Robotics

(Course viewgraphs)

João Silva Sequeira[1]

[1]joao.silva.sequeira@tecnico.ulisboa.pt

Winter P2, 2024-2025

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Differential kinematics – Generic velocity models – Jacobians

- Maps velocity spaces, e.g., from joint space to work space

- It applies both to manipulators and mobile platforms

- Useful to obtain feasible <u>directions of motion</u>

Jacobians

$$y_1 = f_1(x_1, \ldots, x_m)$$

$$\vdots$$

$$y_n = f_n(x_1, \ldots, x_m)$$

$\longrightarrow$ differentiating $\longrightarrow$

$$\delta y_1 = \frac{\partial f_1}{\partial x_1}\delta x_1 + \ldots + \frac{\partial f_1}{\partial x_m}\delta x_m$$

$$\vdots$$

$$\delta y_n = \frac{\partial f_m}{\partial x_1}\delta x_1 + \ldots + \frac{\partial f_m}{\partial x_m}\delta x_m$$

$$f_i \ : \ \mathbb{R}^m \mapsto \mathbb{R}$$

Using matrix notation

$$\begin{bmatrix} \delta y_1 \\ \vdots \\ \delta y_n \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_m} \\ \vdots & & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_m} \end{bmatrix}}_{\text{Jacobian, } J(x_1, \ldots, x_m)} \begin{bmatrix} \delta x_1 \\ \vdots \\ \delta x_m \end{bmatrix}$$

Dividing both sides by δt

$\dot{\mathbf{y}} = J(x_1, \ldots, x_m)\dot{\mathbf{x}} \quad \rightarrow \quad$ Transformation between velocities

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Jacobian

In robotics, for a manipulator with *n* joints

$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}}_{\text{Velocity in the workspace}} = J(\theta_1, \ldots, \theta_n) \underbrace{\begin{bmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_n \end{bmatrix}}_{\text{Velocity in the joint space}}$$

The Jacobian is separable in two parts

$$J(\theta_1, \ldots, \theta_n) = \begin{bmatrix} J_p(\theta_1, \ldots, \theta_n) \\ J_o(\theta_1, \ldots, \theta_n) \end{bmatrix} \begin{matrix} \rightarrow \text{Linear velocity jacobian} \\ \rightarrow \text{Angular velocity jacobian} \end{matrix}$$

$\downarrow$

The number of columns is identical to the number of joints

Jacobians for serial manipulators

- J_p can be computed from
  - differentiation of the direct kinematics equations
  - using the formulas for the propagation of the linear velocities

$$^{i+1}v_{i+1} = {}^{i+1}_{i}R\,{}^{i}v_i + {}^{i+1}_{i}R\,\left({}^{i}\omega_i \times {}^{i}P_{i+1}\right)$$

- J_o can be computed from the formulas for the propagation of the angular velocities

$$^{i+1}\omega_{i+1} = {}^{i+1}_{i}R\,{}^{i}\omega_i + \dot{\theta}_{i+1}\,{}^{i+1}Z_{i+1}$$

- See J. Craig's book for the demonstration of these formulas

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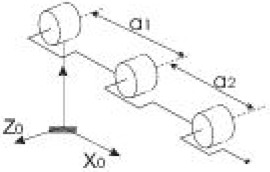
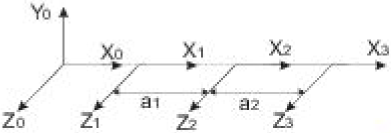
Jacobians – Example – RRR planar manipulator



$$
{}^{0}_{1}T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\qquad
{}^{1}_{2}T = \begin{bmatrix} c_2 & -s_2 & 0 & a_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}_{3}T = \begin{bmatrix} c_3 & -s_3 & 0 & a_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$



$$
{}^{0}_{3}T = \begin{bmatrix} c_{123} & -s_{123} & 0 & a_2 c_{12} + a_1 c_1 \\ s_{123} & c_{123} & 0 & a_2 s_{12} + a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	a_1	0	θ_2
3	0	a_2	0	θ_3

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Jacobians – Example – RRR planar manipulator

$$^{1}\omega_1 = {}^{1}_{0}R\, {}^{0}\omega_0 + \dot{\theta}_1\, {}^{1}Z_1 \qquad {}^{1}v_1 = {}^{1}_{0}R\, \left({}^{0}v_0 + {}^{0}\omega_0 \times {}^{0}P_1\right)$$

$$^{2}\omega_2 = {}^{2}_{1}R\, {}^{1}\omega_1 + \dot{\theta}_2\, {}^{2}Z_2 \qquad {}^{2}v_2 = {}^{2}_{1}R\, \left({}^{1}v_1 + {}^{1}\omega_1 \times {}^{1}P_2\right)$$

$$^{3}\omega_3 = {}^{3}_{2}R\, {}^{2}\omega_2 + \dot{\theta}_3\, {}^{3}Z_3 \qquad {}^{3}v_3 = {}^{3}_{2}R\, \left({}^{2}v_2 + {}^{2}\omega_2 \times {}^{2}P_3\right)$$

Jacobians – Example – RRR planar manipulator

$$
{}^{1}\omega_1 = \begin{bmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}
$$

$$
{}^{1}v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
$$

$$
{}^{2}\omega_2 = \begin{bmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix}
$$

$$
{}^{2}v_2 = \begin{bmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \times \begin{bmatrix} a_1 \\ 0 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} a_1 \dot{\theta}_1 s_2 \\ a_1 \dot{\theta}_1 c_2 \\ 0 \end{bmatrix}
$$

Jacobians – Example – RRR planar manipulator

$$
{}^{3}\omega_{3} = \begin{bmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 + \dot\theta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 + \dot\theta_2 + \dot\theta_3 \end{bmatrix}
$$

$$
{}^{3}v_{3} = \begin{bmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} a_1 s_2 \dot\theta_1 \\ a_1 c_2 \dot\theta_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 + \dot\theta_2 \end{bmatrix} \times \begin{bmatrix} a_2 \\ 0 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} a_1 s_{23} \dot\theta_1 + a_2 s_3 (\dot\theta_1 + \dot\theta_2) \\ a_1 c_{23} \dot\theta_1 + a_2 c_3 (\dot\theta_1 + \dot\theta_2) \\ 0 \end{bmatrix}
$$

Given that, for any joint, ${}^{0}\omega_i = {}^{0}_{i}R\, {}^{i}\omega_i$ and ${}^{0}v_i = {}^{0}_{i}R\, {}^{i}\omega_i$ can be written

$$
{}^{0}\omega_{3} = \begin{bmatrix} c_{123} & -s_{123} & 0 \\ s_{123} & c_{123} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 + \dot\theta_2 + \dot\theta_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 + \dot\theta_2 + \dot\theta_3 \end{bmatrix}
$$

$$
{}^{0}v_{3} = \begin{bmatrix} c_{123} & -s_{123} & 0 \\ s_{123} & c_{123} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_1 s_{23} \dot\theta_1 + a_2 s_3 (\dot\theta_1 + \dot\theta_2) \\ a_1 c_{23} \dot\theta_1 + a_2 c_3 (\dot\theta_1 + \dot\theta_2) \\ 0 \end{bmatrix} = \begin{bmatrix} -a_1 s_1 \dot\theta_1 - a_2 s_{12} (\dot\theta_1 + \dot\theta_2) \\ a_1 c_1 \dot\theta_1 + a_2 c_{12} (\dot\theta_1 + \dot\theta_2) \\ 0 \end{bmatrix}
$$

Jacobians – Example – RRR planar manipulator

$$\begin{bmatrix} {}^0v_3 \\ {}^0\omega_3 \end{bmatrix} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} & 0 \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

The linear velocity Jacobian is

$$J_p = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} & 0 \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The angular velocity jacobian is

$$J_o = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Jacobians – Example – RRR planar manipulator

$$\begin{bmatrix} {}^{0}v_3 \end{bmatrix} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} & 0 \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

The linear velocity Jacobian is

$$J_p = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} & 0 \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Singularities in manipulators

Singularity

$$\Updownarrow$$

Configuration in which the robot looses 1 or more dof, that is these dof(s) do not affect the motion along some directions

$$\text{Singularities} \rightarrow \begin{cases} \text{in position} \\ \text{in orientation} \end{cases} \qquad \text{Singularities} \rightarrow \begin{cases} \text{At the workspace boundary} \\ \text{In the interior of the workspace} \end{cases}$$

Loosing 1 dof means that

- the motion of one joint or
- a combination of movements from multiple joints

does not produces any effect in position and orientation

Position singularities – Example – RRR planar manipulator

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \underbrace{\begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} & 0 \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{J_p} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} \tag{1}$$

- Clearly, $|J_p| = 0$, because the robot can not move along Z
- Only interested in the motion in plane XY and hence only the corresponding submatrix is used. The determinant is

$$\begin{vmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 - a_2 c_{12} & a_2 c_{12} \end{vmatrix} = -a_1 a_2 \sin(\theta_2) = 0$$

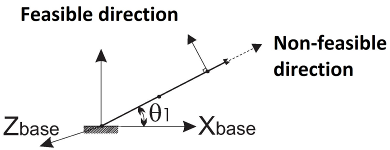
The singular configurations are described by $\theta_2 = k\pi, \; k \in \mathbb{Z}$

Singularities in manipulators

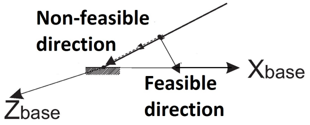
The feasible motion directions in space (x, y, z), when the robot is in a singular configuration can be obtained substituting $\theta_2 = k\pi$ and $\dot{\theta}_2 = 0$ in (1)

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ 0 \end{bmatrix} = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix} \dot{\theta}_1 \begin{cases} (a_1 + a_2) & \text{if } \theta_2 = 2k\pi \\ (a_1 - a_2) & \text{if } \theta_2 = (2k+1)\pi \end{cases}$$

There's no $\dot{\theta}_2$ affecting the output motion directions, i.e., θ_2 is "lost"



Manipulator completely stretched



Manipulator completely folded back

Singularities in manipulators

Orientation singularities
$|J_o| = 0$ since the robot can only be oriented in the subspace $(\alpha, 0, 0)$
The feasible motion directions in space (α, β, γ) are given by

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix}$$

Exercise: What are the singularities for the PUMA 560 ?

Example – Puma 560 like Robots

From the direct kinematics of the Puma 560,

$$\begin{cases} p_x = c_1\left[a_2c_2 + a_3c_{23} - d_4s_{23}\right] - d_3s_1 \\ p_y = s_1\left[a_2c_2 + a_3c_{23} - d_4s_{23}\right] + d_3c_1 \\ p_z = -a_3s_{23} - a_2s_2 - d_4c_{23} \end{cases}$$

For the sake of simplicity consider $a_3 = 0, d_3 = 0$

$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_x \\ \dot{p}_z \end{bmatrix} = \underbrace{\begin{bmatrix} -s_1(a_2c_2 - d_4s_{23}) & -c_1(a_2s_2 + d_4c_{23}) & -c_1d_4c_{23} \\ c_1(a_2c_2 - d - 4s_{23}) & -s_1(a_2s_2 + d_4c_{23}) & -s_1d - 4c_{23} \\ 0 & -a_2c_2 + d_4s_{23} & d_4s_{23} \end{bmatrix}}_{J_p} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

The determinant $|J_p|$ is

$$\begin{aligned} |J_p| &= (a_2c_2 - d_4s_{23})\left[s_1^2d_4c_{23}(a_2c_2 - d_4s_{23}) + c_1^2d_4c_{23}(a_2c_2 - d_4s_{23})\right] + d_4s_{23}\Big[s_1^2(a_2c_2 - d_4s_{23})(a_2s_2 - d_4c_{23}) + \\ &\qquad + c_1^2(a_2c_2 - d_4s_{23})(a_2s_2 - d_4c_{23})\Big] \\ &= d_4(a_2c_2 - d_4s_{23})\left[c_{23}(a_2c_2 - d_4s_{23}) + s_{23}(a_2s_2 + d_4c_{23})\right] \end{aligned}$$

Example – Puma 560 like Robots

The position singularities are the solutions of $|J_p| = 0$,

$$a_2 c_2 - d_4 s_{23} = 0 \qquad \text{Type 1 singularities}$$

$$|J_p| = 0 \quad \begin{matrix} \nearrow \\ \\ \searrow \end{matrix}$$

$$c_{23}(a_2 c_2 - d_4 s_{13}) + s_{23}(a_2 s_2 + d_4 c_{23}) = 0 \qquad \text{Type 2 singularities}$$

Example – Puma 560 like Robots

Type 2 singularities

Working the corresponding expression

$$a_2 c_2 c_{23} - d_2 s_{23} c_{23} + a_2 s_2 s_{23} + d_4 c_{23} s_{23} = a_2 \cos(\theta_2 - (\theta_2 + \theta_3)) = -a_2 \cos(\theta_3) = 0$$

$\boxed{\theta_3 = (2k+1)\pi/2 \quad \rightarrow \quad \text{“elbow stretched/folded”}}$

Type 1 singularites

Working out the corresponding expression

$$a_2 c_2 - d_4 s_{23} = 0$$

$\boxed{a_2 c_2 = d_4 s_{23}}$ Physical meaning?

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Example – Puma 560 like Robots

Substituting $a_2 c_2 - d_4 s_{23} = 0$ in J_p results in

$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{p}_z \end{bmatrix} = \begin{bmatrix} 0 & c_1 a_2 (s_2 - c_2) & -c_1 d_4 c_{23} \\ 0 & -s_1 a_2 (s_2 + c_2) & -s_1 d_4 c_{23} \\ 0 & 0 & d_4 s_{23} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

Substituting $a_2 c_2 - d_4 s_{23} = 0$ in the direct kinematics

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -a_2 s_2 - d_4 c_{23} \end{bmatrix}$$

$\rightarrow$ only motion along Z

- one degree-of-freedom is lost ($\dot{\theta}_1$)
- which motion direction becomes non-feasible ?
- what is the singularity region ?

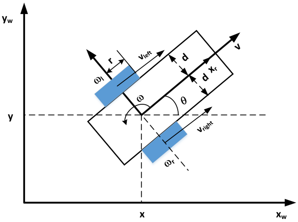
Mobile robot kinematics – Unicycle

Figure 1: Two scout robots (shown next to a quarter for scale). ©2000 by ACM, appeared in Rybski *et al.* (2000).

Miniature scout robot

Unicycle

Mobile robot kinematics – Unicycle



- 2 independent wheels
- 2 free wheels to support the body
- the angular velocity of each wheel is a control variable

From the figure, one can obtain the differential kinematics for the unicycle

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V \\ \omega \end{bmatrix}$$

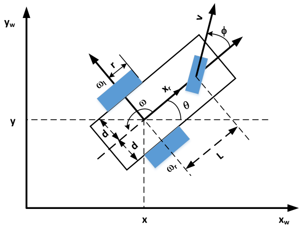
with

$$V = (V_{\text{right}} + V_{\text{left}})/2 = (\omega_{\text{right}} + \omega_{\text{left}})r/2$$

$$\omega = (\omega_{\text{right}} - \omega_{\text{left}})r/(2d)$$

The differential kinematics is a transformation between velocities in frames $\mathcal{F}_R$ and $\mathcal{F}_W$

Mobile robot kinematics – Car Robot



- 2 wheels for traction
- 1 wheel for steering

or

- 2 support free wheels at the back
- 1 wheel for traction and steering

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos(\theta)\cos(\phi) & 0 \\ \sin(\theta)\cos(\phi) & 0 \\ \sin(\phi)/L & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V \\ \omega_s \end{bmatrix}$$

- Control variables are: the steering wheel angular velocity, ω_s, and the linear velocity of the steering wheel, V
- Assume that there is no wheel slippage
- Note that $\omega = V_{\text{tangent}}/L = V\sin(\phi)/L$
- V is the linear velocity of the steering wheel in the robot frame

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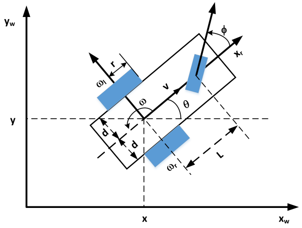
Mobile robot kinematics – Car Robot – Alternative

Now, the linear velocity in the robot frame, V, is the velocity of the frame origin



From the figure, the differential kinematics is given by

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ \tan(\phi)/L & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V \\ \omega_s \end{bmatrix}$$

Where $\omega = V_{\text{tangent}}/L = V\tan(\phi)/L$

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Jacobians in mobile platforms

- The differential models seen before for the kinematics of mobile platforms map velocities in the robot frame into velocities in the world frame.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

- For the unicycle robot the fact that the Jacobian has dimensions 3×2 shows that there are degrees-of-freedom in the joint space (i.e., in the robot frame) that affect multiple dof in the world frame

Holonomic vs. Non-holonomic robots

- **Holonomic robots** – At any configuration there are no constraints on the velocity

- **Non-Holonomic robots** – At any configuration there are non-admissible velocities

- The velocity constraints in a non-holonomic robot can not be “solved” to remove the dependency of the velocities

Example: From the unicycle model

$$\dot{x} = \cos(\theta)\, V$$

$$\dot{y} = \sin(\theta)\, V$$

Combining the two expressions

$$\frac{\dot{y}}{\dot{x}} = \frac{\sin(\theta)}{\cos(\theta)}$$

or

$$\dot{y}\, \cos(\theta) - \dot{x}\, \sin(\theta) = 0$$

Holonomic vs. Non-holonomic robots – Examples

Holonomic robots:

- The serial manipulators that we have been studying ...
- Any robot that can move in any direction at any configuration
  $\dot{\theta} = f_1(\theta)u_1 + \ldots + \ldots + f_n(\theta)u_n$, with $\theta \in \mathbf{R}^n$ and the f_i forming a base in $\mathbf{R}^n$

Non-holonomic robots:

- The unicycle robot, the car robot, ...
- Any robot that has limitations in the directions it can move to at arbitrary configurations

$$\dot{\theta} = f_1(\theta)u_1 + \ldots + f_n(\theta)u_m, \quad \text{with} \quad \theta \in \mathbf{R}^n, \quad m < n$$

This robot can only move in some directions (though it may be possible that it can reach any configuration - if controllable)

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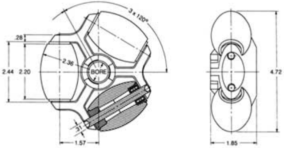
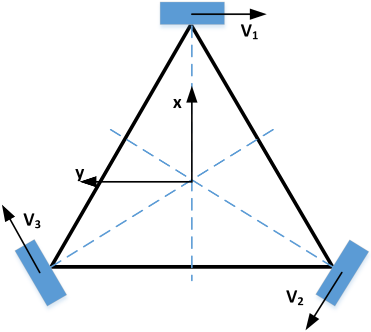
Holonomic robots

- Simple wheels constrain the motion of robots
- Omnidirectional/Mecanum wheels do not constrain the motion



OA

Holonomic robots kinematics – Example



$$\begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} {}^{R}v_1$$

$$\begin{bmatrix} \dot{x}_2 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} {}^{R}v_2$$

$$\begin{bmatrix} \dot{x}_3 \\ \dot{y}_3 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} {}^{R}v_3$$

$${}^{W}v = (\dot{x}, \dot{y}) =$$

$$= \left({}^{W}v_1 + {}^{W}v_2 + {}^{W}v_3 \right) / 3$$

$${}^{W}\omega = \dot{\theta} =$$

$$= {}^{R}\omega = \frac{r}{3L} (\omega_1 + \omega_2 + \omega_3)$$

$${}^{R}v_1 = (0, -\omega_1 \; r)$$

$${}^{R}v_2 = (\cos(-\tfrac{2\pi}{3} - \tfrac{\pi}{2}), \sin(-\tfrac{2\pi}{3} - \tfrac{\pi}{2}))\, \omega_2 \; r = \left(-\tfrac{\sqrt{3}}{2}, \tfrac{1}{2}\right)$$

$${}^{R}v_3 = (\cos(\tfrac{2\pi}{3} - \tfrac{\pi}{2}), \sin(\tfrac{2\pi}{3} - \tfrac{\pi}{2}))\, \omega_3 \; r = \left(\tfrac{\sqrt{3}}{2}, \tfrac{1}{2}\right) \omega_3 r$$

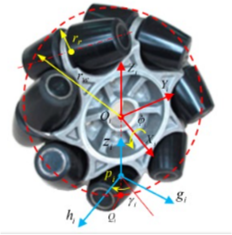
Holonomic robots kinematics – Example

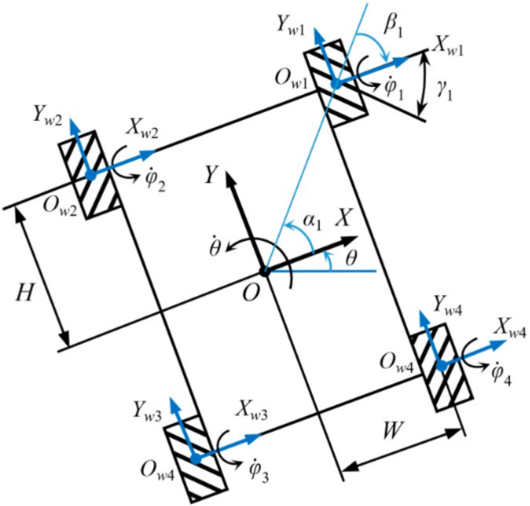
The full kinematic model is

$$\begin{bmatrix} \dot{x} \\ \\ \dot{y} \\ \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{r}{6}\left(\left(\sqrt{3}\mathrm{c}_\theta - \mathrm{s}_\theta\right)\omega_3 - \left(\sqrt{3}\mathrm{c}_\theta + \mathrm{s}_\theta\right)\omega_2 + 2\mathrm{s}_\theta\omega_1\right) \\ \\ \frac{r}{6}\left(\left(\sqrt{3}\mathrm{s}_\theta + \mathrm{c}_\theta\right)\omega_3 - \left(\sqrt{3}\mathrm{s}_\theta - \mathrm{c}_\theta\right)\omega_2 - 2\mathrm{c}_\theta\omega_1\right) \\ \\ \frac{r}{3L}\left(\omega_1 + \omega_2 + \omega_3\right) \end{bmatrix}$$

TÉCNICO LISBOA

An example with Mecanum wheels





Source: "Kinematic Modeling of a Combined System of Multiple Mecanum-Wheeled Robots with Velocity Compensation". Yunwang Li, Shirong Ge, Sumei Dai, Lala Zhao, Xucong Yan, Yuwei Zheng, Yong Shi. MDPI Sensors, 2019

Note that the straight omni wheels would not work in the rectangular arrangement

Sphere: A Cool Non-Holonomic Robot

Source: http://www.gosphero.com/photos-videos/photos/

Tomoki Ohsawa
Geometric Kinematic Control of a Spherical Rolling Robot
Journal of Nonlinear Science (2020) 30:67-91
https://doi.org/10.1007/s00332-019-09568-x