作业5.5的解方程

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$$\begin{cases} d\varepsilon_x = \frac{1+\nu}{E}\cdot\frac{2}{3}d\sigma + d\lambda\frac{2}{3}\sigma \\ \frac{1}{2}d\gamma_{xy} = \frac{1+\nu}{E}d\tau_{xy} + d\lambda\,\tau_{xy} \end{cases}$$ 未知：$d\varepsilon_x, d\tau_{xy}$

其中：$d\tau_{xy} = -\frac{\sigma_x}{3\tau_{xy}}d\sigma_x$，

有：~~先解~~：$d\sigma$

$$\begin{cases} \frac{1}{2}d\gamma_{xy} = \frac{1+\nu}{E}\cdot\frac{-\sigma}{3\tau}d\sigma + d\lambda\cdot\tau \\ d\varepsilon_x = \frac{1+\nu}{E}\cdot\cancel{\frac{2}{3}}d\sigma + d\lambda\cdot\cancel{\frac{2}{3}}\sigma \end{cases}$$

此时：消$d\lambda$：

$$\begin{cases} \frac{\gamma_{xy}}{2\tau} = -\frac{1+\nu}{E\sigma}\cdot\frac{\sigma^2}{3\tau^2}d\sigma + d\lambda \quad ① \\ \frac{3d\varepsilon_x}{2\sigma} = \frac{1+\nu}{E\sigma}\cdot d\sigma + d\lambda \quad ②: \end{cases}$$

②-①

有公式：

$$\frac{1+\nu}{E\sigma}\cdot\left(\frac{\sigma^2}{3\tau^2}+1\right)d\sigma = \frac{3d\varepsilon_x}{2\sigma} - \frac{d\gamma_{xy}}{2\tau}$$

则：$d\sigma = \frac{E\cancel{\sigma}}{1+\nu}\left(\frac{3\tau^2}{\sigma^2-3\tau^2}\right)\left[\cancel{\frac{\sigma d\gamma_{xy}-3\tau d\varepsilon_x}{2\tau\sigma}}\right]$ $\frac{E\sigma}{2(1+\nu)}\left(\frac{3\tau^2}{\sigma^2+3\tau^2}\right)\left[\frac{3d\varepsilon_x}{\sigma}-\frac{d\gamma_{xy}}{\tau}\right]$

$$= \frac{E\sigma}{2(1+\nu)}\left(\frac{3\tau}{\sigma^2+3\tau^2}\right)\left(\frac{3d\varepsilon_x}{\sigma}-\frac{d\gamma_{xy}}{\tau}\right) = \mu\cdot\frac{3\tau\cancel{\sigma}}{\sigma_s^2}\cdot\left(\frac{3\tau d\varepsilon_x-\sigma d\gamma_{xy}}{\cancel{\sigma\tau}}\right)$$

$$= \frac{3\mu}{\sigma_s^2}\,\frac{3\tau d\varepsilon_x-\sigma d\gamma_{xy}}{}\cdot\tau$$

又：$d\tau_{xy} = -\frac{\sigma}{3\tau}d\sigma$

$$\cancel{= -\frac{E}{1+\nu}\cdot\frac{\sigma}{\sigma^2-3\tau^2}(d\gamma_{xy}-3d\varepsilon_x)}$$

$$= \frac{3\mu}{\sigma_s^2}(3\tau^2 d\varepsilon_x - \sigma\tau\, d\gamma_{xy})$$

$$= -\frac{\mu}{\sigma_s}(3\tau\sigma\, d\varepsilon_x - \sigma^2 d\gamma_{xy})$$

注意量纲是 $\frac{E}{\nu}\cdot d\varepsilon$

正解：

(6) 设处于平面应力状态的弹塑性Mises材料，已知：$\sigma_x=\sigma$，$\tau_{xy}=\tau$，$\sigma_y=0$，且满足屈服条件：$\sigma^2+3\tau^2-\sigma_s^2=0$，此时施加应力增量$(d\sigma, d\tau)$，使材料处于加载，使用$E, \sigma_s, \sigma, \tau$、$d\varepsilon_x$、$d\gamma_{xy}$表示$d\sigma, d\tau$，其中有：$\nu=\frac{1}{2}$，

解：①：平面应力状态：有 $\sigma_z=0$，

(1)，则弹性本构关系

$$\begin{cases} \varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) \\ \varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) \\ \varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y) \end{cases}$$

由塑性本构：

$$d\varepsilon_{ij} = \frac{1}{2\mu}dS_{ij} + \frac{1-2\nu}{E}d\sigma_m\delta_{ij} + d\lambda\cdot S_{ij},$$

分别在 σ,τ 方向，有：$dS_z=\frac{2}{3}d\sigma$，$d\sigma_m=\frac{1}{3}d\sigma$，$S_z=\frac{2}{3}\sigma$，

则有

$$\begin{cases} ①\ d\varepsilon_x = \frac{1}{2\mu}\cdot\frac{2}{3}d\sigma + \frac{1-2\nu}{E}\cdot\frac{d\sigma}{3} + \frac{2}{3}\sigma\cdot d\lambda. \\ ②：\frac{1}{2}d\gamma_{xy} = \frac{1}{2\mu}\cdot d\tau + d\lambda\cdot\tau. \end{cases}$$

本构关系 ⇒

此时，有：屈服条件仍然满足，即：

$$\sigma^2+3\tau^2=\sigma_s^2, \xrightarrow[\text{得加载方程}]{\text{两边求导}} \cancel{2d\sigma+6d\tau=0}\quad 2\sigma d\sigma+6\tau d\tau=0$$

求全微分！

$\cancel{d\sigma=-3d\tau}$，③，

联立①②③，

由1)：$$\frac{3}{2}\frac{d\varepsilon_x}{\sigma} = \frac{1}{2\mu\sigma}d\sigma + \frac{1-2\nu}{2E}\cdot\frac{d\sigma}{\sigma} + d\lambda$$

由2)：$$\frac{d\gamma_{xy}}{2\tau} = \frac{1}{2\mu\tau}d\tau + d\lambda$$

消去λ，有：

$$\frac{3}{2}\frac{d\varepsilon_x}{\sigma} - \frac{d\gamma_{xy}}{2\tau} = \frac{d\sigma}{2\mu\sigma} + \cancel{\frac{1-2\nu}{2E}}\cdot\frac{d\sigma}{\sigma} - \frac{d\tau}{2\mu\tau}$$

✓

~~由 $d\sigma=-3d\tau$，代入解 $d\tau$，有：~~ 方程代入错误！

改正：代入 $\boxed{\sigma d\sigma=-3\tau d\tau}$ → 先解 $d\sigma$，则代 $d\tau=-\frac{\sigma}{3\tau}d\sigma$

则有：

$$\frac{3}{2}\frac{d\varepsilon_x}{\sigma} - \frac{d\gamma_{xy}}{2\tau} = \frac{d\sigma}{2\mu\sigma} + \frac{1-2\nu}{2E}\cdot\frac{d\sigma}{\sigma} + \frac{\sigma}{6\mu\tau^2}d\sigma$$

$$= d\sigma\left[\frac{1}{2\mu\sigma} + \frac{\sigma}{6\mu\tau^2} + \frac{1-2\nu}{2E\sigma}\right]$$

$$= \frac{d\sigma}{\sigma}\left[\frac{1}{2\mu} + \frac{\sigma^2}{6\mu\tau^2} + \frac{1-2\nu}{2E}\right]$$

$$\frac{3d\varepsilon_x}{2\sigma} - \frac{d\gamma_{xy}}{2\tau} = \frac{d\sigma}{\sigma}\left[\frac{\sigma_s^2}{6\mu\tau^2} + \frac{1-2\nu}{2E}\right]$$

由于 $\nu=\frac{1}{2}$，需舍去。

$$\left[\frac{3}{2}d\varepsilon_x - \frac{\sigma}{2\tau}d\gamma_{xy}\right] = \frac{(1+\nu)}{3E}\frac{\sigma_s^2}{\tau^2}d\sigma$$

此时：$$d\sigma = \frac{3}{2}\frac{E}{(1+\nu)}\left[3d\varepsilon_x - \frac{\sigma}{\tau}d\gamma_{xy}\right]\cdot\frac{\tau^2}{\sigma_s^2}$$

$$= \frac{3E}{2(1+\nu)\sigma_s^2}\left[3\tau^2 d\varepsilon_x - \sigma\tau d\gamma_{xy}\right]$$

由：$d\tau=-\frac{\sigma}{3\tau}d\sigma$，有：

$$d\tau = -\frac{E}{2(1+\nu)\sigma_s^2}\left[\frac{3\tau}{\sigma}d\varepsilon_x - \sigma^2 d\gamma_{xy}\right]$$