边界为曲线的泰勒展开公式推导

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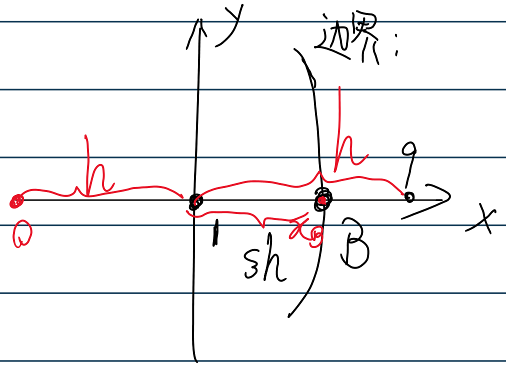

对于边界为曲线的情况：令

$$\begin{cases} q\text{结点: } x_B - \xi h + h \\ 1\text{结点: } x_B - \xi h, \\ 0\text{结点: } x_B - (h + \xi h) \end{cases}$$

利用Taylor展开，有：

$$\Phi_q - \Phi_B = (1-\xi)h\left(\frac{\partial\Phi}{\partial x}\right)_B + \frac{1}{2}(1-\xi)^2h^2\left(\frac{\partial^2\Phi}{\partial x^2}\right)_B \quad ①$$

$$\Phi_1 - \Phi_B = -\xi h\left(\frac{\partial\Phi}{\partial x}\right)_B + \frac{1}{2}\xi^2h^2\left(\frac{\partial^2\Phi}{\partial x^2}\right)_B + \cdots$$

有：③

$$\Phi_0 - \Phi_B = -\frac{\partial\Phi}{\partial x} = -h\left(\frac{\partial\Phi}{\partial x}\right)_B(1+\xi) + \frac{1}{2}h^2(1+\xi)^2\left(\frac{\partial^2\Phi}{\partial x^2}\right)_B$$

其中，高于三次项的应力略去，从上式中消 $\left(\frac{\partial^2\Phi}{\partial x^2}\right)_B$，得到

由①③：

$$\Phi_q = \Phi_B + (1-\xi)h\left(\frac{\partial\Phi}{\partial x}\right)_B + \frac{1}{2}(1-\xi)^2h^2\left(\frac{\partial^2\Phi}{\partial x^2}\right)_B$$

$$\Phi_0 = \Phi_B - (1+\xi)h\left(\frac{\partial\Phi}{\partial x}\right)_B + \frac{1}{2}(1+\xi)^2h^2\left(\frac{\partial^2\Phi}{\partial x^2}\right)_B,$$

$$\Phi_1 = \Phi_B - \xi h\left(\frac{\partial\Phi}{\partial x}\right)_B + \frac{1}{2}\xi^2h^2\left(\frac{\partial^2\Phi}{\partial x^2}\right)_B$$

有：

$$\Phi_q - \frac{(1-\xi)^2}{(1+\xi)^2}\Phi_0 = \Phi_B\left[1 - \frac{(1-\xi)^2}{(1+\xi)^2}\right] + \left[(1-\xi) + \frac{(1-\xi)^2}{1+\xi}\right]h\left(\frac{\partial\Phi}{\partial x}\right)_B.$$

故：

$$\Phi_q = \frac{(1-\xi)^2}{(1+\xi)^2}\Phi_0 + \frac{4\xi}{(1+\xi)^2}\Phi_B + \frac{2(1-\xi)}{1+\xi}h\left(\frac{\partial\Phi}{\partial x}\right)_B \quad (i)$$

$$\Phi_1 - \frac{\xi^2}{(1+\xi)^2}\Phi_0 = \Phi_B\left[1 - \frac{\xi^2}{(1+\xi)^2}\right] + \left[-\xi + \frac{\xi^2}{(1+\xi)^2}\cancel{(1+\xi)}\right]h\cdot\left(\frac{\partial\Phi}{\partial x}\right)_B$$

有：

$$\Phi_1 = \frac{\xi^2}{(1+\xi)^2}\Phi_0 + \frac{1+2\xi}{(1+\xi)^2}\Phi_B + \frac{\cancel{-\xi(1+\xi)}}{(1+\xi)^2}h\left(\frac{\partial\Phi}{\partial x}\right)_B \quad (ii)$$

$$-\xi(1+\xi)^2 + \xi^2(1+\xi) = -\xi - \xi^3 + \xi^2 + \xi^3$$

$$-2\xi^2$$

$$= -\xi - \xi^2$$

即可使用：Φ_0表达Φ_1, Φ_q,

当$\xi=0$，$\Phi_1 = \Phi_B$, $\Phi_q = \Phi_0 + 2h\left(\frac{\partial\Phi}{\partial x}\right)_B$