

Problem 1

1. (10 points) Let \mathbf{r}_1 be the line passing through the point $P = (1, 0, 1)$ and with direction $(1, -1, 0)$. Write the equation of the line in parametric and symmetric form.
2. (10 points) Find the equation of the line \mathbf{r}_2 obtained by intersecting the planes $x + y - z = 2$ and $2x + 2y + z = 3$.
3. (10 points) Find the equation of the plane passing through \mathbf{r}_1 and the line $\mathbf{r}_3(t) = (t, -t, 0)$.
4. (5 points) Consider the line intersecting \mathbf{r}_1 in P and with direction $(1, 1, 1)$. What is the angle of this line with \mathbf{r}_1 ? Is this line orthogonal to the plane you found in part 3?

Problem 2

Consider the function $f : D \subset \mathbb{R}^2 \rightarrow \mathbb{R}$ given by $f(x, y) = \sqrt{x^2 - y^2}$.

1. (15 points) Find and sketch the domain D and the level curves of f in \mathbb{R}^2 .
2. (15 points) Consider the point $P = (1, 0)$. Find the unitary directions \vec{v} along which the partial derivative of f in P has value $\frac{1}{2}$. What are the lowest and highest values of the directional derivatives of f in P ?
3. (5 points) Is f differentiable at the origin?
4. (5 points) Let $A = \{(x, y) | x^2 + y^2 - 6x \leq 10\}$. Find the absolute maximum and minimum of f in A .

Problem 3

1. (10 points) Find the volume of the solid inside the cylinder $(x - 1)^2 + (y - 1)^2 = 1$ and below the paraboloid $z = 30 - 3x^2 - y^2$.
2. (10 points) Find the volume of the solid inside the sphere $x^2 + y^2 + z^2 = 1$ and outside the cone $z = \sqrt{x^2 + y^2}$.
3. (10 points) Use cylindrical coordinates to find the volume of the solid inside the sphere $x^2 + y^2 + z^2 = 1$ and below the plane $z = \frac{\sqrt{2}}{2}$.
4. (5 points) Set up the integral from part 3 in spherical coordinates.