Derivation of high-ordered interpolation basis function

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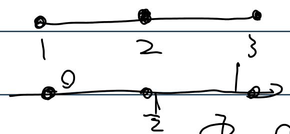
for a line segment with 3 nodes



we firstly set the interpolation function as function of 2^{rd} order

$\Phi_1 = a_1x^2 + b_1x + c_1$

then $\Phi_1(x_1) = 1 \longrightarrow c_1 = 1$, $\Phi_1(x_2) = 0 \quad \frac{a_1}{4} + \frac{b_1}{2} + 1 = 0$, $\Phi_1(x_3) = 0$, $a_1 + b_1 + 1 = 0$

$\Phi_2(x_1) = 0 \longrightarrow c_2 = 0$

$\longrightarrow -\frac{a_1}{4} + \frac{1}{2} = 0 \rightarrow a_1 = 2, b_1 = -3$

$\therefore \Phi_1 = 2x^2 - 3x + 1 = (2x-1)(x-1) = (2\xi - 1)(\xi - 1)$

$\Phi_2(x_2) = 1 \rightarrow \frac{a_2}{4} + \frac{b_2}{2} = 1$

$\Phi_2(x_3) = 0 \rightarrow a_2 + b_2 = 0$

$\Rightarrow -\frac{a_2}{4} = 1$, $\therefore a_2 = -4, b_2 = 4 \quad c_2 = 0$

$\Phi_2 = -4x^2 + 4x$

$= 4x(1-x) \quad (\text{local} \rightarrow 4\xi(1-\xi))$

$\Phi_3(x_1) = 0$, $c_3 = 0$, then

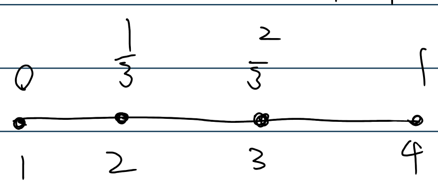
$\Phi_3(x_2) = 0$, $\frac{a_3}{4} + \frac{b_3}{2} = 0$, $\quad \therefore -b_3 = 1 \rightarrow \begin{cases} b_3 = -1 \\ a_3 = 2 \end{cases} \quad c_3 = 0$

then $a_3 + b_3 = 1$,

$\Phi_3 = 2\xi^2 - \xi = \xi(2\xi - 1)$

if we use the 3^{rd} ordered Lagrange Interpolation function

$\xi = \frac{x - x_1}{x_4 - x_1}$ then $\Phi = ax^3 + bx^2 + cx + d$



$\Phi_1(0) = 1$, $d_1 = 1$,

$\Phi_1(\frac{1}{3}) = 0$, $\frac{a_1}{27} + \frac{b_1}{9} + \frac{c_1}{3} = -1$

$\frac{8a_1}{27} + \frac{4}{9}b_1 + \frac{2}{3}c_1 = -1$

$a_1 + b_1 + c_1 = -1$

so $a_1 + b_1 = -1 - c_1 = \frac{11}{2} - 1 = \frac{9}{2}$

$\frac{2}{9}\cancel{\frac{6}{27}}a_1 + \frac{2}{9}b_1 = 1 \quad \rightarrow a_1 + b_1 = \frac{9}{2}$

$\therefore \frac{9}{2} + c_1 = -1$, $\therefore c_1 = -\frac{11}{2}$

$\frac{a}{27} + \frac{b}{9} - \frac{11}{6} = -1 \Rightarrow \frac{a}{27} + \frac{b}{9} = \frac{11}{6} - 1 = \frac{5}{6} \qquad 2\frac{5}{\cancel{6}}\times 27\,9$

$a + 3b = \frac{45}{2}$

$\therefore 2b = \frac{45}{2} - \frac{9}{2} = \frac{36}{2} = 18 \quad \rightarrow b_1 = 9, \quad a_1 = -\frac{9}{2}$

$\therefore a_1 = -\frac{9}{2}, b_1 = 9, c_1 = -\frac{11}{2}, d_1 = 1$

[illegible]

$\therefore\ a_1 = -\frac{9}{2},\ b_1 = 9,\ c_1 = -\frac{11}{2},\ d_1 = 1$

$$\Phi_1 = -\frac{9}{2}x^3 + 9x^2 - \frac{11}{2}x + 1$$

$$\frac{1}{2}(-9\xi^3 + 18\xi^2 - 11\xi + 2)$$

$$\frac{1}{2}(1-3\xi)(2-3\xi)(1-\xi)$$

we don't derive other functions here.