The Pressure distribution of the cylinder under a lifting force

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Firstly, the velocity is given as:

$$V_r = V_\infty \cos\theta \left(1 - \frac{R^2}{r^2}\right)$$

$$V_\theta = -V_\infty \sin\theta \left(1 + \frac{R^2}{r^2}\right) - \frac{\Gamma}{2\pi r},$$

then for $r = R$,

we have:

$$V_r = 0, \quad V_\theta = V = \underline{-2 V_\infty \sin\theta - \frac{\Gamma}{2\pi R}}$$

then using the laplace's equation,

$$P_\infty + \frac{1}{2}\rho V_\infty^2 = P + \frac{1}{2}\rho V^2$$

then

$$C_p = 1 - \left(\frac{V}{V_\infty}\right)^2 = 1 - \left(-2\sin\theta - \frac{\Gamma}{2\pi R V_\infty}\right)^2$$

$$= 1 - \left(2\sin\theta + \frac{\Gamma}{2\pi R V_\infty}\right)^2$$

hence for lifting coefficient,

$$C_l = -\frac{1}{2}\int_0^{2\pi} C_p \sin\theta \, d\theta$$

$$= -\frac{1}{2}\int_0^{2\pi} \left[\cancel{1} - 4\sin^2\theta - \frac{2\Gamma}{\pi R V_\infty}\sin\theta - \cancel{\frac{\Gamma^2}{2\pi R^2 V_\infty}}\right] \sin\theta \, d\theta.$$

$$= \frac{1}{2}\int^{2\pi} \underbrace{4\sin^3\theta}_{0} + \frac{2\Gamma}{\pi \ldots}\sin^2\theta \, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} \overbrace{4\sin^3\theta}^{0} + \frac{2\Gamma}{\pi R V_\infty}\sin^2\theta \; d\theta ,$$

$$= 2\cdot\left[\cancel{4\times\frac{2}{3}} + \frac{2\Gamma}{\pi R V_\infty}\cdot\frac{1}{\cancel{2}}\times\frac{\cancel{\pi}}{2}\right]$$

$$= 2\times\left[\frac{8}{3} + \frac{\Gamma}{2V_\infty R}\right] = \cancel{\frac{16}{3}} + \frac{\Gamma}{V_\infty R} , \quad = \frac{\Gamma}{V_\infty R}$$

for integral from 0 to 2π.

$\int_0^{2\pi}\sin^3\theta \, d\theta = 0$ (can't be projected in $[0,\frac{\pi}{2}]$)