```
clear variables

syms m1 l1 r1 I1 m2 l2 r2 I2 g
syms theta1(t) theta2(t)



x1 = r1*cos(theta1)
```

x1(t) = $r_1 \cos(\theta_1(t))$

```
y1 = r1*sin(theta1)
```

y1(t) = $r_1 \sin(\theta_1(t))$

```
z1 = 0;
x2 = l1*cos(theta1) + r2*cos(theta1+theta2)
```

x2(t) = $l_1 \cos(\theta_1(t)) + r_2 \cos(\theta_1(t) + \theta_2(t))$

```
y2 = l1*sin(theta1) + r2*sin(theta1+theta2)
```

y2(t) = $l_1 \sin(\theta_1(t)) + r_2 \sin(\theta_1(t) + \theta_2(t))$

```
z2 = 0;

% potential energy of the system
P = m1*g*y1 + m2*g*y2
```

P(t) = $g\, m_2\, (l_1 \sin(\theta_1(t)) + r_2 \sin(\theta_1(t) + \theta_2(t))) + g\, m_1\, r_1 \sin(\theta_1(t))$

```


Jv1 = [diff(x1,theta1) diff(x1,theta2)
       diff(y1,theta1) diff(y1,theta2)
       diff(z1,theta1) diff(z1,theta2)]
```

Jv1(t) =

$$\begin{pmatrix} -r_1 \sin(\theta_1(t)) & 0 \\ r_1 \cos(\theta_1(t)) & 0 \\ 0 & 0 \end{pmatrix}$$

```


Jv2 = [diff(x2,theta1) diff(x2,theta2)
       diff(y2,theta1) diff(y2,theta2)
       diff(z2,theta1) diff(z2,theta2)]
```

Jv2(t) =

$$\begin{pmatrix} -l_1 \sin(\theta_1(t)) - r_2 \sin(\theta_1(t) + \theta_2(t)) & -r_2 \sin(\theta_1(t) + \theta_2(t)) \\ l_1 \cos(\theta_1(t)) + r_2 \cos(\theta_1(t) + \theta_2(t)) & r_2 \cos(\theta_1(t) + \theta_2(t)) \\ 0 & 0 \end{pmatrix}$$

```
Jw1 = sym([0 0; 0 0; 1 0])
```

Jw1 =

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}$$

```
Jw2 = sym([0 0; 0 0; 1 1])
```

Jw2 =

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{pmatrix}$$

```
R1 = [cos(theta1) -sin(theta1) 0;
      sin(theta1)  cos(theta1) 0;
      0            0           1]
```

R1(t) =

$$\begin{pmatrix} \cos(\theta_1(t)) & -\sin(\theta_1(t)) & 0 \\ \sin(\theta_1(t)) & \cos(\theta_1(t)) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
R2 = [cos(theta2) -sin(theta2) 0;
      sin(theta2)  cos(theta2) 0;
      0            0           1]
```

R2(t) =

$$\begin{pmatrix} \cos(\theta_2(t)) & -\sin(\theta_2(t)) & 0 \\ \sin(\theta_2(t)) & \cos(\theta_2(t)) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
M = m1*(Jv1.'*Jv1) + Jw1.'*R1*I1*R1.'*Jw1 + ...
    m2*(Jv2.'*Jv2) + Jw2.'*R2*I2*R2.'*Jw2;

M = simplify(M)
```

M(t) =

$$\begin{pmatrix} m_2\, {l_1}^2 + 2\, m_2 \cos(\theta_2(t))\, l_1\, r_2 + m_1\, {r_1}^2 + m_2\, {r_2}^2 + I_1 + I_2 & m_2\, {r_2}^2 + l_1\, m_2 \cos(\theta_2(t))\, r_2 + I_2 \\ m_2\, {r_2}^2 + l_1\, m_2 \cos(\theta_2(t))\, r_2 + I_2 & m_2\, {r_2}^2 + I_2 \end{pmatrix}$$

```
M = M(t);   % evaluate vs. time so we can index the matrix

c11 = 1/2*(diff(M(1,1),theta1) + diff(M(1,1),theta1) - diff(M(1,1),theta1)) * diff(theta1(t),t)
      1/2*(diff(M(1,1),theta2) + diff(M(1,2),theta1) - diff(M(2,1),theta1)) * diff(theta2(t),t)
c12 = 1/2*(diff(M(1,2),theta1) + diff(M(1,1),theta2) - diff(M(1,2),theta1)) * diff(theta1(t),t)
      1/2*(diff(M(1,2),theta2) + diff(M(1,2),theta2) - diff(M(2,2),theta1)) * diff(theta2(t),t)
c21 = 1/2*(diff(M(2,1),theta1) + diff(M(2,1),theta1) - diff(M(1,1),theta2)) * diff(theta1(t),t)
      1/2*(diff(M(2,1),theta2) + diff(M(2,2),theta1) - diff(M(2,1),theta2)) * diff(theta2(t),t)
c22 = 1/2*(diff(M(2,2),theta1) + diff(M(2,1),theta2) - diff(M(1,2),theta2)) * diff(theta1(t),t)
      1/2*(diff(M(2,2),theta2) + diff(M(2,2),theta2) - diff(M(2,2),theta2)) * diff(theta2(t),t)

C = simplify([c11 c12; c21 c22])
```

C(t) =

$$\begin{pmatrix} -l_1\, m_2\, r_2 \sin(\theta_2(t))\, \dfrac{\partial}{\partial t}\, \theta_2(t) & -l_1\, m_2\, r_2 \sin(\theta_2(t)) \left(\dfrac{\partial}{\partial t}\, \theta_1(t) + \dfrac{\partial}{\partial t}\, \theta_2(t) \right) \\ l_1\, m_2\, r_2 \sin(\theta_2(t))\, \dfrac{\partial}{\partial t}\, \theta_1(t) & 0 \end{pmatrix}$$

```
Tg = [diff(P,theta1); diff(P,theta2)];
Tg = simplify(Tg)
```

Tg(t) =

$$\begin{pmatrix} g\, m_2\, (l_1 \cos(\theta_1(t)) + r_2 \cos(\theta_1(t) + \theta_2(t))) + g\, m_1\, r_1 \cos(\theta_1(t)) \\ g\, m_2\, r_2 \cos(\theta_1(t) + \theta_2(t)) \end{pmatrix}$$