

TABLE 7.2 Laplace Transforms

$f(t), t \geq 0$	$F(s)$	ROC
1. $\delta(t)$	1	All s
2. $u(t)$	$\frac{1}{s}$	$\text{Re}(s) > 0$
3. t	$\frac{1}{s^2}$	$\text{Re}(s) > 0$
4. t^n	$\frac{n!}{s^{n+1}}$	$\text{Re}(s) > 0$
5. e^{-at}	$\frac{1}{s+a}$	$\text{Re}(s) > -a$
6. te^{-at}	$\frac{1}{(s+a)^2}$	$\text{Re}(s) > -a$
7. $t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$	$\text{Re}(s) > -a$
8. $\sin bt$	$\frac{b}{s^2 + b^2}$	$\text{Re}(s) > 0$
9. $\cos bt$	$\frac{s}{s^2 + b^2}$	$\text{Re}(s) > 0$
10. $e^{-at} \sin bt$	$\frac{b}{(s+a)^2 + b^2}$	$\text{Re}(s) > -a$
11. $e^{-at} \cos bt$	$\frac{s+a}{(s+a)^2 + b^2}$	$\text{Re}(s) > -a$
12. $t \sin bt$	$\frac{2bs}{(s^2 + b^2)^2}$	$\text{Re}(s) > 0$
13. $t \cos bt$	$\frac{s^2 - b^2}{(s^2 + b^2)^2}$	$\text{Re}(s) > 0$

properties allow additional transform pairs to be derived easily. Also, these properties aid us in solving linear differential equations with constant coefficients.

7.4 LAPLACE TRANSFORM PROPERTIES

In Sections 7.1 through 7.3, two properties were derived for the Laplace transform. These properties are

$$[\text{eq}(7.10)] \quad \mathcal{L}[a_1 f_1(t) + a_2 f_2(t)] = a_1 F_1(s) + a_2 F_2(s)$$

and

$$[\text{eq}(7.20)] \quad \mathcal{L}[e^{-at} f(t)] = F(s) \Big|_{s \leftarrow s+a} = F(s+a).$$

Equation (7.10) is the *linearity* property. Equation (7.20) is sometimes called the *complex shifting* property, since multiplication by e^{-at} in the time domain results in