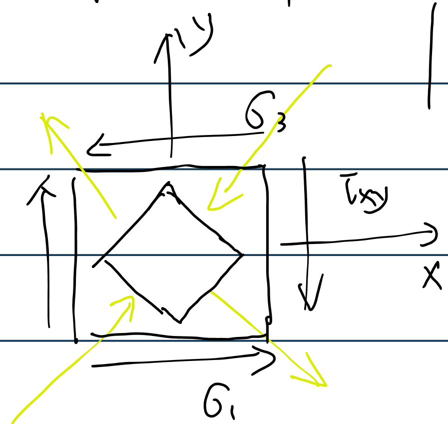
使用形状改变比能推导弹性常数关系式

Wednesday, March 29, 2023 12:14 AM

我们使用纯剪切应力状态下的应变比能证明关系!

$$\boxed{G = \frac{E}{2(1+\nu)}}$$



设：矩形受纯剪切应力作用，

则：由转轴公式：

$$\begin{cases} \sigma_n = \dfrac{\sigma_x+\sigma_y}{2} + \dfrac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \\ \tau_n = \dfrac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \end{cases}$$

代入：则：令 $\theta = 45^\circ$，有

$$45^\circ \begin{cases} \sigma_n = \tau_{xy} \\ \tau_n = 0 \end{cases} \qquad -45^\circ \begin{cases} \sigma_n = -\tau_{xy} \\ \tau_n = 0 \end{cases}$$

用两种方法计算形状改变比能：

$$v_s = \frac{1}{2}\tau\gamma = \frac{\tau^2}{2G}$$

而使用主应力计算：$\sigma_1 = \tau$，$\sigma_2 = 0$，$\sigma_3 = -\tau$

有：
$$\begin{cases} \varepsilon_1 = \dfrac{\tau}{E} + \nu\dfrac{\tau}{E} \\ \varepsilon_2 = 0 \\ \varepsilon_3 = -\dfrac{\tau}{E} - \nu\dfrac{\tau}{E} \end{cases}$$

$$\therefore v_s = \frac{1}{2}\sigma_1\varepsilon_1 + \frac{1}{2}\sigma_3\varepsilon_3 = \frac{(1+\nu)\tau^2}{E}$$

故：有：

$$v_s = \frac{(1+\nu)\cancel{\tau^2}}{E} = \frac{\cancel{\tau^2}}{2G}$$

得：$G = \dfrac{E}{2(1+\nu)}$