Derivation for 2D Jacobian

Saturday, April 15, 2023 11:36 AM

since

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy$$

then we have:

coefficient → since $\frac{\partial x}{\partial \xi}, \frac{\partial y}{\partial \xi}$ has given.

$$\begin{cases} \frac{\partial u}{\partial x}\cdot\left(\frac{\partial x}{\partial \xi}\right) + \frac{\partial u}{\partial y}\left(\frac{\partial y}{\partial \xi}\right) = \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial x}\cdot\left(\frac{\partial x}{\partial \eta}\right) + \frac{\partial u}{\partial y}\left(\frac{\partial y}{\partial \eta}\right) = \frac{\partial u}{\partial \eta} \end{cases}$$

using: Cramer's rule on the linear equations above:

$$\frac{\partial u}{\partial x} = \frac{\begin{vmatrix} \frac{\partial u}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial u}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{vmatrix}}{\begin{vmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{vmatrix}} \qquad \frac{\partial u}{\partial y} = \frac{\begin{vmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial u}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial u}{\partial \eta} \end{vmatrix}}{\begin{vmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{vmatrix}}$$

then $$\frac{\partial}{\partial x} = \frac{1}{J}\left[\frac{\partial}{\partial \xi}\left(\frac{\partial y}{\partial \eta}\right) - \frac{\partial}{\partial \eta}\left(\frac{\partial y}{\partial \xi}\right)\right]$$

$$\frac{\partial}{\partial y} = \frac{1}{J}\left[\frac{\partial}{\partial \eta}\left(\frac{\partial x}{\partial \xi}\right) - \frac{\partial}{\partial \xi}\left(\frac{\partial x}{\partial \eta}\right)\right]$$