Students:
Section 3.1 is a part of 1 assignment:
Reading Assignment 3

Requirements: PA
No due date

3.1 Definition of functions

Survey

The following questions are part of a zyBooks survey to help us improve our content so we can offer the best experience for students. The survey can be taken anonymously and takes just 3-5 minutes. Please take a short moment to answer by clicking the following link.

Link: Student survey

A function maps elements from one set X to elements of another set Y. Many functions are mathematical functions that map numbers to numbers, such as the function x^2, which maps a number to its square. Discrete mathematics is often concerned with functions that map between other kinds of sets, such as binary strings or a set of tasks. Thus, an assignment of people to teams, or of guests to hotel rooms, are also examples of functions. A function from X to Y can be viewed as a subset of $X \times Y$: $(x, y) \in f$ if f maps x to y. It is possible that X and Y are in fact the same set, in which case f is a subset of $X \times X$.

Definition 3.1.1: Functions.

A ***function*** f that maps elements of a set X to elements of a set Y, is a subset of $X \times Y$ such that for every $x \in X$, there is *exactly one* $y \in Y$ for which $(x, y) \in f$.

$f: X \to Y$ is the notation to express the fact that f is a function from X to Y. The set X is called the ***domain*** of f, and the set Y is the ***target*** of f. An alternate word for target that is sometimes used is ***co-domain***. The fact that f maps x to y (or $(x, y) \in f$) can also be denoted as $f(x) = y$.

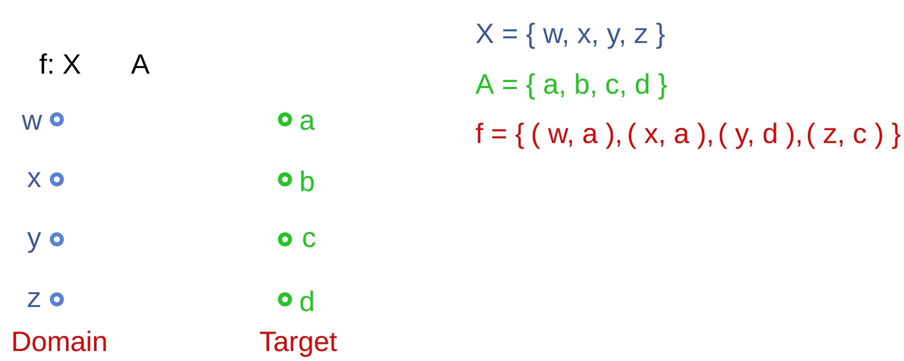
Feedback?

If f maps an element of the domain to zero elements or more than one element of the target, then f is not ***well-defined***.

If f is a function mapping X to Y and X is a finite set, then the function f can be specified by listing the pairs (x, y) in f. Alternatively, a function with a finite domain can be expressed graphically as an arrow diagram. In an ***arrow diagram*** for a function f, the elements of the domain X are listed on the left and the elements of the target Y are listed on the right. There is an arrow from x ∈ X to y ∈ Y if and only if (x, y) ∈ f. Since f is a function, each x ∈ X has exactly one y ∈ Y such that (x, y) ∈ f, which means that in the arrow diagram for a function, there is exactly one arrow pointing out of every element in the domain.

PARTICIPATION ACTIVITY | 3.1.1: Specifying functions with finite domains.

Start 2x speed



Captions

Feedback?

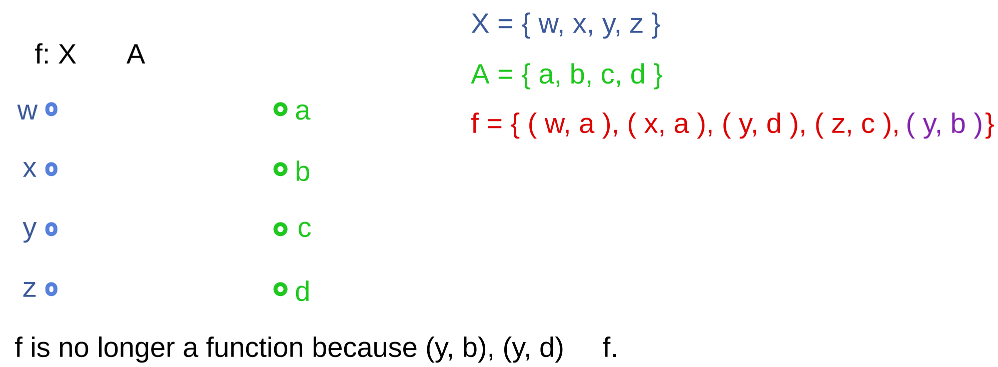
For function f: X → Y, an element y is in the ***range*** of f if and only if there is an x ∈ X such that (x, y) ∈ f. Expressed in set notation:

Range of f = { y: (x, y) ∈ f, for some x ∈ X }

The range of f is a subset of the target but the range is not necessarily equal to the target. In an arrow diagram, the range is the set of elements in the target that have arrows coming into them.

PARTICIPATION ACTIVITY | 3.1.2: Recognizing well-defined functions.

Start 2x speed

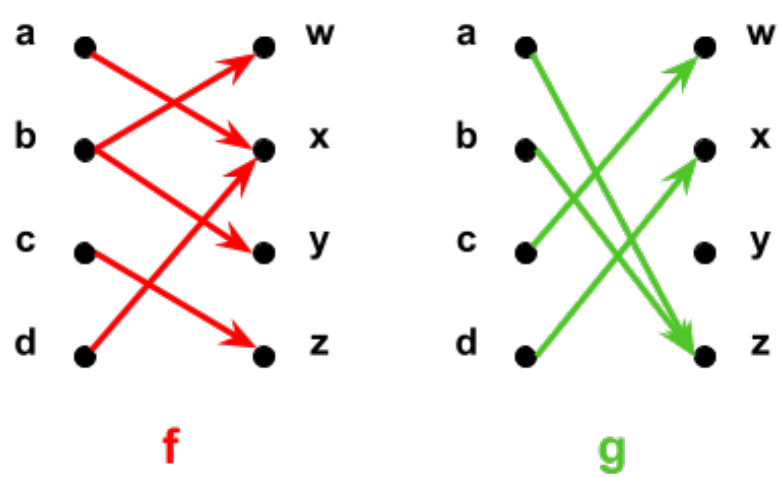


Captions

Feedback?

PARTICIPATION ACTIVITY 3.1.3: Recognizing well-defined functions from arrow diagrams.

Below are two arrow diagrams:



1) Is f a function?

- No. There is an unmapped element in the domain.
- No. An element in the domain is mapped to two different elements in the target.

- Yes.

2) Is g a function?
   - No. There is an unmapped element in the domain.
   - No. An element in the domain is mapped to two different elements.
   - Yes.

Feedback?

PARTICIPATION ACTIVITY | 3.1.4: Function basics.

Sets A and X are defined as:

$$A = \{ a, b, c, d \}$$

$$X = \{ 1, 2, 3, 4 \}$$

A function f: A → X is defined to be

$$f = \{ (a, 3), (b, 1), (c, 4), (d, 1) \}$$

1) What is the target of function f?
   - {a, b, c, d}
   - {1, 2, 3, 4}
   - {1, 3, 4}

2) What is the range of function f?
   - {a, b, c, d}
   - {1, 2, 3, 4}
   - {1, 3, 4}

3) What is f(c)?
   - c
   - 3
   - 4

4) Which of the following sets could be a correct well-defined function definition for g: X → A?

definition for g: X → A?

- { (a, 1), (b, 4), (c, 2), (d, 3) }
- { (1, a), (2, d), (2, b), (4, c) }
- { (1, a), (3, b), (4, c) }
- { (1, a), (2, a), (3, b), (4, b) }

5) Suppose that for function g: X → A, g(2) = a and g(3) = b. Which of the following sets could be the correct function definition for g?

- { (1, a), (2, a), (3, d), (4, d) }
- { (1, a), (2, d), (3, b), (4, c) }
- { (1, d), (2, a), (3, b), (4, c) }

Feedback?

A mathematical function f is often defined by describing how f acts on an input x, as in:

$$f(x) = x^2 - 2.$$

However, the definition is not complete until the domain and target of f are specified. For example:

$$g: \mathbf{R} \to \mathbf{R}, \text{ where } g(x) = |x|.$$

Note that g maps every real number to a real number. However, g does not map any number to a negative number.

PARTICIPATION ACTIVITY 3.1.5: Recognizing well-defined algebraic functions.

Are the expressions below well-defined functions from **R** to **R**?

1)

- Well-defined
- Not well-defined

2)

- Well-defined
- Not well-defined

3)

- Well-defined

Well-defined

Not well-defined

Feedback?

The domain and target set for a function can also be a set of strings. For example, the function f: $\{0, 1\}^3 \to \{0, 1\}^4$ takes as input a 3-bit string and outputs a 4-bit string. Suppose that f is defined so that for any $x \in \{0, 1\}^3$, f(x) = x0. Then for any 3-bit string x, the output of f on input x is obtained by adding a 0 to the end of x. For example f(011) = 0110.

PARTICIPATION ACTIVITY 3.1.6: Functions on sets of strings.

Define f:$\{0,1\}^2 \to \{0, 1\}^3$ such that for $x \in \{0,1\}^2$, f(x) = 1x.

1) What is f(01)?

- 101
- 011
- 01

2) What is the range of f?

- $\{0, 1\}^2$
- { 100, 101, 110, 111 }
- $\{0, 1\}^3$

Feedback?

Function equality

Two functions, f and g, are ***equal*** if f and g have the same domain and target, and f(x) = g(x) for every element x in the domain. The notation f = g is used to denote the fact that functions f and g are equal.

PARTICIPATION ACTIVITY 3.1.7: Function equality.

According to each definition of functions f and g, is it true that f = g?

1) f: $\mathbf{R} \to \mathbf{R}$, $f(x) = x^2 + 1$

g: $\mathbf{R} \to \mathbf{R}^+$, $g(x) = (x + 1)^2 - 2x$

- Equal

Not equal

2) f: **R** → **R**, f(x) = |x|

g: **R** → **R**,

Equal

Not equal

3) X = {0, 1, 2}

f: X → X, $f(x) = (x - 1)^2$

g: X → X, g(0) = 1, g(1) = 0, g(2) = 1

Equal

Not equal

Feedback?

CHALLENGE ACTIVITY 3.1.1: Definition of functions.

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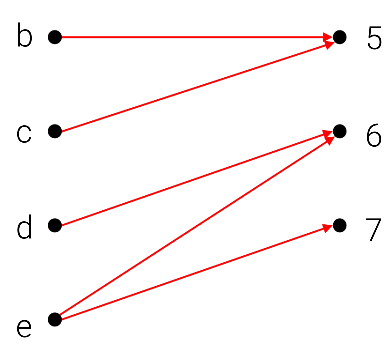
Start

Is f shown in the arrow diagram below a well-defined function?



Pick

1 2 3 4

Check Next

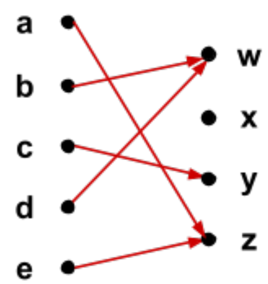
Check Next

Feedback?

Additional exercises

EXERCISE | 3.1.1: Function basics.

The drawing below shows the arrow diagram for a function f.



Give your answers to the questions below using roster notation.

(a) What is the domain of f?

(b) What is the target of f?

(c) What is the range of f?

Feedback?

EXERCISE | 3.1.2: Drawing arrow diagrams.

Draw an arrow diagram for each of the following functions. Give the range of the function using roster notation.

(a) X = {1, 2, 3, 4, 5} and Y = {0, 1, 2, 3, 4}. f: X → Y. f(x) = |x - 2|.

(b) f: $\{0, 1\}^2 \rightarrow \{0, 1\}^3$. For each x $\in$ $\{0, 1\}^2$, f(x) = x0.

(c) f: $\{0, 1\}^2 \rightarrow \{0, 1\}^2$. For each x $\in$ $\{0, 1\}^2$, f(x) is obtained by swapping the two bits in x. For example, f(01) = 10.

Feedback?

EXERCISE | 3.1.3: Recognizing well-defined algebraic functions and their ranges.

Which of the following are functions from **R** to **R**? If f is a function, give its range.

(a)

(b)

(c)

Feedback?

EXERCISE | 3.1.4: Function of an ordered pair.

Let B = {0, 1}. f: B × B → B × B. f(x, y) = (1 - y, 1 - x).

(a) Give the domain of the function f using roster notation. Use ordered pair notation for the Cartesian product.

(b) Draw an arrow diagram for the function .

(c) Give the range of the function f using roster notation. Use ordered pair notation for the Cartesian product.

Feedback?

EXERCISE | 3.1.5: Range of a function.

Express the range of each function using roster notation.

(a) Let A = {2, 3, 4, 5}.
f: A → **Z** such that f(x) = 2x - 1.

(b) Let A = {2, 3, 4, 5}.
f: A → **Z** such that f(x) = x^2.

(c) f: $\{0,1\}^5$ → **Z**. For x ∈ $\{0,1\}^5$, f(x) is the number of times "01" occurs in the string. For example f(01101) = 2 because the string "01" occurs twice in "01101". The first occurrence starts at the first bit. The second occurrence starts in at the fourth bit.

(d) f: $\{0,1\}^5$ → **Z**. For x ∈ $\{0,1\}^5$, f(x) is the number of 1's that occur in x. For example f(01101) = 3, because there are three 1's in the string "01101".

(e) f: $\{0,1\}^3 \rightarrow \{0,1\}^3$. For $x \in \{0,1\}^3$, f(x) is obtained by replacing the last bit with 1. For example f(000) = 001.

(f) Let A = {2, 3, 4, 5}.
f: A × A → **Z**, where f(x,y) = x+y.

(g) Let A = {1, 2, 3}.
f: A × A → **Z**, where f(x,y) = x^y.

(h) Let A = {1, 2, 3}.
f: A × A → **Z**×**Z**, where f(x,y) = (y, x).

(i) Let A = {1, 2, 3}.
f: A × A → **Z**×**Z**, where f(x,y) = (x,y+1).

(j) Let A = {1, 2, 3}.
f: P(A) → **Z**. For X ⊆ A, f(X) = |X|.

(k) Let A = {1, 2, 3}.
f: P(A) → P(A). For X ⊆ A, f(X) = X ∪ {1}.

(l) Let A = {1, 2, 3}.
f: P(A) → P(A). For X ⊆ A, f(X) = X - {1}.

Feedback?

EXERCISE 3.1.6: Determining whether two functions are equal.

Indicate whether the two functions are equal. If the two functions are not equal, then give an element of the domain on which the two functions have different values.

(a) f: **Z** → **Z**, where f(x) = x^2.
g: **Z** → **Z**, where g(x) = $|x|^2$.

(b) f: **Z** → **Z**, where f(x) = x^3.
g: **Z** → **Z**, where g(x) = $|x|^3$.

(c) f: **Z** × **Z** → **Z**, where f(x,y) = |x + y|.
g: **Z** × **Z** → **Z**, where g(x,y) = |x| +| y|.

(d) f: **R** × **R** → **R**, where f(x,y) = x^y.
g: **R** × **R** → **R**, where g(x,y) = y^x.

(e) B = {0, 1}

f: $B \times B \rightarrow B \times B$, where $f(x,y) = (1-y, 1-x)$
g: $B \times B \rightarrow B \times B$, where $g(x,y) = (x,y)$

Feedback?

How was this section?

Provide feedback

Activity summary for assignment: Reading Assignment 3

0 / 90 pts

No due date

0 / 90 pts submitted to canvas

Completion details