Correlation Matrix Computation

1 Introduction

This document illustrates how to compute the correlation matrix for given segments of data and how to average these matrices over multiple segments. Each segment consists of 32 elements, alternating between two series A and B.

2 Segment Definition

Each segment is defined as:

$$\text{Segment} = [A_1, B_1, A_2, B_2, \ldots, A_{16}, B_{16}]$$

3 Correlation Matrix Computation

The correlation matrix C is an 8x8 matrix where each element C_{ij} is computed using the elements from the segment. Specifically, the element C_{ij} is given by:

$$C_{ij} = (A_{i+1} - A_{i+9})(B_{j+1} - B_{j+9})$$

for $i, j \in \{0, 1, \ldots, 7\}$.

3.1 Matrix Elements

For example:

$$\begin{aligned}
C_{00} &= (A_1 - A_9)(B_1 - B_9) \\
C_{01} &= (A_1 - A_9)(B_2 - B_{10}) \\
C_{10} &= (A_2 - A_{10})(B_1 - B_9) \\
&\vdots \\
C_{77} &= (A_8 - A_{16})(B_8 - B_{16})
\end{aligned}$$

4 Averaging the Correlation Matrices

Given N segments, each producing a correlation matrix C_k, the final average correlation matrix $\bar{C}$ is calculated by averaging each corresponding element across all segment matrices:

$$\bar{C}_{ij} = \frac{1}{N} \sum_{k=1}^{N} C_{ij}^{(k)}$$

where $C_{ij}^{(k)}$ is the (i, j)-th element of the k-th segment's correlation matrix and N is the total number of segments.

4.1 Example

For $N = 3$ segments, the average matrix $\bar{C}$ would be:

$$\bar{C}_{ij} = \frac{1}{3} \left(C_{ij}^{(1)} + C_{ij}^{(2)} + C_{ij}^{(3)} \right)$$

5 Summary

The formula for each element of the correlation matrix is:

$$C_{ij} = (A_{i+1} - A_{i+9})(B_{j+1} - B_{j+9})$$

The final average correlation matrix is:

$$\bar{C}_{ij} = \frac{1}{N} \sum_{k=1}^{N} C_{ij}^{(k)}$$

6 Aggregated Correlation Matrix

The correlation matrices for all segments are arranged into an aggregated correlation matrix M, where each column represents a correlation matrix of one segment:

$$M = \begin{bmatrix} C_{00}^{(1)} & C_{00}^{(2)} & \cdots & C_{00}^{(N)} \\ C_{01}^{(1)} & C_{01}^{(2)} & \cdots & C_{01}^{(N)} \\ \vdots & \vdots & \ddots & \vdots \\ C_{77}^{(1)} & C_{77}^{(2)} & \cdots & C_{77}^{(N)} \end{bmatrix}$$

7 Vector for Averaging

The vector $\mathbf{v}$ used for averaging is:

$$\mathbf{v} = \begin{bmatrix} \frac{1}{N} \\ \frac{1}{N} \\ \vdots \\ \frac{1}{N} \end{bmatrix}$$

8 Matrix-Vector Multiplication

The resulting averaged correlation matrix $\bar{C}$ is obtained by multiplying the aggregated correlation matrix M by the vector $\mathbf{v}$:

$$\bar{C} = M\mathbf{v}$$

This can be illustrated as:

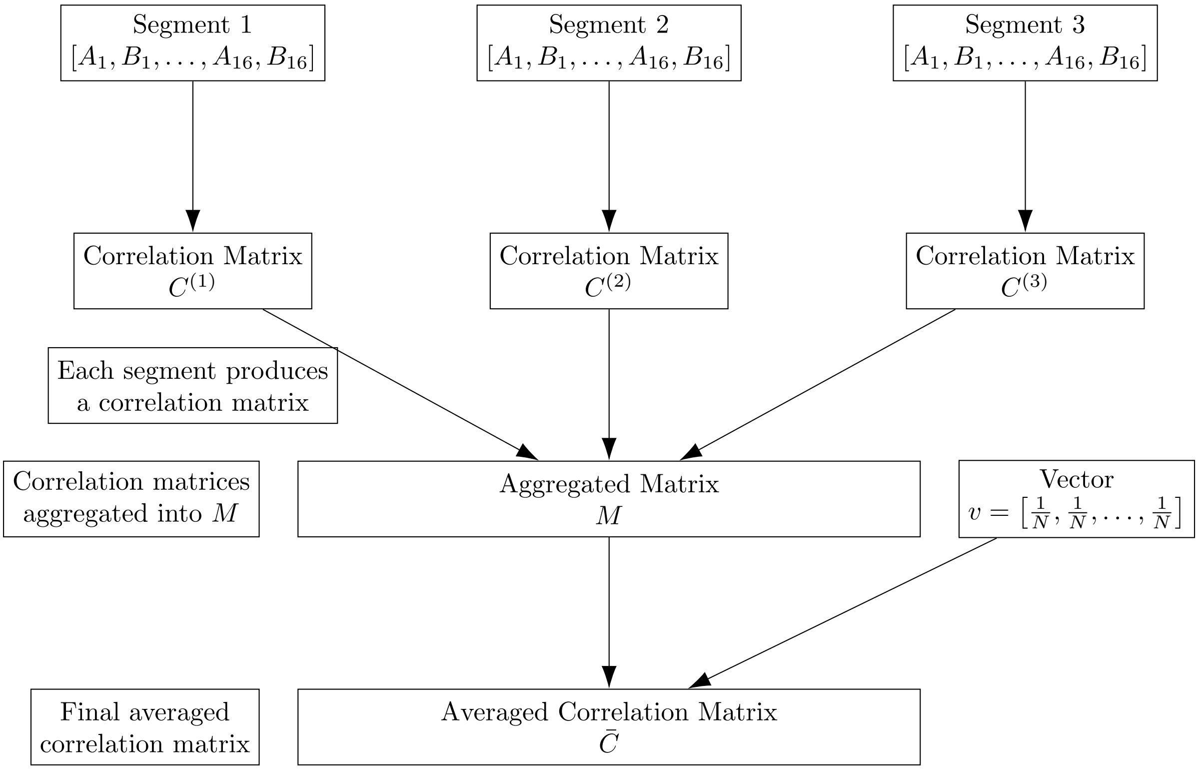
$$\bar{C} = \begin{bmatrix} C_{00}^{(1)} & C_{00}^{(2)} & \cdots & C_{00}^{(N)} \\ C_{01}^{(1)} & C_{01}^{(2)} & \cdots & C_{01}^{(N)} \\ \vdots & \vdots & \ddots & \vdots \\ C_{77}^{(1)} & C_{77}^{(2)} & \cdots & C_{77}^{(N)} \end{bmatrix} \begin{bmatrix} \frac{1}{N} \\ \frac{1}{N} \\ \vdots \\ \frac{1}{N} \end{bmatrix}$$

9 Resulting Averaged Correlation Matrix

Each element of the final averaged correlation matrix $\bar{C}$ is:

$$\bar{C}_{ij} = \sum_{k=1}^{N} \frac{1}{N} C_{ij}^{(k)}$$

10 Workflow



Advanced: Magnon Correlation g_2 Formula

The magnon correlation g_2 matrix is then given by:

The magnon correlation g_2 matrix term G_{ijmn} can be defined as:

$$G_{ijmn} = \frac{\sum [(A_i - A_{i+8})(A_j - A_{j+8})(B_m - B_{m+8})(B_n - B_{n+8})]}{\left(\sum [(A_i - A_{i+8})(A_j - A_{j+8})]\right)\left(\sum [(B_m - B_{m+8})(B_n - B_{n+8})]\right)}$$

where $i, j, m, n \in \{1, 2, \ldots, 8\}$.

The sums are over multiple segments.

11 Magnon Correlation g_2 Computation

The magnon correlation g_2 is a measure of the correlation between two series of data, A and B. This section describes the steps to compute the g_2 magnon correlation matrix.

11.1 AutoCorrelation Matrices

For each segment, we compute the auto-correlation matrices for A and B. Each segment contains 16 elements for A and 16 elements for B, represented as:

$$\text{Segment} = [A_1, B_1, A_2, B_2, \ldots, A_{16}, B_{16}]$$

11.1.1 AutoCorrelation Matrix for Series A

The auto-correlation matrix C_A for series A is an 8x8 matrix where each element is computed as:

$$(C_A)_{ij} = (A_i - A_{i+8})(A_j - A_{j+8})$$

for $i, j \in \{1, 2, \ldots, 8\}$.

11.1.2 AutoCorrelation Matrix for Series B

Similarly, the auto-correlation matrix C_B for series B is computed as:

$$(C_B)_{ij} = (B_i - B_{i+8})(B_j - B_{j+8})$$

for $i, j \in \{1, 2, \ldots, 8\}$.

11.2 Aggregated AutoCorrelation Matrices

Given N segments, we aggregate the auto-correlation matrices for series A and B into two large matrices, M_A and M_B. Each column in these matrices corresponds to the flattened auto-correlation matrix of one segment.

11.2.1 Aggregated AutoCorrelation Matrix for Series A

$$M_A = \begin{bmatrix} (C_A)_{11}^{(1)} & (C_A)_{11}^{(2)} & \cdots & (C_A)_{11}^{(N)} \\ (C_A)_{12}^{(1)} & (C_A)_{12}^{(2)} & \cdots & (C_A)_{12}^{(N)} \\ \vdots & \vdots & \ddots & \vdots \\ (C_A)_{88}^{(1)} & (C_A)_{88}^{(2)} & \cdots & (C_A)_{88}^{(N)} \end{bmatrix}$$

11.2.2 Aggregated AutoCorrelation Matrix for Series B

$$M_B = \begin{bmatrix} (C_B)_{11}^{(1)} & (C_B)_{11}^{(2)} & \cdots & (C_B)_{11}^{(N)} \\ (C_B)_{12}^{(1)} & (C_B)_{12}^{(2)} & \cdots & (C_B)_{12}^{(N)} \\ \vdots & \vdots & \ddots & \vdots \\ (C_B)_{88}^{(1)} & (C_B)_{88}^{(2)} & \cdots & (C_B)_{88}^{(N)} \end{bmatrix}$$

11.3 Cross Correlation C_{AB} Matrix

The cross correlation between the auto-correlation matrices A and B is calculated by multiplying the aggregated auto-correlation matrices M_A and M_B:

$$M_{AB} = M_A \times M_B^T$$

where M_{AB} is a 64x64 matrix.

11.4 Auto Correlation Matrix Reduction

We also reduce the matrices M_A and M_B by multiplying them by a vector of ones to sum over all segments, resulting in reduced matrices $M_A^{reduced}$ and $M_B^{reduced}$:

$$M_A^{reduced} = M_A \times \mathbf{1}$$

$$M_B^{reduced} = M_B \times \mathbf{1}$$

where $\mathbf{1}$ is a column vector of ones. $M_A^{reduced}$ and $M_B^{reduced}$ are 8x8 matrices.

11.5 Elements of the g_2 Matrix

The element G_{ijmn} of the g_2 matrix is given by:

$$G_{ijmn} = \frac{\sum [(A_i - A_{i+8})(A_j - A_{j+8})(B_m - B_{m+8})(B_n - B_{n+8})]}{\left(\sum [(A_i - A_{i+8})(A_j - A_{j+8})]\right)\left(\sum [(B_m - B_{m+8})(B_n - B_{n+8})]\right)} = \frac{M_{AB_{ijmn}}}{(M_A^{\text{reduced}})_{ij}(M_B^{\text{reduced}})_{mn}}$$

where $i, j, m, n \in \{1, 2, \ldots, 8\}$.