密绕弹簧受拉的变形公式推导

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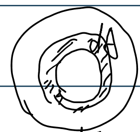
①：对于在 F 下，弹簧伸长 λ

则有 $W=\frac{1}{2}F\lambda$，而弹簧的应变能 $V_\varepsilon=\frac{1}{2}\frac{\tau^2}{G}=\frac{1}{2}\cdot\frac{\left(\frac{16FD\rho}{\pi d^4}\right)^2}{G}$

(单位体积应变能) $=\frac{128F^2D^2\rho^2}{G\pi^2d^8}$



设：dA 表示弹簧丝横截面积，ds 为沿弹簧丝线的微长度，(s 从 0 到 l)

则 $dV=dA\cdot ds=2\pi\rho d\rho\cdot ds$，设弹簧由 n 圈组成，则 $l=n2\pi R=n\pi D$

则有：

$$W=\int_V\frac{128F^2D^2\rho^2}{G\pi^2d^8}dV=\int\frac{128F^2D^2\rho^2}{G\pi^2d^8}2\pi\rho d\rho\cdot ds$$

$$=\int_0^r n\cancel{\pi}D\cdot\frac{256F^2D^2\rho^3}{G\cancel{\pi}d^8}d\rho=\int_0^r n\cdot\frac{256F^2D^3\rho^3}{Gd^8}d\rho=\frac{64nF^2D^3r^4}{Gd^8}=\frac{4F^2D^3}{Gd^4}n$$

则有两者相等，即：

$$\frac{4F^2D^3}{Gd^4}\cdot n=\frac{1}{2}\cancel{F}\lambda$$

此时：$\lambda=\frac{8FD^3n}{Gd^4}=\frac{64FR^3n}{Gd^4}$