谐波平衡推导Duffing方程自由振动

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③ 谐波平衡方法对Duffing系统自由振动推导：

对Duffing方程自由振动：

$$\ddot{x} + \omega_0^2(x+\varepsilon x^3) = 0$$

取一次谐波项：

$$x = A_c\cos\omega t + A_s\sin\omega t$$ ，代入：

$$\ddot{x} = -\omega^2 A_c\cos\omega t - \omega^2 A_s\sin\omega t$$ ，则有：

$$(\omega_0^2-\omega^2)A_c\cos\omega t + (\omega_0^2-\omega^2)A_s\sin\omega t + \omega_0^2\varepsilon x^3\cos\omega t \cos\omega t + \omega_0^2\varepsilon x^3\sin\omega t \sin\omega t$$

g_{c1} g_{s1}

而：将 $\omega_0^2\varepsilon x^3$ 部分取平均，有：

$$= \omega_0^2\varepsilon\frac{1}{T}\int_0^T (A_c\cos\omega t + A_s\sin\omega t)^3$$

$$g_{c1} = \frac{\omega_0^2\varepsilon}{T}\int_0^T (A_c\cos\omega t + A_s\sin\omega t)^3 \cos\omega t \, dt = 0$$

$$= \frac{\omega_0^2\varepsilon}{T}\int_0^T \left(A_c^3\cos^4\omega t + \cancel{3A_c^2A_s\cos^3\omega t\sin\omega t} + 3A_cA_s^2\sin^2\omega t\cos^2\omega t + \cancel{A_s^3\sin^3\omega t\cos\omega t}\right)dt$$

$$= \frac{\omega_0^2\varepsilon}{T}\int_0^T \left[A_c^3\cos^4\omega t + 3A_cA_s^2\sin^2\omega t\cos^2\omega t\right]dt$$

$$= \omega_0^2\varepsilon\left[\frac{2}{\pi}A_c^3\times\frac{3}{4}\times\frac{1}{2}\times\frac{\pi}{2} + \frac{2}{\pi}\int_0^{\frac{\pi}{2}} 3A_cA_s^2(\cos^2\omega t - \cos^4\omega t)\,dt\right]$$

$$= \omega_0^2\varepsilon\left[\frac{3}{8}A_c^3 + 3A_cA_s^2\times\left(\frac{\pi}{4} - \frac{3}{4}\times\frac{1}{2}\times\frac{\pi}{2}\right)\times\frac{2}{\pi}\right]$$

$$g_{c1} = \omega_0^2\varepsilon\left[\frac{3}{8}A_c^3 + \frac{3A_cA_s^2}{8}\right] = \frac{3}{8}A_c\omega_0^2\varepsilon(A_c^2+A_s^2)$$

同理：

$$g_{s1} = \frac{3}{8}A_s\omega_0^2\varepsilon(A_c^2+A_s^2)$$

则：有：

AS ①：$\cos\omega t$ 系数为： $$(\omega_0^2-\omega^2)A_{c1} + g_{c1} = 0$$

$$\text{①}: \cos\omega t \text{ 系数为}: \quad (\omega_0^2-\omega^2)A_{c1} + g_{c1} = 0$$

$$\text{②}: \sin\omega t \text{ 系数为}: \quad (\omega_0^2-\omega^2)A_{s1} + g_{s1} = 0$$

得： $(\omega_0^2-\omega^2) + \frac{3}{8}\omega_0^2\varepsilon(A_c^2+A_s^2) = 0$

$\therefore$ 有： $A_c^2+A_s^2 = A^2 = \frac{8(\omega^2-\omega_0^2)}{3\varepsilon\omega_0^2}$ 则有： $\omega^2 = \omega_0^2\left(1+\frac{3\varepsilon}{8}A^2\right)$