Journées–Séminaire

Sur

Traitements Algébriques et Informatiques des Séries Formelles Non Commutatives

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Organisateurs :

G. Jacob
LIFL U.A. 369 du CNRS/Université de Lille I

et

C. Reutenauer
LITP U.A. 248 du CNRS/Université Paris VII

Université des Sciences et Techniques de Lille Flandres Artois
U.F.R. d'I.E.E.A. Bât. M3, 59655 Villeneuve d'Ascq Cedex. Tel. 20 43 44 92.

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on

"Traitements Algébriques et Informatiques des Séries Formelles Non Commutatives".

University of Lille, december 12-13th, 1988.

Abstracts of the conferences:

M.P. DELEST (Bordeaux) **Utilisation de MACSYMA dans l'Enumération des Polyominos.**

We present some applications of formal calculus to the enumeration of planar pictures called polyominoes.

Using the Schutzenberger's methodology (coding words of algebraic languages and taking commutative images), we get results about the number of polyominoes according to the perimeter. Moreover, we use the notation of attribute grammar for the computation of q-analog functional equations which solutions are generating functions of polyominoes according to the area.

All along this talk, we point out some problems we met working with MACSYMA.

Gérard DUCHAMP (Paris) **Projecteur orthogonal sur l'Algèbre de Lie libre.**

As an answer to a question of A. Garsia, we give a universal formula in a "normal form" for the orthogonal projection of the free associative algebra onto the free Lie Algebra.

M. GOLDWURM (Milano) **Counting functions and formal series in noncommuting variables.**

In this work we study the complexity of certain counting functions related to formal series in noncommuting variables. For every $r \in Z\langle\langle\Sigma\rangle\rangle$, we consider the function $f_r : \{1\}^* \longrightarrow Z$ such that $f_r(1^n) = \sum_{|w|=n} (r,w)$, for every $n \in N$. In particular, if r is the characteristic function of a language L on the alphabet Σ, then $f_r(1^n)$ is the number of strings of length n in L. The complexity of this counting function is an interested subject related to classical problems of combinatorial enumeration and other topics of computational complexity. In particular, counting functions are related to the ranking problem, i.e. given a language L on Σ, compute the function $r_L(x) = \#\{y \in L / y \leq x\}$, where $\leq$ denotes the lexicographic ordering on Σ^*.

We prove that, for every algebraic formal series $r \in Z\langle\langle\Sigma\rangle\rangle$, the problem of computing f_r is NC^1-reducible to integer division. Therefore, for every unambiguous context-free language L on the alphabet Σ, the function f_L defined by $f_L(1^n) = \#\{x \in L / |x| = n\}$

Christiane HESPEL (Rennes) **Approximation des systèmes dynamiques non linéaires par des séries rationnelles.**

Given an analytic system, we compute a bilinear system of minimal dimension which approximates it up to order k (i.e. the outputs of these two systems have the same Taylor expansion up to order k).

The algorithm is based on noncommutative series computation : call s the generating series of the analytic system ; then a rational series g is constructed, whose coefficients are equal to those of s, for words of length not greater than k. These words of length not greater than k are encoded, which allows to deal more easily with the Hankel matrices of s and g. Then with g we associate a bilinear system, which is a solution of our problem.

Another method may be used for computing a bilinear system which approximates a given analytical system (S) : with (S) we associate a vector fields R-automaton and we build the truncated automaton by cancelling all the states which have the following property : the length of the shortest successful path labelled by a word that gets through this state is > k . Then, the number of states of this truncated automaton gives a dimension (not necessarily minimal) of the state space.

Vincel HOANG NGOC MINH (Lille) **Eléments d'un calcul symbolique pour les systèmes dynamiques non linéaires. (Transformation de Jacob).**

Given a nonlinear analytical dynamic system (affine with respect to the input), one can view its output function as a signal, parametrized by the primitives of the input functions. This signal can be formally described by its generating series, that is a power series in noncommuting variables. Hence we obtain a symbolic transform that generalizes Laplace transform of signals that depends only on the time. We develop here the basic tools of that symbolic calculus. We prove a correspondance theorem between certain convolutions of signals and Cauchy products of generating power series.

Gérard JACOB(Lille) **Calcul du produit de mélange de deux séries formelles données par leur description matricielle.**

It is known that the input/output relation of an analytical dynamic system (affine in the input) is completely described by its (noncommutative) "generating power series". On the other hand, any generating series can be written as an infinite sequence of arrow matrices, each of which describes a linear form on some tensor power of a "symbolic" vector space.

In this paper, we give a complete recurrent definition, in matrix form, of the shuffle product of two noncommutative formal power series.

Pierre Vincent KOSELEFF (IBM-Paris) **Jeux de mots dans les algèbres de Lie libres : quelques bases et formules.**

We intent to compare efficiency of several methods allowing computations in free Lie Algebras. There are different ways to construct a basis of a finite generated free Lie Algebra. We use here, respectively, Hall basis and Lyndon basis, in order to compute some Campbell-Hausdorf formulas. We have written several implementations of these computations in Lisp, and recently in SCRATCHPAD II.

Daniel KROB (Rouen) **Quelques exemples de séries formelles utilisées en algèbre non commutative.**

We study Hankel properties of different formal series used in non commutative algebra. First we recall an old result we proved: it deals with an Hankel characterization for pseudo-differential formal operators which are rational differential. Secondly, we look for a same characterization for K[X;s]. Finally we explore Hankel results for Malcev series : we begin by two negative results and we end by an Hankel result for Malcev series on a commutative group.

Nour-éddine OUSSOUS (Lille) **Calcul, sur MACSYMA, de la représentation différentielle minimale des polynômes non commutatifs.**

We present here a package of Macsyma programs, allowing the manipulation of words, and noncommutative power series over some finite alphabet.

On the basis of M. Fliess and C. Reutenauer, concerning the local minimal realization of analytical systems, we present an algorithm allowing to compute the local and minimal realization of finite generating power series. We describe that algorithm in the algebraic computation language MACSYMA.

Bruno SALVY et Paul ZIMMERMANN (INRIA, Rocquencourt)
Lambda-Upsilon-Oméga : an assistant algorithms analyser.

Lambda-Upsilon-Omega, $\Lambda\Upsilon O$, is a system designed to perform automatic analysis of well-defined classes of algorithms operating over "decomposable" data structures.

It consists of an 'Algebraic Analyser' System that compiles algorithms specifications into generating functions of average costs, and an 'Analytic Analyser' System that extracts asymptotic informations on coefficients of generating functions. The algebraic part relies on recent methodologies in combinatorial analysis based on systematic correspondances between structural type definitions and counting generating functions. The analytic part makes use of partly classical and partly new correspondences between singularities of analytic functions and the growth of their Taylor coefficients.

The current version $\Lambda\Upsilon O_0$ of $\Lambda\Upsilon O$ implements as basic data types, term trees as encountered in symbolic algebra systems. The analytic analyser can treat large classes of functions with explicit expressions. In this way, $\Lambda\Upsilon O_0$ can generate in the current stage about a dozen non-trivial average case analyses of algorithms like: formal differentiation, some algebraic simplifications and matching algorithms. Its analytic analyser can determine asymptotic expansions for large classes of generating functions arising in the analysis of algorithms.

The outline of a design for a full system is also discussed here. The long term goal is to include a fairly rich set of data structuring mechanisms including some general recursive definitions, and have the analytic analyser treat wide classes of functional equations as may be encountered in combinatorial analysis and the analysis of algorithms.

Gérard VIENNOT **A combinatorial approach to nonlinear functionnal expansions : an introduction with an example.**

In this paper, we present a new approach to causal functionals. We introduce combinatorial interpretations of the solution of nonlinear equations with forcing terms. This theory parallels the algebraic approach with formal power series in noncommuting variables developed by Fliess, Lamnabhi, and Lamnabhi-Lagarrigue.

This theory make use of certain combinatorial objects called *weighted increasing trees*, *weighted paths* and *histories*. We can deduce very efficient algorithms for the computation of the corresponding Volterra kernels.

In this paper, we just give an introduction to our combinatorial theory. An example with a nonlinear circuit will give the flavor of our approach. The complete proofs and general theory will be exposed in another paper.