交流电下的RC微分关系推导

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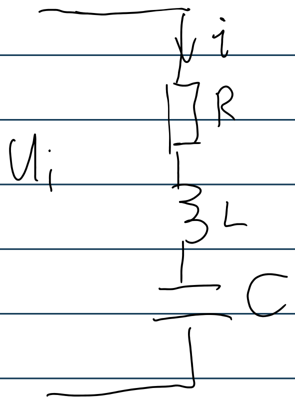
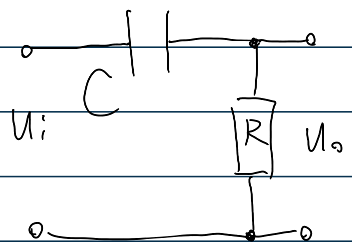


① 首先：对RLC串联电路：

$$U_i = U_R + U_L + U_C \quad \rightarrow Q = CU$$

$$= iR + L\frac{di}{dt} + \frac{Q}{C} = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt.$$

则：微分电路：



$u_o = iR$, $u_i = \frac{1}{C}\int i\,dt + iR$,

∴ 若RC极小：显然 iR忽略 $\rightarrow u_i = \frac{1}{C}\int i\,dt$

$\rightarrow u_o \approx RC\frac{du_i}{dt}$.

实际关系：假设 $\dot{I} = I_m e^{j\omega t}$

则：

$$\dot{U}_i = \frac{1}{jC\omega} I_m e^{j\omega t} + I_m R e^{j\omega t} = -j\frac{1}{C\omega}\dot{I} + \dot{I}R$$

而 $\dot{U}_o = \dot{I}R$,

$$\therefore \frac{\dot{U}_o}{\dot{U}_i} = \frac{\cancel{\dot{I}}R}{\cancel{\dot{I}}(R - jX_L)} = \frac{R(R+jX_L)}{R^2+X_L^2} = \frac{R^2 + j\frac{R}{\omega C}}{R^2 + \frac{1}{C^2\omega^2}} = \frac{C^2R^2\omega^2 + jCR\omega}{C^2R^2\omega^2 + 1}$$

由 $\tau = RC$，得：

$$\frac{\omega^2\tau^2 + j\omega\tau}{\omega^2\tau^2+1} = \frac{\omega\tau\sqrt{1+\omega^2\tau^2}\,e^{j\varphi}}{1+\omega^2\tau^2}$$，其中 $\varphi = \arctan\frac{1}{\omega\tau}$

$\omega\tau(\omega\tau + j)$

得：$$\frac{\dot{U}_o}{\dot{U}_1} = \frac{\omega\tau}{\sqrt{1+\omega^2\tau^2}}e^{j\varphi},\quad \varphi = tg^{-1}\left(\frac{1}{\omega\tau}\right)$$

② 对于积分电路：

$$\dot{U}_o = \frac{1}{C}\int i\omega t = -jX_L\cdot\dot{I} = -j\frac{1}{\omega C}\dot{I}$$

而 $\dot{U}_i = \dot{I}(R - jX_L)$

$$\therefore 实际比例 \frac{\dot{U}_o}{\dot{U}_1} = \frac{-jX_L}{R-jX_L} = \frac{-jX_L(R+jX_L)}{R^2+X_L^2} = \frac{-j\cdot\omega C(R+j\cdot\frac{1}{\omega C})}{R^2\omega^2C^2+1} = \frac{1-\tau\omega j}{1+\tau^2\omega^2}$$

$$= \frac{1}{\cdots}e^{j\varphi},\quad \varphi = tg^{-1}(-\omega\tau)$$

$$= \frac{1}{\sqrt{1+\tau^2\omega^2}} e^{j\varphi}, \quad \varphi = tg^{-1}(-\omega\tau)$$