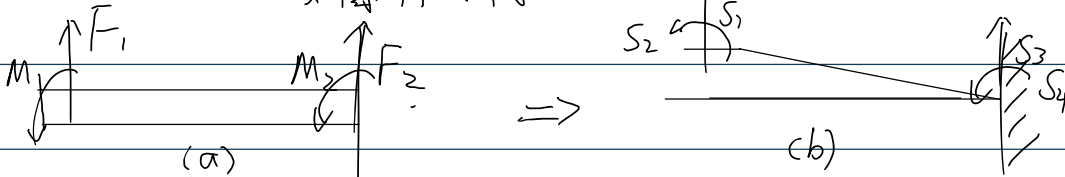


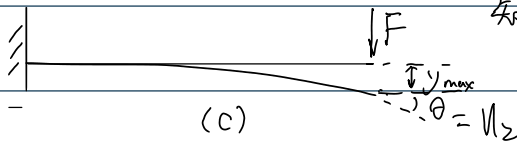
梁的刚度矩阵的求出

Monday, March 13, 2023 9:33 AM

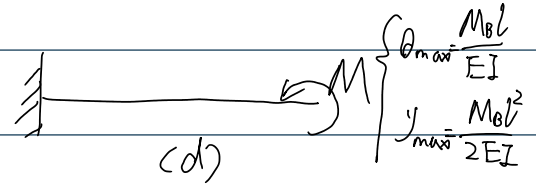
如图所示的梁单元, 可简化为右图所示:



由材料力学变形公式:

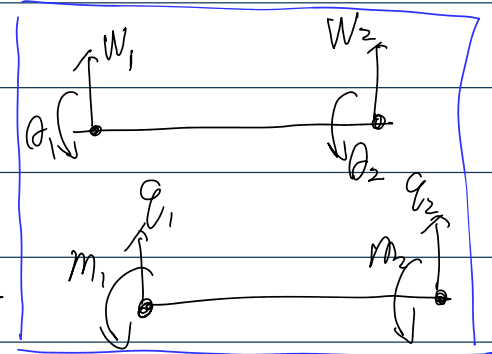


$$\text{知: } \begin{cases} \theta = \frac{Fl^2}{2EI} \\ W_{max} = \frac{Fl^3}{3EI} \end{cases}$$



则: 对于图 (b) 中的梁有:

$$\begin{cases} W = \frac{Fl^3}{3EI} - \frac{Ml}{2EI} = \frac{S_1 l^3}{3EI} - \frac{S_2 l^2}{2EI} = U_1 \\ \theta = \frac{Ml}{EI} - \frac{Fl^2}{2EI} = \frac{S_2 l}{EI} - \frac{S_1 l^2}{2EI} = U_2 \end{cases}$$



$$= 2U_1 + U_2 l = \frac{2S_1 l^3}{3EI} - \frac{S_1 l^3}{2EI} = \frac{S_1 l^3}{6EI}$$

此时: 令 $U_1 = 1$, 得: $S_1 = \frac{12EI}{l^3} = a_{11}$

代入: 有: $4 - \frac{S_2 l^2}{2EI} = 1$, $\rightarrow \frac{S_2 l^2}{2EI} = 3$, 则: $S_2 = \frac{6EI}{l^2} = a_{21}$

显然由平衡方程: $S_3 = -S_1 = -\frac{12EI}{l^3} = a_{31}$

$$S_4 = S_1 l - S_2 = \frac{12EI}{l^2} - \frac{6EI}{l^2} = \frac{6EI}{l^2} = a_{41}$$

又: 令 $U_2 = 1$, 得: $\begin{cases} \frac{S_1 l^3}{3EI} = \frac{S_2 l^2}{2EI} \\ \frac{S_2 l}{EI} = \frac{S_1 l^2}{2EI} + 1 \end{cases}$ 有: $\frac{S_1 l^3}{3EI} = \frac{S_1 l^2}{4EI} + \frac{1}{2}$

$$\therefore S_1 = \frac{6EI}{l^2} = a_{12}$$

而: $S_2 = \frac{2}{3} S_1 l = \frac{4EI}{l} = a_{22}$, $S_3 = -S_1 = -\frac{6EI}{l^2} = a_{32}$

故可得到一半的矩阵:

$$S_4 = S_1 l - S_2 = \frac{2EI}{l} = a_{42}$$

另一半由对称性求出; 故:

$$[K] = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

对称: $U_3 = 1$ 时: 对 θ_2 影响是相反的
W 影响相同

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -0.6 & 1.2 \\ 2.6 & -0.6 \end{bmatrix}$$

W 子矩阵

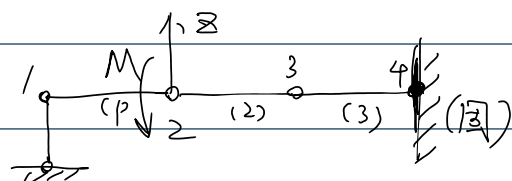
将上式分块矩阵记为:

$$[K]^e = \begin{bmatrix} k_{ii} & k_{ij} \\ k_{ji} & k_{jj} \end{bmatrix}, \text{ 即得: } \begin{Bmatrix} p_i \\ p_j \end{Bmatrix}^e = \begin{bmatrix} k_{ii} & k_{ij} \\ k_{ji} & k_{jj} \end{bmatrix}^e \begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix}^e$$

使用增广矩阵法求解: 即: 对于右图杆系: 有:

$$[K] \{ \delta \} = \{ Q \}, \text{ 或:}$$

$$K = \sum_{e=1}^m [K]^e \quad \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix} = \begin{bmatrix} k_{11}^1 & k_{12}^1 & & \\ k_{21}^1 & k_{22}^1 + k_{22}^2 & k_{23}^2 & \\ & k_{32}^2 & k_{33}^2 + k_{33}^3 & k_{43}^3 \\ & & k_{43}^3 & k_{44}^3 \end{bmatrix}$$



(3)、边界约束情况: 上式为 8 个方程联立,

8 个位移: $f_1 = f_4 = \theta_4 = 0$, 而: Z_1, Z_4, M_4 为约束反力, 未知:

而: 其余 5 个分量为已知或为 0,