TABLE 5.1 Fourier Transform Properties

Operation	Time Function	Fourier Transform
Linearity	$af_1(t) + bf_2(t)$	$aF_1(\omega) + bF_2(\omega)$
Time shift	$f(t - t_0)$	$F(\omega)e^{-j\omega t_0}$
Time reversal	$f(-t)$	$F(-\omega)$
Time scaling	$f(at)$	$\frac{1}{\lvert a\rvert}F\left(\frac{\omega}{a}\right)$
Time transformation	$f(at - t_0)$	$\frac{1}{\lvert a\rvert}F\left(\frac{\omega}{a}\right)e^{-j\omega t_0/a}$
Duality	$F(t)$	$2\pi f(-\omega)$
Frequency shift	$f(t)e^{j\omega_0 t}$	$F(\omega-\omega_0)$
Convolution	$f_1(t)*f_2(t)$	$F_1(\omega)F_2(\omega)$
Modulation (Multiplication)	$f_1(t)f_2(t)$	$\frac{1}{2\pi}F_1(\omega)*F_2(\omega)$
Integration	$\int_{-\infty}^{t} f(\tau)d\tau$	$\frac{1}{j\omega}F(\omega) + \pi F(0)\delta(\omega)$
Differentiation in time	$\frac{d^n[f(t)]}{dt^n}$	$(j\omega)^n F(\omega)$
Differentiation in Frequency	$(-jt)^n f(t)$	$\frac{d^n[F(\omega)]}{d\omega^n}$
Symmetry	$f(t)$ real	$F(-\omega) = F^*(\omega)$

Linearity

Because the Fourier transform (5.1) is an integral of $f(t)$ and its inverse (5.2) is an integral of $F(\omega)$, and because integration is a linear operation, it can be reasoned that the Fourier transform is a linear operation. The linearity property of the Fourier transform states that if we are given the transform pairs

$$f_1(t) \overset{\mathscr{F}}{\longleftrightarrow} F_1(\omega) \quad \text{and} \quad f_2(t) \overset{\mathscr{F}}{\longleftrightarrow} F_2(\omega),$$

then

$$[af_1(t) + bf_2(t)] \overset{\mathscr{F}}{\longleftrightarrow} [aF_1(\omega) + bF_2(\omega)], \tag{5.10}$$

where a and b are constants. In words, the principle of superposition applies to the Fourier transform.

EXAMPLE 5.3 **The linearity property of the Fourier transform**

We can make use of the property of linearity to find the Fourier transforms of some types of waveforms. For example, consider

$$f(t) = B \cos \omega_0 t.$$