

Derivation of Nonlinear steady problem

Sunday, March 26, 2023 10:28 AM

the initial equation of the problem is:

$$\alpha \frac{\partial u}{\partial x^2} - u \frac{\partial u}{\partial x} = -P \quad \text{firstly we write it as:}$$

where u is the speed of fluid

$$\int_{\Omega} \left[\alpha \frac{\partial^2 u}{\partial x^2} - \frac{\partial}{\partial x} \left(\frac{1}{2} u^2 \right) + P \right] \delta u \, d\Omega = 0$$

considering

$$\int_a^b \delta u \frac{\partial^2 u}{\partial x^2} dx = \left(\frac{\partial u}{\partial x} \delta u \right) \Big|_a^b - \int_a^b \frac{\partial (\delta u)}{\partial x} \frac{\partial u}{\partial x} dx$$

then eq becomes:

$$\int_{\Omega} \left(\alpha \frac{\partial (\delta u)}{\partial x} \frac{\partial u}{\partial x} - P \delta u \right) d\Omega = \alpha \left(\frac{\partial u}{\partial x} \delta u \right) \Big|_a^b - \int_{\Omega} \frac{\partial}{\partial x} \left(\frac{1}{2} u^2 \right) d\Omega$$

also we set Γ_2 as the natural boundary condition,

→ then on Γ_1 , $\frac{\partial u}{\partial x} = 0$,

partial integral

$$\text{then we have } \int_{\Omega} \delta u \frac{\partial}{\partial x} \left(\frac{1}{2} u^2 \right) d\Omega = \left. \frac{1}{2} u^2 \delta u \right|_a^b - \int_{\Omega} \frac{1}{2} u^2 \frac{\partial (\delta u)}{\partial x} d\Omega$$

hence:

$$\int_{\Omega} \left(\alpha \frac{\partial (\delta u)}{\partial x} \frac{\partial u}{\partial x} - \frac{1}{2} u^2 \frac{\partial (\delta u)}{\partial x} - P \delta u \right) d\Omega = \left(\alpha \frac{\partial u}{\partial x} \delta u - \frac{1}{2} u^2 \delta u \right) \Big|_a^b$$

the right term is equal to → $\int_{\Gamma} \left(\alpha \frac{\partial u}{\partial x} + \frac{1}{2} u^2 \right) \delta u \, d\Gamma$ (???)

then the weak form is:

$$\int_{\Omega} \left(\alpha \frac{\partial (\delta u)}{\partial x} \frac{\partial u}{\partial x} - \frac{1}{2} u^2 \frac{\partial (\delta u)}{\partial x} - P \delta u \right) d\Omega = \int_{\Gamma} \left(\alpha \frac{\partial u}{\partial x} - \frac{1}{2} u^2 \right) \delta u \, d\Gamma$$

we substitute

$$u^{(e)} = \Phi_i u_i^{(e)}, \quad \text{pay attention that } u^2 = \Phi_i \Phi_j u_i^{(e)} u_j^{(e)}$$

$$\int_{\Omega} \left(\alpha \frac{\partial \Phi_i}{\partial x} \frac{\partial \Phi_k}{\partial x} u_i^{(e)} \delta u_k^{(e)} - \frac{1}{2} \Phi_i \Phi_j \frac{\partial \Phi_k}{\partial x} u_i^{(e)} u_j^{(e)} \delta u_k^{(e)} - P \Phi_k \delta u_k^{(e)} \right) d\Omega = \int_{\Gamma} \left(\alpha \frac{\partial \Phi_i}{\partial x} - \frac{1}{2} \Phi_i \Phi_j u_i^{(e)} u_j^{(e)} \right) \Phi_k \delta u_k^{(e)} d\Gamma$$

that is:

$$u_i^{(e)} \left[\int_{\Omega} \alpha \frac{\partial \Phi_i}{\partial x} \frac{\partial \Phi_k}{\partial x} d\Omega - \int_{\Gamma_2} \alpha \frac{\partial \Phi_i}{\partial x} \Phi_k d\Gamma \right] + u_i^{(e)} u_j^{(e)} \left[\int_{\Omega} \frac{1}{2} \Phi_i \Phi_j \frac{\partial \Phi_k}{\partial x} d\Omega + \int_{\Gamma_2} \frac{1}{2} \Phi_i \Phi_j \Phi_k d\Gamma \right] = \int_{\Omega} P \Phi_k d\Omega$$

negative

we set

$$A_{ik}^{(e)} = \int_{\Omega} \alpha \frac{\partial \Phi_i}{\partial x} \frac{\partial \Phi_k}{\partial x} d\Omega - \int_{\Gamma_2} \alpha \frac{\partial \Phi_i}{\partial x} \Phi_k d\Gamma$$

$$B_{ijk}^{(e)} = - \int_{\Omega} \frac{1}{2} \Phi_i \Phi_j \frac{\partial \Phi_k}{\partial x} d\Omega + \int_{\Gamma_2} \frac{1}{2} \Phi_i \Phi_j \Phi_k d\Gamma$$

$$\text{and } f_k = \int_{\Omega} P \Phi_k d\Omega$$

then the equation becomes:

$$A_{ik}^{(e)} u_i^{(e)} + B_{ijk}^{(e)} u_i^{(e)} u_j^{(e)} = f_k$$

② 计算??