Several properties of the Laplace transform have been developed. These properties are useful in generating tables of Laplace transforms and in applying the Laplace transform to the solutions of linear differential equations with constant coefficients. Because we prefer to model continuous-time physical systems with linear differential equations with constant coefficients, these properties are useful in both the analysis and design of linear time-invariant physical systems. Table 7.3 gives the derived properties for the Laplace transform, plus some additional properties. The derivations of some of these additional properties are given as problems at the end of this chapter, or are derived later when the properties are used.

TABLE 7.3 Laplace Transform Properties

Name	Property
1. Linearity, (7.10)	$\mathcal{L}[a_1 f_1(t) + a_2 f_2(t)] = a_1 F_1(s) + a_2 F_2(s)$
2. Derivative, (7.15)	$\mathcal{L}\left[\dfrac{df(t)}{dt}\right] = sF(s) - f(0^+)$
3. nth-order derivative, (7.29)	$\mathcal{L}\left[\dfrac{d^n f(t)}{dt^n}\right] = s^n F(s) - s^{n-1} f(0^+)$
	$- \cdots - sf^{(n-2)}(0^+) - f^{(n-1)}(0^+)$
4. Integral, (7.31)	$\mathcal{L}\left[\displaystyle\int_0^t f(\tau)d\tau\right] = \dfrac{F(s)}{s}$
5. Real shifting, (7.22)	$\mathcal{L}[f(t - t_0)u(t - t_0)] = e^{-t_0 s}F(s)$
6. Complex shifting, (7.20)	$\mathcal{L}[e^{-at}f(t)] = F(s + a)$
7. Initial value, (7.36)	$\lim\limits_{t \to 0^+} f(t) = \lim\limits_{s \to \infty} sF(s)$
8. Final value, (7.39)	$\lim\limits_{t \to \infty} f(t) = \lim\limits_{s \to 0} s\,F(s)$
9. Multiplication by t, (7.34)	$\mathcal{L}[tf(t)] = -\dfrac{dF(s)}{ds}$
10. Time transformation, (7.42) ($a > 0; b \geqq 0$)	$\mathcal{L}[f(at - b)u(at - b)] = \dfrac{e^{-sb/a}}{a} F\left(\dfrac{s}{a}\right)$
11. Convolution	$\mathcal{L}^{-1}[F_1(s)F_2(s)] = \displaystyle\int_0^t f_1(t - \tau)f_2(\tau)\,d\tau$
	$= \displaystyle\int_0^t f_1(\tau)f_2(t - \tau)d\tau$
12. Time periodicity	$\mathcal{L}[f(t)] = \dfrac{1}{1 - e^{-sT}} F_1(s)$, where
$[f(t) = f(t + T)], t \geqq 0$	$F_1(s) = \displaystyle\int_0^T f(t)e^{-st}\,dt$