where, V_w is the velocity of the wheel, θ is the reference wheel angle, V_{in} is the induced velocity on wheel, ϕ is the reference body velocity angle, and V_b is the body velocity of robot.

Now $\mathbf{V_{in}}$ and $\mathbf{V_w}$ are always orthogonal:

$$\mathbf{V_b}^2 = \mathbf{V_w}^2 + \mathbf{V_{in}}^2 \tag{1}$$

Also:

$$\mathbf{V_{in}}^2 = \mathbf{V_b}^2 + \mathbf{V_w}^2 - 2\,\mathbf{V_w}\,\mathbf{V_b}\cos(\theta - \phi)$$

$$= \mathbf{V_b}^2 + \mathbf{V_w}^2 - 2\,\mathbf{V_w}\,\mathbf{V_b}(\cos\theta\cos\phi + \sin\theta\sin\phi) \tag{2}$$

Substituting (2) into (1), we may obtain:

$$\mathbf{V_w} = \mathbf{V_b}(\cos\theta\cos\phi + \sin\theta\sin\phi) \tag{3}$$

For a given rotational velocity of the centre of mass, $\dot{\Psi}$, each wheel must apply velocity:

$$V_w = R\dot{\Psi}\,, \tag{4}$$

where R is the distance of the wheel from the centre of mass. Thus, for each wheel:

$$V_w = V_b(\cos\theta\cos\phi + \sin\theta\sin\phi) + R\dot{\Psi} \tag{5}$$

This is a general equation that is independent of the number of wheels. Consider a three wheeled omni-directional vehicle with wheels arranged at angles of 0°, 120° and 240°, equation (5) yields:

Wheel 1 ($\theta = 0°$): $$V_{w1} = V_b\cos\phi + R\dot{\Psi} \tag{6}$$

Wheel 2 ($\theta = 120°$): $$V_{w2} = V_b(\frac{-1}{2}\cos\phi + \frac{\sqrt{3}}{2}\sin\phi) + R\dot{\Psi} \tag{7}$$

Wheel 3 ($\theta = 240°$): $$V_{w3} = V_b(\frac{-1}{2}\cos\phi - \frac{\sqrt{3}}{2}\sin\phi) + R\dot{\Psi} \tag{8}$$

Similar equations for this three-wheeled case appear in [6]. The translational component only appears also in [2]. As pointed out by Pin and Killough [6], if we separate V_b into x and y components, where $V_{bx} = V_b\cos\phi$, and $V_{by} = V_b\sin\phi$, this becomes a linear relation. We may invert the matrix for any n-wheeled system. Consider, for example, the three-wheeled system above: