ADDIS ABABA EDUCATION BUREAU

2013/2021 GRADE 12 MATHEMATICS MODEL EXAMINATIOS

TIME ALLOWED: 3HOURS

GENERAL DIRECTIONS

THIS BOOKLET CONTAINS MATHEMATICS EXAMINATION FOR THE NATURAL SCIENCE CANDIDATES ONLY. IN THIS EXAMINATION, THERE ARE A TOTAL OF 65MULTIPLE CHOICE QUESTIONS.

THERE IS ONLY ONE BEST ANSWER FOR EACH QUESTION. CHOOSE THE BEST ANSWER FROM THE SUGGESTED OPTIONS AND BLACKEN THE LETTER OF YOUR CHOICE ON THE ANSWER SHEET. USE ONLY PENCILE TO MARK YOUR ANSWERS.

YOU WILL BE ALLOWED TO WORK ON THE EXAM FOR 3 HOURS. WHEN TIME IS CALLED, YOU MUST IMMEDIATELY STOP WORKING, PUT YOUR PENCIL DOWN, AND WAIT FOR FURTHER INSTRUCTIONS.

ANY FORM OF CHEATING OR AN ATTEMT TO CHEAT IN THE EXAMINATION WILL RESULT IN AN AUTOMATIC DISMISSAL FROM THE EXAMINATION HALL AND CANCELLATION OF YOUR SCORE(S).

PLEASE MAKE SURE THAT YOU HAVE WRITTEN ALL THE QEQUIRED INFORMATION ON THE ANSWER SHEET BEFORE YOU START TO WORK ON THE EXAMINATION.

DIRECTIONS: For each of the following problems, choose the best answer from the given alternatives and carefully blacken the letter of your best choice on the separate answer sheet provided.

1. Which one of the following is true about the sequence $a_n = \frac{4n-3}{n+3}$?
   A. Its glb is 4
   B. It is bounded monotonic sequence.
   C. Its lub is ¼
   D. It is a null sequence
2. Let $f(x) = 5x + 2$ and $g(x) = 2x - 1$, then what is the value of k such that $f(g(k)) = 17$?
   A. 2 B. 4 C. 5 D. 7
3. Let $\{a_n\}$ be a sequence such that $a_1 = a_2 = 1$ and $a_{n+2} = a_n + 3a_{n+1}$ for $n \geq 1$, then the sixth term is equal to:_____
   A. 40 B. 43 C. 142 D. 129
4. Which one of the following is true about the function $f(x) = x|x|$?
   A. $f'(x) = 2x\ ,\ \forall x \in \mathbb{R}$
   B. $f'(x) = -2x,\ \forall x \in \mathbb{R}$
   C. $f'(x) = 2|x|,\ \forall x \in \mathbb{R}$
   D. $f'(0)\ does\ not\ exist$
5. What is the sum of whole numbers that are less than 100 and leave remainder 2 when divided by 5?
   A. 988 B. 980 C. 992 D. 990
6. The simplified form of $\frac{4-3i}{3+4i} + \overline{1-2i}$ is equal to:_____
   A. $1 - i$
   B. $1 + i$
   C. $1 - 3i$
   D. $1 + 3i$
7. Let $f(x) = \frac{x^2}{x^2+1}$ and $f^{-1}(2a) = 1$ then, what is the value of a?
   A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. 1 D. 2
8. What is the solution set of $\frac{1-\frac{1}{x}}{1-\frac{1}{x^2}} = 3x^2 - \frac{x}{1+\frac{1}{x}}$?
   A. $\left\{-1, 0, \frac{1}{3}\right\}$
   B. $\left\{-1,\ \frac{1}{3}\right\}$
   C. $\left\{\frac{1}{3}\right\}$
   D. $\left\{-\frac{1}{3}\right\}$
9. In which interval the function $f(x) = \sqrt{1 - x^2}$ is differentiable?
   A. $(-\infty, -1) \cup (1,\ \infty)$
   B. $[1,\ \infty)$
   C. $(-1,\ 1)$
   D. $(-\infty,\ \infty)$

10. Which one of the following is not true about the graph of $f(x) = \frac{3x^2}{x^2-x-2}$?

A. The graph of f crosses its horizontal asymptote.

B. The range of f is the set of $\mathbb{R}/\{3\}$

C. $x = -1 \ and \ x = 2$ are its vertical asymptote

D. As $x \to \infty, \ f(x) \to 3$

11. What is the equation of the tangent line to the circle $(x+3)^2 + (y-1)^2 = 29$ at a point $P(2,3)$?

A. $2y + 5x = 16$ C. $2y - 5x = 4$

B. $5y + 2x = 19$ D. $5y - 2x = 11$

12. What is the focus and directrix of the parabola $y^2 + 8x + 4y - 20 = 0$?

A. $(3,-4), \quad y = 0$ C. $(5,-2), \quad x = 1$

B. $(1,-2), \quad x = 5$ D. $(3, \ 0), \quad y = -4$

13. Which one of the following is an asymptote of the hyperbola $y^2 - x^2 + 4x - 5 = 0$?

A. $y = x + 2$ C. $y = -x - 2$

B. $y = x - 2$ D. $y = x$

14. If $B = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 0 & 4 & 5 \end{pmatrix}$ and $AB = 2I$, where A is 3×3 matrix and I is identity matrix of order 3, then $det(A^T)$ is equal to:_____

A. 1 B. 2 C. 4 D. 8

15. Which of the following is true about the function given by

$$f(x) = \begin{cases} 2x - 1, & if \ \ x < -1 \\ x^2 + 1, & if -1 \le x \le 1 \\ x + 1, & if \ \ x > 1 \end{cases}?$$

A. f is continuous everywhere except at $x = 1$

B. f is continuous every where

C. f is continuous everywhere except at $x = -1 \ and \ x = 1$

D. f is continuous everywhere except at $x = -1$

16. The floor of a conference hall is made in a shape of ellipse. If the largest chord has length $20m$ and the smallest chord has length $16m$, what is the eccentricity of this conference hall?

A. $\frac{3}{5}$ B. $\frac{4}{5}$ C. $\frac{5}{4}$ D. $\frac{5}{3}$

17. Which one of the following is invalid logical argument?

A. $p, p \Rightarrow q,\ q \Rightarrow r \vdash r$ C. $p \Rightarrow q, q \Rightarrow r \vdash p \Rightarrow r$

B. $p \wedge q, p \Rightarrow q \vdash p \vee q$ D. $\neg p \wedge \neg q,\ (\neg p \Rightarrow r) \Rightarrow p \vdash r$

18. The value of $\int_0^1 (x+1)e^x dx$ is equal to:_____

A. 0 B. 1 C. $2e$ D. e

19. A group of five students in mathematics class have a mean average score of 80. If a sixth student with score of 74 is added to the group, what is the mean score of all students?

A. 76 B. 79 C. 78 D. 82

20. If $g(x) = f(2x+4),\ f^n(4) = 2\ then\ , g^n(0)$ is equal to:_____

A. 2^{n+1} *B.* 2^n C. 0 D. 2^{n-1}

21. A bag contains 4 red, 5 black and 3 white balls. If three balls are drawn one after the other, then what is the probability of getting red in the first draw, black in the second and white balls in the third draw, if the balls are drawn without replacement?

A. $\frac{5}{144}$ B. 1 C. $\frac{1}{22}$ D. $\frac{1}{144}$

22. In how many ways can a committee of 7 students can be selected from 5 boys and 5 girls, if the committee consists of at least 3 girls?

A. 50 B. 120 C. 100 D. 110

23. If $\lim_{x\to\infty}\left(\frac{x+k}{x-k}\right)^x = 5,$ then the value of k is equal to:_____

A. $ln5$ B. $ln(\sqrt{2})$ C. $ln(\sqrt{5})$ D. 2

24. If the third term of a geometric progression is 5 and its sixth term is -40, then which one of the following is **not true**?

A. The common ratio is -2

B. The first term $G_1 = \frac{5}{4}$

C. The n[th] term $G_n = 5(-2)^{n-3}$

D. The sum of the first five term is $\frac{55}{4}$

25. Which one of the following is the set of all critical numbers of $f(x) = \frac{1}{3}x^3 - |4x-1|$?

A. $\left\{\frac{1}{4}, 2\right\}$ B. $\left\{-2, \frac{1}{4}, 2\right\}$ C. $\{-2, 2\}$ D. $\left\{\frac{1}{4}\right\}$

26. A ball is thrown vertically from the ground up to a height of $16m$. Each time it drops h meters and it rebounds $0.8h$ meters. What is the total distance traveled by the ball?

A. $144m$ B. $160m$ C. $40m$ D. $80m$

27. If $f(x) = (8 - x^3)\left(\sqrt{2 - x}\right)$, what is the slope of the tangent line to the graph of f at $x = -2$?

A. 32 B. 0 C. -16 D. -28

28. If $G(x) = f(x^2 + 2x),\ f'(-1) = 1 and\ f''(-1) = -1$, then $G''(-1) =$ _____

A. 10 B. -10 C. -2 D. 2

29. Which of the following is not true?

A. If f is a rational function and $a \in domain\ of\ f$, then $\lim_{x \to a} f(x)$ exist and is real number.

B. For a function f, if $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$, then the limit of f at a exists.

C. If a point is an element of the domain of a function f, then either $\lim_{x \to a^+} f(x)$ or $\lim_{x \to a^-} f(x)$ exist.

D. Every function has a unique limit at a point in its domain, provided that the limit exists.

30. Which one of the following is a true statement on the set of real number?

A. $(\forall x)(\exists y)(x^2 + y^2 = 9)$ C. $(\forall x)(\forall y)(x + y^2 + 1 = 0)$

B. $(\exists x)(\forall y)(x + y = y)$ D. $(\exists x)(x^2 + x + 1 = 0)$

31. The following is the frequency distribution of a grouped data

$Class$	$10 - 14$	$15 - 19$	$20 - 24$	$25 - 29$
$Frequency$	7	6	10	1

What are the median and the range of the distribution respectively?

A. 17.83 and 19 C. 17.83 and 20

B. 18.67 and 20 D. 18.67 and 19

32. If $f'(x) = 2x + 1\ \ and\ \ f(1) = -2$, then what is the formula for $f(x)$?

A. $x^2 + x$ C. $x^2 + x - 2$

B. $2x^2 - x + 1$ D. $x^2 + x - 4$

33. Which one of the following is equal to $\int_{-2}^{1} (x+1)\sqrt{x+2}\,dx$?

A. $6\sqrt{3}$ B. $9\sqrt{3}$ C. $12\sqrt{3}$ D. $\frac{8}{5}\sqrt{3}$

34. $\int \frac{6x}{\left(x^2+1\right)^2} dx$ is equal to:_____

A. $\frac{1}{(x^2+1)^3} + c$
C. $\frac{3}{x^2+1} + c$
B. $\frac{-x}{x^2+1} + c$
D. $-\frac{3}{x^2+1} + c$

35. What is the $\lim_{x\to 0}\left(\frac{1}{x\sqrt{1+x}} - \frac{1}{x}\right)$?

A. 0
C. $-\frac{1}{2}$
B. $-\frac{2}{3}$
D. $does\ not\ exist$

36. What is the polar form of the complex number $z = \frac{1+\sqrt{3}i}{1+i}$?

A. $\sqrt{2}[cos(15^0) + isin(15^0)]$
B. $\sqrt{2}[cos(60^0) + isin(60^0)]$
C. $2[cos(30^0) + isin(30^0)]$
D. $\sqrt{2}[cos(105^0) + isin(105^0)]$

37. If $\begin{pmatrix} 3 & y \\ x & 2 \end{pmatrix}$ is the inverse of $\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$, then the value of $x\ and\ y$ respectively are:_____

A. $5\ and\ 1$
C. $5\ and\ 4$
B. $8\ and\ 4$
D. $-5\ and\ 1$

38. $\lim_{x\to 1}\left(\frac{x^2-1}{x^2+3x-4}\right)$ is equal to:_____

A. $\frac{1}{5}$
B. $\frac{2}{5}$
C. $\frac{3}{4}$
D. $\frac{4}{5}$

39. Which one of the following is the solution set of the system of equation

$$\begin{cases} x - 3y + z = -1 \\ 2x + y - 4z = -1 \\ 6x - 7y + 8z = 7 \end{cases}$$

A. $\{(1,\ 1, 1)\}$
C. $\{(2,\ 2, 2)\}$
B. $\{(2,\ 1, 0)\}$
D. $\{(0,\ 2, 1)\}$

40. Which of the following is an anti-derivative of the function $f(x) = \frac{x+2}{x+1}$?

A. $ln\left|\frac{x+1}{x+2}\right|$
C. $x + ln|x + 1|$
B. $1 + ln|x + 1|$
D. $x + ln|x + 2|$

41. Let $f(x) = \begin{cases} xcot(4x), & if\ x < 0 \\ \frac{2}{a} + 2x, & if\ x \geq 0 \end{cases}$ what is the value of a if f is continuous at $x = 0$?

A. 7 B. 8 C. 6 D. 3

42. $\lim_{x\to 0}\left(\frac{|x+1|+|x-1|-2}{x}\right)$ is equal to:_____

A. 0
B. -1
C. 2
D. $Does\ not\ exist$

43. If $f(x) = e^x cos(x^2)$, then $f'(x)$ is equal to:_____

A. $-e^x sin(x^2)$
B. $e^x\left(cos(x^2) + 2xsin(x^2)\right)$
C. $-2xe^x sin(x^2)$
D. $e^x\left(cos(x^2) - 2xsin(x^2)\right)$

44. Let $f(x) = e^x lnx + xe^x$, then $\lim_{h\to 0}\left(\frac{f(h+1) - f(1)}{h}\right)$ is equal to:_____

A. 2e B. 4e C. 3e D. e

45. Which of the following is false about the function $f(x) = x^3 - 6x - 1$?

A. The graph of $f(x)$is concave upward in $(-\infty, 0)$
B. $\left(-\infty, -\sqrt{2}\right] \cup \left[\sqrt{2},\ \infty\right)$ is an interval where $f(x)$ is increasing.
C. $f'(x)$ is increasing in $(0,\ \infty)$
D. $(0, -1)$ is inflection point of $f(x)$

46. An open top square base rectangular box container is to have a volume of $32cm^3$, what should be the height of box to obtain the minimum possible surface area?

A. $2cm$ B. $4cm$ C. $8cm$ D. $6cm$

47. A right circular cone whose altitude h is related with base radius r by $h = r$ has its radius increasing at a rate of $2cm/min$, what is the rate of change of its volume when $r = 6cm$?

A. $36\pi cm^3/min$
B. $216\pi cm^3/min$
C. $\pi cm^3/min$
D. $72\pi cm^3/min$

48. What is the derivative of $\int_1^{x^2+1} \ln(t)\,dt$?

A. $\frac{2x}{x^2+1}$
B. $\frac{2xln(x^2+1)}{x^2+1}$
C. $2xln(x^2+1)$
D. $\int_1^{2x} \ln(t)$

49. What is the area of the region enclosed by the graph of $f(x) = 2 - x^2$ and $g(x) = x$?

A. $\frac{9}{2} sq.units$ C. $\frac{10}{3} sq.units$

B. $\frac{7}{6} sq.units$ D. $\frac{27}{2} sq.units$

50. The following grouped data shows marks of 36 students obtained in mathematics examination out of 60

Mark	$30-34$	$35-39$	$40-44$	$45-49$	$50-54$	$55-60$
Number of students	7	12	3	5	3	6

If students in the top 25% are able to be awarded certificate of "Best in mathematics", then what is the minimum mark to awarded certificate?

A. 35.3 B. 36.08 C. 59.5 D. 49.5

51. What is the exact value of $tan\left(cos^{-1}\left(\frac{3}{5}\right)\right)$?

A. $\frac{3}{4}$ B. $\frac{4}{5}$ C. $\frac{4}{3}$ D. $\frac{3}{5}$

52. Which one of the following is false about the graph of the function $f(x) = -3sin\left(\frac{1}{2}x - \frac{\pi}{3}\right) + 4$?

A. $[0, 2\pi]$ is its one complete cycle.

B. $\theta = \frac{2}{3}\pi$ is the phase shift.

C. Its amplitude is 3

D. Its period is 4π

53. Given vectors $\vec{v} = -i + j$ and $\vec{u} = 2i + j$, then the unit vector in the direction of $\vec{z} = \vec{u} + 2\vec{v} - 3i + j$ is equal to:

A. $\left(\frac{3}{5}, -\frac{4}{5}\right)$ C. $\left(-\frac{3}{5}, \frac{4}{5}\right)$

B. $\left(-\frac{3}{5}, -\frac{4}{5}\right)$ D. $\left(\frac{3}{5}, \frac{4}{5}\right)$

54. If a translation takes $x^2 + (y-5)^2 = 4$ to $x^2 + y^2 - 2x - 2y - 2 = 0$, then what is the image of the point $(3, -1)$ under this transformation?

A. $(-1, 3)$ B. $(2, 3)$ C. $(1,6)$ D. $(4, -5)$

55. Which one of the following is the simplified form of $cos\left(x - \frac{\pi}{2}\right) cotx$?

A. $-sinx$ B. $cosx$ C. $sinx$ D. $-cosx$

56. In the interval $[0, \pi]$, what is the solution set of the equation

$$2sin^2(x) - cosx - 1 = 0?$$

A. $\{0, \pi\}$ B. $\left\{\frac{\pi}{3}, \pi\right\}$ C. $\left\{\frac{\pi}{3}, \frac{2}{3}\pi\right\}$ D. $\{\pi\}$

57. What is the image of the point $P(-1, 5)$, when reflected about the line $y - x = 2$?

A. $(3, 1)$ B. $(1, -5)$ C. $(2, 3)$ D. $(-2, 5)$

58. Let $\vec{a}$ and $\vec{b}$ be vectors such that $\|\vec{a}\| = 4$ and $\|2\vec{a} - \vec{b}\| = \|\vec{a} + \vec{b}\|$. Which of the following is equal to $\vec{a} \cdot \vec{b}$?

A. 24 B. 10 C. 12 D. 8

59. Suppose that the equation $x^2 + y^2 + z^2 - 2x + 4y + 4 = 0$ represents a sphere. Where is the point $A(1, -2, -1)$ located relative to the sphere?

A. Inside the sphere
B. On the sphere
C. Outside the sphere
D. At the center of the sphere

60. To prove $p \Longrightarrow q$, you can prove $\neg q \Longrightarrow \neg p$ this type of proof is called:_____

A. Direct proof
B. Proof by contradiction
C. Proof by exhaustion
D. proof by contra positive

61. Vectors U and V makes an angle $\theta = \pi/6$. If $\|U\| = \sqrt{3}$ and $\|V\| = 2$, then the value of $\|U + V\|$ is:_____

A. 27 B. 7 C. $\sqrt{7}$ D. $3\sqrt{3}$

62. An observer on a level of ground is at a distance $10\sqrt{3}m$ from a building. The angle of elevation to the bottom of the windows on the second and third floors are 30^0 and 60^0, respectively. What is the distance between the bottoms of the windows?

A. $20m$
B. $15m$
C. $32m$
D. $15\sqrt{3}m$

63. Which one of the following is the image of the point $P(-3, 2)$ rotated through 180^0 about the origin?

A. $(-3, 2)$
B. $(-4, 3)$
C. $(3, -2)$
D. $(3, -4)$

64. The parametric vector equation of a line l is given by $(x, y) = (2,5) + t(1, -2)$ the rectangular equation of l is:_____

A. $3y - 4x = 3$
B. $y = -2x + 9$
C. $y = 3x + 5$
D. $x - 3y = 3$

65. What is the value of k for which the two vectors $\vec{u} = (1, k, -3)$ and $\vec{v} = (2k, -5, 4)$ are perpendicular?

A. 4 B. -4 C. 3 D. -3