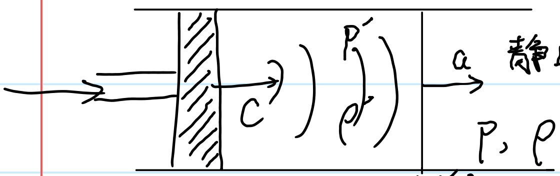
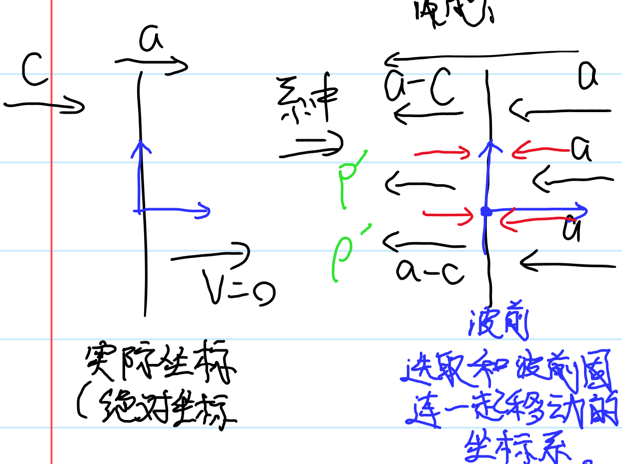
声速的推导过程

Saturday, September 2, 2023 2:26 PM

① : 我们设活塞以速度c向右运动，并产生速度a向右的波，则分析波前前后两侧的压力等情况。



有：设左侧p'，则 实际情况中，右侧气体速度为0，因而取随波前移动坐标系左图，



从而有：

左侧气体速度 a-C，右侧 a，则由连续方程：

$$\rho'(a-C) = \rho a, \quad ① \qquad P_2 - P_1 = F\Delta t$$

由动量定理，单位时间动量变化等于合力，则气体

单位时间：$\Delta P = \rho'(a-c)^2 A - \rho \cdot a^2 A = (p-p')A$

则：$$p'-p = \rho a^2 - \rho'(a-c)^2 \quad ②$$

有：由①②，消c，则有：得到波前速度a表达式，

$$p'-p = \rho a^2 - \rho a \cdot \frac{\rho a}{\rho'}$$

则：$$\left(\rho - \frac{\rho^2}{\rho'}\right) a^2 = p'-p$$

$$= \left(\frac{p'-p}{\rho}\right) \cdot \left(\frac{\rho'}{\rho'-\rho}\right)$$

故：$\boxed{a^2 = \frac{\rho'}{\rho}\left(\frac{p'-p}{\rho'-\rho}\right)}$ 即：$a = \left(\frac{\rho'}{\rho} \frac{p'-p}{\rho'-\rho}\right)^{\frac{1}{2}}$ ☆

$a = \frac{\cdots}{\rho}\left(\frac{\cdots}{\rho'-\rho}\right)$ ... $\left(\frac{\cdots}{\rho}\frac{\cdots}{\rho'-\rho}\right)$ ☆

对于理想气体的定熵过程:

有: $pv^k = const$, 即: $\frac{dp}{p} + k\frac{dv}{v} = 0$

此时: $\frac{\partial p}{\partial v} = -k\frac{p}{v}$ ③

我们指出, 声波振幅很小, 因而近似有: $\frac{\rho'}{\rho} = 1$

而: $a = \left(\frac{p'-p}{\rho'-\rho}\right)^{\frac{1}{2}} = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s} \Rightarrow \sqrt{\left(\frac{\partial p}{\partial v}\right)_s\left(\frac{\partial v}{\partial \rho}\right)_s} = \sqrt{-\frac{1}{\rho^2}\left(\frac{\partial p}{\partial v}\right)_s}$

看成偏导. 视为定熵过程, 由 $v = \frac{1}{\rho}$, 则 $\frac{\partial v}{\partial \rho} = -\frac{1}{\rho^2}$

则: 代③, 得: $a = \sqrt{-\frac{1}{\rho^2}\left(-k\frac{p}{v}\right)} = \sqrt{kpv} = \sqrt{\frac{kp}{\rho}}$

我们由 $pv = RT$, 得: $a = \sqrt{kRT}$

其中: k为比热比, 有时用 γ 表示