



# Control System Training

## MODULE 4 – Combinatorial Logic

# Copyright Notice

These training materials, including the samples, exercises, and solutions, are copyrighted materials. Any reproduction, or use of any kind without the specific written approval of the author is strictly prohibited.

Permission for extra-curricular use by First FRC teams for FRC related training is granted, provided the original copyright and acknowledgements are retained.

© Jim Simpson, 2018

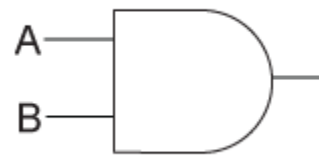
# Combinatorial Logic

- **Boolean – ZERO / ONE – Logic**
- **Outcome depends only on the CURRENT inputs.**
  - No “time” dependency.

# AND Gate

AND GATE

Input		Output
A	B	
0	0	0
0	1	0
1	0	0
1	1	1



- **Output is true when all inputs are true.**
- **Boolean algebra written as  $AB$  or  $A \cdot B$**

# OR Gate

OR GATE

Input		Output
A	B	
0	0	0
0	1	1
1	0	1
1	1	1

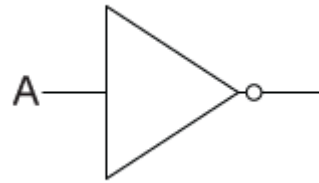


- **Output is true when any input is true**
- **Boolean algebra written as  $A+B$**

# NOT Gate

NOT GATE

INPUT	Output
0	1
1	0

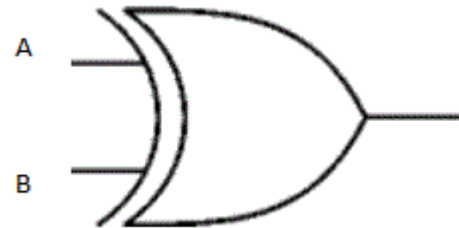


- Output is the opposite of the input.
- Boolean algebra written as  $\neg A$  or  $\overline{A}$

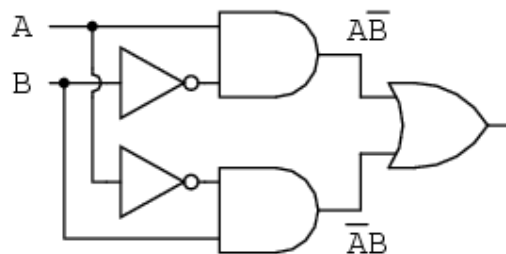
# XOR (Exclusive OR) Gate

XOR GATE

Input		Output
A	B	
0	0	0
0	1	1
1	0	1
1	1	0

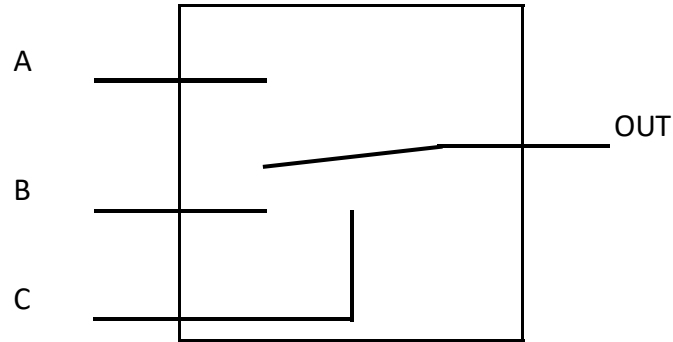


- Output is true when inputs are different.
- Boolean algebra written as  $A \oplus B$
- Equivalent to:



# Special - Digital Switch

Digital Switch			
Input			
A	B	C	Output
?	?	0	A
?	?	1	B



- ❑ **Switches between inputs A and B based on C.**
- ❑ **Doesn't care what the values of A or B are.**
- ❑ **Could also be implemented with “normal” gates.**




# Special – Analog Test Switch

- **EQUAL MONITOR** – Output is true when analog value  $A = B$  (Best used with integer values!)
- **HI MONITOR** – Output is true when analog value  $A \geq B$
- **LO MONITOR** – Output is true when analog value  $B \leq A$
- **HI-LO MONITOR** – Output is true when analog value  $\geq A$  and  $\leq B$ . For the output to ever be true  $B$  must be  $> A$ .
- These functions often have a deadband value to reduce chatter.

# Boolean Algebra

- **Write out as regular algebra**
- **Do this as part of the simplification process**
- **Symbols**
  - + means OR
  - \* means AND
  - • means AND
  - Line over term or ^ means NOT
  - $\oplus$  means XOR (exclusive OR)
  - = means equals
- **Samples**
  - $OUT = PERM \cdot INPUT$
  - $E = A \cdot B + C \cdot D$

# Boolean Algebra - Rules

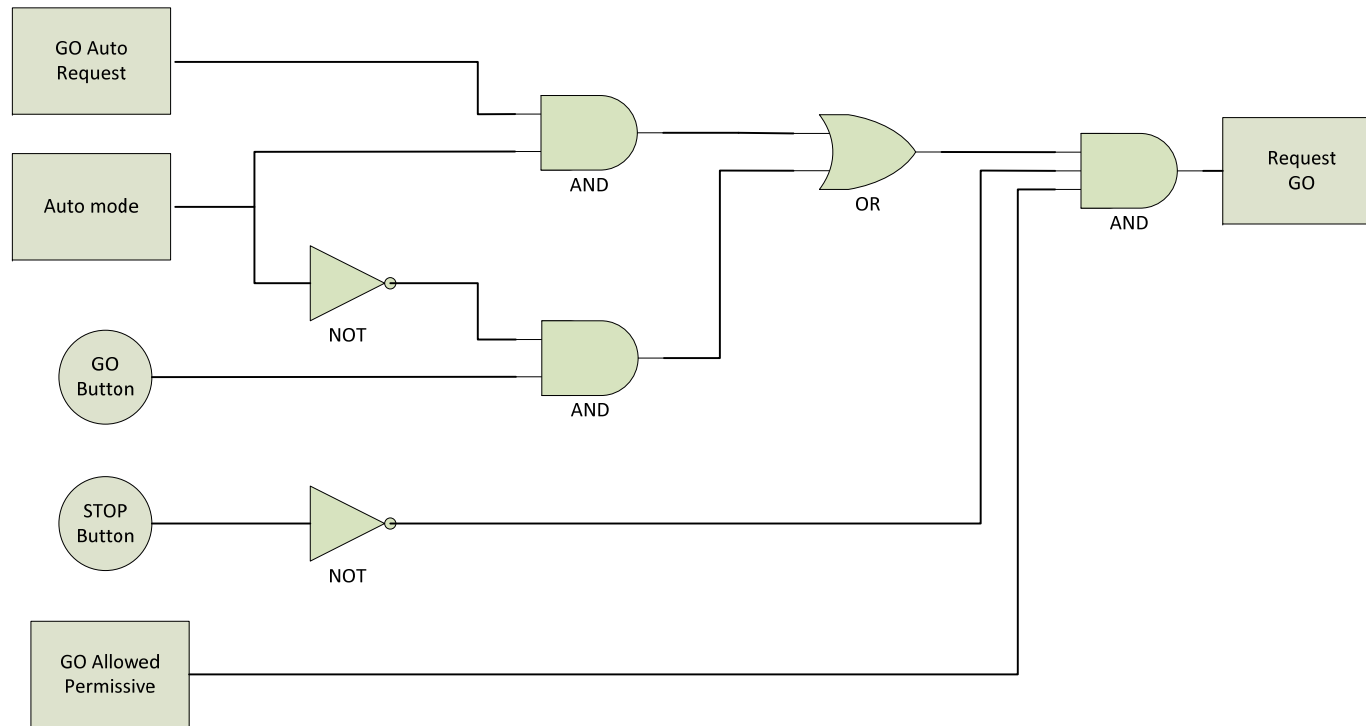
1a. $X \cdot 0 = 0$	1b. $X + 1 = 1$	Annulment Law
2a. $X \cdot 1 = X$	2b. $X + 0 = X$	Identity Law
3a. $X \cdot X = X$	3b. $X + X = X$	Idempotent Law
4a. $X \cdot \bar{X} = 0$	4b. $X + \bar{X} = 1$	Complement Law
5. $\bar{\bar{X}} = X$		Double Negation Law
6a. $X \cdot Y = Y \cdot X$	6b. $X + Y = Y + X$	Commutative Law
7a. $X(YZ) = (XY)Z = (XZ)Y = XYZ$		Associative Law
7b. $X + (Y + Z) = (X + Y) + Z = (X + Z) + Y = X + Y + Z$		Associative Law
8a. $X \cdot (Y + Z) = XY + XZ$	8b. $X + YZ = (X + Y) \cdot (X + Z)$	Distributive Law
9a. $\overline{X \cdot Y} = \bar{X} + \bar{Y}$	9b. $\overline{X + Y} = \bar{X} \cdot \bar{Y}$	de Morgan's Theorem 
10a. $X \cdot (X + Y) = X$	10b. $X + X \cdot Y = X$	Absorption Law
11a. $(X + Y) \cdot (X + \bar{Y}) = X$	11b. $X \cdot Y + X \cdot \bar{Y} = X$	Redundancy Law
12a. $(X + \bar{Y}) \cdot Y = XY$	12b. $X \cdot \bar{Y} + Y = X + Y$	Redundancy Law
13a. $(X + Y) \cdot (\bar{X} + Z) \cdot (Y + Z) = (X + Y) \cdot (\bar{X} + Z)$		Consensus Law
13b. $XY + \bar{X}Z + YZ = XY + \bar{X}Z$		Consensus Law
14a. $X \oplus Y = (X + \bar{Y}) \cdot (\bar{X} + Y)$	14b. $X \oplus Y = \bar{X}Y + X\bar{Y}$	XOR Gate
15a. $X \odot Y = (X + Y) \cdot (\bar{X} \cdot \bar{Y})$	15b. $X \odot Y = \bar{X}\bar{Y} + XY$	XNOR Gate
15c. $X \odot Y = (X + Y) \cdot (\bar{X} + \bar{Y})$		XNOR Gate

□ There are others.

# Boolean Algebra – Simplification

- **Simplify to Product of Sums or Sum of Products**
  - Often for control systems, “product of sums” is better than “sum of products”.
  - Conditions AND permissives
- **There is much, much, more about this online.**

# Sample 4.1



$$\text{GO\_Request} = \text{GO\_Perm} \cdot \text{GO\_allowed} \cdot \neg \text{STOP\_Button} \cdot (\text{GO\_Auto} \cdot \text{Auto\_mode} + \text{GO\_Button} \cdot \neg \text{Auto\_Mode})$$

## Exercise 4.1 – Ball shooter size detector

- **Floor contains 3 sizes of balls. Only the middle size can be shot correctly.**
  - The small balls have 80% the diameter of the middle ball
  - The large balls have 120% the diameter of the middle ball.
- **Have three digital sensors.**
  - One at front of ball.
  - One at 97% diameter of middle ball
  - One at 103% diameter of middle ball
- **The balls roll along a belt.**
- **If robot is disabled, reject all the balls.**
- **Create a circuit to reject all but the middle balls. Draw the logic diagram.**

## Exercise 4.2 – List Programming Objects

- **Create a list of potential objects from this chapter to program.**