

混合重要抽样的密度推导

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我们知道

$$P_f = \int_{R_n} I_F(x) \frac{f_x(x)}{h_x(x)} h_x(x) dx$$

由于我们使用

$$h_x(x) = \sum_{k=1}^m \alpha_k h_x^{(k)}(x)$$

则代入有:

$$\begin{aligned} P_f &= \sum_{k=1}^m \int_{R_n} I_F(x) \frac{f_x(x)}{h_x^{(k)}(x)} \alpha_k h_x^{(k)}(x) dx \\ &= \sum_{k=1}^m E \left(I_F(x) \frac{f_x(x)}{h_x^{(k)}(x)} \alpha_k \right) \end{aligned}$$

样本均值计算:

我们取 N_k 个样本, 则

$$\hat{P}_f = \sum_{k=1}^m \frac{1}{N_k} \sum_{l=1}^{N_k} I_F(X_{kl}) \frac{f_x(X_{kl})}{h_x^{(k)}(X_{kl})} \alpha_k$$

对应的均值显然:

$$E(\hat{P}_f) = E \left[\sum_{k=1}^m \frac{1}{N_k} \sum_{l=1}^{N_k} I_F(X_{kl}) \frac{f_x(X_{kl})}{h_x^{(k)}(X_{kl})} \alpha_k \right]$$

样本均值收敛于总体均值, 则:

$$= \sum_{k=1}^m \frac{1}{N_k} \sum_{l=1}^{N_k} I_F(X_{kl}) \frac{f_x(X_{kl})}{h_x^{(k)}(X_{kl})} \alpha_k = \hat{P}_f$$

$$\text{同样地: } V(\hat{P}_f) = V \left[\sum_{k=1}^m \frac{1}{N_k} \sum_{l=1}^{N_k} I_F(X_{kl}) \frac{f_x(X_{kl})}{h_x^{(k)}(X_{kl})} \alpha_k \right]$$

$$= \frac{1}{N_k} \sum_{k=1}^m V \left[I_F(X_{kl}) \frac{f_x(X_{kl})}{h_x^{(k)}(X_{kl})} \alpha_k \right]$$

$$\text{考虑到 } S^2(x) = \frac{1}{N-1} \left[\sum_{i=1}^{N_k} X_i^2 + N \bar{X}^2 \right]$$

$$= \frac{1}{N_k(N_k-1)} \sum_{k=1}^m \left[\sum_{l=1}^{N_k} \left(I_F(X_{kl}) \frac{f_x(X_{kl})}{h_x^{(k)}(X_{kl})} \alpha_k \right)^2 + N_k \left(\frac{1}{N_k} \sum_{l=1}^{N_k} I_F(X_{kl}) \frac{f_x(X_{kl})}{h_x^{(k)}(X_{kl})} \alpha_k \right)^2 \right]$$