Midterm Exam

EECS 461, Winter 2023

Available: midnight Friday, March 10, 2023

Due: Canvas upload 11:59 PM Saturday, March 11, 2023

Name:.............SOLUTIONS..............

- 6 questions weighted as shown.
- Reasons should be stated clearly and concisely to receive maximum credit. Your writing must be legible for me to grade the solutions!
- This is a take-home exam, so obviously it's open book, open notes. You may use Matlab.
- **You must submit your completed exam electronically by the deadline.** You do not have to submit your answers on the exam, but make sure your solutions are neat and well-organized. Make sure you submit your exam by the deadline. You have more than enough time, so there's no excuse to miss the deadline. Period.
- **The exam is an individual effort. Regardless of whether you submit your answers on the exam or on a separate sheet, you must sign the statement of the honor code. I will not grade your exam if you do not sign the honor code.**

1. .... (12 points)
2. .... (6 points)
3. .... (9 points)
4. .... (9 points)
5. .... (15 points)
6. .... (6 points)

Sign the Honor Code:
"I have neither given nor received aid on this exam, nor have I concealed any violation of the honor code."

Signature:...

1. Consider the block diagram illustrated in Figure 1.

   (a) Find the transfer function from the input, $u(t)$ to the output $y(t)$.

   (b) Find the transfer function from the disturbance $d(t)$ to the output $y(t)$.

   (c) What are the values of the damping coefficient, ζ, undamped natural frequency (radians/second), ω_n and damped natural frequency (radians per second), ω_d of the system?

   (d) What are the characteristic roots of the system? Estimate the 10–90% rise time in seconds and the percent overshoot.

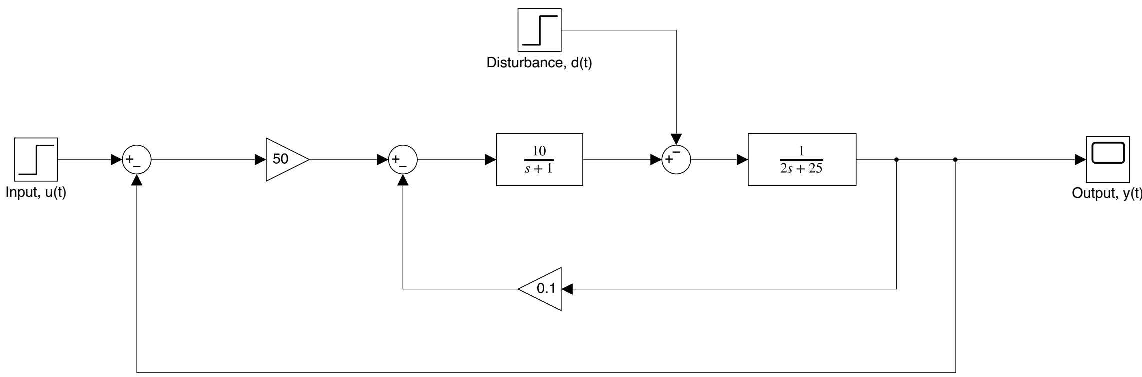

Figure 1: Block Diagram for Problem 1

SOLUTIONS FOR PROBLEM 1

1. Consider the block diagram illustrated in Figure 1.

   (a) The transfer function from the input, $u(t)$ to the output $y(t)$:

   $$\frac{Y(s)}{U(s)} = \frac{250}{s^2 + 13.5s + 263}$$

   (b) Find the transfer function from the disturbance $d(t)$ to the output $y(t)$.

   $$\frac{Y(s)}{D(s)} = \frac{-(s+1)}{s^2 + 13.5s + 263}$$

   (c) What are the values of the damping coefficient, ζ, undamped natural frequency (radians/second), ω_n and damped natural frequency (radians per second), ω_d of the system?

   $$\omega_n = \sqrt{263} = 16.2173$$

   $$\zeta = \frac{13.5}{2 \times 16.2173} = 0.4162$$

   $$\omega_d = \omega_n \sqrt{1 - \zeta^2} = 14.746$$

   (d) What are the characteristic roots of the system? Estimate the 10–90% rise time in seconds and the percent overshoot.

   $$s_{1,2} = -\zeta\omega_n \pm j\omega_n \sqrt{1 - \zeta^2} = -6.7500 \pm j14.7458$$

   $$t_R \approx \frac{1.8}{\omega_n} = 0.11$$

   $$\%\text{O.S.} = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \approx 23\%$$

2. Systems that require multiple independent coordinates to describe their motion are said to have multiple degrees of freedom. For simple mechanical systems such as the spring-mass systems we have studied, the number of degrees of freedom equals the number of masses in the system times the number of possible types of motion of each mass. Consider two wheels coupled by a spring and damper, as shown in figure 2. The differential equations describing this system are given by:

$$\ddot{\theta}_z + \frac{b}{J_z}\dot{\theta}_z + \frac{k}{J_z}\theta_z = \frac{b}{J_z}\dot{\theta}_w + \frac{k}{J_z}\theta_w$$

$$\ddot{\theta}_w + \frac{b}{J_w}\dot{\theta}_w + \frac{k}{J_w}\theta_w = \frac{b}{J_w}\dot{\theta}_z + \frac{k}{J_w}\theta_z$$

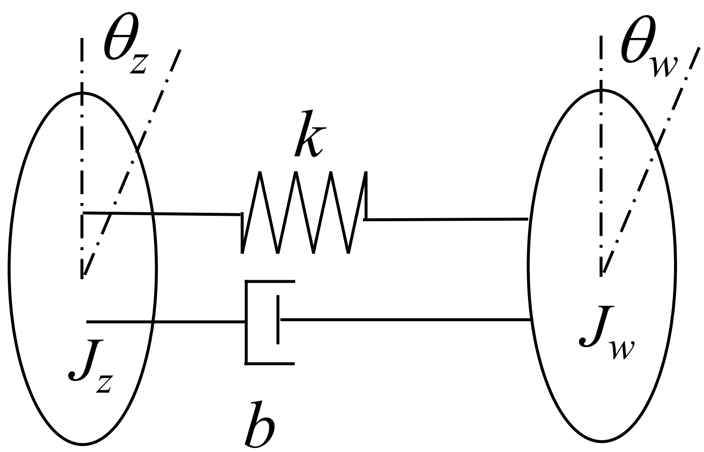

Figure 2: Two degree of freedom system

(a) Define state variables $x_1 = \theta_z$, $x_2 = \dot{\theta}_z$, $x_3 = \theta_w$, $x_4 = \dot{\theta}_w$. Derive a system of state equations of the form $\dot{x} = Ax$.

(b) Suppose we approximate the system in discrete time using the Forward Euler approximation with sample period T. Specify the discrete system matrix A_d.

SOLUTIONS FOR PROBLEM 2

(a) Change of variables gives four states:

$$\begin{aligned} x_1 &= \theta_z \\ x_2 &= \dot{\theta}_z \\ x_3 &= \theta_w \\ x_4 &= \dot{\theta}_w \end{aligned}$$

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \ddot{\theta}_z \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \ddot{\theta}_w \end{aligned}$$

Thus

$$\dot{x} = Ax \tag{1}$$

where:

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k}{J_z} & \frac{-b}{J_z} & \frac{k}{J_z} & \frac{b}{J_z} \\ 0 & 0 & 0 & 1 \\ \frac{k}{J_w} & \frac{b}{J_w} & \frac{-k}{J_w} & \frac{-b}{J_w} \end{bmatrix} x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

(b) Applying $x(k+1) = (I + TA)x(k)$ yields

$$A_d = \begin{bmatrix} 1 & T & 0 & 0 \\ \frac{-Tk}{J_z} & 1 - \frac{Tb}{J_z} & \frac{Tk}{J_z} & \frac{Tb}{J_z} \\ 0 & 0 & 1 & T \\ \frac{Tk}{J_w} & \frac{Tb}{J_w} & \frac{-Tk}{J_w} & 1 - \frac{Tb}{J_w} \end{bmatrix}$$

3. Three FlexTimer clock cycles are required to process each rising or falling edge of a quadrature signal. The FlexTimer clock is set to 10MHz.

   (a) Recall that the EECS 461 lab encoder is 1000 CPR. What is the maximum rate at which the haptic wheel may turn, in revolutions/second, before the FlexTimer fails to process all edges?

   (b) Suppose that the haptic wheel is turning at the maximum rate allowed by the FlexTimer, as computed in part (a). What is the slowest rate at which the CPU can read the FlexTimer counter register without losing track of the direction of rotation? Recall that the FlexTimer uses a 16 bit counter register.

SOLUTIONS FOR PROBLEM 3

(a) Recall that the EECS 461 lab encoder is 1000 CPR. What is the maximum rate at which the haptic wheel may turn, in revolutions/second, before the FlexTimer fails to process all edges?

The FlexTimer clock runs at 10MHz $= 10 \times 10^6$ cycles/second. Since the FlexTimer takes three clock cycles to process each edge, the time required to do so is (3 cycles/edge) / (10×10^6 cycles/second) $= 3 \times 10^{-7}$ seconds/edge (equivalently, 0.3 μsec/edge). It follows that the maximum number of edges that may be processed in one second is $(10/3) \times 10^6$, and thus that the maximum rate at which edges are processed is equal to $(10/3) \times 10^6 = 3.33 \times 10^6$ edges/second. (If we need to process edges more frequently than this, then the FlexTimer will not be able to do so.) With our encoder one wheel revolution produces 4000 edges, and thus the maximum number of wheel revolutions/second that may be processed is $(10/3) \times 10^6$ (edges/second)/(4000 edges/revolution) $=(10/12) \times 10^3 = 833.3$ revolutions/second.

(b) Suppose that the haptic wheel is turning at the maximum rate allowed by the FlexTimer, as computed in part (a). What is the slowest rate at which the CPU can read the FlexTimer counter register without losing track of the direction of rotation? Recall that the FlexTimer uses a 16 bit counter register.

As calculated in part (a), if the wheel is turned at the maximum rate of 833.3 revs/second, then edges are processed at the maximum rate of 3.33×10^6 edges/second. Since the counter is incremented by one count/edge, it follows that the the counter is incremented at the rate 3.33×10^6 counts/second. To prevent loss of direction information, the counter must be read before it gets halfway to rollover. For a 16-bit counter, (0x8000 counts)/(3.33×10^6 counts/second) = (2^{15} counts)/(3.33×10^6 counts/second)= 0.0098 seconds (101.6 Hz).

4. A second-order Butterworth filter similar to the one used in the lab has a transfer function:

$$\frac{Y(s)}{U(s)} = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2} \tag{2}$$

where

$$\zeta = \frac{1}{\sqrt{2}}$$

and ω_n is the bandwidth of the filter in radians/second defined as the frequency at

$$\frac{A_0}{A_i} = -3 \text{ dB}$$

(a) Specify ω_n such that the magnitude of an input signal $u = \sin(2\pi t)$ is attenuated by 90%. You may wish to use the Matlab script `Midterm_Problem_4a.m`.

(b) Suppose a second-order Butterworth anti-aliasing filter has the frequency response illustrated in figure 3. A noisy signal $u(t) = \sin(4.8\pi t) + \sin(20\pi t)$ is passed through the filter and then sampled at a rate of 5 Hz. Will the sampled signal be aliased? Carefully explain why or why not.

(c) Will the sampled signal of part (b) exhibit beating? Carefully explain why or why not. If the sampled signal exhibits beats, what is the beat frequency?

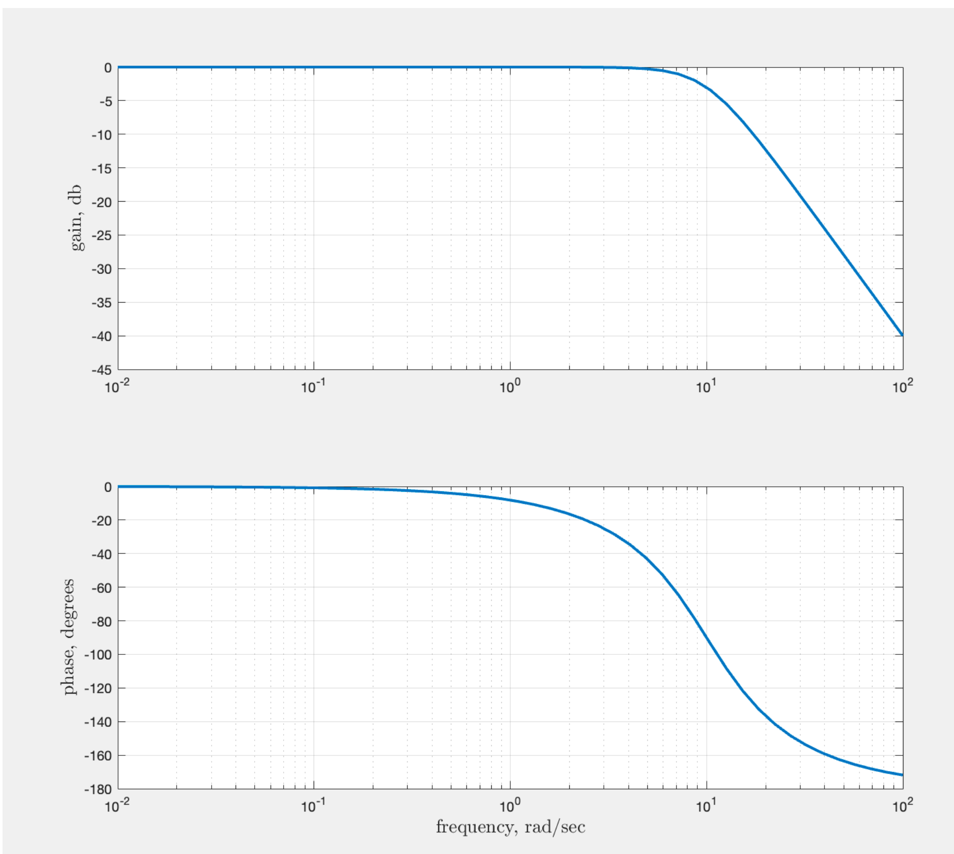

Figure 3: Second Order Butterworth Anti-aliasing Filter

SOLUTIONS FOR PROBLEM 4

(a) Specify ω_n such that the magnitude of an input signal $u = \sin(2\pi t)$ is attenuated by 90%. You may wish to use the Matlab script `Midterm_Problem_4a.m`.

The semi-logarithmic frequency response magnitude plot of a second order system declines asymptotically from the bandwidth frequency at -40 dB/decade. Consequently, if we want the signal $u = \sin(2\pi t)$ attenuated by 90%, then we must place the geometric midpoint of $\{\omega_n \; 10\omega_n\}$ at 2π radians/second corresponding to attenuation of -20 dB.

Logarithmic interpolation between ω_n and $10\omega_n$ gives us

$$\frac{\log 2\pi - \log \omega_n}{\log 10\omega_n - \log \omega_n} = 0.5 \rightarrow 2\pi = \sqrt{10\omega_n^2}$$

Hence, $\omega_n = 1.9869$ radians/second, see figure 4. Alternatively, you could simply choose values of ω_n and plot the resulting frequency response until you got sufficiently close to the specification. The solution is confirmed by the time response plot, figure 5, which illustrates that the output signal converges to 10% of the input with roughly 150° phase lag.

(b) Suppose a second-order Butterworth anti-aliasing filter has the frequency response illustrated in figure 3. A noisy signal $u(t) = \sin(4.8\pi t) + \sin(20\pi t)$ is passed through the filter and then sampled at a rate of 5 Hz. Will the sampled signal be aliased? Carefully explain why or why not.

It is clear from the frequency response, figure 3, that the 10 Hz (20π radians/second) component of the signal will be attenuated by at least -30 dB which, for engineering purposes, means we are sampling only the 2.4 Hz (4.8π radians/second) signal. Since 5 Hz $> 2 \times 2.4$ Hz, the Shannon sampling theorem is satisfied and the sampled signal will not be aliased (it will, however, be attenuated and phase shifted).

(c) Will the sampled signal of part (b) exhibit beating? Carefully explain why or why not.

Since the sampling frequency is $f_s = 5$ Hz, the Nyquist frequency is $f_N = f_s/2 = 2.5$ Hz, and the sampling period is equal to $T = 1/f_s = 0.2$ seconds. In this case the Shannon sampling theorem is satisfied, but the signal frequency $f_0 = 2.4$ Hz is only a little smaller than $f_N = 2.5$ Hz. Hence we expect to see beats with frequency $f_{\text{beat}} = 2(f_N - f_0) = 0.2$ Hz. The sampled signal is illustrated in figure 6.

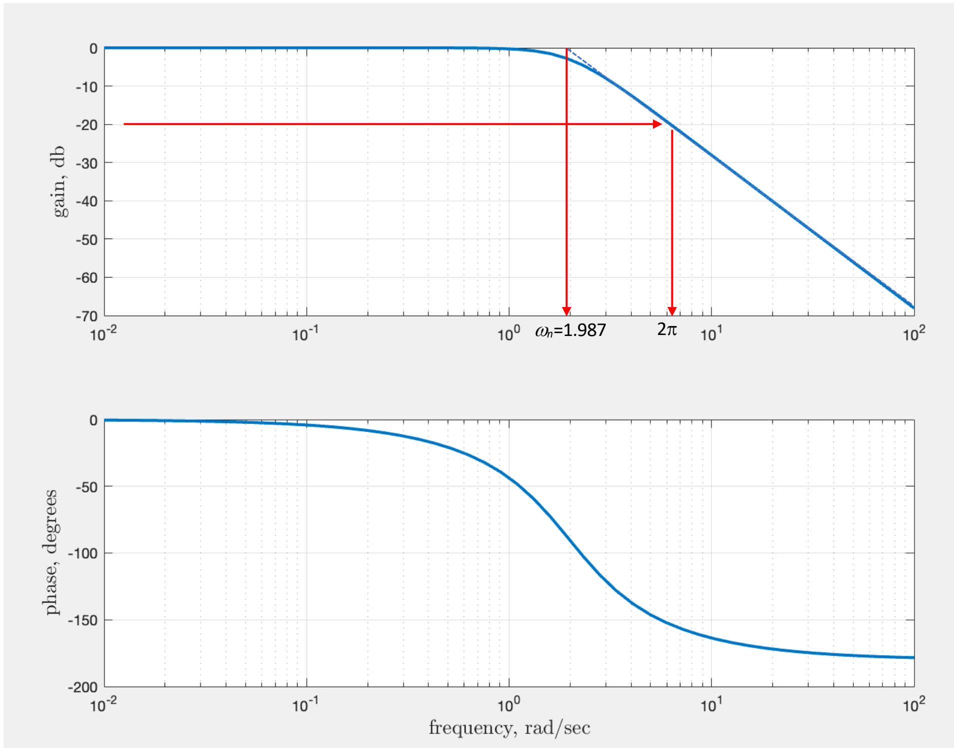

Figure 4: Butterworth filter frequency response

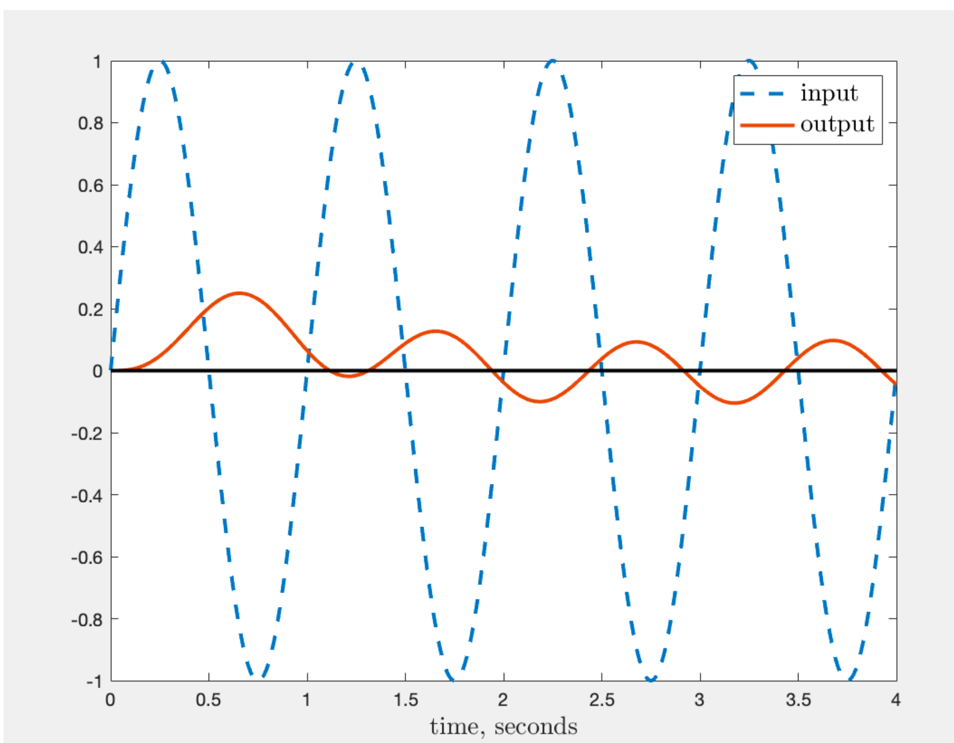

Figure 5: Butterworth filter time response

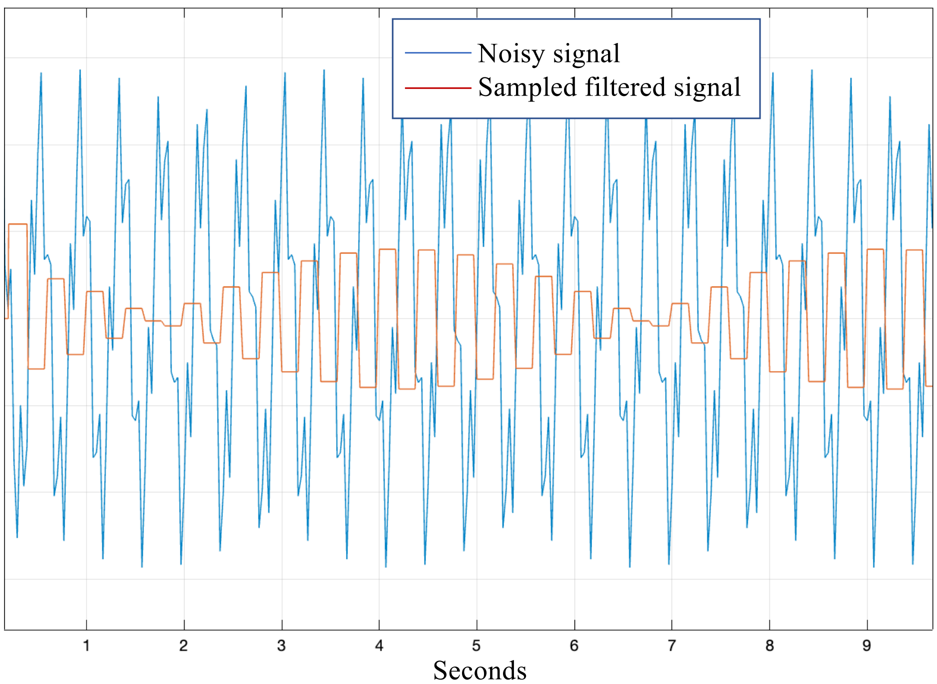

Figure 6: Sampled signal time response

5. Consider a CAN network with 8 lab stations working in pairs to implement 4 virtual walls. Each lab station will read the encoder every T seconds, and write this value to the network. The partner lab station will take the encoder value, use it to compute a reaction torque, and transmit the reaction torque to the network. The torque value will be received by the original lab station, and used to update the PWM duty cycle that controls the motor. With 4 virtual walls implemented, it will thus be necessary to transmit 8 messages on the CAN bus every T seconds.

   Recall that a CAN message containing the maximum 64 bits of data will actually be 111 bits long due to overhead bits. Furthermore, with high speed CAN (500 kbit/sec) it takes $2\mu s = 0.002$ msec to transmit 1 bit on the CAN bus.

   If the effects of bit-stuffing are considered, there are additional bits that need to be added after every sequence of 5 bits of identical polarity, except in the CRC delimiter, ACK, EOF, and INT fields. Hence, there is a worst case maximum of $\lfloor \frac{34+\#\text{databits}-1}{4} \rfloor$ stuff bits in the CAN message [1]. The numerator of this formula follows because only 34 of the non-data bits are potentially eligible for bit-stuffing. The denominator is equal to 4 because each stuff bit may itself be the first bit of a 5 bit sequence.

   (a) Suppose that all 8 CAN messages use the maximum 64 bits of data. How much time is required to transmit these messages? Consider the best case (no stuff bits) and worst case (maximum number of stuff bits) scenarios.

   (b) If each CAN message uses only 32 bits of data, how much time will it take to transmit all 8 messages? Consider the best case (no stuff bits) and worst case (maximum number of stuff bits) scenarios.

   (c) Again suppose that the 8 CAN messages each use 32 bits of data. What is the minimum sample period T that will achieve 25% bus utilization assuming the worst case message length? Round your answer up to the nearest integer value.

   (d) Suppose that the sample period must satisfy $T \leq 4$ msec in order to achieve a good virtual wall. What is the maximum number of virtual walls that can be implemented on the network with no more than a 25% bus utilization? Assume 32 bits of data per CAN message and the worst case message length.

   (e) Suppose instead of using standard CAN messages with 11 bit identifiers, we instead use extended messages with 29 bit identifiers. Extended messages have two additional bits, and are thus 20 bits longer than a standard CAN message. With a sample time given by $T = 4$ msec, what is the utilization required to implement 4 virtual walls with extended CAN messages using 32 bits of data? Assume no stuff bits for this calculation.

[1] $\lfloor\rfloor$ denotes the floor function: the largest integer less than or equal to the given argument.

SOLUTIONS FOR PROBLEM 5

(a) If there are no stuff bits: 8 messages $\times$111 bits/message $\times$ 0.002 msec/bit = 1.776 msec. The maximum number of stuff bits is $\lfloor \frac{34+\#\text{databits}-1}{4} \rfloor = 24$ stuff bits $\rightarrow$ messages with the maximum number of stuff bits are 135 bits long. Hence 8 messages $\times$ 135 bits/message $\times$ 0.002 msec/bit = 2.16 msec.

(b) If there are no stuff bits: 8 messages $\times$ 79 bits/message $\times$ 0.002 msec/bit = 1.264 msec. Messages with the maximum number of stuff bits are are 95 bits long. Hence 8 messages $\times$ 95 bits/message $\times$ 0.002 msec/bit = 1.52 msec.

(c) $1.52/T < 0.25 \rightarrow T > 6.08$ msec. For an integer value, set T = 7 msec.

(d) We have seen that, with worst case messages, 4 walls require 7 msec. For three walls, which require 6 messages we may compute as follows: 6 messages $\times$ 95 bits/message $\times$ 0.002 msec/bit = 1.14 msec, and thus $1.14/4 = 0.285 < 0.25$, which also violates the utilization constraint. Hence we can implement at most 2 walls (which require 4 messages) under these conditions. Check: 4 messages $\times$ 95 bits/message $\times$ 0.002 msec/bit = 0.76 msec, and thus $0.76/4 = 0.19 < 0.25$, which satisfies the utilization constraint.

(e) With 32 bits of data an extended CAN message has 99 total bits. 8 messages $\times$ 99 bits/message $\times$ 0.002 msec/bit = 1.584 msec. Bus utilization is $1.584/4 = 0.396$, or 39.6%.

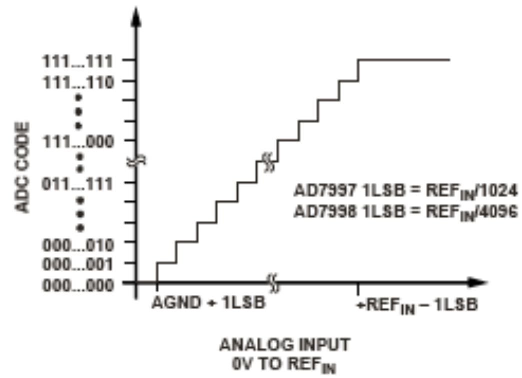

Figure 7: Analog Devices AD7997 Specification

6. The Analog Devices AD7997 is an 8 channel, 10-bit, high speed, low power, successive-approximation ADC. Figure 7 is the complete ADC technical specification from the Analog Devices website. Assume $REF_{IN} = 5\text{V}$, $AGND = 0\text{V}$.

   (a) What is the quantization error of the ADC?

   (b) What is the maximum value in volts that can be measured by this ADC?

SOLUTIONS FOR PROBLEM 6

(a) From the figure, 1 LSB $= \frac{\text{REF}_{\text{IN}}}{1024} = \frac{5}{1024} = 0.0049$

(b) From the figure, Vmax $= (\text{REF}_{\text{IN}} - 1\text{ LSB}) = (5 - 0.0049) = 4.995$ volts.