Problem 1

Consider the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by $f(x, y) = x^2 + (y - 1)^2$.

1. *(10 points)* Identify the level curves of this function and sketch them in $\mathbb{R}^2$.
2. *(10 points)* Identify the graph as a quadric in $\mathbb{R}^3$ and give a surface parametrization.
3. *(10 points)* Find the equation of the plane tangent to the graph in $(3, 2)$.
4. *(10 points)* Find the absolute maximum and minimum of this function in the domain

$$D = \{(x, y) |\, (x - 1)^2 + y^2 \leq 1\}$$

Problem 2

Evaluate the following integrals:

1. *(10 points)* $\iint_S \boldsymbol{F} \cdot dS$ for S the closed surface bounding the region

$$\{(x, y, z) |\, z \geq x^2 + y^2,\ z \leq 4\}$$

   and $\boldsymbol{F}(x, y, z) = (\cos(z) + xy^2, xe^{-z}, \sin(y) + x^2 z)$.
2. *(10 points)* $\iint_S \mathrm{curl}\boldsymbol{F} \cdot dS$ for S the part of the surface $z = x^2 + y^2$ that lies below the plane $z = 1$ and $\boldsymbol{F}(x, y, z) = (y^2, x, z^2)$.
3. *(10 points)* The volume of the region

$$E = \{(x, y, z) |\, x^2 + y^2 \leq 4,\ z \leq \sqrt{x^2 + y^2},\ z \geq 0\}$$

Problem 3

Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be the vector field

$$\boldsymbol{F}(x, y) = (e^{x-y} + \frac{y^3}{3})\boldsymbol{i} + (xy^2 - e^{x-y})\boldsymbol{j}$$

and γ be the curve starting at $(0, 1)$, following the parabola $x = y^2 - 1$ until $(0, -1)$ and then following the unit circle counterclockwise to the point $(1, 0)$.

1. *(10 points)* Parametrize γ piecewise and set up the integral for its arclength.
2. *(10 points)* Prove whether $\boldsymbol{F}$ is conservative or not.

3. *(10 points)* Evaluate $\int_\gamma \boldsymbol{F} \cdot d\gamma$.
4. *(5 points)* Consider $\boldsymbol{G}(x, y) = \boldsymbol{F}(x, y) + (2y, 3x)$. Prove if $\boldsymbol{G}$ is conservative or not.
5. *(5 points)* Evaluate $\int_\gamma \boldsymbol{G} \cdot d\gamma$.