

例题1.4

Friday, March 31, 2023 2:41 PM

如图, 两均质细杆 AB, BC 长 l , 质量均为 m , 静止地悬挂在铅垂位置。此时, BC 杆中点 D 处受一碰撞, 冲量为 S , 求碰撞结束时, AB, BC 角速度。

解:

如图: 系统自由度 2

使用碰撞的 Lagrange 方程:

$$\Delta P_j = Q_j$$

$$\text{其中: } \Delta P_j = \left. \frac{\partial T}{\partial \dot{q}_j} \right|_{t_1+\Delta t} - \left. \frac{\partial T}{\partial \dot{q}_j} \right|_{t_1}$$

$$\hat{Q}_j = \sum_{i=1}^n S_i \frac{\partial r_{i0}}{\partial q_j}$$

有: 对应于 θ_1 和 θ_2 的广义动量分别为:

$$\begin{cases} P_1 = \frac{\partial T}{\partial \dot{\theta}_1} \\ P_2 = \frac{\partial T}{\partial \dot{\theta}_2} \end{cases}$$

注意: 计算公式是 $\frac{1}{2} I \omega^2$ 而 $I = \frac{1}{3} m l^2$ $I = \frac{1}{12} m l^2$

注意: 带有牵连速度的杆件,

只能先计算出质心速度再使用质心的转动惯量计算

$$\text{即: } T = \frac{1}{2} \left(\frac{1}{3} m l^2 \right) \dot{\theta}_1^2 + \frac{1}{2} m_2 \left(\dot{\theta}_1 l + \dot{\theta}_2 \frac{l}{2} \right)^2 + \frac{1}{2} \left(\frac{1}{2} m_2 l^2 \right) \dot{\theta}_2^2$$

$$= \frac{1}{6} m l^2 \dot{\theta}_1^2 + \frac{1}{2} m \left(\dot{\theta}_1^2 l^2 + \dot{\theta}_1 \dot{\theta}_2 l^2 + \frac{\dot{\theta}_2^2 l^2}{4} \right) + \frac{1}{24} m l^2 \dot{\theta}_2^2$$

$$= \frac{2}{3} m l^2 \dot{\theta}_1^2 + \frac{1}{2} m l^2 \dot{\theta}_1 \dot{\theta}_2 + \frac{1}{6} m \dot{\theta}_2^2 l^2$$

$$\text{则有: } \begin{cases} P_1 = \frac{4}{3} m l^2 \dot{\theta}_1 + \frac{1}{2} m l^2 \dot{\theta}_2 \\ P_2 = \frac{1}{2} m l^2 \dot{\theta}_1 + \frac{1}{3} m l^2 \dot{\theta}_2 \end{cases}$$

$$\begin{cases} P_1 = \frac{4}{3} m l^2 \dot{\theta}_1 + \frac{1}{2} m l^2 \dot{\theta}_2 \\ P_2 = \frac{1}{2} m l^2 \dot{\theta}_1 + \frac{1}{3} m l^2 \dot{\theta}_2 \end{cases}$$

由于显然 $P_1|_{t=0} = P_2|_{t=0} = 0$, 则:

$$\Delta P_1 = \frac{4}{3} m l^2 \omega_1 + \frac{1}{2} m l^2 \omega_2$$

$$\Delta P_2 = \frac{1}{2} m l^2 \omega_1 + \frac{1}{3} m l^2 \omega_2$$

$$\text{由 } \hat{Q}_j = \sum_{i=1}^n S_i \frac{\partial r_{i0}}{\partial q_j}$$

$$\text{则 } \hat{Q}_j \delta q_j = \sum_{i=1}^n S_i \delta r_{i0} = \Delta P \cdot \delta q_j$$

$$\text{有: } S \cdot \delta r_{D0} = \hat{Q}_j \delta q_j = S \left(l \delta \theta_1 + \frac{1}{2} l \delta \theta_2 \right) \quad \star \text{ 难点: 将 } \delta r_{D0} \text{ 计为 } \delta A, \delta A$$

有: $\delta \cdot \delta r_D = \hat{Q}_j \delta q_j = \delta (l \delta \theta_1 + \frac{1}{2} l \delta \theta_2)$ ★ 难点: 将 δr_D 表达为 $\delta \theta_1, \delta \theta_2$

则: $\cancel{S l \delta \theta_1} = (\frac{4}{3} m l^2 \omega_1 + \frac{1}{2} m l^2 \omega_2) \delta \theta_1$

\therefore 有: ①: $\frac{4}{3} m l^2 \omega_1 + \frac{1}{2} m l^2 \omega_2 = S l$

又: $\delta \cdot \frac{1}{2} l \delta \theta_2 = (\frac{1}{2} m l^2 \omega_1 + \frac{1}{3} m l^2 \omega_2) \delta \theta_2$

②: $m l^2 \omega_1 + \frac{2}{3} m l^2 \omega_2 = S l$

联①②:

有:

$$\begin{cases} \frac{8}{3} m l^2 \omega_1 - \frac{3}{2} m l^2 \omega_2 = \frac{S}{2} l \\ \text{即: } \frac{7}{6} m l^2 \omega_1 = \frac{S}{2} l \end{cases}$$

即: $\frac{7}{6} m l^2 \omega_1 = \frac{S}{2} l$

有: $\boxed{\omega_1 = \frac{3S}{7ml}}$

即: $\frac{2}{3} m l^2 \omega_2 = \frac{2S}{7} l$

$\boxed{\omega_2 = \frac{6S}{7ml}}$

故得到:

$$\omega_1 = \frac{3S}{7ml}, \omega_2 = \frac{6S}{7ml}$$