```
clear variables

syms m1 l1 r1 I1 m2 l2 r2 I2 g
syms alpha1(t) alpha2(t)

x1 = r1*cos(alpha1)
```

x1(t) = $r_1 \cos(\alpha_1(t))$

```
y1 = r1*sin(alpha1)
```

y1(t) = $r_1 \sin(\alpha_1(t))$

```
z1 = 0;
x2 = l1*cos(alpha1) + r2*cos(alpha2)
```

x2(t) = $l_1 \cos(\alpha_1(t)) + r_2 \cos(\alpha_2(t))$

```
y2 = l1*sin(alpha1) + r2*sin(alpha2)
```

y2(t) = $l_1 \sin(\alpha_1(t)) + r_2 \sin(\alpha_2(t))$

```
z2 = 0;

% potential energy of the system
P = m1*g*y1 + m2*g*y2
```

P(t) = $g\, m_2\, (l_1 \sin(\alpha_1(t)) + r_2 \sin(\alpha_2(t))) + g\, m_1\, r_1 \sin(\alpha_1(t))$

```
Jv1 = [diff(x1,alpha1) diff(x1,alpha2)
       diff(y1,alpha1) diff(y1,alpha2)
       diff(z1,alpha1) diff(z1,alpha2)]
```

Jv1(t) =

$$\begin{pmatrix} -r_1 \sin(\alpha_1(t)) & 0 \\ r_1 \cos(\alpha_1(t)) & 0 \\ 0 & 0 \end{pmatrix}$$

```
Jv2 = [diff(x2,alpha1) diff(x2,alpha2)
       diff(y2,alpha1) diff(y2,alpha2)
       diff(z2,alpha1) diff(z2,alpha2)]
```

Jv2(t) =

$$\begin{pmatrix} -l_1 \sin(\alpha_1(t)) & -r_2 \sin(\alpha_2(t)) \\ l_1 \cos(\alpha_1(t)) & r_2 \cos(\alpha_2(t)) \\ 0 & 0 \end{pmatrix}$$

```
Jw1 = sym([0 0; 0 0; 1 0])
```

Jw1 =

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}$$

```
Jw2 = sym([0 0; 0 0; 0 1])
```

Jw2 =

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$$

```
R1 = [cos(alpha1) -sin(alpha1) 0;
      sin(alpha1)  cos(alpha1) 0;
      0            0           1]
```

R1(t) =

$$\begin{pmatrix} \cos(\alpha_1(t)) & -\sin(\alpha_1(t)) & 0 \\ \sin(\alpha_1(t)) & \cos(\alpha_1(t)) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
R2 = [cos(alpha2) -sin(alpha2) 0;
      sin(alpha2)  cos(alpha2) 0;
      0            0           1]
```

R2(t) =

$$\begin{pmatrix} \cos(\alpha_2(t)) & -\sin(\alpha_2(t)) & 0 \\ \sin(\alpha_2(t)) & \cos(\alpha_2(t)) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
M = m1*(Jv1.'*Jv1) + Jw1.'*R1*I1*R1.'*Jw1 + ...
    m2*(Jv2.'*Jv2) + Jw2.'*R2*I2*R2.'*Jw2;

M = simplify(M)
```

M(t) =

$$\begin{pmatrix} m_2\, l_1{}^2 + m_1\, r_1{}^2 + I_1 & l_1\, m_2\, r_2 \cos(\alpha_1(t) - \alpha_2(t)) \\ l_1\, m_2\, r_2 \cos(\alpha_1(t) - \alpha_2(t)) & m_2\, r_2{}^2 + I_2 \end{pmatrix}$$

```
M = M(t);   % evaluate vs. time so we can index the matrix

c11 = 1/2*(diff(M(1,1),alpha1) + diff(M(1,1),alpha1) - diff(M(1,1),alpha1)) *
diff(alpha1(t),t) + ...
```

```
       1/2*(diff(M(1,1),alpha2) + diff(M(1,2),alpha1) - diff(M(2,1),alpha1)) *
diff(alpha2(t),t);
 c12 = 1/2*(diff(M(1,2),alpha1) + diff(M(1,1),alpha2) - diff(M(1,2),alpha1)) *
diff(alpha1(t),t) + ...
       1/2*(diff(M(1,2),alpha2) + diff(M(1,2),alpha2) - diff(M(2,2),alpha1)) *
diff(alpha2(t),t);
 c21 = 1/2*(diff(M(2,1),alpha1) + diff(M(2,1),alpha1) - diff(M(1,1),alpha2)) *
diff(alpha1(t),t) + ...
       1/2*(diff(M(2,1),alpha2) + diff(M(2,2),alpha1) - diff(M(2,1),alpha2)) *
diff(alpha2(t),t);
 c22 = 1/2*(diff(M(2,2),alpha1) + diff(M(2,1),alpha2) - diff(M(1,2),alpha2)) *
diff(alpha1(t),t) + ...
       1/2*(diff(M(2,2),alpha2) + diff(M(2,2),alpha2) - diff(M(2,2),alpha2)) *
diff(alpha2(t),t);

 C = simplify([c11 c12; c21 c22])
```

C(t) =

$$\begin{pmatrix} 0 & l_1\, m_2\, r_2 \sin(\alpha_1(t) - \alpha_2(t)) \frac{\partial}{\partial t}\, \alpha_2(t) \\ l_1\, m_2\, r_2 \sin(\alpha_2(t) - \alpha_1(t)) \frac{\partial}{\partial t}\, \alpha_1(t) & 0 \end{pmatrix}$$

```
 Tg = [diff(P,alpha1); diff(P,alpha2)];
 Tg = simplify(Tg)
```

Tg(t) =

$$\begin{pmatrix} g \cos(\alpha_1(t))\ (l_1\, m_2 + m_1\, r_1) \\ g\, m_2\, r_2 \cos(\alpha_2(t)) \end{pmatrix}$$