Homework 1

July 11, 2019

Problem 1

Let $\vec{a}$,$\vec{b}$ be two vectors in $\mathbb{R}^3$.

1. Show that the vector $\vec{b} - \text{proj}_a b$ is orthogonal to $\vec{a}$.
2. Under what condition do we have $||\text{proj}_a b|| = ||\text{proj}_b a||$?
3. Under what condition do we have $\text{proj}_a b = \text{proj}_b a$?

Problem 2

Let $\vec{a} = (1, 1, -1)$, $\vec{b} = (1, -1, 1)$.

1. Find a ***unitary*** vector $\vec{n}$ which is orthogonal to both $\vec{a}$ and $\vec{b}$. How many vectors satisfy such properties? Draw the one you found.
2. Find the volume of the parallelepiped spanned by $\vec{a}, \vec{b}$ and $\vec{n}$.
3. Find the area of the triangle spanned by $\vec{a}$ and $\vec{b}$.

Problem 3

1. Find the equation in parametric form and in symmetric form of the line $\boldsymbol{r_1}$ in $\mathbb{R}^3$ passing through the points $P_1 = (6, 1, -3)$ and $P_2 = (2, 4, 5)$.
2. Find the equation of a line which intersects $\boldsymbol{r_1}$ in one point and show that it is intersecting.
3. Consider the line $\boldsymbol{r_2}(t) = (3 + 8t, 3 - 6t, -16)$. Write its symmetric form. Is it intersecting, skew or parallel to $\boldsymbol{r_1}$?
4. Determine whether the following lines are skew, parallel or intersecting:

$$\boldsymbol{r_3} : \frac{x-1}{2} = \frac{y-3}{2} = \frac{z-2}{-1}$$
$$\boldsymbol{r_4} : \frac{x-2}{1} = \frac{y-6}{-1} = \frac{z+2}{3}$$

Problem 4

1. Find the plane through the point $P_1 = (2, 5, -1)$ and containing the line
$$\boldsymbol{r}(t) = (4 - t, 2, 3 + 2t)$$
.

2. Find the plane that is parallel to the plane $x + y - z = 1$ and contains the point $P_2 = (1, 0, -1)$.

3. Given the plane $x + y - z = 1$, find whether the line with symmetric equations
$$\frac{x-2}{1} = \frac{y-6}{-1} = \frac{z+2}{3}$$
is parallel, orthogonal or neither.

4. *Bonus question*: Consider the generic equation of a plane $ax+by+cz = d$, where at least one coefficient is nonzero. Prove that the vector normal to the plane is $\vec{n} = (a, b, c)$.

Problem 5

Consider the function $f : D_1 \subset \mathbb{R}^3 \to \mathbb{R}$ given by
$$f(x, y, z) = \frac{\sqrt{x^2 + y^2 - 4z^2}}{x^2 + y^2}$$

1. Find the domain D of f.

2. Describe the domain in terms of quadrics and use the trace method to identify it.

3. Consider $g : D_2 \subset \mathbb{R}^2 \to \mathbb{R}$ defined as $g(x, y) = f(x, y, 1)$ and write an expression for it. Find the domain D_2, draw it on the plane and calculate $\lim_{(x,y)\to\infty} g(x, y)$.