Derivation for the divergence and curl for source & vortex flow

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since source flow velocity is:

$$V_r = \frac{C}{r}, \quad V_\theta = 0,$$

then

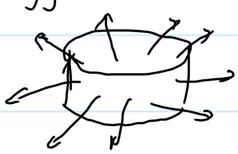
$$\nabla \cdot V = \frac{1}{r}\frac{\partial}{\partial r}(rV_r) + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta}$$

$$= \frac{1}{r}\frac{\partial}{\partial r}(C) = 0,$$

$$\nabla \times V = \frac{1}{r}\begin{vmatrix} \vec{e_r} & r\vec{e_\theta} & \vec{e_z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ \frac{C}{r} & 0 & 0 \end{vmatrix} \longrightarrow \text{appearantly is zero},$$

however, we discuss the situation for the origin point, for source flow, we set a infinitesimal circle and use the divergence theorem, which gives:

$$\iint V \cdot dS = \iiint_V \nabla \cdot V \cdot dV \xrightarrow[\text{depth}]{\text{for unit}} \quad V \cdot 2\pi r = \nabla \cdot V \cdot \pi r^2.$$



that is: $\nabla \cdot V = \frac{2V}{r}$.

for $r \to 0$, $\nabla \cdot V \to \infty$.

Also for vortex flow,

$$V_r = 0, \quad V_\theta = \frac{C}{r},$$

$$\nabla \cdot V = \frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{C}{r}\right) = 0$$

$$\nabla \times V = \frac{1}{r}\begin{vmatrix} \vec{e_r} & r\vec{e_\theta} & \vec{e_z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ 0 & C & 0 \end{vmatrix} = 0,$$

for calculation on the origion point, using stokes' equation:

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$$\iint_S (\nabla \times V)\, dS = \oint_S V\, ds = V \cdot 2\pi r = -\Gamma$$

relation of circulation to vorticity,

$\nabla \times V \cdot \pi r^2$ → we consider it's a constant at infinitsimal circle

then $|\nabla \times V| = -\dfrac{\Gamma}{\pi r^2}$ → for $r \to 0$, $|\nabla \times V| \to \infty$

also in the text book, it is derived as: $\nabla \times V = \dfrac{2\pi C}{dS} = \dfrac{2\pi \times -\frac{\Gamma}{2\pi}}{\pi r^2} = -\dfrac{\Gamma}{\pi r^2}$