高阶线性方程组推导参考

高阶偏微分方程 `→` 可以转化为一阶方程组

$$a\frac{\partial^2 f}{\partial x^2}+b\frac{\partial^2 f}{\partial x\partial y}+c\frac{\partial^2 f}{\partial y^2}=d$$

我们取

$$u=\frac{\partial f}{\partial x} \qquad v=\frac{\partial f}{\partial y}$$

则原方程化为一阶方程组:

$$\boxed{\begin{cases} a\dfrac{\partial u}{\partial x}+b\dfrac{\partial u}{\partial y}+c\dfrac{\partial v}{\partial y}=d \\ \dfrac{\partial u}{\partial y}=\dfrac{\partial v}{\partial x} \end{cases}}$$

然后我们将等式左右两边单位化， 则上式化为

$$\frac{\partial}{\partial x}\begin{bmatrix} u \\ v \end{bmatrix}+\begin{bmatrix} b/a & c/a \\ -1 & 0 \end{bmatrix}\frac{\partial}{\partial y}\begin{bmatrix} u \\ v \end{bmatrix}=\begin{bmatrix} d/a \\ 0 \end{bmatrix}$$

其中系数矩阵称为A, 并对A求解特征值:

$$A=\begin{bmatrix} b/a & c/a \\ -1 & 0 \end{bmatrix} \qquad |\lambda I-A|=0->a\lambda^2-b\lambda+c=0$$

此时:

$$\begin{cases} b^2-4ac>0 & \text{hyperbolic} \\ b^2-4ac=0 & \text{parabolic} \\ b^2-4ac<0 & \text{elliptic} \end{cases}$$