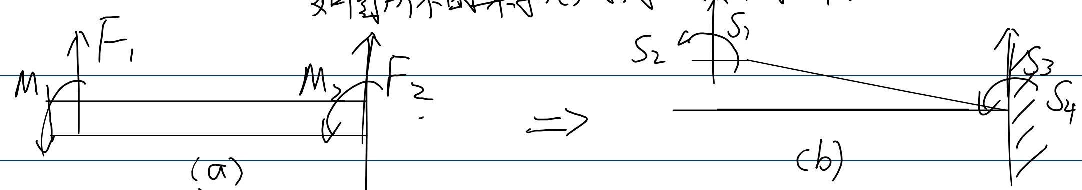
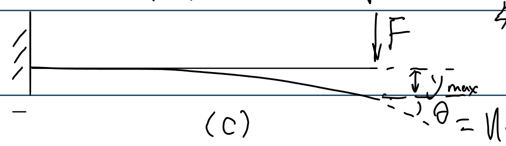
梁的刚度矩阵的求出

Monday, March 13, 2023 9:33 AM

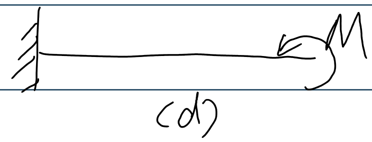
如图所示的梁单元,可等化为右图所示:



由材力知变形公式:



知: $\theta = \frac{Fl^2}{2EI}$, $W_{max} = \frac{Fl^3}{3EI}$.



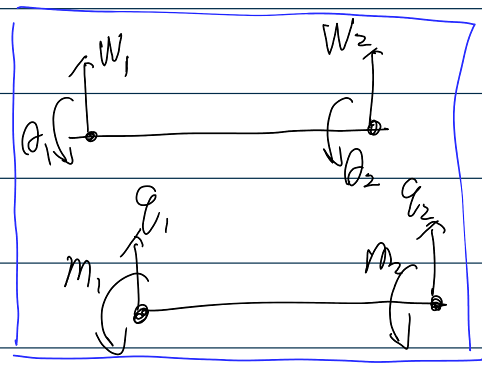
$\theta_{max} = \frac{M_B l}{EI}$, $y_{max} = \frac{M_B l^2}{2EI}$

则:对于图(b)中的梁有:

$$\begin{cases} W = \frac{Fl^3}{3EI} - \frac{Ml}{2EI} = \frac{S_1 l^3}{3EI} - \frac{S_2 l^2}{2EI} = u_1 \\ \theta = \frac{Ml}{EI} - \frac{Fl^2}{2EI} = \frac{S_2 l}{EI} - \frac{S_1 l^2}{2EI} = u_2 \end{cases}$$

S_2 验证

利用单元刚度矩阵: $\{P\}^e = [k]^e \{\delta\}^e$



$\therefore 2u_1 + u_2 l = \frac{2S_1 l^3}{3EI} - \frac{S_1 l^3}{2EI} = \frac{S_1 l^3}{6EI}$

此时:令 $u_1 = 1$, 特0 得: $S_1 = \frac{12EI}{l^3} = a_{11}$

代入:有: $4 - \frac{S_2 l^2}{2EI} = 1$, $\rightarrow \frac{S_2 l^2}{2EI} = 3$, 则: $S_2 = \frac{6EI}{l^2} = a_{21}$

显然由平衡方程: $S_3 = -S_1 = -\frac{12EI}{l^3} = a_{31}$

$S_4 = S_1 l - S_2 = \frac{12EI}{l^2} - \frac{6EI}{l^2} = \frac{6EI}{l^2} = a_{41}$.

又:令 $u_2 = 1$, $u_1 = 0$ 得: $\begin{cases} \frac{S_1 l^3}{3EI} = \frac{S_2 l^2}{2EI} \\ \frac{S_2 l}{EI} = \frac{S_1 l^2}{2EI} + 1 \end{cases}$ 有 $\Rightarrow \frac{S_1 l^3}{3EI} = \frac{S_1 l^3}{4EI} + \frac{l}{2}$

$\therefore S_1 = \frac{6EI}{l^2} = a_{12}$

而: $S_2 = \frac{2}{3} S_1 l = \frac{4EI}{l} = a_{22}$, $S_3 = -S_1 = -\frac{6EI}{l^2} = a_{32}$

$S_4 = S_1 l - S_2 = \frac{2EI}{l} = a_{42}$

故可得到一半的矩阵:

另一半可由对称性求出,故:

$$[k] = \frac{EI}{l^3} \left[\begin{array}{cc:cc} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ \hdashline -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{array}\right]$$

对称: $u_3 = 1$ 时: 对 θ_2 影响是相反的, W 影响相同。

利用节点分块表示:

$$[k] \quad l \begin{bmatrix} & -6l & 12 & -6l \\ & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

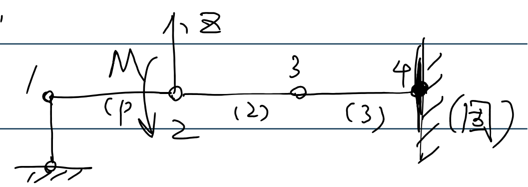
w 和θ相同

利用节点分块表示；

将上式分块矩阵记为：

$$[k]^e = \begin{bmatrix} k_{ii} & k_{ij} \\ k_{ji} & k_{jj} \end{bmatrix}，即得：\begin{Bmatrix} P_i \\ P_j \end{Bmatrix}^e = \begin{bmatrix} k_{ii} & k_{ij} \\ k_{ji} & k_{jj} \end{bmatrix}^e \begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix}^e$$

使用增广矩阵法求解；即：对于右图杆系；有：



$$[K]\{\delta\} = \{Q\}，或：$$

$$K = \sum_{e=1}^{m} [k]^e \qquad \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix} = \begin{bmatrix} K_{11}^1 & K_{12}^1 & & \\ K_{21}^1 & K_{22}^1 + K_{22}^2 & K_{23}^2 & \\ & K_{23}^2 & K_{33}^2 + K_{33}^3 & K_{43}^3 \\ & & K_{43}^3 & K_{44}^3 \end{bmatrix}$$

(3)、边界约束情况：上式为 8个方程联立，

8个位移：$f_1 = f_4 = \theta_4 = 0$，　而：Z_1, Z_4, M_4 为约束反力，未知；

而：其余5个分量为已知或为0，方程可解