

BIG O NOTATION - TIME COMPLEXITY

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MERGE SORT IS AN EFFICIENT, COMPARISON-BASED SORTING ALGORITHM THAT LEVERAGES THE DIVIDE-AND- CONQUER STRATEGY TO MINIMIZE THE NUMBER OF COMPARISONS REQUIRED TO SORT AN ARRAY. IT OUTPERFORMS SIMPLER SORTING ALGORITHMS SUCH AS BUBBLE SORT, INSERTION SORT, AND SELECTION SORT, WHICH HAVE A TIME COMPLEXITY OF $O(N^2)$. HERE IS A REFINED EXPLANATION OF MERGE SORT'S TIME COMPLEXITY

Best Case:
 $O(n \log n)$

Average Case:
 $O(n \log n)$

Worst Case:
 $O(n \log n)$

REGARDLESS OF THE INPUT, MERGE SORT CONSISTENTLY OPERATES IN $O(N \log N)$ TIME COMPLEXITY. THIS CONSISTENT PERFORMANCE IS DUE TO THE ALGORITHM'S STRUCTURE, WHICH INVOLVES TWO MAIN PHASES: DIVIDING AND MERGING.

Phases of Merge Sort

01

Dividing Phase:

- The algorithm recursively splits the unsorted array into smaller subarrays until each subarray contains only one element.
- The time complexity for this phase is $O(\log n)$, as the array is halved at each recursive step.

02

Merging Phase:

- The algorithm then merges these subarrays back together in a sorted manner.
- The time complexity for this phase is $O(n)$, as each element is processed once during the merge.

THE TOTAL TIME COMPLEXITY OF MERGE SORT IS THE PRODUCT OF THESE TWO PHASES, RESULTING IN $O(n \log n)$.

Space Complexity

- Merge Sort requires additional space for the temporary arrays used during the merge phase, leading to a space complexity of $O(n)$. This additional memory usage makes Merge Sort less memory-efficient compared to in-place algorithms like Quick Sort, which typically has a space complexity of $O(\log n)$.

Scalability

- Merge Sort is highly scalable and well-suited for large datasets due to its predictable $O(n \log n)$ performance. Its stable nature ensures that the relative order of equal elements is maintained, which can be crucial for certain applications.

OVERALL, MERGE SORT'S EFFICIENT TIME COMPLEXITY AND STABLE SORTING CHARACTERISTICS MAKE IT A RELIABLE CHOICE FOR SORTING LARGE DATASETS, DESPITE ITS HIGHER MEMORY REQUIREMENTS. ITS CONSISTENT PERFORMANCE ACROSS ALL CASES—BEST, AVERAGE, AND WORST FURTHER SOLIDIFIES ITS UTILITY IN VARIOUS COMPUTATIONAL SCENARIOS.