

Mathematics Step by Step

Show $[x^n]' = nx^{n-1}$ by using first principles.

$$\begin{aligned}f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\&= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^n - (x)^n}{\Delta x} \\&= \lim_{\Delta x \rightarrow 0} \frac{\binom{n}{0}x^n\Delta x^0 + \binom{n}{1}x^{n-1}\Delta x^1 + \cdots + \binom{n}{n}x^0\Delta x^n - x^n}{\Delta x} \\&= \lim_{\Delta x \rightarrow 0} \frac{1x^n(1) + nx^{n-1}\Delta x^1 + \cdots + 1(1)\Delta x^n - x^n}{\Delta x} \\&= \lim_{\Delta x \rightarrow 0} \frac{x^n + nx^{n-1}\Delta x + \cdots + \Delta x^n - x^n}{\Delta x} \\&= \lim_{\Delta x \rightarrow 0} \frac{\cancel{x^n}(nx^{n-1} + \cdots + \Delta x^{n-1})}{\cancel{\Delta x}}\end{aligned}$$