Example 3.1(3)

Tuesday, February 7, 2023 10:07 PM

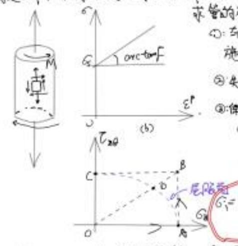
前两问的解题如下：

例题3.1. 如图为一薄壁圆管，其拉伸强化曲线如(b)，讨论下列三种加载路径求管的轴向应变 ε_z 和切向应变 $\gamma_{z\theta}$

①：轴向拉伸至 σ_s，然后保持 $\sigma_z=\sigma_s$，施加 $\tau_{z\theta}=\frac{\sigma_s}{\sqrt{3}}$ (OAB)

②：先施加：$\tau_{z\theta}=\frac{\sigma_s}{\sqrt{3}}$ 保持并加载 $\sigma_z=\sigma_s$

③：保持 $\sigma_z:\tau_{z\theta}=\sqrt{3}:1$，加载至 $\sigma_z=\sigma_s$，$\tau_{z\theta}=\frac{\sigma_s}{\sqrt{3}}$



由于 $\sigma_1=\sigma_z$，$\sigma_2=\tau$，$\sigma_3=-\tau$.

$$\sigma_i=\frac{1}{\sqrt{2}}\sqrt{(\sigma_z+\tau)^2+(\sigma_z-\tau)^2+(2\tau)^2}$$

$$=\frac{1}{\sqrt{2}}\sqrt{2\sigma_z^2+6\tau^2}=\sqrt{\sigma^2+3\tau^2}$$

解：①：(1) 弹性阶段：由：本构关系

$$d\varepsilon_{ij}=\frac{1}{2\mu}dS_{ij}+\frac{1-2\nu}{E}\sigma_m\delta_{ij}$$

$$d\varepsilon_{ij}=d\varepsilon_x=\frac{1}{2\mu}S_x+\frac{1-2\nu}{E}\sigma_m$$

其中：$S_x=\frac{2}{3}\sigma_s$，$\sigma_m=\frac{\sigma_s}{3}$

$$\therefore \varepsilon_x=\frac{\sigma_s}{3\mu}+\frac{1-2\nu}{E}\cdot\frac{\sigma_s}{3}\quad 代入：\mu=\frac{E}{2(1+\nu)}$$

$$=\frac{2(1+\nu)}{3E}\sigma_s+\frac{1-2\nu}{3E}\sigma_s=\frac{\sigma_s}{E}$$

(2) 塑性阶段

$$由\ d\varepsilon_{ij}=\cancel{\frac{1}{2\mu}}dS_{ij}+\cancel{\frac{1-2\nu}{E}\sigma_m\delta_{ij}}+d\lambda\cdot S_{ij}$$

$$\to d\lambda=\frac{3\,d\sigma_i}{2F'\sigma_i}$$

由于塑性：$dS_{ij}=0$，另有：$\nu=\frac{1}{2}$，第二项为 0

故：$d\varepsilon_{ij}=d\varepsilon_{ij}^p=d\lambda\cdot S_{ij}$.

其中 $d\lambda$ 满足强化关系：

$$d\lambda=\frac{d\varepsilon_{ij}^p}{S_{ij}}=\frac{\sqrt{\frac{3}{2}d\varepsilon_i^{p2}}}{\sqrt{\frac{2}{3}\sigma_i\sigma_i}}=\frac{3d\varepsilon_i^p}{2\sigma_i}$$

其中：$\frac{d\sigma_i}{d\varepsilon_i^p}=F'$ 故：$d\lambda=\frac{3\,d\sigma_i}{2F'\sigma_i}$

其中：σ_i 由 $\sigma_s\to\sqrt{\sigma_s^2+3\tau^2}=\sqrt{2}\sigma_s$

$$\int d\varepsilon_p=\int d\lambda\, S_{ij}=\int\frac{3\,d\sigma_i}{2F'\sigma}S_{ij}$$

此处注意

τ_θ,τ_r 的使用正应力计算 $\to S_\theta, S_r$ 而 $S_{r\theta}$ 为切应力

有：第二阶段：由 $\sigma_1=\sigma_z$. $\sigma_2=\tau$，$\sigma_3=-\tau$

$$\begin{cases} dS_z=0, & \sigma_i\ 由\ \sigma_s\to\sqrt{2}\sigma_s \\ dS_r=dS_\theta=0, \\ S_r=S_\theta=-\frac{\sigma_s}{3} \\ S_z=\frac{2\sigma_s}{3}\end{cases}$$

$$S_{r\theta}=\frac{1}{\sqrt{3}}\tau$$

代换

使用应力表达式求 $\frac{d\sigma_i}{\sigma_i}$

由 $\sigma_i=\sqrt{3\tau_{z\theta}^2+\sigma_z^2}$

$$有：d\sigma_i=\frac{\partial\sigma_i}{\partial\tau_{z\theta}}d\tau_{z\theta}=\frac{1}{2}\frac{6\tau_{z\theta}\,d\tau_{z\theta}}{\sqrt{3\tau_{z\theta}^2+\sigma_z^2}}=\frac{3\tau_{z\theta}}{\sqrt{3\tau_{z\theta}^2+\sigma_z^2}}d\tau_{z\theta}$$

$$\frac{d\sigma_i}{\sigma_i}=\frac{3\tau_{z\theta}}{3\tau_{z\theta}^2+\sigma_z^2}$$

$$\int\frac{\frac{1}{2}}{\tau_{r\theta}^2+\left(\frac{\sigma_s}{\sqrt{3}}\right)^2}d(\tau_{r\theta}^2)$$

代入增量本构并积分：

$$则第二段：\int d\varepsilon_z=\int\frac{3}{2}\frac{d\sigma_i}{F'\sigma_i}S_z=\int\frac{\sigma_s\,3\tau_{z\theta}}{F'(3\tau_{z\theta}^2+\sigma_z^2)}d\tau_{z\theta}=\frac{\sigma_s}{F'}\int\frac{3\tau_{r\theta}}{3\tau_{r\theta}^2+\sigma_z^2}d\tau_{r\theta}=\frac{\sigma_s}{F'}\cdot\int_0^{\frac{\sigma_s}{\sqrt{3}}}\frac{\frac{1}{2}}{\tau_{r\theta}^2+\left(\frac{\sigma_s}{\sqrt{3}}\right)^2}d\tau_{r\theta}^2$$

$$=\frac{\sigma_s}{2F'}\left[\ln\left(\frac{\sigma_s^2}{3}+\frac{\sigma_s^2}{3}\right)-\ln\left(\frac{\sigma_s^2}{3}\right)\right]$$

$$=\frac{\sigma_s}{2F'}\ln 2$$

$$第二段的\int d\gamma_{r\theta}=\int_0^{\frac{\sigma_s}{\sqrt{3}}}\frac{3\sigma_s}{F'}\cdot\frac{3\tau_{r\theta}}{3\tau_{z\theta}^2+\sigma_z^2}\tau_{r\theta}\,d\tau_{r\theta}$$

$$=\frac{3\sigma_s}{2F'}\cdot\int_0^{\frac{\sigma_s}{\sqrt{3}}}\frac{1}{1+\left(\frac{1}{\sqrt{3}}\frac{\sigma_s}{\tau_{z\theta}}\right)^2}d\tau_{r\theta}\to\frac{\sigma_s}{\sqrt{3}}\cdot d\left(\frac{\sqrt{3}\tau_{r\theta}}{\sigma_s}\right)$$

$$由\int\frac{1}{1+x^2}dx=\arctan x+C$$

$$\int_0^{\frac{\sigma_s}{\sqrt{3}}}\frac{1}{1+\left(\frac{1}{\sqrt{3}}\frac{\sigma_s}{\tau_{r\theta}}\right)^2}d\tau_{r\theta}\quad 令\ t=\frac{\tau_{r\theta}}{\sigma_s/\sqrt{3}}$$

$$由：\int_0^1\frac{1}{1+\frac{1}{t^2}}dt\cdot\quad 则\ d\tau_{r\theta}=dt\cdot\frac{\sigma_s}{\sqrt{3}}$$

$$=\int_0^1\left(\frac{t^2}{1+t^2}\right)dt\cdot\frac{\sigma_s}{\sqrt{3}}$$

$$=\int_0^1\left(1-\frac{1}{1+t^2}\right)dt\cdot\frac{\sigma_s}{\sqrt{3}}$$

$$=(t-\arctan t)\frac{\sigma_s}{\sqrt{3}}$$

$$=\frac{\sigma_s}{\sqrt{3}}(1-\arctan 1)=\left(1-\frac{\pi}{4}\right)\frac{\sigma_s}{\sqrt{3}}$$

$$d\gamma_{r\theta}=\frac{1}{\mu}dS_{r\theta}+\frac{3}{F'}\frac{d\sigma_i}{\sigma_i}\tau_{z\theta}\quad(缺\tfrac{1}{2})$$

$$\therefore\ \gamma_{r\theta}=\frac{1}{\mu}\times\frac{\sigma_s}{\sqrt{3}}+\frac{\sigma_s}{\sqrt{3}}\left(1-\frac{\pi}{4}\right).$$

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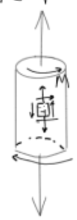
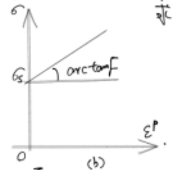
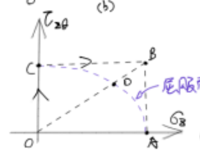




ⓑ 先施加：$\tau_{z\theta}=\frac{\sigma_s}{\sqrt{3}}$，保持并加载 $\sigma_z=\sigma_s$

ⓐ：先施加：$\tau_{z\theta}=\frac{\sigma_s}{\sqrt{3}}$ 时 → 由 $\sigma_i=\sqrt{3J_2}$，∴ $\sqrt{\frac{1}{2}[(\sigma_r-\sigma_\theta)^2+(\sigma_\theta-\sigma_z)^2+(\sigma_z-\sigma_r)^2+6(\tau_{r\theta}^2+\tau_{\theta z}^2+\tau_{rz}^2)]}$

$$\sigma_i=\sqrt{\sigma_z^2+3\tau_{z\theta}^2}$$



(1)：由增量本构关系有：

$$d\varepsilon_{ij}=\frac{1}{2\mu}dS_{ij}+\frac{1-2\nu}{E}d\sigma_m\delta_{ij}+d\lambda S_{ij}$$

因此有：

①：弹性阶段：$dS_z=0\Rightarrow d\sigma_z=0$，

故：$d\varepsilon_z=0$，

由剪切胡克定律：$\frac{1}{2}\gamma=\frac{1}{2\mu}\cdot\frac{\sigma_s}{\sqrt{3}}$，则：$\gamma=\frac{1}{\mu}\cdot\frac{\sigma_s}{\sqrt{3}}=\frac{\sigma_s}{\sqrt{3}}\frac{2(1+\nu)}{E}=\frac{2\sigma_s(1+\nu)}{\sqrt{3}E}$　$\mu=G$

第一阶段 $\begin{cases}\varepsilon_z=0\\ \gamma_{z\theta}=\frac{2\sigma_s(1+\nu)}{\sqrt{3}E}\end{cases}$

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② 塑性阶段：

$$d\varepsilon_{ij}=\frac{1}{2\mu}dS_{ij}+\frac{1-2\nu}{E}d\sigma_m\delta_{ij}+d\lambda S_{ij}$$

$\frac{1}{2\mu}\to\frac{1+\nu}{E}$

其中：使用强化材料本构：$d\lambda=\frac{3\,d\sigma_i}{2F\sigma}$　以σ代替σ

其中：z方向：

有：$\int dS_z=\frac{2}{3}\sigma_s$　而：$\int d\sigma_m=\frac{\sigma_s}{3}$

则：$\int d\varepsilon_z=\frac{1+\nu}{E}\cdot\frac{2}{3}\sigma_s+\frac{\sigma_s}{3}\cdot\frac{1-2\nu}{E}+\frac{3d\sigma_i}{2F\sigma_i}S_z$

$$=\frac{2(1+\nu)\sigma_s+(1-2\nu)\sigma_s}{3E}+\int\frac{3d\sigma_i}{2F\sigma_i}\cdot S_z$$

$=\frac{\sigma_s}{E}$　$S_z=\frac{2}{3}\sigma_z$

其中 $\int\frac{3S_z}{2F}\cdot d\sigma_i=\int_0^{\sigma_s}\frac{1}{F}\frac{\sigma_z^2}{\sigma_z^2+3\tau_{\theta z}^2}d\sigma_z$　$\int_0^{\sqrt{3}}\frac{1}{F}\frac{\frac{\sigma_z}{\tau_{\theta z}}}{\frac{\sigma_z^2}{\tau_{\theta z}^2}+3}\tau_{\theta z}d\left(\frac{\sigma_z}{\tau_{\theta z}}\right)$

$\sigma_i=\sqrt{\sigma_z^2+3\tau_{\theta z}^2}$　$\frac{1}{2}\frac{2\sigma_z}{\sigma_z^2+3\tau_{\theta z}^2}$

$$=\frac{\sigma_s}{\sqrt{3}}\cdot\int_0^{\sqrt{3}}\frac{1}{F}\cdot\frac{t}{t^2+3}dt=\cancel{\frac{\sigma_s}{2\sqrt{3}F}\int_0^3\frac{1}{\sqrt{x+3}}dx}$$

$$\cancel{=\frac{1}{2\sqrt{3}}2\sqrt{x+3}\Big|_0^3\frac{\sigma_s}{F}=\sqrt{\frac{6}{3}}\frac{\sigma_s}{F}}$$

$$\cancel{=\sqrt{2}\frac{\sigma_s}{F}}.$$

因此：第二阶段：

$\varepsilon_z=\frac{\sigma_s}{E}+\sqrt{2}\frac{\sigma_s}{F}$.

单独计算积分部分：注意：

第一个塑性变形部分为：

$\int_0^{\sigma_s}\frac{\sigma_z^2}{\sigma_z^2+3\tau_{\theta z}^2}d\sigma_z=\int_0^{\sigma_s}1-\frac{3\tau_{\theta z}^2}{\sigma_z^2+3\tau_{\theta z}^2}d\sigma_z$　代入：$\tau_{\theta z}=\frac{\sigma_s}{\sqrt{3}}\to 3\tau_{\theta z}^2=\sigma_s^2$

$=\int_0^{\sigma_s}\left(1-\frac{\sigma_s^2}{\sigma_z^2+\sigma_s^2}\right)d\sigma_z$

$=\sigma_s-\sigma_s\int_0^1\frac{1}{1+\left(\frac{\sigma_z}{\sigma_s}\right)^2}d\left(\frac{\sigma_z}{\sigma_s}\right)$

$=\sigma_s-\sigma_s\left(\arctan x\Big|_0^1\right)$

$=\sigma_s\left(1-\frac{\pi}{4}\right)$.

第二个塑性变形部分：

$d\varepsilon^p=\frac{3\sigma_s}{2\sqrt{3}F}\int_0^{\sigma_s}\frac{\sigma_z}{\sigma_z^2+3\tau_{\theta z}^2}d\sigma_z$

$=\frac{\sqrt{3}}{2}\frac{\sigma_s}{F}\cdot\frac{1}{2}\int_0^{\sigma_s^2}\frac{1}{\sigma_z^2+3\tau_{\theta z}^2}d\sigma_z^2$

$=\frac{\sqrt{3}}{4}\frac{\sigma_s}{F}\cdot\ln(\sigma_z^2+\sigma_s^2)\Big|_0^{\sigma_s^2}$

$=\frac{\sqrt{3}}{4}\frac{\sigma_s}{F}(\ln(2\sigma_s^2/\sigma_s^2))=\frac{\sqrt{3}\sigma_s}{4F}\ln 2$

第二阶段中的剪切塑性变形：

因于：$dS_{r\theta}=0$，$d\sigma_m\delta_{ij}=0$，只需计算：$d\varepsilon_p=d\lambda\cdot S_{ij}=\frac{3d\sigma_i}{2F}\cdot\tau_{r\theta}$

其中：$\tau_{r\theta}=\frac{1}{\sqrt{3}}\sigma_s$；则：$\int d\varepsilon_p=\cancel{\frac{3\sigma_s}{2\sqrt{3}F}\cdot\int\frac{\sigma_z}{\sqrt{\sigma_z^2+3\tau_{r\theta}^2}}d\sigma_z}=\frac{1}{2}\int_0^{\sigma_s}\frac{1}{\sqrt{\sigma_z^2+3\tau_{r\theta}^2}}d\sigma_z^2=\cancel{\frac{1}{2}2\sqrt{t+3\tau_{r\theta}^2}\Big|_0^{\sigma_s}}$

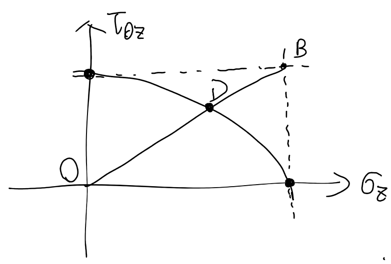
$\sigma_z^2+3\tau_{r\theta}^2$　$3\times\frac{\sigma_s^2}{3}$

$=2\sqrt{2}\sigma_s$

有：$\varepsilon_p=\sqrt{6}\frac{\sigma_s}{F}=\frac{1}{2}\gamma$

因此有：$\begin{cases}\varepsilon_z=\frac{\sigma_s}{E}+\cancel{\sqrt{2}\frac{\sigma_s}{F}}+\sigma_s\left(1-\frac{\pi}{4}\right)\\ \gamma_{r\theta}=\frac{2\sigma_s(1+\nu)}{\sqrt{3}E}+\cancel{2\sqrt{6}\frac{\sigma_s}{F}}+\frac{\sqrt{3}\sigma_s}{2F}\ln 2\end{cases}$

(3)、加载路径为：ODB 路径时：有 $\sigma_z:\tau_{rz}=\sqrt{3}:1$，此时：需要考虑屈服曲面前后的值：



由：$\sigma_i=\sqrt{\frac{1}{2}\left[(\sigma_x-\sigma_y)^2+\cdots+(\sigma_y-\sigma_z)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)\right]}=\sqrt{3J_2}$ 其中使用柱面坐标系；

∴ 得 $\sigma_i=\sqrt{\sigma_z^2+3\tau_{\theta z}^2}=\sigma_s$

此时保持 $\sigma_z=\sqrt{3}\tau_{\theta z}$，有 $\sqrt{6\tau_{\theta z}^2}=\sigma_s\Rightarrow\tau_{\theta z}^2=\frac{\sigma_s^2}{6}$

则：$\tau=\frac{\sigma_s}{\sqrt{6}}$，$\sigma_z=\frac{\sigma_s}{\sqrt{2}}$

∴在积分中，直接代换即可：

由本构方程：$\varepsilon_{ij}=\frac{1}{2\mu}S_i+\frac{1-2\nu}{E}\sigma_{ij}\delta_{ij}+\lambda\cdot S_{ij}$ | 此时第一阶段总的变形：

其中：由强化本构关系 $3d\sigma_i$　$3d\sigma$　$\varepsilon_z=\sigma_s$　$\sqrt{2}$

由本构方程：$\varepsilon_{ij} = \frac{1}{2\mu} S_i + \frac{1-2\nu}{E} \sigma_{ij}\delta_{ij} + \lambda \cdot S_{ij}$

其中：由强化本构关系：$\lambda = \frac{3 d\sigma_i}{2H\sigma_i} = \frac{3d\sigma_i}{2F\sigma_i}$

有：①：第一阶段（屈服前）有：$\lambda = 0$

$$\varepsilon_{ij} = \frac{1}{2\mu} S_{ij} + \frac{1-2\nu}{E}\sigma_{ij}\delta_{ij}$$

代入：

即 $d\varepsilon_z = \frac{1}{2\mu} dS_z + \frac{1-2\nu}{E} d\sigma_z$，其中：$\int dS_z = \frac{2}{3}\sigma_s \times \frac{\sqrt{2}}{2}$

得 $\varepsilon_z = \frac{1}{3\mu}\sigma_s + \frac{1-2\nu}{E}\sigma_s = \frac{2(1+\nu)}{3E}\sigma_s + \frac{1-2\nu}{E}\sigma_s = \frac{\sigma_s}{E} \Rightarrow \sigma_z = \frac{\sqrt{2}}{2}\frac{\sigma_s}{E}$

此时：$\gamma_{\theta z} = \cancel{2} \cdot \left(\frac{1}{\cancel{2}\mu} dS_{\theta z} + \frac{1-2\nu}{E} \cdot 0\right)$

$$= \frac{\cancel{2}(1+\nu)}{E} \cdot \frac{\sqrt{6}}{3\cancel{2}}\sigma_s = \frac{\sqrt{6}(1+\nu)}{3E}$$

此时第一阶段总的变形：

$$\varepsilon_z = \frac{\sigma_s}{E} \times \frac{\sqrt{2}}{2}$$

$$\gamma_{\theta z} = \frac{\sqrt{6}(1+\nu)}{3E}\sigma_s$$

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②. 塑性阶段：

由本构方程：$d\varepsilon_{ij} = \frac{1}{2\mu} dS_{ij} + \frac{1-2\nu}{E}\sigma_{ij} + d\lambda \cdot S_{ij}$ 其中 $d\lambda = \frac{3d\sigma_i}{2F\sigma_i}$

对于 z 方向：有：$\sigma_z : \frac{\sigma_s}{\sqrt{2}} \longrightarrow \sigma_s$

(1). z 方向形变量：

$$\tau_{\theta z} : \frac{\sigma_s}{\sqrt{6}} \longrightarrow \frac{\sigma_s}{\sqrt{3}}$$

其中：$d\sigma_i = d\sqrt{\sigma_z^2 + 3\tau_{\theta z}^2}$，由 $\sigma_z = \sqrt{3}\tau_{\theta z}$

此时可得：$= \sqrt{2}\, d\sigma_z$，而 $\sigma_i = \sqrt{2}\sigma_z$

$$\int d\varepsilon_z = \int_{\frac{\sqrt{2}}{2}}^{1} \left[\frac{1+\nu}{E} dS_z + \frac{1-2\nu}{E} d\sigma_m \delta_{ij} + \frac{3}{\cancel{2}F} \cdot \frac{\cancel{\sqrt{2}} d\sigma_z}{\cancel{\sqrt{2}\sigma_z}} \cdot \frac{\cancel{2}}{3}\cancel{\sigma_z}\right]$$

$$\int d\varepsilon_z = \frac{1+\nu}{E} \times \frac{2}{3}\left(1-\frac{\sqrt{2}}{2}\right)\sigma_s + \frac{1-2\nu}{E} \cdot \frac{\sigma_s}{3}\left(1-\frac{\sqrt{2}}{2}\right) + \frac{1}{F} \cdot \left(1-\frac{\sqrt{2}}{2}\right)\sigma_s$$

注意别漏乘 S_{ij}。

$$= \frac{\sigma_s}{E}\left(1-\frac{\sqrt{2}}{2}\right) + \frac{3}{2F} \cdot \frac{1}{2}\ln 2 = \left(1-\frac{\sqrt{2}}{2}\right)\frac{\sigma_s}{E} + \frac{\sigma_s}{F}\left(1-\frac{\sqrt{2}}{2}\right)$$

$$\frac{d\sigma_i}{\sigma_i} = \frac{d\sigma_z}{\sigma_z}$$

同理 $\frac{d\sigma_i}{\sigma_i} = \frac{d\tau_{\theta z}}{\tau_{\theta z}}$

(2)：θz 方向的夹角改变：

$$d\varepsilon_{\theta z} = \frac{1+\nu}{E} d\tau_{\theta z} + \frac{3}{2F} \cdot \frac{d\tau_{\theta z}}{\cancel{\tau_{\theta z}}} \cdot \cancel{\tau_{\theta z}}$$

$$\varepsilon_{\theta z} = \int d\varepsilon_{\theta z} = \frac{(1+\nu)\sigma_s}{E}\left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}}\right) + \frac{3}{2F} \cdot \sigma_s\left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}}\right) \times 2$$

$$= \frac{\sqrt{6}}{3} \frac{(1+\nu)\sigma_s}{\sqrt{6}E}(\sqrt{2}-1) + \frac{\sqrt{6}}{\cancel{\sqrt{3}}}\frac{\cancel{3}\sigma_s}{\cancel{2\sqrt{6}}F}(\sqrt{2}-1)$$

$\frac{3}{\sqrt{6}}\frac{\sigma_s}{F}(\sqrt{2}-1) \to \frac{\sqrt{6}}{2}\frac{\sigma_s}{F}(\sqrt{2}-1)$

$\gamma_{\theta z} = \frac{3}{F}\left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}}\right) = \frac{\sqrt{3}}{F}\left(1 - \frac{1}{\sqrt{2}}\right)$

得到第二阶段的变形量表达：

$$\varepsilon_z = \sigma_s\left(1-\frac{\sqrt{2}}{2}\right) \cdot \left(\frac{1}{E} + \frac{1}{F}\right)$$

$$\gamma_{\theta z} = \left(\frac{\sqrt{6}\,\sigma_s}{3E}(1+\nu) + \frac{\sqrt{6}\,\sigma_s}{2F}\right)(\sqrt{2}-1)$$

$$= \sigma_s\left(\frac{\sqrt{6}}{3E}(1+\nu) + \frac{\sqrt{6}\,\sigma_s}{2F}\right)(\sqrt{2}-1)$$

两式相加：有：

$$\varepsilon_z = \frac{\sigma_s}{E} \times \frac{\sqrt{2}}{2} + \frac{\sigma_s}{E}\left(1-\frac{\sqrt{2}}{2}\right) + \frac{\sigma_s}{F}\left(1-\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sigma_s}{E} + \left(1-\frac{\sqrt{2}}{2}\right)\frac{\sigma_s}{F}.$$

$$\gamma_{\theta z} = \cancel{\frac{\sqrt{6}(1+\nu)}{3E}}\sigma_s + \frac{\sqrt{6}(1+\nu)}{3E}(\sqrt{2}-1)\sigma_s + \frac{\sqrt{6}}{2F}\sigma_s(\sqrt{2}-1)$$

$$= \frac{2\sqrt{3}(1+\nu)}{3E}\sigma_s + \frac{\sqrt{6}\,\sigma_s}{2F}(\sqrt{2}-1) \quad 由于\ \frac{1}{\mu} = \frac{2(1+\nu)}{E}$$

$$= \frac{\sigma_s}{\sqrt{3}\mu} + \frac{\sqrt{3}}{F}\left(1-\frac{1}{\sqrt{2}}\right)$$