Homework 4

August 1, 2019

Problem 1

Let γ be the curve obtained by intersecting the cylinder $x^2 + z^2 = 1$ with the plane $x = -y$.

1. Find a parametric representation of the curve.
2. Show that the curve lies on the ellipsoid $x^2 + y^2 + 2z^2 = 2$.
3. Find the *unit* tangent vector of γ at each point.
4. Write an expression for its arclength.

Problem 2

For each of the following surfaces, write a parametrization as a surface of revolution, express its grid curves and find its unit normal vector.

1. $x^2 - y^2 + z^2 = 0$.
2. $x^2 + y^2 - z^2 = 1$.
3. $z^2 - y^2 - x^2 = 1$.
4. $x = z^2 + y^2$.

Problem 3

Let S be the surface obtained by intersecting the cone S_1 given by the equation $z = 2 - \sqrt{x^2 + y^2}$ with the paraboloid S_2 of equation $z = x^2 + y^2$. Evaluate the surface area of S.

Problem 4

Let $\boldsymbol{F}(x, y) = (x^2 - y)\boldsymbol{i} + (x + y^2)\boldsymbol{j}$.

1. Compute the line integral
$$\int_\gamma \boldsymbol{F} \cdot d\boldsymbol{r}$$
for γ the segment from $(0, -1)$ to $(0, 1)$.
2. Compute the same integral for γ the unit semicircle for $x \geq 0$ from $(0, -1)$ to $(0, 1)$.
3. Is $\boldsymbol{F}$ conservative? Explain.

Problem 5

Consider the integral given by

$$\int_{\gamma} \boldsymbol{F} \cdot d\boldsymbol{r}$$

where $\boldsymbol{F}(x,y) = \frac{x}{x^2+y^2}\boldsymbol{i} + y\frac{1-x^2-y^2}{x^2+y^2}\boldsymbol{j}$ and γ is a curve in the domain of F.

1. What is the domain of $\boldsymbol{F}$? Is it simply connected?
2. Is $\boldsymbol{F}$ conservative? If so, what is its potential?
3. Evaluate the integral for γ the circle of radius 1 in $\mathbb{R}^2$.
4. Evaluate the integral when γ is the parabola $y = 1 - x^2$ starting at $(-1, 0)$ and ending at $(1, 0)$.