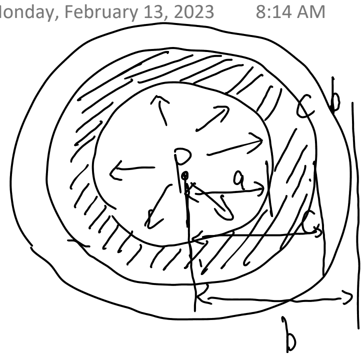
厚壁球壳屈服后分析

Monday, February 13, 2023 8:14 AM



此时，设内部塑性区半径为C，在r=C处，恰好屈服

则边界条件为：$P|_{r=c} = P_E$. ⟵ 注意：不是$\sigma_{r=c}=\sigma_s$，而是Mises屈服条件。

由应力公式知：

$$\sigma_r = \frac{\left(\frac{b}{r}\right)^3 - 1}{\left(\frac{b}{c}\right)^3 - 1}\cdot -P$$

此处，球壳内半径为C

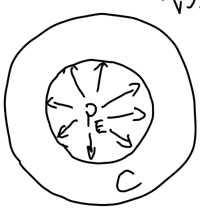
其中：$P_E = \frac{2}{3}\sigma_s\left(1-\frac{c^3}{b^3}\right)$

则：代入：内压

$$P = \frac{2}{3}\sigma_s\left(1-\frac{c^3}{b^3}\right)$$

注意屈服条件是 $\sigma_\theta - \sigma_r = \sigma_s$

即 => $P = P_E = \frac{2}{3}\sigma_s\left(1-\frac{a^3}{b^3}\right)$



代入：$P|_{r=c} = P_E$.（直接代内压公式）

$$\text{得：}\sigma_r = \frac{\left(\frac{b}{r}\right)^3 - 1}{\left(\frac{b}{c}\right)^3 - 1}\cdot -\frac{2}{3}\sigma_s\left(1-\frac{c^3}{b^3}\right)$$

$$= \frac{\left(\frac{c}{b}\right)^3\left[\left(\frac{b}{r}\right)^3 - 1\right]}{\cancel{1-\left(\frac{c}{b}\right)^3}}\cdot -\frac{2}{3}\left[\cancel{1-\left(\frac{c}{b}\right)^3}\right]\sigma_s$$

$$= \boxed{-\frac{2}{3}\sigma_s\cdot\left[\left(\frac{c}{r}\right)^3 - \left(\frac{c}{b}\right)^3\right]}$$

$$\sigma_\theta = \frac{P_c}{(b/c)^3 - 1}\left(\frac{b^3}{2r^3} - 1\right) \xrightarrow{\text{代入}P_c}$$

$$= \frac{\frac{2}{3}\sigma_s\left[1-\frac{c^3}{b^3}\right]}{\frac{b^3}{c^3}-1}\cdot\left[\frac{b^3}{2r^3}-1\right]$$

$$= \frac{2}{3}\frac{c^3}{b^3}\cdot\sigma_s\cdot\left[\frac{b^3}{2r^3}-1\right] = \boxed{\frac{2}{3}\sigma_s\left[\frac{c^3}{2r^3} - \frac{c^3}{b^3}\right]}$$

弹性区：

$$\begin{cases} \sigma_r = -\frac{2}{3}\sigma_s\left[\left(\frac{c}{r}\right)^3 - \left(\frac{c}{b}\right)^3\right] \\ \sigma_\theta = \frac{2}{3}\sigma_s\left[\frac{c^3}{2r^3} - \frac{c^3}{b^3}\right] \end{cases}$$

对塑性区，应力既要满足屈服条件

平衡方程（球面系适用）

$$\boxed{\begin{cases} \sigma_\theta - \sigma_r = \sigma_s \\ \frac{d\sigma_r}{dr} + 2\frac{\sigma_r - \sigma_\theta}{r} = 0 \end{cases}}$$

则：$\frac{d\sigma_r}{dr} = +\frac{2\sigma_s}{r}$，则：$-\frac{1}{2\sigma_s}\cdot\sigma_r = \ln r$

此时有：

$$\frac{1}{2\sigma_s}\cdot\sigma_r\Big|_r^c = \ln r\Big|_r^c$$

$$\sigma_r\Big|_{r=c} = -\frac{2}{3}\sigma_s\left(1-\frac{c^3}{b^3}\right)$$

弹塑性分界应力连续条件，

$$\sigma_r\Big|_{r=c} - \sigma_r\Big|_{r=r} = \ln\frac{c}{r}\cdot +2\sigma_s$$

$$\therefore \sigma_r = -2\sigma_s\ln\frac{c}{r} + \sigma_r\Big|_{r=c} \longrightarrow \sigma_r = -2\sigma_s\ln\frac{c}{r} - \frac{2}{3}\sigma_s\left(1-\frac{c^3}{b^3}\right)$$

或利用边界条件：

$\sigma_r|_{r=a} = -P$，则：$+\frac{\sigma_r}{2\sigma_s}\Big|_a^r = \ln r\Big|_a^r$

考虑：塑性区半径C和内压P的关系：

或利用边界条件：

$\sigma_r|_{r=a} = -p$，则：$+\frac{\sigma_r}{2\sigma_s}\Big|_a^r = \ln r\Big|_a^r$

此时：有：

$$\sigma_r + p = +2\sigma_s \ln\left(\frac{r}{a}\right)$$

则：$\boxed{\sigma_r = -p + 2\sigma_s\ln\left(\frac{r}{a}\right)}$ ☆

因此：对于内部屈服后的球壳整体：

因 $\sigma_r|_{r=a}=-p$，$\sigma_r|_{r=c} = -\frac{2}{3}\sigma_s\left(1-\frac{c^3}{b^3}\right)$

则：$\sigma_r|_{r=c} = -p+2\sigma_s\ln\left(\frac{c}{a}\right) = -\frac{2}{3}\sigma_s\left(1-\frac{c^3}{b^3}\right)$

此时有：

$p = \frac{2}{3}\sigma_s\left(1-\frac{c^3}{b^3}\right)+2\sigma_s\ln\left(\frac{c}{a}\right)$，代入☆得：

$$\sigma_r = -\frac{2}{3}\sigma_s\left(1-\frac{c^3}{b^3}\right) - 2\sigma_s\ln\frac{c}{a} + 2\sigma_s\ln\left(\frac{r}{a}\right)$$

$$= -\frac{2}{3}\sigma_s\left(1-\frac{c^3}{b^3}\right) - 2\sigma_s\ln\left(\frac{c}{r}\right)$$

$$\boxed{= -\frac{2}{3}\sigma_s\left(1-\frac{c^3}{b^3}+\ln\left(\frac{c^3}{r^3}\right)\right)}$$

$$\sigma_\theta = \sigma_s+\sigma_r$$

$$= \frac{1}{3}\sigma_s + \frac{2c^3}{3b^3}\sigma_s - \frac{2}{3}\sigma_s\ln\left(\frac{c^3}{r^3}\right)$$

$$\boxed{= \frac{\sigma_s}{3}\left[1+2\frac{c^3}{b^3} - 2\ln\left(\frac{c^3}{r^3}\right)\right]}$$

考虑：塑性区半径c和内压p的关系：

由：由 $\sigma_r = -p+2\sigma_s\ln\left(\frac{r}{a}\right)$，令 $\left\{ r=c \atop \sigma_r = -\frac{2}{3}\sigma_s\left(1-\frac{c^3}{b^3}\right) \right.$

则：$p = 2\sigma_s\ln\left(\frac{c}{a}\right) + \frac{2}{3}\sigma_s\left(1-\frac{c^3}{b^3}\right)$

$$\boxed{p = \frac{2\sigma_s}{3}\left(1-\frac{c^3}{b^3}+\ln\frac{c^3}{a^3}\right)}$$

该式确定了内压p与塑性半径c之间的关系

则：令c=b，则得到塑性极限压力：

$$p_p = \frac{2\sigma_s}{3}\cdot\ln\frac{b^3}{a^3} = \boxed{2\sigma_s\ln\frac{b}{a}}$$

此时：代入：则：

$$\boxed{\begin{cases} \sigma_r = -p+2\sigma_s\ln\frac{r}{a} = 2\sigma_s\ln\left(\frac{r}{b}\right) \\ \sigma_\theta = \sigma_s\left(1+2\ln\frac{r}{b}\right) \end{cases}}$$

为完全屈服时的应力