

## Problem 1

Consider the following limits and prove if they exist or not. If they do, find the value of the limit.

1.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^2 + y^2}$$

2.

$$\lim_{(x,y) \rightarrow \infty} \frac{x^2 y}{x^2 + y^2}$$

3.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{2x^2 y}{3x^4 + y^2}$$

## Problem 2

Let  $f(x, y) = \sqrt{16 - x^2 - y^2}$ .

1. Find the domain of the function and sketch it on  $\mathbb{R}^2$
2. Sketch the graph of the function and describe the relationship between the graph and the level curves. What is the range of the function?

## Problem 3

Consider the function defined by

$$f(x, y) = \begin{cases} \frac{x^2 y^4}{x^\alpha + y^{2\alpha}} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

where  $\alpha$  is an even positive integer.

1. Consider  $\alpha = 2$ . Study continuity, derivability and differentiability of the function.
2. Consider  $\alpha = 4$ . Study continuity, derivability and differentiability of the function.

### Problem 4

Consider the function  $f(x, y) = xe^{xy} + y$

1. Find the plane tangent to the graph of the function in  $P = (2, 0)$  and calculate the linear approximation of the function in  $(1.9, 0.1)$ .
2. Find the directional derivative of the function at point  $P$  in the direction of  $\vec{w} = (1, -2)$ .
3. Find the direction of maximal change for the function at point  $P$ .
4. Is there any direction  $\vec{v}$  such that the directional derivative at point  $P$  in direction  $\vec{v}$  is equal to 7? Explain your answer.

### Problem 5

Consider the function  $f(x, y) = (x^2 + y^2 - 1)(3x^2 + y^2 - 4)$

1. Find the critical points of  $f$  and classify them.
2. *Bonus question:* restrict your function to the domain  $\{(x, y) \in \mathbb{R}^2 | x^2 + y^2 \leq 16\}$ . Find the absolute maximum and absolute minimum of the function in this domain.