Homework 3

July 24, 2019

Problem 1

Let D be the domain in $\mathbb{R}^2$ bounded by the curves $y = 4 - x^2$, $y = x + 2$ and $y = 2 - x$.

1. Find the center of mass of D for a constant density.
2. Evaluate
$$\iint_D f(x,y)dA$$
for $f(x,y) = 2y^3x$.

Problem 2

Consider the domain D in $\mathbb{R}^2$ inside the circle of radius 1 centered at the origin and below the line $y = \frac{1}{2}$. Let $f(x,y) = \sin(x^2 + y^2)$.

1. Set up the integral as a union of type I domains, without computing it.
2. Set up the integral as a union of type II domains, without computing it.
3. Set up the integral in polar coordinates, without computing it.

Problem 3

Let E be the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere of radius 1 centered in the origin. Let P be the center of mass of E.

1. Set up the integrals for the (x, y, z) coordinates of P in cylindrical coordinates, without computing them.
2. Set up the integrals for the (x, y, z) coordinates of P in spherical coordinates, without computing them.

Problem 4

Consider the integral given by
$$\int_0^{2\pi} \int_0^1 \int_0^{2-r} r\, dz\, dr\, d\theta$$

1. Sketch a domain in $\mathbb{R}^3$ on which we are calculating the integral.

2. Rewrite the domain in $dr\,dz\,d\theta$ (meaning you should write the boundaries of r as a function of z).

3. Evaluate the integral. Is this the volume of a solid?