理想变压器的变比关系推导

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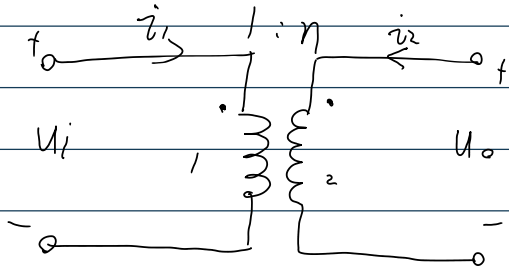

我们设有如图的理想变压器，则：

由 $u_i = u_{i1} + u_{i2}$

$u_o = u_{o1} + u_{o2}$

显然：$u_{i1} = \frac{d\psi_{i1}}{dt} = L_1 \frac{di_1}{dt} = N_1 \frac{d\phi_{i1}}{dt}$

由于线性为高导磁：$d\phi_{i1} = d\phi_{o1}$，$d\phi_{i2} = d\phi_{o2}$，则：

$$\left.\begin{aligned} u_{in} &= L_1 \frac{di_1}{dt} + M\frac{di_2}{dt} = N_1\frac{d\phi_1}{dt} + N_1\frac{d\phi_{i2}}{dt} = N_1\frac{d\phi_1}{dt} \\ u_{out} &= M\frac{di_1}{dt} + L_2\frac{di_2}{dt} = N_2\frac{d\phi_{o1}}{dt} + N_2\frac{d\phi_{o2}}{dt} = N_2\frac{d\phi_2}{dt} \end{aligned}\right\} \text{故 } \frac{u_{out}}{u_{in}} = \frac{N_2}{N_1}$$

$\Rightarrow$ 同时有功率关系 $u_{in} I_1 + u_{out} I_2 = 0$，则有关系

$$\boxed{I_2 = -\frac{1}{n} I_1}$$ 电流关系

即此变压器：

$$u_{out} = \frac{N_2}{N_1} u_{in} = n u_{in}$$

根据自感系数和互感系数的定义有：

$\frac{d\psi}{dt} = N\frac{d\phi}{dt} = L\frac{di}{dt}$（一般电感）

$= \sqrt{\frac{\phi_{12}\phi_{21}}{\phi_1\phi_2}} \Rightarrow$ 由于 $\phi_{12} = \phi_2$ 是显然的，$\phi_{21} = \phi_1$

耦合电感 $\begin{cases} u_{in} = j\omega L_1 \dot{I}_1 + j\omega M \dot{I}_2 \\ u_{out} = j\omega M \dot{I}_1 + j\omega L_2 \dot{I}_2 \end{cases} \Rightarrow$ 根据 $K = \frac{M}{\sqrt{L_1 L_2}} = 1$，则有 $M = \sqrt{L_1 L_2}$，

则 $u_{in} \cdot I_1 + u_{out} \cdot I_2 = 0$

$\rightarrow j\omega L_1 I_1^2 + j\omega M I_1 I_2 + j\omega L_2 I_2^2 = 0 \Rightarrow$ 有：$L_1 I_1^2 + 2\sqrt{L_1 L_2}\, I_1 I_2 + L_2 I_2^2 = 0$

$\because$ 由于 $I_2 = -\frac{1}{n} I_1$，则 $I_1 = -n I_2$，同除 I_2^2，有：$n^2 L_1 - 2n\sqrt{L_1 L_2} + L_2 = 0$

即：$\left(n\sqrt{L_1} - \sqrt{L_2}\right)^2 = 0 \rightarrow$ 有：$n\sqrt{\frac{L_1}{L_2}} = 1$，即：$\sqrt{\frac{L_1}{L_2}} = \frac{1}{n} = \frac{N_1}{N_2}$，得：$\sqrt{\frac{L_1}{L_2}} = \frac{N_1}{N_2}$

有关等：

另外，$\Psi = KN \cdot I$，$\rightarrow L = KN^2 = N\frac{\Psi}{I}$ 为电感；

则：$L_1 = N_1 \frac{\Psi_{11}}{i_1} = \frac{N_1}{N_2} \cdot \underbrace{\frac{N_2 \Psi_{21}}{i_1}}_{M} = \frac{1}{n} M$

$L_2 = N_2 \frac{\Psi_{22}}{i_2} = \frac{N_2}{N_1} \cdot \frac{N_1 \Psi_{12}}{i_2} = n \cdot M$

因为 $\frac{L_1}{L_2} = \frac{1}{n^2} = \frac{N_1^2}{N_2^2}$

$\rightarrow \sqrt{\frac{L_1}{L_2}} = \frac{N_1}{N_2}$