

作业5.5的解方程

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$$\begin{cases} d\varepsilon_x = \frac{1+\nu}{E} \cdot \frac{2}{3} d\sigma + d\lambda \cdot \frac{2}{3} \sigma \end{cases}$$

$$\begin{cases} \frac{1}{2} d\gamma_{xy} = \frac{1+\nu}{E} d\tau_{xy} + d\lambda \tau_{xy} \end{cases} \quad \text{未知: } d\varepsilon_x, d\tau_{xy}$$

其中: $d\tau_{xy} = -\frac{\sigma_x}{3\tau_{xy}} d\sigma_x$

有: $d\sigma$

$$\begin{cases} \frac{1}{2} d\gamma_{xy} = \frac{1+\nu}{E} \cdot \frac{\sigma}{3\tau} d\sigma + d\lambda \cdot \tau \end{cases}$$

$$\begin{cases} d\varepsilon_x = \frac{1+\nu}{E} \cdot \frac{2}{3} d\sigma + d\lambda \cdot \frac{2}{3} \sigma \end{cases}$$

此时: 消去 $d\lambda$:

$$\begin{cases} \frac{\gamma_{xy}}{2\tau} = \frac{1+\nu}{E} \cdot \frac{\sigma}{3\tau^2} d\sigma + d\lambda \quad (1) \\ \frac{3d\varepsilon_x}{2\sigma} = \frac{1+\nu}{E\sigma} d\sigma + d\lambda \quad (2) \end{cases}$$

有公式:

$$\frac{1+\nu}{E} \cdot \left(\frac{\sigma}{3\tau^2} - \frac{1}{\sigma} \right) d\sigma = \frac{d\gamma_{xy}}{2\tau} - \frac{3d\varepsilon_x}{2\sigma}$$

$$\begin{aligned} \text{则: } d\sigma &= \frac{E}{1+\nu} \left(\frac{3\tau^2}{\sigma^2 - 3\tau^2} \right) \left[\frac{2d\gamma_{xy} - 3d\varepsilon_x}{2\tau\sigma} \right] \\ &= \frac{E}{1+\nu} \left(\frac{3\tau}{\sigma^2 - 3\tau^2} \right) (d\gamma_{xy} - 3d\varepsilon_x) \end{aligned}$$

$$= \frac{E}{1+\nu} \left(\frac{3\tau}{\sigma^2 - 3\tau^2} \right) (d\gamma_{xy} - 3d\epsilon_x)$$

$$\text{又: } d\tau_{xy} = - \frac{\sigma}{3\tau} d\sigma$$

$$= - \frac{E}{1+\nu} \cdot \frac{\sigma}{\sigma^2 - 3\tau^2} (d\gamma_{xy} - 3d\epsilon_x)$$