平面应力应变状态下的本构方程推导

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① : 原始形式的广义胡克定律:

$$\varepsilon_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \nu\theta\delta_{ij}$$

其中: $\theta = 3\sigma_m = \sigma_x+\sigma_y+\sigma_z$.

(1). 平面应力状态: $\sigma_x, \sigma_y \neq 0$, $\sigma_z = 0$,

$$\boxed{\begin{cases} \varepsilon_x = \frac{1}{E}[\sigma_x - \nu\sigma_y] \\ \varepsilon_y = \frac{1}{E}[\sigma_y - \nu\sigma_x] \\ \varepsilon_z = -\frac{\nu}{E}[\sigma_x+\sigma_y] \end{cases}} \quad ①.$$

$$\boxed{\begin{cases} \sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) \\ \sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) \end{cases}} \quad ②$$

应力-应变关系:

$$\begin{cases} E\varepsilon_x = \sigma_x - \nu\sigma_y \\ E\varepsilon_y = \sigma_y - \nu\sigma_x, \end{cases}$$

则: $E\varepsilon_x = \sigma_x - \nu(E\varepsilon_y + \nu\sigma_x)$

$\therefore E\varepsilon_x = \sigma_x(1-\nu^2) - \nu E\varepsilon_y$

即: $\sigma_x = \frac{E(\varepsilon_x + \nu\varepsilon_y)}{1-\nu^2}$

同: $\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$

(2). 平面应变状态: $\varepsilon_z = 0$,

$$\begin{cases} \varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) \\ \varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) \\ \varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) = 0 \end{cases}$$

则: $\sigma_z = \nu(\sigma_x+\sigma_y)$

代入得到:

$$\varepsilon_x = \frac{1}{E}\left[(1-\nu^2)\sigma_x - \frac{(\nu+\nu^2)}{E}\sigma_y\right]$$

即:

$$\boxed{\begin{aligned} \varepsilon_x &= \frac{1-\nu^2}{E}\left[\sigma_x - \frac{\nu}{1-\nu}\sigma_y\right] \\ \varepsilon_y &= \frac{1-\nu^2}{E}\left[\sigma_y - \frac{\nu}{1-\nu}\sigma_x\right] \end{aligned}}$$

||

或

$$\boxed{\begin{cases} \varepsilon_x = \frac{1+\nu}{E}[(1-\nu)\sigma_x - \nu\sigma_y] \\ \varepsilon_y = \frac{1+\nu}{E}[(1-\nu)\sigma_y - \nu\sigma_x] \end{cases}}$$

使用应力-应变关系式,则:

$$\begin{cases} \sigma_x - \frac{\nu}{1-\nu}\sigma_y = \frac{E\varepsilon_x}{1-\nu^2} \\ \sigma_y - \frac{\nu}{1-\nu}\sigma_x = \frac{E\varepsilon_y}{1-\nu^2}. \end{cases} \longrightarrow \sigma_x = \frac{E\varepsilon_x}{1-\nu^2} + \frac{\nu}{1-\nu}\left(\frac{E\varepsilon_y}{1-\nu^2} + \frac{\nu}{1-\nu}\sigma_x\right)$$

$$\sigma_x\left[1 - \frac{\nu^2}{(1-\nu)^2}\right] = \frac{E}{\cancel{1-\nu^2}}\left(\varepsilon_x + \frac{\nu}{1-\nu}\varepsilon_y\right)$$

$$\sigma_x \frac{2\nu+1}{\cancel{1-\nu^2}} = \cdots$$

$$\therefore \sigma_x = \frac{E}{1+2\nu}\left(\varepsilon_x + \frac{\nu}{1-\nu}\varepsilon_y\right) = \frac{E}{(1-\nu)(1+2\nu)}\left[(1-\nu)\varepsilon_x + \nu\varepsilon_y\right]$$

所得到应力-应变式:

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$$\boxed{\begin{aligned}
\sigma_x &= \frac{E}{(1-\nu)(1+2\nu)}\left[(1-\nu)\varepsilon_x+\nu\varepsilon_y\right]\\
\sigma_y &= \frac{E}{(1-\nu)(1+2\nu)}\left[\nu\varepsilon_x+(1-\nu)\varepsilon_y\right]\\
\sigma_z &= \nu(\sigma_x+\sigma_y)=\frac{E}{(1-\nu)(1+2\nu)}\left[\nu\varepsilon_x+\nu\varepsilon_y\right]
\end{aligned}}$$