2024 IEEE International Conference on
Autonomous Robot Systems and Competitions (ICARSC)
Paredes de Coura, Portugal – May 2-3, 2024

2024 IEEE International Conference on Autonomous Robot Systems and Competitions (ICARSC) | 979-8-3503-5234-4/24/$31.00 ©2024 IEEE | DOI: 10.1109/ICARSC61747.2024.10535925

Cooperative 3D Exploration and Mapping using Distributed Multi-Robot Teams

André Ribeiro
Instituto de Sistemas e Robótica
Instituto Superior Técnico
Lisbon, Portugal
andre.m.ribeiro.2000@tecnico.ulisboa.pt

Meysam Basiri
Instituto de Sistemas e Robótica
Instituto Superior Técnico
Lisbon, Portugal
meysam.basiri@tecnico.ulisboa.pt

***Abstract*—Exploring and mapping unknown environments pose significant challenges in robotics, especially with large and intricate landscapes. In this paper, we present a novel framework for distributed Multi-Robot Systems (MRS) that leverages the 3D movement capabilities of Unmanned Aerial Vehicles (UAVs) equipped with 3D LiDAR sensors for the rapid exploration of unknown environments. Our approach uniquely integrates the strengths of frontier-based exploration and Next-Best-View (NBV) planning, offering a comprehensive strategy for MRS. Through experiments conducted in a simulated environment with up to three UAVs, our exploration planner and cooperative strategy have been validated. The results showcase successful exploration of complex 3D environments using both single and multiple UAVs. Furthermore, they demonstrate a significant reduction in exploration time and individual travel distance when employing multiple robots, thereby leading to optimized battery resource usage. This work contributes to the expanding field of cooperative MRS, offering valuable insights and a robust basis for future investigations.**

***Index Terms*—Multi-Robot Systems, UAVs, Autonomous 3D Exploration, Cooperative Mapping, Frontier-based Exploration, NBV Planning**

I. Introduction

In the field of robotics, the exploration and mapping of unknown environments is a crucial task. This becomes more challenging when the environment is large, complex, or potentially hazardous. Single Robot Systems (SRS) have made significant progress in exploring and mapping applications such as disaster response [1], environmental monitoring [2], inspection [3] and planetary exploration [4]. However, these systems often face limitations in speed, coverage, and resilience. To overcome these limitations, there has been growing interest in the use of Multi-Robot Systems (MRS). By deploying a team of robots, it is possible to explore and map an environment more quickly, thoroughly, and robustly than a SRS could achieve alone [5].

Regarding exploration, two effective strategies are frontier-based exploration (FBE) [6] and Next-Best-View (NBV) planning [7]. FBE directs robots towards the boundary between known and unknown areas, while NBV planning optimizes the information gained from each observation. Our research proposes a hybrid approach that combines these methods for efficient and robust exploration and mapping. However, challenges in MRS exploration, such as coordinating robot movements, data sharing and merging, and dealing with real-world variability, remain.

In this article, we introduce a new framework for MRS equipped with 3D Light Detection and Ranging (LiDAR) sensors. This active, distributed, and dynamic framework is designed for the autonomous 3D exploration and mapping of unknown environments. The key contributions of our work are: 1) Novelty in Approach: We propose a unique 3D exploration and mapping strategy for MRS, combining FBE with NBV planning; 2) Optimization of Resources: Through rigorous experiments in simulated environments, we have validated our approach, demonstrating significant improvements in exploration time, travel distance, and battery resource optimization; 3) Leveraging UAV Features: Our framework uniquely exploits the 3D movement capabilities of Unmanned Aerial Vehicles (UAVs) in 3D environments. These contributions underscore the strength and potential of our proposed framework, paving the way for future research in this domain.

The remainder of this article is organized as follows: Section II reviews recent research and developments in multi-robot exploration and mapping, Section III discusses the proposed framework, Section IV presents the results and analysis of the simulations performed, and Section V concludes the article and summarizes the key points.

II. Related Work

The field of robotic exploration and mapping has been a subject of extensive research over the last several years. This section provides an overview of the key advancements and methodologies that have shaped this field, particularly focusing on SRS, MRS, FBE, NBV planning, and the challenges associated with these strategies.

Regarding 2D SRS exploration, significant progress has been made, leading to the development of advanced methods for constructing structured 2D maps of static physical environments. The exploration problem in this setting is typically addressed using reactive [6], [8] or greedy strategies [9], [10]. These strategies are often employed in environments that can

This work was partially supported by LARSyS funding (DOI: 10.54499/LA/P/0083/2020, 10.54499/UIDP/50009/2020 and 10.54499/UIDB/50009/2020) and Aero.Next project (PRR - C645727867-00000066)

2024 IEEE International Conference on
Autonomous Robot Systems and Competitions (ICARSC)
Paredes de Coura, Portugal – May 2-3, 2024

be perceived as a plane, where the robot can move forward, backward, and turn, but cannot move up or down. Additionally, a common technique employed by SRS is Simultaneous Localization and Mapping (SLAM) [11], [10].

On the other hand, 3D SRS extends the exploration problem to three dimensions, adding the ability to move up and down. This added complexity presents a new set of challenges. Despite these challenges, the field has seen notable advancements, such as the development of precise occupancy grid mapping [12] and autonomous exploration strategies, namely FBE and NBV planning. These strategies are particularly relevant in environments with varying elevations, underwater, or aerial exploration. However, these systems often encounter constraints in terms of speed, coverage, and resilience.

Recognizing these limitations, researchers have turned their attention to MRS. Using a team of robots allows for faster, more thorough, and more robust exploration and mapping. MRS can cover larger areas in less time, provide redundancy in case of failures, and potentially handle more complex tasks. Recent studies have proposed the use of MRS for exploration to reduce mission time and increase scalability [5]. This introduces the challenge of maintaining efficient cooperation among the fleet members while preserving communication. Existing approaches to this challenge can be centralized, with one robot in the fleet responsible for assigning targets [13], or distributed, where each robot selects its own target [14], [15].

On the subject of FBE, first introduced in [6], has been widely adopted due to its simplicity and effectiveness. It involves directing a robot or a team of robots towards the ”frontier” - the boundary between known and unknown areas of the environment. This strategy is particularly effective in a MRS, where each robot can be tasked with exploring a different part of the environment [16]. In the context of autonomous exploration using UAVs, frontier-based approaches continue to be a popular choice [17]. The next best goal in these approaches is typically the closest frontier or the frontier that minimizes the velocity change to maintain a consistently high flight speed. Additionally, the high computational cost associated with frontier exploration using LiDAR scanners is mitigated by utilizing the multi-resolution capabilities of 3D occupancy grid maps, such as the OctoMap data structure, to detect frontiers at a coarse level [12]. This demonstrates the adaptability and efficiency of frontier-based approaches in handling the complexities of 3D environments.

Conversely, NBV planning was first introduced in [18], and has seen more recent developments in the area of 3D exploration [7], [19]. Instead of simply moving towards unknown areas, NBV planning involves evaluating potential viewpoints and choosing the one that is expected to add the most valuable information to the map. This strategy is particularly effective in complex or cluttered environments, where the optimal viewpoints may not be immediately obvious from the frontier areas. NBV approaches aim to determine a minimal sequence of robot sensor viewpoints for complete exploration. These viewpoints, typically sampled near the frontier or randomly, are evaluated for potential information gain.

Several studies have attempted to combine NBV with FBE to enhance the efficiency of the mapping process [20]. In a MRS, this hybrid strategy allows each robot to independently calculate its next best goal, share this information with the team, and coordinate movements to avoid redundant observations [14].

The proposed approach extends this hybrid strategy by leveraging the benefits of a MRS and the 3D mobility of Unmanned Aerial Vehicles (UAVs) for 3D exploration. It involves the use of a local OctoMap and frontier for each robot when they are outside the communication range, and merging of OctoMaps and unification of frontiers within the communication range. The global best frontiers are sequentially assigned to the robots and evaluated using a cost-utility approach. This MRS exploration planner, designed to operate online, expedites the 3D exploration process.

III. Proposed Approach

Our system comprises a team of autonomous robots, each equipped with LiDAR sensors for environmental perception. These robots are capable of communicating with each other through a decentralized network, enabling cooperative behavior. Furthermore, they operate without any pre-existing knowledge of the environment and autonomously explore different regions at the same time. The exploration planner uses a hybrid strategy between FBE and NBV planning by directing each robot toward the frontier point that optimally balances benefit and cost.

The proposed decentralized multi-robot autonomous exploration and mapping framework, executed on robot i, is represented in Fig. 1.

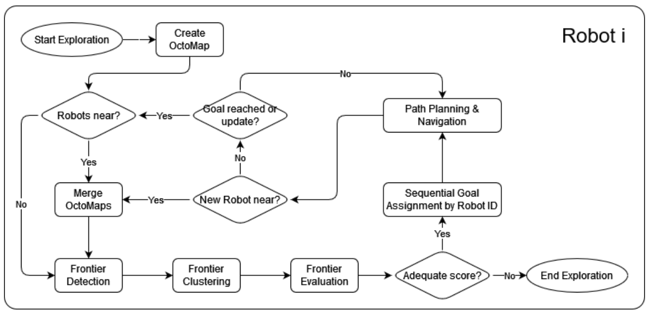

Fig. 1. Proposed framework deployed in robot i.

In the exploration process, each robot begins by creating a local OctoMap. The system then checks if there are any robots within a predefined distance suitable for information sharing.

If there are robots within this distance, their OctoMaps are consolidated to form a more comprehensive map. If not, each OctoMap is processed individually. The system then detects frontier points and clusters them to form frontier candidates. These candidates are evaluated and scored based on their potential for exploration. If the score surpasses a specific threshold, the system proceeds to the decision-making step where sequential goal assignment by Robot ID occurs. If the score is not adequate, the exploration concludes.

Once the goal is assigned, it is relayed to the path planning module, and navigation commences. The process to identify the next waypoint and initiate a new cycle occurs under three conditions: when a new robot enters the communication range, when any robot reaches its current waypoint, or based on the system's update rate which determines how often new cycles begin. This process continues until the exploration of the environment is complete.

Our system assumes each robot has global positioning data, enabling effective distribution across unexplored environments. Our exploration strategy operates independently of the SLAM algorithm used, instead generating an OctoMap from sensor data. This OctoMap creates a coarse map, aids in identifying unexplored areas, and ensures collision-free navigation. Our approach integrates with any suitable path planning and following algorithms [21], [22], emphasizing system adaptability and robustness.

A. OctoMap Creation and Merging

The algorithm under consideration relies on OctoMap, a probabilistic occupancy grid mapping framework predicated on octrees, for the representation of 3D environments [12]. OctoMap uses a grid composed of voxels v, which represent a specific cubic volume in the environment $V \subset \mathbb{R}^3$. The occupancy status of each voxel, either free, occupied, or unknown, defines three distinct subspaces: the free space $V_{free} \subset V$, the occupied space $V_{occ} \subset V$, and the unknown space $V_{un} \subset V$. These subspaces consist of free, occupied, and unknown voxels, respectively. The entirety of the space is the union of these three subspaces, represented as $V \equiv V_{free} \cup V_{occ} \cup V_{un}$.

An environment is fully explored when $V_{un} = \emptyset$. However, areas inaccessible to sensors remain unknown, leading to an occupancy grid that is an approximation of the actual environment, not an exact representation. This approximation is due to sensor limitations and unobservable areas.

Building on this, each robot uses its LiDAR sensor to measure distances to nearby obstacles. These measurements are subsequently used to update each local OctoMap. The grid map is shared among the robots, enabling them to keep track of explored areas and identify new frontiers.

B. Cooperative Strategy

Our cooperative strategy is based in a distributed and sequential waypoint assignment. In this system, each robot, when within a suitable distance of others, shares its unique ID, global position, and individual OctoMap. These local OctoMaps are then shared among all robots within a predefined range, thereby updating their respective local maps. Frontier points are subsequently identified and processed individually.

The waypoint assignment process is independent for each robot, with the sequence determined by the robot's ID. The local frontier candidate list, which contains all frontier candidates from the local map, serves as the source for the next waypoint. Once a waypoint is selected by a robot, it is removed from the candidate list of the remaining UAVs. Additionally, any frontier candidate points within a certain proximity of the selected waypoint are also eliminated to prevent overlap in exploration.

The selection process is based on certain criteria such as proximity, information gain, and previously explored areas, which will be addressed in Section III-E. This approach ensures a balanced distribution of exploration tasks among the robots, prevents the selection of identical or nearby waypoints, and operates without the need for a central controlling entity.

C. Frontier Detection

In this study, the MRS operates within a 3D voxel grid environment, where each voxel, or cell, is denoted by its centroid. A LiDAR scanner, in conjunction with Octomap, is used to label voxels as either unknown, occupied, or free. Based on this labeling, a frontier, F, represents the boundary between known and unexplored regions for the considered OctoMap and is defined in (1) as a set of voxels v_F with the following property, adapted from [20]:

$$\begin{aligned} F = \{v_F \in V_{free} : & \exists neighbor(v_f) \in V_{un} \\ & \wedge \nexists neighbor(v_f) \in V_{occ}\} \end{aligned} \tag{1}$$

In other words, a frontier consists of free voxels with at least one unknown adjacent voxel and none occupied. This approach provides a safety buffer for potential frontier candidates, which serves as a measure for collision avoidance. Additionally, the center of a frontier voxel is often referred to as a frontier point. The computation of frontier points is carried out independently, leveraging the local OctoMap of each robot.

D. Frontier Clustering

In autonomous exploration, clustering algorithms streamline path planning by grouping frontier points. This work proposes employing a mean-shift clustering, first introduced in [23], on frontier points, aggregating nearby ones based on spatial proximity without assuming a fixed number of clusters. This process yields cluster centers, each representing a group of frontiers, and identifies the nearest frontier point to each center as a candidate. The most computationally intensive aspect of mean-shift is identifying a point's 3D neighbors, influenced by the kernel (mean of points within a radius) and bandwidth (radius of the kernel). Selecting an optimal bandwidth is crucial for balancing computational efficiency with desired results, considering factors like environment size and resolution to ensure real-time operation.

E. Frontier Evaluation

After clustering, each robot evaluates the frontier candidates based on their potential information gain and the cost of reaching them. In conventional frontier selection-based methodologies, the selection of the next target can be determined in various ways, such as randomly, or proximity-based. However, in this work, this is done using a strategy similar to the NBV, selecting the frontier that maximizes the expected information gain, given the cost of reaching that view. To this end, an utility function is employed for each robot i, to assess which voxels

2024 IEEE International Conference on
Autonomous Robot Systems and Competitions (ICARSC)
Paredes de Coura, Portugal – May 2-3, 2024

could expedite the exploration process. This utility function, proposed in [20] and adapted from [19], assigns a score to each candidate $v_c \in F_c$ and is defined in (2):

$$S_i\left(v_c\right) = \frac{I\left(v_c\right)}{e^{\lambda L\left(p_i, p_{v_c}\right)}} \tag{2}$$

The positive constant λ balances the importance of robot motion cost against the expected information gain. A smaller λ prioritizes information gain, while a larger λ prioritizes distance. The distance $L(p_i, p_{v_c})$ is the Euclidean distance between the robot's position p_i and p_{v_c}. The information gain $I(v_c)$ estimates the proportion of unknown voxels within a cube centered around v_c, simplifying the computational load compared to ray tracing. The size of the cube is determined relative to the range of the sensor in use.

Each robot calculates this for each frontier candidate and selects as the next waypoint w the one with the highest information gain to cost ratio:

$$w_i = \underset{v_c \in F_c}{\arg\max} S_i(v_c). \tag{3}$$

As previously discussed in III-B, our system uses a cooperative strategy where robots sequentially select and remove their chosen waypoint and any nearby frontier points from the list of the remaining UAVs to prevent overlap in exploration. The frontier selection process is represented in Fig. 2.

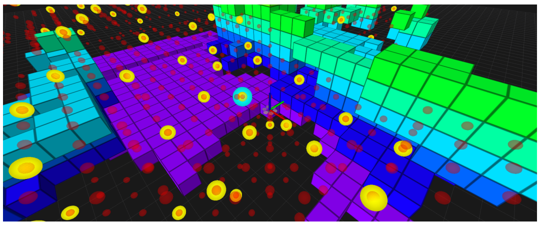

Fig. 2. An instance from a simulation experiment illustrating, the frontier points, frontier candidates, and selected waypoint for robot i (red, yellow and cyan spheres, respectively).

F. Path Planning

Once the next waypoint is selected, each robot plans a path to that frontier point and navigates using the MRS SubT Planner [22], accounting for the occupancy grid map. To address local minima, the path planning algorithm adjusts the score of unreachable waypoints by halving it in subsequent iterations. This systematic reduction incentivizes exploration towards alternative paths and promotes the selection of reachable waypoints with higher scores, aiding in breaking out of local minima traps. The exploration continues until no waypoints with a score above the minimum threshold are left, indicating full environment exploration and approximate map completion.

IV. Simulation-based Evaluation

The evaluation of the proposed approach has been conducted through five simulation studies for each n-UAV exploration scenario, within an unknown Gazebo environment, as shown in Fig. 3.

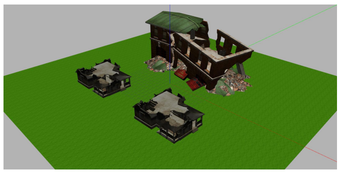

Fig. 3. Gazebo simulation world with collapsed buildings used in the exploration experiments, simulating a disaster response scenario.

The MRS UAV System [24] was chosen for these simulations due to its realism and wide use in Gazebo/ROS robotics simulations. It accurately simulates the DJI F550 hexacopter and allows for detailed testing of UAV behaviors. The UAVs, equipped with an Ouster OS0-128 LiDAR sensor, explore an unknown area and construct a grid map in the simulation. The sensor data range was limited to 20 meters to balance realism and computational efficiency. Additionally, a maximum velocity of $1m/s$ was set to ensure precise and stable UAV movement for accurate data collection and mapping. This setup provides realistic and valuable data for analysis. The remaining parameters set for these simulations are listed in Table I.

TABLE I
COMMON PARAMETRES

Collision avoidance distance (m)	2.0
Environment dimension (m^3)	60 * 60 * 8
Information sharing range (m)	20
Mean-shift bandwidth	1.0
Update rate (Hz)	1.0
Voxel size (m)	1.0
λ	0.13

A. One-UAV Exploration

In the one-UAV experiment, the objective is comprehensive spatial exploration, serving as a testbed for assessing the efficacy and efficiency of the proposed exploration planner.

The performance metrics used in this experiment include the distance and time taken to explore the environment, and the completeness of the map. The results of one of the five simulations are represented in Figs. 4 and 5, as well as the mean and standard deviation values for all experiments in Table II.

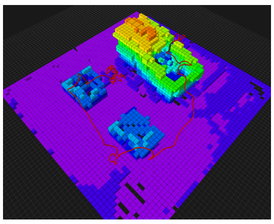

Fig. 4. OctoMap and 3D trajectory illustrating the path traversed by a single UAV during its exploration mission.

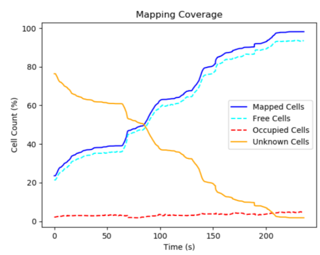

Fig. 5. Mapping coverage in function time taken.

2024 IEEE International Conference on
Autonomous Robot Systems and Competitions (ICARSC)
Paredes de Coura, Portugal – May 2-3, 2024

TABLE II
ONE-UAV EXPLORATION RESULTS

Distance travelled (m)	231.8 ± 17.1
Exploration time (s)	223.0 ± 13.9
Space coverage $(\%)$	98.6 ± 0.8

In this scenario, the UAV covered a distance of 231.8 meters, as per the mean outcome of five experiments. The exploration and mapping process was typically completed in 223.0 seconds. The UAV managed to map 98.6% of the environment within this duration. These values, representative of the central tendency, indicate an efficient process and validate the effectiveness of our exploration planner in a single UAV setup. These results provide a promising basis for further experiments involving multiple UAVs.

B. Two-UAV Exploration

In the two-UAVs experiment, a second UAV was added into the environment. The UAVs started 10 meters apart and a minimum distance of 15 meters between waypoints was set for the sequential waypoint assignment. The same performance metrics were used in this scenario and the results of one of the five simulations are represented in Figs. 6 and 7, as well as the mean and standard deviation values for all experiments in Table III.

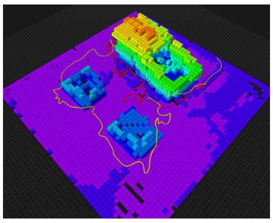

Fig. 6. OctoMap and 3D trajectory illustrating the path traversed by UAV1 (red), and UAV2 (yellow) during their exploration mission.

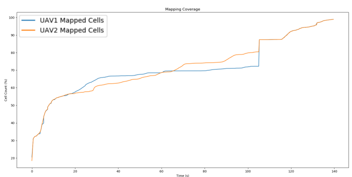

Fig. 7. Mapping coverage for UAV1, and UAV2's local map, and shared global map in function time taken.

TABLE III
TWO-UAV EXPLORATION RESULTS

	UAV1	UAV2
Distance travelled (m)	134.5 ± 39.5	153.2 ± 53.6
Exploration time (s)	145.8 ± 53.3	
Space coverage $(\%)$	96.0 ± 2.9	

The results presented here are the average of five experiments. In the two-UAV setup, both UAVs completed the task, mapping 96.0% of the environment in 145.8 seconds, a significant improvement over the single UAV's 223.0 seconds.

Key differences were observed between single and dual UAV experiments. The exploration time decreased by approximately 34.6% with the addition of a second UAV. The total distance covered increased slightly, but the distance each UAV traveled was reduced, indicating more efficient coverage.

Mapping completeness decreased slightly by 2.6%, possibly due to the complexity of coordinating multiple UAVs. Despite this, the two UAVs effectively mapped most of the environment. The large standard deviation in the data can be attributed to infrequent information sharing between the UAVs, GPS inaccuracies, and sensor errors. These factors can cause inconsistencies in the UAVs' perceived positions, leading to variations in their performance.

C. Three-UAV Exploration

In the three-UAVs experiment, an additional UAV was introduced to the system. The same minimum distance between waypoints and performance metrics were used in this scenario and the results of one of the five simulations are represented in Figs. 8 and 9, as well as the mean and standard deviation values for all experiments in Table IV.

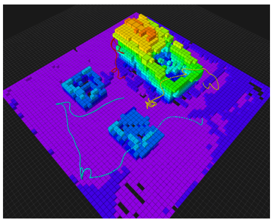

Fig. 8. OctoMap and 3D trajectory illustrating the path traversed by UAV1 (red), UAV2 (yellow), and UAV3 (cyan) during their exploration mission.

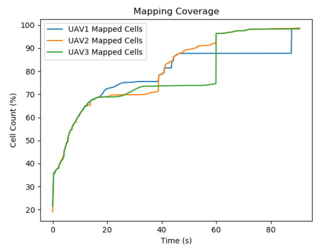

Fig. 9. Mapping coverage for each UAV's local map in function time taken.

TABLE IV
THREE-UAV EXPLORATION RESULTS

	UAV1	UAV2	UAV3
Distance travelled (m)	74.6 ± 10.2	88.1 ± 9.3	102.6 ± 20.0
Exploration time (s)		87.8 ± 10.1	
Space coverage $(\%)$		98.6 ± 0.4	

The following analysis is based on the average results from five experiments conducted with a three-UAV setup. All UAVs successfully completed the task, achieving a mapping coverage of 98.6% of the environment in a time span of 87.8 seconds, a notable enhancement compared to the two-UAV setup.

When comparing the two-UAV and three-UAV experiments, several key distinctions were noted. With the introduction of a third UAV, the exploration time saw a further reduction. Although the total distance covered by all UAVs was 265.3 meters, this was less than the total distance covered by two UAVs in the previous experiment, indicating an increase in coverage efficiency.

The mapping completeness with three UAVs increased to 98.6%, the same level achieved with a single UAV. This improvement, along with the decrease in the standard deviation in the data, could be attributed to more frequent information sharing among the UAVs. More consistent data between the UAVs due to frequent information sharing can reduce discrepancies, thereby lowering the standard deviation. This observation suggests that improving the communication frequency among UAVs could be a crucial factor in enhancing the efficiency and accuracy of multi-UAV systems. Despite the inherent challenges, the overall performance of the three-UAV setup surpassed that of both the single and dual UAV setups.

V. Conclusion

In this work, we introduced a cooperative strategy for MRS in 3D exploration and mapping tasks. Our approach, based on distributed and sequential waypoint assignment, demonstrated significant improvements in exploration time and energy conservation. Our exploration planner played a crucial role in these results. It effectively managed the assignment of waypoints, ensuring optimal coverage of the environment and efficient use of resources. The planner's ability to dynamically adjust to changes in the environment and robot team composition was key to its success.

In the simulated environment, the effectiveness of our exploration planner was validated with a single UAV setup. The introduction of a second UAV optimized the use of battery resources by reducing the exploration time and individual travel distance. However, it also led to a slight decrease in mapping coverage and an increase in standard deviation. These changes could be attributed to the challenges of coordinating multiple UAVs. As the number of UAVs in a system increases, so does the complexity of coordination and the challenges of collision avoidance, which may affect performance. The addition of a third UAV not only improved efficiency but also improved mapping completeness. The increased frequency of information sharing among the UAVs might have contributed to this improved performance, reducing discrepancies.

These findings highlight the importance of communication frequency in multi-UAV systems and indicate that more research is needed to fully understand these dynamics. The careful consideration of these factors is crucial when increasing the number of UAVs in a system due to potential trade-offs.

In conclusion, our work provides a solid foundation for future investigations in cooperative MRS. The exploration planner presented in this study has potential applications in a broad spectrum of MRS, underscoring the relevance of our work in this field. Future research could explore the scalability of this approach with larger UAV teams and the impact of different cooperative strategies on task performance.

References

[1] M. Basiri, J. Gonçalves, J. Rosa, R. Bettencourt, A. Vale, and P. Lima, "A multipurpose mobile manipulator for autonomous firefighting and construction of outdoor structures," *Field Robotics*, vol. 1, no. 1, Oct. 2021.

[2] G. Hitz, A. Gotovos, F. Pomerleau, M.- Garneau, C. Pradalier, A. Krause, and R. Y. Siegwart, "Fully autonomous focused exploration for robotic environmental monitoring," in *2014 IEEE International Conference on Robotics and Automation (ICRA)*, May 2014, pp. 2658–2664, iSSN: 1050-4729.

[3] M. Pimentel and M. Basiri, "A bimodal rolling-flying robot for micro level inspection of flat and inclined surfaces," *IEEE Robotics and Automation Letters*, vol. 7, no. 2, pp. 5135–5142, 2022.

[4] J. Bares, M. Hebert, T. Kanade, E. Krotkov, T. Mitchell, R. Simmons, and W. Whittaker, "Ambler: an autonomous rover for planetary exploration," *Computer*, vol. 22, no. 6, pp. 18–26, Jun. 1989.

[5] W. Burgard, M. Moors, C. Stachniss, and F. Schneider, "Coordinated multi-robot exploration," *IEEE Transactions on Robotics*, vol. 21, no. 3, pp. 376–386, Jun. 2005.

[6] B. Yamauchi, "A frontier-based approach for autonomous exploration," in *IEEE International Symposium on Computational Intelligence in Robotics and Automation CIRA'97. 'Towards New Computational Principles for Robotics and Automation'*, Jul. 1997, pp. 146–151.

[7] A. Bircher, M. Kamel, K. Alexis, H. Oleynikova, and R. Siegwart, "Receding Horizon "Next-Best-View" Planner for 3D Exploration," in *2016 IEEE International Conference on Robotics and Automation (ICRA)*, May 2016, pp. 1462–1468.

[8] F. Bourgault, A. Makarenko, S. Williams, B. Grocholsky, and H. Durrant-Whyte, "Information based adaptive robotic exploration," in *IEEE/RSJ International Conference on Intelligent Robots and Systems*, vol. 1, Sep. 2002, pp. 540–545 vol.1.

[9] S. Koenig, C. Tovey, and W. Halliburton, "Greedy mapping of terrain," in *Proceedings 2001 ICRA. IEEE International Conference on Robotics and Automation (Cat. No.01CH37164)*, vol. 4, May 2001, pp. 3594–3599 vol.4, iSSN: 1050-4729.

[10] R. Sim and N. Roy, "Global A-Optimal Robot Exploration in SLAM," in *Proceedings of the 2005 IEEE International Conference on Robotics and Automation*, Apr. 2005, pp. 661–666, iSSN: 1050-4729.

[11] R. Bettencourt, J. Lewis, R. Serra, M. Basiri, A. Vale, and P. U. Lima, "Geers: Georeferenced enhanced ekf using point cloud registration and segmentation," *IEEE Robotics and Automation Letters*, vol. 9, no. 2, pp. 1803–1810, 2024.

[12] A. Hornung, K. M. Wurm, M. Bennewitz, C. Stachniss, and W. Burgard, "OctoMap: an efficient probabilistic 3D mapping framework based on octrees," *Autonomous Robots*, vol. 34, no. 3, pp. 189–206, Apr. 2013.

[13] A. Marjovi, J. G. Nunes, L. Marques, and A. de Almeida, "Multi-robot exploration and fire searching," in *2009 IEEE/RSJ International Conference on Intelligent Robots and Systems*, Oct. 2009, pp. 1929–1934, iSSN: 2153-0866.

[14] B. Zhou, H. Xu, and S. Shen, "RACER: Rapid Collaborative Exploration with a Decentralized Multi-UAV System," Sep. 2022, arXiv:2209.08533.

[15] J. Lewis, P. U. Lima, and M. Basiri, "Collaborative 3D Scene Reconstruction in Large Outdoor Environments Using a Fleet of Mobile Ground Robots," *Sensors*, vol. 23, no. 1, p. 375, Jan. 2023, number: 1 Publisher: Multidisciplinary Digital Publishing Institute.

[16] B. Yamauchi, "Frontier-based exploration using multiple robots," in *Proceedings of the second international conference on Autonomous agents - AGENTS '98*. Minneapolis, Minnesota, United States: ACM Press, 1998, pp. 47–53.

[17] B. Zhou, Y. Zhang, X. Chen, and S. Shen, "FUEL: Fast UAV Exploration Using Incremental Frontier Structure and Hierarchical Planning," *IEEE Robotics and Automation Letters*, vol. 6, no. 2, pp. 779–786, Apr. 2021.

[18] C. Connolly, "The determination of next best views," in *1985 IEEE International Conference on Robotics and Automation Proceedings*, vol. 2, Mar. 1985, pp. 432–435.

[19] H. H. González-Baños and J.-C. Latombe, "Navigation Strategies for Exploring Indoor Environments," *The International Journal of Robotics Research*, vol. 21, no. 10-11, pp. 829–848, Oct. 2002.

[20] A. Batinović, T. Petrović, A. Ivanovic, F. Petric, and S. Bogdan, "A Multi-Resolution Frontier-Based Planner for Autonomous 3D Exploration," *IEEE Robotics and Automation Letters*, vol. 6, no. 3, pp. 4528–4535, Jul. 2021, arXiv:2011.02182 [cs].

[21] M. A. Basiri, S. Chehelgami, E. Ashtari, M. T. Masouleh, and A. Kalhor, "Synergy of deep learning and artificial potential field methods for robot path planning in the presence of static and dynamic obstacles," in *2022 30th International Conference on Electrical Engineering (ICEE)*, 2022, pp. 456–462.

[22] V. Krátký, P. Petráček, T. Báča, and M. Saska, "An autonomous unmanned aerial vehicle system for fast exploration of large complex indoor environments," *Journal of Field Robotics*, vol. 38, no. 8, pp. 1036–1058, 2021, _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1002/rob.22021.

[23] K. Fukunaga and L. Hostetler, "The estimation of the gradient of a density function, with applications in pattern recognition," *IEEE Transactions on Information Theory*, vol. 21, no. 1, pp. 32–40, 1975.

[24] T. Baca, M. Petrlik, M. Vrba, V. Spurny, R. Penicka, D. Hert, and M. Saska, "The MRS UAV System: Pushing the Frontiers of Reproducible Research, Real-world Deployment, and Education with Autonomous Unmanned Aerial Vehicles," *Journal of Intelligent & Robotic Systems*, vol. 102, no. 1, p. 26, Apr. 2021.