FRBNY DSGE Model Documentation

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1 General Structure

The FRBNY DSGE model is a medium scale, one-sector dynamic stochastic general equilibrium model which is based on the New Keynesian model with financial frictions used in Del Negro et al. (2015). The core of the model is based on the work of Smets and Wouters (2007) (henceforth SW) and Christiano et al. (2005): It builds on the neo-classical growth model by adding nominal wage and price rigidities, variable capital utilization, costs of adjusting investment, habit formation in consumption. The model also includes credit frictions as in the *financial accelerator* model developed by Bernanke et al. (1999b) where the actual implementation of credit frictions follows closely Christiano et al. (2014), and accounts for forward guidance in monetary policy by including anticipated policy shocks as in Laseen and Svensson (2011).

The current version of the model has several features that improve upon the version presented in the FRBNY Staff Report no. 647. It features both a deterministic and a stochastic trend in productivity and allows for exogenous movements in risk premia; the inflation target is time-varying, following Del Negro and Schorfheide (2013); households preferences are non-separable in consumption and leisure; the Dixit-Stiglitz aggregator of intermediate goods has been replaced by the more flexible Kimball aggregator; we include indexation in the price and wage adjustment processes.

Here is a brief overview. The model economy is populated by eight classes of agents: 1) a continuum of households, who consume and supply differentiated labor; 2) competitive labor aggregators that combine labor supplied by individual households; 3) competitive final good-producing firms that aggregate the intermediate goods into a final product; 4) a continuum of monopolistically competitive intermediate good producing firms; 5) competitive capital producers that convert final goods into capital; 6) a continuum of entrepreneurs who purchase capital using both internal and borrowed funds and rent it to intermediate good producing firms; 7) a representative bank collecting deposits from the households and lending funds to the entrepreneurs; and finally 8) a government, composed of a monetary authority that sets short-term interest rates and a fiscal authority that sets public spending and collects taxes.

2 DSGE Model Specification

Growth in the economy is driven by technological progress. We specify a process for technology Z_t^* which includes both a deterministic and a stochastic trend, and a stationary component:

$$Z_t^* = e^{\frac{1}{1-\alpha}\tilde{z}_t} Z_t^p e^{\gamma t}, \tag{1}$$

where γ is the steady state growth rate of the economy, Z_t^p is a stochastic trend and $\tilde{z}_t$ is the stationary component.

The *production function* is

$$Y_t(i) = \max\{e^{\tilde{z}_t} K_t(i)^\alpha \left(L_t(i) e^{\gamma t} Z_t^p\right)^{1-\alpha} - \Phi Z_t^*, 0\}, \tag{2}$$

where ΦZ_t^* is a fixed cost.

Trending variables are divided by Z_t^* to express the model's equilibrium conditions in terms of the stationary variables. In what follows we present a summary of the log-linearized equilibrium conditions, where all variables are expressed in log deviations from their non-stochastic steady state.

2.1 The log-linear equilibrium conditions

The stationary component of productivity $\tilde{z}_t$ evolves as:

$$\tilde{z}_t = \rho_z \tilde{z}_{t-1} + \sigma_z \varepsilon_{z,t}. \tag{3}$$

Since Z_t^p is a non stationary process, we define its growth rate as $z_t^p = log(Z_t^p / Z_{t-1}^p)$ and assume that it follows an AR(1) process:

$$z_t^p = \rho_{z^p} z_{t-1}^p + \sigma_{z^p} \epsilon_{z^p,t},\ \epsilon_{z^p,t} \sim N(0,1). \tag{4}$$

It follows that

$$z_t \equiv log(Z_t^* / Z_{t-1}^*) - \gamma = \frac{1}{1-\alpha}(\rho_z - 1)\tilde{z}_{t-1} + \frac{1}{1-\alpha}\sigma_z \epsilon_{z,t} + z_t^p, \tag{5}$$

where γ is the steady-state growth rate of the economy. Steady-state values are denoted by

$*$-subscripts, and steady-state formulas are provided in the technical appendix of Del Negro and Schorfheide (2013), which is available online.

The *optimal allocation of consumption* satisfies the following consumption Euler equation:

$$c_t = -\frac{(1-he^{-\gamma})}{\sigma_c(1+he^{-\gamma})}\left(R_t - \mathbb{E}_t[\pi_{t+1}] + b_t\right) + \frac{he^{-\gamma}}{(1+he^{-\gamma})}\left(c_{t-1} - z_t\right) + \frac{1}{(1+he^{-\gamma})}\mathbb{E}_t\left[c_{t+1} + z_{t+1}\right] + \frac{(\sigma_c - 1)}{\sigma_c(1+he^{-\gamma})}\frac{w_* L_*}{c_*}\left(L_t - \mathbb{E}_t[L_{t+1}]\right), \quad (6)$$

where c_t is consumption, L_t is labor supply, R_t is the nominal interest rate, and π_t is inflation. The exogenous process b_t drives a wedge between the intertemporal marginal utility of consumption and the riskless real return $R_t - \mathbb{E}_t[\pi_{t+1}]$, and is meant to capture risk-premium shocks.[1] This shock follows an AR(1) process with parameters ρ_b and σ_b. The parameters σ_c and h capture the degree of relative risk aversion and the degree of habit persistence in the utility function, respectively.

The *optimal investment decision* satisfies the following relationship between the level of investment i_t, measured in terms of consumption goods, and the value of capital in terms of consumption q_t^k:

$$i_t = \frac{q_t^k}{S'' e^{2\gamma}(1+\bar{\beta})} + \frac{1}{1+\bar{\beta}}\left(i_{t-1} - z_t\right) + \frac{\bar{\beta}}{1+\bar{\beta}}\mathbb{E}_t\left[i_{t+1} + z_{t+1}\right] + \mu_t. \quad (7)$$

This relationship shows that investment is affected by investment adjustment costs (S'' is the second derivative of the adjustment cost function) and by an exogenous process μ_t, which we call "marginal efficiency of investment", that alters the rate of transformation between consumption and installed capital (see Greenwood et al. (1998)). The shock μ_t follows an AR(1) process with parameters ρ_μ and σ_μ. The parameter $\bar{\beta}$ depends on the intertemporal discount rate in the household utility function, β, on the degree of relative risk aversion σ_c, and on the steady-state growth rate γ: $\bar{\beta} = \beta e^{(1-\sigma_c)\gamma}$.

The *capital stock*, $\bar{k}_t$, which we refer to as "installed capital", evolves as

$$\bar{k}_t = \left(1 - \frac{i_*}{\bar{k}_*}\right)\left(\bar{k}_{t-1} - z_t\right) + \frac{i_*}{\bar{k}_*} i_t + \frac{i_*}{\bar{k}_*} S'' e^{2\gamma}(1+\bar{\beta})\mu_t, \quad (8)$$

[1] In the code, the b_t shock is normalized to be in the same units as consumption, i.e., we estimate the shock $\tilde{b}_t = -\frac{(1-he^{-\gamma})}{\sigma_c(1+he^{-\gamma})} b_t$.

where $i_*/\bar{k}_*$ is the steady state investment to capital ratio.

Capital is subject to variable capacity utilization u_t; *effective capital* rented out to firms, k_t, is related to $\bar{k}_t$ by:

$$k_t \;=\; u_t - z_t + \bar{k}_{t-1}. \tag{9}$$

The optimality condition determining the *rate of capital utilization* is given by

$$\frac{1-\psi}{\psi} r_t^k = u_t, \tag{10}$$

where r_t^k is the rental rate of capital and ψ captures the utilization costs in terms of foregone consumption.

Real marginal costs for firms are given by

$$mc_t \;=\; w_t + \alpha L_t - \alpha k_t, \tag{11}$$

where w_t is the real wage and α is the income share of capital (after paying mark-ups and fixed costs) in the production function.

From the optimality conditions of goods producers it follows that all firms have the same *capital-labor ratio*:

$$k_t \;=\; w_t - r_t^k + L_t. \tag{12}$$

We include financial frictions in the model, building on the work of Bernanke et al. (1999a), Christiano et al. (2003), De Graeve (2008), and Christiano et al. (2014). We assume that banks collect deposits from households and lend to entrepreneurs who use these funds as well as their own wealth to acquire physical capital, which is rented to intermediate goods producers. Entrepreneurs are subject to idiosyncratic disturbances that affect their ability to manage capital. Their revenue may thus turn out to be too low to pay back the loans received by the banks. The banks therefore protect themselves against default risk by pooling all loans and charging a spread over the deposit rate. This spread may vary as a function of entrepreneurs' leverage and riskiness.

The *realized return on capital* is given by:

$$\tilde{R}_t^k - \pi_t = \frac{r_*^k}{r_*^k + (1-\delta)} r_t^k + \frac{(1-\delta)}{r_*^k + (1-\delta)} q_t^k - q_{t-1}^k, \tag{13}$$

where $\tilde{R}_t^k$ is the gross nominal return on capital for entrepreneurs, r_*^k is the steady state value of the rental rate of capital r_t^k, and δ is the depreciation rate.

The *excess return on capital* (the spread between the expected return on capital and the riskless rate) can be expressed as a function of the entrepreneurs' leverage (i.e. the ratio of the value of capital to nominal net worth) and exogenous fluctuations in the volatility of entrepreneurs' idiosyncratic productivity:

$$E_t\left[\tilde{R}^k_{t+1} - R_t\right] = b_t + \zeta_{sp,b}\left(q_t^k + \bar{k}_t - n_t\right) + \tilde{\sigma}_{\omega,t}, \tag{14}$$

where n_t is entrepreneurs' net worth, $\zeta_{sp,b}$ is the elasticity of the credit spread to the entrepreneurs' leverage $(q_t^k + \bar{k}_t - n_t)$, and $\tilde{\sigma}_{\omega,t}$ captures mean-preserving changes in the cross-sectional dispersion of ability across entrepreneurs (see Christiano et al. (2014)). $\tilde{\sigma}_{\omega,t}$ follows an AR(1) process with parameters ρ_{σ_ω} and σ_{σ_ω}.

Entrepreneurs' net worth n_t evolves according to:

$$\begin{aligned} n_t \;=\; & \zeta_{n,\tilde{R}^k}\left(\tilde{R}^k_t - \pi_t\right) - \zeta_{n,R}\left(R_{t-1} - \pi_t + b_{t-1}\right) + \zeta_{n,qK}\left(q^k_{t-1} + \bar{k}_{t-1}\right) + \zeta_{n,n} n_{t-1} \\ & -\gamma_* \tfrac{v_*}{n_*} z_t - \tfrac{\zeta_{n,\sigma_\omega}}{\zeta_{sp,\sigma_\omega}} \tilde{\sigma}_{\omega,t-1}, \end{aligned} \tag{15}$$

where the ζ's denote elasticities, that depend among others on the entrepreneurs' steady-state default probability $F(\bar{\omega})$, where γ_* is the fraction of entrepreneurs that survive and continue operating for another period, and where v_* is the entrepreneurs' real equity divided by Z_t^*, in steady state.

The *production function* is

$$y_t = \Phi_p\left(\alpha k_t + (1-\alpha) L_t\right), \tag{16}$$

where $\Phi_p = \frac{y_* + \Phi}{y_*}$, and the *resource constraint* is:

$$y_t = g_* g_t + \frac{c_*}{y_*} c_t + \frac{i_*}{y_*} i_t + \frac{r_*^k k_*}{y_*} u_t. \tag{17}$$

where $g_t = log(\frac{G_t}{Z_t^* y_* g_*})$ and $g_* = 1 - \frac{c_* + i_*}{y_*}$.

Government spending g_t is assumed to follow the exogenous process:

$$g_t = \rho_g g_{t-1} + \sigma_g \varepsilon_{g,t} + \eta_{gz} \sigma_z \varepsilon_{z,t}.$$

The *price and wage Phillips curves* are, respectively:

$$\pi_t = \kappa\ mc_t + \frac{\iota_p}{1+\iota_p\bar{\beta}}\pi_{t-1} + \frac{\bar{\beta}}{1+\iota_p\bar{\beta}}I\!\!E_t[\pi_{t+1}] + \lambda_{f,t}, \tag{18}$$

and

$$\begin{aligned} w_t = &\frac{(1-\zeta_w\bar{\beta})(1-\zeta_w)}{(1+\bar{\beta})\zeta_w((\lambda_w-1)\epsilon_w+1)}\left(w_t^h - w_t\right) - \frac{1+\iota_w\bar{\beta}}{1+\bar{\beta}}\pi_t + \frac{1}{1+\bar{\beta}}\left(w_{t-1} - z_t + \iota_w\pi_{t-1}\right) \\ &+ \frac{\bar{\beta}}{1+\bar{\beta}}I\!\!E_t\left[w_{t+1} + z_{t+1} + \pi_{t+1}\right] + \lambda_{w,t}, \end{aligned} \tag{19}$$

where $\kappa = \frac{(1-\zeta_p\bar{\beta})(1-\zeta_p)}{(1+\iota_p\bar{\beta})\zeta_p((\Phi_p-1)\epsilon_p+1)}$, the parameters ζ_p, ι_p, and ϵ_p are the Calvo parameter, the degree of indexation, and the curvature parameter in the Kimball aggregator for prices, and ζ_w, ι_w, and ϵ_w are the corresponding parameters for wages. w_t^h measures the household's marginal rate of substitution between consumption and labor, and is given by:

$$w_t^h = \frac{1}{1-he^{-\gamma}}\left(c_t - he^{-\gamma}c_{t-1} + he^{-\gamma}z_t\right) + \nu_l L_t, \tag{20}$$

where ν_l characterizes the curvature of the disutility of labor (and would equal the inverse of the Frisch elasticity in the absence of wage rigidities). The mark-ups $\lambda_{f,t}$ and $\lambda_{w,t}$ follow exogenous ARMA(1,1) processes:

$$\lambda_{f,t} = \rho_{\lambda_f}\lambda_{f,t-1} + \sigma_{\lambda_f}\varepsilon_{\lambda_f,t} - \eta_{\lambda_f}\sigma_{\lambda_f}\varepsilon_{\lambda_f,t-1},$$

and

$$\lambda_{w,t} = \rho_{\lambda_w}\lambda_{w,t-1} + \sigma_{\lambda_w}\varepsilon_{\lambda_w,t} - \eta_{\lambda_w}\sigma_{\lambda_w}\varepsilon_{\lambda_w,t-1},$$

respectively.

Finally, the monetary authority follows a generalized *policy feedback rule*:

$$\begin{aligned} R_t \;=\; &\rho_R R_{t-1} + (1-\rho_R)\Big(\psi_1(\pi_t - \pi_t^*) + \psi_2(y_t - y_t^f)\Big) \\ &+\psi_3\Big((y_t - y_t^f) - (y_{t-1} - y_{t-1}^f)\Big) + r_t^m. \end{aligned} \tag{21}$$

where y_t^f is the flexible price/wage output, obtained from solving the version of the model without nominal rigidities and markup shocks (that is, Equations (6) through (20) with

$\zeta_p = \zeta_w = 0$, and $\lambda_{f,t} = \lambda_{w,t} = 0$), and the residual r^m_t follows an AR(1) process with parameters ρ_{r^m} and σ_{r^m}.

In this version of the model we have replaced a constant inflation target with a time-varying inflation target π^*_t, to capture the rise and fall of inflation and interest rates in the estimation sample. Although time-varying target rates have been frequently used for the specification of monetary policy rules in DSGE model (e.g., Erceg and Levin (2003) and Smets and Wouters (2003), among others), we follow the approach of Aruoba and Schorfheide (2008) and Del Negro and Eusepi (2011) and include data on long-run inflation expectations as an observable for the estimation of the model. At each point in time, long-run inflation expectations essentially determine the level of the target inflation rate. To the extent that long-run inflation expectations at the forecast origin contain information about the central bank's objective function, e.g. the desire to stabilize inflation at 2%, this information is automatically included in the forecast.

The time-varying *inflation target* evolves according to:

$$\pi^*_t = \rho_{\pi^*}\pi^*_{t-1} + \sigma_{\pi^*}\epsilon_{\pi^*,t}, \tag{22}$$

where $0 < \rho_{\pi^*} < 1$ and $\epsilon_{\pi^*,t}$ is an iid shock. We model π^*_t as a stationary process, although our prior for ρ_{π^*} will force this process to be highly persistent. The assumption that the changes in the target inflation rate are exogenous is, to some extent, a short-cut. For instance, the learning models of Sargent (1999) or Primiceri (2006) imply that the rise in the target inflation rate in the 1970's and the subsequent drop is due to policy makers learning about the output-inflation trade-off and trying to set inflation optimally. We are abstracting from such a mechanism in our specification.

2.2 Anticipated Policy Shocks

This section describes the introduction of anticipated policy shocks in the model, which follows Laseen and Svensson (2011). We modify the exogenous component of the policy rule (21) as follows:

$$r^m_t = \rho_{r^m} r^m_{t-1} + \epsilon^R_t + \sum_{k=1}^{K} \epsilon^R_{k,t-k}, \tag{23}$$

where ϵ_t^R is the usual contemporaneous policy shock, and $\epsilon_{k,t-k}^R$ is a policy shock that is known to agents at time $t-k$, but affects the policy rule k periods later, that is, at time t. We assume that $\epsilon_{k,t-k}^R \sim N(0, \sigma_{k,r}^2)$, $i.i.d.$

In order to solve the model we need to express the anticipated shocks in recursive form. For this purpose, we augment the state vector s_t (described below) with K additional states ν_t^R,...,ν_{t-K}^R whose law of motion is as follows:

$$\begin{aligned}
\nu_{1,t}^R &= \nu_{2,t-1}^R + \epsilon_{1,t}^R \\
\nu_{2,t}^R &= \nu_{3,t-1}^R + \epsilon_{2,t}^R \\
&\vdots \\
\nu_{K,t}^R &= \epsilon_{K,t}^R
\end{aligned}$$

and rewrite the exogenous component of the policy rule (23) as[2]

$$r_t^m = \rho_{r^m} r_{t-1}^m + \epsilon_t^R + \nu_{1,t-1}^R.$$

2.3 Adding COVID-19 Shocks

Some of the model modifications needed to capture the COVID-19 shock, at least within the narrow framework on this one sector DSGE model, amount to adding i.i.d. shocks. These shocks are i.i.d. because the economic disruptions related to COVID-19, such as the lockdown of productive capacity and the impossibility to consume some goods/services, are temporary. However, even i.i.d. shocks may have lasting effects on the economy via the model's dynamics. Moreover, some of the COVID shocks hit the economy for more than one period. As a result, the sequence of shocks affecting the economy is not i.i.d., even though their impulse responses reflect their lack of exogenous persistence. In addition, we assume that some of these shocks are anticipated. For instance, in 2020Q1 agents expect that a set of disturbances twice the size of those affecting the economy in the current quarter will also hit it in the following quarter. In 2020Q2 a new set of disturbances will hit the economy on top of the shocks that were anticipated in the previous quarter.

We introduce two new shocks to capture the macroeconomic effects of the pandemic: a so-called "discount factor" shock $\tilde{\beta}_t$ and a "labor supply" shock $\hat{\varphi}_t$. The first one enters as

[2] It is easy to verify that $\nu_{1,t-1}^R = \sum_{k=1}^K \epsilon_{k,t-k}^R$, that is, $\nu_{1,t-1}^R$ is a "bin" that collects all anticipated shocks that affect the policy rule in period t.

a stochastic addition to the discount rate β, and the second as a labor (dis)utility shifter. These shocks modify the Euler equation and the intratemporal condition as follows:

$$\begin{aligned}\hat{c}_t = &-\frac{(1-he^{-z_*^*})}{\sigma_c(1+he^{-z_*^*})}\left(\hat{R}_t - \mathbb{E}_t[\hat{\pi}_{t+1}]\right) + \frac{he^{-z_*^*}}{(1+he^{-z_*^*})}\left(\hat{c}_{t-1}-\hat{z}_t^*\right) + \hat{b}_t + \hat{\beta}_t \\ &+ \frac{1}{(1+he^{-z_*^*})}\mathbb{E}_t\left[\hat{c}_{t+1}+\hat{z}_{t+1}^*\right] + \frac{(\sigma_c-1)}{\sigma_c(1+he^{-z_*^*})}\frac{w_*L_*}{c_*}\left(\hat{L}_t - \mathbb{E}_t[\hat{L}_{t+1}]\right) \\ &+ \frac{(\sigma_c-1)}{\sigma_c(1+he^{-z_*^*})}\frac{w_*L_*}{c_*}\left(\hat{\varphi}_t - \mathbb{E}_t[\hat{\varphi}_{t+1}]\right), \end{aligned} \tag{24}$$

and

$$\frac{1}{1-he^{-z_*^*}}\left(\hat{c}_t - he^{-z_*^*}\hat{c}_{t-1} + he^{-z_*^*}\hat{z}_t^*\right) + \nu_l\hat{L}_t + \nu_l\hat{\varphi}_t = \hat{w}_t^h. \tag{25}$$

Note that φ_t enters the wage Phillips curve in the same way as a wage mark-up shock via $\hat{w}_t^h$. However, differently from $\hat{\lambda}_{w,t}$ it also enters the Euler equation.

In addition, we add a *stationary* i.i.d. productivity disturbance $\check{z}_t$. As a consequence total productivity growth becomes:

$$\hat{z}_t^* = \frac{1}{1-\alpha}(\tilde{z}_t - \tilde{z}_{t-1}) + z_t^p + \frac{1}{1-\alpha}(\check{z}_t - \check{z}_{t-1}). \tag{26}$$

All the shocks are i.i.d. (that is, $\rho_\beta = \rho_\varphi = \rho_{\check{z}} = 0$). As mentioned, some of the scenarios feature anticipated shocks:

$$\begin{aligned}\check{z}_t &= \rho_{\check{z}}\check{z}_{t-1} + \sigma_{\check{z}}\epsilon_{\check{z},t} + \sum_{k=1}^{K}\sigma_{\check{z},k}\varepsilon_{k,t-k}^{\check{z}} \\ \hat{\beta}_t &= \rho_\beta\hat{\beta}_{t-1} + \sigma_\beta\varepsilon_{\beta,t} + \sum_{k=1}^{K}\sigma_{\beta,k}\varepsilon_{k,t-k}^{\beta} \\ \hat{\varphi}_t &= \rho_\varphi\hat{\varphi}_{t-1} + \sigma_\varphi\varepsilon_{\varphi,t} + \sum_{k=1}^{K}\sigma_{\varphi,k}\varepsilon_{k,t-k}^{\varphi}.\end{aligned}$$

We use $K = 1$ (only one anticipated shock) and set the anticipated shock to be a proportion ϕ of the current shock, e.g. $\sigma_{\check{z},1}\varepsilon_{1,t}^{\check{z}} = \phi\sigma_{\check{z}}\epsilon_{\check{z},t}$.

From 2020Q1 onwards, the measurement error for Core PCE is replaced by a negatively-autocorrelated error term. This allows the model to capture transitory, mean reverting changes in the price level associated with the pandemic. From 2021Q1 to 2022Q2, the standard deviation is set to half of its value in earlier quarters. This error also enters the measurement equation for the GDP Deflator, although this equation includes an iid measurement error as well. Hence, the Core PCE measurement equation is now:

$$\text{Core PCE}_t = \pi_t + 100(\pi^* - 1) + e_{\pi,t} - e_{\pi,t-1}$$

where

$$e_{\pi,t} = \rho_{m\pi} e_{\pi,t-1} + \epsilon_{m\pi}$$

$\epsilon_{m\pi}$ is normally distributed with mean 0 and standard deviation $\sigma_{m\pi}$. $\rho_{m\pi}$ and $\sigma_{m\pi}$ are both estimated.

2.4 Introducing Flexible AIT

To reflect the change in the FOMC monetary policy strategy announced on August 2020, as described in the 2020 Statement on Longer-Run Goals and Monetary Policy Strategy, we replaced the historical reaction function with a new one that captures the flavor of flexible average inflation targeting (AIT). In our implementation of flexible AIT, the interest rate responds to a moving average (MA) of deviations of inflation from the FOMC longer-run goal of 2 percent. It also reacts to an MA of deviations of output growth from its steady state value in light of the Federal Reserve's dual mandate, consistent with the "flexible" nature of AIT. Finally, the new reaction function features inertia, like the old one. Formally, the flexible AIT rule is

$$R_t = \rho_R R_{t-1} + (1-\rho_R)(1-\rho_p)\varphi_p pgap_t + (1-\rho_R)(1-\rho_y)\varphi_y ygap_t, \tag{27}$$

where R_t is the interest rate. The inflation and output gaps are computed as

$$pgap_t = (\pi_t - 2) + \rho_p pgap_{t-1} \tag{28}$$

and

$$ygap_t = (\Delta y_t + z_t - \gamma) + \rho_y ygap_{t-1}$$

where π_t is (core PCE) inflation and $\Delta y_t + z_t - \gamma$ is the growth rate of real GDP in deviation from its steady state value γ (z_t is the growth rate of technology in deviations from the trend). This means that the inflation gap is constructed by cumulating the past shortfalls of inflation from 2 percent, with a discount factor that approximates the length of the window over which the "average" in AIT is computed. The output gap similarly cumulates shortfalls of GDP from trend.[3]

We set ρ_p and ρ_y equal to 0.93, which corresponds to a half life of 10 quarters. Therefore, $pgap$ roughly approximates a 5-year average of inflation shortfalls. We also set $pgap_0 = -0.125$ and $ygap_0 = -12$ percent in 2020Q2. The latter reflects the fall in output through the second quarter of 2020. The remaining parameters are chosen so that the rule implies an expected path for the federal funds rate in line with current FOMC communication to the extent possible. To this effect, we are currently setting the inertia parameter ρ_R to .9. The feedback parameters are set at $\varphi_p = 4$ and $\varphi_y = 3$. We should stress that this specific formulation of the rule, as well as the parameter choices, are just a way of implementing flexible AIT in the context of the model and they do not reflect the views of the FOMC.

Monetary policy in rational expectations DSGE models affects economic outcomes through two channels. The first channel is direct stimulus via lower interest rates. The second channel is through its effect on expectations. In implementing the new policy strategy, we partially shut down the latter. We do so by assuming that the new reaction function is only partially reflected in agents' expectations, to capture the fact that households and firms may catch on only slowly to the introduction of the new policy strategy. Specifically, expectations are formed using a convex combination of the implied law of motion of the economy under the old and the new policy functions. Mechanically, agents at time t attach a probability p to the event that the central bank will pursue the new strategy in the future, and the complementary probability $1 - p$ to the event that the central bank will instead continue to pursue the old strategy. When the interest rate is at the ZLB, both strategies imply the same interest rate at time t. When $p = 1$, the approach collapses to the standard rational expectation solution under the new policy. When $p = 0$, agents disregard any announced change in the policy rule when forming expectations, regardless of the central bank's actions. Probabilities in $(0, 1)$ imply that expectations are formed as a convex combination of these two cases, a situation we call "imperfect awareness." The appendix provides further detail.

We further assume that this awareness increases slowly over time, reflecting the fact that agents in the economy may incorporate the new framework in their expectations only

[3] As a result, NGDP targeting is a special case of this framework that obtains when the policy rate responds with the same coefficient to the two gaps and there is no discount in the cumulation.

gradually. Currently, we assume that p increases linearly starting from 0 in 2020Q4 (the period in which the new framework takes effect) and reaching 1 over a period of 24 quarters.

Given the settings outlined above, flexible AIT implies that the interest rate would fall below zero for an extended period of time. Under the historical rule, we used anticipated shocks to impose the zero bound (in expectations). Under flexible AIT, the ZLB is implemented with the piece-wise linear approach of Kulish and Pagan (2017) and Cagliarini and Kulish (2013), as described in the appendix.

2.5 State Space Representation

The equilibium conditions can be written in compact form as:

$$\Gamma_0 s_t = \Gamma_c + \Gamma_1 z_{t-1} + \Psi \varepsilon_t + \Pi \eta_t, \tag{29}$$

where s_t is a vector containing all endogenous and exogenous variables, *including* expectations via expectational equations of the type $x_t = E_{t-1}[x_t] + \eta_t$ for a generic variable x_t (see Sims, 2002). We will discuss at the end of this section how this system differs from the way we usually write it, and also how to possibly include expectational variables.

We use the method in Sims (2002) to solve the system of log-linear approximate equilibrium conditions and obtain the transition equation, which summarizes the evolution of the states s_t:

$$s_t = \mathcal{T}(\theta) s_{t-1} + \mathcal{R}(\theta) \epsilon_t. \tag{30}$$

where θ is a vector collecting all the DSGE model parameters and ϵ_t is a vector of all the structural shocks. The state-space representation for our vector of observables y_t, which we describe in the next section, is composed of the transition equation (30), and a system of measurement equations:

$$y_t = \mathcal{D}(\theta) + \mathcal{Z}(\theta) s_t, \tag{31}$$

mapping the states into the observables, which we describe in detail in the section 3. We assume that some of the variables are measured with “error”, that is, the observed value equals the model implied value plus an exogenous process, which evolves as an AR(1). We add this exogenous process to the vector of states s_t.

3 Data

The estimation of the model is based on data on real output growth (including both GDP and GDI measures), consumption growth, investment growth, real wage growth, hours worked, inflation (measured by core PCE and GDP deflators), short- and long- term interest rates, 10-year inflation expectations, credit spreads, and total factor productivity. Measurement equations relate the model variables that appear in Section 2 to the observables as follows:

$$
\begin{aligned}
\text{GDP growth} &= 100\gamma + (y_t - y_{t-1} + z_t) + e_t^{gdp} - \mathcal{C}_{me} e_{t-1}^{gdp} \\
\text{GDI growth} &= 100\gamma + (y_t - y_{t-1} + z_t) + e_t^{gdi} - \mathcal{C}_{me} e_{t-1}^{gdi} \\
\text{Consumption growth} &= 100\gamma + (c_t - c_{t-1} + z_t) \\
\text{Investment growth} &= 100\gamma + (i_t - i_{t-1} + z_t) \\
\text{Real Wage growth} &= 100\gamma + (w_t - w_{t-1} + z_t) \\
\text{Hours} &= \bar{L} + L_t \\
\text{Core PCE Inflation} &= \pi_* + \pi_t + e_t^{pce} \\
\text{GDP Deflator Inflation} &= \pi_* + \delta_{gdpdef} + \gamma_{gdpdef} * \pi_t + e_t^{gdpdef} \\
\text{FFR} &= R_* + R_t \\
\text{10y Nominal Bond Yield} &= R_* + I\!E_t \left[\tfrac{1}{40} \textstyle\sum_{k=1}^{40} R_{t+k} \right] + e_t^{10y} \\
\text{10y Infl. Expectations} &= \pi_* + I\!E_t \left[\tfrac{1}{40} \textstyle\sum_{k=1}^{40} \pi_{t+k} \right] \\
\text{Spread} &= SP_* + I\!E_t \left[\tilde{R}_{t+1}^k - R_t \right] \\
\text{TFP growth, demeaned} &= z_t + \tfrac{\alpha}{1-\alpha} (u_t - u_{t-1}) + e_t^{tfp}.
\end{aligned}
\tag{32}
$$

All variables are measured in percent. All the e_t^* processes follow exogenous AR(1) specifications, and can be thought of either measurement errors or some other unmodeled source of discrepancy between the model and the data (e.g., risk premia for the long term nominal rate). However, we introduce correlation in the measurement errors for GDP and GDI, which evolve as follows:

$$
\begin{aligned}
e_t^{gdp} &= \rho_{gdp} \cdot e_{t-1}^{gdp} + \sigma_{gdp} \epsilon_t^{gdp}, \;\; \epsilon_t^{gdp} \sim i.i.d.N(0,1) \\
e_t^{gdi} &= \rho_{gdi} \cdot e_{t-1}^{gdi} + \varrho_{gdp} \cdot \sigma_{gdp} \epsilon_t^{gdp} + \sigma_{gdi} \epsilon_t^{gdi}, \;\; \epsilon_t^{gdi} \sim i.i.d.N(0,1).
\end{aligned}
$$

We assume that $\mathcal{C}_{me} = 1$. The measurement errors for GDP and GDI are thus stationary in *levels*, and enter the observation equation in first differences (e.g. $e_t^{gdp} - e_{t-1}^{gdp}$ and $e_t^{gdp} - e_{t-1}^{gdp}$). GDP and GDI are also cointegrated as they are driven by a comment stochastic trend. The terms π_* and R_* measure respectively the net steady-state inflation rate and short-term nominal interest rate, expressed in percentage terms, and $\bar{L}$ captures the mean of hours (this variable is measured as an index). The 10-year inflation expectations contain information about low-frequency movements of inflation and are obtained from the Blue Chip Economic Indicators survey and the Survey of Professional Forecasters. As spread variable we use a Baa Corporate Bond Yield spread over the 10-Year Treasury Note Yield at constant maturity. Details on the construction of the data set are provided in Appendix A.

In order to estimate the importance of anticipated shocks and their effect on the variables of interest, we follow Del Negro and Schorfheide (2013) and augment the measurement equation (31) with the expectations for the policy rate:

$$\begin{aligned} FFR^e_{t,t+1} &= R_* + \mathcal{Z}(\theta)_{R,\cdot}\mathcal{T}(\theta)^1 s_t, \\ &\vdots \\ FFR^e_{t,t+K} &= R_* + \mathcal{Z}(\theta)_{R,\cdot}\mathcal{T}(\theta)^K s_t, \end{aligned} \tag{33}$$

where $FFR^e_{t,t+k}$ are the market's expectations for the FFR k quarters ahead, and $\mathcal{Z}(\theta)_{R,\cdot}$ is the row of $\mathcal{Z}(\theta)$ corresponding to the interest rate.

4 Inference, Prior and Posterior Parameter Estimates

We use Bayesian techniques for estimation, which require the specification of a prior distribution for the model parameters. For most of the parameters we use the same marginal prior distributions as Smets and Wouters (2007), with two important exceptions. First, the original prior for the quarterly steady state inflation rate π_* used by Smets and Wouters (2007) is tightly centered around 0.62% (which is about 2.5% annualized) with a standard deviation of 0.1%. We favor a looser prior, one that has less influence on the model's forecasting performance, that is centered at 0.75% and has a standard deviation of 0.4%. Second, for the financial frictions mechanism we specify priors for the parameters SP_*, $\zeta_{sp,b}$, ρ_{σ_ω}, and σ_{σ_ω}, while we fix the parameters corresponding to the steady state default probability and the survival rate of entrepreneurs, respectively. In turn, these parameters imply values for the parameters of (15).

Information on the priors is provided in Table 1, in the Appendix.

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A Data Construction

Data on real GDP (GDPC), the GDP deflator (GDPDEF), core PCE inflation (PCEPILFE), nominal personal consumption expenditures (PCEC), and nominal fixed private investment (FPI) are produced at a quarterly frequency by the Bureau of Economic Analysis, and are included in the National Income and Product Accounts (NIPA). Average weekly hours of production and nonsupervisory employees for total private industries (AWHNONAG), civilian employment (CE16OV), and the civilian non-institutional population (LNSINDEX) are produced by the Bureau of Labor Statistics (BLS) at a monthly frequency. The first of these series is obtained from the Establishment Survey, and the remaining from the Household Survey. Both surveys are released in the BLS Employment Situation Summary. Since our models are estimated on quarterly data, we take averages of the monthly data. Compensation per hour for the non-farm business sector (COMPNFB) is obtained from the Labor Productvity and Costs release, and produced by the BLS at a quarterly frequency. All data are transformed following Smets and Wouters (2007). The federal funds rate is obtained from the Federal Reserve Board's H.15 release at a business day frequency. We take quarterly averages of the annualized daily data and divide by four. Let Δ denote the temporal difference operator. Then:

$$
\begin{array}{lcl}
\text{Output growth} & = & 100 * \Delta LN((GDPC)/LNSINDEX) \\
\text{Consumption growth} & = & 100 * \Delta LN((PCEC/GDPDEF)/LNSINDEX) \\
\text{Investment growth} & = & 100 * \Delta LN((FPI/GDPDEF)/LNSINDEX) \\
\text{Real wage growth} & = & 100 * \Delta LN(COMPNFB/GDPDEF) \\
\text{Hours worked} & = & 100 * LN((AWHNONAG * CE16OV/100)/LNSINDEX) \\
\text{GDP Deflator Inflation} & = & 100 * \Delta LN(GDPDEF) \\
\text{Core PCE Inflation} & = & 100 * \Delta LN(PCEPILFE) \\
\text{FFR} & = & (1/4) * FEDERAL\ FUNDS\ RATE
\end{array}
$$

Long-run inflation expectations are obtained from the Blue Chip Economic Indicators survey and the Survey of Professional Forecasters available from the FRB Philadelphia's Real-Time Data Research Center. Long-run inflation expectations (average CPI inflation over the next 10 years) are available from 1991Q4 onward. Prior to 1991Q4, we use the 10-year expectations data from the Blue Chip survey to construct a long time series that begins in 1979Q4. Since the Blue Chip survey reports long-run inflation expectations only twice a year, we treat these expectations in the remaining quarters as missing observations and adjust the measurement equation of the Kalman filter accordingly. Long-run inflation expectations $\pi_t^{O,40}$ are therefore measured as

$$\text{10y Infl Exp} = (\text{10-year average CPI inflation forecast} - 0.50)/4.$$

where 0.50 is the average difference between CPI and GDP annualized inflation from the beginning of the sample to 1992. We divide by 4 to express the data in quarterly terms.

We measure *Spread* as the annualized Moody's Seasoned Baa Corporate Bond Yield spread over the 10-Year Treasury Note Yield at Constant Maturity. Both series are available from the Federal Reserve Board's H.15 release. Like the federal funds rate, the spread data are also averaged over each quarter and measured at a quarterly frequency. This leads to:

$$\text{Spread} = (1/4) * (Baa\ Corporate\ -\ 10\ year\ Treasury).$$

Similarly,

$$\text{10y Bond yield} = (1/4) * (10\ year\ Treasury).$$

Lastly, TFP growth is measured using John Fernald's TFP growth series, unadjusted for changes in utilization. That series is demeaned, divided by 4 to express it in quarterly growth rates, and divided by Fernald's estimate of $(1-\alpha)$ to convert it in labor augmenting terms:

$$\text{TFP growth, demeaned } = (1/4)*(\text{Fernald's TFP growth, unadjusted, demeaned})\ /(1-\alpha).$$

B Solving linear rational expectation methods with anticipated policy changes

This solution method uses the approach in Kulish and Pagan (2017) and Cagliarini and Kulish (2013). Write the DSGE model as:

$$\tilde{\Gamma}_2 \mathbb{E}_t[z_{t+1}] + \tilde{\Gamma}_0 z_t = \tilde{\Gamma}_c + \tilde{\Gamma}_1 z_{t-1} + \tilde{\Psi}\varepsilon_t, \tag{34}$$

where z_t includes all endogenous and exogenous variables, but not expectational equations of the type $y_t = E_{t-1}[y_t] + \eta_t$. We will discuss at the end of this section how this system differs from the way we usually write it—that is, expression (29)—and also how to possibly include expectational variables.

The model for $t > \bar{H}$ is

$$\tilde{\Gamma}_{2,s} \mathbb{E}_t[z_{t+1}] + \tilde{\Gamma}_{0,s} z_t = \tilde{\Gamma}_{c,s} + \tilde{\Gamma}_{1,s} z_{t-1} + \tilde{\Psi}_{2,s}\varepsilon_t, \qquad t > \bar{H} \tag{35}$$

where the subscript s refers to "stationary": for $t \geq \bar{H}+1$ the system is time invariant. This time invariant solution for $t \geq \bar{H}+1$ can be computed as usual via `gensys`, and is:

$$z_t = \mathcal{C}_s + \mathcal{T}_s z_{t-1} + \mathcal{R}_s \varepsilon_t. \tag{36}$$

For $t \leq \bar{H}$ the equilibrium conditions are:

$$\tilde{\Gamma}_{2,t} \mathbb{E}_t[z_{t+1}] + \tilde{\Gamma}_{0,t} z_t = \tilde{\Gamma}_{c,t} + \tilde{\Gamma}_{1,t} z_{t-1} + \tilde{\Psi}_t\varepsilon_t, \quad t \leq \bar{H}. \tag{37}$$

where in principle we let these conditions be different for each t. If there are only two regimes — that is, for all $t \leq \bar{H}$ the regime is the same — then these matrices would not change.

Write the (time-varying) solution of this model as

$$z_t = \mathcal{C}_t + \mathcal{T}_t z_{t-1} + \mathcal{R}_t \varepsilon_t. \tag{38}$$

(Note that by writing the solution this way we have ruled out sunspots solutions by assumption). We need to compute $\mathcal{C}_t$, $\mathcal{T}_t$, and $\mathcal{R}_t$. By construction, for $t > \bar{H}$

$$\mathcal{C}_t = \mathcal{C}_s,\ \mathcal{T}_t = \mathcal{T}_s,\ \mathcal{R}_t = \mathcal{R}_s,\ \text{for } t > \bar{H}. \tag{39}$$

For $t \leq \bar{H}$ we will compute $\mathcal{C}_t$, $\mathcal{T}_t$, and $\mathcal{R}_t$ recursively. Substitute for $\mathbb{E}_t[z_{t+1}]$ the expectations generated from (38):

$$\tilde{\Gamma}_{2,t}\mathcal{C}_{t+1} + \tilde{\Gamma}_{2,t}\mathcal{T}_{t+1}z_t + \tilde{\Gamma}_{0,t}z_t = \tilde{\Gamma}_{c,t} + \tilde{\Gamma}_{1,t}z_{t-1} + \tilde{\Psi}_t\varepsilon_t, \tag{40}$$

implying

$$\begin{aligned} z_t = \left(\tilde{\Gamma}_{2,t}\mathcal{T}_{t+1} + \tilde{\Gamma}_{0,t}\right)^{-1}\left(\tilde{\Gamma}_{c,t} - \tilde{\Gamma}_{2,t}\mathcal{C}_{t+1}\right) + \left(\tilde{\Gamma}_{2,t}\mathcal{T}_{t+1} + \tilde{\Gamma}_{0,t}\right)^{-1}\tilde{\Gamma}_{1,t}z_{t-1} \\ + \left(\tilde{\Gamma}_{2,t}\mathcal{T}_{t+1} + \tilde{\Gamma}_{0,t}\right)^{-1}\tilde{\Psi}_t\varepsilon_t, \text{ for } t \leq \bar{H}. \end{aligned} \tag{41}$$

Equating (38) and (41) we have:

$$\begin{aligned} \mathcal{C}_t = \left(\tilde{\Gamma}_{2,t}\mathcal{T}_{t+1} + \tilde{\Gamma}_{0,t}\right)^{-1}\left(\tilde{\Gamma}_{c,t} - \tilde{\Gamma}_{2,t}\mathcal{C}_{t+1}\right), \ \mathcal{T}_t = \left(\tilde{\Gamma}_{2,t}\mathcal{T}_{t+1} + \tilde{\Gamma}_{0,t}\right)^{-1}\tilde{\Gamma}_{1,t}, \\ \mathcal{R}_t = \left(\tilde{\Gamma}_{2,t}\mathcal{T}_{t+1} + \tilde{\Gamma}_{0,t}\right)^{-1}\tilde{\Psi}_t, \text{ for } t = 1, \ldots, \bar{H}. \end{aligned} \tag{42}$$

We start the recursion at $t = \bar{H}$ with $\mathcal{T}_{t+1} = \mathcal{T}_s$, $\mathcal{C}_{t+1} = \mathcal{C}_s$.

Now let us discuss how to write the equilibrium conditions. Imagine we have an equation of the type:

$$\alpha_2 E_t[y_{t+1}] + \alpha_0 y_t + ... = 0$$

and that the vector s_t in `gensys` is $s_t = [y_t \ ... \ E_t[y_{t+1}]]$ (that is, the expectational variable is ordered last). One approach is to eliminate the expectational variables along with the expectational equations. Therefore z_t, unlike the s_t vector in `gensys`, would *not* include expectations. So for instance

$$\tilde{\Gamma}_2 = \begin{bmatrix} \alpha_2 & ... \\ ... & ... \end{bmatrix}, \ \tilde{\Gamma}_0 = \begin{bmatrix} \alpha_0 & ... \\ ... & ... \end{bmatrix}, ...$$

and $E_t[y_{t+1}]$ is dropped as a variable from s_t. In other words, the matrices $\tilde{\Gamma}_c$, $\tilde{\Gamma}_0$, $\tilde{\Gamma}_1$, and $\tilde{\Psi}$ correspond to the Γ_c, Γ_0, Γ_1, and Ψ matrices in `gensys`, except that the rows/columns associated with the expectational equations/variables are cut off. The matrix $\tilde{\Gamma}_2$ contains the coefficients on the expectations (which in `gensys` are part of Γ_0), and these coefficients are aligned with the variable itself (that the coefficient in Γ_0 corresponding to $E_t[y_{t+1}]$ is associated with y_t). The expectational errors η_t and the corresponding matrix Π are missing from this formulation. Similarly, $\mathcal{C}$, $\mathcal{T}$, and $\mathcal{R}$ are not quite the C, T, and R in `gensys` solution, because z_t is of a different size relative to s_t. We can write the state s_t in `gensys` as $s_t' = [s_{1,t}' s_{2,t}']'$ where $s_{1,t} = z_t$ and $s_{2,t}$ collects the expectation terms $\mathbb{E}_t[z_{t+1}]$. Partition

$$T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix},\ R = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix},\ C = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}.$$

If $T_{12} = 0$, then $\mathcal{T} = T_{11}$, $\mathcal{C} = C_1$ and $\mathcal{R} = R_1$.

An alternative approach is to keep the expectational variables (so $z_t = s_t$) but add the expectational equations $E_t[y_{t+1}]$. So for instance (if the expectational equation is the last equation)

$$\tilde{\Gamma}_2 = \begin{bmatrix} \alpha_2 & ... & 0 \\ ... & ... & ... \\ -1 & 0 & 0 \end{bmatrix},\ \tilde{\Gamma}_0 = \begin{bmatrix} \alpha_0 & ... & 0 \\ ... & ... & ... \\ 0 & 0 & 1 \end{bmatrix}, ...$$

In fact, it may not even be necessary to modify the Γ_0 and Γ_2 matrices at all (!)–other than adding the identity for the expectations:

$$\tilde{\Gamma}_2 = \begin{bmatrix} 0 & ... & 0 \\ ... & ... & ... \\ 1 & 0 & 0 \end{bmatrix},\ \tilde{\Gamma}_0 = \begin{bmatrix} \alpha_0 & ... & \alpha_2 \\ ... & ... & ... \\ 0 & 0 & -1 \end{bmatrix}, ...$$

C Imperfect Awareness

When modeling a change in policy we may want to allow for the fact that the policy change may only be partially incorporated in how agents form expectations. This section describes an approach for implementing this feature.

Imagine P policy rules indexed by p. Call $\tilde{\Gamma}_2^{(p)}$, $\tilde{\Gamma}_0^{(p)}$, $\tilde{\Gamma}_c^{(p)}$, $\tilde{\Gamma}_1^{(p)}$, and $\tilde{\Psi}^{(p)}$ the matrices characterizing the equilibrium conditions under each policy (recall these are not exactly the usual Gensys $\Gamma_0^{(p)}$, $\Gamma_c^{(p)}$, $\Gamma_1^{(p)}$, ...; section B shows how to transform these into the $\tilde{\Gamma}$'s). These matrices differ only for the policy equation. Now call $\mathcal{C}^{(p)}$, $\mathcal{T}^{(p)}$, and $\mathcal{R}^{(p)}$ the reduced form transition equation matrices associated with each of these rules — that is, the law of motion of the economy if these rule were to be in place *forever*.

Now imagine that in period t policy $\tilde{p}$ is in place so that the economy evolves according to

$$\tilde{\Gamma}_2^{(\tilde{p})} I\!E_t[z_{t+1}] + \tilde{\Gamma}_0^{(\tilde{p})} z_t = \tilde{\Gamma}_c^{(\tilde{p})} + \tilde{\Gamma}_1^{(\tilde{p})} z_{t-1} + \tilde{\Psi}_2^{(\tilde{p})} \varepsilon_t. \tag{43}$$

Imagine also that in period t agents continue forming their expectations under the assumption that from period $t+1$ onward the economy will switch into regime (p) with probability $\pi^{(p)}$, and then remain in regime (p) *forever* from then on. This implies:

$$I\!E_t[z_{t+1}] = \sum_p \pi^{(p)} \left(\mathcal{C}^{(p)} + \mathcal{T}^{(p)} z_t \right) = \bar{\mathcal{C}}(\pi) + \bar{\mathcal{T}}(\pi) z_t, \tag{44}$$

where

$$\bar{\mathcal{C}}(\pi) = \sum_p \pi^{(p)} \mathcal{C}^{(p)}, \quad \bar{\mathcal{T}}(\pi) = \sum_p \pi^{(p)} \mathcal{T}^{(p)}. \tag{45}$$

In practice, consider the case where P is be equal to 2, with $p = \tilde{p}$ and $p = p^{old}$ being the "new" and the "old" regimes, respectively. In the extreme where $\pi^{(\tilde{p})} = 0$, agents continue forming expectations using the "old" policy regime (p^{old}) in spite of the fact that regime $(\tilde{p})$ is currently in place. For $0 < \pi^{(\tilde{p})} < 1$ one can imagine that this setting captures a situations where agents have not fully internalized the new regime in forming expectations.

Equation (45) implies that in period t the economy evolves according to:

$$\tilde{\Gamma}_2^{(\tilde{p})} \bar{\mathcal{C}}(\pi) + \tilde{\Gamma}_2^{(\tilde{p})} \bar{\mathcal{T}}(\pi) z_t + \tilde{\Gamma}_0^{(\tilde{p})} z_t = \tilde{\Gamma}_c^{(\tilde{p})} + \tilde{\Gamma}_1^{(\tilde{p})} z_{t-1} + \tilde{\Psi}^{(\tilde{p})} \varepsilon_t, \tag{46}$$

where $\tilde{p}$ is the policy actually being carried out in period t (in other words, the ongoing period t "regime"). This implies the following reduced form law of motion for the economy:

$$\begin{aligned} z_t = \left(\tilde{\Gamma}_2^{(\tilde{p})} \bar{\mathcal{T}}(\pi) + \tilde{\Gamma}_0^{(\tilde{p})} \right)^{-1} \left(\tilde{\Gamma}_c^{(\tilde{p})} - \tilde{\Gamma}_2^{(\tilde{p})} \bar{\mathcal{C}}(\pi) \right) + \left(\tilde{\Gamma}_2^{(\tilde{p})} \bar{\mathcal{T}}(\pi) + \tilde{\Gamma}_0^{(\tilde{p})} \right)^{-1} \tilde{\Gamma}_1^{(\tilde{p})} z_{t-1} \\ + \left(\tilde{\Gamma}_2^{(\tilde{p})} \bar{\mathcal{T}}(\pi) + \tilde{\Gamma}_0^{(\tilde{p})} \right)^{-1} \tilde{\Psi}^{(\tilde{p})} \varepsilon_t. \end{aligned} \tag{47}$$

This can be rewritten as

$$z_t = \mathcal{C}^{(ia)} + \mathcal{T}^{(ia)} z_{t-1} + \mathcal{R}^{(ia)} \varepsilon_t. \tag{48}$$

where $\mathcal{C}^{(ia)}$, $\mathcal{T}^{(ia)}$ and $\mathcal{R}^{(ia)}$ (ia stands for "imperfect awareness") are constructed as

$$\mathcal{C}^{(ia)} = \left(\tilde{\Gamma}_2^{(\tilde{p})}\bar{\mathcal{T}}(\pi) + \tilde{\Gamma}_0^{(\tilde{p})}\right)^{-1}\left(\tilde{\Gamma}_c^{(\tilde{p})} - \tilde{\Gamma}_2^{(\tilde{p})}\bar{\mathcal{C}}(\pi)\right),\ \mathcal{T}^{(ia)} = \left(\tilde{\Gamma}_2^{(\tilde{p})}\bar{\mathcal{T}}(\pi) + \tilde{\Gamma}_0^{(\tilde{p})}\right)^{-1}\tilde{\Gamma}_1^{(\tilde{p})},$$
$$\mathcal{R}^{(ia)} = \left(\tilde{\Gamma}_2^{(\tilde{p})}\bar{\mathcal{T}}(\pi) + \tilde{\Gamma}_0^{(\tilde{p})}\right)^{-1}\tilde{\Psi}^{(\tilde{p})}. \quad (49)$$

Note that these computations are much easier than those in the previous section, as they involve no recursion. In fact, the whole thing amounts to two steps: (i) compute the $\bar{\mathcal{C}}(\pi)$ and $\bar{\mathcal{T}}(\pi)$ matrices from (45); (ii) compute the transition eq. matrices from (49).

C.1 Time Varying Imperfect Awareness

In principle the probabilities in equation (45) could vary over time, reflecting the fact that the public's awareness of a given policy could vary over time (due to central bank communication, or other reasons). This implies that equation (45) becomes

$$\bar{\mathcal{C}}(\pi_t) = \sum_p \pi_t^{(p)}\mathcal{C}^{(p)}, \quad \bar{\mathcal{T}}(\pi_t) = \sum_p \pi_t^{(p)}\mathcal{T}^{(p)}, \quad (50)$$

and that in period t the economy evolves according to:

$$\tilde{\Gamma}_2^{(\tilde{p})}\bar{\mathcal{C}}(\pi_t) + \tilde{\Gamma}_2^{(\tilde{p})}\bar{\mathcal{T}}(\pi_t)z_t + \tilde{\Gamma}_0^{(\tilde{p})}z_t = \tilde{\Gamma}_c^{(\tilde{p})} + \tilde{\Gamma}_1^{(\tilde{p})}z_{t-1} + \tilde{\Psi}^{(\tilde{p})}\varepsilon_t. \quad (51)$$

The reduced form law of motion for the economy is therefore:

$$z_t = \left(\tilde{\Gamma}_2^{(\tilde{p})}\bar{\mathcal{T}}(\pi_t) + \tilde{\Gamma}_0^{(\tilde{p})}\right)^{-1}\left(\tilde{\Gamma}_c^{(\tilde{p})} - \tilde{\Gamma}_2^{(\tilde{p})}\bar{\mathcal{C}}(\pi_t)\right) + \left(\tilde{\Gamma}_2^{(\tilde{p})}\bar{\mathcal{T}}(\pi) + \tilde{\Gamma}_0^{(\tilde{p})}\right)^{-1}\tilde{\Gamma}_1^{(\tilde{p})}z_{t-1}$$
$$+ \left(\tilde{\Gamma}_2^{(\tilde{p})}\bar{\mathcal{T}}(\pi_t) + \tilde{\Gamma}_0^{(\tilde{p})}\right)^{-1}\tilde{\Psi}^{(\tilde{p})}\varepsilon_t. \quad (52)$$

implying that the $\mathcal{C}^{(ia)}$, $\mathcal{T}^{(ia)}$ and $\mathcal{R}^{(ia)}$ matrices in (48) are now time-varying and given by:

$$\mathcal{C}_t^{(ia)} = \left(\tilde{\Gamma}_2^{(\tilde{p})}\bar{\mathcal{T}}(\pi_t) + \tilde{\Gamma}_0^{(\tilde{p})}\right)^{-1}\left(\tilde{\Gamma}_c^{(\tilde{p})} - \tilde{\Gamma}_2^{(\tilde{p})}\bar{\mathcal{C}}(\pi_t)\right),\ \mathcal{T}_t^{(ia)} = \left(\tilde{\Gamma}_2^{(\tilde{p})}\bar{\mathcal{T}}(\pi_t) + \tilde{\Gamma}_0^{(\tilde{p})}\right)^{-1}\tilde{\Gamma}_1^{(\tilde{p})},$$
$$\mathcal{R}_t^{(ia)} = \left(\tilde{\Gamma}_2^{(\tilde{p})}\bar{\mathcal{T}}(\pi_t) + \tilde{\Gamma}_0^{(\tilde{p})}\right)^{-1}\tilde{\Psi}^{(\tilde{p})}. \quad (53)$$

C.2 Accounting for the ZLB with Imperfect Awareness

The "imperfect awareness" solution can be combined with the results in section B to account for the ZLB. For instance, it may well be that the dynamics implied by system (48) involve breaching the ZLB in periods $1, .., \bar{H}$, or that keeping the interest rate at the ZLB for an extended period is simply part of the current policy strategy $(\tilde{p})$. In this case, the reduced form motion of the economy can be computed as follows:

1. Using the recursive system of section B, and in particular expression (42), compute the matrices $\mathcal{C}_t^{(\tilde{p})}$, $\mathcal{T}_t^{(\tilde{p})}$, and $\mathcal{R}_t^{(\tilde{p})}$ for $t \leq \bar{H}$ using as terminal conditions

$$\mathcal{C}_{\bar{H}+1}^{(\tilde{p})} = \mathcal{C}^{(p_1)},\ \mathcal{T}_{\bar{H}+1}^{(\tilde{p})} = \mathcal{T}^{(p_1)},\ \mathcal{R}_{\bar{H}+1}^{(\tilde{p})} = \mathcal{R}^{(p_1)}, \tag{54}$$

   where p_1 is the alternative policy (e.g., AIT), Interpret these matrices as the law of motion if agents are to fully believe the new regime, which involves the policy rate at the ZLB until $\bar{H}$, and policy rule p_1 thereafter.

2. For generality, assume that awareness is time varying. Because of the ZLB the law of motion under policy $(\tilde{p})$ (and possibly under any alternative policy) is also time varying so that expression (50) becomes:

$$\bar{\mathcal{C}}(\pi_t) = \sum_p \pi_t^{(p)} \mathcal{C}_{t+1}^{(p)}, \quad \bar{\mathcal{T}}(\pi_t) = \sum_p \pi_t^{(p)} \mathcal{T}_{t+1}^{(p)}. \tag{55}$$

   For concreteness, in the case where $P = 2$, and where the "old" policy (p^{old}) does not involve any time variation, this boils down to:

$$\bar{\mathcal{C}}(\pi_t) = \pi^{(p_1)} \mathcal{C}_{t+1}^{(\tilde{p})} + (1 - \pi_t^{(\tilde{p})}) \mathcal{C}^{(p^{old})}, \quad \bar{\mathcal{T}}(\pi_t) = \pi_t^{(\tilde{p})} \mathcal{T}_{t+1}^{(\tilde{p})} + (1 - \pi_t^{(\tilde{p})}) \mathcal{T}^{(p^{old})}, \tag{56}$$

   where p^{old} is the "historical" policy, and $\mathcal{C}_t^{(\tilde{p})}$, $\mathcal{T}_t^{(\tilde{p})}$ are taken from the previous step.

3. In period t the economy evolves according to:

$$\tilde{\Gamma}_{2,t}^{(\tilde{p})} \bar{\mathcal{C}}(\pi_t) + \tilde{\Gamma}_{2,t}^{(\tilde{p})} \bar{\mathcal{T}}(\pi_t) z_t + \tilde{\Gamma}_{0,t}^{(\tilde{p})} z_t = \tilde{\Gamma}_{c,t}^{(\tilde{p})} + \tilde{\Gamma}_{1,t}^{(\tilde{p})} z_{t-1} + \tilde{\Psi}_t^{(\tilde{p})} \varepsilon_t, \tag{57}$$

   where

$$\tilde{\Gamma}_{2,t}^{(\tilde{p})} = \tilde{\Gamma}_2^{(p1)},\ \tilde{\Gamma}_{0,t}^{(\tilde{p})} = \tilde{\Gamma}_0^{(p1)},\ \tilde{\Gamma}_{1,t}^{(\tilde{p})} = \tilde{\Gamma}_1^{(p1)},\ \tilde{\Psi}_t^{(\tilde{p})} = \tilde{\Psi}^{(p1)},\ \text{for } t > \bar{H} \tag{58}$$

and

$$\tilde{\Gamma}^{(\tilde{p})}_{2,t} = \tilde{\Gamma}^{(zlb)}_{2},\ \tilde{\Gamma}^{(\tilde{p})}_{0,t} = \tilde{\Gamma}^{(zlb)}_{0},\ \tilde{\Gamma}^{(\tilde{p})}_{1,t} = \tilde{\Gamma}^{(zlb)}_{1},\ \tilde{\Psi}^{(\tilde{p})}_{t} = \tilde{\Psi}^{(zlb)},\ \text{for } t \le \bar{H}. \tag{59}$$

This implies that the reduced form law of motion for the economy is:

$$\begin{aligned} z_t = \left(\tilde{\Gamma}^{(\tilde{p})}_{2,t}\bar{\mathcal{T}}(\pi_t) + \tilde{\Gamma}^{(\tilde{p})}_{0,t}\right)^{-1}\left(\tilde{\Gamma}^{(\tilde{p})}_{c,t} - \tilde{\Gamma}^{(\tilde{p})}_{2,t}\bar{\mathcal{C}}(\pi_t)\right) + \left(\tilde{\Gamma}^{(\tilde{p})}_{2,t}\bar{\mathcal{T}}(\pi) + \tilde{\Gamma}^{(\tilde{p})}_{0,t}\right)^{-1}\tilde{\Gamma}^{(\tilde{p})}_{1,t} z_{t-1} \\ + \left(\tilde{\Gamma}^{(\tilde{p})}_{2,t}\bar{\mathcal{T}}(\pi_t) + \tilde{\Gamma}^{(\tilde{p})}_{0,t}\right)^{-1}\tilde{\Psi}^{(\tilde{p})}_{t}\varepsilon_t. \end{aligned} \tag{60}$$

In other words the economy evolves according to

$$z_t = \mathcal{C}^{(ia)}_t + \mathcal{T}^{(ia)}_t z_{t-1} + \mathcal{R}^{(ia)}_t \varepsilon_t. \tag{61}$$

where

$$\begin{aligned} \mathcal{C}^{(ia)}_t &= \left(\tilde{\Gamma}^{(\tilde{p})}_{2,t}\bar{\mathcal{T}}(\pi_t) + \tilde{\Gamma}^{(\tilde{p})}_{0,t}\right)^{-1}\left(\tilde{\Gamma}^{(\tilde{p})}_{c,t} - \tilde{\Gamma}^{(\tilde{p})}_{2,t}\bar{\mathcal{C}}(\pi_t)\right), \\ \mathcal{T}^{(ia)}_t &= \left(\tilde{\Gamma}^{(\tilde{p})}_{2,t}\bar{\mathcal{T}}(\pi_t) + \tilde{\Gamma}^{(\tilde{p})}_{0,t}\right)^{-1}\tilde{\Gamma}^{(\tilde{p})}_{1,t}, \\ \mathcal{R}^{(ia)}_t &= \left(\tilde{\Gamma}^{(\tilde{p})}_{2,t}\bar{\mathcal{T}}(\pi_t) + \tilde{\Gamma}^{(\tilde{p})}_{0,t}\right)^{-1}\tilde{\Psi}^{(\tilde{p})}_{t}. \end{aligned} \tag{62}$$

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	Dist	Mean	Std Dev		Dist	Mean	Std Dev
Policy Parameters							
ψ_1	Normal	1.50	0.25	ρ_{r^m}	Beta	0.50	0.20
ψ_2	Normal	0.12	0.05	σ_{r^m}	InvG	0.10	2.00
ψ_3	Normal	0.12	0.05	$\sigma_{r^m}, reg2$	InvG	1.00	2.00
ρ_R	Beta	0.75	0.10	σ_{ant1}	InvG	0.20	4.00
Nominal Rigidities Parameters							
ζ_p	Beta	0.50	0.10	ζ_w	Beta	0.50	0.10
ι_p	Beta	0.50	0.15	ι_w	Beta	0.50	0.15
ϵ_p	-	10.00	fixed	ϵ_w	-	10.00	fixed
Other Endogenous Propagation and Steady State Parameters							
100γ	Normal	0.40	0.10	S''	Normal	4.00	1.50
α	Normal	0.30	0.05	ψ	Beta	0.50	0.15
$100(\beta^{-1}-1)$	Gamma	0.25	0.10	π_*	-	0.50	fixed
σ_c	Normal	1.50	0.37	γ_{gdpdef}	Normal	1.00	2.00
h	Beta	0.70	0.10	δ_{gdpdef}	Normal	0.00	2.00
ν_l	Normal	2.00	0.75	$\bar{L}$	Normal	-45.00	5.00
δ	-	0.03	fixed	λ_w	-	1.50	fixed
Φ_p	Normal	1.25	0.12	g_*	-	0.18	fixed
Financial Frictions Parameters							
$F(\bar{\omega})$	-	0.03	fixed	$\zeta_{sp,b}$	Beta	0.05	0.00
SP_*	Gamma	2.00	0.10	γ_*	-	0.99	fixed
Exogenous Process Parameters							
ρ_g	Beta	0.50	0.20	σ_μ	InvG	0.10	2.00
ρ_b	Beta	0.50	0.20	$\sigma_\mu, reg2$	InvG	0.10	1.00
ρ_μ	Beta	0.50	0.20	$\sigma_{\tilde{z}}$	InvG	0.10	2.00
$\rho_z til$	Beta	0.50	0.20	$\sigma_{\tilde{z}}, reg2$	InvG	0.10	1.00
ρ_{σ_ω}	Beta	0.75	0.15	σ_{σ_ω}	InvG	0.05	4.00

Note: For Inverse Gamma prior mean and SD, τ and ν reported.

σ_{ant1} through σ_{ant6} all have the same distribution.

	Dist	Mean	Std Dev		Dist	Mean	Std Dev
ρ_{π_*}	-	0.99	fixed	$\sigma_{\sigma_\omega}, reg2$	InvG	0.05	2.00
ρ_{z^p}	Beta	0.50	0.20	σ_{π_*}	InvG	0.03	6.00
ρ_{λ_f}	Beta	0.50	0.20	$\sigma_{\pi_*}, reg2$	InvG	0.03	3.00
ρ_{λ_w}	Beta	0.50	0.20	σ_{z^p}	InvG	0.10	2.00
η_{λ_f}	Beta	0.50	0.20	σ_{λ_f}	InvG	0.10	2.00
η_{λ_w}	Beta	0.50	0.20	$\sigma_{\lambda_f}, reg2$	InvG	0.10	1.00
σ_g	InvG	0.10	2.00	σ_{λ_w}	InvG	0.10	2.00
$\sigma_g, reg2$	InvG	0.10	1.00	$\sigma_{\lambda_w}, reg2$	InvG	0.10	1.00
σ_b	InvG	0.10	2.00	η_{gz}	Beta	0.50	0.20
$\sigma_b, reg2$	InvG	0.10	1.00				
Measurement Error Parameters							
$\mathcal{C}_{me}$	-	1.00	fixed	σ_{10y}	InvG	0.75	2.00
ρ_{gdp}	Normal	0.00	0.20	$\sigma_{10y}, reg2$	InvG	0.75	1.00
ρ_{gdi}	Normal	0.00	0.20	σ_{tfp}	InvG	0.10	2.00
ρ_{10y}	Beta	0.50	0.20	$\sigma_{tfp}, reg2$	InvG	0.10	1.00
ρ_{tfp}	Beta	0.50	0.20	σ_{gdpdef}	InvG	0.10	2.00
ρ_{gdpdef}	Beta	0.50	0.20	$\sigma_{gdpdef}, reg2$	InvG	1.00	2.00
ρ_{pce}	Beta	0.50	0.20	σ_{pce}	InvG	0.10	2.00
ϱ_{gdp}	Normal	0.00	0.40	$\sigma_{pce}, reg2$	-	0.00	fixed
σ_{gdp}	InvG	0.10	2.00	$\rho_{meas\pi}$	-	0.23	fixed
$\sigma_{gdp}, reg2$	InvG	0.10	1.00	$\rho_{meas\pi}, reg2$	Beta	0.50	0.20
σ_{gdi}	InvG	0.10	2.00	$\sigma_{meas\pi}$	-	0.10	fixed
$\sigma_{gdi}, reg2$	InvG	0.10	1.00	$\sigma_{meas\pi}, reg2$	InvG	0.40	2.00
COVID-19 Parameters							
$\sigma_{z,iid}$	-	0.00	fixed	$\sigma_{\varphi}, reg3$	InvG	4.00	80.00
$\sigma_{z,iid}, reg2$	InvG	5.01	10.00	$\sigma_{\varphi,ant1}$	-	0.00	fixed
$\sigma_{z,iid}, reg3$	InvG	0.05	10.00	$\sigma_{biidc,ant1}$	-	0.00	fixed
$\sigma_{b,iid,c}$	-	0.00	fixed	$\sigma_{ziid,ant1}$	-	0.00	fixed
$\sigma_{b,iid,c}, reg2$	InvG	4.01	8.00	σ^{prop}_{biidc}	-	0.00	fixed
$\sigma_{b,iid,c}, reg3$	InvG	0.04	8.00	σ^{prop}_{φ}	-	0.00	fixed
σ_{φ}	-	0.00	fixed	σ^{prop}_{ziid}	-	0.00	fixed

Note: For Inverse Gamma prior mean and SD, τ and ν reported.

	Dist	Mean	Std Dev		Dist	Mean	Std Dev
$\sigma_{\varphi}, reg2$	InvG	400.00	80.00				

Note: For Inverse Gamma prior mean and SD, τ and ν reported.