We now derive an additional transform as an example.

EXAMPLE 11.3   *z-transform of the unit ramp function*

The z-transform of the unit ramp function, $f[n] = n$, is now derived.

$$\mathcal{Z}[n] = \sum_{n=0}^{\infty} nz^{-n} = 0 + z^{-1} + 2z^{-2} + 3z^{-3} + \cdots.$$

We have, from Appendix C, the summation formula

$$\sum_{n=0}^{\infty} na^n = \frac{a}{(1-a)^2}; \quad |a| < 1.$$

Hence, letting $a = z^{-1}$, we have the z-transform

$$\mathcal{Z}[n] = \sum_{n=0}^{\infty} nz^{-n} = \frac{z^{-1}}{(1 - z^{-1})^2} = \frac{z}{(z-1)^2}$$

TABLE 11.2   *z*-Transforms

$f[n], n \geqq 0$	$F(z)$	ROC
1. $\delta[n]$	1	All z
2. $\delta[n - n_0]$	z^{-n_0}	$z \neq 0$
3. $u[n]$	$\dfrac{z}{z - 1}$	$|z| > 1$
4. n	$\dfrac{z}{(z - 1)^2}$	$|z| > 1$
5. n^2	$\dfrac{z(z + 1)}{(z - 1)^3}$	$|z| > 1$
6. a^n	$\dfrac{z}{z - a}$	$|z| > |a|$
7. na^n	$\dfrac{az}{(z - a)^2}$	$|z| > |a|$
8. n^2a^n	$\dfrac{az(z + a)}{(z - a)^3}$	$|z| > |a|$
9. $\sin bn$	$\dfrac{z \sin b}{z^2 - 2z \cos b + 1}$	$|z| > 1$
10. $\cos bn$	$\dfrac{z(z - \cos b)}{z^2 - 2z \cos b + 1}$	$|z| > 1$
11. $a^n \sin bn$	$\dfrac{az \sin b}{z^2 - 2az \cos b + a^2}$	$|z| > |a|$
12. $a^n \cos bn$	$\dfrac{z(z - a \cos b)}{z^2 - 2az \cos b + a^2}$	$|z| > |a|$