例1. 线性系统平衡点不变子空间

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例1. 给出线性系统的不变子空间为:

(1)

$$A_1 = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad 则: |\lambda E - A| = \begin{bmatrix} \lambda-1 & -2 & \\ -1 & \lambda & \\ & & \lambda \end{bmatrix}$$

故有:

$$(\lambda-1)\lambda^2 - \lambda\cdot(-1)(-2) = 0$$

$$(\lambda-1)\lambda^2 - 2\lambda = 0$$

$$\lambda[\lambda(\lambda-1)-2] = \lambda(\lambda^2-\lambda-2) = \lambda(\lambda-2)(\lambda+1)$$

有特征值: $\lambda_1 = 0,\ \lambda_2 = 2,\ \lambda_3 = -1$. 对应特征向量:

求法:

首先: 代入: $\lambda_1 = 0$, $[\lambda E - A]x = 0$,

$\lambda_2 = 0 \longleftarrow$ 大小排序

$$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \therefore x_2 = -\frac{1}{2}x_1,\ x_1 = 0,\ x_3 = -x_1 - x_2 = 0, \longrightarrow x_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\lambda_1 = 2$

$$\begin{bmatrix} 1 & -2 & \\ -1 & 2 & \\ & & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad x_1 = 2x_2,\ x_3 = 0 \quad x = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

$\lambda_3 = -1$

$$\begin{bmatrix} -2 & -2 & \\ -1 & -1 & \\ & & -1 \end{bmatrix} \quad x_1 = -x_2,\ x_3 = 0 \quad x_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix},$$

故

$$\begin{cases} E^s = span[1, -1, 0]^T \\ E^c = span[0, 0, 0]^T \\ E^u = span[2, 1, 0]^T \end{cases}$$

(2)

$$A_2 = \begin{bmatrix} -1 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

有:

不带λ!

$$|\lambda E - A_2| = \begin{vmatrix} \lambda+1 & 1 & \\ -1 & \lambda+1 & \\ & & \lambda-2 \end{vmatrix} = (\lambda+1)^2(\lambda-2) - (\lambda-2)\cdot(-1)$$

$$= (\lambda-2)[(\lambda^2+2\lambda+1) + 1] = 0$$

$$= (\lambda-2)(\lambda^2+2\lambda+2) = 0$$

在此种情况下,有2个特征值,但其实有3个

$$2(\lambda-2)(\lambda+1) = 0$$

$$\lambda_1 = 2,\ \lambda_2 = -1$$

$\lambda^2+2\lambda+2 = 0 \quad \therefore (\lambda+1)^2 = -1 \longrightarrow \lambda = -1 \pm i$

$2(\lambda-2)(\lambda+1)=0$ $\qquad = (\lambda-2)(\lambda^2+2\lambda+2)=0$

$\lambda_1=2,\ \lambda_2=-1$ $\qquad \lambda^2+2\lambda+2=0 \ \therefore (\lambda+1)^2=-1 \longrightarrow \lambda=-1\pm i$

则：$\lambda_1=2,\ \lambda_2=-1+i\quad \lambda_3=-1-i$ 。

此时：代入λ计算张成的子空间：注意：⟹ 两个共轭复根一块算子空间.

此时：

$\lambda=-1-i$ 对应：$\begin{vmatrix} -i & 1 & \\ -1 & -i & \\ & & -3-i \end{vmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ $\quad -ia+b=0 \quad \therefore b=ia,\quad c=0.$ $\quad \begin{bmatrix} 1 \\ i \\ 0 \end{bmatrix}$ ①,

$\lambda=-1+i$ 对应：$\begin{vmatrix} i & 1 & \\ -1 & i & \\ & & -3+i \end{vmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ $\quad \therefore -a+ib=0 \quad a=ib$ $\quad \begin{bmatrix} 1 \\ -i \\ 0 \end{bmatrix}$ ②

$\lambda=2$ 对应：$\begin{vmatrix} 3 & 1 & \\ -1 & 3 & \\ & & 0 \end{vmatrix}$ $\longrightarrow$ 注意：C系数为0：则对应向量 $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

在计算张成子空间时：将两个复根合并为一个（这是由于实部均为负）

$E^s=\mathrm{span}\left[[1,0,0]^T,\ [0,1,0]^T\right),\ E^u=\mathrm{span}\left([0,0,1]^T\right)$

实际是 $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\pm i\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$