

Standard Imports

Import Graph Theory tools to commence assessment on Characteristic Path Length.

Characteristic Path Length and Global Efficiency

Characteristic Path Length is a distance metric that provides insight into network analysis.[1,2,3,6,7] It is used in many fields such as World-Wide-Web, the Internet, biological, electrical, and many other giving insights into many things. The number of connections needed to transverse from one connection to another is the measure that is given by the characteristic path length. Here is the equation for Characteristic Path Length often called average shortest path length. Futhermore it said to be the average of the all the distances over the pairs of edges.

Where l_i is average path length.

Where V is the set of nodes in a graph.

Where $q = |V|$ represents its order.

$$\frac{1}{q(q-1)} \sum_{i \in V} l_i$$

[6]

Much of this is based on the dynamics of the small networks. [3] Much of this is done to networks that are between regularity, and disorder in a region called small world networks. [3] Some uses that comes from this for example is shown in how neural circuits in the brain are heavily impacted by the various types of lesions that can form, storkes and other damages that can occur to them. [4] Using Graph Theory one can see the impact of the importance of the measure. In reality this measure shows how information can easily be spread from one area to another such as the case of the Major Depressive Disorder Patients in a study that used a network/graph to treat patients. [5]

Global Efficiency is given in the following equation which looks very similar to the Characteristic Path Length Equation.

Where l_{ij} is average path length.

Where i and j are two nodes that are in the graph.

Where V is the set of nodes in a graph.

Where $q = |V|$ represents its order.

$$\frac{1}{q(q-1)} \sum_{i \in V} \frac{1}{l_{ij}}$$

The global efficiency is the average of the inverse shortest path length for all pairs of nodes.[1,2,3,6,7] The main focus of this network is the how it shows how two nodes communicate thus being a valuable connection between characteristic path length giving insight to how information speards across a network.

Work Cited

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In [1]: import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import networkx as nx
from IPython.display import display
```

```
In [2]: class AdjtoGraph:
def __init__(self,nodes):
    self.nodes = nodes
def matrix_creation(self):
    matrix = np.empty((self.nodes,self.nodes))
    for i in range(self.nodes):
        for j in range(self.nodes):
            matrix[i][j] = np.random.randint(0,self.nodes)
    G = nx.from_numpy_array(matrix)
    return G, matrix
```

```
In [4]: ASPL_ls = []
matrix_ls = []
graph_ls = []
for i in range(4):
    Graph1 = AdjtoGraph(nodes=4)
    G, matrix = Graph1.matrix_creation()
    ASPL_ls.append(nx.average_shortest_path_length(G))
    matrix_ls.append(matrix)
    graph_ls.append(G)
    print(f"Global Efficiency is {nx.global_efficiency(G)}")
```

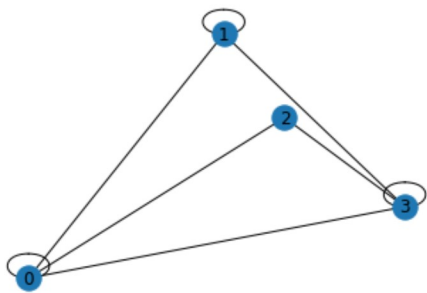
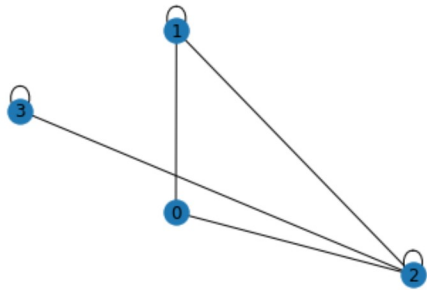
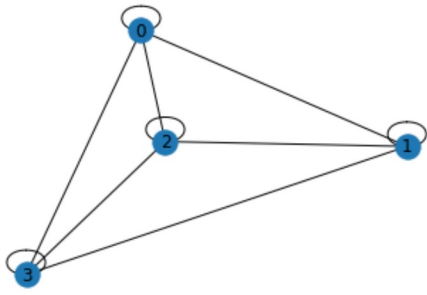
```
Global Efficiency is 1.0
Global Efficiency is 1.0
Global Efficiency is 0.8333333333333334
Global Efficiency is 0.9166666666666666
```

0		0	1	2	3
0	2.0	3.0	1.0	3.0	
1	2.0	0.0	3.0	3.0	
2	1.0	1.0	3.0	0.0	
3	3.0	3.0	2.0	3.0	
1		0	1	2	3
0	3.0	3.0	3.0	1.0	
1	2.0	3.0	3.0	2.0	
2	1.0	0.0	2.0	3.0	
3	2.0	0.0	0.0	2.0	
2		0	1	2	3
0	0.0	0.0	2.0	0.0	
1	2.0	2.0	3.0	0.0	
2	1.0	2.0	3.0	1.0	
3	0.0	0.0	3.0	1.0	
3		0	1	2	3
0	2.0	2.0	0.0	3.0	
1	1.0	3.0	0.0	3.0	
2	2.0	0.0	0.0	3.0	
3	0.0	2.0	2.0	1.0	

```

graph TD
    0((0)) --- 1((1))
    1 --- 2((2))
    2 --- 3((3))
    3 --- 0
    0 --- 2
    1 --- 3
    0 -- self-loop --> 0
    1 -- self-loop --> 1
    2 -- self-loop --> 2
    3 -- self-loop --> 3

```



```
In [6]: nx.draw_circular(G, with_labels = True)
```

