EECS 461, Winter 2023, Problem Set 3: SOLUTIONS[1]

1. (a) SOLUTION: The plot is in Figure 1.

   (b) SOLUTION: We see from Figure 1 that $|H(j20)| = 0.1$ (-20 db) and $\angle H(j20) = -85°$.

   (c) SOLUTION: By definition the signal u_1 has zero crossings at those times for which $\sin(\omega_0 t) = 0$, and thus for which $\omega_0 t$ is an integer multiple of 2π. Hence the zero crossings are at times $t_1 = N\pi/\omega_0, N = 0, \pm 1, \pm 2, \ldots$. The signal u_2 has zero crossings at those times for which $\omega_0 t + \phi$ is an integer multiple of 2π. Hence the zero crossing for $u_2(t)$ are at times $t_2 = (N\pi - \phi)/\omega_0, N = 0, \pm 1, \pm 2, \ldots$. Hence the difference between successive zero crossings may be obtained by comparing t_1 and t_2 for the same value of N, yielding $\Delta t = t_2 - t_1 = -\phi/\omega_0$. Note that u_2 will *lag* u_1 if $\phi < 0$, and will *lead* u_1 if $\phi > 0$.

   (d) SOLUTION: The time response is plotted in Figure 2. Note that, after an initial transient which decays with the time constant of the filter, the steady state output response has an amplitude of 0.1 and lags the input by almost 90°. These values do agree with the frequency response plots.

   (e) SOLUTION: Frequency and time response plots for $\tau = 0.1$ are shown in Figures 3-4 and for $\tau = 10$ in Figures 5-6. For $\tau = 0.1$, the filter has a higher bandwidth, and thus does not cause as much attenuation or phase lag. For $\tau = 10$, the filter has lower bandwidth, and causes much greater attenuation.

[1] Revised November 5, 2022.

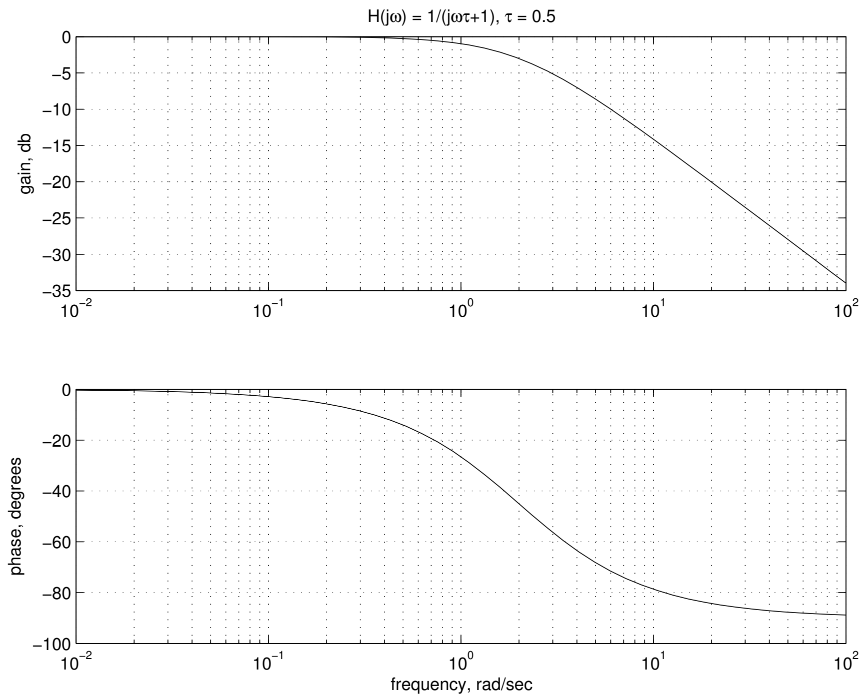

Figure 1: Frequency Response for $H(j\omega) = 1/(j\omega\tau + 1), \tau = 0.5$.

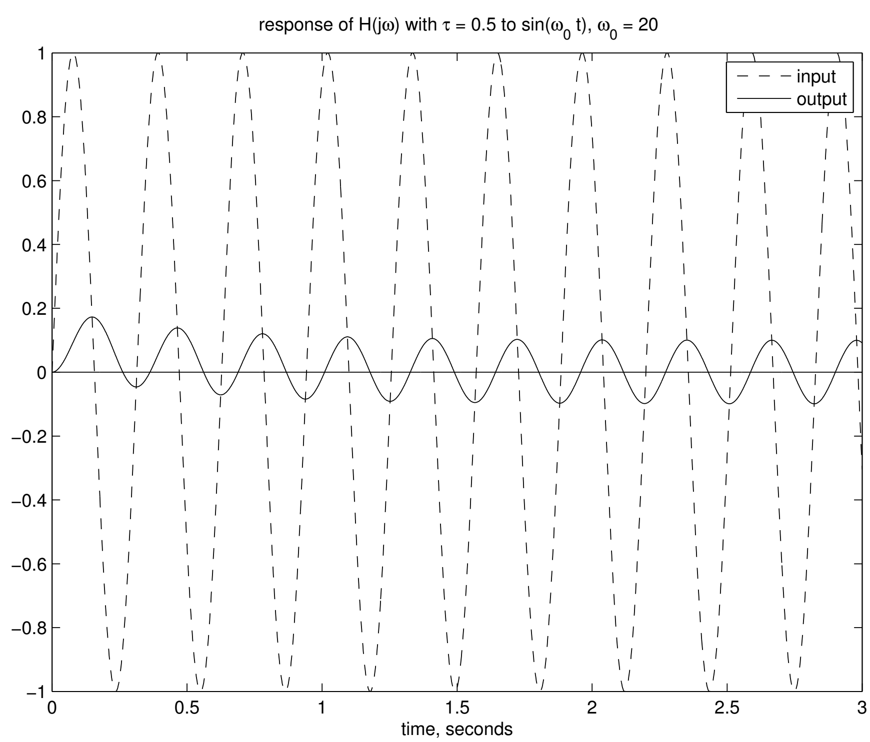

Figure 2: Time Response with $\tau = 0.5$ for Input $u(t) = \sin 20t$.

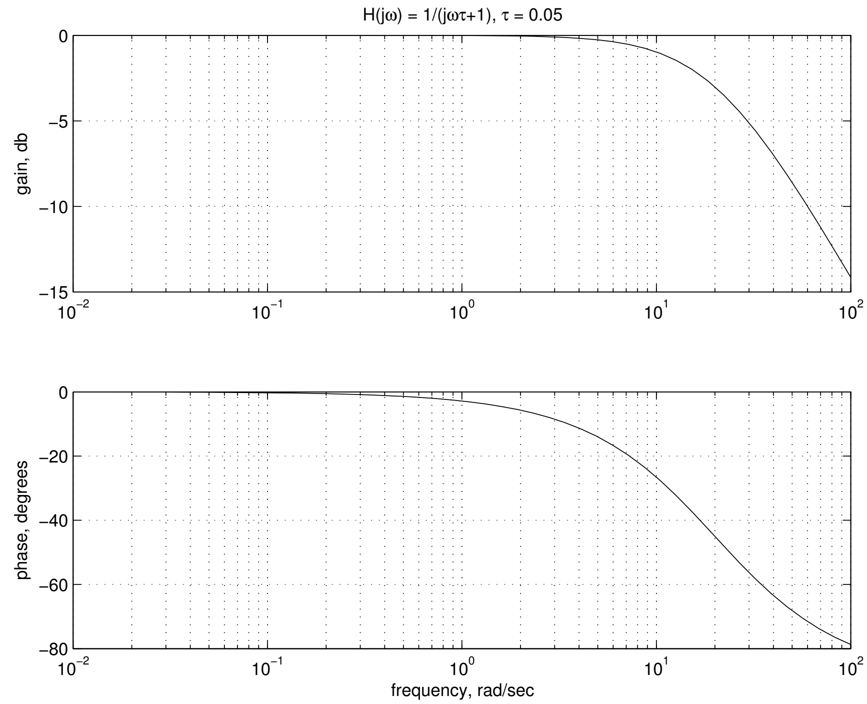

Figure 3: Frequency Response for $H(j\omega) = 1/(j\omega\tau + 1), \tau = 0.05$.

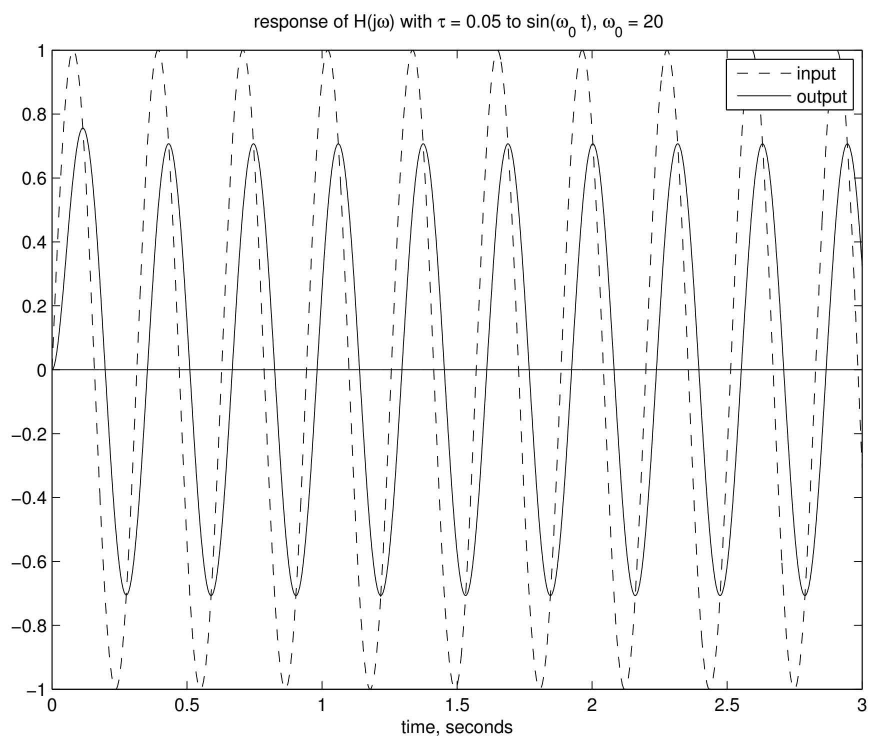

Figure 4: Time Response with $\tau = 0.05$ for Input $u(t) = \sin 20t$.

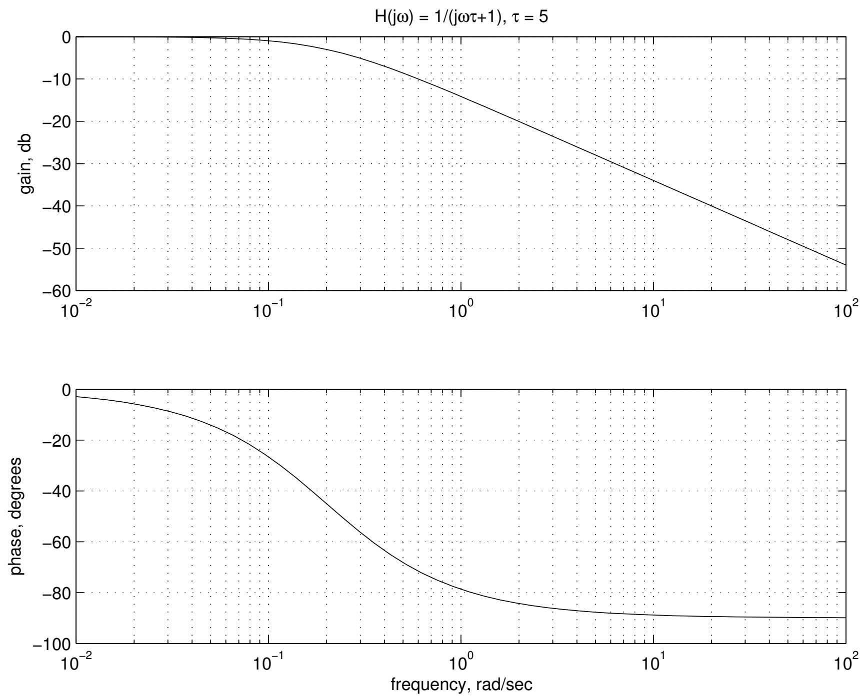

Figure 5: Frequency Response for $H(j\omega) = 1/(j\omega\tau + 1), \tau = 5$.

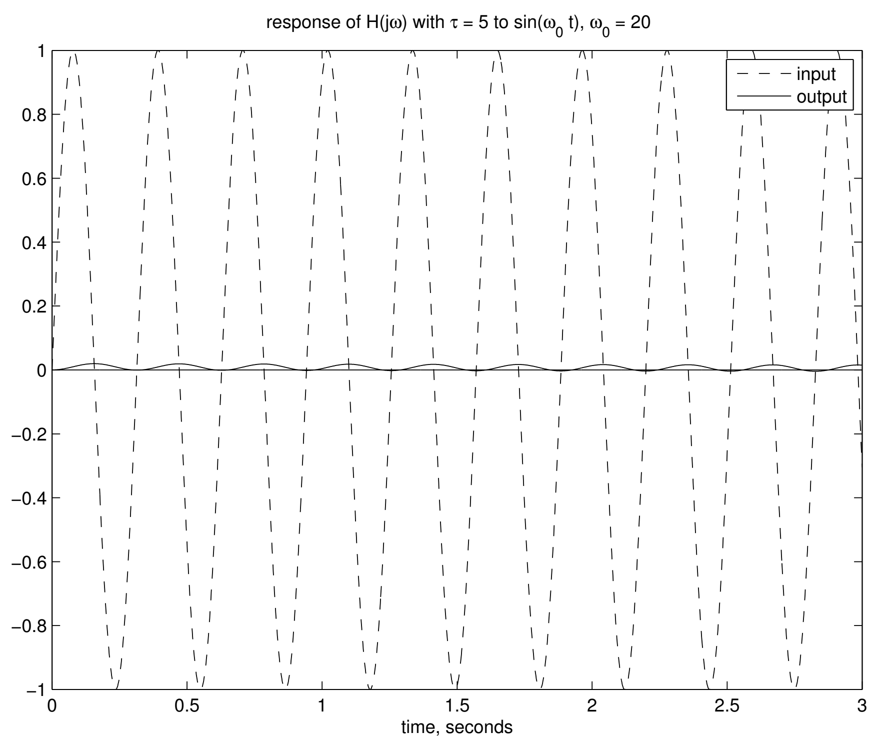

Figure 6: Time Response with $\tau = 5$ for Input $u(t) = \sin 20t$.

2. (a) SOLUTION: The torque-speed curves do not depend upon either the shaft inertia, J, or the armature inductance, L because these parameters only affect the transient response of the motor. They determine how long it takes the motor to achieve steady state, but not the values of steady state speed and torque.

   (b) SOLUTION: Figure 7 shows several values of steady state speed and torque as the load torque is varied.

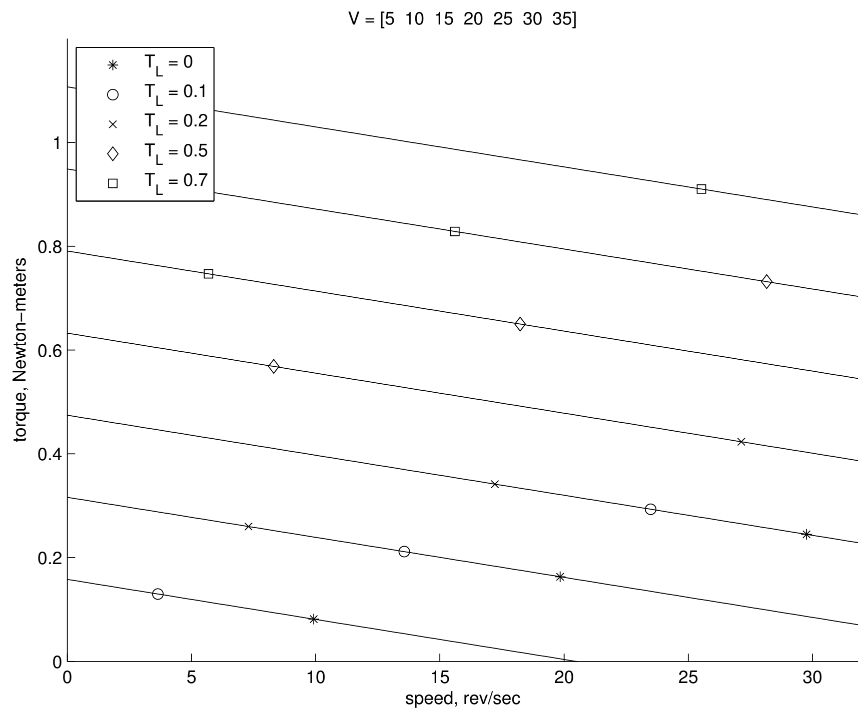

Figure 7: Torque-Speed Curves

   (c) SOLUTION: The frequency response plots are shown in Figure 8, and the magnitude is plotted in absolute units in Figure 9. We see that the DC gain of the speed response is 1.98 (rev/sec)/V, and the DC gain of the torque response is 0.0163 N-m/V. Hence the steady state response to an input of one volt will be $\Omega = 1.98$ (rev/sec) and $T_M = 0.0163$ N-m. These values together with the stated values of K_M, K_V, and R may be used to show that the torque-speed equation is satisfied (remember to convert Ω back to units of rad/sec).

   (d) SOLUTION: In Figure 10 we plot the response of the motor to a load torque applied after the system has already achieved a steady state with no load in response to a constant input of 10 V. Using the Matlab command "ginput", we determine that the unloaded steady state response satisfies $\Omega = 20$ rev/sec, and $T_M = 0.164$ N-m. With a load torque $T_L = 0.2$ N-m, the steady state response becomes $\Omega = 7.3$ rev/sec, $T_M = 0.26$ N-m. These values are in agreement with the torque-speed curves in Figure 7.

   (e) SOLUTION: It follows from Figure 9 that the DC gain of the speed response is equal to 1.98 (rev/sec)/V, the gain at 25kHz is 9.8×10^{-5} (rev/sec)/V, and the ratio of these gains is thus 4.9×10^{-5}. Similarly, the DC gain of the torque response is 0.017 (N-m/V) and the gain at 25kHz is 6.8×10^{-4} (N-m/V). It follows that the relative attenuation of the 25kHz component of the PWM input (with respect to the gain of the DC component) is an order of magnitude greater for the speed response than for the torque response. This is reasonable physically, because the speed signal is filtered through the mechanical dynamics as well as the electrical dynamics, and thus receives additional filtering. As a consequence, we see essentially no switching in the speed response of Figure 11, but we do see some

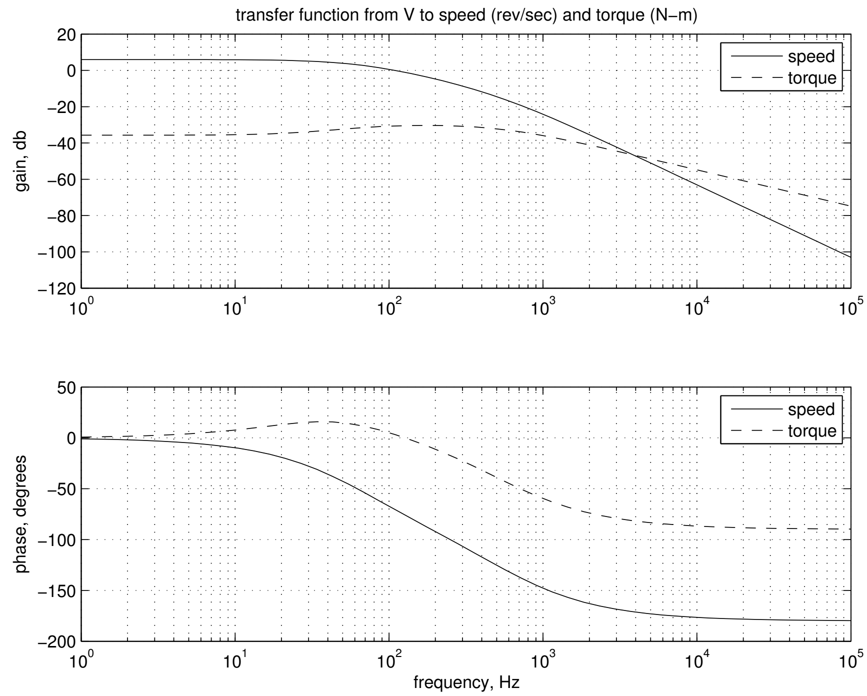

Figure 8: Frequency Response of DC Motor

switching noise in the torque response (we plot 100 times the torque response so we can plot both responses on the same axes.)

(f) SOLUTION: The steady state response of speed is about 4.95 rev/sec, and that of torque is about 0.041 N-m. We may compute these responses as follows. A PWM signal with amplitude 5 and a 100% duty cycle corresponds (approximately) to a step input of amplitude 5. Since the speed response of the motor has DC gain equal to 1.98 (rev/sec)/V, such a full duty cycle will result in a steady state speed of 9.9 rev/sec. More generally, a $100x\%$ ($0 \leq x \leq 1$) duty cycle will thus result (approximately) in a steady state speed of $9.9x$ rad/sec. For example, a 50% duty cycle will result in a steady state shaft speed of about 4.95 rev/sec, which is consistent with Figure 11. Similar remarks apply to the torque response.

To show that these responses satisfy the torque-speed equation, we merely compute that $0.041 + (K_M K_V/R) 4.95 \times 2\pi = 0.0791$, which in turn equals $(K_M/R) \times 2.5$. Alternately, we may redraw the torque-speed curves as shown in Figure 12

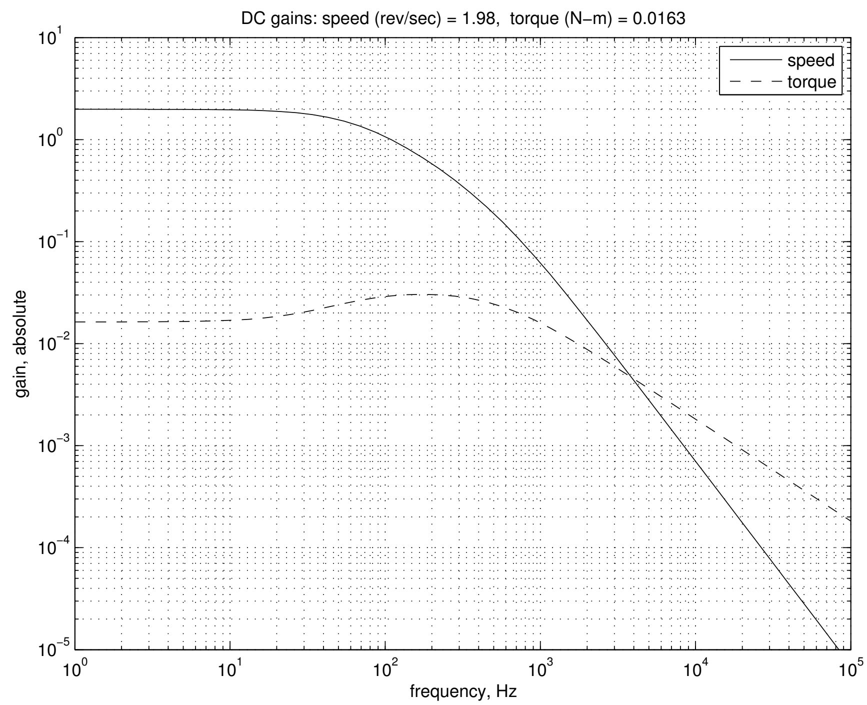

Figure 9: Magnitude Response of DC Motor, absolute units

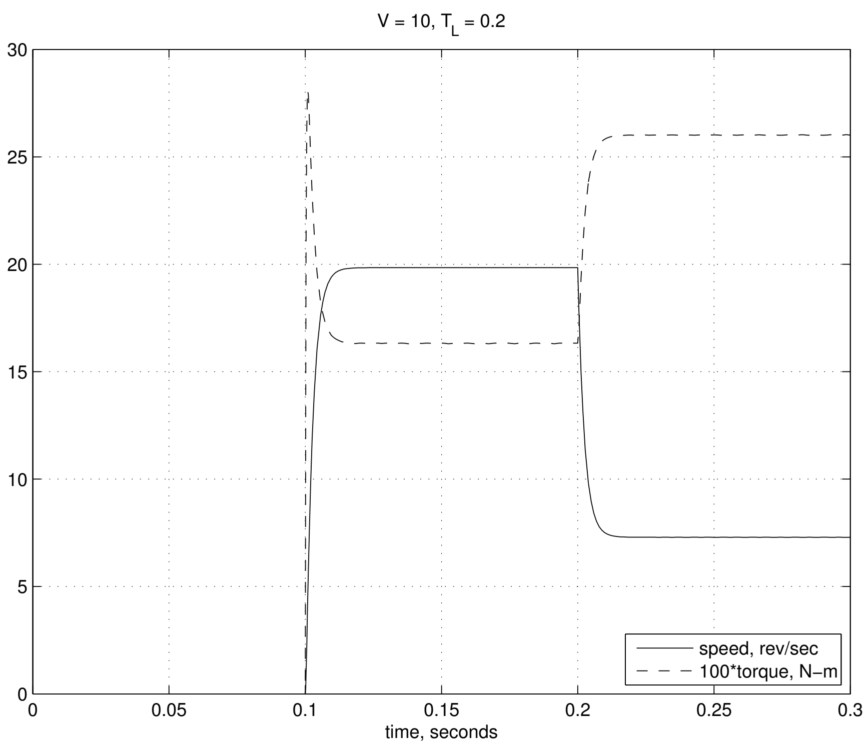

Figure 10: Response of Speed and Torque to a Load

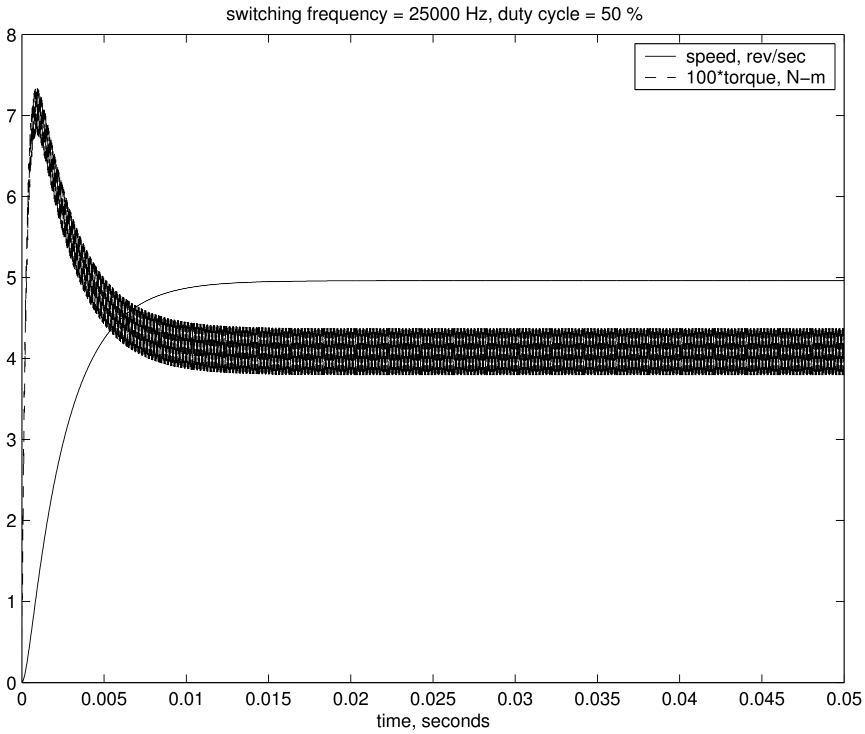

Figure 11: PWM with 25 kHz Switching Frequency

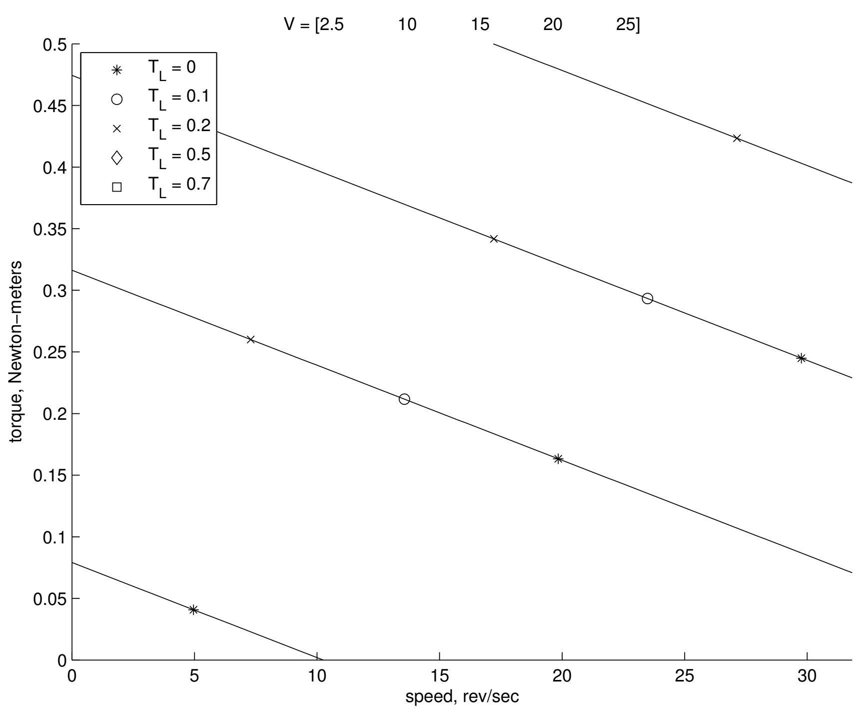

Figure 12: Torque-Speed Curves with $V = 2.5$.

3. (a) To compute the transfer functions from load torque to speed and motor torque, we simply set $V = 0$ and solve for Ω and T_M in terms of T_L. The following three equations may be obtained directly from the block diagram with $V = 0$:

$$\Omega(s) = \left(\frac{1}{sJ + B}\right)(-T_L(s) + T_M(s)) \tag{1}$$

$$T_M(s) = \left(\frac{K_M}{sL + R}\right)(0 - V_B(s)) \tag{2}$$

$$V_B(s) = K_V \Omega(s). \tag{3}$$

Substituting (3) and (2) into (1) yields

$$\Omega(s) = \left(\frac{-1}{sJ + B}\right) T_L(s) + \left(\frac{-K_M K_V}{(sJ + B)(sL + R)}\right)\Omega(s). \tag{4}$$

Solving (4) for Ω yields

$$\Omega(s) = \frac{\frac{-1}{(sJ+B)}}{1 + \frac{K_M K_V}{(sL+R)(sJ+B)}} T_L(s). \tag{5}$$

Similar manipulations yield

$$T_M(s) = \frac{\frac{K_M K_V}{(sL+R)(sJ+B)}}{1 + \frac{K_M K_V}{(sL+R)(sJ+B)}} T_L(s). \tag{6}$$

(b) The plots are shown in Figures 13-14. Note from the phase plot in Figure 13 that the DC gain of the response of speed to load torque is negative because the phase at $\omega = 0$ is equal to $-180°$. (The DC gain stated at the top of Figure 14 is actually the absolute value of the DC gain.)

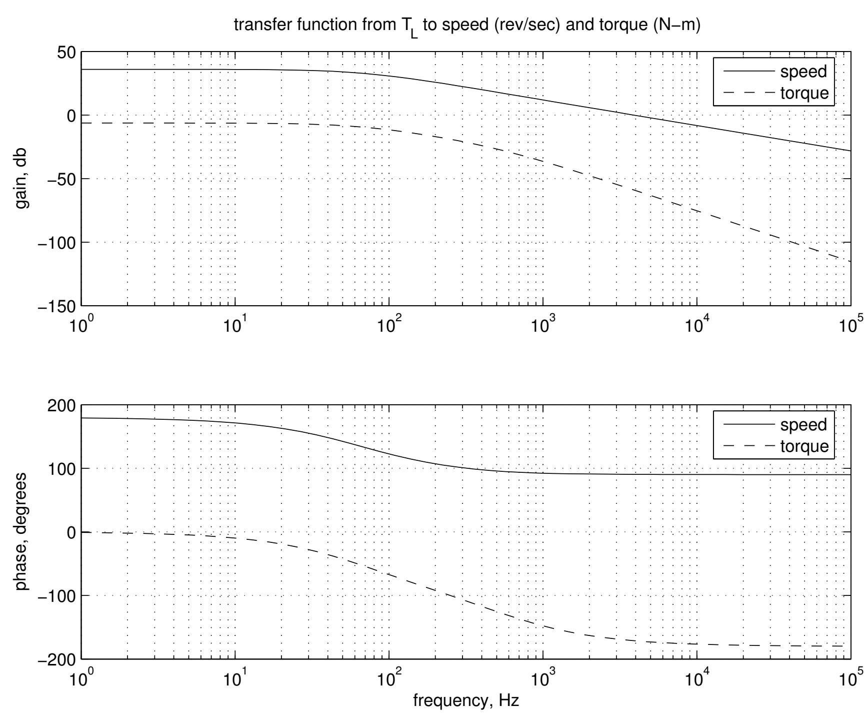

Figure 13: Frequency Response of DC Motor Response to Load Torque

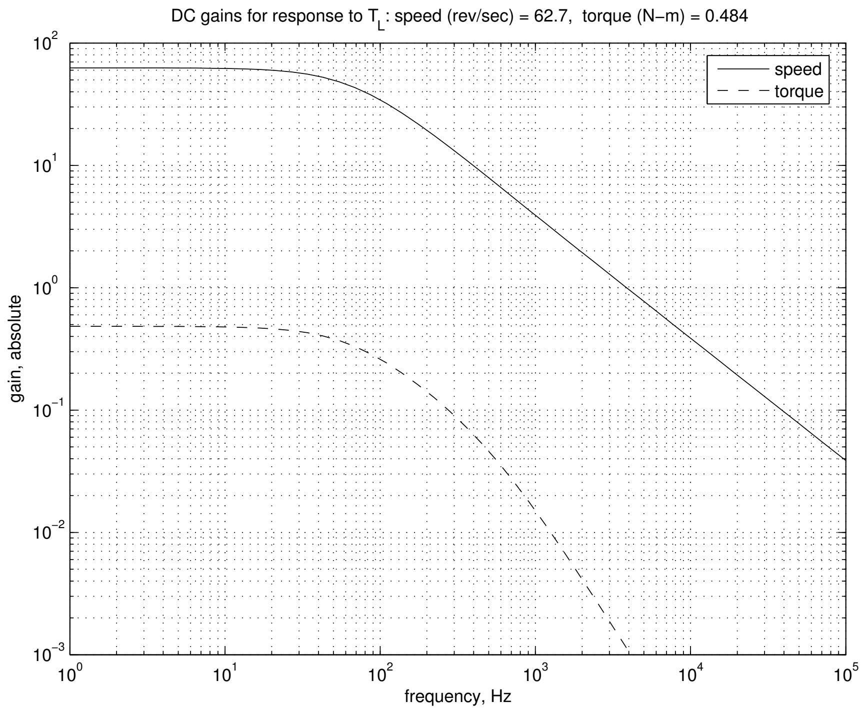

Figure 14: Magnitude Response of DC Motor Response to Load Torque, absolute units

(c) Based on Figures 13-14 and corresponding figures for the response to input voltage V, we have that

$$H_{\Omega V}(0) = 1.98, \qquad H_{T_M V}(0) = 0.0163 \tag{7}$$

$$H_{\Omega T_L}(0) = -62.7, \qquad H_{T_M T_L}(0) = 0.484 \tag{8}$$

Hence the steady state response to an input voltage $V = 10$ and a load torque $T_L = .2$ N-m should satisfy

$$\Omega_{ss} = 1.98 \times 10 - 62.7 \times .2 = 7.26 \tag{9}$$

$$T_{Mss} = 0.0163 \times 10 + .484 \times .2 = 0.26 \tag{10}$$

These values agree with the torque speed curve in Figure 15 for $V = 10$ and $T_L = 0.2$, and the time response plots in Figure 16 after steps in both voltage and load torque have been applied[2]. Agreement is guaranteed because the torque speed curves and the transfer functions evaluated at $\omega = 0$ both yield the same information: the steady state response to a constant input.

[2]Note that you may need to run the time simulation longer than depicted in Figure 16 until speed and motor torque settle to their final values. You may also need to use more significant digits for the DC gains in equations (9)-(10) to see exact agreement.

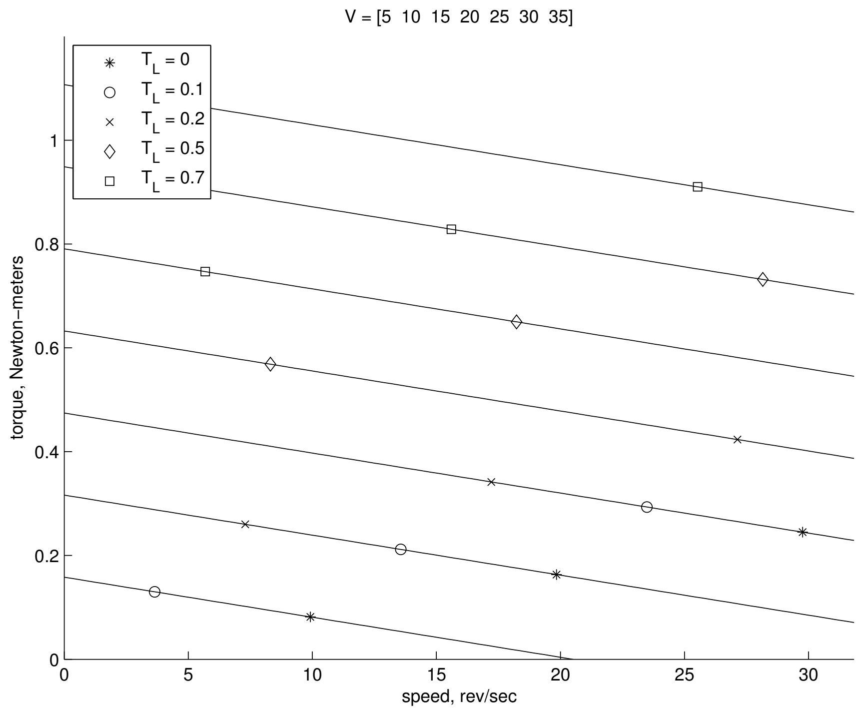

Figure 15: Torque Speed Curves

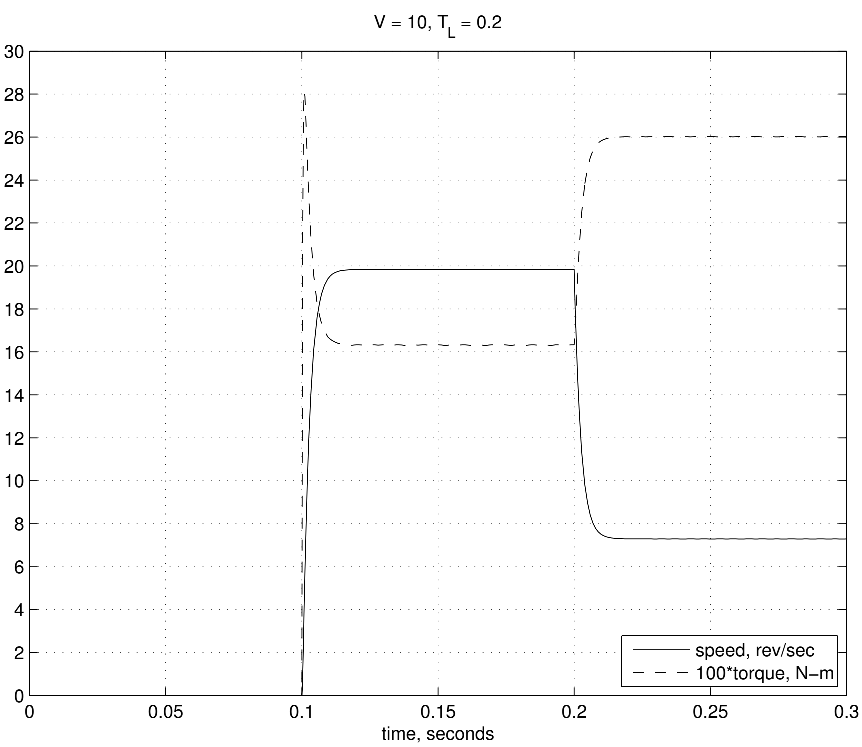

Figure 16: Response of Speed and Torque to Input Voltage and Load

4. SOLUTION:

(a) The period of the displayed signal is 10 seconds, which corresponds to a frequency $f_0 = 0.1$ Hz. Due to the possibility of aliasing, this could correspond to any signal with frequency $f_0 = 0.1 + kf_s$ Hz, where $k = 0, \pm 1, \pm 2, \ldots$.

(b) According to the handout "Sampling, Beats, and the Software Oscilloscope", $f_{beat} = 2\Delta = 2(f_N - f_0)$. Hence, with $f_{beat} = 1/50 = .02$ Hz and $f_N = 0.5$ Hz, we have that $f_0 = f_N - f_{beat}/2 = 0.49$ Hz.

(c) For $T_{beat} = 7$ seconds, we need $f_{beat} = 1/T_{beat} = 0.1429$ Hz. With $f_1 = 1$ Hz, this implies that $f_2 = 1.1429$ Hz, as shown in Figure 17.

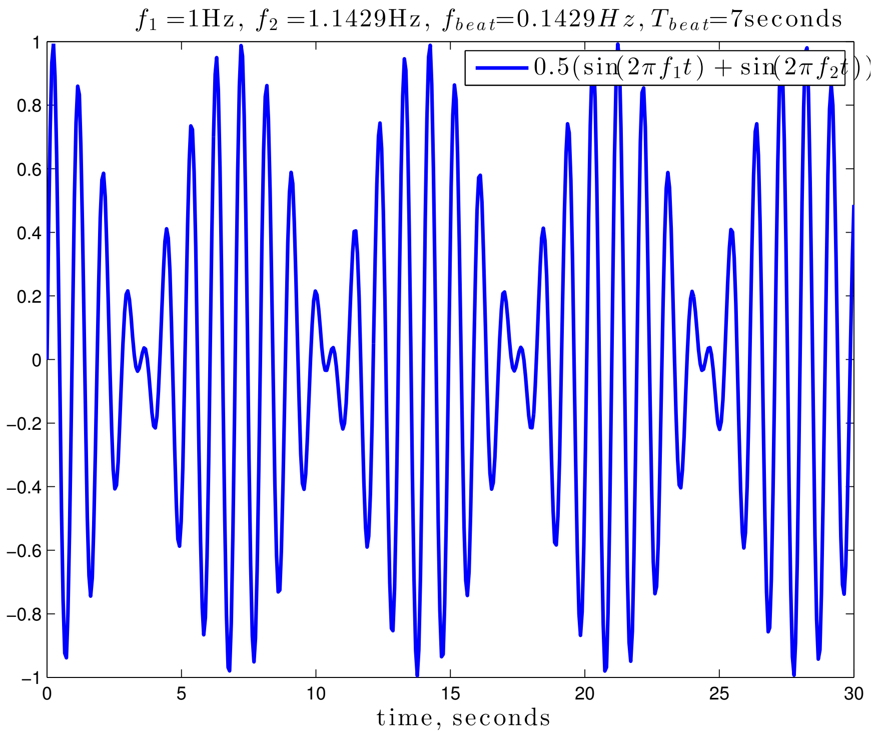

Figure 17: Beats with period 7 seconds.