$$u[-n + 10] = \begin{cases} 1, & n \leqq 10 \\ 0, & n > 10 \end{cases}.$$

A second example is given in Figure 11.14(b).

**3.** A function $f[n]$ is *two sided* if it is neither right sided nor left sided. For example, $\cos(n)$ is two sided.

**4.** A function is of *finite duration* if it is both right sided and left sided. For example, $(u[n] - u[n - 10])$ is of finite duration. A second example is given in Figure 11.14(c).

We find these definitions useful when working with bilateral transforms.

### Bilateral Transforms

In Chapter 7, a procedure was given for finding bilateral Laplace transforms from unilateral Laplace-transform tables. An equivalent procedure can be developed for finding bilateral $z$-transforms from unilateral $z$-transform tables. However, this procedure is complex and prone to error. Instead, a table of bilateral $z$-transforms is given as Table 11.5. A procedure is now given for using this table.

**TABLE 11.5** Bilateral $z$-Transform

| $f[n]$ | $F(z)$ | ROC |
|---|---|---|
| 1. $\delta[n]$ | $1$ | All $z$ |
| 2. $\delta[n - n_0]$ | $z^{-n_0}$ | $z \neq 0, n_0 \geqq 0$<br>$z \neq \infty, \ n_0 < 0$ |
| 3. $u[n]$ | $\dfrac{z}{z - 1}$ | $\lvert z\rvert > 1$ |
| 4. $nu[n]$ | $\dfrac{z}{(z - 1)^2}$ | $\lvert z\rvert > 1$ |
| 5. $a^n u[n]$ | $\dfrac{z}{z - a}$ | $\lvert z\rvert > \lvert a\rvert$ |
| 6. $na^n u[n]$ | $\dfrac{az}{(z - a)^2}$ | $\lvert z\rvert > \lvert a\rvert$ |
| 7. $a^n \sin(bn)u[n]$ | $\dfrac{az \sin b}{z^2 - 2az \cos b + a^2}$ | $\lvert z\rvert > \lvert a\rvert$ |
| 8. $a^n \cos(bn)u[n]$ | $\dfrac{z(z - a \cos b)}{z^2 - 2az \cos b + a^2}$ | $\lvert z\rvert > \lvert a\rvert$ |
| 9. $-u[-n - 1]$ | $\dfrac{z}{z - 1}$ | $\lvert z\rvert < 1$ |
| 10. $-a^n u[-n - 1]$ | $\dfrac{z}{z - a}$ | $\lvert z\rvert < \lvert a\rvert$ |
| 11. $-na^n u[-n - 1]$ | $\dfrac{az}{(z - a)^2}$ | $\lvert z\rvert < \lvert a\rvert$ |
| 12. $a^{\lvert n\rvert}, \lvert a\rvert < 1$ | $\dfrac{z}{z - a} - \dfrac{z}{z - 1/a}$ | $\lvert a\rvert < z < \lvert 1/a\rvert$ |