned in a well-controlled way. It is thus natural to argue about the possibility of observing resonant activation with the same type of experimental setup.

In this paper, we theoretically demonstrate the phenomenon of resonant activation in a generic rate–equations model for a two-mode semiconductor laser under modulation of the bias current. The basic ingredients that act in the theoretical descriptions are a fluctuating potential barrier and some activating noise. In the laser system, the latter is basically provided by spontaneous emission while current fluctuations, that appear additively into the rate equations, effectively act multiplicatively if a suitable separation of time scales holds [20]. In a previous paper [21], we have explicitly demonstrated such multiplicative–noise effects on the mode–hopping dynamics. This was shown by a reduction to a bistable one–dimensional potential system with both multiplicative and additive stochastic forces. Several predictions drawn from such a simplified model are in good agreement with the experimental observations carried out for a bulk, Edge-Emitting Laser (EEL) [21]. In the present context, we will show that this reduced description is of great help in the interpretation of simulation data.

The outline of the paper is the following. In Sec. II we recall the model for a two-mode semiconductor laser. In Sec. III we present the numerical simulation for two physically distinct cases displaying resonant activation. These results are discussed and interpreted by comparing with the reduced one–dimensional Langevin model mentioned above (Sec. IV). We draw our conclusions in Sec. V.

II. RATE EQUATIONS

Our starting point is a stochastic rate-equation model for a semiconductor laser that may operate in two longi-

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tudinal modes whose complex amplitudes are denoted by $E_\pm$. Both of them interact with a single carrier density N that provides the necessary amplification. The two modes have very similar linear gains, provided that their wavelengths are almost equal and they are close to the gain peak. Let $J(t)$ denote the bias (injection) current, the model can be written as [21]

$$\dot{E}_+ = \frac{1}{2}\Big[(1+i\alpha)g_+ - 1\Big]E_+ + \sqrt{2D_{sp}N}\,\xi_+ \qquad (1a)$$

$$\dot{E}_- = \frac{1}{2}\Big[(1+i\alpha)g_- - 1\Big]E_- + \sqrt{2D_{sp}N}\,\xi_- \qquad (1b)$$

$$\dot{N} = \gamma\Big[J(t) - N - g_+|E_+|^2 - g_-|E_-|^2\Big] \qquad (1c)$$

where γ is carrier density relaxation rate, α is the linewidth enhancement factor [22]. The modal gains read

$$g_\pm = \frac{N \pm \varepsilon(N - N_c)}{1 + s|E_\pm|^2 + c|E_\mp|^2} , \qquad (2)$$

where ε determines the difference in differential gain among the two modes while N_c defines the carrier density where the unsaturated modal gains are equal. The parameters s and c are respectively the self- and cross-saturation coefficients. The $\xi_\pm$ are two independent, complex white noise processes with zero mean [$\langle\xi_\pm(t)\rangle = 0$] and unit variance [$\langle\xi_i(t)\xi_j^*(t')\rangle = \delta_{ij}\delta(t-t')$] that model spontaneous emission. The noise terms in Eqs. (1a) and (1b) are gauged by the spontaneous emission coefficient D_{sp}.

All quantities are expressed in suitable dimensionless units. In particular, time is normalized to the photons' lifetime, which for semiconductor laser is typically of the order of a few picoseconds or less (see e.g. [22, 23, 24])

A detailed analysis of the stationary solutions of Eqs. (1) is reported in Ref. [25]. For a constant bias current $J(t) = J_0$ and $D_{sp} = 0$, Eqs. (1) admit four different steady state solutions: the trivial one $E_\pm = 0$, two single-mode solutions — $E_+ \neq 0$, $E_- = 0$ and viceversa — and a solution where both modes are lasing, $E_\pm \neq 0$. For $N_c > 1$, and $c > s$, there exist a finite interval of J_0 values for which the two single–mode solutions coexist and are stable while the $E_\pm \neq 0$ is unstable (bistable region). Here, for $D_{sp} > 0$ the laser performs stochastic mode-hopping, with the total emitted intensity remains almost constant while each mode switches on and off alternately at random times. We point out that the emission in each mode is nonvanishing even in the "off" state, as the average power spontaneously emitted in each mode at any time is given by $4D_{sp}N$ [recall that Eqs. (1) are usually interpreted in Itô sense [23]]. Observation of this behaviour has been reported in several experimental works on EELs [26, 27, 28].

We remark that while Eqs. (1) aim at modeling EELs, the results presented henceforth would apply also to polarization switching in Vertical Cavity Surface Emitting Lasers (VCSELs). Indeed, experimental data [29] show strong similarities between this phenomenon and the longitudinal mode dynamics. On the theoretical side, this analogy is supported by the fact that the polarization dynamics in VCSELs is described by models that are mathematically similar to the one discussed here [30, 31, 32].

In the following, we will focus on the effect of the externally imposed fluctuation/modulation of the injected current. This situation is modeled by letting

$$J(t) = J_0 + \delta J(t) . \qquad (3)$$

The DC value J_0 sets the working point and will be always chosen to be in the bistability region. We focus on the case in which δJ is a Ornstein-Uhlenbeck process with zero average $\langle\delta J(t)\rangle = 0$ and correlation time τ:

$$\dot{\delta J} = -\frac{\delta J}{\tau} + \sqrt{\frac{2D_J}{\tau}}\,\xi_J \qquad (4)$$

that means

$$\langle\delta J(t)\delta J(0)\rangle = D_J \exp(-|t|/\tau) . \qquad (5)$$

This choice is suitable to model a finite-bandwith noise generator. Notice that τ and the variance of fluctuations $D_J = \langle\delta J^2\rangle$ can be fixed independentely.

Another case of experimental interest that we will consider is using the current modulation

$$\delta J = A\sin\Omega t \qquad (6)$$

To assess the nature of the stochastic process at hand, it is important to introduce the relevant time scales. We define first of all the switching or relaxation time T_R as the typical time for the emission to change from one mode to the other. The main quantities we are interested in are the Kramers or residence times $T_\pm$ defined as the average times for which the emission occurs in each mode. In semiconductor lasers $T_\pm$ are generally much larger than T_R. Typically, $T_R \sim 1 - 10ns$ while residence times may range between 0.1 and 100 μs [29, 33]. The third time-scale is of course given by the characteristic time of the external driving, namely, τ and $2\pi/\Omega$ respectively.

In the following, we will study how the hopping dynamics changes upon varying these latter parameters as well as the strength of the perturbation.

III. NUMERICAL SIMULATIONS

In this Section we present the outcomes of a series of numerical simulation of Eqs. (1). In Ref. [21] it was observed that the sensitivity of each of the $T_\pm$ on the imposed current fluctuations may be notably different depending on the parameters' choice. This is a typical signature of the multiplicative nature of the stochastic process. In particular, one can argue [21] that such

"simmetry-breaking" effects mostly depend on the ratio $\varepsilon\sigma/\delta$ where

$$\sigma \;=\; \frac{c+s}{2}, \qquad \delta \;=\; \frac{c-s}{2} \quad . \qquad (7)$$

The parameter σ represents the gain saturation induced by the total power in the laser, while δ describes the reduction in gain saturation due to partitioning of the power between the two modes.

The possibility of obtaining qualitatively different responses depending on the actual parameters corresponds to the different experimental observations reported for both EELs [21, 28, 33] and VCSELs [17, 29]. Those two classes of lasers were indeed found to display markedly different simmetry-breaking effects under current modulation. To account for those features, we consider two different sets of phenomenological parameters. For definiteness, in both cases we fix $\varepsilon = 0.1$, $s = 1.0$, $N_c = 1.1$, $\gamma = 0.01$ and change the values of c and D_{sp} (see Table I). The first set ($\delta = 0.05$) corresponds to the case in which added modulation changes the hopping time scale in an almost symmetric way. On the contrary, in the second case ($\delta = 0.15$) the asymmetry effect of the noise is stronger [21]. We can thus consider the two as representative of the VCSELs and EELs case respectively. The value of J_0 has been empirically adjusted to yield $T_+ \simeq T_- \equiv T_s$ and an almost symmetric distribution of intensities in absence of modulation. The actual values are about 10% above the laser threshold. The spontaneous emission coefficient D_{sp} has been chosen to yield a value of the residence times of the same order of magnitude of the experimental ones.

In the following, we decide to set $\alpha = 0$ which is appropriate for our EEL model where the phase dynamics is not relevant [21]. This choice may however not be fully justified for the VCSEL case. In this respect, the simulations presented below are representative of the VCSEL dynamics only in a qualitative sense. Nonetheless, it should be pointed out that a 1D Langevin model independent of α describes also the VCSEL case [30, 31]. Since resonant activation is mainly due to the multiplicative noise effect described by such equations [see Eq. (9) below] we consider this as an indirect proof that phenomenology we will report below should be observable also in the VCSEL case.

The largest part of the simulations were performed with Euler method with time steps 0.01-0.05 for times in the range $10^7 - 10^8$ time units depending on the values of τ and Ω. For comparison, some checks with Heun method [34] have also been carried on. Within the statistical accuracy, the results are found to be insensitive the the choice of the algorithm.

A. Stochastic modulation

Let us start illustrating the results in the case of stochastic current modulation (Eq. (4)). In Fig. 1 and

TABLE I: The parameter values used in the two series of simulations of Eqs. (1), the other values are given in the text.

c	D_{sp}	J_0	δ	σ
1.1	0.7×10^{-5}	1.197	0.05	1.05
1.3	1.5×10^{-5}	1.194	0.15	1.15

2 we report the measured dependence of the residence times $T_\pm$ on the correlation time τ for the two parameter sets given in Table I and different values of the noise variance D_J. In all cases, the curves display well-pronounced minima at an optimal value of τ. This is the typical signature of resonant activation. The minima are almost located between the relaxation time T_R and the hopping time T_s (marked by the vertical dashed lines). The values of T_R reported in the figures have been estimated from the reduced model discussed in the next Section, see Eq. (14) below.

The effect manifest in a different way for the second parameter set. In the case of Fig. 1 *both* times attain a minimum, albeit with different values. On the contrary the data of Fig. 2 show that one of the two times is hardly affected from the external perturbation regardless of the value of τ. In other terms, we can tune the current correlation in such a way that emission along only one of the two modes is strongly reduced (about a factor 10 in the simulation discussed here).

B. Periodic modulation

Let us now turn to the case of sinusoidal current modulation (Eq. 6). In Fig. 3 and 4 we report the measured dependence of the residence times $T_\pm$ on the frequency Ω for the two parameter sets given in Table I and different values of the amplitude A. For comparison with the previous case we choose A such that the RMS value of (6) is roughly equal to the variance of (4), i.e. $A \simeq \sqrt{2D_J}$.

As in the previous case, the curves display resonant activation at an optimal value of Ω. For the second set of parameters, one of the two hopping times is more reduced than the other (compare Fig. 4 with Fig. 2). It should be also noticed that the data in Fig. 2 display some statistical fluctuations while the curves for the periodic modulation are smoother.

IV. INSIGHTS FROM A REDUCED MODEL

In order to better understand the activation phenomenon it is useful to reduce the five–dimensional dynamical system (1) to an effective one-dimensional system. This has been accomplished in Ref. [21]. For completeness, we only recall here some basic steps of the derivation. In the first place, we introduce the change of
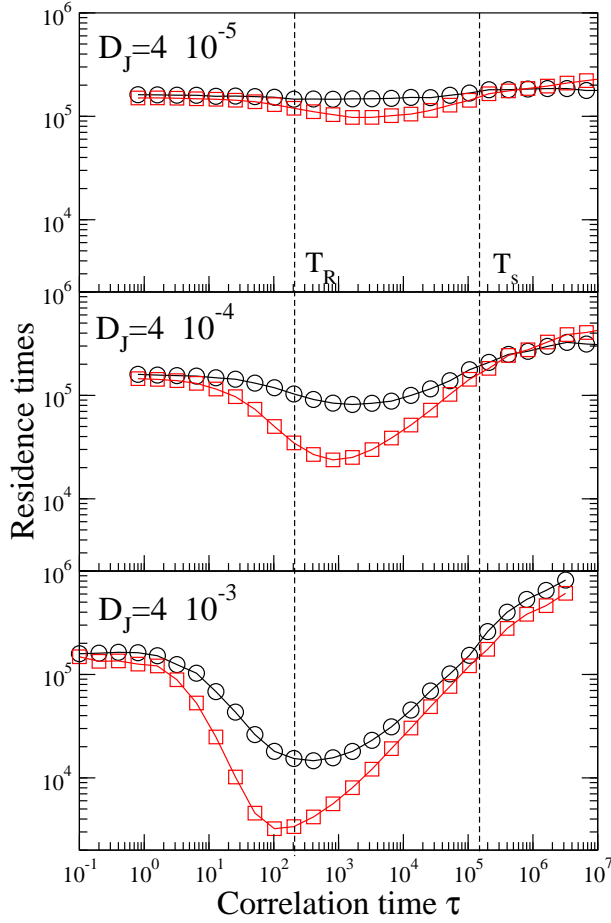
FIG. 1: (Color online) Simulations of the rate equations with Ornstein-Uhlenbeck current fluctuations, parameter set with $c = 1.1$ (see text and Table I): residence times T_+ (squares) and T_- (circles) for increasing values of the current variance D_J. The values of the relaxation time T_R and the hopping time T_s (in absence of modulation) are marked by the vertical dashed lines.
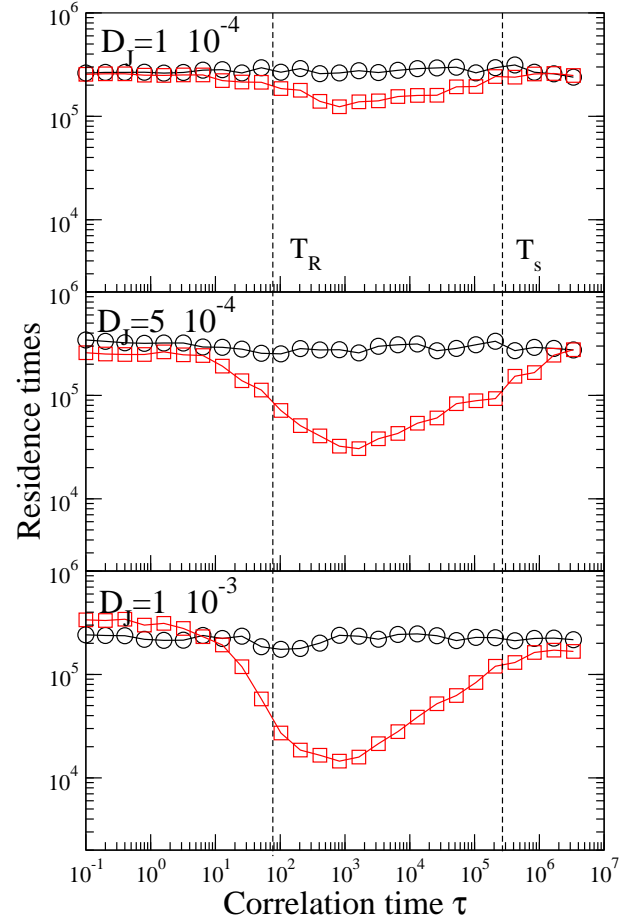
FIG. 2: (Color online) Simulations of the rate equations with Ornstein-Uhlenbeck current fluctuations, parameter set with $c = 1.3$ (see text and Table I): residence times T_+ (squares) and T_- (circles) for increasing values of the current variance D_J.

coordinates

$$E_+ = r \cos \phi \exp i\psi_+, \qquad E_- = r \sin \phi \exp i\psi_- . \quad (8)$$

In these new variables, r^2 is the total power emitted by the laser, and ϕ determines how this power is partitioned among the two modes. The values $\phi = 0, \pi/2$ correspond to pure emission in mode $+$ and $-$ respectively. The phases $\psi_\pm$ do not influence the evolution of the modal amplitudes and carrier density and can be ignored.

In order to simplify the analysis, we assume that (i) The difference between modal gains is very small, i.e., $N_c \gtrsim 1$, $\varepsilon \ll 1$, $c \gtrsim s$; (ii) the laser operates close enough to threshold, so that $r^2 \ll 1$ and the saturation term is small: in this limit, r and N decouple to leading order from ϕ; (iii) r and N can be adiabatically eliminated and (iv) only their fluctuations around the equilibrium values due to J are retained. This last assumption holds for weak spontaneous noise and amounts to say that r and

N are stochastic processes given by nonlinear transformations of J (see Eqs. (16) in Ref. [21]). This requires that J does not change too fast. For example, in the case of the Orstein–Uhlenbeck process, Eq. (4), τ should be larger than the relaxation time of the total intensity. The validity of the above reduction has been carefully checked against simulations of the complete model [21]. For the scope of the present work, we performed a further check by comparing the spectrum of fluctuations of r^2 with the imposed one, Eq. (4). Indeed, the behaviour is the same for $\tau > T_R$ while for shorter τ some differences are detected. This means that the reduced description discussed below becomes less and less accurate. On the other hand, in this regime spontaneous fluctuation should dominate and this limitation become less relevant for our purposes.

Altogether, the hopping dynamics is effectively one-dimensional and is described by the slow variable ϕ. Its
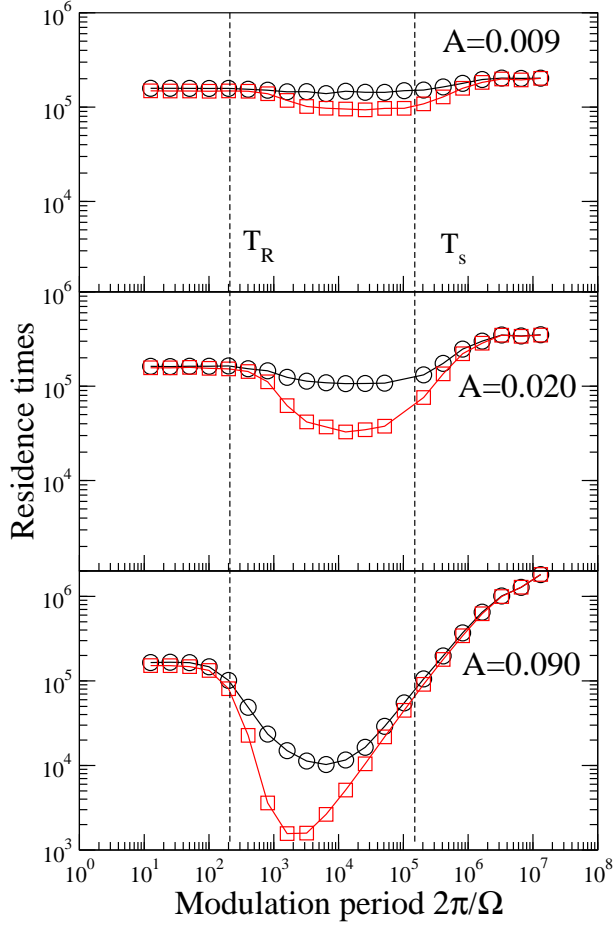
FIG. 3: (Color online) Simulations of the rate equations with sinusoidal modulation of the current, parameter set with $c = 1.1$ (see text and Table I): residence times T_+ (squares) and T_- (circles) for increasing values of modulation amplitude A.
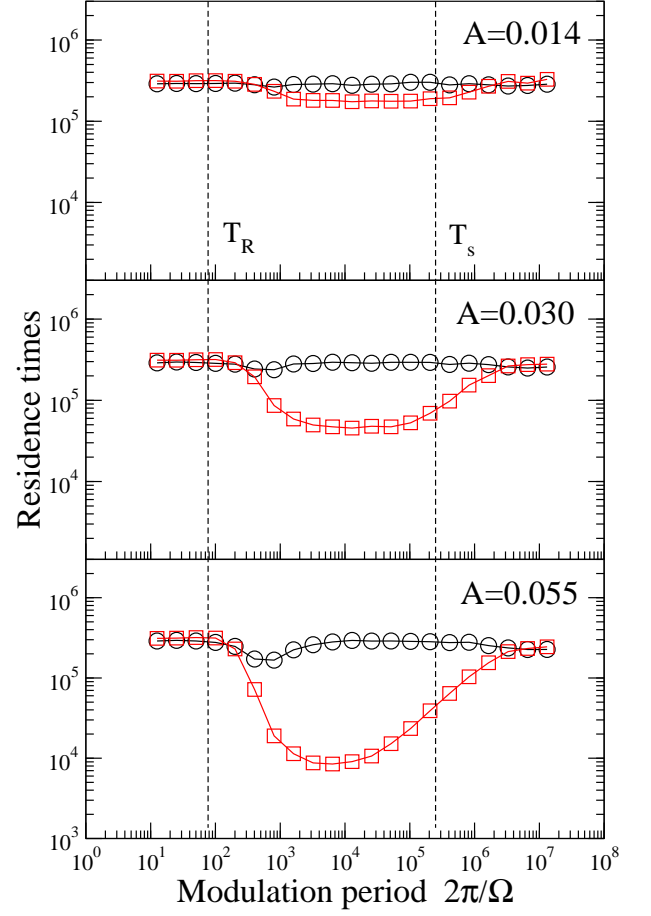

FIG. 4: (Color online) Simulations of the rate equations with sinusoidal modulation of the current, parameter set with $c = 1.3$ (see text and Table I): residence times T_+ (squares) and T_- (circles) for increasing values of modulation amplitude A.

evolution is ruled by the effective Langevin equation

$$\dot{\phi} = -\frac{1}{2}\Big[a\cos 2\phi + b\Big]\sin 2\phi + \frac{2D_\phi}{\tan 2\phi} + \sqrt{2D_\phi}\,\xi_\phi \quad (9)$$

where, together with (7) we have defined the new set of parameters

$$J_s = \frac{(1+\sigma)N_c - 1}{\sigma} \quad (10)$$

$$a = \frac{\delta}{1+\sigma}(J-1) \quad (11)$$

$$b = \frac{\varepsilon\sigma}{1+\sigma}(J - J_s) \quad (12)$$

$$D_\phi = \frac{(1+\sigma J)^2}{(1+\sigma)(J-1)}D_{sp} \quad . \quad (13)$$

We remind in passing that the same equation (9) has been derived by Willemsen et al. [30, 31] to describe polarization switches in VCSELs (see also Ref. [35] for a similar reduction). The starting point of their derivation

is the San Miguel-Feng-Moloney model [36]. The physical meaning of the variable ϕ is different from here as it represents the polarization angle of emitted light. This supports the above claim that, upon a suitable reinterpretation of variables and parameters, many of the results presented henceforth may apply also to the dynamics of VCSELs.

In absence of modulation ($\delta J = 0$), Eq. (9) is bistable in an interval of current values where it admits two stable stationary solutions $\phi_\pm$ and an unstable one ϕ_0 (double-well). This regime correspond to the bistability region of model (1). Notice that for $J_0 = J_s$, $b = 0$ the hopping between the two modes occurs at the same rate. The above definitions allows an estimate of relaxation time T_R defined above. This is is the inverse of the curvature of the potential in ϕ_0. For $J_0 = J_s$ this is straightforwardly evaluated to be

$$T_R \simeq \frac{(1+\sigma)}{\delta(J_s - 1)} \quad (14)$$

For the two parameter sets given in Table I one finds

$T_R = 210$ $T_R = 77.0$, respectively. These are the values emploied to draw the leftmost vertical lines in Figs. 1-4.

The effect of a time-dependent current is to make the coefficients a, b and D_ϕ fluctuating. It can be shown [21] that the effect on D_ϕ can be recasted as a renormalization of the intensity of the spontaneous-emission noise. However, for the parameters employed in the present work it turns out that this correction is pretty small and will be neglected henceforth by simply considering D_ϕ as constant [38]. For simplicity, we also disregard the dependence of D_ϕ on δJ in the drift term of Eq. (9). Under those further simplifications the Langevin equation can be rewritten as

$$\dot{\phi} = -U'(\phi) - V'(\phi)\,\delta J + \sqrt{2D_\phi}\,\xi_\phi \qquad (15)$$

where we have express the force term as derivatives of the "potentials"

$$U(\phi) = -\frac{\delta(J_0 - 1)}{16(1+\sigma)}\cos 4\phi - \frac{\varepsilon\sigma(J_0 - J_s)}{4(1+\sigma)}\cos 2\phi$$
$$- D_\phi \ln \sin 2\phi \qquad (16)$$
$$V(\phi) = -\frac{\delta}{16(1+\sigma)}\cos 4\phi - \frac{\varepsilon\sigma}{4(1+\sigma)}\cos 2\phi. \quad (17)$$

Langevin equations of the form (15) with (4) have been thoroughly studied in the literature (see e.g. [7, 8, 9, 10, 11] and references therein) as prototypical examples of the phenomenon of activated escape over a fluctuating barrier. In view of their non-Markovian nature, their full analytical solution for arbitrary τ is not generally feasible. Several approximate results can be provided in some limits.

For an arbitrary choice of the parameters, V has a different symmetry with respect to U meaning that the effective amplitude of multiplicative noise is different within the two potential wells. If this difference is large enough, current fluctuation will remove the degeneracy between the two stationary solutions. This is best seen by computing the istantaneous potential barriers $\Delta U_\pm(t)$ close to the symmetry point $J_0 = J_s$. For weak noise and $\delta J \ll (J_s - 1)$, they are given to first-order in $\delta J(t)$ by

$$\Delta U_\pm(t) \simeq \frac{\delta}{8(1+\sigma)}(J_s - 1) + \frac{\delta \pm 2\varepsilon\sigma}{8(1+\sigma)}\delta J(t). \quad (18)$$

Obviously, this last expression makes sense only when the fluctuating term is sub-threshold i.e. whenever the system is bistable. In the case of periodic modulation, formula (18) allows estimating the range of amplitude values for a sub-threshold driving

$$A < \frac{\delta(J_s - 1)}{\delta \pm 2\varepsilon\sigma} \quad . \qquad (19)$$

Using this condition, along with the parameter values at hand, we deduce that the cases displayed in lower panels of Figs. 3 and 4 correspond to superthreshold driving.

However, while the minima are much more pronounced than in the other panels, there is no qualitative difference in the system response. In the case of stochastic modulation, the same remark applies in a probabilistic sense for the last panels of Figs. 1 and 2.

Altogether, the mode switching can be seen as an activated escape over fluctuating barriers given by Eq. (18). The statistical properties of the latter process is controlled by the current fluctuations. We now discuss the properties of various regimes. For simplicity, we refer to the case of stochastic modulations. Most of the remarks and formulas reported in the following Subsection should apply also to the periodic case by replacing τ and D_J with $2\pi/\Omega$ and $A^2/2$ whenever appropriate.

A. Fast barrier fluctuations: $\tau < T_R \ll T_\pm$

As we already pointed out, in this regime the reduction to Eq. (15) is not justified. We may thus only expect some qualitative insight on the behaviour of the rate-equations. From a mathematical point of view, some analytical approximations for equations like (15) are feasible in this limit (see e.g. Ref. [8] for the stochastic case). For our purposes, it is sufficient to note that in this regime the effect of δJ is hardly detected for both types of driving (see again Figs. 1-4). Note also that working at D_J fixed means that for $\tau \to 0$ the fluctuation become negligible.

B. Resonant activation: $T_R < \tau \ll T_\pm$

If $T_R < \tau$ we are in the colored noise case. The problem is amenable of a kinetic description which amounts to neglect intrawell motion and reduce to a rate model describing the statistical transitions in terms of transition rates. If we consider τ as a time scale of the external driving we can follow the terminology of Ref. [39] and refer to this situation as the "semiadiabatic" limit of Eq. (15).

In this regime, the residence time is basically the shortest escape time, which in turn correspond to the lowest value of the barrier (the noise is approximatively constant in the current range considered henceforth). For the case of interest, $\delta < 2\varepsilon\sigma$ we can use (18) to infer that the minimal values of $\Delta U_\pm$ should be attained for $\delta J \propto \mp\sqrt{D_J}$ respectively. This yields

$$T_\pm \simeq T_s \exp\left[-K\frac{2\varepsilon\sigma \pm \delta}{1+\sigma}\frac{\sqrt{D_J}}{D_\phi}\right] \qquad (20)$$

where K is a suitable numerical constant. Notice that δ controls the asymmetry level: if $\delta \ll 2\varepsilon\sigma$ the two residence times decrease at approximatively the same rate. This prediction is verified in the simulations and also in the experiment [21].

As a further argument in support of the above reasoning, we also evaluated the probability distributions

of the residence times obtained from the simulation of the rate equations. In Fig. 5, we show two representative cumulative distributions. The data are well fitted by a Poissonian $P(T) = 1 - \exp(-T/T_\pm)$ for both the stochastic and periodic modulation cases. This confirms that hopping occours preferentially when a given (minimal) barrier occurs.
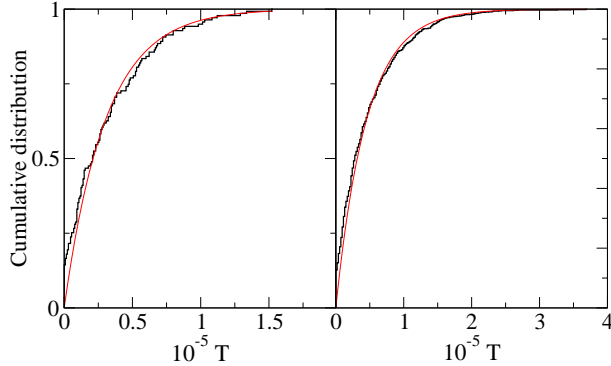

FIG. 5: (Color online) Cumulative distributions of the residence times in the resonant activation region, parameter set with $c = 1.3$ (see text and Table I). Left panel: stochastic modulation with $D_J = 5 \times 10^{-4}$, $\tau = 1.638 \times 10^3$. Right panel: periodic modulation with $A = 0.03$ and period 1.286×10^4. We report only the histograms for the times whose averages are denoted by T_+ in the text. Solid line is the cumulative Poissonian distribution with the same average.

C. Slow barrier, frequent hops: $T_R \ll T_\pm \ll \tau$

This corresponds to the adiabatic limit in which the time scale of the external driving is slower than the intrinsic dynamics of the system [39]. To a first approximation we can here treat current fluctuations in a parametric way. Correction terms may be evaluated by means of a suitable perturbation expansion in the small parameter $1/\tau$ [10]. If δJ is small enough for the expression (18) to make sense, the escape time can be estimated as the average of escape times over the distribution of barrier fluctuations, i.e. $\langle T_\pm \rangle_{\delta J}$. For the case of Eq. (4), the variable δJ is Gaussian and we can use the identity $\langle \exp \beta z \rangle = \exp(\beta^2 \langle z^2 \rangle /2)$ to obtain [11]

$$T_\pm \simeq T_s \exp\left[\frac{2(\delta \pm 2\varepsilon\sigma)^2}{(1+\sigma)^2 D_\phi^2} D_J\right]. \quad (21)$$

This reasoning implies that for large τ the residence times should approach two different constant values. A closer inspection of the graphs (in linear scale) reveals that this is not fully compatible with the data of Fig.1 even for the smallest value of D_J. In several cases, $T_\pm$ continue to increase with τ and no convincing evidence of saturation is observed. We note that the same type of behaviour was already observed in the analog simulations data of

Ref. [11]. There, an increase of hopping times duration at large τ was found. The Authors of Ref. [11] explained this as an effect of a too large value of the noise fluctuation forcing the system to jump roughly every τ. We argue that the same explanation holds for our case. This is also consistent with the fact that the exponential factors in Eq. (21) evaluated with the simulation parameters turn out to be much larger than unity.

V. CONCLUSIONS

In this paper, we have explored numerically and analytically the effects of external current fluctuations on the mode-hopping dynamics in a model of a bistable semiconductor laser. To the best of our knowledge, this setup provides the first theoretical evidence of resonant activation in a laser system. As the phenomenon has hardly received any experimental confirmation in optics, we believe that our study may open the way to future research in this subfield.

The model we investigated is based on a rate-equation description, where the bias current enters parametrically into the evolution of the modal amplitudes. We considered, two kinds of current flutuations, namely, a stochastic process ruled by an Orstein-Uhlenbeck statistics, and a coherent, sinusoidal modulation. These choices are motivated by the aim of proposing a suitable setup for an experimental verification of our results. Upon varying the characteristic time-scale of the imposed fluctuations, we have shown that the residence times attain a minimum for a well-defined value, which is the typical signature of resonant activation. The magnitude of the effect can be different depending on the parameters of the model. Moreover, the response of the system appears very much similar for both periodic and random modulations.

The reduction of the rate equations to a one-dimensional Langevin equation allowed us to recast the problem as an activated escape over a fluctuating barrier. To first approximation, the fluctuating barrier (multiplicative term) is mainly controlled by current modulations while the spontaneous noise act as an additive source. This simplified description has allowed us to draw some predictions (e.g. the dependence of residence times on noise strength) and to better understand the role of the physical parameters. Given the generality of the description, our results should apply to a broad class of multimode lasers, including both Edge Emitting and Vertical Cavity Lasers.

From an experimental point of view, driving the laser in a orders-of-magnitude wide range of time-scales is more feasible in the case of a sinusoidal modulation than for a colored, high frequency noise. However, given the evidence of a resonant activation phenomenon for such modulation, our results indicate that it occurs almost for the same parameters in the case of colored noise, provided that the RMS of the modulations equals the amplitude of the added noise. Thus, the phenomenon could be fully

exploited along those lines. Since the reported experimental evidences of the phenomenon are so far scarce, we hope that the present work could suggest a detailed characterization in optical systems that allows for both very precise measurements and careful control of parameters.

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