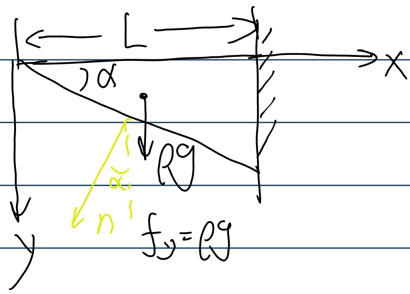
第三次作业

Thursday, March 30, 2023 12:49 PM

对于如图楔形体，受重力作用：设应力函数



$\Phi = Ax^3+Bx^2y+Cxy^2+Dy^3$

①：先验证相容方程：$\nabla^4\varphi=0$，显然成立

②：对有体力情况：

$$\begin{cases}\sigma_x = \dfrac{\partial^2\Phi}{\partial y^2} - f_x x \\ \sigma_y = \dfrac{\partial^2\Phi}{\partial x^2} - f_y y \\ \tau_{xy} = -\dfrac{\partial^2\Phi}{\partial x\partial y}\end{cases}$$

$l=0,\ m=-1$

$$\begin{cases}\sigma_x = 2Cx+6Dy \\ \sigma_y = 6Ax+2By-\rho g y \\ \tau_{xy} = -2Bx-2Cy\end{cases}$$

此时有：边界条件：$y=0$ $\begin{cases}\tau_{xy}=0 \\ -\sigma_y = \rho g y\end{cases}$

$y=0 \rightarrow \sigma_y = -\rho g y = 0 \Rightarrow A=0$　　$y=0,\ \tau_{xy}=0 \rightarrow B=0$

$y=x\tan\alpha \longrightarrow \begin{cases}-\sigma_x\sin\alpha+\tau_{xy}\cos\alpha = f_x + lf_x x = 0 \\ -\tau_{xy}\sin\alpha+\sigma_y\cos\alpha = f_y+mf_y y = \cos\alpha\cdot\rho g x\tan\alpha\end{cases}$

分别代入，有：

$-(2Cx\sin\alpha+6Dx\sin\alpha\tan\alpha)+\cos\alpha(-2Bx-2Cx\tan\alpha)=0$

$-4C\cancel{x\sin\alpha}-6D\cancel{x\sin\alpha}\tan\alpha=0$

得到：$4C-6D\tan\alpha=0$

而：

$(0+2Cx\tan\alpha)\sin\alpha + (-\rho g x\tan\alpha)\cos\alpha = \rho g x\sin\alpha$

有：$2C\dfrac{\sin^2\alpha}{\cos\alpha} = 2\rho g\ \cancel{\sin\alpha}$

则：

$C = \rho g\cot\alpha$　　则：$D = \dfrac{2C}{3}\dfrac{1}{\tan\alpha} = -\dfrac{2\rho g}{3}\cot^2\alpha$

显有：$\begin{cases}\sigma_x = 2\rho g x\cot\alpha - 4\rho g\cot^2\alpha \\ \sigma_y = -\rho g y \\ \tau_{xy} = -2\rho g\cot\alpha\end{cases}$

说明：对于带体力问题的边界条件：

由平衡微分方程：有：

$\{ \frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}+f_x=0$　　不忽略体力，我们令

由平衡微分方程，有：

$$\begin{cases} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + f_x = 0 \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + f_y = 0 \end{cases}$$

不忽略体力，我们令

有体力情况下，有相容方程应为：

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(\sigma_x + \sigma_y) = -(1+\nu)\left(\frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y}\right) \quad (P33)$$

全完按照 P35 的推导过程

进行通解与特解的叠加：

得：$\sigma_x = \frac{\partial^2 \Phi}{\partial y^2} - f_x x$，$\sigma_y = \frac{\partial^2 \Phi}{\partial x^2} - f_y y$　$\tau_{xy} = -\frac{\partial^2 \Phi}{\partial x \partial y}$

(P37, 2-24)

我们在解题中设应力函数为 Φ，并设 $\frac{\partial^2 \Phi}{\partial y^2} = \sigma_x'$　$\frac{\partial^2 \Phi}{\partial x^2} = \sigma_y'$，$-\frac{\partial^2 \Phi}{\partial x \partial y} = \tau_{xy}' = \tau_{xy}$

有 $\begin{cases} \sigma_x = \sigma_x' - f_x x, \\ \sigma_y = \sigma_y' - f_y y, \end{cases}$　代入边界条件 $\begin{cases} l\sigma_x + m\tau_{xy} = \bar{p}_x \\ l\tau_{xy} + m\sigma_y = \bar{p}_y \end{cases}$

得：$\boxed{\begin{cases} l\sigma_x' + m\tau_{xy}' = \bar{p}_x + l f_x x \\ l\tau_{xy} + m\sigma_y' = \bar{p}_y + l f_y y \end{cases}}$　为新的边界条件，

应当注意：

此时通过 Φ 求 σ_x，σ_y 公式也应变为：

$$\boxed{\begin{cases} \sigma_x' = \frac{\partial^2 \Phi}{\partial y^2} \\ \sigma_y' = \frac{\partial^2 \Phi}{\partial x^2} \\ \tau_{xy}' = -\frac{\partial^2 \Phi}{\partial x \partial y} \end{cases}}.$$