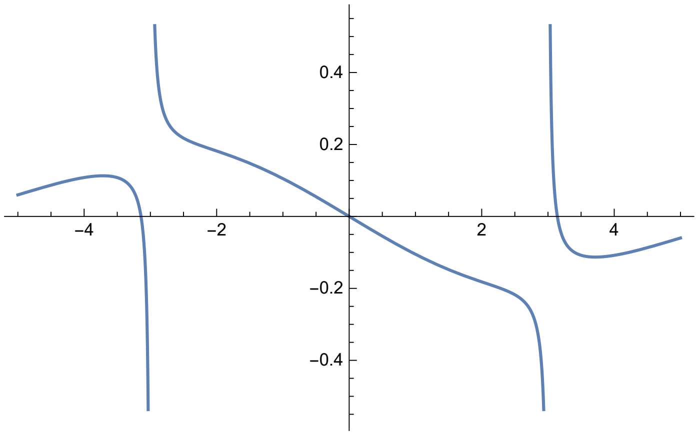
In[1]:= `b[x_] := Sin[x] / (x^2 - 9)`

In[2]:= `Plot[b[x], {x, -5, 5}]`

Out[2]=



(a). Domain all real numbers except positive, and negative 3:

In[3]:= `FunctionDomain[b[x], x]`

Out[3]= $x < -3 \;||\; -3 < x < 3 \;||\; x > 3$

(b). Y-Int, occurs at the point (0,0)

In[4]:= `b[0]`

Out[4]= 0

(c). X-Int or zeroes of function

In[74]:= `Solve[b[x] == 0 && x ∈ Interval[{-5, 5}], x]`

Out[74]= $\{\{x \to \{0\}\}, \{x \to \{-\pi\}\}, \{x \to \{\pi\}\}\}$

(d.) Vertical, and Horizontal Asymptotes

In[6]:= `Limit[b[x], x → Infinity]`

Out[6]= 0

In[7]:= `Limit[b[x], x → -Infinity]`

Out[7]= 0

There is a horizontal asymptote at y=0.

In[10]:= `Solve[Denominator[b[x]] == 0, x]`

Out[10]= $\{\{x \to -3\}, \{x \to 3\}\}$

In[11]:= `Limit[b[x], x → 3]`

Out[11]= ∞

In[12]:= `Limit[b[x], x → 3, Direction → 1]`

Out[12]= $-\infty$

In[13]:= `Limit[b[x], x → 3, Direction → -1]`

Out[13]= ∞

In[14]:= `Limit[b[x], x → -3]`

Out[14]= ∞

In[15]:= `Limit[b[x], x → -3, Direction → 1]`

Out[15]= $-\infty$

In[16]:= `Limit[b[x], x → -3, Direction → -1]`

Out[16]= ∞

The graph appears to approach a vertical asymptote at x=3, and at x=-3.
(e.) Intervals of increasing and decreasing

In[36]:= $\text{Reduce}\left[\frac{\text{Cos}[x]}{-9+x^2} - \frac{2\,x\,\text{Sin}[x]}{\left(-9+x^2\right)^2} > 0\ \&\&\ x \in \text{Interval}[\{-5, 5\}], x\right]$

Out[36]= $-5 \le x_1 < \text{Root}\left[\left\{-9\,\text{Cos}[\#1] - 2\,\text{Sin}[\#1]\,\#1 + \text{Cos}[\#1]\,\#1^2\ \&, -3.7156898500460495465\right\}\right]\ ||\ \text{Root}\left[\left\{-9\,\text{Cos}[\#1] - 2\,\text{Sin}[\#1]\,\#1 + \text{Cos}[\#1]\,\#1^2\ \&, 3.7156898500460495465\right\}\right] < x_1 \le 5$

In[37]:= `N[%]`

Out[37]= $-5. \le x_1 < -3.71569\ ||\ 3.71569 < x_1 \le 5.$

In[38]:= `Solve[b'[x] == 0 && x ∈ Interval[{-5, 5}], x]`

Out[38]= $\left\{\left\{x \to \left\{\text{Root}\left[\left\{-9\,\text{Cos}[\#1] - 2\,\text{Sin}[\#1]\,\#1 + \text{Cos}[\#1]\,\#1^2\ \&, -3.7156898500460495465\right\}\right]\right\}\right\}, \left\{x \to \left\{\text{Root}\left[\left\{-9\,\text{Cos}[\#1] - 2\,\text{Sin}[\#1]\,\#1 + \text{Cos}[\#1]\,\#1^2\ \&, 3.7156898500460495465\right\}\right]\right\}\right\}\right\}$

In[39]:= `N[%]`

Out[39]= $\{\{x \to \{-3.71569\}\}, \{x \to \{3.71569\}\}\}$

In[40]:= `Reduce[b'[x] < 0 && x ∈ Interval[{-5, 5}], x]`

Out[40]= $\text{Root}\left[\left\{-9\,\text{Cos}[\#1] - 2\,\text{Sin}[\#1]\,\#1 + \text{Cos}[\#1]\,\#1^2\ \&, -3.7156898500460495465\right\}\right] < x_1 < -3\ ||\ -3 < x_1 < 3\ ||\ 3 < x_1 < \text{Root}\left[\left\{-9\,\text{Cos}[\#1] - 2\,\text{Sin}[\#1]\,\#1 + \text{Cos}[\#1]\,\#1^2\ \&, 3.7156898500460495465\right\}\right]$

In[41]:= `N[%]`

Out[41]= $-3.71569 < x_1 < -3.\ ||\ -3. < x_1 < 3.\ ||\ 3. < x_1 < 3.71569$

The graph is increasing on an open interval from -5 to -3.71569, and 3.71569 to 5. The graph is also decreasing from an open interval -3.71569 to -3, and 3 to 3.71569, and as well as in -3 to 3.
(f.) Local Extrema

In[42]:= `b[-3.71569]`

Out[42]= 0.112992

In[43]:= `b[3.71569]`

Out[43]= -0.112992

The local maximum occurs at (-3.71569,0.112992), and minimum occurs at (3.71569,-0.112992).

(g.) Concavity

```
In[44]:= Reduce[b''[x] < 0 && x ∈ Interval[{-5, 5}], x]

Out[44]= Root[{63 Sin[#1] - 36 Cos[#1] #1 - 24 Sin[#1] #1^2 + 4 Cos[#1] #1^3 + Sin[#1] #1^4 &,
            -4.9568766852431779463}] < x1 < -3 ||
          Root[{63 Sin[#1] - 36 Cos[#1] #1 - 24 Sin[#1] #1^2 + 4 Cos[#1] #1^3 + Sin[#1] #1^4 &,
            -2.0612340828679723452}] < x1 < 0 ||
          Root[{63 Sin[#1] - 36 Cos[#1] #1 - 24 Sin[#1] #1^2 + 4 Cos[#1] #1^3 + Sin[#1] #1^4 &,
            2.0612340828679723452}] < x1 < 3 ||
          Root[{63 Sin[#1] - 36 Cos[#1] #1 - 24 Sin[#1] #1^2 + 4 Cos[#1] #1^3 + Sin[#1] #1^4 &,
            4.9568766852431779463}] < x1 ≤ 5

In[45]:= N[%]

Out[45]= -4.95688 < x1 < -3. || -2.06123 < x1 < 0. || 2.06123 < x1 < 3. || 4.95688 < x1 ≤ 5.
```

The graph is concave down on the given intervals given.

```
In[48]:= Reduce[b''[x] > 0 && x ∈ Interval[{-5, 5}], x]

Out[48]= -5 ≤ x1 < Root[{63 Sin[#1] - 36 Cos[#1] #1 - 24 Sin[#1] #1^2 + 4 Cos[#1] #1^3 + Sin[#1] #1^4 &,
            -4.9568766852431779463}] ||
          -3 < x1 < Root[{63 Sin[#1] - 36 Cos[#1] #1 - 24 Sin[#1] #1^2 + 4 Cos[#1] #1^3 + Sin[#1] #1^4 &,
            -2.0612340828679723452}] ||
          0 < x1 < Root[{63 Sin[#1] - 36 Cos[#1] #1 - 24 Sin[#1] #1^2 + 4 Cos[#1] #1^3 + Sin[#1] #1^4 &,
            2.0612340828679723452}] ||
          3 < x1 < Root[{63 Sin[#1] - 36 Cos[#1] #1 - 24 Sin[#1] #1^2 + 4 Cos[#1] #1^3 + Sin[#1] #1^4 &,
            4.9568766852431779463}]

In[49]:= N[%]

Out[49]= -5. ≤ x1 < -4.95688 || -3. < x1 < -2.06123 || 0. < x1 < 2.06123 || 3. < x1 < 4.95688
```

On the intervals given the graph is concave up.

(h.) Inflection Points

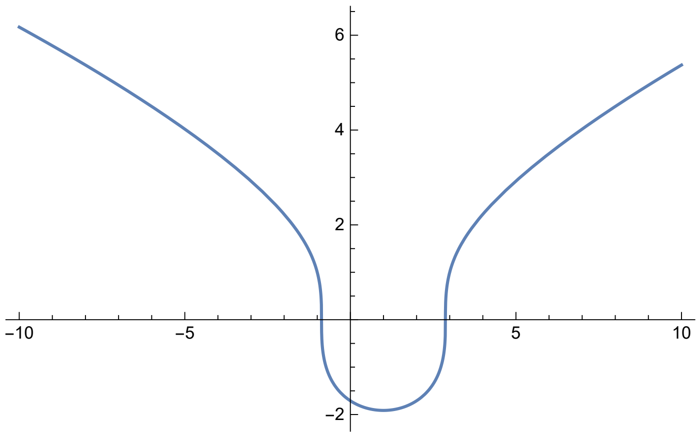
```
In[50]:= Solve[b''[x] == 0 && x ∈ Interval[{-5, 5}], x]

Out[50]= {{x → {0}},
          {x → {Root[{63 Sin[#1] - 36 Cos[#1] #1 - 24 Sin[#1] #1^2 + 4 Cos[#1] #1^3 + Sin[#1] #1^4 &,
              -4.9568766852431779463}]}},
          {x → {Root[{63 Sin[#1] - 36 Cos[#1] #1 - 24 Sin[#1] #1^2 + 4 Cos[#1] #1^3 + Sin[#1] #1^4 &,
              -2.0612340828679723452}]}},
          {x → {Root[{63 Sin[#1] - 36 Cos[#1] #1 - 24 Sin[#1] #1^2 + 4 Cos[#1] #1^3 + Sin[#1] #1^4 &,
              2.0612340828679723452}]}},
          {x → {Root[{63 Sin[#1] - 36 Cos[#1] #1 - 24 Sin[#1] #1^2 + 4 Cos[#1] #1^3 + Sin[#1] #1^4 &,
              4.9568766852431779463}]}}}

In[51]:= N[%]

Out[51]= {{x → {0.}}, {x → {-4.95688}}, {x → {-2.06123}}, {x → {2.06123}}, {x → {4.95688}}}
```

The inflection points occur at the points listed above.

In[68]:=
```
u[x_] := CubeRoot[2 x^2 - 4 x - 5]
```

In[69]:=
```
Plot[u[x], {x, -10, 10}]
```

Out[69]=



(a) Domain is all real numbers since the cube root can be taken of negative numbers, and the inner function is a polynomial which is continuous on all reals.
(b)Y-Int, which occurs at (0,-1.70998)

In[70]:=
```
u[0]
```

Out[70]= $-5^{1/3}$

In[71]:= $N\left[-5^{1/3}\right]$

Out[71]= -1.70998

(c) X-Intercepts or zeroes

In[72]:=
```
Solve[u[x] == 0, x]
```

Out[72]= $\left\{\left\{x \to \frac{1}{2}\left(2-\sqrt{14}\right)\right\}, \left\{x \to \frac{1}{2}\left(2+\sqrt{14}\right)\right\}\right\}$

In[73]:=
```
N[%]
```

Out[73]= {{x → -0.870829}, {x → 2.87083}}

The x-intercepts occur at (-0.870829,0), and (2.87083,0).
(d) Vertical and Horizontal Asymptotes

In[75]:=
```
Limit[u[x], x → Infinity]
```

Out[75]= ∞

In[76]:=
```
FullForm[∞]
```

Out[76]//FullForm= DirectedInfinity[1]

In[77]:=
```
Limit[u[x], x → -Infinity]
```

Out[77]= ∞

The function seems to increasing without bound as its end behavior.

(e) Intervals of Increasing and Decreasing

In[86]:= `Reduce[u'[x] > 0, x]`

Out[86]= $1 < x < \frac{1}{2}\left(2+\sqrt{14}\right) \,||\, x > \frac{1}{2}\left(2+\sqrt{14}\right)$

In[87]:= `N[%]`

Out[87]= $1. < x < 2.87083 \,||\, x > 2.87083$

The interval for which the function is increasing.

In[84]:= `Reduce[u'[x] < 0, x]`

Out[84]= $\frac{1}{2}\left(2-\sqrt{14}\right) < x < 1 \,||\, x < \frac{1}{2}\left(2-\sqrt{14}\right)$

In[85]:= `N[%]`

Out[85]= $-0.870829 < x < 1. \,||\, x < -0.870829$

The interval for which the function is increasing.

(f) Local Extrema

In[80]:= `Solve[u'[x] == 0, x]`

Out[80]= $\{\{x \to 1\}\}$

In[81]:= `u[1]`

Out[81]= $-7^{1/3}$

In[82]:= `N[%]`

Out[82]= -1.91293

The point at which the function has a minimum is at (1,-1.91293)

(g) Concavity

In[88]:= `Reduce[u''[x] > 0, x]`

Out[88]= $\frac{1}{2}\left(2-\sqrt{14}\right) < x < \frac{1}{2}\left(2+\sqrt{14}\right)$

In[89]:= `N[%]`

Out[89]= $-0.870829 < x < 2.87083$

The for which the function is concave up is given.

In[90]:= `Reduce[u''[x] < 0, x]`

Out[90]= $x > \frac{1}{2}\left(2+\sqrt{14}\right) \,||\, x < \frac{1}{2}\left(2-\sqrt{14}\right)$

In[91]:= `N[%]`

Out[91]= $x > 2.87083 \,||\, x < -0.870829$

The function is concave down on these intervals.

(h) Points of Inflection

In[92]:= `Solve[u''[x] == 0, x]`

Out[92]= $\left\{\left\{x \to \frac{1}{2}\left(2 - i\sqrt{42}\right)\right\}, \left\{x \to \frac{1}{2}\left(2 + i\sqrt{42}\right)\right\}\right\}$

In[93]:= `N[%]`

Out[93]= $\{\{x \to 1. - 3.24037\, i\}, \{x \to 1. + 3.24037\, i\}\}$

There are no points of inflection in the graph.

Overall both functions exhibited weird behavior like the first b[x] looked like a tangent function for part of its domain, and the u[x] looked like a bell graph sort of.