hw7

Exercise. *Find a closed form formula for* $E[\max\{e^M - e^N, 0\}]$ *where* M *and* N *are jointly normal.*

Hint. $E[\max\{e^M - e^N, 0\}] = E[e^M \max\{1 - e^{N-M}\}, 0\}]$.

Let $S_t = se^{-\sigma^2 t/2 + \sigma B_t}$ for $t \geq 0$ where (B_t) is standard Browinian motion. The value of a call at time t with strike k expiring at T in an arbitrage-free model is $\bar{V}_t = E_t[\max\{S_T - k, 0\}]$.

Let $\bar{v}(s, k, t) = \bar{V}_0$, where $s = S_0$. Note $\bar{v}$ is a function, not a random variable.

Exercise. *Show* $\bar{V}_t = S_t \bar{v}(S_t, k, T - t)$.

Note $\bar{v}(S_t, k, T - t)$ is a random variable.

Hint. $V_t = E_t[\max\{S_t e^{-\sigma^2 (T-t)/2 + \sigma(B_T - B_t} - k, 0\}]$

If a trader puts on a position (M_t, N_t) at time t it has value $V_t = M_t + N_t S_t$ at t and $V_u = M_t + N_t S_u$ at time $u \geq t$ if the position is constant over $[t, u]$ This is not, in general, the same as the model value $\bar{V}_u$.

Exercise. *Find a general formula for* $V_u - \bar{V}_u$.