孔边集中应力的剩余步骤

Friday, February 24, 2023 7:46 PM

$$\cos2\theta\times\left[f^{(4)}(r)+\frac{1}{r}f^{(3)}(r)-\frac{6}{r^2}f^{(2)}(r)+\frac{18}{r^3}f'(r)-\frac{24}{r^4}f(r)\right]$$

$$+\frac{1}{r}f^{(3)}(r)+\frac{1}{r^2}f^{(2)}(r)-\frac{5}{r^3}f'(r)+\frac{8}{r^4}f(r)$$

$$-\frac{4}{r^2}f^{(2)}(r)-\frac{4}{r^3}f'(r)+\frac{16}{r^4}f(r)$$

$$=\cos2\theta\times\left[f^{(4)}(r)+\frac{2}{r}f^{(3)}(r)-\frac{9}{r^2}f^{(2)}(r)+\frac{9}{r^3}f'(r)\right]=0$$

$dt=\frac{1}{r}dr$

解方程: $f^{(4)}(r)+\frac{2}{r}f^{(3)}(r)-\frac{9}{r^2}f^{(2)}(r)+\frac{9}{r^3}f'(r)=0$

★ 令 $r=e^t$, 则: $t=\ln r$, 照抄:

利用 $\frac{df}{dt}=\frac{df}{dr}\cdot\frac{dr}{dt}=r\frac{df}{dr}\rightarrow\frac{df}{dr}=\frac{1}{r}\frac{df}{dt}$

即有: $r^4f^{(4)}(r)+2r^3f^{(3)}(r)-9r^2f^{(2)}(r)+9rf'(r)=0$

$\frac{d^2f}{dr^2}$

$$\frac{d^2\varphi}{dr^2}=-\frac{1}{r^2}\frac{d\varphi}{dt}+\frac{1}{r^2}\frac{d^2\varphi}{dt^2}=\frac{1}{r^2}\left(\frac{d^2\varphi}{dt^2}-\frac{d\varphi}{dt}\right)$$

$$\frac{d^3\varphi}{dr^3}=-\frac{2}{r^3}\left(\frac{d^2\varphi}{dt^2}-\frac{d\varphi}{dt}\right)+\frac{1}{r^3}\left(\frac{d^3\varphi}{dt^3}-\frac{d^2\varphi}{dt^2}\right)=\frac{1}{r^3}\left(\frac{d^3\varphi}{dt^3}-\frac{3d^2\varphi}{dt^2}+\frac{2d\varphi}{dt}\right)$$

$$\frac{d^4\varphi}{dr^4}=\frac{1}{r^4}\left[\left(\frac{d^4\varphi}{dt^4}-\frac{3d^3\varphi}{dt^3}+\frac{2d^2\varphi}{dt^2}\right)-3\left(\frac{d^3\varphi}{dt^3}-\frac{3d^2\varphi}{dt^2}+\frac{2d\varphi}{dt}\right)\right]$$

$$=\frac{1}{r^4}\left[\frac{d^4\varphi}{dt^4}-\frac{6d^3\varphi}{dt^3}+\frac{11d^2\varphi}{dt^2}-\frac{6d\varphi}{dt}\right]$$

代入得: $\frac{d^4\varphi}{dt^4}-\frac{6d^3\varphi}{dt^3}+\frac{11d^2\varphi}{dt^2}-\frac{6d\varphi}{dt}+2\left(\frac{d^3\varphi}{dt^3}-3\frac{d^2\varphi}{dt^2}+\frac{2d\varphi}{dt}\right)-9\left[\frac{d^2\varphi}{dt^2}-\frac{d\varphi}{dt}\right]+9\frac{d\varphi}{dt}$

其中: $\varphi=f$,

即: $\underbrace{\frac{d^4f}{dt^4}-4\frac{d^3f}{dt^3}-4\frac{d^2f}{dt^2}+16\frac{df}{dt}=0}$.

$$\lambda^4-4\lambda^3-4\lambda^2+16\lambda=0\rightarrow\begin{cases}\lambda_1=0\\ \lambda_{2,3}=\pm2\\ \lambda_4=4\end{cases}\Rightarrow \text{即有:}$$

$$\lambda^3-4\lambda^2-4\lambda+16=0$$

$$(\lambda^2-4)(\lambda-4)=0$$

$\lambda_1=4,\ \lambda_2=2,\ \lambda_3=0,\ \lambda_4=-2,$

即: $f(t)=Ae^{4t}+Be^{2t}+C+De^{-2t}$

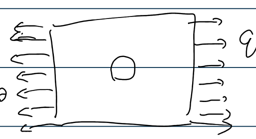
由: $r=e^t$, $t=\ln r$, $\frac{dt}{dr}=\frac{1}{r}$

$$f(r)=Ar^4+Br^2+C+\frac{D}{r^2}$$

则有: $\phi=f(r)\cos2\theta=\left(Ar^4+Br^2+C+\frac{D}{r^2}\right)\cos2\theta$

③: 已经得出 $\phi=\left(Ar^4+Br^2+C+\frac{D}{r^2}\right)\cos2\theta$，推导受拉q的平面体的应力分布情况.

应力函数



有:

$$\sigma_r=\frac{1}{r}\frac{\partial\varphi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2}=\frac{1}{r}\left(4Ar^3+2Br-\frac{2D}{r^3}\right)\cos2\theta-\frac{4}{r^2}\left(Ar^4+Br^2+C+\frac{D}{r^2}\right)\cos2\theta$$

$$\sigma_\theta=\frac{\partial^2\phi}{\partial r^2}\qquad=\left[\cancel{4Ar^2}+2B-\frac{2D}{r^4}-\cancel{4Ar^2}-4B-\frac{4C}{r^2}-\frac{4D}{r^4}\right]\cos2\theta$$

$$=\underbrace{\left[-2B-\frac{4C}{r^2}-\frac{6D}{r^4}\right]\cos2\theta}=-\left(2B+\frac{4C}{r^2}+\frac{6D}{r^4}\right)\cos2\theta$$

$$\sigma_\theta=\frac{\partial^2\phi}{\partial r^2}=(12Ar^2+2B)\cos2\theta.$$

$$\tau_{r\theta}=-\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\varphi}{\partial\theta}\right)=\frac{\partial}{\partial r}\left(2Ar^3+2Br+\frac{2C}{r}+\frac{2D}{r^3}\right)\sin2\theta$$

$$\begin{cases} \sigma_\theta = \frac{\partial^2\varphi}{\partial r^2} = (12Ar^2+2B)\cos 2\theta, \\ \tau_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \varphi}{\partial \theta}\right) = \frac{\partial}{\partial r}\left(2Ar^3+2Br+\frac{2C}{r}+\frac{2D}{r^3}\right)\sin 2\theta \end{cases}$$

$$= \left(6Ar^2+2B-\frac{2C}{r^2}-\frac{6D}{r^4}\right)\sin 2\theta$$

此时,有:利用边界条件,即:

$$\begin{cases} \sigma_r\big|_{r=b} = \frac{q}{2}\cos 2\theta & \text{有:} \quad 2B+\frac{4C}{b^2}+\frac{6D}{b^4} = -\frac{q}{2} \quad ① \\ \tau_{r\theta}\big|_{r=b} = -\frac{q}{2}\sin 2\theta & \text{而:} \quad 6Ab^2+2B-\frac{2C}{b^2}-\frac{6D}{b^4} = -\frac{q}{2} \quad ② \end{cases}$$

而 $\sigma_r\big|_{r=a} = 0$ 即: $\begin{cases} 2B+\frac{4C}{a^2}+\frac{6D}{a^4} = 0 & ③ \\ 6Aa^2+2B-\frac{2C}{a^2}-\frac{6D}{a^4} = 0 & ④ \end{cases}$ 联①②③④解A,B,C,D

$\tau_{r\theta}\big|_{r=a} = 0$,

解出 $A=0$ $B=-\frac{q}{4}$, $C=qa^2$, $D=-\frac{qa^4}{4}$,

代入: $$\begin{cases} \sigma_r = \left(\frac{q}{2}-\frac{4qa^2}{r^2}+\frac{3qa^4}{2r^4}\right)\cos 2\theta \\ \sigma_\theta = \left(-\frac{q}{2}-\frac{3qa^4}{2r^4}\right)\cos 2\theta \\ \tau_{r\theta} = \left(-\frac{q}{2}-\frac{2qa^2}{r^2}+\frac{3qa^4}{2r^4}\right)\sin 2\theta \end{cases}$$

后续不算了,