第一类拉格朗日方程的推导

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①：首先对受理想约束非自由质点，有约束方程

1)完整：$f_s(x_1,y_1,z_1 \cdots x_n,y_n,z_n,t)=0 \quad (s=1,2,\cdots l)$

2)非完整：$\sum_{i=1}^{n}\left(a_{ri}\dot{x}_i+b_{ri}\dot{y}_i+c_{ri}\dot{z}_i\right)+e_r=0 \quad (r=1,2,\cdots m)$

则：对①求时间偏导：

$$\frac{\partial f_s}{\partial x_1}\dot{x}_1+\frac{\partial f_s}{\partial y_1}\dot{y}_1+\cdots+\frac{\partial f_s}{\partial t}=0$$

$$=\sum_{i=1}^{n}\left(\frac{\partial f_s}{\partial x_i}\dot{x}_i+\frac{\partial f_s}{\partial y_i}\dot{y}_i+\frac{\partial f_s}{\partial z_i}\dot{z}_i\right)+\frac{\partial f_s}{\partial t}=0 \quad (s=1,2,\cdots l)$$

写成变分方程的形式：

$$\begin{cases} a_{ri}\delta x_i+b_{ri}\delta y_i+c_{ri}\delta z_i=0 \quad (r=1,2,\cdots m) \\ \left(\frac{\partial f_s}{\partial x}\delta x_i+\frac{\partial f_s}{\partial y}\delta y_i+\frac{\partial f_s}{\partial z}\delta z_i\right)=0 \end{cases}$$

代入动力学普遍方程

3)引入Lagrange乘子 λ_r,μ_r 并添加到方程中，则 $\sum_{i=1}^{n}(F_i-m_i a_i)\delta r_i=0$

$$m_i\ddot{x}_i\delta x_i=\left(F_{xi}-\sum_{r=1}^{m}\lambda_r a_{ri}-\sum_{r=1}^{m}\mu_r\frac{\partial f_s}{\partial x}\right)\delta x_i$$

故：

$$\begin{cases} m_i\ddot{x}_i=F_{xi}-\sum_{r=1}^{m}\lambda_r a_{ri}-\sum_{r=1}^{m}\mu_r\frac{\partial f_s}{\partial x} \\ m_i\ddot{y}_i=F_{yi}-\sum_{r=1}^{m}\lambda_r b_{ri}-\sum_{r=1}^{m}\mu_r\frac{\partial f_s}{\partial y} \\ m_i\ddot{z}_i=F_{zi}-\sum_{r=1}^{m}\lambda_r c_{ri}-\sum_{r=1}^{m}\mu_r\frac{\partial f_s}{\partial z} \end{cases}$$