非共振受迫振动的推导

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1) Duffing 方程的非共振受迫振动，

即：$\ddot{x} + \omega_0^2(x + \varepsilon x^3) = F\cos\omega t$

可化为：$\ddot{x} + \omega_0^2 x = F(t) + \varepsilon f(x,\dot{x})$，其中 ε 为小量

则改方程为：

$$\begin{cases} \ddot{x} + \omega_0^2 x_0 = F(t) \\ \ddot{x}_1 + \omega_0^2 x_1 = f(x_0, \dot{x}_0) \\ \ddot{x}_2 + \omega_0^2 x_2 = x_1 \dfrac{\partial f(x_0,\dot{x}_0)}{\partial x} + \dot{x}_1 \dfrac{\partial f(x_0,\dot{x}_0)}{\partial \dot{x}} \end{cases}$$

则 $x(t,\varepsilon) = X(t) + \varepsilon x_1(t) + \varepsilon^2 x_2(t) + \cdots$

因而将上式分解：

$$\ddot{x} + \omega_0^2 x = F\cos\omega t - \varepsilon\omega_0^2 x^3$$

∴有：$x_1 \propto -\omega_0^2 x_0^3$，

则：得出为：

$$\ddot{x}_0 + \omega_0^2 x_0 = F\cos\omega t$$
$$\ddot{x}_1 + \omega_0^2 x_1 = -\omega_0^2 x_0^3$$
$$\ddot{x}_2 + \omega_0^2 x_2 = -3\omega_0^2 x_0^2 x_1$$

示零次近似方程为线性系统的受迫振动方程：为；

$$x_0 = A_0\cos(\omega t + \theta_0) + A\cos\omega t,$$

$A = \dfrac{F}{\omega_0^2 - \omega^2}$

第一项：自由振动

第二项：受迫振动，