# Task 3: Reconstructing the Trajectory in Cartesian Space from Accelerometer Data

To reconstruct the trajectory of the sensor, we need to numerically integrate the accelerometer data twice: once to get velocity and once again to get position. We assume the accelerometer data is in $\mathrm{m/s^2}$ after conversion.

Step 1: Numerical Integration of Acceleration to Obtain Velocity

First, we integrate the acceleration data $a_x(t), a_y(t), a_z(t)$ along each axis to get the velocity components:

$$v_x(t) = \int a_x(t)\,dt$$

$$v_y(t) = \int a_y(t)\,dt$$

$$v_z(t) = \int a_z(t)\,dt$$

The velocity is computed numerically using discrete time steps:

$$v_x(t) = \sum_{i=1}^{N} a_x(t_i) \cdot \Delta t$$

Where $\Delta t$ is the time difference between measurements.

Step 2: Numerical Integration of Velocity to Obtain Position

Next, we integrate the velocity components to get the position components:

$$x(t) = \int v_x(t)\,dt$$

$$y(t) = \int v_y(t)\,dt$$

$$z(t) = \int v_z(t)\,dt$$

We use the same numerical integration technique to calculate the position:

$$x(t) = \sum_{i=1}^{N} v_x(t_i) \cdot \Delta t$$

# Task 4: Reconstructing the Trajectory in Orientation Space from Rate-Gyro Data

The rate-gyro provides angular velocities along the X, Y, and Z axes, denoted by $\omega_x(t)$, $\omega_y(t)$, and $\omega_z(t)$, respectively. To reconstruct the orientation of the sensor, we integrate the angular velocities to obtain the Euler angles, which describe the sensor's orientation in 3D space.

Step 1: Euler Angles (Roll, Pitch, Yaw) The Euler angles are three angles that represent the orientation of a rigid body in 3D space. In this case, we are using three angles: **roll** ($\alpha$), **pitch** ($\beta$), and **yaw** ($\gamma$).

The angular velocities ($\omega_x$, $\omega_y$, and $\omega_z$) are the time derivatives of the Euler angles:

$$\alpha(t) = \int \omega_x(t)\, dt$$

$$\beta(t) = \int \omega_y(t)\, dt$$

$$\gamma(t) = \int \omega_z(t)\, dt$$

Where: - $\alpha(t)$ is the **roll** angle, representing rotation around the X-axis. - $\beta(t)$ is the **pitch** angle, representing rotation around the Y-axis. - $\gamma(t)$ is the **yaw** angle, representing rotation around the Z-axis.

Step 2: Numerical Integration To obtain the Euler angles, we need to integrate the angular velocities numerically. Since the angular velocities are typically measured at discrete time intervals, we perform a **numerical integration** (e.g., using the **cumulative sum** or the trapezoidal rule) for each axis:

$$\alpha(t) = \sum_{i=1}^{N} \omega_x(t_i)\Delta t$$

$$\beta(t) = \sum_{i=1}^{N} \omega_y(t_i)\Delta t$$

$$\gamma(t) = \sum_{i=1}^{N} \omega_z(t_i)\Delta t$$

Where: - $\omega_x(t)$, $\omega_y(t)$, and $\omega_z(t)$ are the angular velocities along the X, Y, and Z axes, respectively. - $\Delta t$ is the time difference between successive measurements.

Step 3: Conversion to Euler Angles Once the angular velocities are integrated over time, the Euler angles $\alpha(t)$, $\beta(t)$, and $\gamma(t)$ describe the orientation of the sensor at each time step.

$$\alpha(t) = \int \omega_x(t)\, dt \quad \text{(Roll)}$$

$$\beta(t) = \int \omega_y(t)\, dt \quad \text{(Pitch)}$$

$$\gamma(t) = \int \omega_z(t)\, dt \quad \text{(Yaw)}$$

The integration process assumes that the initial orientation is known or assumed to be zero at $t = 0$, depending on the context.