

## Supplementary Material

### A Derivation from feature interaction component in NFM to Aggregated pairwise analysis component in GDCF

This appendix is going to show the details of the derivation in GDCF paper.

$$\begin{aligned}
& MLP(f_{BI}) \\
&= MLP\left(\sum_{i=1}^N \sum_{j=i+1}^N (x_i \mathbf{v}_i \odot x_j \mathbf{v}_j)\right) \\
&= \mathbf{W}_L(\dots(\mathbf{W}_1\left(\sum_{i=1}^N \sum_{j=i+1}^N (x_i \mathbf{v}_i \odot x_j \mathbf{v}_j)\right) + b_1)\dots) + b_L \\
&= \mathbf{W}_L \dots \mathbf{W}_1 \sum_{i=1}^N \sum_{j=i+1}^N (x_i \mathbf{v}_i \odot x_j \mathbf{v}_j) + (\mathbf{W}_L \dots \mathbf{W}_2 b_1) + \dots + b_L \\
&= \sum_{i=1}^N \sum_{j=i+1}^N (\mathbf{W}_L \dots \mathbf{W}_1 (x_i \mathbf{v}_i \odot x_j \mathbf{v}_j)) + (\mathbf{W}_L \dots \mathbf{W}_2 b_1) + \dots + b_L \\
&= \sum_{i=1}^N \sum_{j=i+1}^N (\mathbf{W}_L \dots \mathbf{W}_1 (x_i \mathbf{v}_i \odot x_j \mathbf{v}_j) + (\mathbf{W}_L \dots \mathbf{W}_2 b_1) + \dots + b_L) - C \\
&= \sum_{i=1}^N \sum_{j=i+1}^N (\mathbf{W}_L(\dots(\mathbf{W}_1(x_i \mathbf{v}_i \odot x_j \mathbf{v}_j) + b_1)\dots) + b_L) - C \\
&\equiv \sum_{i,j \in R_k, j > i} (MLP_{pairwise}(x_i \mathbf{v}_i \odot x_j \mathbf{v}_j)) - C \\
&= \sum_{i,j \in R_k, j > i} (MLP_{pairwise}((\mathbf{v}_i \odot \mathbf{v}_j) e_{ij})) - C \\
&= \sum_{i,j \in R_k, j > i} (\phi_p(e_{ij}, \mathbf{v}_i, \mathbf{v}_j)) - C,
\end{aligned}$$

where  $C = (\frac{N(N-1)}{2} - 1)((\mathbf{W}_L \dots \mathbf{W}_2 b_1) + \dots + b_L)$ .