正规摄动方法推导

Monday, June 16, 2025 2:11 PM

对正规摄动法,

$$f(x)=f(x_0)+\varepsilon f'+\frac{\varepsilon^2}{2}f''$$

$$\ddot{x}+\omega_0^2 x = F(t)+\varepsilon f(x,\dot{x})$$

展: $x(t,\varepsilon)=x_0+\varepsilon x_1+\varepsilon^2 x_2+\cdots$

$$\ddot{x}_0+\varepsilon\ddot{x}_1+\varepsilon^2\ddot{x}_2+\omega_0^2 x_0+\varepsilon\omega_0^2 x_1+\varepsilon^2\omega_0^2 x_2$$

$$=F(t)+\varepsilon\Big[f(x_0,\dot{x}_0)+\frac{\partial f}{\partial x}(x,\dot{x})(\varepsilon x_1+\varepsilon^2 x_2+\cdots)+$$

$$+\frac{\partial f}{\partial \dot{x}}(x,\dot{x})(\varepsilon \dot{x}_1+\varepsilon^2 \dot{x}_2+\cdots)$$

$$+\frac{1}{2}\frac{\partial^2 f}{\partial x^2}(x,\dot{x})(\varepsilon x_1+\varepsilon^2 x_2+\cdots)^2$$

$$+2\frac{\partial^2 f}{\partial x\partial \dot{x}}(x,\dot{x})(\varepsilon x_1+\varepsilon^2 x_2+\cdots)(\varepsilon \dot{x}_1+\cdots)$$

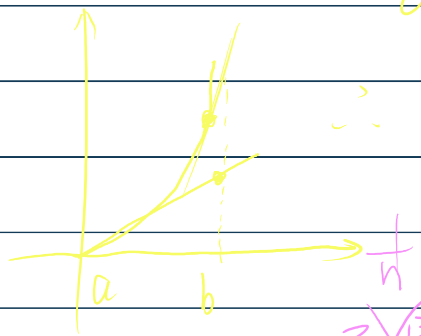
$$+\frac{1}{2}\frac{\partial^2 f}{\partial \dot{x}^2}(x,\dot{x})(\varepsilon \dot{x}_1+\varepsilon^2 \dot{x}_2+\cdots)^2\Big]$$

另:二阶展开公式说明:

$$f(x,y)=f(x_0,y_0)+f_x dx+f_y dy+\frac{1}{2}f_{xx}+f_{xy}+\frac{1}{2}f_{yy}$$

①:先理解一元:

$f'(x)=\frac{df}{dx}$, 由 $f'(b)=f'(a)+(b-a)f''(a)$



$$\therefore \int_a^b f'(x)\,dx \approx \frac{1}{2}(f'(a)+f'(b))(b-a)$$

$$=f(a)(b-a)+\frac{1}{2}f''(a)(b-a)^2.$$

命实际是n阶函数的近似

因此,由ε阶系数对应,有

$$\ddot{x}_0+\omega_0^2 x_0=F(t)$$

$$\ddot{x}_1+\omega_0^2 x_1=f(x_0,\dot{x}_0)$$

$$\ddot{x}_2+\omega_0^2 x_2=\frac{\partial f}{\partial x}(x,\dot{x})x_1+\frac{\partial f}{\partial \dot{x}}(x,\dot{x})\dot{x}_1$$

$$\ddot{x}_2 + \omega_0^2 x_2 = \frac{\partial f}{\partial x}(x,\dot{x})\, x_1 + \frac{\partial f}{\partial \dot{x}}(x,\dot{x})\, \dot{x}_1$$