1章弹簧滑块例题

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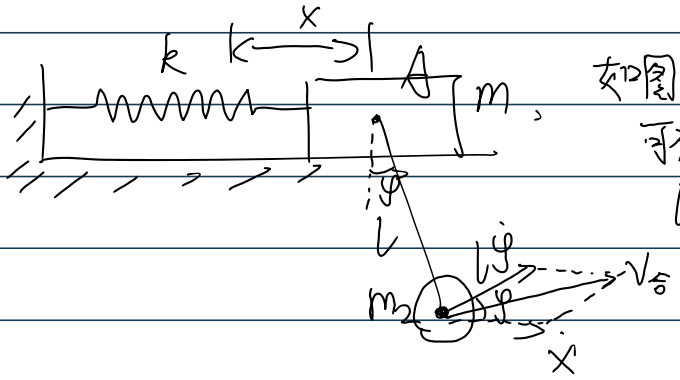

如图：与刚度k弹簧相连的滑块A，质量m_1，可在光滑水平面上滑动，A滑块连一质量m_2长为l的单摆

分析：2自由度，x，φ为广义坐标

有：$T=\frac{1}{2}m\dot{x}^2+\frac{1}{2}m_2\left[(\dot{x}+l\dot{\varphi}\cos\varphi)^2+(l\dot{\varphi}\sin\varphi)^2\right]$

而势能以y=0为势能零点，有：

$$V=\frac{1}{2}kx^2-m_2g\cdot l\cos\varphi$$

得到：$L=T-V=\frac{1}{2}m_1\dot{x}^2+\frac{1}{2}m_2\left[(\dot{x}+l\dot{\varphi}\cos\varphi)^2+(l\dot{\varphi}\sin\varphi)^2\right]-\frac{1}{2}kx^2+m_2gl\cos\varphi$

显然由

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{x}}\right)-\frac{\partial L}{\partial x}=0,$$

即：

$$m_1\ddot{x}+\frac{d}{dt}\left[m_2(\dot{x}+l\dot{\varphi}\cos\varphi)\right]+kx=0$$

$$m_1\ddot{x}+m_2(\ddot{x}+l\ddot{\varphi}\cos\varphi-l\dot{\varphi}^2\sin\varphi)-kx=0 \quad ①$$

$$m_2\frac{d}{dt}\left[(\dot{x}+l\dot{\varphi}\cos\varphi)\cdot l\cos\varphi+l^2\sin^2\varphi\cdot\dot{\varphi}\right]+m_2gl\sin\varphi=0$$

注意：先进行化简

$$m_2\frac{d}{dt}(\dot{x}l\cos\varphi+l^2\cos^2\varphi\,\dot{\varphi}+l^2\sin^2\varphi\,\dot{\varphi})+m_2gl\sin\varphi=0$$

$$\ddot{x}\cos\varphi+l\ddot{\varphi}+g\sin\varphi=0$$

即：有：

① $(m_1+m_2)\ddot{x}+m_2l\ddot{\varphi}\cos\varphi-m_2l\dot{\varphi}^2\sin\varphi+kx=0$

② $\ddot{x}\cos\varphi+l\ddot{\varphi}+g\sin\varphi=0$

即为运动方程。