Calculation of lift and drag from pressure distribution around cylinder

Refer to figure 1, which shows a cross section of a circular cylinder. The cylinder radius is r, and a pressure p acts at a location θ around the cylinder surface. For convenience we define θ as positive measured clockwise from the leading edge of the cylinder. The (x, y) coordinate system has its origin at the centre of the circle. We wish to calculate the lift and drag force on the cylinder due to the pressure. Pressure is a scalar and acts equally in all directions, but the force due to a pressure acts normally to the surface. Therefore at a location (x, y) on the surface the pressure force per unit span acting towards the centre of the circle is

$$dF = p\, r\, d\theta \tag{1}$$

since $rd\theta$ is the incremental length of the surface that the force acts over. We resolve this force into x and y components, so

$$dF_x = p\, r\, \cos\theta\, d\theta \tag{2}$$

and

$$dF_y = -\, p\, r\, \sin\theta\, d\theta \tag{3}$$

where the forces are per unit span.

To evaluate the net forces in the x and y directions we integrate around the entire surface, and the force in the x direction is the pressure drag D_p, and the force in the y direction is the lift L (per unit span). Therefore

$$D_p = \int_0^{2\pi} p\, r\, \cos\theta\, d\theta \tag{4}$$

and

$$L = -\int_0^{2\pi} p\, r\, \sin\theta\, d\theta \tag{5}$$

We define the pressure coefficient C_p as

$$C_p = \frac{p - p_\infty}{q_\infty} \tag{6}$$

where q_∞ is the free stream dynamic pressure $q_\infty = \frac{1}{2}\rho U^2$. We know that $\int_0^{2\pi} \sin\theta = 0$ and that $\int_0^{2\pi} \cos\theta = 0$, so it is straightforward to then work out that

$$C_{D_p} = \frac{1}{2}\int_0^{2\pi} C_p \cos\theta\, d\theta \tag{7}$$

and

$$C_L = -\frac{1}{2}\int_0^{2\pi} C_p \sin\theta\, d\theta \tag{8}$$

where $C_{D_p} = \frac{D}{q_\infty d}$ is the pressure drag coefficient and $C_L = \frac{L}{q_\infty d}$ is the lift coefficient, with d the cylinder chord length (diameter).

We could calculate the drag and lift for the various cylinder portions by changing the integration limits appropriately.

Important note: the above equations are specific to the case of the circular cylinder since the body geometry was included in equation 1.

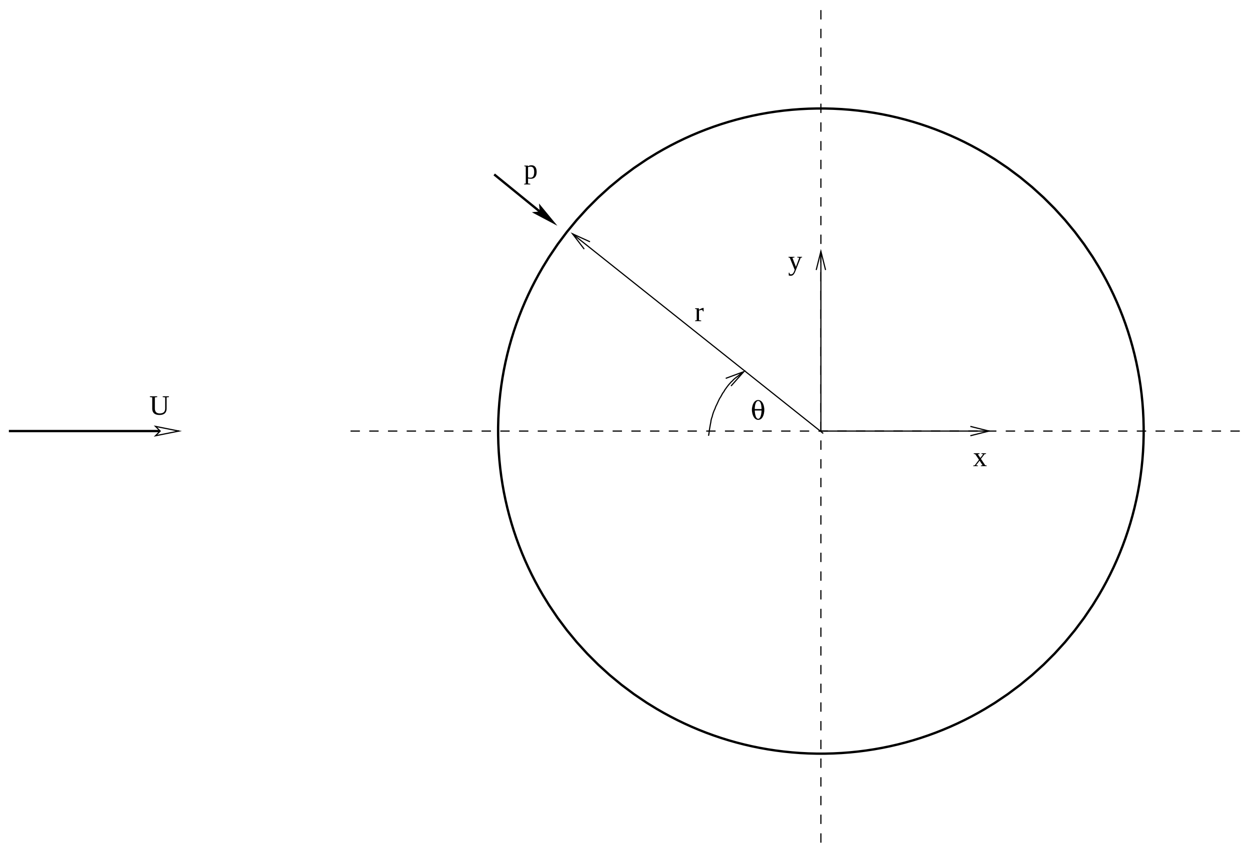

Figure 1: Notation for calculating pressure drag and lift around a circular cylinder from the pressure distribution.