Proof of the correctness of 1-D FEM problem

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$$\int_0^1 w_{,x} u_{,x}\, dx = \int_0^1 w l\, dx + w(0) h$$

for all $w \in \mathcal{V}$,

for $\int_0^1 u_{,x}\, dw = u_{,x} w_{,x} \Big|_0^1 - \int u_{,xx} w\, dx$

$\therefore$ then $\boxed{\int_0^1 w(u_{,xx}+l)\, dx + w(0)\left[h + u_{,x}(0)\right] = 0}$ (1.4.4)

for proving the u is a solution of (S), it suffice to show that the equation above implies

i; $u_{,xx} + l = 0$ on Ω

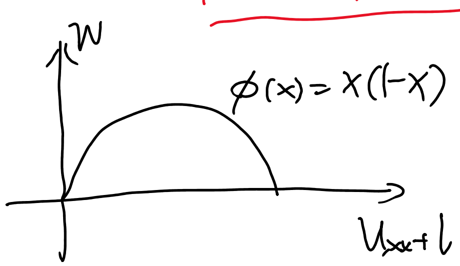
ii. $u_{,x}(0) + h = 0$, $\leftarrow$ $w(0)\left[h + u_{,x}(0)\right] = 0$

firstly we prove (i):

we define w in the equation above by:

$\boxed{w = \phi \cdot (u_{,xx} + l)}$ where ϕ is smooth

and $\phi(0) = \phi(1) = 0$ (1.4.5)



thus we can substitute the eq (1.4.5) into the equation (1.4.4)

the the second part automatically become zero and the equation becomes

$$\int_0^1 \phi \underbrace{(u_{xx}+l)^2}_{\geq 0} dx + 0 = 0$$ then $u_{xx} + l = 0,$ (i)

then we will proove the equation (ii), namely

$$\boxed{0 = w(0)\left[u_x(0)+h\right]}$$

since the boundary condition puts no restricts on $W(0)$, then we may assume that $W(0) \neq 0$,

then we have $\left[u_{,x}(0) + h = 0,\right]$ (ii)