二端口网络的参数关系和互易对称条件推导

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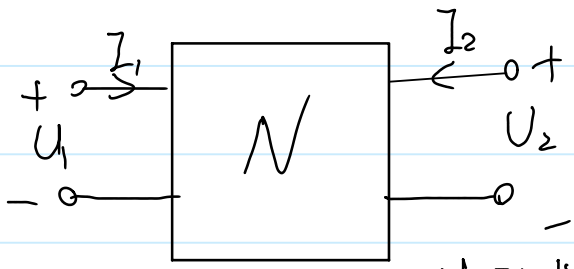
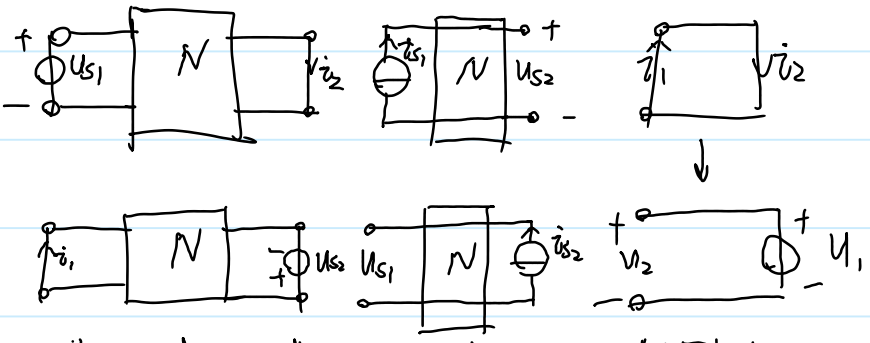
对二端口网络，有参数 U_1, U_2, I_1, I_2，则：



$$\begin{cases} \dot{U}_1 = \dot{I}_1 Z_{11} + \dot{I}_2 Z_{12} \\ \dot{U}_2 = \dot{I}_1 Z_{21} + \dot{I}_2 Z_{22} \end{cases}$$

我们首先推导互易性：

1）互易性网络有三种形式：（同时成立）



其中，第二种形式有：对成比例的激励有：

$i_{s1} = k i_{s2}$ 时，$u_{s2} = k u_{s1}$ 由齐次定理，

$$\left.\frac{\dot{U}_{s1}}{\dot{I}_{s2}}\right|_{I_1=0} = \left.\frac{\dot{U}_{s2}}{\dot{I}_{s1}}\right|_{I_2=0} \Rightarrow \text{有：} Z_{12} = Z_{21}\text{，即互易定理满足}$$

对称时：$Z_{11} = Z_{22}$ 同时满足。

② 对于 Y 方程，有：

$$\begin{cases} \dot{I}_1 = y_{11}\dot{U}_1 + y_{12}\dot{U}_2 \\ \dot{I}_2 = y_{21}\dot{U}_1 + y_{22}\dot{U}_2 \end{cases}$$

从而有：$\begin{cases} U_{s1} = k U_{s2} \text{ 时，} \\ i_2 = k i_1 \end{cases}$

$$\therefore \left.\frac{I_1}{U_{s2}}\right|_{u_{s1}=0} = \left.\frac{I_2}{U_{s1}}\right|_{u_{s2}=0}$$

从而有：$y_{12} = y_{21}$

③ 由互参数推 A 参数：

$$A: \begin{cases} U_1 = a_{11}U_2 + a_{12}(-I_2) \\ I_1 = a_{21}U_2 + a_{22}(-I_2) \end{cases} \text{从而有：} \dot{I}_1 = \frac{\dot{U}_2 - Z_{22}\dot{I}_2}{Z_{21}}$$

$$\begin{cases} U_1 = a_{11}U_2 + a_{12}(-I_2) \\ I_1 = a_{21}U_2 + a_{22}(-I_2) \end{cases}$$ 从而有：$I_1 = \dfrac{U_2}{Z_{21}} - \dfrac{Z_{22}}{Z_{21}}I_2$

$$\therefore U_1 = \frac{Z_{11}}{Z_{21}}\left[U_2 - Z_{22}I_2\right] + Z_{12}I_2$$

从而有关系：

$$a_{11} = Z_{11}/Z_{21} \qquad a_{12} = -Z_{12} + \frac{Z_{11}}{Z_{21}}Z_{22} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}$$

$$a_{21} = \frac{1}{Z_{21}} \qquad a_{22} = \frac{Z_{22}}{Z_{21}}$$

当网络互易时：由 $Z_{12} = Z_{21}$

显然：$a_{11}a_{22} = \dfrac{Z_{11}Z_{22}}{Z_{21}^2}$，$a_{12}a_{21} = \dfrac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}^2}$

有：$a_{11}a_{12} - a_{12}a_{21} = \dfrac{Z_{12}}{Z_{21}} = 1$ √ 即：$|A| = 1$.

其余方程可类似导出。