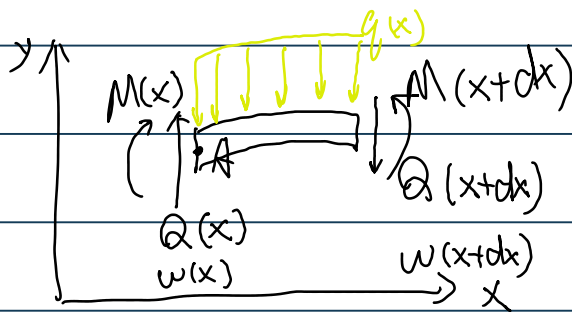


梁的横向振动推导过程

Wednesday, May 17, 2023 9:56 AM

(不考虑压杆稳定问题)



y方向: $dm = \rho A dx$

设梁上的每一点挠度为 $w(x)$

有: y方向方程: \rightarrow 认为在 dx 内, q 为常数

$$ma = \rho A dx \cdot \ddot{w}(x) = Q(x) - Q(x+dx) - q(x) dx$$

$$\text{展开得 } \rho A \ddot{w}(x) = -\frac{\partial Q}{\partial x} dx - q(x) dx \quad (1)$$

$$\text{而: 对 A 点列矩: 有: } M(x) + q(x) dx \frac{dx}{2} + Q(x+dx) dx - M(x+dx) = 0$$

$$\text{代入: 有: } -\frac{\partial M}{\partial x} dx + \frac{1}{2} q(x) dx^2 + Q dx + \frac{\partial Q}{\partial x} dx^2 = 0$$

略去高阶项

$$\therefore -\frac{\partial M}{\partial x} + Q = 0$$

$$\text{则: } Q = \frac{\partial M}{\partial x} \quad (2)$$

将 (2) 代入 (1): 有:

$$\rho A \ddot{w}(x) + \frac{\partial^2 M}{\partial x^2} = -q(x) \quad , \text{ 因材料性质有 } M = EI \frac{\partial^2 w}{\partial x^2}$$

代入得到

$$\rho A \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} = -q(x)$$