失效概率函数相关推导

8:36 PM

①: AK-MCS方法的问题转化

$$\int y^{\alpha_k} \hat{f}_{Y|\theta_j}(y)\, dy = M_Y^{\alpha_k}$$

$$\text{mini}: -\int f_{Y|\theta_j}(y) \cdot -\sum_{k=0}^{m} \lambda_k y^{\alpha_k}\, dy$$

$$= \underline{\sum_{k=0}^{m} \lambda_k \int f_{Y|\theta_j}(y)\, y^{\alpha_k} dy}$$

$$= +\lambda_0 \int f_{Y|\theta_j}(y)\, y^0\, dy + \sum_{k=1}^{m} \lambda_k \cdot M_Y^{\alpha_k}$$

$$\xrightarrow[\text{代入 } \alpha_0,\ \lambda_0 \text{ 则}]{\text{mini}} = \underbrace{\int f_{Y|\theta_j}(y)\, dy}_{M_0 \text{ 零阶矩为1}} \cdot \ln\left[\int_Y \exp\left(-\sum_{k=1}^{m} \lambda_k y^{\alpha_k}\right) dy\right] + \sum_{k=1}^{m} \lambda_k M_Y^{\alpha_k}$$

$$\longrightarrow \text{mini}\ \ln\left[\int_Y \exp\left(-\sum_{k=1}^{m} \lambda_k y^{\alpha_k}\right) dy\right] + \sum_{k=1}^{m} \lambda_k M_Y^{\alpha_k}$$

②.乘法降维:

首先, 由 $\varphi(x) = \ln|g(x)|$, 代入近似有:

$$\varphi(x) \approx \sum_{i=1}^{n} \ln|g(x_i, \mu_{\sim x_i})| - (n-1)\ln|g(x_0)|$$

两边取指数:

$$\exp(\varphi(x)) = g(x) = \frac{\prod_{i=1}^{n} g(x_i, \mu_{\sim x_i})}{g(x_0)^{(n-1)}}$$

整理 $g(x) = g(x_0)^{1-n} \cdot \prod_{i=1}^{n} g(x_i, \mu_{\sim x_i})$

$$\xrightarrow{\text{整理}} g(x) = g(x_0)^{(1-n)} \cdot \prod_{i=1}^{n} g(x_i, \mu_{\sim x_i})$$

由定义Y分数矩为

$$M_Y^{\alpha_k} = \int y^{\alpha_k} f_{Y|\theta_j}(y)\, dy$$

$\Rightarrow$ 其中 $Y = g(X)$，则：

$$M_{Y|\theta_j}^{\alpha_k} = \int_{X|\theta_j} g(x)^{\alpha_k} f_{X|\theta_j}(x)\, dx \text{，代入：}$$

$$= \int_X g(x_0)^{\alpha_k(1-n)} \cdot \prod_{i=1}^{n} \left(g(x_i, \mu_{\sim x_i})\right)^{\alpha_k} f_{X|\theta_j}(x)\, dx$$

$$\xrightarrow{x_0 = \mu_x} \approx g(\mu_x)^{\alpha_k(1-n)} \cdot \prod_{i=1}^{n} \int_{x_i} f_{X|\theta_j}(x) \cdot \left(g(x_i, \mu_{\sim x_i})\right)^{\alpha_k} dx$$

即变成了n个单变量积分连乘

而积分可使用高斯积分公式近似求解