1 Chapter 4.7 Variation of Parameters

1.1 Number 1.

$$y'' + y = \sec(x) \tag{1}$$

Then using the auxiliary equation one gets the following when solving for the associated homogeneous equation:

$$\begin{aligned} m^2 + 1 &= 0 \\ m^2 &= -1 \\ m &= \pm\sqrt{-1} \\ m &= \pm i \end{aligned}$$

Once we have the powers of Euler's number we can plug it in and get the following:

$$y_h = c_1 \cos(x) + c_2 \sin(x) \tag{2}$$

After this one begins to set up a Wronskian for the problem from the known solutions:

$$W = \begin{vmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{vmatrix} \tag{3}$$

$$\det(W) = (\cos(x)) * (\cos(x)) - (\sin(x)) * (-\sin(x)) = 1 \tag{4}$$

$$u_1' = \frac{-y_2 f(x)}{W} = \frac{-\sin(x)\sec(x)}{1} \tag{5}$$

Then one integrates $u_1'(x)$

$$\begin{aligned} u_1 &= -\int \frac{\sin(x)}{\cos(x)} dx \\ &= \int \frac{1}{u} du \\ &= \ln|\cos(x)| \end{aligned}$$

Then to get the other u one has to do the following:

$$u_2' = \frac{y_1 f(x)}{W} = \frac{\cos(x)\sec(x)}{1} = 1 \tag{6}$$

Then one can integrate $u_2'(x)$

$$u_2 = \int 1 dx = x \tag{7}$$

Then we would have to do the following which is $y_p = y_1 u_1 + y_2 u_2$

$$y_p = (\cos(x))(\ln(|\cos(x)|) + (\sin(x))(x) \tag{8}$$

The final solution to the problem is the following:

$$\begin{aligned} y &= y_h + y_p \qquad (9) \\ &= c_1 \cos(x) + c_2 \sin(x) + \cos(x) \ln(|\cos(x)|) + x \sin(x) \qquad (10) \end{aligned}$$