混合重要抽样的密度推导

Wednesday, March 13, 2024 11:39 AM

我们知道

$$P_f = \int_{R_n} I_F(x) \frac{f_X(x)}{h_X(x)} h_X(x)\, dx$$

由于我们使用

$$h_X(x) = \sum_{k=1}^{m} \alpha_k h_X^{(k)}(x)$$

则代入有:

$$P_f = \sum_{k=1}^{m} \int_{R_n} I_F(x) \frac{f_X(x)}{h_X^{(k)}(x)} \alpha_k h_X^{(k)}(x)\, dx$$

$$= \sum_{k=1}^{m} E\left(I_F(x) \frac{f_X(x)}{h_X^{(k)}(x)} \alpha_k\right)$$

样本均值计算:

我们取 N_k 个样本，则

$$\hat{P}_f = \sum_{k=1}^{m} \frac{1}{N_k} \sum_{l=1}^{N_k} I_F(x_l) \frac{f_X(x_l)}{h_X^{(k)}(x_l)} \alpha_k$$

对应的均值显然:

$$E(\hat{P}_f) = E\left[\sum_{k=1}^{m} \frac{1}{N_k} \sum_{l=1}^{N_k} I_F(x_l) \frac{f_X(x_l)}{h_X^{(k)}(x_l)} \alpha_k\right]$$

样本均值收敛于总体均值，则:

$$= \sum_{k=1}^{m} \frac{1}{N_k} \sum_{l=1}^{N_k} I_F(x_l) \frac{f_X(x_l)}{h_X^{(k)}(x_l)} \alpha_k] = \hat{P}_f$$

(独立变量)

$$V(a+b) = V(a) + V(b)$$

同样地: $V(\hat{P}_f) = V\left[\sum_{k=1}^{m} \frac{1}{N_k} \sum_{l=1}^{N_k} I_F(x_l) \frac{f_X(x_l)}{h_X^{(k)}(x_l)} \alpha_k\right]$

$$\frac{1}{N_k} \to \frac{1}{N_k^2}.$$

提出 $\sum_{l=1}^{N_k} \to \frac{1}{N_k}$

$$= \frac{1}{N_k} \sum_{k=1}^{m} V\left[I_F(x_l) \frac{f_X(x_l)}{h_X^{(k)}(x_l)} \alpha_k\right]$$

考虑到 $S^2(x) = \frac{1}{N-1}\left[\sum_{i=1}^{N_k} X_i^2 - N\bar{X}^2\right]$

$$= \frac{1}{N_k(N_k-1)} \sum_{k=1}^{m} \left[\sum_{l=1}^{N_k}\left(I_F(x_l) \frac{f_X(x_l)}{h_X^{(k)}(x_l)} \alpha_k\right)^2 - N_k\left(\frac{1}{N_k}\sum_{l=1}^{N_k} I_F(x_l) \frac{f_X(x_l)}{h_X^{(k)}(x_l)} \alpha_k\right)^2\right]$$