Frank Schorfheide ANU Course on Bayesian Analysis of DSGE Models

Consider the numerical illustration of the Kalman and particle filters in the lecture notes, using the same parameters for the matrices of the state-space system:

$$\begin{aligned} y_t &= \Gamma + \Psi s_t + u_t \\ s_t &= \Phi s_{t-1} + \epsilon_t. \end{aligned}$$

where $I\!E[u_t u_t'] = H$ and $I\!E[\epsilon_t \epsilon_t'] = \Sigma$. Let the dimension of y_t be 2×1 and s_t be 1×1.

1. Generate 50 observations from the state-space model. You can set $s_0 = 0$ and initialize the filter based on $s_0 \sim N(0, .75^2)$.

2. Run the Kalman filter and store the mean and variance of $p(s_t|Y_{1:t})$ as well as the likelihood contributions $\ln p(y_t|Y_{1:t-1})$.

3. Plot $\mathbb{E}[s_t|Y_{1:t}]$ as well as 90% credible sets based on $p(s_t|Y_{1:t})$. Overlay the "true" states s_t.

4. Run the particle filter as described in the lecture slides for various choices of the number of particles. Replicate the figures in the slides that compare the KF and PF output. Compute the average of $(\mathbb{E}_{PF}[s_t|Y_{1:t}] - \mathbb{E}_{KF}[s_t|Y_{1:t}])^2$ as a function of the number of particles.

5. Consider a version of the particle filter with and without re-sampling. Compare the effective sample size (ESS) for the two versions of the particle filter.

6. Now multiply the twenty-fifth observation y_{25} in your sample by a factor of 5 (or 10). Assess the accuracy of the particle filter compared to the Kalman filter.

7. Modify the particle filter as follows: instead of generating s_t^i from the distribution $p(s_t|s_{t-1}^i)$, generate it from the distribution $p(s_t|Y_{1:T}, s_{t-1}^i)$. For each particle i you can use the updating step of the Kalman filter to compute the mean and variance of $p(s_t|Y_{1:T}, s_{t-1}^i)$. Take a look at the slides to see how to modify the particle weights under this alternative proposal distribution.

8. Repeat the comparison between particle and Kalman filter. Do you notice the increase in accuracy?