楔形体受重力和液体压力的求解

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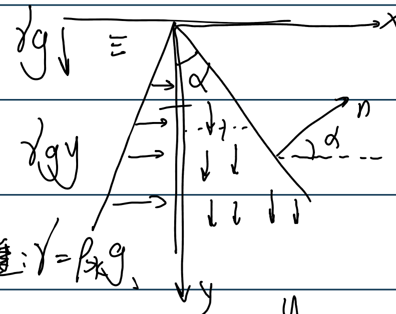
对于楔形体受重力和液体压力部分的求解，既可以看成是半逆解法，也可看成逆解法。

即：有应力方程：$\phi = ax^3 + bx^2y + cxy^2 + dy^3$



$$\begin{cases} \sigma_x = \frac{\partial^2\phi}{\partial y^2} - f_x x = 2cx + 6dy \\ \sigma_y = \frac{\partial^2\phi}{\partial x^2} - f_y y = 2by + 6ax - \rho g y \\ \tau_{xy} = -\frac{\partial^2\phi}{\partial x \partial y} = -2bx - 2cy \end{cases}$$

体力项

其中 γ 为比重：$\gamma = \rho_水 g$

代入边界条件：有：① $\sigma_x\big|_{x=0} = -\gamma y$，$\tau_{xy}\big|_{x=0} = 0$，$\rightarrow c=0$ （先使用比重求解）

$6d = -\gamma$，$c = 0$

即：$d = -\frac{\gamma}{6}$

②：$y = \frac{x}{\tan\alpha}$ $\begin{cases} \sigma_x \cos\alpha - \tau_{xy}\sin\alpha = 0 \\ \tau_{xy}\cos\alpha - \sigma_y \sin\alpha = 0 \end{cases}$

由 $l = \cos\alpha$，$m = -\sin\alpha$

即有：代入 $\begin{cases} \left(6d \cdot \frac{x}{\tan\alpha}\right)\cos\alpha + 2bx \cdot \sin\alpha = 0 \\ (-2bx)\cos\alpha - \left(2b\frac{x}{\tan\alpha} + 6ax - \rho g \frac{x}{\tan\alpha}\right)\sin\alpha = 0 \end{cases}$

∴有：$\begin{cases} b\sin\alpha + 3d\frac{\cos\alpha}{\sin\alpha} = 0 \\ b\cos\alpha + b\cos\alpha + 3a\sin\alpha - \frac{\rho g}{2}\cos\alpha = 0 \end{cases}$

两边除 $-2x$

∴ $b = -3d \cdot \frac{\cos^2\alpha}{\sin^2\alpha} = +\frac{\gamma}{2} \times \frac{1}{\tan^2\alpha}$ 注意符号

$c = 0$，$d = -\frac{\gamma}{6}$，

而：$3a = \frac{\rho g}{2\tan\alpha} - \frac{2b}{\tan\alpha}$，$\rightarrow a = \frac{1}{6}\frac{\rho g}{\tan\alpha} - \frac{2b}{3\tan\alpha}$

整理，得到常数：

$d = -\frac{\gamma}{6}$，$b = -\frac{\gamma}{2}\cot^2\alpha$　$c = 0$，$a = \frac{1}{6}\rho g \cot\alpha - \frac{\gamma}{3}\cot^3\alpha$

此处 γ 为水的比重，

若使用密度表示，则：$d = -\frac{\gamma g}{6}$，$b = -\frac{\gamma g}{2}\cot^2\alpha$，$a = \frac{\rho g}{6}\cot\alpha - \frac{\gamma g}{3}\cot^3\alpha$

代入应力函数中，有解答：

$$\begin{cases}\sigma_x = 6dy \\ \sigma_y = 2by + 6ax - \rho g y \\ \tau_{xy} = -2bx\end{cases} \xrightarrow{\text{代入}} \begin{cases}\sigma_x = -\gamma g y \\ \sigma_y = (+\gamma g \cot^2\alpha)\, y + (\rho g \cot\alpha - 2\gamma g \cot^3\alpha)\, x - \rho g y \\ \tau_{xy} = \gamma g \cot^2\alpha \cdot x\end{cases}$$

$$\downarrow$$

$$= \underline{(\rho g \cot\alpha - 2\gamma g \cot^3\alpha)\, x + (\gamma g \cot^2\alpha - \rho g)\, y}$$

$\boxed{\text{为Levy解答}}$ (楔形体问题)