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Evaluating Limits
Objective: Analyze and compute limits either graphically, numerically, or algebraically. Reenforce the concept of continuity by providing a rigorous definition that is dependent on limits, and it being defined there. Also to see how continuity plays a key role in many theorems.
Commands:Limit, Direction, FullForm, Plot, CubeRoot, N, PlotRange, Automatic, FindRoot

1. Given all these several functions in each part a, b, and c limits were calculated by Wolfram Mathematica, and then the answers were evaulated.

(**a**) Given the function f1[x] = 1 / (x – 10)

```
In[1]:= f1[x] := 1 / (x - 10)
```

The limit of the function, f1[x], as x approached 10 was calculated.

```
In[4]:= Limit[f1[x], x → 10]
Out[4]= ∞
```

There were three outputs before this one which were simple mistakes that were made while inputting the exact input. The output of the limit is infinity, because the function when it was calculated approaches infinity from the left and right; however, from the right it approached positive infinity, and left negative infinity when approached by hand. By hand the limit does not exist, and does not have a special does not exist condition such as infinity, because it approaches two separate infinities. The domain of the function, f1[x], is D: (-∞,10) ∪ (10, ∞). The limit to this can be approached intuitively, by using one sided limits, and then we can see values of the limit approaches from the left, and then from the right.

The limit was calculated from numbers less than 10, and there were a couple of errors while trying out the new command. The one-sided limit yielded negative infinity.

```
In[12]:= Limit[f1[x], x → 10, Direction → 1]
Out[12]= -∞
```

The limit was calculated from numbers greater than 10, and this one-sided limit yielded positive infinity.

```
In[11]:= Limit[f1[x], x → 10, Direction -> -1]
Out[11]= ∞
```

The limit from the left (In[12]), and the limit from the right (In[11]) do not equal hence the limit does not exist because both limits do not equal each other.
Another function was used and that function name is FullForm, and it simplfiies the output into a form without symbols.

```
In[6]:= FullForm[Limit[f1[x], x → 10]]
Out[6]//FullForm= DirectedInfinity[1]
```

There was one output which was done before, and it was the exact input as input 6, but because I saw it was a new command I thought it was wrong. This new function seems to like show the output in a simple notation, without symbols.

(b) Given the function f2[x]=Sin[1/(x+Pi)].

In[13]:=
```
f2[x_] := Sin[1 / (x - Pi)]
```

The limit was then calculated as the function, f2[x], approached Pi.

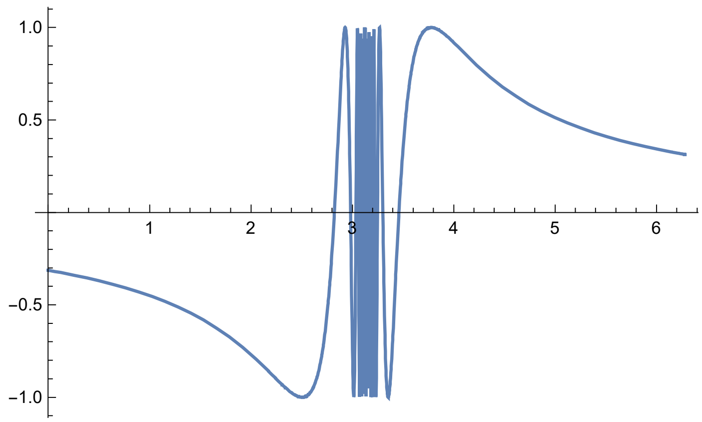
In[14]:=
```
Limit[f2[x], x → Pi]
```

Out[14]=
```
Interval[{-1, 1}]
```

The limit of the function was evaluated and it appears to oscillates between two values as x approaches Pi, and those values are -1, and 1. One way to evaluate this limit is to do it graphically. The function is plotted at In[18].

In[18]:=
```
Plot[f2[x], {x, 0, 2 * Pi}]
```

Out[18]=



Here in Out[18] we see that the graph oscillates between -1, and 1 as it approaches x=Pi. As we can see the limit does not exist. The domain of the function of the all real numbers except x=Pi. In other words (-∞,Pi)U(Pi,∞)

(c).

Given the function f3[x] =(Sqrt[6-x]-2)/(Sqrt[3-x]-1)

In[19]:=
```
f3[x_] := (Sqrt[6 - x] - 2) / (Sqrt[3 - x] - 1)
```

The limit of this function as x approaches 2 was calculated.

In[20]:=
```
Limit[f3[x], x → 2]
```

Out[20]= $\frac{1}{2}$

The limit of the function appears to be .5. Using algebraic properties, one could multiply by the conjugate of the denominator. Then Limit Laws would suffice to finish. The domain of the function is all real numbers except numbers greater than 3 and 3. The domain of the function is from (-∞,3).

2. Given the function f[x_]:=(x^2-9)/(2x^2+7x+3)

In[21]:=
```
f[x_] := (x^2 - 9) / (2 x^2 + 7 x + 3)
```

Then f[x], was factored.

In[22]:= `Factor[f[x]]`

Out[22]= $\frac{-3+x}{1+2x}$

The factored form of f[x], was then used to define g[x].

In[23]:= `g[x_] := Out[22]`

The limit of g[x] was calculated.

In[24]:= `Limit[g[x], x → -3]`

Out[24]= $\frac{6}{5}$

The limit of f[x] was calculated.

In[25]:= `Limit[f[x], x → -3]`

Out[25]= $\frac{6}{5}$

f[x] does not equal g[x] because f[x] is not defined at x=-3, and g[x] is defined. f[x] is defined (-∞,-3)∪(-3, -.5)U(-.5,∞), and g[x] is defined from (-∞,-.5)∪(-.5,∞). Yes the limit is the same because when we calculate the limit of f[x], we can be able to reduce the function into g[x] as long as its in the limit, and then we can use Direct Substitution Property and evaluate the limit. Thus seeing that they are equal.

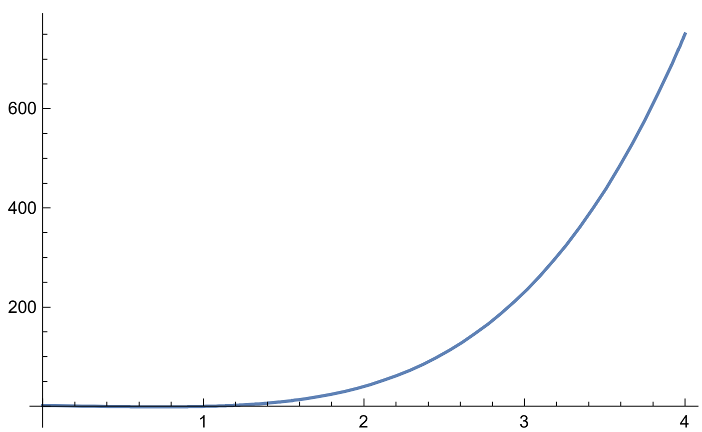
3. Given the function m[x]= 3x^4-5x+CubeRoot[x^2+4] the definition of continuity, and limit methods will be used to show that m(x) is continuous at a=2.

In[46]:= `m1[x_] := 3 x^4 - 5 x + CubeRoot[x^2 + 4]`

To show that the function is continuous that means that the one sided limit as x approaches a from the left, and right exists, the limit as x approaches a exists as well, and the function is defined at that point. The graph of m[x] was plotted

In[47]:= `Plot[m1[x], {x, 0, 4}]`

Out[47]=



The left handed limit is calculated.

In[48]:= `Limit[m1[x], x → 2, Direction → 1]`

Out[48]= 40

Then the right handed limit is then calculated.

```
In[49]:= Limit[m1[x], x → 2, Direction → -1]
Out[49]= 40
```

The limit of the function was then calculated as a whole.

```
In[50]:= Limit[m1[x], x → 2]
Out[50]= 40
```

m1[2] was also evaluated.

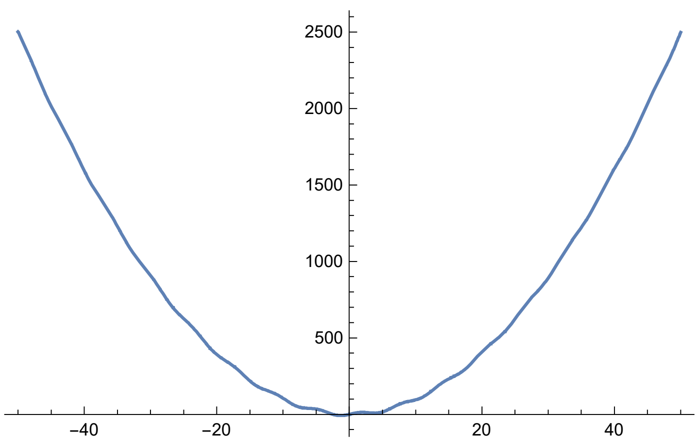
```
In[51]:= m1[2]
Out[51]= 40
```

After several attempts, I found out that I had made an error within the code which caused why so many outputs, and even the change of variable. I found out that a mistake had occurred when I was defining the function, and I had forgot the underscore. The function appears at the point m1[2] is defined. So looking at all the criterion of how a function is continuous it is made known through how the limit from the left, and right-hand existed, and were equal. So did the limit overall of m1[x] as it approached 2, existed, and so m1[2] was defined. All these three criterion being valid mean that the function is continuous at a=2.

Given the function h[x]:=x^2+10sin(x), and plotting the function.

```
In[57]:= h[x_] := x^2 + 10 Sin[x]
         Plot[h[x], {x, -50, 50}, PlotRange → Automatic]
```

Out[58]=



The previous plot shows 2 values of which the function appears to be 1000. Using the definition of Intermediate value theorem setting the interval from x=20, to x=40.

h[20] was then evaluated.

```
In[59]:= h[20]
Out[59]= 400 + 10 Sin[20]
```

The numerical value was then calculated to make analyzing things easier.

```
In[60]:= N[400 + 10 Sin[20]]
Out[60]= 409.129
```

h[40] was then evaluated.

```
In[61]:= h[40]
Out[61]= 1600 + 10 Sin[40]
```

The numerical value was then calculated to make analyzing things easier.

```
In[62]:= N[1600 + 10 Sin[40]]
Out[62]= 1607.45
```

Seeing the value of the functions are not equivalent at the end points. Then we can say since the function is continuous that there exists a number c, 1000, in (20,40) such that h(c)=1000.

Given the function h1[x]:= h[x]-1000.

```
In[63]:= h1[x_] := h[x] - 1000
```

Then using the command FindRoot the functions zeroes were found.

```
In[64]:= FindRoot[h1[x] == 0, {x, 20, 40}]
Out[64]= {x → 31.5947}
```

The root is appears to be at x=31.5947.

The overall problem showed how it is possible to achieve the N which was evaluated at the beginning of the problem of the, using a vertical shift.

The project taught how to use the limit function, and the expanded limit function which is used to calculate one-sided limits. The function also revealed how the Direction command within the expanded limit function intuitively inverts the original thought directions of left, and right with 1, and -1. There was how an arsenal of functions such as Limits, and plot, and full form which could be used to understand the out puts which are given from Mathematica. The intermediate value theorem was also used from the software using root functions, and intuition.