Derivation for the flow pattern of source-sink pair

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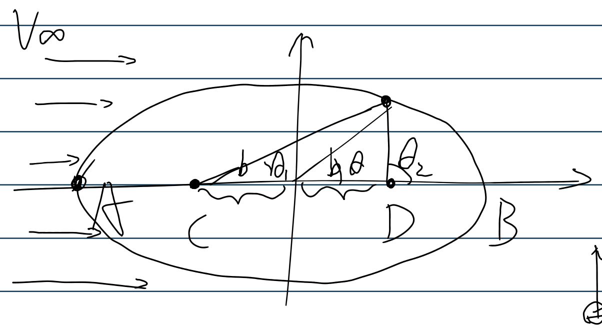

We consider a source-sink pair placed at the middle of the uniform flow

for the source flow, we have:

$$\frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{\Lambda}{2\pi r}, \text{ then} \rightarrow \psi = \frac{\Lambda\theta}{2\pi},$$

and the uniform flow gives:

$$\frac{\partial\psi}{\partial y} = u, \qquad \psi = u\cdot r\sin\theta = V_\infty r\sin\theta$$

then the sink flow is: $\psi = -\frac{\Lambda\theta}{2\pi}$.

so we have the

$$\psi = \frac{\Lambda\theta_1}{2\pi} + V_\infty r\sin\theta - \frac{\Lambda\theta_2}{2\pi}, \text{ or } \boxed{\psi = V_\infty r\sin\theta - \frac{\Lambda}{2\pi}(\theta_1-\theta_2)} \quad ①$$

also we calculate the location of the point A and B, that is:

$$V_\infty - \frac{\Lambda}{2\pi(r-b)} + \frac{\Lambda}{2\pi(r+b)} = 0, \text{ then } V_\infty = \frac{\Lambda}{2\pi}\left(\frac{1}{r-b} - \frac{1}{r+b}\right) = \frac{\Lambda\cdot\cancel{2}b}{\cancel{2}\pi(r^2-b^2)} = \frac{\Lambda b}{\pi(r^2-b^2)}$$

so we have:

$$r^2-b^2 = \frac{\Lambda b}{\pi V_\infty} \quad \therefore r = \sqrt{b^2 - \frac{\Lambda b}{\pi V_\infty}} \xrightarrow{\text{that is}}; \quad OA = OB = \sqrt{b^2 - \frac{\Lambda b}{\pi V_\infty}}$$

so the equation of streamlines is given by eq ①.

that is, $\psi = V_\infty r\sin\theta - \frac{\Lambda}{2\pi}(\theta_1-\theta_2) = \text{const}$

we note at the point A and B, $\theta_1 = \theta_2 = \pi = \theta$, so $\psi_1 = \psi_2 = 0$, then we know that the stream function along a streamline is constant, so the stagnation streamline is given by:

$$\boxed{\psi = V_\infty r\sin\theta - \frac{\Lambda}{2\pi}(\theta_1-\theta_2) = 0}$$

this equation can be approved as an oval.

which is called Rankine Oval