Proof of the positive definite of the Stiffness Matrix

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Definition: An $n \times n$ matrix A is said to be positive defite if

i. $c^T A c \geq 0$ for all n-vector c;

ii. $c^T A c = 0$ implies $\boxed{c = 0}$

Theorem: the $n \times n$ stiff matrix A is positive defitive;

Proof:

1. we let $c_A, A = 1, 2, \cdots n$ be the components of $\{c_A\}$, which is an arbitary vector, and use c_A's to construct a member of $\mathcal{V}^h$, $w^h = \sum_{A=1}^{n} c_A N_A$. where N_A's are basis functions for $\mathcal{V}^h$, then we have. $c^T K c = \sum_{A,B=1}^{n} c_A K_{AB} c_B$.

$$= \sum_{A,B=1}^{n} c_A \, a(N_A, N_B) c_B$$

using the linear properties, the equation becomes:

$$c^T K c = a\left(\sum_{A=1}^{n} c_A N_A, \sum_{B=1}^{n} c_B N_B\right) = a(w^h, w^h)$$

N_B and N_A is the same set but use different index to show the dummy variables

$$= \int (w_{h,x})^2 dx \geq 0.$$

2: also, if $\int (w_{h,x})^2 = 0$, then $w_{h,x} = 0$,

since $w_{h,x} = \sum_{A=1}^{n} c_A N_A \longrightarrow w_{h,x} = 0$ is possible only when $c_A = 0$, $\xrightarrow{\text{then}} \{c_A\} \neq 0$,

so the stiffness matrix is:

| 1 Symmetric.

So the stiffness matrix is:

1. Symmetric
2. Banded
3. Positive-definite.