## SUMMARY

In this chapter, we define the Fourier transform (5.1) and the inverse Fourier transform (5.2).

The sufficient conditions for the existence of the integral (5.1) are called the Dirichlet conditions. In general, the Fourier transform of $f(t)$ exists if it is reasonably well behaved (if we could draw a picture of it) and if it is absolutely

**TABLE 5.3** Key Equations of Chapter 5

| Equation Title | Equation Number | Equation |
|---|---|---|
| Fourier transform | (5.1) | $\mathscr{F}\{f(t)\} = F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$ |
| Inverse Fourier transform | (5.2) | $f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega = F^{-1}\{F(\omega)\}$ |
| Linearity property | (5.10) | $af_1(t) + bf_2(t) \xleftrightarrow{\mathscr{F}} aF_1(\omega) + bF_2(\omega)$ |
| Time-transformation property | (5.15) | $f(at - t_0) \xleftrightarrow{\mathscr{F}} \frac{1}{\lvert a\rvert}F\left(\frac{\omega}{a}\right)e^{-jt_0(\omega/a)}$ |
| Duality property | (5.16) | $F(t) \xleftrightarrow{\mathscr{F}} 2\pi f(-\omega)$ when $f(t) \xleftrightarrow{\mathscr{F}} F(\omega)$ |
| Convolution property | (5.17) | $f_1(t)*f_2(t) \xleftrightarrow{\mathscr{F}} F_1(\omega)F_2(\omega)$ |
| Multiplication property | (5.18) | $f_1(t)f_2(t) \xleftrightarrow{\mathscr{F}} \frac{1}{2\pi}F_1(\omega)*F_2(\omega)$ |
| Frequency-shifting property | (5.19) | $x(t)e^{j\omega_0 t} \xleftrightarrow{\mathscr{F}} X(\omega - \omega_0)$ |
| Fourier transform of periodic signal | (5.36) | $\sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t} \xleftrightarrow{\mathscr{F}} 2\pi\sum_{k=-\infty}^{\infty} C_k\delta(\omega - k\omega_0)$ or |
| | | $\sum_{k=-\infty}^{\infty} g(t - kT_0) \xleftrightarrow{\mathscr{F}} \omega_0\sum_{k=-\infty}^{\infty} G(n\omega_0)\delta(\omega - k\omega_0), \omega_0 = \frac{2\pi}{T_0}$ |
| Frequency response function | (5.43) | $H(\omega) = \mathbb{F}\{h(t)\}, \quad H(\omega) = \frac{Y(\omega)}{X(\omega)}$ |
| Energy spectral density | (5.45) | $\mathscr{E}_f(\omega) = \frac{1}{\pi}\lvert F(\omega)\rvert^2 = \frac{1}{\pi}F(\omega)F^*(\omega)$ |
| Signal energy | (5.46) | $E = \int_0^{\infty}\mathscr{E}_f(\omega)d\omega$ |
| Power spectral density | (5.49) | $\mathscr{P}_f(\omega) = \lim_{T\to\infty}\frac{1}{T}\lvert F_T(\omega)\rvert^2$ |
| Signal power | (5.50) | $P = \frac{1}{\pi}\int_0^{\infty}\mathscr{P}_f(\omega)d\omega$ |
| Energy and | (5.55) | $\mathscr{E}_g(\omega) = \lvert H(\omega)\rvert^2\mathscr{E}_f(\omega)$ |
| Power transmission | (5.56) | $\mathscr{P}_g(\omega) = \lvert H(\omega)\rvert^2\mathscr{P}_f(\omega)$ |