Homework 4

July 2020

1 QR decompositions (20 points)

Consider the following matrix:

$$A = \begin{bmatrix} 1 & 3 & 5 \\ -1 & -3 & 1 \\ 0 & 2 & 3 \\ 1 & 5 & 2 \\ 1 & 5 & 8 \end{bmatrix}$$

1. Justify why this matrix admits a QR decomposition and find it.
2. Does this matrix have a left pseudo-inverse or a right pseudo-inverse? Justify your answer.
3. Find the first column of the pseudo-inverse.

2 Least square methods (20 points)

For each of the systems, answer the following questions.

$$\begin{cases} 4x_1 + x_3 = 9 \\ x_1 - 5x_2 + x_3 = 0 \\ 6x_1 + x_2 = 0 \\ x_1 - x_2 - 5x_3 = 0 \end{cases} \tag{1}$$

$$\begin{cases} x_1 + x_2 = 2 \\ x_1 - x_3 = 5 \\ x_2 + x_3 = 6 \\ -x_1 + x_2 + 2x_3 = 6 \end{cases} \tag{2}$$

1. Prove that the system does not admit a solution.
2. What type of system does the least square solution solve? Does it admit solutions?
3. If the system from part 2 admits solutions, solve it.

3 Diagonalization (20 points)

For each of the following matrices, prove if they are diagonalizable or not and in case they are, find the diagonalization.

$$\begin{bmatrix} 3 & -1 \\ 1 & 5 \end{bmatrix}, \qquad \begin{bmatrix} 5 & -2 & 3 \\ 0 & 1 & 0 \\ 6 & 7 & -2 \end{bmatrix}, \qquad \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}.$$

4 Multiplicities (20 points)

Consider the following matrix

$$A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & h & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1. Find the eigenvalues of A and their corresponding algebraic multiplicity. Do they depend on h?
2. Find the geometric multiplicity of each eigenvalue. Does this depend on h?
3. For the values of h for which all the algebraic multiplicites are equal to the corresponding geometric multiplicity, find a diagonalization of A.

5 Eigenvalues of transformations in $\mathbb{R}^2$ (20 points)

For each of the following transformations in $\mathbb{R}^2$, find the eigenvalues, eigenvectors and, if it's possible and the eigenvalues are real, a diagonalization:

1. A horizontal shear transformation by $\frac{3}{2}$.
2. A counter-clockwise rotation by $\frac{\pi}{3}$.
3. A reflection across the line $y = -x$.