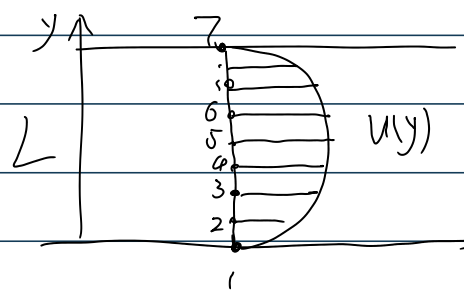
PPTp66

Thursday, April 13, 2023 9:49 PM



For a couette flow between two fixed planes which is a steady and viscid flow, we have the equation:

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{\partial P}{\partial x} = C$$ and boundary condition: $u(y=0) = u(y=L) = 0$

we can discretize the equation as:

$$\frac{u_{j+1} + u_{j-1} - 2u_{ij}}{\Delta y^2} = \frac{1}{\mu}\frac{P_{i+1} - P_i}{\Delta X} = \text{constant}.$$

Solve this partial differential equation by Mathematica:

$$\text{DSolve}[\{u''[y] == C, u[0] == 0, u[l] == 0\}, u[y], y]$$

$$\left\{\left\{u[y] \to \frac{1}{2}(-Cly + Cy^2)\right\}\right\}$$

Then we can discretize the solution by setting the matrix by the following code :

$$\text{dx} = \text{length}/(\text{xelem} - 1)\,;\ \text{dy} = l/(\text{yelem} - 1) \qquad \frac{l}{6}$$

coef
= 1/dy^2
* Table[If[$j - i <= 1 \&\& j - i >= -1, 1 *$ If[$i == j, -2, 1$], 0], {i, 1, yelem}, {j, 1, yelem}]
× (*构造系数矩阵*)

coef//MatrixForm

$$\begin{pmatrix} -\frac{72}{l^2} & \frac{36}{l^2} & 0 & 0 & 0 & 0 & 0 \\ \frac{36}{l^2} & -\frac{72}{l^2} & \frac{36}{l^2} & 0 & 0 & 0 & 0 \\ 0 & \frac{36}{l^2} & -\frac{72}{l^2} & \frac{36}{l^2} & 0 & 0 & 0 \\ 0 & 0 & \frac{36}{l^2} & -\frac{72}{l^2} & \frac{36}{l^2} & 0 & 0 \\ 0 & 0 & 0 & \frac{36}{l^2} & -\frac{72}{l^2} & \frac{36}{l^2} & 0 \\ 0 & 0 & 0 & 0 & \frac{36}{l^2} & -\frac{72}{l^2} & \frac{36}{l^2} \\ 0 & 0 & 0 & 0 & 0 & \frac{36}{l^2} & -\frac{72}{l^2} \end{pmatrix}$$

we have boundary condition $u_1 = 0$, $u_7 = 0$, and also: for inner points.

$$\rightarrow \frac{u_{i-1} - 2u_i + u_{i+1}}{2\Delta y} = C$$

where $\Delta y = \frac{l}{6}$

coef[[1,1]] = 1; coef[[1,2]] = 0; coef[[yelem, yelem − 1]] = 0; coef[[yelem, yelem]] = 1;

right = Table[If[$i == 1 || i ==$ yelem, 0, c], {i, 1, yelem}]

result = Inverse[coef]. right

$$\left\{0, -\frac{5cl^2}{72}, -\frac{cl^2}{9}, -\frac{cl^2}{8}, -\frac{cl^2}{9}, -\frac{5cl^2}{72}, 0\right\}$$

result = Inverse[coef]. right

$$\left\{0, -\frac{5cl^2}{72}, -\frac{cl^2}{9}, -\frac{cl^2}{8}, -\frac{cl^2}{9}, -\frac{5cl^2}{72}, 0\right\}$$

u_1 u_2 u_3 u_4 u_5 u_6 u_7

ListLinePlot[{0, (−5 ∗ c ∗ l^2)/72, −((c ∗ l^2)/9), −((c ∗ l^2)/8), −((c ∗ l^2)/9), (−5 ∗ c ∗ l^2)/72, 0}]