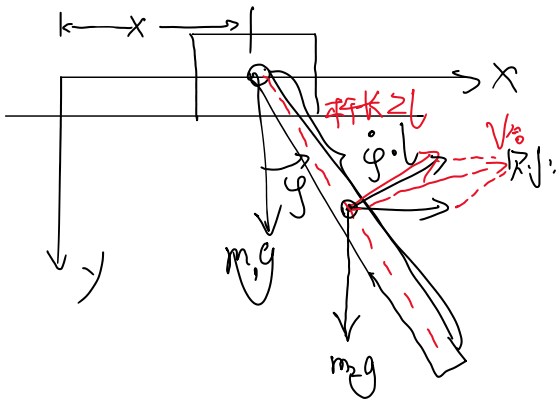
例题1.3

Saturday, March 18, 2023 10:09 AM

如图：系统由滑块A，均质细杆AB构成，其中滑块A质量m，且可沿光滑水平面自由移动，而细杆AB由圆柱铰链连接于A上，AB质量m_2，长2l，列系统运动微分方程



分析：图中为非保守系统，

使用x，φ作为广义坐标，列Lagrange平衡方程

2自由度；

$$L=T-V$$

$$\begin{cases}\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}}\right)-\dfrac{\partial L}{\partial x}=0\\[2ex]\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{\varphi}}\right)-\dfrac{\partial L}{\partial \varphi}=0\end{cases}$$

系整体系统：$T=\frac{1}{2}m_1\dot{x}^2+\frac{1}{2}m_2(V_a)^2+\frac{1}{2}I_2\omega_2^2$

注意速度是整体合速度　转动动能、

则：$T=\frac{1}{2}m_1\dot{x}^2+\frac{1}{2}m_2\left[(\dot{\varphi}l\cos\varphi+\dot{x})^2+(\dot{\varphi}l\sin\varphi)^2\right]+\frac{1}{2}\left[\frac{1}{12}\cdot m_2(2l)^2\cdot\dot{\varphi}^2\right]$

$=\frac{1}{2}m_1\dot{x}^2+\frac{1}{2}m_2\left[\dot{\varphi}^2l^2+\dot{x}^2+2\dot{x}\dot{\varphi}l\cos\varphi\right]+\frac{1}{6}m_2l^2\dot{\varphi}^2$ ✓

系势能$V=-m_2gl\cos\varphi$

则由Lagrange方程，有：$L=T-V=\frac{1}{2}m_1\dot{x}^2+\frac{1}{2}m_2[\dot{\varphi}^2l^2+\dot{x}^2+2\dot{x}\dot{\varphi}l\cos\varphi]+\frac{1}{6}m_2l^2\dot{\varphi}^2+m_2gl\cos\varphi$

系：代入方程：

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x}=0$，$\Rightarrow$ $m_1\ddot{x}+m_2\ddot{x}+\frac{d}{dt}(m_2\dot{\varphi}l\cos\varphi)$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)=(m_1+m_2)\ddot{x}+m_2\ddot{\varphi}l\cos\varphi-m_2\dot{\varphi}^2l\sin\varphi=0$

$\frac{\partial L}{\partial x}=0$ ↗

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right)-\frac{\partial L}{\partial \varphi}=0$，$\Rightarrow$ $m_2\ddot{\varphi}l^2+\frac{d}{dt}m_2\dot{x}l\cos\varphi+\frac{1}{3}m_2l^2\ddot{\varphi}+\cancel{m_2\dot{x}\dot{\varphi}l\sin\varphi}+m_2gl\sin\varphi=0$

$\Rightarrow$ $\frac{4}{3}m_2l^2\ddot{\varphi}+m_2gl\sin\varphi=0$

$+m_2\ddot{x}l\cos\varphi$补

→ $\frac{4}{3}m_2l^2\ddot{\varphi}+m_2\ddot{x}l\cos\varphi-\cancel{m_2\dot{x}l\sin\varphi\cdot\dot{\varphi}}+\cancel{m_2\dot{x}l\sin\varphi\cdot\dot{\varphi}}+m_2gl\sin\varphi=0$、

注意这样对

即：运动微分方程：

$$\begin{cases}(m_1+m_2)\ddot{x}+m_2\ddot{\varphi}l\cos\varphi-m_2\dot{\varphi}^2l\sin\varphi=0\\ m_2\ddot{x}l\cos\varphi+\frac{4}{3}m_2l^2\ddot{\varphi}+m_2gl\sin\varphi=0\end{cases}$$

为解答、

$m_2\ddot{x}l\cos\varphi + \frac{4}{3}m_2l^2\ddot{\varphi} + m_2gl\sin\varphi = 0$

$4l\ddot{\varphi} + 3g\sin\varphi + 3\ddot{x}\cos\varphi$

为解答、