厚壁圆筒受压应力公式推导

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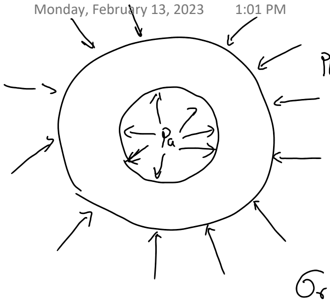

设圆筒受内压为 P_a，外压 P_b，

$$\begin{cases}\sigma_r = \dfrac{A}{r^2}+2C\\[2ex] \sigma_\theta = -\dfrac{A}{r^2}+2C\end{cases}$$

边界条件：

$\sigma_r|_{r=a} = -P_a$，

$\sigma_r|_{r=b} = -P_b$

则：
$$\begin{cases}\dfrac{A}{a^2}+2C=-P_a\\[2ex] \dfrac{A}{b^2}+2C=-P_b\end{cases}$$

$$P_b-P_a = A\left(\frac{1}{a^2}-\frac{1}{b^2}\right)$$

$$\therefore A = \frac{P_b-P_a}{\frac{1}{a^2}-\frac{1}{b^2}} = \frac{a^2b^2(P_b-P_a)}{b^2-a^2} = \frac{a^2(P_b-P_a)}{1-\frac{a^2}{b^2}}$$

而：
$$2C = -\frac{A}{a^2}-P_a = -\frac{b^2(P_b-P_a)}{b^2-a^2}-P_a = \frac{a^2P_a-b^2P_b}{b^2-a^2} = \frac{\left(\frac{a^2}{b^2}\right)P_a-P_b}{1-\frac{a^2}{b^2}}$$

$\therefore$ 代入：

$$\sigma_r = \frac{A}{r^2}+2C = \frac{\frac{a^2}{r^2}(P_b-P_a)}{1-\frac{a^2}{b^2}} + \frac{\left(\frac{a^2}{b^2}\right)P_a-P_b}{1-\frac{a^2}{b^2}} = \frac{\left(\frac{a^2}{b^2}-\frac{a^2}{r^2}\right)}{1-\frac{a^2}{b^2}}P_a + \frac{\frac{a^2}{r^2}-1}{1-\frac{a^2}{b^2}}P_b$$

$$= \boxed{-\frac{\frac{b^2}{r^2}-1}{\frac{b^2}{a^2}-1}P_a - \frac{1-\frac{a^2}{r^2}}{1-\frac{a^2}{b^2}}P_b}$$ 为内应力公式

$$\sigma_\theta = -\frac{A}{r^2}+2C = \frac{-\frac{a^2}{r^2}(P_b-P_a)}{1-\frac{a^2}{b^2}} + \frac{\left(\frac{a^2}{b^2}\right)P_a-P_b}{1-\frac{a^2}{b^2}} = \frac{\frac{a^2}{r^2}+\frac{a^2}{b^2}}{1-\frac{a^2}{b^2}}P_a + \frac{-\frac{a^2}{r^2}-1}{1-\frac{a^2}{b^2}}P_b$$

$$= \boxed{\frac{\frac{b^2}{r^2}+1}{\frac{b^2}{a^2}-1}P_a - \frac{1+\frac{a^2}{r^2}}{1-\frac{a^2}{b^2}}P_b}$$

特殊情况：

① $a=0$，$P_a=0$ 时：
$$\begin{cases}\sigma_r = -P_b\\ \sigma_\theta = -P_b\end{cases}$$

② $q_b=0$ 时：空心受内压：
$$\begin{cases}\sigma_r = -\dfrac{\frac{b^2}{r^2}-1}{\frac{b^2}{a^2}-1}P_a\\[3ex] \sigma_\theta = \dfrac{\frac{b^2}{r^2}+1}{\frac{b^2}{a^2}-1}P_a\end{cases}$$

③ 空心，外压：
$$\begin{cases}\sigma_r = -\dfrac{1-\frac{a^2}{r^2}}{1-\frac{a^2}{b^2}}P_b\\[3ex] \sigma_\theta = -\dfrac{1+\frac{a^2}{r^2}}{1-\frac{a^2}{b^2}}P_b\end{cases}$$

④ $b\to\infty$，$P_b=0$（无限大弹性体）
$$\begin{cases}\sigma_r = -\dfrac{a^2}{r^2}P_a\\[2ex] \sigma_\theta = \dfrac{a^2}{r^2}P_a\end{cases}$$