Lecture 8: Recurrent Neural Networks

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Today's Roadmap

Today we'll cover neural models for sequences:

- Recurrent neural networks.
- Backpropagation through time.
- Neural language models.
- The vanishing gradient problem.
- Gated units: LSTMs and GRUs.
- Bidirectional LSTMs.
- Example: ELMO representations.
- From sequences to trees: recursive neural networks.
- Other deep auto-regressive models: PixelRNNs.

Outline

Recurrent Neural Networks

Much interesting data is sequential in nature:

- ✓ Words in text
- ✓ DNA sequences
- ✓ Stock market returns
- ✓ Samples of sound signals
- ✓ ...

How to deal with sequences of arbitrary length?

Feed-forward vs Recurrent Networks

- Feed-forward neural networks:

$$\begin{aligned} \boldsymbol{h} &= \boldsymbol{g}(\boldsymbol{V}\boldsymbol{x} + \boldsymbol{c}) \\ \widehat{\boldsymbol{y}} &= \boldsymbol{W}\boldsymbol{h} + \boldsymbol{b} \end{aligned}$$

Feed-forward vs Recurrent Networks

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- Recurrent neural networks (RNN) (Elman, 1990):

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\boldsymbol{h}_t &= \boldsymbol{g}(\boldsymbol{V}\boldsymbol{x}_t + \boldsymbol{U}\boldsymbol{h}_{t-1} + \boldsymbol{c}) \\
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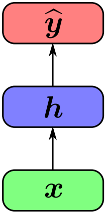
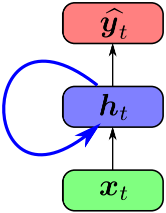
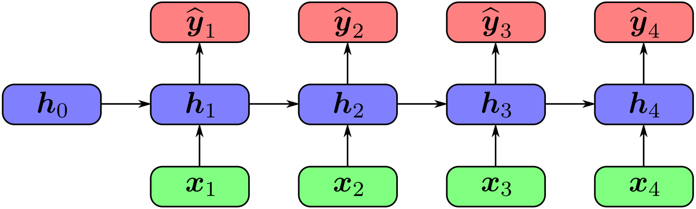




- An RNN is a dynamical system

Unrolling the Graph



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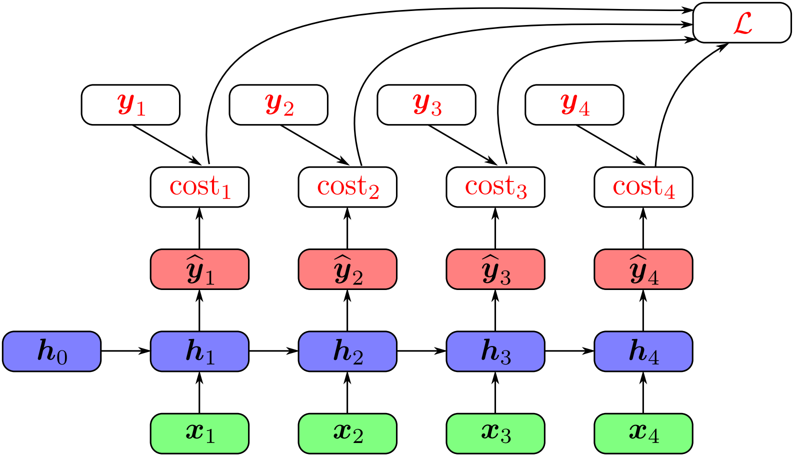
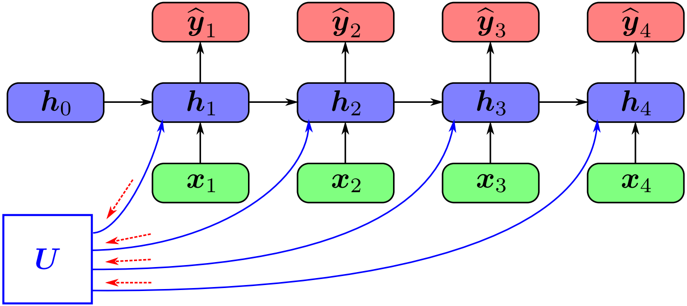


How do We Train the RNN Parameters?

- The unrolled graph is a correct (directed and acyclic) computation graph: gradient backpropagation can be used
- Parameters are tied/shared accross “time”
- Derivatives are aggregated across time steps
- This is called backpropagation through time (BPTT).

Parameter Tying



$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{U}} = \sum_{t=1}^{4} \frac{\partial \boldsymbol{h}_t}{\partial \boldsymbol{U}} \frac{\partial \mathcal{L}}{\partial \boldsymbol{h}_t}$$

- Same idea as when learning the filters in convolutional neural networks

Three Standard Applications of RNNs

1. **Sequence generation:** generates symbols sequentially with an auto-regressive model (e.g. language modeling).
2. **Sequence tagging:** takes a sequence as input, and returns a label for every element in the sequence; e.g., *part of speech* (POS) tagging.
3. **Pooled classification:** takes a sequence as input, and returns a single label by pooling the RNN states; e.g., text classification.

Outline

Recap: Full History Model

$$\mathbb{P}(\text{START}, y_1, y_2, \ldots, y_L, \text{STOP}) = \prod_{t=1}^{L+1} \mathbb{P}(y_t | y_{t-1}, \ldots, y_0)$$

- Generating each word depends on all the previous words.
- Huge expressive power!

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- But, too many parameters to estimate! (quiz: how many?)
- ... thus, may not generalize well, specially for long sequences.

Can We Have Unlimited Memory?

- Markov models avoid the full history by using limited memory:

$$\mathbb{P}(y_t \mid \underbrace{y_{t-1}, \ldots, y_0}_{\text{grows with } t}) = \mathbb{P}(y_t \mid \underbrace{y_{t-1}, \ldots, y_{t-m}}_{\text{fixed length}}),$$

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- RNNs do this!

Auto-Regressive Models

Key ideas:

- Feed the previous output as input to the current step:

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Let's see each of these steps in detail

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p(y_t = i | y_{t-1}, \dots, y_0) &= \frac{\exp((\boldsymbol{z}_t)_i)}{\sum_j \exp((\boldsymbol{z}_t)_j)} \\
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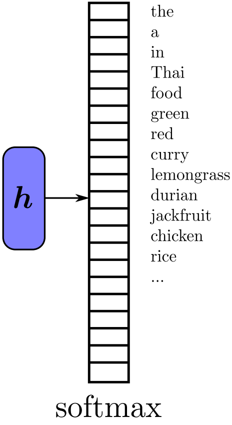

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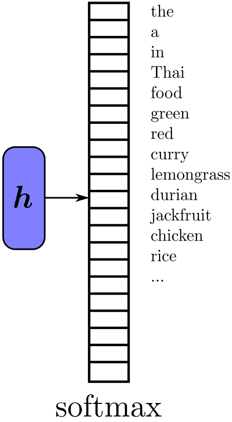
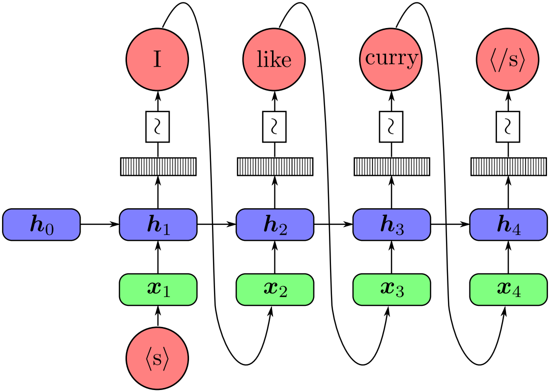


Language Modeling: Auto-Regression



$$
\begin{aligned}
\mathbb{P}(y_1, \ldots, y_L) &= \mathbb{P}(y_1) \times \mathbb{P}(y_2 \mid y_1) \times \ldots \times \mathbb{P}(y_L \mid y_{L-1}, \ldots, y_1) \\
&= (\mathrm{softmax}(\boldsymbol{W}\boldsymbol{h}_1 + \boldsymbol{b}))_{y_1} \times (\mathrm{softmax}(\boldsymbol{W}\boldsymbol{h}_2 + \boldsymbol{b}))_{y_2} \times \ldots \\
&\quad \times (\mathrm{softmax}(\boldsymbol{W}\boldsymbol{h}_L + \boldsymbol{b}))_{y_L}
\end{aligned}
$$

Three Problems for Sequence-Generating RNNs

Algorithms are needed for:

- Sampling sequences from the probability distribution the RNN defines.
- Obtaining the most probable sequence.
- Training the RNN (learning $\boldsymbol{W}, \boldsymbol{U}, \boldsymbol{V}, \boldsymbol{b}, \boldsymbol{c}$)

Sampling a Sequence

This is easy!

(Notation: $\boldsymbol{y}_t$ is the embedding of word y_t)

- Compute $\boldsymbol{h}_1$ from $\boldsymbol{x}_1 = \mathrm{START}$;
- Sample $y_1 \sim \mathrm{softmax}(\boldsymbol{W}\boldsymbol{h}_1 + \boldsymbol{b})$;
- Compute $\boldsymbol{h}_2$ from $\boldsymbol{h}_1$ and $\boldsymbol{x}_2 = \boldsymbol{y}_1$;
- Sample $y_2 \sim \mathrm{softmax}(\boldsymbol{W}\boldsymbol{h}_2 + \boldsymbol{b})$;
- And so on ...

Obtaining the Most Probable Sequence

Unfortunately, this is hard!

- Find the sequence $y_1, y_2, \ldots$ that jointly maximize the product

$$(\mathrm{softmax}(\boldsymbol{W}\boldsymbol{h}_1 + \boldsymbol{b}))_{y_1} \times (\mathrm{softmax}(\boldsymbol{W}\boldsymbol{h}_2 + \boldsymbol{b}))_{y_2} \times \ldots$$

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- More later, when discussing sequence-to-sequence models.

Training the RNN

- Sequence-generating RNNs are typically trained with maximum likelihood estimation.

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- In other words, training uses the log-loss (cross-entropy):

$$\mathcal{L}(\Theta; y_1, ..., y_L) = -\frac{1}{L+1}\sum_{t=1}^{L+1} \log \mathbb{P}_\Theta(y_t \mid y_0, \ldots, y_{t-1})$$

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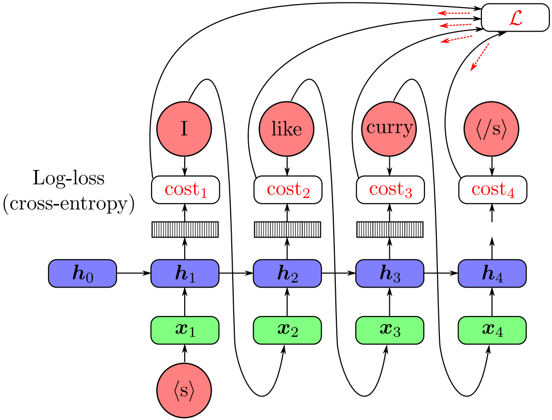
  where $\Theta = (\boldsymbol{W}, \boldsymbol{U}, \boldsymbol{V}, \boldsymbol{b}, \boldsymbol{c})$

- This is equivalent to minimizing perplexity: $\exp(\mathcal{L}(\Theta, y_{1:L}))$

- Intuition: $-\log \mathbb{P}_\Theta(y_t \mid y_0, \ldots, y_{t-1})$

  measures how "perplexed" (or "surprised") the model is when the t-th word is revealed

Training the RNN



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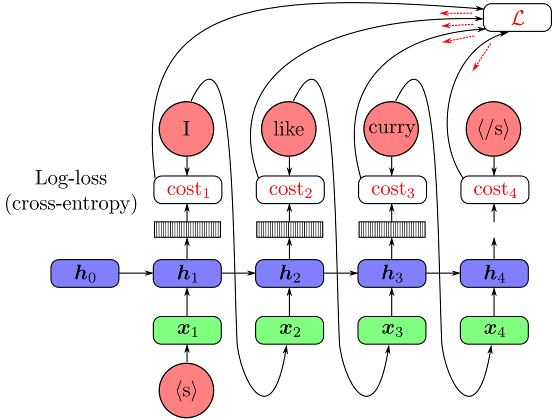
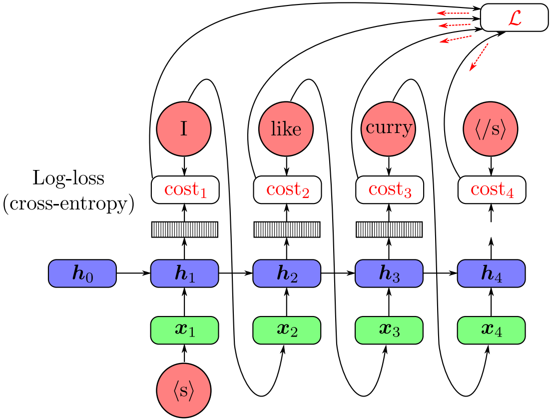


- Unlike Markov (*n*-gram) models, RNNs never forget!

Training the RNN



- Unlike Markov (n-gram) models, RNNs never forget!
- However, they may struggle to learn to use their memories (more soon...)

Teacher Forcing and Exposure Bias

Conditioning is on the **true history**, not on the model predictions!

This is known as teacher forcing.

Teacher forcing causes exposure bias: at run time, the model has trouble recovering from mistakes, as it generates histories never seen in training.

Character-Level Language Models

We can also have an RNN over characters instead of words!

Advantage 1: can generate any combination of characters, not just words in a fixed vocabulary.

Advantage 2: much smaller set of output symbols.

Disadvantage: need to remember much deeper back in history!

A Character-Level RNN Generating Fake Shakespeare

PANDARUS: Alas, I think he shall be come approached and the day When little srain would be attain'd into being never fed, And who is but a chain and subjects of his death, I should not sleep.

Second Senator: They are away this miseries, produced upon my soul, Breaking and strongly should be buried, when I perish The earth and thoughts of many states.

DUKE VINCENTIO: Well, your wit is in the care of side and that.

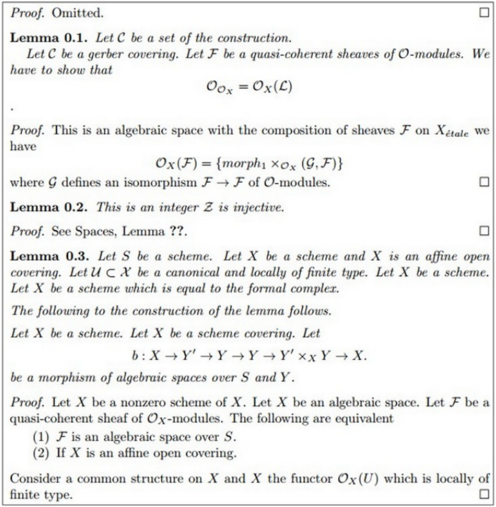
Second Lord: They would be ruled after this chamber, and my fair nues begun out of the fact, to be conveyed, Whose noble souls I'll have the heart of the wars.

Clown: Come, sir, I will make did behold your worship.

VIOLA: I'll drink it.

(Credits: Andrej Karpathy)

A Char-Level RNN Generating a Math Paper



Proof. Omitted. □

Lemma 0.1. *Let $\mathcal{C}$ be a set of the construction.*

Let $\mathcal{C}$ be a gerber covering. Let $\mathcal{F}$ be a quasi-coherent sheaves of $\mathcal{O}$-modules. We have to show that

$$\mathcal{O}_{\mathcal{O}_X} = \mathcal{O}_X(\mathcal{L})$$

.

Proof. This is an algebraic space with the composition of sheaves $\mathcal{F}$ on $X_{étale}$ we have

$$\mathcal{O}_X(\mathcal{F}) = \{morph_1 \times_{\mathcal{O}_X} (\mathcal{G}, \mathcal{F})\}$$

where $\mathcal{G}$ defines an isomorphism $\mathcal{F} \to \mathcal{F}$ of $\mathcal{O}$-modules. □

Lemma 0.2. *This is an integer $\mathcal{Z}$ is injective.*

Proof. See Spaces, Lemma **??**. □

Lemma 0.3. *Let S be a scheme. Let X be a scheme and X is an affine open covering. Let $\mathcal{U} \subset \mathcal{X}$ be a canonical and locally of finite type. Let X be a scheme. Let X be a scheme which is equal to the formal complex.*

The following to the construction of the lemma follows.

Let X be a scheme. Let X be a scheme covering. Let

$$b : X \to Y' \to Y \to Y \to Y' \times_X Y \to X.$$

be a morphism of algebraic spaces over S and Y.

Proof. Let X be a nonzero scheme of X. Let X be an algebraic space. Let $\mathcal{F}$ be a quasi-coherent sheaf of $\mathcal{O}_X$-modules. The following are equivalent

(1) $\mathcal{F}$ is an algebraic space over S.
(2) If X is an affine open covering.

Consider a common structure on X and X the functor $\mathcal{O}_X(U)$ which is locally of finite type. □

(Credits: Andrej Karpathy)

A Char-Level RNN Generating C++ Code

```
/*
 * Increment the size file of the new incorrect UI_FILTER group information
 * of the size generatively.
 */
static int indicate_policy(void)
{
  int error;
  if (fd == MARN_EPT) {
    /*
     * The kernel blank will coeld it to userspace.
     */
    if (ss->segment < mem_total)
      unblock_graph_and_set_blocked();
    else
      ret = 1;
    goto bail;
  }
  segaddr = in_SB(in.addr);
  selector = seg / 16;
  setup_works = true;
  for (i = 0; i < blocks; i++) {
    seq = buf[i++];
    bpf = bd->bd.next + i * search;
    if (fd) {
      current = blocked;
    }
  }
  rw->name = "Getjbbregs";
  bprm_self_clearl(&iv->version);
  regs->new = blocks[(BPF_STATS << info->historidac)] | PFMR_CLOBATHINC_SECONDS << 12;
  return segtable;
}
```

(Credits: Andrej Karpathy)

Note: these examples are from $6 \sim 7$ years ago.

There are now much better language generators/models;

e.g., GPT-3, GPT-3.5, GPT-4, Llamma, Gemma, Gemini, Mistral, EuroLLM, and 1000s of others.

Instead of RNNs, the current language generators use transformers.

We will cover transformers in a later lecture!

Three Applications of RNNs

1. **Sequence generation:** generates symbols sequentially with an auto-regressive model; e.g., language modeling; ✓
2. **Sequence tagging:** takes a sequence as input, and returns a label for every element in the sequence; e.g., part of speech (POS) tagging;
3. **Pooled classification:** takes a sequence as input, and returns a single label by pooling the RNN states; e.g., sequence classification.

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Sequence Tagging with RNNs

- In sequence tagging, we are given an input sequence $x_1, \ldots, x_L$,
- ... the goal is to assign a tag to each element of the sequence, yielding an output sequence $y_1, \ldots, y_L$.

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- **Examples:** POS tagging, named entity recognition

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- **Examples:** POS tagging, named entity recognition
- Differences with respect to sequence generation:
  - The input and output are distinct (no need for auto-regression)
  - The length of the output is known (same as that of the input)

Example: POS Tagging

- Map **sentences** to sequences of **part-of-speech tags.**

Time	flies	like	an	arrow	.
noun	verb	prep	det	noun	.

Example: POS Tagging

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Time	flies	like	an	arrow	.
noun	verb	prep	det	noun	.

- Need to predict a morphological tag for each word of the sentence

- High correlation between adjacent words!
  (Ratnaparkhi, 1999; Brants, 2000; Toutanova et al., 2003)

An RNN-Based POS Tagger

- The inputs $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_L \in \mathbb{R}^{E \times L}$ are word embeddings (found by looking up rows in an $V \times E$ embedding matrix, possibly pre-trained).

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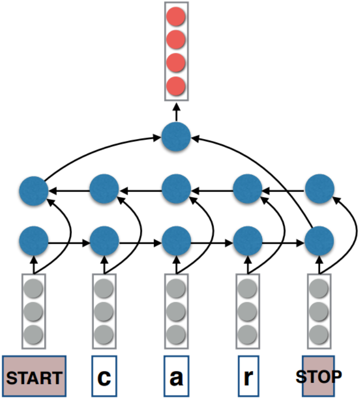
- This model can be improved, e.g., by using bidirectionality (next)

Bidirectional RNNs

- We can read a sequence from left to right to obtain a representation
- Or we can read it from right to left
- Or we can read it in both directions and combine the representations
- More later...



(Slide credit: Chris Dyer)

Example: Named Entity Recognition

From **sentences** extract **named entities.**

- Identify segments referring to entities (person, organization, location)
- Typically done with sequence models and **B-I-O** tagging:
  - ✓ B = Beginning; I = Inside; O = Other
  - ✓ PER = Person; LOC = Location; ORG = Organization

Example:

Louis	Elsevier	was	born	in	Leuven	.
B-PER	I-PER	O	O	O	B-LOC	.

(Zhang and Johnson, 2003; Ratinov and Roth, 2009)

RNN-Based NER

- The model we described for POS tagging works just as well for NER
- However, NER has constraints about tag transitions: e.g., we cannot have I-PER after B-LOC
- The RNN tagger model we described exploits input structure (via the states encoded in the recurrent layer) but lacks output structure...

Three Applications of RNNs

1. **Sequence generation:** generates symbols sequentially with an auto-regressive model (e.g. language modeling) ✓

2. **Sequence tagging:** takes a sequence as input, and returns a label for every element in the sequence (e.g., POS tagging) ✓

3. **Pooled classification:** takes a sequence as input, and returns a single label by pooling the RNN states.

Outline

Pooled Classification

- What we have seen so far assumes we want to output a sequence of labels (either to generate tags a full sequence).
- What about predicting a single label for the whole sequence?

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- What about predicting a single label for the whole sequence?
- We can still use an RNN to capture the input sequential structure.
- Just pool the RNNs states, *i.e.*, map them to a single vector.
- Use a single softmax to output the final label.

Pooling Strategies

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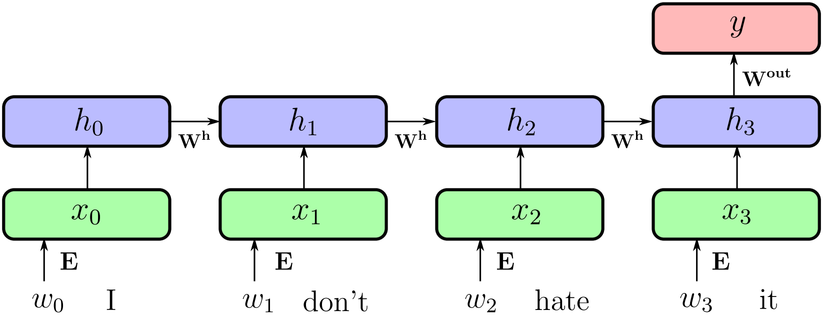
Pooling Strategies

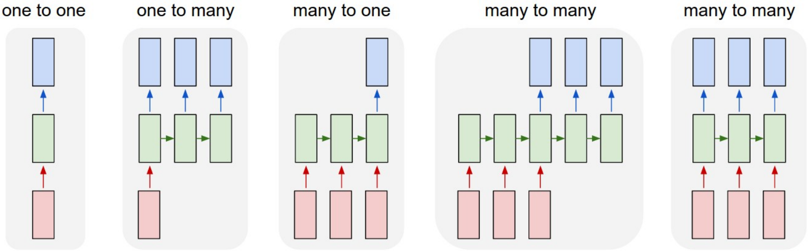
- The simplest strategy is just to use the last RNN state.
- This state results from traversing the full sequence left-to-right, hence it has information about the whole sequence.
- Disadvantage: for long sequences, the influence the earliest words may vanish
- Other pooling strategies:
  - ✓ Use a bidirectional RNN and combine both last states of the left-to-right and right-to-left RNN.
  - ✓ Average pooling.
  - ✓ Others...

Example: Sentiment Analysis



(Slide credit: Ollion & Grisel)

Recurrent Neural Networks are Very Versatile



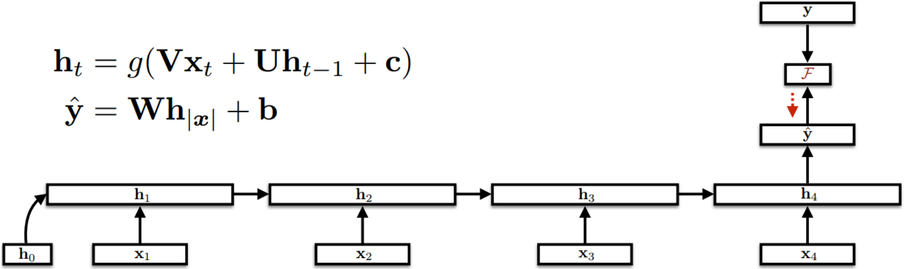
See Andrej Karpathy's blog post: "The Unreasonable Effectiveness of Recurrent Neural Networks"
(`http://karpathy.github.io/2015/05/21/rnn-effectiveness/`).

Outline

Training the RNN: Backpropagation Through Time

What happens to the gradients as we go back in time?

$$\mathbf{h}_t = g(\mathbf{V}\mathbf{x}_t + \mathbf{U}\mathbf{h}_{t-1} + \mathbf{c})$$
$$\hat{\mathbf{y}} = \mathbf{W}\mathbf{h}_{|\boldsymbol{x}|} + \mathbf{b}$$



(Slide credit: Chris Dyer)

Backpropagation Through Time

What happens to the gradients as we go back in time?

$$\frac{\partial \mathcal{F}}{\partial \boldsymbol{h}_1} = \underbrace{\frac{\partial \boldsymbol{h}_2}{\partial \boldsymbol{h}_1}\frac{\partial \boldsymbol{h}_3}{\partial \boldsymbol{h}_2}\frac{\partial \boldsymbol{h}_4}{\partial \boldsymbol{h}_3}}_{\prod_{t=2}^{4} \frac{\partial \boldsymbol{h}_t}{\partial \boldsymbol{h}_{t-1}}}\frac{\partial \widehat{\boldsymbol{y}}}{\partial \boldsymbol{h}_4}\frac{\partial \mathcal{F}}{\partial \widehat{\boldsymbol{y}}}$$

where (with $\boldsymbol{z}_t$ denoting the pre-activation)

$$\prod_t \frac{\partial \boldsymbol{h}_t}{\partial \boldsymbol{h}_{t-1}} = \prod_t \frac{\partial \boldsymbol{h}_t}{\partial \boldsymbol{z}_t}\frac{\partial \boldsymbol{z}_t}{\partial \boldsymbol{h}_{t-1}} = \prod_t \mathrm{Diag}(\boldsymbol{g}'(\boldsymbol{z}_t))\boldsymbol{U}$$

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Three cases:

- largest eigenvalue of $\boldsymbol{U}$ exactly 1: gradient propagation is stable
- largest eigenvalue of $\boldsymbol{U} < 1$: gradient vanishes (exponential decay)
- largest eigenvalue of $\boldsymbol{U} > 1$: gradient explodes (exponential growth)

Vanishing and Exploding Gradients

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Vanishing and Exploding Gradients

- **Exploding gradients** can be dealt with by gradient clipping (truncating the gradient if it exceeds some magnitude)
- **Vanishing gradients** are more frequent and harder to mitigate. In practice: long-range dependencies are difficult to learn
- **Solutions:**
  - Better optimizers (second order methods).
  - Normalization to keep the gradient norms stable across time.
  - Clever initialization to start with good spectra (e.g., start with random orthonormal matrices).
  - Alternative parameterizations: LSTMs and GRUs.

Gradient Clipping

- **Norm clipping:**

$$\tilde{\nabla} \leftarrow \begin{cases} \frac{c}{\|\nabla\|}\nabla & \text{if } \|\nabla\| \geq c \\ \nabla & \text{otherwise.} \end{cases}$$

- **Elementwise clipping:**

$$\tilde{\nabla}_i \leftarrow \min\{c, |\nabla_i|\} \times \text{sign}(\nabla_i), \ \forall i$$

Alternative RNNs

- Gated recurrent unit (GRU)
  (Cho et al., 2014)

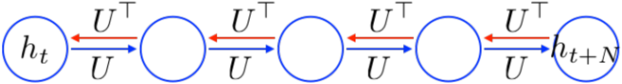
- Long short-term memorie (LSTM)
  (Hochreiter and Schmidhuber, 1997)

Intuition: instead of multiplying across time (which leads to exponential growth), we want the error to be approximately constant

They solve the vanishing gradient problem, but still have exploding gradients (still need gradient clipping)

Gated Recurrent Units (Cho et al., 2014)

- Recall the problem: the error must backpropagate through all the intermediate nodes:



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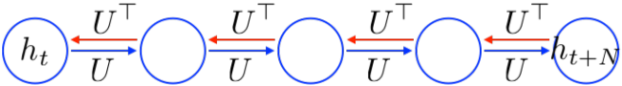
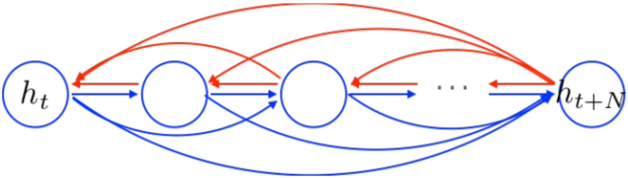


- **Idea:** create some kind of shortcut connections:



(Image credit: Thang Luong, Kyunghyun Cho, Chris Manning)

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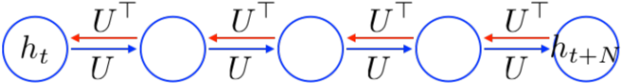

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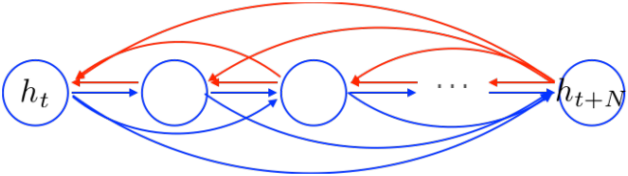
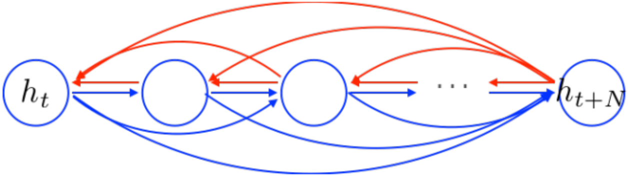


(Image credit: Thang Luong, Kyunghyun Cho, Chris Manning)

- Create adaptive shortcuts controlled by special gates

Gated Recurrent Units (Cho et al., 2014)



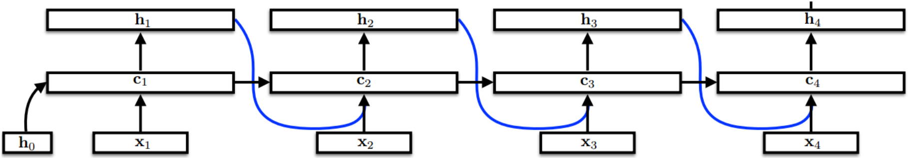
(Image credit: Thang Luong, Kyunghyun Cho, Chris Manning)

$$\boxed{\boldsymbol{h}_t = \boldsymbol{u}_t \odot \tilde{\boldsymbol{h}}_t + (1 - \boldsymbol{u}_t) \odot \boldsymbol{h}_{t-1}}$$

- **Candidate update:** $\tilde{\boldsymbol{h}}_t = \boldsymbol{g}(\boldsymbol{V}\boldsymbol{x}_t + \boldsymbol{U}(\boldsymbol{r}_t \odot \boldsymbol{h}_{t-1}) + \boldsymbol{b})$
- **Reset gate:** $\boldsymbol{r}_t = \sigma(\boldsymbol{V}_r\boldsymbol{x}_t + \boldsymbol{U}_r\boldsymbol{h}_{t-1} + \boldsymbol{b}_r)$
- **Update gate:** $\boldsymbol{u}_t = \sigma(\boldsymbol{V}_u\boldsymbol{x}_t + \boldsymbol{U}_u\boldsymbol{h}_{t-1} + \boldsymbol{b}_u)$

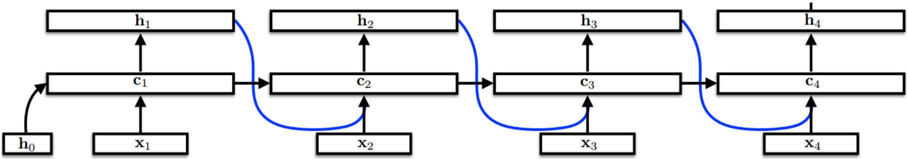
Long Short-Term Memories (Hochreiter and Schmidhuber, 1997)

- **Key idea:** use memory cells $\boldsymbol{c}_t$
- To avoid the multiplicative effect, information flows *additively* through these cells
- Control the flow with special input, forget, and output gates



(Image credit: Chris Dyer)

Long Short-Term Memories

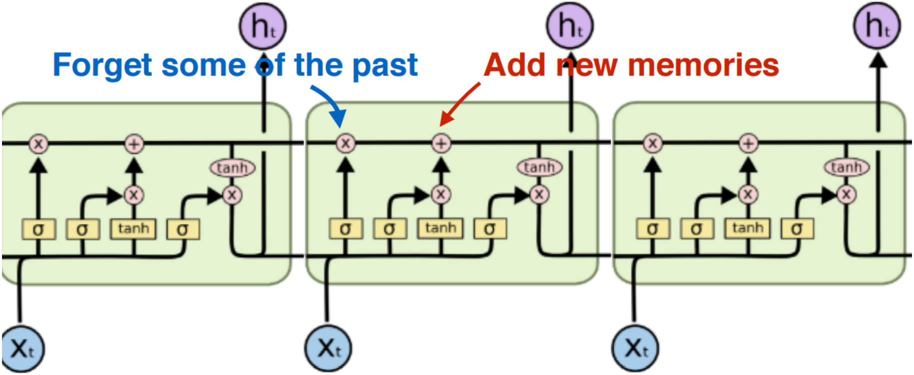


(Image credit: Chris Dyer)

$$\boxed{\boldsymbol{c}_t = \boldsymbol{f}_t \odot \boldsymbol{c}_{t-1} + \boldsymbol{i}_t \odot \boldsymbol{g}(\boldsymbol{V}\boldsymbol{x}_t + \boldsymbol{U}\boldsymbol{h}_{t-1} + \boldsymbol{b}), \qquad \boldsymbol{h}_t = \boldsymbol{o}_t \odot \boldsymbol{g}(\boldsymbol{c}_t)}$$

- **Forget gate:** $\boldsymbol{f}_t = \sigma(\boldsymbol{V}_f \boldsymbol{x}_t + \boldsymbol{U}_f \boldsymbol{h}_{t-1} + \boldsymbol{b}_f)$
- **Input gate:** $\boldsymbol{i}_t = \sigma(\boldsymbol{V}_i \boldsymbol{x}_t + \boldsymbol{U}_i \boldsymbol{h}_{t-1} + \boldsymbol{b}_i)$
- **Output gate:** $\boldsymbol{o}_t = \sigma(\boldsymbol{V}_o \boldsymbol{x}_t + \boldsymbol{U}_o \boldsymbol{h}_{t-1} + \boldsymbol{b}_o)$

Long Short-Term Memories

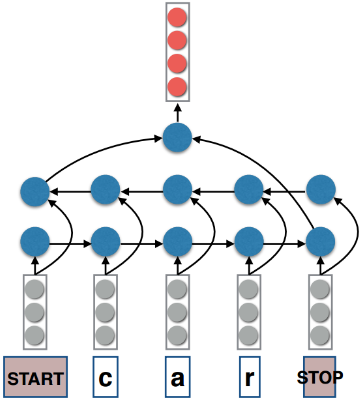


(Slide credit: Christopher Olah)

For a detailed explanation, see
`colah.github.io/posts/2015-08-Understanding-LSTMs/`

Bidirectional LSTMs

- Same idea as a bidirectional RNNs, but with LSTMs instead of standard RNNs.



(Slide credit: Chris Dyer)

LSTMs and BILSTMs: Some Success Stories

- Time series prediction (Schmidhuber et al., 2005)
- Speech recognition (Graves et al., 2013)
- Named entity recognition (Lample et al., 2016)
- Machine translation (Sutskever et al., 2014)
- ELMo (deep contextual) word representations (Peters et al., 2018)
- ... and many others.

Summary

- Better gradient propagation is possible if we use additive rather than multiplicative/highly non-linear recurrent dynamics.
- Other variants of LSTMs exist which tie/simplify some of the gates
- Extensions for *non-sequential* structured inputs/outputs (e.g. trees):
  - ✓ recursive neural networks (Socher et al., 2011),
  - ✓ PixelRNN (Oord et al., 2016).

Outline

Outline

From Sequences to Trees

- So far we've talked about recurrent neural networks, designed to capture sequential structure.

- What about other kinds of structure? For example, trees?

- It is also possible to handle these structures with recursive computation, via RNNs.

Recursive Neural Networks

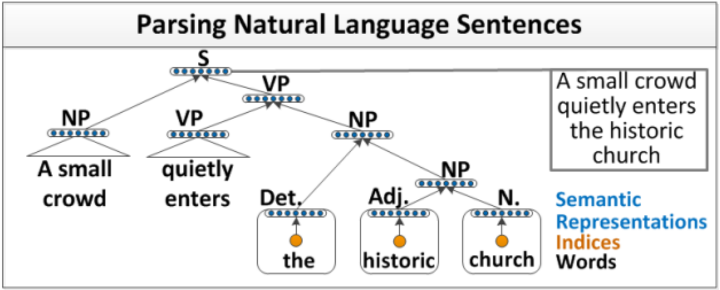
- Proposed by Socher et al. (2011) for parsing images and text.
- Assume a binary tree (each node except the leaves has two children).
- Propagate states bottom-up in the tree, computing the parent state $\boldsymbol{p}$ from the children states $\boldsymbol{c}_1$ and $\boldsymbol{c}_2$:

$$\boldsymbol{p} = \tanh\left(\boldsymbol{W}\begin{bmatrix} \boldsymbol{c}_1 \\ \boldsymbol{c}_2 \end{bmatrix} + \boldsymbol{b}\right)$$

- Use the same parameters $\boldsymbol{W}$ and $\boldsymbol{b}$ at all nodes.
- Can compute scores at the root or at each node by appending a softmax output layer at these nodes.

Compositionality in Text

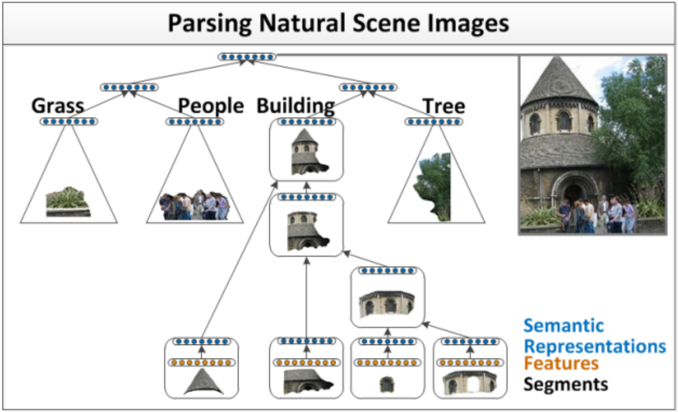
Uses a recurrent net to build a bottom-up parse tree for a sentence.



(Credits: Socher et al. (2011))

Compositionality in Images

Same idea for images.



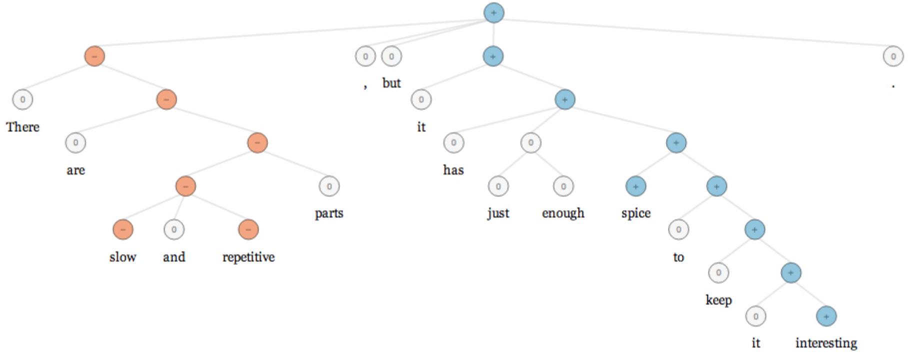
(Credits: Socher et al. (2011))

Tree-LSTMs

- Extend recursive neural networks the same way LSTMs extend RNNs, with a few more gates to account for the left and right child.

- Extensions exist for non-binary trees.

Fine-Grained Sentiment Analysis



(Taken from Stanford Sentiment Treebank.)

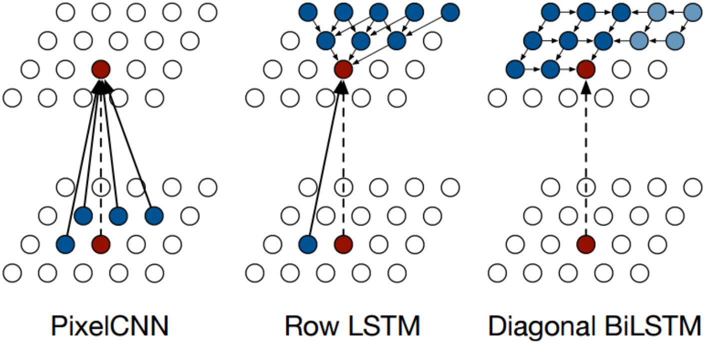
Outline

What about Images?

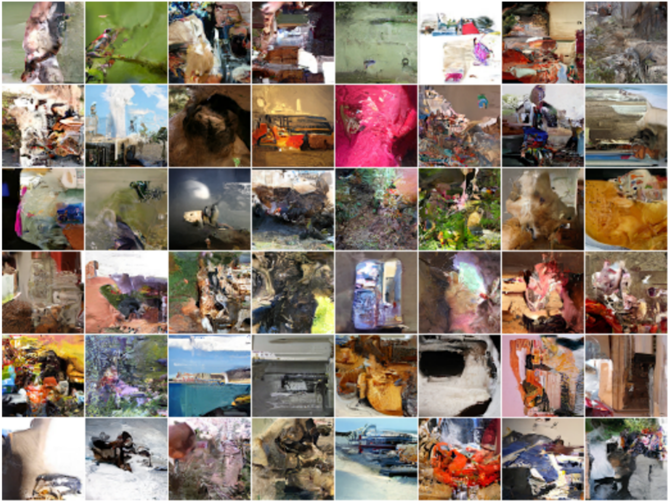
- While sequences are 1D, images are 2D.
- PixelRNNs are 2D extensions of RNNs.
- They can be used as auto-regressive models to generate images, i.e., pixels in a particular order, conditioning on neighboring pixels.
- Several variants...

RNNs for Generating Images

- Input-to-state and state-to-state mappings for PixelCNN and two PixelRNN models (Oord et al., 2016):

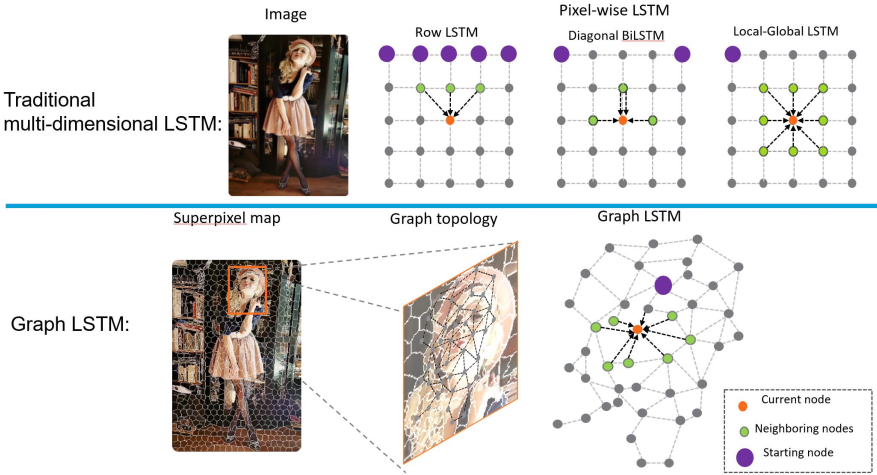


RNNs for Generating Images



(Oord et al., 2016)

Even More General: Graph LSTMs



(Credits: Xiaodan Liang)

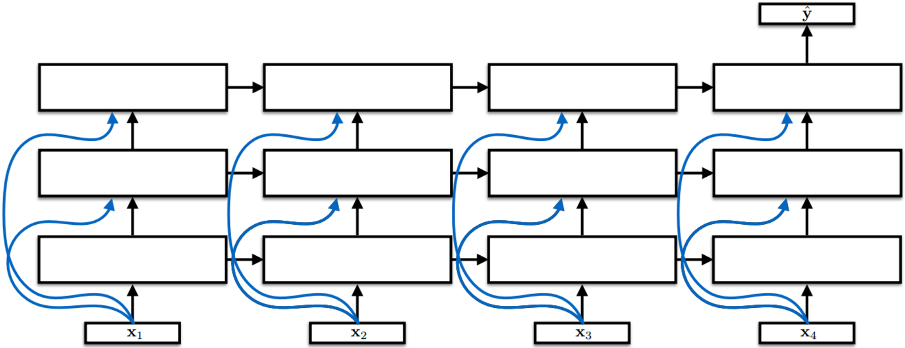
Outline

More Tricks of the Trade

- Depth
- Dropout
- Implementation tricks
- Mini-batching

Deep RNNs/LSTMs/GRUs

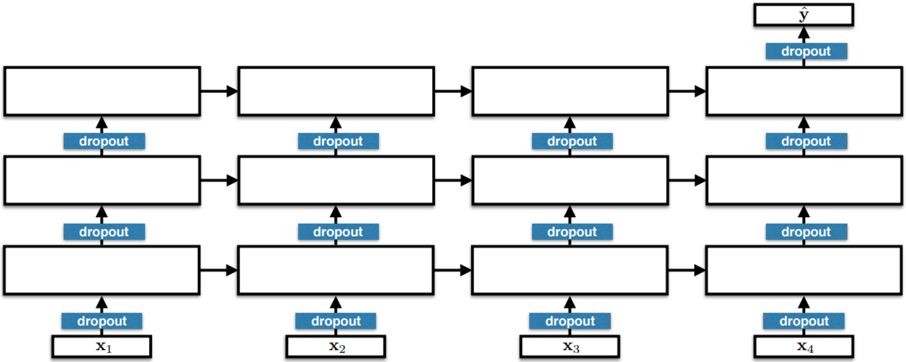
- Depth in recurrent layers helps in practice ($2 \sim 8$ layers are standard)
- Input connections may or may not be used



(Slide credit: Chris Dyer)

Dropout in Deep RNNs/LSTMs/GRUs

- Apply dropout between layers, but not on the recurrent connections
- ... or use the same mask for all recurrent connections (Gal and Ghahramani, 2015)



(Slide credit: Chris Dyer)

Implementation Tricks

For speed:

- Use diagonal matrices instead of full matrices (esp. for gates).
- Concatenate parameter matrices for all gates and do a single matrix-vector multiplication.
- Use optimized implementations (e.g., from NVIDIA).
- Use GRUs or reduced-gate variant of LSTMs.

For learning speed and performance:

- Initialize so that the bias on the forget gate is large (intuitively: at the beginning of training, the signal from the past is unreliable).
- Use random orthogonal matrices to initialize the square matrices.

Mini-Batching

- RNNs, LSTMs, GRUs all consist of many element-wise operations (addition, multiplication, nonlinearity), and many matrix-vector products.

- Mini-batching: convert many matrix-vector products into a single matrix-matrix multiplication.

- Batch across instances, not across time.

- The challenge with working with mini batches of sequences is that sequences are of different lengths.

- This usually means you bucket training instances based on similar lengths, and pad with zeros.

- Be careful when padding not to back propagate a non-zero value!

Outline

Conclusions

- Recurrent neural networks take advantage of sequential input structure.
- They can be used to generate, tag, and classify sequences, and are trained with backpropagation through time.
- Standard RNNs suffer from vanishing and exploding gradients.
- LSTMs and other gated units are more complex variants of RNNs that avoid vanishing gradients.
- They can be extended to other structures, *e.g.*, trees, images, graphs.

Thank you!

Questions?

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