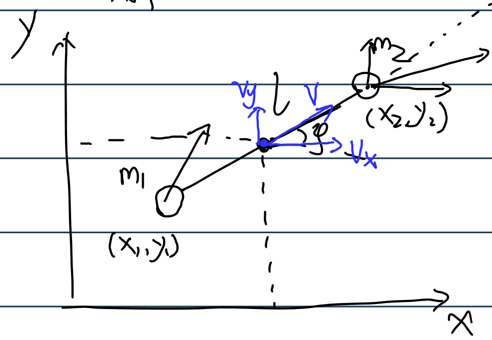
例题1.7

Friday, March 31, 2023　　3:33 PM

两个质量均m质点、M_1, M_2，由一长度为l的刚性杆相连，其质量忽略，运动时杆中点速度必须沿杆向，建立系统运动数学模型。

解：　　$k=3$，广义坐标 x_c, y_c, θ，而坐标 $(x_1,y_1)(x_2,y_2)$



约束：

①：$(y_2-y_1)^2+(x_2-x_1)^2=l^2$　　（完整）

②：$\dfrac{\dot y_2+\dot y_1}{2}\dfrac{1}{\tan\varphi}=\dfrac{\dot x_2+\dot x_1}{2}$

代入：$\tan\varphi=\dfrac{y_2-y_1}{x_2-x_1}$

得：$(\dot y_2+\dot y_1)(x_2-x_1)-(\dot x_2+\dot x_1)(y_2-y_1)=0$　　②

将约束方程①②进行改写，变分形式为：

分别对各坐标求取变分

$2(y_2-y_1)\delta y_2-2(y_2-y_1)\delta y_1+2(x_2-x_1)\delta x_2-2(x_2-x_1)\delta x_1=0$

$(x_2-x_1)(\delta x_2-\delta x_1)+(y_2-y_1)(\delta y_2-\delta y_1)=0$　① （完整约束）

以及：$(\delta y_2+\delta y_1)(x_2-x_1)-(\delta x_2+\delta x_1)(y_2-y_1)=0$　② （非完整约束）

此时有：由第一类 Lagrange 方程。

先将上式化为：$\dfrac{\partial f_s}{\partial x_1}$　$\dfrac{\partial f_s}{\partial x_2}$

$$\begin{cases}(x_1-x_2)\delta x_1+(x_2-x_1)\delta x_2+(y_1-y_2)\delta y_1+(y_2-y_1)\delta y_2=0\\ (y_1-y_2)\delta x_1+(y_1-y_2)\delta x_2+(x_2-x_1)\delta y_1+(x_2-x_1)\delta y_2=0\end{cases}$$

a_{11}

故：由：$\dfrac{d}{dt}\left(\dfrac{\partial T}{\partial \dot q_j}\right)-\dfrac{\partial T}{\partial q_j}=Q_j+\sum_{r=1}^{m}\lambda_r a_{rj}+\sum_{s=1}^{l}\mu_s\dfrac{\partial f_s}{\partial x_j}$

由 $T=\frac{1}{2}m_1(\dot x_1^2+\dot y_1^2)+\frac{1}{2}m_2(\dot x_2^2+\dot y_2^2)=\frac{m}{2}(\dot x_1^2+\dot y_1^2+\dot x_2^2+\dot y_2^2)$

$=m(\dot x_c^2+\dot y_c^2)+\frac{1}{2}\left(m\left(\frac{l}{2}\right)^2+m\left(\frac{l}{2}\right)^2\right)\cdot\dot\theta^2$

$$\begin{cases}m_1\ddot x_1=\lambda_1(y_1-y_2)+\mu_1(x_1-x_2)\\ m_2\ddot x_2=\lambda_1(y_1-y_2)-\mu_1(x_1-x_2)\\ m_1\ddot y_1=-\lambda_1(x_1-x_2)+\mu_1(y_1-y_2)-m_1g\end{cases}$$

非完整　完整

此六式为描述系统运

$$m_2\ddot{x}_2 = \lambda_1(y_1-y_2)\ \mu_1(x_1-x_2)$$

$$m_1\ddot{y}_1 = -\lambda_1(x_1-x_2)+\mu_1(y_1-y_2)-m_1 g$$

$$m_2\ddot{y}_2 = -\lambda_1(x_1-x_2)-\mu_1(y_1-y_2)-m_2 g$$

铅垂面

和约束方程

$$(x_1-x_2)^2+(y_1-y_2)^2 = l^2$$

$$(\dot{x}_1+\dot{x}_2)(y_1-y_2) = (\dot{y}_1+\dot{y}_2)(x_1-x_2)$$

此六式为描述系统运动的数学模型