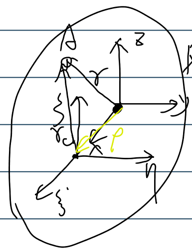
第三章的定理证明

Saturday, April 29, 2023   10:19 PM

1、移心公式的证明



①：对移心公式：

刚体上任取一点A，设质量m，

则 $[J_0] = -\sum m[\tilde{\gamma}]^2$

有：我们取质心系下的矢径为γ_c，有：

$\vec{\gamma} = \vec{\gamma}_c + \vec{\rho}$，写矩阵形式即：

并在相对坐标系表示

$\{\tilde{\gamma}\} = \{\tilde{\gamma}_c\} + \{\tilde{\rho}\}$，

及：$[\tilde{\gamma}] = [\tilde{\gamma}_c] + [\tilde{\rho}]$，

由：$J_0 = -\sum m[\tilde{\gamma}]^2$

$= -\sum$

2、动量矩第二种表达方式：

对：动量矩：

$\{H\} = m\vec{\gamma}\times\vec{v} = m\vec{\gamma}\times(\vec{\omega}\times\vec{\gamma})$

$= -m(\vec{\gamma}\times\vec{\gamma})\times\vec{\omega}$，$= -m[\tilde{\gamma}]^2\{\omega\}$

我们考虑$[\tilde{\gamma}]^2$的表达，

有：

$$\begin{bmatrix} & -\gamma_3 & +\gamma_2 \\ \gamma_3 & & -\gamma_1 \\ -\gamma_2 & \gamma_1 & \end{bmatrix}\begin{bmatrix} & -\gamma_3 & +\gamma_2 \\ \gamma_3 & & -\gamma_1 \\ -\gamma_2 & \gamma_1 & \end{bmatrix} = \begin{bmatrix} -\gamma_2^2-\gamma_3^2 & \gamma_1\gamma_2 & \gamma_1\gamma_3 \\ \gamma_1\gamma_2 & -\gamma_1^2-\gamma_3^2 & \gamma_2\gamma_3 \\ \gamma_1\gamma_3 & \gamma_2\gamma_3 & -\gamma_1^2-\gamma_2^2 \end{bmatrix}$$

我们使用：$\{\gamma\}\{\gamma\}^T = \begin{bmatrix}\gamma_1\\ \gamma_2\\ \gamma_3\end{bmatrix}[\gamma_1, \gamma_2, \gamma_3] = \begin{bmatrix} \gamma_1^2 & \gamma_1\gamma_2 & \gamma_1\gamma_3 \\ \gamma_1\gamma_2 & \gamma_2^2 & \gamma_2\gamma_3 \\ \gamma_1\gamma_3 & \gamma_2\gamma_3 & \gamma_3^2 \end{bmatrix}$

∴有

$[\tilde{\gamma}]^2 = -(\gamma_1^2+\gamma_2^2+\gamma_3^2)I + \{\gamma\}\{\gamma\}^T$

即：$J = -m[\tilde{\gamma}]^2 = \underline{m\left[\gamma^2[I] - \{\gamma\}\{\gamma\}^T\right]}$