Show $[x^n]' = nx^{n-1}$ by using first principles.

$$
\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^n - (x)^n}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\binom{n}{0}x^n\Delta x^0 + \binom{n}{1}x^{n-1}\Delta x^1 + \cdots + \binom{n}{n}x^0\Delta x^n - x^n}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{1x^n(1) + nx^{n-1}\Delta x^1 + \cdots + 1(1)\Delta x^n - x^n}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{x^n + nx^{n-1}\Delta x + \cdots + \Delta x^n - x^n}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(nx^{n-1} + \cdots + \Delta x^{n-1})}{\cancel{\Delta x}} \\
&= nx^{n-1}
\end{aligned}
$$