冲激函数性质证明

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对$\delta(t)$有:

(1). 证明偶函数: $\delta(t)=\delta(-t)$.

证明: 设 $f(t)$为任意有界函数:

$$\therefore \int_{-\infty}^{+\infty} \delta(t) f(t)\,dt = f(0)$$

代换有: $\int_{-\infty}^{+\infty} \delta(-t) f(-t)\,d(-t) = f(0) = \int_{-\infty}^{+\infty} \delta(-t) f(-t)\,dt$

即: $\int_{-\infty}^{+\infty} \delta(-t) f(-t)\,d(-t) = f(0)$ 显然成立

$\Longrightarrow$ 从而 $\int_{-\infty}^{+\infty} \delta(-t) f(t)\,dt \xlongequal{t'=-t} \int_{+\infty}^{-\infty} \delta(t') f(-t')\,-d(t')$

$$\therefore = \int_{-\infty}^{+\infty} \delta(t') f(-t')\,dt' = f(0)$$

即: $\int_{-\infty}^{+\infty} \delta(t) f(t)\,dt' = \int_{-\infty}^{+\infty} \delta(t) f(t)\,dt$.

应有 $\delta(-t)=\delta(t)$ 成立.

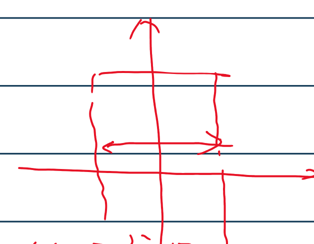
② 微分关系证明:

显然 $\int_{-\infty}^{x} \delta(t)\,dt = 0$, $x<0$,

而 $\int_{-\infty}^{x} \delta(t)\,dt = 1$, $x>0$

故: $\int_{-\infty}^{t} \delta(\tau)\,d\tau = \begin{cases} 0 & t<0 \\ 1 & t>0 \end{cases} = U(t)$

而: $\frac{dU(t)}{dt} = \lim_{\Delta t\to 0} \frac{U(t+\Delta t)-U(t)}{\Delta t} = \frac{U(t+\frac{\Delta t}{2})-U(t-\frac{\Delta t}{2})}{\Delta t}$



$= \frac{1}{\Delta t} G_{\Delta t}(t) \Rightarrow$ 高$\frac{1}{\Delta t}$, 宽Δt的门信号.