第十二次习题课作业5重解

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对于薄壁圆管，进入塑性状态后：

$\varepsilon_{ij}^{p} = \lambda \cdot S_{ij}$,

①：两端开口：

$S_1 = \frac{2}{3}\sigma,\ S_2 = -\frac{1}{3}\sigma \quad S_3 = -\frac{1}{3}\sigma$,

则 $\varepsilon_1 : \varepsilon_2 : \varepsilon_3 = 2 : -1 : -1$

②：两端闭口：$\sigma_1 = \frac{pr}{t} = 2\sigma \quad \sigma_2 = \frac{pr}{2t} = \sigma,\ \sigma_3 = 0$

$S_1 = \sigma,\ S_2 = 0,\ S_3 = -\sigma$

则：$\varepsilon_1 : \varepsilon_2 : \varepsilon_3 = 1 : 0 : -1$

③：带拉扭开口：$\sigma_\theta = \frac{pr}{t} \quad \sigma_z = \frac{T}{2\pi r t},\ \sigma_3 = 0$

由 $\varepsilon_\theta = 0$, $\quad \sigma_m = \frac{pr}{3t} + \frac{T}{6\pi r t}$

则：$\underbrace{\frac{1+\nu}{E}\left(\frac{2pr}{3t} - \frac{T}{6\pi r t}\right) + \frac{1-2\nu}{E}\left(\frac{pr}{3t} + \frac{T}{6\pi r t}\right)}_{} = 0$

$\therefore = \frac{1}{E} \cdot \frac{pr}{t} - \frac{T}{E\,2\pi r t} = 0$ 按弹性计算：

$\therefore \frac{pr}{t} = \frac{T}{2\pi r t}$

注：若以塑性本构关系计，有：

$d\lambda \cdot S_\theta = 0 \Rightarrow S_\theta \quad 2pr \quad T$

$d\lambda$: $S_\theta = 0$, 而: $S_\theta = \frac{2Pr}{3t} - \frac{T}{2\cdot 3\pi r t}$

而: $\frac{T}{2\pi r t} = \frac{2Pr}{t}$

则: 代入: $S_\theta = \frac{2Pr}{3t} - \frac{T}{6\pi r t} = 0$

$$S_z = \frac{T}{3\pi r t} - \frac{Pr}{3t} = \frac{4Pr}{3t} - \frac{Pr}{3t}$$

则有: $\boxed{S_z = \frac{Pr}{t}}$, $S_r = -\frac{Pr}{3t} - \frac{T}{6\pi r t} = -\frac{Pr}{t}$,

则 $\varepsilon_1 : \varepsilon_2 : \varepsilon_3 = 0 : 1 : -1$

此时:

由: $\sigma_1 = \frac{2Pr}{t}$, $\sigma_2 = \frac{Pr}{t}$, $\sigma_3 = 0$,

则: $\sigma_i = \sqrt{\left(\frac{4P^2r^2}{t^2} + \frac{2P^2r^2}{t^2}\right)\times\frac{1}{2}} = \frac{\sqrt{3}Pr}{t} = \sigma_s$

$\boxed{即: P = \frac{\sigma_s t}{\sqrt{3} r},}$ ①

④ 拉+闭口:

$\sqrt{3}r,$

④ 拉+闭口：

$$\sigma_r=0,\ \sigma_\theta=\frac{pr}{t},\ \sigma_z=\frac{T}{2\pi r t}+\frac{pr}{2t}$$

$$\therefore \sigma_m=\frac{pr}{2t}+\frac{T}{6\pi r t},$$

按塑性计算 $S_\theta=\frac{pr}{2t}-\frac{T}{6\pi r t}=0$

$$\therefore \frac{T}{2\pi r t}=\frac{3pr}{2t},$$

代入：$\sigma_r=0,\ \sigma_\theta=\frac{pr}{t},\ \sigma_z=\frac{2pr}{t},$

$$\therefore \sigma_i=\sqrt{\frac{1}{2}(4+1+1)\frac{p^2r^2}{t^2}}=\frac{\sqrt{3}pr}{t}=\sigma_s,\quad p=\frac{\sigma_s t}{\sqrt{3}r}$$

故：$p_1=p_2=\frac{\sigma_s t}{\sqrt{3}r},$