跨越率方法有关定理推导过程

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①. 证明公式：

$$\max_{t_0 \le \tau \le t_s} P_f(\tau,\tau) \le P_f(t_0,t_s) \le P_f(t_0,t_0) + P\{N^+(t_0,t_s) > 0\}$$

1) 证明：先证 $\max P_f(\tau,\tau) \le P_f(t_0,t_s)$

由时变失效概率知：

$P_f(t_0,t_s) = P\{\bigcup_{t_0 \le \tau \le t_s} g(X,\tau) \le 0\}$，显然有：$\max P_f(\tau,\tau) \subseteq P_f(t_0,t_s)$

$$\max_{t_0 \le \tau \le t_s} P_f(\tau,\tau) = \max_{t_0 \le \tau \le t_s} P\{g(X,\tau) \le 0\} \le P\{\bigcup_{t_0 \le \tau \le t_s} g(X,\tau) \le 0\}$$

2). 证：$P_f(t_0,t_s) \le P_f(t_0,t_0) + P\{N^+(t_0,t_s) > 0\}$

由 $P_f(t_0,t_s) = P\{\{g(X,t_0) \le 0\} \cup \{N^+(t_0,t_s) > 0\}\}$

$$= P\{g(X,t_0) \le 0\} + P\{N^+(t_0,t_s) > 0\} - P\{\{g(X,t_0) \le 0\} \cap \{N^+(t_0,t_s) > 0\}\}$$

$$\le P\{g(X,t_0) \le 0\} + P\{N^+(t_0,t_s) > 0\},$$

其中：$P\{g(X,t_0) \le 0\} = P_f(t_0,t_0)$ 为该时刻失效概率.

②、跨越率法的失效概率计算

考虑到：假设跨越次数服从 Pission 分布，则从安全域向失效域跨越次数概率密度为：

$$P\{X=k\} = \frac{\lambda^k e^{-\lambda}}{k!}$$

其中取 k 为 t_0 到 t_s 跨越率的积分，即：$\lambda = \int_{t_0}^{t_s} \nu^+(\tau)\,d\tau$

因此：失效概率由 $P\{k \ge 1\}$ 计算；而有效时 k 必须为 0.

$$P_f = 1 - P\{X=0\}$$

$$= 1 - \exp\left(-\int_{t_0}^{t_s} \nu^+(\tau)\,d\tau\right)$$

另外，考虑到初始 $P_f(t_0,t_0) < 0$ 有一定概率，

则：

有效：$P_s = (1 - P_f(t_0,t_0)) \exp\left(-\int_{t_0}^{t_s} \nu^+(\tau)\,d\tau\right)$

有效: $P_s = \left(1 - P_f(t_0, t_0)\right) \exp\left(-\int_{t_0}^{t_s} \nu^+(\tau)\, d\tau\right)$

失效: $P_f = 1 - \left(1 - P_f(t_0, t_s)\right) \exp\left(-\int_{t_0}^{t_s} \nu^+(\tau)\, d\tau\right)$