二端口特性参数推导

Monday, December 4, 2023 9:19 AM

对二端口网络(互易)，我们已经得到：

$$Z_{in}=\frac{a_{11}Z_{C2}+a_{12}}{a_{21}Z_{C2}+a_{22}}=Z_{C1}\quad(1),\qquad Z_{out}=\frac{a_{22}Z_{C1}+a_{12}}{a_{21}Z_{C1}+a_{11}}=Z_{C2}\quad(2)$$

求解，有：

$$Z_{C1}=\frac{\dfrac{a_{11}a_{22}Z_{C1}+a_{11}a_{12}}{a_{21}Z_{C1}+a_{11}}+a_{12}}{\dfrac{a_{21}a_{22}Z_{C1}+a_{21}a_{12}}{a_{21}Z_{C1}+a_{11}}+a_{22}}$$

$$\cancel{a_{11}a_{22}Z_{C1}}+a_{11}a_{12}+\cancel{a_{12}a_{21}Z_{C1}}+a_{12}a_{11}$$
$$=Z_{C1}^2a_{21}a_{22}+Z_{C1}a_{21}a_{12}+Z_{C1}^2a_{21}a_{22}+\cancel{Z_{C1}a_{11}a_{12}}$$

故有关系：$\cancel{2}Z_{C1}^2a_{21}a_{22}=\cancel{2}a_{12}a_{11}$

即：$Z_{C1}=\sqrt{\dfrac{a_{11}a_{12}}{a_{21}a_{22}}}$ ①

此时：将(1)代入(2)，则：

$$Z_{C2}=\frac{\dfrac{a_{11}a_{22}Z_{C2}+a_{22}a_{12}}{a_{21}Z_{C2}+a_{22}}+a_{12}}{\dfrac{a_{21}a_{11}Z_{C2}+a_{21}a_{12}}{a_{21}Z_{C2}+a_{22}}+a_{11}}$$，从而：

$$a_{21}a_{11}Z_{C2}^2+\cancel{a_{21}a_{12}Z_{C2}}+a_{11}a_{21}Z_{C2}^2+\cancel{a_{22}a_{11}Z_{C2}}$$
$$=\cancel{a_{11}a_{22}Z_{C2}}+a_{22}a_{12}+\cancel{a_{12}a_{21}Z_{C2}}+a_{12}a_{22}$$

$\therefore Z_{C2}^2=\dfrac{a_{22}a_{12}}{a_{11}a_{21}}$，$Z_{C2}=\sqrt{\dfrac{a_{12}a_{22}}{a_{11}a_{21}}}$ ②

对于传播系数：

$\gamma=\ln\sqrt{\dfrac{\dot U_1\dot I_1}{\dot U_2\dot I_2}}$，有：当网络互易时

$\sqrt{U_2 I_2}$

$$\begin{cases} \dot U_1 = a_{11}\dot U_2 - a_{12}\dot I_2 \\ \dot I_1 = a_{21}\dot U_2 - a_{22}\dot I_2 \end{cases} \longrightarrow \sqrt{\frac{\dot U_1 \dot I_1}{\dot U_2 \dot I_2}}$$

而传播系数是在特性阻抗下求得，从而有：

※ $\dot I_2 = -\dfrac{\dot U_2}{Z_{c2}}$ 从而：$\dot U_2 \dot I_2 = \dot U_2^2 / Z_{c2}$,

a参数

而：$\dot U_1 \dot I_1 = \left(a_{11}\dot U_2 + a_{12}\dfrac{\dot U_2}{Z_2}\right)\left(a_{21}\dot U_2 + a_{22}\dfrac{\dot U_2}{Z_2}\right)$

$$= \dot U_2^2\left(a_{11} + \frac{a_{12}}{Z_2}\right)\left(a_{21} + \frac{a_{22}}{Z_2}\right)$$

从而有：$\dfrac{\dot U_1 \dot I_1}{\dot U_2 \dot I_2} = (a_{11}Z_{c2} + a_{12})\left(a_{21} + \dfrac{a_{22}}{Z_{c2}}\right)$

我们已经求出：$Z_{c2} = \sqrt{\dfrac{a_{12}a_{22}}{a_{11}a_{21}}}$

则 $= a_{11}a_{21}Z_{c2} + a_{12}a_{21} + \dfrac{a_{12}a_{22}}{Z_{c2}} + a_{11}a_{22}$

$$= \sqrt{a_{11}a_{21}a_{12}a_{22}} + \sqrt{a_{12}a_{22}a_{11}a_{21}} + a_{12}a_{21} + a_{11}a_{22}$$

$$\frac{U_1 I_1}{U_2 I_2} = \left(\sqrt{a_{11}a_{22}} + \sqrt{a_{12}a_{21}}\right)^2$$

因此：

$$\gamma = \ln\sqrt{\frac{U_1 I_1}{U_2 I_2}} = \ln\left(\sqrt{a_{11}a_{22}} + \sqrt{a_{12}a_{21}}\right)$$

同样：因 $\sqrt{\dfrac{U_2 I_2}{U_1 I_1}} = \dfrac{1}{\left(\sqrt{a_{11}a_{22}} + \sqrt{a_{12}a_{21}}\right)}$ $= a_{11}a_{22} - a_{12}a_{21}$

从而 $\gamma' = \ln\sqrt{\dfrac{U_2 I_2}{U_1 I_1}} = \ln\left(\dfrac{1}{\sqrt{a_{11}a_{22}} + \sqrt{a_{12}a_{21}}}\right) = -\ln\left(\sqrt{a_{11}a_{22}} + \sqrt{a_{12}a_{21}}\right)$

$= -\gamma$