

SerraJoseAssignment3

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Hoeffding's Bound Inequality

$$\mathbb{P}[|E_{in}(g) - E_{out}(g)| > \epsilon] \leq 2Me^{-2\epsilon N} \quad (1)$$

#Question 1

0.1 (a)

1 Equation 1: Error Bound

$$\sqrt{\frac{1}{2N} \cdot \ln\left(\frac{2M}{\delta}\right)} \quad (2)$$

2 Substitution

$$\sqrt{\frac{1}{2(600)} \cdot \ln\left(\frac{2 \cdot (1000)}{(0.05)}\right)} = 9.397 \cdot 10^{-2} \quad (3)$$

The full Error Bar is given as follows:

$$E_{out} \leq E_{in} + \sqrt{\frac{1}{2(600)} \cdot \ln\left(\frac{2 \cdot (1000)}{(0.05)}\right)} \quad (4)$$

(b) This will cause the variance to rise which is not what we want. We want to find balance and examine the error where every parameter is at its minimum.

#Question 2

2.1 (a)

3 Equation 2 : Growth Function

- $X = \mathbb{R}$ (one dimensional)
- \mathcal{H} contains h , where each $h(x) = \text{sign}(x - a)$ for threshold a
- one dichotomy for $a \in (x_n, x_{n+1})$:

$$m_H(N) \leq N + 1 \quad (5)$$

which is the maximum number of dichotomies for positive rays

OR

$$m_H(N) \leq N + 1 \quad (6)$$

which is the maximum number of dichotomies for negative rays

Considering both positive or negative rays the maximum number of dichotomies:

$$m_H(N) \leq (N + 1) + (N + 1) \quad (7)$$

$$\leq (-\infty, a] \cup [a, \infty) \quad (8)$$

$$\leq \text{set}_W + \text{set}_V; \text{ where } \text{set}_W = (-\infty, a] \text{ and } \text{set}_V = [a, \infty) \quad (9)$$

$$\leq \text{set}_W + \text{set}_V - (\text{set}_W \cap \text{set}_V) \quad (10)$$

$$\leq (-\infty, a] + [a, \infty) - [(-\infty, a] \cap [a, \infty)] \quad (11)$$

$$\leq (-\infty, \infty) \quad (12)$$

$$\leq 2(N + 1) \quad (13)$$

3.1 (b)

The circumstance that one is given in the problem is the Real Number Line therefore the VC-dimension is infinity. $m_h(N) = 2^N$ for all N , then $d_{vc}(H) = \infty$

4 Question 3

```
[14]: import numpy as np
import matplotlib.pyplot as plt

# Define the function
def f(x, y):
    return x**2 + 2*y**2 + 2*np.sin(2*np.pi*x)*np.sin(2*np.pi*y)
def gradient_descent_Jose(learning_rate, num_iterations):
    x = np.linspace(0, .1, num=num_iterations)
    y = np.linspace(0, .1, num=num_iterations)
    X, Y = np.meshgrid(x, y)
    df_dx, df_dy = np.gradient(f(X,Y), x, y )
    z = []
    for i in range(num_iterations):
        #x -= learning_rate * df_dx
        #y -= learning_rate * df_dy
        # Update x and y using gradient descent
        x -= learning_rate * df_dx[0]
        y -= learning_rate * df_dy[0]
        z.append(f(x, y))

    return x, y, z
learning_rate_1 = 0.01
```

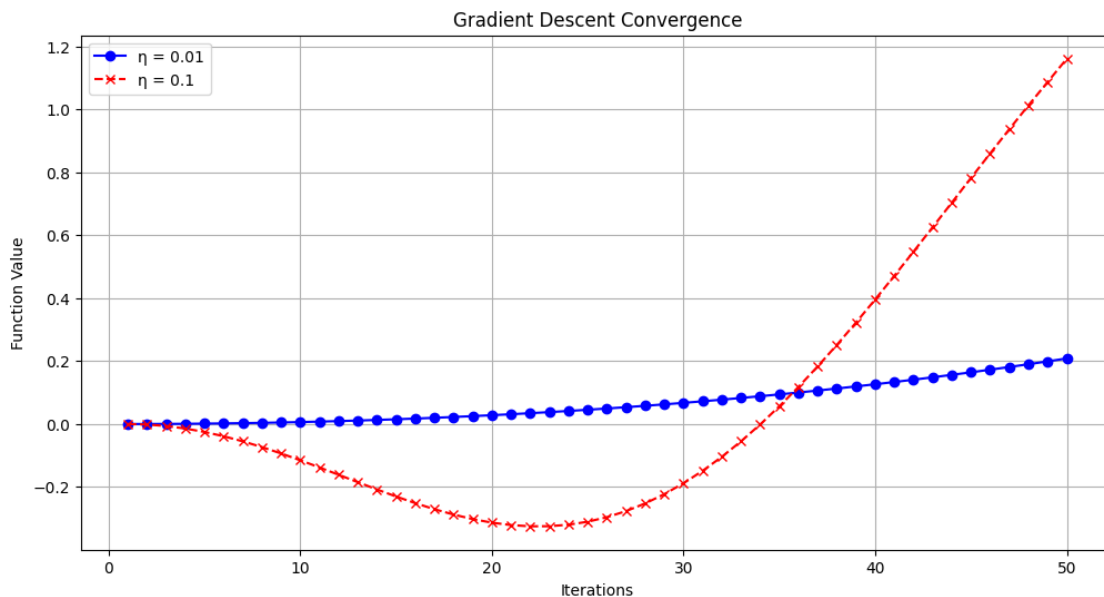
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learning_rate_2 = 0.1
num_iterations = 50

# Perform gradient descent
x1, y1, history1 = gradient_descent_Jose(learning_rate_1, num_iterations)
x2, y2, history2 = gradient_descent_Jose(learning_rate_2, num_iterations)

# Plot the convergence
iterations = list(range(1, num_iterations + 1))
plt.figure(figsize=(12, 6))
plt.plot(iterations, history1[0], label='  $\eta$  = 0.01', marker='o', linestyle='-', color='b')
plt.plot(iterations, history2[0], label='  $\eta$  = 0.1', marker='x', linestyle='--', color='r')
plt.xlabel('Iterations')
plt.ylabel('Function Value')
plt.legend()
plt.title('Gradient Descent Convergence')
plt.grid(True)
plt.show()
plt.figure(figsize=(12, 6))

```



[14]: <Figure size 1200x600 with 0 Axes>

<Figure size 1200x600 with 0 Axes>

```
[15]: # Gradient of the function
def gradient_f(x, y):
    df_dx = 2*x + 4*np.pi*np.cos(2*np.pi*x)*np.sin(2*np.pi*y)
    df_dy = 4*y + 4*np.pi*np.sin(2*np.pi*x)*np.cos(2*np.pi*y)
    return df_dx, df_dy
# Define a function for gradient descent
def gradient_descent_with_starting_point(learning_rate, num_iterations,
    ↪start_point):
    x, y = start_point
    history = [] # Store function values for plotting

    for i in range(num_iterations):
        df_dx, df_dy = gradient_f(x, y)
        x -= learning_rate * df_dx
        y -= learning_rate * df_dy
        history.append(f(x, y))

    return x, y, history

# List of initial points
initial_points = [(0.1, 0.1), (1, 1), (-0.5, -0.5), (-1, -1)]

# Perform gradient descent for each initial point
results = []
for start_point in initial_points:
    x, y, history = gradient_descent_with_starting_point(learning_rate_1,
    ↪num_iterations, start_point)
    min_value = f(x, y)
    results.append((start_point, (x, y), min_value))

# Print results in a table
print("Initial Point      Minimum Location      Minimum Value")
for start_point, (x, y), min_value in results:
    print(f"{start_point}          ({x:.4f}, {y:.4f})          {min_value:.4f}")
```

Initial Point	Minimum Location	Minimum Value
(0.1, 0.1)	(0.2438, -0.2379)	-1.8201
(1, 1)	(1.2181, 0.7128)	0.5933
(-0.5, -0.5)	(-0.7314, -0.2379)	-1.3325
(-1, -1)	(-1.2181, -0.7128)	0.5933

5 Question 4

The bias variance trade off allows one to note that the increasing of the validation set test size one can better approximate the out-of-sample error since bias nears 0.