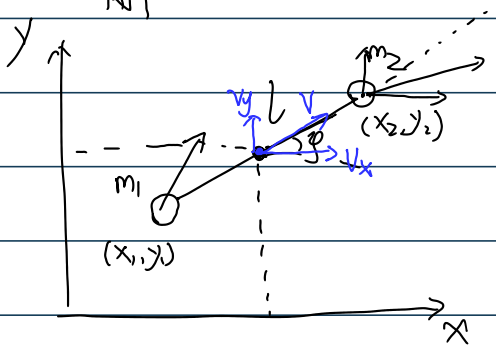
例题1.7

Friday, March 31, 2023 3:33 PM

两个质量均m质点 M_1, M_2, 由一长度为l的刚性杆相连, 其质量忽略, 运动时杆中点速度必须沿杆向, 建立系统运动数学模型.

解:



$k=3$, 广义坐标 x_c, y_c, θ, 动坐标 $(x_1,y_1)(x_2,y_2)$

约束:

①: $(y_2-y_1)^2+(x_2-x_1)^2=l^2$ (完整)

②: $\frac{y_2+y_1}{2}\cdot\frac{1}{\tan\varphi}=\frac{x_2+x_1}{2}$

代入: $\tan\varphi=\frac{y_2-y_1}{x_2-x_1}$

得: $(\dot{y}_2+\dot{y}_1)(x_2-x_1)-(\dot{x}_2+\dot{x}_1)(y_2-y_1)=0$ ②

将约束方程①②进行改写, 变分形式为:

分别对坐标取变分 $\rightarrow$ $2(y_2-y_1)\delta y_2-2(y_2-y_1)\delta y_1+2(x_2-x_1)\delta x_2-2(x_2-x_1)\delta x_1=0$

$$\begin{cases}(x_2-x_1)(\delta x_2-\delta x_1)+(y_2-y_1)(\delta y_2-\delta y_1)=0 & ①\\ (\delta y_2+\delta y_1)(x_2-x_1)-(\delta x_2+\delta x_1)(y_2-y_1)=0 & ②\end{cases}$$

以及:

此时有: 由第一类 Lagrange 方程.

先将上式化为: $\frac{\partial f_s}{\partial x_2}$, $\frac{\partial f_s}{\partial x_1}$

$$\begin{cases}(x_1-x_2)\delta x_1+(x_2-x_1)\delta x_2+(y_1-y_2)\delta y_1+(y_2-y_1)\delta y_2=0\\ a_{11}\rightarrow (y_1-y_2)\delta x_1+(y_1-y_2)\delta x_2+(x_2-x_1)\delta y_1+(x_2-x_1)\delta y_2=0\end{cases}$$

故: 由: $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)-\frac{\partial T}{\partial q_j}=Q_j+\sum_{r=1}^{m}\lambda_r a_{rj}+\sum_{s=1}^{t}\mu_s\frac{\partial f_s}{\partial x_j}$

由 $T=\frac{1}{2}m_1(\dot{x}_1^2+\dot{y}_1^2)+\frac{1}{2}m_2(\dot{x}_2^2+\dot{y}_2^2)=\frac{m}{2}(\dot{x}_1^2+\dot{y}_1^2+\dot{x}_2^2+\dot{y}_2^2)$

$=m(\dot{x}_c^2+\dot{y}_c^2)+\frac{1}{2}\left(m\left(\frac{l}{2}\right)^2+m\left(\frac{l}{2}\right)^2\right)\cdot\dot{\theta}^2$

$$\begin{cases}m_1\ddot{x}_1=\lambda_1(y_1-y_2)+\mu_1(x_1-x_2)\\ m_2\ddot{x}_2=\lambda_1(y_1-y_2)-\mu_1(x_1-x_2)\\ m_1\ddot{y}_1=-\lambda_1(x_1-x_2)+\mu_1(y_1-y_2)-m_1g\end{cases}$$ 约束力

此六式为描述系统运

$$
\begin{cases}
m_1\ddot{x}_1 = -\lambda_1(x_1-x_2)+\mu_1(y_1-y_2)-m_1g \\
m_2\ddot{x}_2 = -\lambda_1(x_1-x_2)-\mu_1(y_1-y_2)-m_2g
\end{cases}
\text{钟摆}
$$

和约束方程 $\Rightarrow$

$$
\begin{cases}
(x_1-x_2)^2+(y_1-y_2)^2 = l^2 \\
(\dot{x}_1+\dot{x}_2)(y_1-y_2) = (\dot{y}_1+\dot{y}_2)(x_1-x_2)
\end{cases}
$$

此六式为 描述系统运动的数学模型