

相对通频带与半功率点推导

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9:42 AM

$$\text{有: } I = \frac{U_s}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \quad Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$

$$I_0 = \frac{U_s}{R}, \text{ 则: } \frac{I}{I_0} = \frac{1}{\sqrt{1 + \left[\frac{1}{R}(\omega L - \frac{1}{\omega C})\right]^2}} \quad \text{此时:}$$

$$\text{①: 当 } \frac{1}{R}(\omega L - \frac{1}{\omega C}) = \pm 1 \text{ 时}$$

$$\text{有: } \omega L - \frac{1}{\omega C} = \pm R, \text{ 则}$$

$$\text{即 } \omega_1 L - \frac{1}{\omega_1 C} = R \quad \text{或} \quad \omega_2 L - \frac{1}{\omega_2 C} = -R$$

从而有:

$$\frac{\omega_1}{\omega_0} \sqrt{\frac{L}{C}} - \frac{\omega_0}{\omega_1} \sqrt{\frac{L}{C}} = R \rightarrow \frac{\omega_1}{\omega_0} - \frac{\omega_0}{\omega_1} = \frac{1}{Q}$$

$$\text{同样: } \frac{\omega_2}{\omega_0} - \frac{\omega_0}{\omega_2} = -\frac{1}{Q}$$

$$\text{联立: } \frac{\omega_2}{\omega_0} - \frac{\omega_0}{\omega_2} = \frac{\omega_0}{\omega_1} - \frac{\omega_1}{\omega_0}$$

$$\frac{\omega_1 + \omega_2}{\omega_0} = \omega_0 \left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2} \right) \Rightarrow \omega_1 \omega_2 = \omega_0^2 \quad \text{①}$$

$$\text{又: } \frac{\omega_1 - \omega_2}{\omega_0} - \omega_0 \left(\frac{\omega_2 - \omega_1}{\omega_1 \omega_2} \right) = \frac{2}{Q}$$

$$\frac{2(\omega_1 - \omega_2)}{\omega_0} = \frac{2}{Q} \Rightarrow \Delta\omega = \boxed{\omega_1 - \omega_2 = \frac{1}{Q} \omega_0}$$

为相对通频带

对应地, 半功率点处:

$$P_1 = P_2 = \frac{U^2}{R} = I^2 R \xrightarrow{I = \frac{1}{\sqrt{2}} I_0} = \frac{1}{2} I_0^2 R = \frac{1}{2} P$$