Due 2pm on 9/8/2022

Exercise 1. If X is a random variable with mean m and variance s^2 show $(X - m)/s$ has mean 0 and variance 1.

Recall the *cumulant* of a random variable X is $\kappa^X(s) = \log E[e^{sX}]$.

Exercise 2. *Let F be any positive random variable where $\log F$ has finite mean and variance. Show $F = fe^{-\kappa^X(s)+sX}$ where $f = E[F]$, $s^2 = \operatorname{Var}(\log F)$, and X has mean 0 and variance 1.*

Hint: $\log F = m + sX$ where X has mean 0 and variance 1.

Exercise 3. *If $F = fe^{-\kappa(s)+sX}$ show $F \leq k$ if and only if $X \leq (\log k/f + \kappa(s))/s$.*

Hint: Assume $s > 0$.

We call the functions $x(k; f, s) = (\log k/f + \kappa(s))/s$ the *moneyness* at k.

The Fischer Black model assumes X is normal and $s = \sigma\sqrt{t}$.

Exercise 4. *Find the formula for moneyness in the Black model.*

Recall if N is a normally distributed random variable then $E[e^N] = e^{E[N]+\operatorname{Var}(N)/2}$.