3.1 Recursion: Introduction

An ***algorithm*** is a sequence of steps for solving a problem. For example, an algorithm for making lemonade is:

Figure 3.1.1: Algorithms are like recipes.

Make lemonade:

- Add sugar to pitcher
- Add lemon juice
- Add water
- Stir

Some problems can be solved using a recursive algorithm. A ***recursive algorithm*** is an algorithm that breaks the problem into smaller subproblems and applies the same algorithm to solve the smaller subproblems.

Figure 3.1.2: Mowing the lawn can be broken down into a recursive process.

- Mow the lawn
  - Mow the frontyard
    - Mow the left front
    - Mow the right front
  - Mow the backyard
    - Mow the left back
    - Mow the right back

The mowing algorithm consists of applying the mowing algorithm on smaller pieces of the yard and thus is a recursive algorithm.

At some point, a recursive algorithm must describe how to actually do something, known as the ***base case***. The mowing algorithm could thus be written as:

- *Mow* the lawn
    - *If lawn is less than 100 square meters*
        - Push the lawnmower left-to-right in adjacent rows
    - *Else*
        - Mow one half of the lawn
        - Mow the other half of the lawn

PARTICIPATION ACTIVITY 3.1.1: Recursion.

Which are recursive definitions/algorithms?

1) Helping N people:

If N is 1, help that person.
Else, help the first N/2 people, then
help the second N/2 people.

○ True

○ False

2) Driving to the store:

Go 1 mile.
Turn left on Main Street.
Go 1/2 mile.

○ True

○ False

3) Sorting envelopes by zipcode:

If N is 1, done.
Else, find the middle zipcode. Put all
zipcodes less than the middle zipcode
on the left, all greater ones on the
right. Then sort the left, then sort the
right.

○ True

○ False

3.2 Recursive methods

A method may call other methods, including calling itself. A method that calls itself is a ***recursive method***.

PARTICIPATION ACTIVITY 3.2.1: A recursive method example.

Animation captions:

1. The first call to countDown( ) method comes from main. Each call to countDown( ) effectively creates a new "copy" of the executing method, as shown on the right.
2. Then, the countDown( ) function calls itself. countDown(1) similarly creates a new "copy" of the executing method.
3. countDown( ) method calls itself once more.
4. That last instance does not call countDown( ) again, but instead returns. As each instance returns, that copy is deleted.

Each call to countDown() effectively creates a new "copy" of the executing method, as shown on the right. Returning deletes that copy.

The example is for demonstrating recursion; counting down is otherwise better implemented with a loop.

Recursion may be direct, such as f() itself calling f(), or indirect, such as f() calling g() and g() calling f().

PARTICIPATION ACTIVITY 3.2.2: Thinking about recursion.

Refer to the above countDown example for the following.

1) How many times is countDown() called if main() calls CountDown(5)?

Check **Show answer**

2) How many times is countDown() called if main()

calls CountDown(0)?

Check Show answer

3) Is there a difference in how we define the parameters of a recursive versus non-recursive method? Answer yes or no.

Check Show answer

CHALLENGE ACTIVITY 3.2.1: Calling a recursive method.

Write a statement that calls the recursive method backwardsAlphabet() with parameter startingLetter.

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```
import java.util.Scanner;

public class RecursiveCalls {
   public static void backwardsAlphabet(char currLetter) {
      if (currLetter == 'a') {
         System.out.println(currLetter);
      }
      else {
         System.out.print(currLetter + " ");
         backwardsAlphabet((char)(currLetter - 1));
      }
   }

   public static void main (String [] args) {
      Scanner scnr = new Scanner(System.in);
      char startingLetter;

```

Run

View your last submission

3.3 Recursive algorithm: Search

Recursive search (general)

Consider a guessing game program where a friend thinks of a number from 0 to 100 and you try to guess the number, with the friend telling you to guess higher or lower until you guess correctly. What algorithm would you use to minimize the number of guesses?

A first try might implement an algorithm that simply guesses in increments of 1:

- Is it 0? Higher
- Is it 1? Higher
- Is it 2? Higher

This algorithm requires too many guesses (50 on average). A second try might implement an algorithm that guesses by 10s and then by 1s:

- Is it 10? Higher
- Is it 20? Higher
- Is it 30? Lower
- Is it 21? Higher
- Is it 22? Higher
- Is it 23? Higher

This algorithm does better but still requires about 10 guesses on average: 5 to find the correct tens digit and 5 to guess the correct ones digit. An even better algorithm uses a binary search. A ***binary search*** algorithm begins at the midpoint of the range and halves the range after each guess. For example:

- Is it 50 (the middle of 0-100)? Lower
- Is it 25 (the middle of 0-50)? Higher
- Is it 38 (the middle of 26-50)? Lower
- Is it 32 (the middle of 26-38)?

After each guess, the binary search algorithm is applied again, but on a smaller range, i.e., the algorithm is recursive.

PARTICIPATION ACTIVITY 3.3.1: Binary search: A well-known recursive algorithm.

Animation content:

```
undefined
```

Animation captions:

1. A friend thinks of a number from 0 to 100 and you try to guess the number, with the friend telling you to guess higher or lower until you guess correctly.
2. Using a binary search algorithm, you begin at the midpoint of the lower range. (highVal + lowVal) / 2 = (100 + 0) / 2, or 50.
3. The number is lower. The algorithm divides the range in half, then chooses the midpoint of that range.
4. After each guess, the binary search algorithm is applied, halving the range and guessing the midpoint or the corresponding range.
5. A recursive function is a natural match for the recursive binary search algorithm. A function GuessNumber(lowVal, highVal) has parameters that indicate the low and high sides of the guessing range.

Recursive search method

A recursive method is a natural match for the recursive binary search algorithm. A method guessNumber(lowVal, highVal, scnr) has parameters that indicate the low and high sides of the guessing range and a Scanner object for getting user input. The method guesses at the midpoint of the range. If the user says lower, the method calls guessNumber(lowVal, midVal, scnr). If the user says higher, the method calls guessNumber(midVal + 1, highVal, scnr)

The recursive method has an if-else statement. The if branch ends the recursion, known as the ***base case***. The else branch has recursive calls. Such an if-else pattern is common in recursive methods.

Figure 3.3.1: A recursive method carrying out a binary search algorithm.

```
import java.util.Scanner;

public class NumberGuessGame {
   public static void guessNumber(int lowVal, int highVal, Scanner scnr)
{
      int midVal;            // Midpoint of low..high
      char userAnswer;       // User response

      midVal = (highVal + lowVal) / 2;

      // Prompt user for input
      System.out.print("Is it " + midVal + "? (l/h/y): ");
      userAnswer = scnr.next().charAt(0);

      if ((userAnswer != 'l') && (userAnswer != 'h')) { // Base case:
found number
         System.out.println("Thank you!");
      }
      else {                                             // Recursive
case: split into lower OR upper half
         if (userAnswer == 'l') {                        // Guess in lower
half
            guessNumber(lowVal, midVal, scnr);             // Recursive
call
         }
         else {                                          // Guess in upper
half
            guessNumber(midVal + 1, highVal, scnr);            //
Recursive call
         }
      }
   }

   public static void main(String[] args) {
      Scanner scnr = new Scanner(System.in);

      // Print game objective, user input commands
      System.out.println("Choose a number from 0 to 100.");
      System.out.println("Answer with:");
      System.out.println("   l (your num is lower)");
      System.out.println("   h (your num is higher)");
      System.out.println("   any other key (guess is right).");

      // Call recursive function to guess number
      guessNumber(0, 100, scnr);
   }
}
```

```
Choose a number from 0 to 100.
Answer with:
   l (your num is lower)
   h (your num is higher)
   any other key (guess is right).
Is it 50? (l/h/y): l
Is it 25? (l/h/y): h
Is it 38? (l/h/y): l
Is it 32? (l/h/y): y
Thank you!
```

Calculating the middle value

Because midVal has already been checked, it need not be part of the new window, so midVal + 1 rather than midVal is used for the window's new low side, or midVal - 1 for the window's new high side. But the midVal - 1 can have the drawback of a non-intuitive base case (i.e., midVal < lowVal, because if the current window is say 4..5, midVal is 4, so the new window would be 4..4-1, or 4..3). rangeSize == 1 is likely more intuitive, and thus the algorithm uses midVal rather than midVal - 1. However, the algorithm uses midVal + 1 when searching higher, due to integer rounding. In particular, for window 99..100, midVal is 99 ((99 + 100) / 2 = 99.5, rounded to 99 due to truncation of the fraction in integer division). So the next window would again be 99..100, and the algorithm would repeat with this window forever. midVal + 1 prevents the problem, and doesn't miss any numbers because midVal was checked and thus need not be part of the window.

PARTICIPATION ACTIVITY 3.3.2: Binary search tree tool.

The following program guesses the hidden number known by the user. Assume the hidden number is 63.

```
import java.util.Scanner;

public class NumberGuessGame {
   public static void guessNumber(int lowVal, int highVal, Scanner scnr) {
      int midVal;              // Midpoint of low..high
      char userAnswer;         // User response

      midVal = (highVal + lowVal) / 2;

      System.out.print("Is it " + midVal + "? (l/h/y): ");
      userAnswer = scnr.next().charAt(0);

      if ((userAnswer != 'l') && (userAnswer != 'h')) { // Base case:
         System.out.println("Thank you!");               // Found number
      }
      else { // Recursive case: split into lower OR upper half
         if (userAnswer == 'l') {                    // Guess in lower half
            guessNumber(lowVal, midVal, scnr);       // Recursive call
         }
         else {                                      // Guess in upper half
            guessNumber(midVal + 1, highVal, scnr);  // Recursive call
         }
      }
      return;
   }

   public static void main (String[] args) {
      Scanner scnr = new Scanner(System.in);
      System.out.println("Choose a number from 0 to 100.");
      System.out.println("Answer with:");
      System.out.println("   l (your num is lower)");
      System.out.println("   h (your num is higher)");
      System.out.println("   any other key (guess is right).");

      guessNumber(0, 100, scnr);

      return;
   }
}
```

→main()

```
public static void main (String[] args) {
   Scanner scnr = new Scanner(System.in);
   System.out.println("Choose a number from 0 to 100.");
   System.out.println("Answer with:");
   System.out.println("   l (your num is lower)");
   System.out.println("   h (your num is higher)");
   System.out.println("   any other key (guess is right).");

   guessNumber(0, 100, scnr);

   return;
}
```

Recursively searching a sorted list

Search is commonly performed to quickly find an item in a sorted list stored in an array or ArrayList. Consider a list of attendees at a conference, whose names have been stored in alphabetical order in an array or ArrayList. The following quickly determines whether a particular person is in attendance.

findMatch() restricts its search to elements within the range lowVal to highVal. main() initially passes a range of the entire list: 0 to (list size - 1). findMatch() compares to the middle element, returning that element's position if matching. If not matching, findMatch() checks if the window's size is just one element, returning -1 in that case to indicate the item was not found. If neither of those two base cases are satisfied, then findMatch() recursively searches either the lower or upper half of the range as appropriate.

Figure 3.3.2: Recursively searching a sorted list.

```
import java.util.Scanner;
import java.util.ArrayList;

public class NameFinder {
   /* Finds index of string in vector of strings, else -1.
      Searches only with index range low to high
      Note: Upper/lower case characters matter
   */
   public static int findMatch(ArrayList<String> stringList, String
itemMatch,
                                int lowVal, int highVal) {
      int midVal;        // Midpoint of low and high values
      int itemPos;       // Position where item found, -1 if not found
      int rangeSize;     // Remaining range of values to search for match

      rangeSize = (highVal - lowVal) + 1;
      midVal = (highVal + lowVal) / 2;

      if (itemMatch.equals(stringList.get(midVal))) {          // Base
case 1: item found at midVal position
         itemPos = midVal;
      }
      else if (rangeSize == 1) {                               // Base
case 2: match not found
         itemPos = -1;
      }
      else {                                                   //
Recursive case: search lower or upper half
         if (itemMatch.compareTo(stringList.get(midVal)) < 0) { // Search
lower half, recursive call
            itemPos = findMatch(stringList, itemMatch, lowVal, midVal);
         }
         else {                                                // Search
upper half, recursive call
            itemPos = findMatch(stringList, itemMatch, midVal + 1,
highVal);
         }
      }

      return itemPos;
   }

   public static void main(String[] args) {
      Scanner scnr = new Scanner(System.in);
      ArrayList<String> attendeesList = new ArrayList<String>(); // List
of attendees
      String attendeeName;                                        // Name
of attendee to match
      int matchPos;                                               //
Matched position in attendee list

      // Omitting part of program that adds attendees
      // Instead, we insert some sample attendees in sorted order
      attendeesList.add("Adams, Mary");
      attendeesList.add("Carver, Michael");
      attendeesList.add("Domer, Hugo");
      attendeesList.add("Fredericks, Carlos");
      attendeesList.add("Li, Jie");

      // Prompt user to enter a name to find
      System.out.print("Enter person's name: Last, First: ");
      attendeeName = scnr.nextLine(); // Use nextLine() to allow space in
name

      // Call function to match name, output results
      matchPos = findMatch(attendeesList, attendeeName, 0,
attendeesList.size() - 1);
      if (matchPos >= 0) {
         System.out.println("Found at position " + matchPos + ".");
      }
      else {
         System.out.println("Not found.");
      }
   }
}
```

```
Enter person's name: Last, First: Meeks, Stan
Not found.

...

Enter person's name: Last, First: Adams, Mary
Found at position 0.

...

Enter person's name: Last, First: Li, Jie
Found at position 4.
```

PARTICIPATION ACTIVITY | 3.3.3: Recursive search algorithm.

Consider the above findMatch() method for finding an item in a sorted list.

1) If a sorted list has elements 0 to 50 and the item being searched for is at element 6, how many times will findMatch() be called?

Check Show answer

2) If an alphabetically ascending list has elements 0 to 50, and

the item at element 0 is "Bananas", how many calls to findMatch() will be made during the failed search for "Apples"?

Check Show answer

PARTICIPATION ACTIVITY 3.3.4: Recursive calls.

A list has 5 elements numbered 0 to 4, with these letter values: 0: A, 1: B, 2: D, 3: E, 4: F.

1) To search for item C, the first call is findMatch(0, 4). What is the second call to findMatch()?

- ○ findMatch(0, 0)
- ○ findMatch(0, 2)
- ○ findMatch(3, 4)

2) In searching for item C, findMatch(0, 2) is called. What happens next?

- ○ Base case 1: item found at midVal.
- ○ Base case 2: rangeSize == 1, so no match.
- ○ Recursive call: findMatch(2, 2)

CHALLENGE ACTIVITY 3.3.1: Enter the output of binary search.

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Start

Type the program's output

```
import java.util.Scanner;

public class NumberSearch {
   public static void findNumber(int number, int lowVal, int highVal) {
      int midVal;

      midVal = (highVal + lowVal) / 2;
      System.out.print(number);
      System.out.print(" ");
      System.out.print(midVal);

      if (number == midVal) {
         System.out.println(" l");
      }
      else {
         if (number < midVal) {
            System.out.println(" m");
            findNumber(number, lowVal, midVal);
         }
         else {
            System.out.println(" n");
            findNumber(number, midVal + 1, highVal);
         }
      }
   }

   public static void main(String[] args) {
      Scanner scnr = new Scanner(System.in);
      int number;

      number = scnr.nextInt();
      findNumber(number, 0, 8);
   }
}
```

Input

```
0
```

Output

```
0 4
0 2
0 1
0 0
```

Exploring further:

- Binary search from GeeksforGeeks.org

3.4 Adding output statements for debugging

Recursive methods can be particularly challenging to debug. Adding output statements can be helpful. Furthermore, an additional trick is to indent the print statements to show the current depth of recursion. The following program adds a parameter indent to a findMatch() method that searches a sorted list for an item. All of findMatch()'s print statements start with `System.out.print(indentAmt + ...);`. Indent is typically some number of spaces. main() sets indent to three spaces. Each recursive call *adds* three more spaces. Note how the output now clearly shows the recursion depth.

Figure 3.4.1: Output statements can help debug recursive methods, especially if indented based on recursion depth.

```
import java.util.Scanner;
import java.util.ArrayList;

public class NameFinder {
   /* Finds index of string in vector of strings, else -1.
      Searches only with index range low to high
      Note: Upper/lower case characters matter
   */
   public static int findMatch(ArrayList<String> stringList, String
itemMatch,
                               int lowVal, int highVal, String indentAmt)
{ // indentAmt used for print debug
      int midVal;        // Midpoint of low and high values
      int itemPos;       // Position where item found, -1 if not found
      int rangeSize;     // Remaining range of values to search for match

      System.out.println(indentAmt + "Find() range " + lowVal + " " +
highVal);
      rangeSize = (highVal - lowVal) + 1;
      midVal = (highVal + lowVal) / 2;

      if (itemMatch.equals(stringList.get(midVal))) {           // Base
case 1: item found at midVal position
         System.out.println(indentAmt + "Found person.");
         itemPos = midVal;
      }
      else if (rangeSize == 1) {                                 // Base
case 2: match not found
         System.out.println(indentAmt + "Person not found.");
         itemPos = -1;
      }
      else {                                                     //
Recursive case: search lower or upper half
         if (itemMatch.compareTo(stringList.get(midVal)) < 0) { // Search
lower half, recursive call
            System.out.println(indentAmt + "Searching lower half.");
            itemPos = findMatch(stringList, itemMatch, lowVal, midVal,
indentAmt + "   ");
         }
         else {                                                  // Search
upper half, recursive call
            System.out.println(indentAmt + "Searching upper half.");
            itemPos = findMatch(stringList, itemMatch, midVal + 1,
highVal, indentAmt + "   ");
         }
      }

      System.out.println(indentAmt + "Returning pos = " + itemPos + ".");
      return itemPos;
   }
   public static void main(String[] args) {
      Scanner scnr = new Scanner(System.in);
      ArrayList<String> attendeesList = new ArrayList<String>(); // List
of attendees
      String attendeeName;                                     // Name
of attendee to match
      int matchPos;                                            //
Matched position in attendee list

      // Omitting part of program that adds attendees
      // Instead, we insert some sample attendees in sorted order
      attendeesList.add("Adams, Mary");
      attendeesList.add("Carver, Michael");
      attendeesList.add("Domer, Hugo");
      attendeesList.add("Fredericks, Carlos");
      attendeesList.add("Li, Jie");

      // Prompt user to enter a name to find
      System.out.print("Enter person's name: Last, First: ");
      attendeeName = scnr.nextLine(); // Use nextLine() to allow space in
name

      // Call function to match name, output results
      matchPos = findMatch(attendeesList, attendeeName, 0,
attendeesList.size() - 1, "   ");
      if (matchPos >= 0) {
         System.out.println("Found at position " + matchPos + ".");
      }
      else {
         System.out.println("Not found.");
      }
   }
}
```

```
Enter person's name: Last, First: Meeks, Stan
   Find() range 0 4
   Searching upper half.
      Find() range 3 4
      Searching upper half.
         Find() range 4 4
         Person not found.
         Returning pos = -1.
      Returning pos = -1.
   Returning pos = -1.
Not found.

...

Enter person's name: Last, First: Adams, Mary
   Find() range 0 4
   Searching lower half.
      Find() range 0 2
      Searching lower half.
         Find() range 0 1
         Found person.
         Returning pos = 0.
      Returning pos = 0.
   Returning pos = 0.
Found at position 0.
```

Some programmers like to leave the output statements in the code, commenting them out with "//" when not in use. The statements actually serve as a form of comment as well.

PARTICIPATION ACTIVITY 3.4.1: Recursive debug statements.

Refer to the above code using indented output statements.

1) The above debug approach requires an extra parameter be passed to indicate the amount of indentation.
   - ○ True
   - ○ False

2) Each recursive call should add a few spaces to the indent parameter.
   - ○ True
   - ○ False

3) The method should remove a few spaces from the indent parameter before returning.
   - ○ True
   - ○ False

zyDE 3.4.1: Output statements in a recursive function.

- Run the recursive program, and observe the output statements for debugging, a the person is correctly not found.
- Introduce an error by changing `itemPos = -1` to `itemPos = 0` in the range base case.
- Run the program, notice how the indented print statements help isolate the errc person incorrectly being found.

Load default template...

Run

```

import java.util.Scanner;
import java.util.ArrayList;

public class NameFinder {
   /* Finds index of string in vector of s
      Searches only with index range low t
      Note: Upper/lower case characters ma
   */
   public static int findMatch(ArrayList<S
                               int lowVal,
      int midVal;        // Midpoint of lo
      int itemPos;       // Position where
      int rangeSize;     // Remaining range

      System.out.println(indentAmt + "Find
      rangeSize = (highVal - lowVal) + 1;
```

CHALLENGE ACTIVITY 3.4.1: Adding output statements for debugging.

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Start

Type the program's output

```
import java.util.Scanner;

public class NumberSearch {
   public static void findNumber(int number, int lowVal, int highVal, String indentAmt) {
      int midVal;

      midVal = (highVal + lowVal) / 2;
      System.out.print(indentAmt);
      System.out.print(midVal);

      if (number == midVal) {
         System.out.println(" a");
      }
      else {
         if (number < midVal) {
            System.out.println(" b");
            findNumber(number, lowVal, midVal, indentAmt + " ");
         }
         else {
            System.out.println(" c");
            findNumber(number, midVal + 1, highVal, indentAmt + " ");
         }
      }

      System.out.println(indentAmt + "d");
   }

   public static void main(String[] args) {
      Scanner scnr = new Scanner(System.in);
      int number;
```

3.5 Creating a recursive method

Creating a recursive method can be accomplished in two steps.

- **Write the base case** -- Every recursive method must have a case that returns a value without performing a recursive call. That case is called the ***base case***. A programmer may write that part of the method first, and then test. There may be multiple base cases.
- **Write the recursive case** -- The programmer then adds the recursive case to the method.

The following illustrates a simple method that computes the factorial of N (i.e. N!). The base case is N = 1 or 1! which evaluates to 1. The base case is written as `if (N <= 1) { fact = 1; }`. The recursive case is used for N > 1, and written as `else { fact = N * NFact( N - 1 ); }`.

PARTICIPATION ACTIVITY | 3.5.1: Writing a recursive method for factorial: First write the base case, then add the recursive case.

Animation captions:

1. The base case, which returns a value without performing a recursive call, is written and tested first. If N is less than or equal to 1, then the nFact() method returns 1.
2. Next the recursive case, which calls itself, is written and tested. If N is greater than 1, then the nFact() method returns N * nFact(N - 1).

A common error is to not cover all possible base cases in a recursive method. Another common error is to write a recursive method that doesn't always reach a base case. Both errors may lead to infinite recursion, causing the program to fail.

Typically, programmers will use two methods for recursion. An "outer" method is intended to be called from other parts of the program, like the method `int calcFactorial(int inVal)`. An "inner" method is intended only to be called from that outer method, for example a method `int calcFactorialHelper(int inVal)`. The outer method may check for a valid input value, e.g., ensuring inVal is not negative, and then calling the inner method. Commonly, the inner method has parameters that are mainly of use as part of the recursion, and need not be part of the outer method, thus keeping the outer method more intuitive.

PARTICIPATION ACTIVITY 3.5.2: Creating recursion.

1) Recursive methods can be accomplished in one step, namely repeated calls to itself.

- True
- False

2) A recursive method with parameter N counts up from any negative number to 0. An appropriate base case would be `N == 0`.

- True
- False

3) A recursive method can have two base cases, such as `N == 0` returning 0, and `N == 1` returning 1.

- True
- False

Before writing a recursive method, a programmer should determine:

1. Does the problem naturally have a recursive solution?

2. Is a recursive solution better than a non-recursive solution?

For example, computing N! (N factorial) does have a natural recursive solution, but a recursive solution is not better than a non-recursive solution. The figure below illustrates how the factorial computation can be implemented as a loop. Conversely, binary search has a natural recursive solution, and that solution may be easier to understand than a non-recursive solution.

Figure 3.5.1: Non-recursive solution to compute N!

```
for (i = inputNum; i > 1; --i)
{
   facResult = facResult * i;
}
```

PARTICIPATION ACTIVITY 3.5.3: When recursion is appropriate.

1) N factorial (N!) is commonly implemented as a recursive method due to being easier to understand and executing faster than a loop implementation.

- True
- False

zyDE 3.5.1: Output statements in a recursive function.

Implement a recursive method to determine if a number is prime. Skeletal code is pro
the isPrime method.

Load default template...

Run

```

public class PrimeChecker {
// Returns 0 if value is not prime, 1 if va
   public static int isPrime(int testVal, i
      // Base case 1: 0 and 1 are not prime

      // Base case 2: testVal only divisib

      // Recursive Case
         // Check if testVal can be evenly
         // Hint: use the % operator

         // If not, recursive call to isPr

   }

   public static void main(String[] args) {
```

CHALLENGE ACTIVITY

3.5.1: Recursive method: Writing the base case.

Write code to complete doublePennies()'s base case. Sample output for below program with inputs 1 and 10:

```
Number of pennies after 10 days: 1024
```

Note: If the submitted code has an infinite loop, the system will stop running the code after a few seconds, and report "Program end never reached." The system doesn't print the test case that caused the reported message.

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```
import java.util.Scanner;

public class CalculatePennies {
// Returns number of pennies if pennies are doubled numDays times
   public static long doublePennies(long numPennies, int numDays) {
      long totalPennies;

      /* Your solution goes here  */

      else {
```

Run

View your last submission

CHALLENGE ACTIVITY | 3.5.2: Recursive method: Writing the recursive case.

Write code to complete printFactorial()'s recursive case. Sample output if input is 5:

```
5! = 5 * 4 * 3 * 2 * 1 = 120
```

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```
import java.util.Scanner;

public class RecursivelyPrintFactorial {
   public static void printFactorial(int factCounter, int factValue) {
      int nextCounter;
      int nextValue;

      if (factCounter == 0) { // Base case: 0! = 1
         System.out.println("1");
      }
      else if (factCounter == 1) { // Base case: Print 1 and result
         System.out.println(factCounter + " = " + factValue);
      }
      else { // Recursive case
         System.out.print(factCounter + " * ");
         nextCounter = factCounter - 1;
         nextValue = nextCounter * factValue;
```

Run

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3.6 Recursive math methods

Fibonacci sequence

Recursive methods can solve certain math problems, such as computing the Fibonacci sequence. The ***Fibonacci sequence*** is 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, etc.; starting with 0, 1, the pattern is to compute the next number by adding the previous two numbers.

Below is a program that outputs the Fibonacci sequence values step-by-step, for a user-entered number of steps. The base case is that the program has output the requested number of steps. The recursive case is that the program needs to compute the number in the Fibonacci sequence.

Figure 3.6.1: Fibonacci sequence step-by-step.

```
import java.util.Scanner;

public class FibonacciSequence {
   /* Output the Fibonacci sequence step-by-step.
      Fibonacci sequence starts as:
      0 1 1 2 3 5 8 13 21 ... in which the first
      two numbers are 0 and 1 and each additional
      number is the sum of the previous two numbers
   */
   public static void computeFibonacci(int fibNum1,
int fibNum2, int runCnt) {
      System.out.println(fibNum1 + " + " + fibNum2 +
" = " +
                         (fibNum1 + fibNum2));

      if (runCnt <= 1) { // Base case: Ran for user
specified
                         // number of steps, do
nothing
      }
      else {             // Recursive case: compute
next value
         computeFibonacci(fibNum2, fibNum1 + fibNum2,
runCnt - 1);
      }
   }

   public static void main(String[] args) {
      Scanner scnr = new Scanner(System.in);
      int runFor;     // User specified number of
values computed

      // Output program description
      System.out.println("This program outputs the\n"
+
                         "Fibonacci sequence step-by-
step,\n" +
                         "starting after the first 0
and 1.\n");

      // Prompt user for number of values to compute
      System.out.print("How many steps would you
like? ");
      runFor = scnr.nextInt();

      // Output first two Fibonacci values, call
recursive function
      System.out.println("0\n1");
      computeFibonacci(0, 1, runFor);
   }
}
```

```
This program outputs
the
Fibonacci sequence
step-by-step,
starting after the
first 0 and 1.

How many steps would
you like? 10
0
1
0 + 1 = 1
1 + 1 = 2
1 + 2 = 3
2 + 3 = 5
3 + 5 = 8
5 + 8 = 13
8 + 13 = 21
13 + 21 = 34
21 + 34 = 55
34 + 55 = 89
```

zyDE 3.6.1: Recursive Fibonacci.

Complete computeFibonacci() to return F_N, where F_0 is 0, F_1 is 1, F_2 is 1, F_3 is 2, F_4 is continuing: F_N is $F_{N-1} + F_{N-2}$. Hint: Base cases are N == 0 and N == 1.

Load default ter

```java

public class FibonacciSequence {
   public static int computeFibonacci(int N) {

      System.out.println("FIXME: Complete this method.");
      System.out.println("Currently just returns 0.");

      return 0;
   }

   public static void main(String[] args) {
      int N;      // F_N, starts at 0

      N = 4;

      System.out.println("F_" + N + " is " + computeFibonacci(N));
   }
```

Run

Greatest common divisor (GCD)

Recursion can solve the greatest common divisor problem. The ***greatest common divisor*** (GCD) is the largest number that divides evenly into two numbers, e.g. GCD(12, 8) = 4. One GCD algorithm (described by Euclid around 300 BC) subtracts the smaller number from the larger number until both numbers are equal. Ex:

- GCD(12, 8): Subtract 8 from 12, yielding 4.
- GCD(4, 8): Subtract 4 from 8, yielding 4.
- GCD(4, 4): Numbers are equal, return 4

The following recursively computes the GCD of two numbers. The base case is that the two numbers are equal, so that number is returned. The recursive case subtracts the smaller number from the larger number and then calls GCD with the new pair of numbers.

Figure 3.6.2: Calculate greatest common divisor of two numbers.

```
import java.util.Scanner;

public class GCDCalc {
   /* Determine the greatest common divisor
      of two numbers, e.g. GCD(8, 12) = 4
   */
   public static int gcdCalculator(int inNum1, int
inNum2) {
      int gcdVal;     // Holds GCD results

      if (inNum1 == inNum2) {   // Base case: Numbers
are equal
         gcdVal = inNum1;       // Return value
      }
      else {                    // Recursive case:
subtract smaller from larger
         if (inNum1 > inNum2) { // Call function with
new values
            gcdVal = gcdCalculator(inNum1 - inNum2,
inNum2);
         }
         else { // n1 is smaller
            gcdVal = gcdCalculator(inNum1, inNum2 -
inNum1);
         }
      }

      return gcdVal;
   }

   public static void main (String[] args) {
      Scanner scnr = new Scanner(System.in);
      int gcdInput1;     // First input to GCD calc
      int gcdInput2;     // Second input to GCD calc
      int gcdOutput;     // Result of GCD

      // Print program function
      System.out.println("This program outputs the
greatest \n" +
                         "common divisor of two
numbers.");

      // Prompt user for input
      System.out.print("Enter first number: ");
      gcdInput1 = scnr.nextInt();

      System.out.print("Enter second number: ");
      gcdInput2 = scnr.nextInt();

      // Check user values are > 1, call recursive
GCD function
      if ((gcdInput1 < 1) || (gcdInput2 < 1)) {
         System.out.println("Note: Neither value can
```

```
This program outputs
the greatest
common divisor of two
numbers.
Enter first number:
12
Enter second number:
8
Greatest common
divisor = 4

...

This program outputs
the greatest
common divisor of two
numbers.
Enter first number:
456
Enter second number:
784
Greatest common
divisor = 8

...

This program outputs
the greatest
common divisor of two
numbers.
Enter first number: 0
Enter second number:
10
Note: Neither value
can be below 1.
```

```
be below 1.");
      }
      else {
         gcdOutput = gcdCalculator(gcdInput1,
gcdInput2);
         System.out.println("Greatest common divisor
= " +  gcdOutput);
      }
   }
}
```

PARTICIPATION ACTIVITY 3.6.1: Recursive GCD example.

1) How many calls are made to gcdCalculator() method for input values 12 and 8?

- ○ 1
- ○ 2
- ○ 3

2) What is the base case for the GCD algorithm?

- ○ When both inputs to the method are equal.
- ○ When both inputs are greater than 1.
- ○ When inNum1 > inNum2.

Exploring further:

- Fibonacci number from Wolfram.
- Greatest Common Divisor from Wolfram.

CHALLENGE ACTIVITY 3.6.1: Writing a recursive math method.

Write code to complete raiseToPower(). Sample output if userBase is 4 and userExponent is 2 is shown below. Note: This example is for practicing recursion; a non-recursive method, or using the built-in method pow(), would be more common.

```
4^2 = 16
```

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```
import java.util.Scanner;

public class ExponentMethod {
   public static int raiseToPower(int baseVal, int exponentVal) {
      int resultVal;

      if (exponentVal == 0) {
         resultVal = 1;
      }
      else {
         resultVal = baseVal * /* Your solution goes here  */;
      }

      return resultVal;
   }

   public static void main (String [] args) {
```

Run

View your last submission

3.7 Recursive exploration of all possibilities

Recursion is a powerful technique for exploring all possibilities, such as all possible reorderings of a word's letters, all possible subsets of items, all possible paths between cities, etc. This section provides several examples.

Word scramble

Consider printing all possible combinations (or "scramblings") of a word's letters. The letters of abc can be scrambled in 6 ways: abc, acb, bac, bca, cab, cba. Those possibilities can be listed by making three choices: Choose the first letter (a, b, or c), then choose the second letter, then choose the third letter. The choices can be depicted as a tree. Each level represents a choice. Each node in the tree shows the unchosen letters on the left, and the chosen letters on the right.

PARTICIPATION ACTIVITY | 3.7.1: Exploring all possibilities viewed as a tree of choices.

Animation captions:

1. Consider printing all possible combinations of a word's letters. Those possibilities can be listed by choosing the first letter, then the second letter, then the third letter.
2. The choices can be depicted as a tree. Each level represents a choice.
3. A recursive exploration function is a natural match to print all possible combinations of a string's letters. Each call to the function chooses from the set of unchosen letters, continuing until no unchosen letters remain.

The tree guides creation of a recursive exploration method to print all possible combinations of a string's letters. The method takes two parameters: unchosen letters, and already chosen letters. The base case is no unchosen letters, causing printing of the chosen letters. The recursive case calls the method once for each letter in the unchosen letters. The above animation depicts how the recursive algorithm traverses the tree. The tree's leaves (the bottom nodes) are the base cases.

The following program prints all possible ordering of the letters of a user-entered word.

Figure 3.7.1: Scramble a word's letters in every possible way.

```
import java.util.Scanner;

public class WordScrambler {
   /* Output every possible combination of a word.
      Each recursive call moves a letter from
      remainLetters" to scramLetters".
   */
   public static void scrambleLetters(String remainLetters,  // Remaining
letters
                                      String scramLetters) { // Scrambled
letters
      String tmpString;      // Temp word combinations
      int i;                 // Loop index

      if (remainLetters.length() == 0) { // Base case: All letters used
         System.out.println(scramLetters);
      }
      else {                             // Recursive case: move a letter
from
                                         // remaining to scrambled
letters
         for (i = 0; i < remainLetters.length(); ++i) {
            // Move letter to scrambled letters
            tmpString = remainLetters.substring(i, i + 1);
            remainLetters = removeFromIndex(remainLetters, i);
            scramLetters = scramLetters + tmpString;

            scrambleLetters(remainLetters, scramLetters);

            // Put letter back in remaining letters
            remainLetters = insertAtIndex(remainLetters, tmpString, i);
            scramLetters = removeFromIndex(scramLetters,
scramLetters.length() - 1);
         }
      }
   }

   // Returns a new String without the character at location remLoc
   public static String removeFromIndex(String origStr, int remLoc) {
      String finalStr;      // Temp string to extract char

      finalStr = origStr.substring(0, remLoc);                    //
Copy before location remLoc
      finalStr += origStr.substring(remLoc + 1, origStr.length()); //
Copy after location remLoc

      return finalStr;
   }

   // Returns a new String with the character specified by insertStr
   // inserted at location addLoc
   public static String insertAtIndex(String origStr, String insertStr,
int addLoc) {
      String finalStr;      // Temp string to extract char

      finalStr = origStr.substring(0, addLoc);                 // Copy
before location addLoc
      finalStr += insertStr;                                   // Copy
character to location addLoc
      finalStr += origStr.substring(addLoc, origStr.length()); // Copy
after location addLoc

      return finalStr;
   }

   public static void main(String[] args) {
      Scanner scnr = new Scanner(System.in);
      String wordScramble;      // User defined word to scramble

      // Prompt user for input
      System.out.print("Enter a word to be scrambled: ");
      wordScramble = scnr.next();

      // Call recursive method
      scrambleLetters(wordScramble, "");
   }
}
```

```
Enter a word to be scrambled: cat
cat
cta
act
atc
tca
tac
```

PARTICIPATION ACTIVITY 3.7.2: Letter scramble.

1) What is the output of scrambleLetters("xy", "")? Determine your answer by manually tracing the code, not by running the program.

- yx xy
- xx yy xy yx
- xy yx

Shopping spree

Recursion can find all possible subsets of a set of items. Consider a shopping spree in which a person can select any 3-item subset from a larger set of items. The following program prints all possible 3-item subsets of a given larger set. The program also prints the total price of each subset.

shoppingBagCombinations() has a parameter for the current bag contents, and a parameter for the remaining items from which to choose. The base case is that the current bag already has 3 items, which prints the items. The recursive case moves one of the remaining items to the bag, recursively calling the method, then moving the item back from the bag to the remaining items.

Figure 3.7.2: Shopping spree in which a user can fit 3 items in a shopping bag.

GroceryItem.java:

```java
public class GroceryItem {
   public String itemName;   // Name of item
   public int priceDollars; // Price of item
}
```

```
Milk  Belt  Toys  = $45
Milk  Belt  Cups  = $38
Milk  Toys  Belt  = $45
Milk  Toys  Cups  = $33
Milk  Cups  Belt  = $38
Milk  Cups  Toys  = $33
Belt  Milk  Toys  = $45
Belt  Milk  Cups  = $38
Belt  Toys  Milk  = $45
Belt  Toys  Cups  = $55
Belt  Cups  Milk  = $38
Belt  Cups  Toys  = $55
Toys  Milk  Belt  = $45
Toys  Milk  Cups  = $33
Toys  Belt  Milk  = $45
Toys  Belt  Cups  = $55
Toys  Cups  Milk  = $33
Toys  Cups  Belt  = $55
Cups  Milk  Belt  = $38
Cups  Milk  Toys  = $33
Cups  Belt  Milk  = $38
Cups  Belt  Toys  = $55
Cups  Toys  Milk  = $33
Cups  Toys  Belt  = $55
```

ShoppingSpreeCombinations.java:

```java
import java.util.ArrayList;

public class ShoppingSpreeCombinations {
   public static final int MAX_SHOPPING_BAG_SIZE = 3; // Max number of
items in shopping bag

   /* Output every combination of items that fit
      in a shopping bag. Each recursive call moves
      one item into the shopping bag.
   */
   public static void shoppingBagCombinations(ArrayList<GroceryItem>
currBag,          // Bag contents
                                              ArrayList<GroceryItem>
remainingItems) { // Available items
      int bagValue;                 // Cost of items in shopping bag
      GroceryItem tmpGroceryItem; // Grocery item to add to bag
      int i;                        // Loop index

      if (currBag.size() == MAX_SHOPPING_BAG_SIZE) {   // Base case:
Shopping bag full
         bagValue = 0;
         for (i = 0; i < currBag.size(); ++i) {
            bagValue += currBag.get(i).priceDollars;
            System.out.print(currBag.get(i).itemName + "  ");
         }
         System.out.println("= $" + bagValue);
      }
      else {                                                // Recursive
case: move one
         for (i = 0; i < remainingItems.size(); ++i) { // item to bag
            // Move item into bag
            tmpGroceryItem = remainingItems.get(i);
            remainingItems.remove(i);
            currBag.add(tmpGroceryItem);

            shoppingBagCombinations(currBag, remainingItems);

            // Take item out of bag
            remainingItems.add(i, tmpGroceryItem);
            currBag.remove(currBag.size() - 1);
         }
      }
   }

   public static void main(String[] args) {
      ArrayList<GroceryItem> possibleItems = new
ArrayList<GroceryItem>(); // Possible shopping items
      ArrayList<GroceryItem> shoppingBag = new
ArrayList<GroceryItem>();   // Current shopping bag
      GroceryItem tmpGroceryItem;
// Temp item

      // Populate grocery with different items
      tmpGroceryItem = new GroceryItem();
      tmpGroceryItem.itemName = "Milk";
      tmpGroceryItem.priceDollars = 2;
      possibleItems.add(tmpGroceryItem);

      tmpGroceryItem = new GroceryItem();
      tmpGroceryItem.itemName = "Belt";
      tmpGroceryItem.priceDollars = 24;
      possibleItems.add(tmpGroceryItem);

      tmpGroceryItem = new GroceryItem();
      tmpGroceryItem.itemName = "Toys";
      tmpGroceryItem.priceDollars = 19;
      possibleItems.add(tmpGroceryItem);

      tmpGroceryItem = new GroceryItem();
      tmpGroceryItem.itemName = "Cups";
      tmpGroceryItem.priceDollars = 12;
      possibleItems.add(tmpGroceryItem);

      // Try different combinations of three items
      shoppingBagCombinations(shoppingBag, possibleItems);
   }
}
```

PARTICIPATION ACTIVITY | 3.7.3: All letter combinations.

1) When main() calls shoppingBagCombinations(), how many items are in the remainingItems list?

- None
- 3
- 4

2) When main() calls shoppingBagCombinations(), how many items are in currBag list?

- None
- 1
- 4

3) After main() calls shoppingBagCombinations(), what happens first?

- The base case prints Milk, Belt, Toys.
- The method bags one item, makes recursive call.
- The method bags 3 items, makes recursive call.

4) Just before shoppingBagCombinations() returns back to main(), how many items are in the remainingItems list?

- None
- 4

5) How many recursive calls occur before the first combination is printed?

- None
- 1
- 3

6) What happens if main() only put 2, rather than 4, items in the possibleItems list?

- ○ Base case never executes; nothing printed.
- ○ Infinite recursion occurs.

Traveling salesman

Recursion is useful for finding all possible paths. Suppose a salesman must travel to 3 cities: Boston, Chicago, and Los Angeles. The salesman wants to know all possible paths among those three cities, starting from any city. A recursive exploration of all travel paths can be used. The base case is that the salesman has traveled to all cities. The recursive case is to travel to a new city, explore possibilities, then return to the previous city.

Figure 3.7.3: Find distance of traveling to 3 cities.

```
import java.util.ArrayList;

public class TravelingSalesmanPaths {
   public static final int NUM_CITIES = 3;
// Number of cities
   public static int[][] cityDistances = new int[NUM_CITIES][NUM_CITIES];
// Distance between cities
   public static String[] cityNames = new String[NUM_CITIES];
// City names

   /* Output every possible travel path.
      Each recursive call moves to a new city.
   */
   public static void travelPaths(ArrayList<Integer> currPath,
                                  ArrayList<Integer> needToVisit) {
      int totalDist;     // Total distance given current path
      int tmpCity;       // Next city distance
      int i;             // Loop index

      if ( currPath.size() == NUM_CITIES ) { // Base case: Visited all
cities
         totalDist = 0;                      // Return total path
distance
         for (i = 0; i < currPath.size(); ++i) {
            System.out.print(cityNames[currPath.get(i)] + "   ");

            if (i > 0) {
               totalDist += cityDistances[currPath.get(i -
1)][currPath.get(i)];
            }
         }

         System.out.println("= " + totalDist);
      }
      else {                                 // Recursive case: pick next
city
         for (i = 0; i < needToVisit.size(); ++i) {
            // add city to travel path
            tmpCity = needToVisit.get(i);
            needToVisit.remove(i);
            currPath.add(tmpCity);

            travelPaths(currPath, needToVisit);

            // remove city from travel path
            needToVisit.add(i, tmpCity);
            currPath.remove(currPath.size() - 1);
         }
      }
   }

   public static void main (String[] args) {
      ArrayList<Integer> needToVisit = new ArrayList<Integer>(); //
Cities left to visit
      ArrayList<Integer> currPath = new ArrayList<Integer>();    //
Current path traveled

      // Initialize distances array
      cityDistances[0][0] = 0;
      cityDistances[0][1] = 960;  // Boston-Chicago
      cityDistances[0][2] = 2960; // Boston-Los Angeles
      cityDistances[1][0] = 960;  // Chicago-Boston
      cityDistances[1][1] = 0;
      cityDistances[1][2] = 2011; // Chicago-Los Angeles
      cityDistances[2][0] = 2960; // Los Angeles-Boston
      cityDistances[2][1] = 2011; // Los Angeles-Chicago
      cityDistances[2][2] = 0;

      cityNames[0] = "Boston";
      cityNames[1] = "Chicago";
      cityNames[2] = "Los Angeles";

      needToVisit.add(0); // Boston
      needToVisit.add(1); // Chicago
      needToVisit.add(2); // Los Angeles

      // Explore different paths
      travelPaths(currPath, needToVisit);
   }
}
```

```
Boston   Chicago   Los Angeles   = 2971
Boston   Los Angeles   Chicago   = 4971
Chicago   Boston   Los Angeles   = 3920
Chicago   Los Angeles   Boston   = 4971
Los Angeles   Boston   Chicago   = 3920
Los Angeles   Chicago   Boston   = 2971
```

PARTICIPATION ACTIVITY 3.7.4: Recursive exploration.

1) You wish to generate all possible 3-letter subsets from the letters in an N-letter word (N>3). Which of the above recursive methods is the closest?

- ○ shoppingBagCombinations
- ○ scrambleLetters
- ○ main()

CHALLENGE ACTIVITY 3.7.1: Enter the output of recursive exploration.

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Start

Type the program's output

```
import java.util.Scanner;
import java.util.ArrayList;

public class NumScrambler {
   public static void scrambleNums(ArrayList<Integer> remainNums,
                                   ArrayList<Integer> scramNums) {
      ArrayList<Integer> tmpRemainNums;
      int tmpRemovedNum;
      int i;

      if (remainNums.size() == 0) {
         System.out.print(scramNums.get(0));
         System.out.print(scramNums.get(1));
         System.out.println(scramNums.get(2));
      }
      else {
         for (i = 0; i < remainNums.size(); ++i) {
            tmpRemainNums = new ArrayList<Integer>(remainNums); // Make a copy.
            tmpRemovedNum = tmpRemainNums.remove(i);
            scramNums.add(tmpRemovedNum);
            scrambleNums(tmpRemainNums, scramNums);
            scramNums.remove(scramNums.size() - 1);
         }
      }
   }

   public static void main(String[] args) {
      Scanner scnr = new Scanner(System.in);
      ArrayList<Integer> numsToScramble = new ArrayList<Integer>();
      ArrayList<Integer> resultNums = new ArrayList<Integer>();

      numsToScramble.add(5);
      numsToScramble.add(4);
      numsToScramble.add(0);

      scrambleNums(numsToScramble, resultNums);
   }
}
```

```
54
50
45
40
05
04
```

1 2

Check Next

Exploring further:

- Recursive Algorithms from khanacademy.org

3.8 Stack overflow

Recursion enables an elegant solution to some problems. But, for large problems, deep recursion can cause memory problems. Part of a program's memory is reserved to support function calls. Each method call places a new ***stack frame*** on the stack, for local parameters, local variables, and more method items. Upon return, the frame is deleted.

Deep recursion could fill the stack region and cause a ***stack overflow***, meaning a stack frame extends beyond the memory region allocated for stack. Stack overflow usually causes the program to crash and report an error like: stack overflow error or stack overflow exception.

PARTICIPATION ACTIVITY | 3.8.1: Recursion causing stack overflow.

Animation captions:

1. Deep recursion may cause stack overflow, causing a program to crash.

The animation showed a tiny stack region for easy illustration of stack overflow.

The number (and size) of parameters and local variables results in a larger stack frame. Large ArrayLists, arrays, or Strings declared as local variables can lead to faster stack overflow.

A programmer can estimate recursion depth and stack size to determine whether stack overflow might occur. Sometimes a non-recursive algorithm must be developed to avoid stack overflow.

PARTICIPATION ACTIVITY | 3.8.2: Stack overflow.

1) A memory's stack region can store at most one stack frame.
   - True
   - False

2) The size of the stack is unlimited.
   - True
   - False

3) A stack overflow occurs when the stack frame for a method call extends past the end of the stack's memory.
   - True

○ False

4) The following recursive method will result in a stack overflow.

```
int recAdder(int inValue) {
   return recAdder(inValue +
1);
}
```

○ True

○ False

3.9 Java example: Recursively output permutations

zyDE 3.9.1: Recursively output permutations.

The below program prints all permutations of an input string of letters, one permutat
line. Ex: The six permutations of "cab" are:

```
cab
cba
acb
abc
bca
bac
```

Below, the permuteString method works recursively by starting with the first characte
permuting the remainder of the string. The method then moves to the second charac
permutes the string consisting of the first character and the third through the end of
and so on.

1. Run the program and input the string "cab" (without quotes) to see that the abo
   is produced.
2. Modify the program to print the permutations in the opposite order, and also to
   permutation count on each line.
3. Run the program again and input the string cab. Check that the output is revers
4. Run the program again with an input string of abcdef. Why did the program tak
   to produce the results?

Load default ter

```java
import java.util.Scanner;

public class Permutations {
   // FIXME: Use a static variable to count permutations. Why must it be stati

   public static void permuteString(String head, String tail) {
      char current;
      String newPermute;
      int len;
      int i;

      current = '?';
      len = tail.length();

      if (len <= 1) {
         // FIXME: Output the permutation count on each line too
         System.out.println(head + tail);
```

```
cab
```

Run

zyDE 3.9.2: Recursively output permutations (solution).

Below is the solution to the above problem.

Load default ter

```
import java.util.Scanner;

public class PermutationsSolution {
   static int permutationCount = 0;

   public static void permuteString(String head, String tail) {
      char current;
      String newPermute;
      int len;
      int i;

      current = '?';
      len = tail.length();

      if (len <= 1) {
         ++permutationCount;
         System.out.println(permutationCount + ") " + head + tail);
```

```
cab
abcdef
```

Run

3.10 Merge sort

Merge sort is a sorting algorithm that divides a list into two halves, recursively sorts each half, and then merges the sorted halves to produce a sorted list. The recursive partitioning continues until a list of 1 element is reached, as list of 1 element is already sorted.

PARTICIPATION ACTIVITY | 3.10.1: Merge sort recursively divides the input into two halves, sorts each half, and merges the lists together.

Animation captions:

1. Merge sort recursively divides the list into two halves.
2. The list is divided until a list of 1 element is found.
3. A list of 1 element is already sorted.
4. At each level, the sorted lists are merged together while maintaining the sorted order.

The merge sort algorithm uses three index variables to keep track of the elements to sort for each recursive method call. The index variable i is the index of first element in the list, and the index variable k is the index of the last element. The index variable j is used to divide the list into two halves. Elements from i to j are in the left half, and elements from j + 1 to k are in the right half.

PARTICIPATION ACTIVITY | 3.10.2: Merge sort partitioning.

Determine the index j and the left and right partitions.

1) numbers = {1 2 3 4 5}, i = 0, k = 4

`j =`

Check **Show answer**

2) numbers = {1 2 3 4 5}, i = 0, k = 4

`Left partition  = {`
`}`

Check **Show answer**

3) numbers = {1 2 3 4 5}, i = 0, k = 4

`Right partition  = {`
`}`

Check **Show answer**

4) numbers = {34 78 14 23 8 35}, i = 3, k = 5

j =

Check Show answer

5) numbers = {34 78 14 23 8 35}, i = 3, k = 5

Left partition = { }

Check Show answer

6) numbers = {34 78 14 23 8 35}, i = 3, k = 5

Right partition = { }

Check Show answer

Merge sort merges the two sorted partitions into a single list by repeatedly selecting the smallest element from either the left or right partition and adding that element to a temporary merged list. Once fully merged, the elements in the temporary merged list are copied back to the original list.

PARTICIPATION ACTIVITY 3.10.3: Merging partitions: Smallest element from left or right partition is added one at a time to a temporary merged list. Once merged, temporary list is copied back to the original list.

Animation captions:

1. Create a temporary list for merged numbers. Initialize mergePos, leftPos, and rightPos to the first element of each of the corresponding list.
2. Compare the element in the left and right partitions. Add the smallest value to the temporary list and update the relevant indices.
3. Continue to compare the elements in the left and right partitions until one of the partitions is empty.
4. If a partition is not empty, copy the remaining elements to the temporary list. The elements are already in sorted order.
5. Lastly, the elements in the temporary list are copied back to the original list.

PARTICIPATION ACTIVITY | 3.10.4: Tracing merge operation.

Trace the merge operation by determining the next value added to mergedNumbers.

14	18	35	17	38	49
0	1	2	3	4	5

1) leftPos = 0, rightPos = 3

Check Show answer

2) leftPos = 1, rightPos = 3

Check Show answer

3) leftPos = 1, rightPos = 4

Check Show answer

4) leftPos = 2, rightPos = 4

Check Show answer

5) leftPos = 3, rightPos = 4

Check Show answer

6) leftPos = 3, rightPos = 5

Check Show answer

Figure 3.10.1: Merge sort algorithm.

```
public class MergeSort {
   public static void merge(int [] numbers, int i, int j, int k) {
      int mergedSize = k - i + 1;       // Size of merged partition
      int mergedNumbers [] = new int[mergedSize]; // Temporary array for
merged numbers
      int mergePos;                     // Position to insert merged
number
      int leftPos;                      // Position of elements in left
partition
      int rightPos;                     // Position of elements in right
partition

      mergePos = 0;
      leftPos = i;                      // Initialize left partition
position
      rightPos = j + 1;                 // Initialize right partition
position

      // Add smallest element from left or right partition to merged
numbers
      while (leftPos <= j && rightPos <= k) {
         if (numbers[leftPos] < numbers[rightPos]) {
            mergedNumbers[mergePos] = numbers[leftPos];
            ++leftPos;
         }
         else {
            mergedNumbers[mergePos] = numbers[rightPos];
            ++rightPos;
         }
         ++mergePos;
      }

      // If left partition is not empty, add remaining elements to merged
numbers
      while (leftPos <= j) {
         mergedNumbers[mergePos] = numbers[leftPos];
         ++leftPos;
         ++mergePos;
      }

      // If right partition is not empty, add remaining elements to
merged numbers
      while (rightPos <= k) {
         mergedNumbers[mergePos] = numbers[rightPos];
         ++rightPos;
         ++mergePos;
      }

      // Copy merge number back to numbers
      for (mergePos = 0; mergePos < mergedSize; ++mergePos) {
         numbers[i + mergePos] = mergedNumbers[mergePos];
      }
   }
   public static void mergeSort(int [] numbers, int i, int k) {
      int j;

      if (i < k) {
         j = (i + k) / 2;  // Find the midpoint in the partition

         // Recursively sort left and right partitions
         mergeSort(numbers, i, j);
         mergeSort(numbers, j + 1, k);

         // Merge left and right partition in sorted order
         merge(numbers, i, j, k);
      }
   }

   public static void main(String [] args) {
      int [] numbers = {10, 2, 78, 4, 45, 32, 7, 11};
      int i;

      System.out.print("UNSORTED: ");
      for (i = 0; i < numbers.length; ++i) {
         System.out.print(numbers[i] + " ");
      }
      System.out.println();

      /* initial call to merge sort with index */
      mergeSort(numbers, 0, numbers.length - 1);

      System.out.print("SORTED: ");
      for (i = 0; i < numbers.length; ++i) {
         System.out.print(numbers[i] + " ");
      }
      System.out.println();
   }
}
```

```
UNSORTED: 10 2 78 4 45 32 7 11
SORTED: 2 4 7 10 11 32 45 78
```

The merge sort algorithm's runtime is O(N log N). Merge sort divides the input in half until a list of 1 element is reached, which requires log N partitioning levels. At each level, the algorithm does about N comparisons selecting and copying elements from the left and right partitions, yielding N * log N comparisons.

Merge sort requires O(N) additional memory elements for the temporary array of merged elements. For the final merge operation, the temporary list has the same number of elements as the input. Some sorting algorithms sort the list elements in place and require no additional memory, but are more complex to write and understand.

PARTICIPATION ACTIVITY 3.10.5: Merge sort runtime and memory complexity.

1) How many recursive partitioning levels are required for a list of 8 elements?

Check Show answer

2) How many recursive partitioning levels are required for a list of 2048 elements?

Check Show answer

3) How many elements will the temporary merge list have for merging two partitions with 250 elements each?

Check Show answer

3.11 LAB: All permutations of names

Write a program that lists all ways people can line up for a photo (all permutations of a list of Strings). The program will read a list of one word names into ArrayList `nameList` (until -1), and use a recursive method to create and output all possible orderings of those names separated by a comma, one ordering per line.

When the input is:

```
Julia Lucas Mia -1
```

then the output is (must match the below ordering):

```
Julia, Lucas, Mia
Julia, Mia, Lucas
```

```
Lucas, Julia, Mia
Lucas, Mia, Julia
Mia, Julia, Lucas
Mia, Lucas, Julia
```

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LAB ACTIVITY | 3.11.1: LAB: All permutations of names

7 / 10

PhotoLineups.java

Load default template...

```java
import java.util.Scanner;
import java.util.ArrayList;

public class PhotoLineups {

   // TODO: Write method to create and output all permutations of the list of names.
//   public static void printAllPermutations(ArrayList<String> permList, ArrayList<String>

//   }

//   public static void stringPermute(ArrayList<String> nameList, String str, String ans)
   public static void printAllPermutations(ArrayList<String> nameList, String str, String a
   {
      int asInt;

      if (str.length() == 0) {
         //System.out.print(ans + " ");
```

Develop mode **Submit mode**

Run your program as often as you'd like, before submitting for grading. Below, type any needed input values in the first box, then click **Run program** and observe the program's output in the second box.

Enter program input (optional)

```
Julia Lucas Mia -1
```

Run program

Input (from above) → **PhotoLineups.java** (Your program)

Program output displayed here

Coding trail of your work What is this?

```
9/8 R-------- F-------------------0---------------3--------
- S-- U---------------------------------------7,7,0,0,7
 min:160
```

3.12 LAB: Number pattern

Write a recursive method called printNumPattern() to output the following number pattern.

Given a positive integer as input (Ex: 12), subtract another positive integer (Ex: 3) continually until a negative value is reached, and then continually add the second integer until the first integer is again reached. For this lab, do not end output with a newline.

Ex. If the input is:

```
12
3
```

the output is:

```
12 9 6 3 0 -3 0 3 6 9 12
```

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LAB ACTIVITY 3.12.1: LAB: Number pattern 10 / 10

NumberPattern.java Load default template...

```java
import java.util.Scanner;

public class NumberPattern {
   // TODO: Write recursive printNumPattern() method
    public static void printNumPattern(int n1, int n2) {
         if ( n1 < 0 ) {
             System.out.print(n1 + " ");
             return;
         }
        System.out.print(n1 + " ");
        printNumPattern(n1-n2, n2);
        System.out.print(n1 + " ");

    }

   public static void main(String[] args) {
```

Develop mode Submit mode

Run your program as often as you'd like, before submitting for grading. Below, type any needed input values in the first box, then click **Run program** and observe the program's output in the second box.

Enter program input (optional)

```
12 3
```

Run program

Input (from above) → **NumberPattern.java** (Your program) →

Program output displayed here

Coding trail of your work What is this?

```
9/16 F-----7,7,0,10 min:18
```

3.13 Lab 3 - Review Linear Recursion

Module 2: Lab 3 - Review Linear Recursion

Linear Recursion

This lab is made to help you review/learn basic recursion. The next lab will get a little trickier as you move from linear to branching recursion.

What is recursion?

Linear recursion is when a function calls itself without branching out into multiple function calls. A good example of this would be a factorial function, $f(n) = n!$

Why is factorial linear in a recursive sense? Lets take f(5), for example. To compute this we need to do: $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$

In other words we are doing $f(4) \cdot 5$ or $f(3) \cdot 4 \cdot 5$ and so on until we get to $f(0)$, which is considered to be the **base case**.

This function will only have one recursive call to itself, meaning if you draw out the stack trace each call will point to another in a linear fashion.

This displays two important definitions: the **recursive call** and the **base case.**

A **recursive call** is when a function in code calls itself, and the **base case** is a condition in which causes the function to eventually terminate. Without a base case, a recursive function will go indefinitely until you get what is called a **stack overflow** (too many recursive calls for the stack to handle). Java will throw a runtime error if you create a recursive function which overflows.

Lab Assignment

The best way to get an idea of how recursion works is to practice. In the code given to you, complete the unfinished methods and run them with the given test cases.

The following .java files are included in this lab:

L3
├── LinearRecursion.java
Download starter jar L3.jar to work on the lab in another environment.

The Factorial Method

The first method we are going to modify is the Factorial Method.

TO DO:
1. Write this method so that it returns the factorial for any input integer *n* . n! is n * (n - 1) * (n - 2) * ... * 1.

The Summation Method

The next method we are going to modify is the Summation Method. This method is similar to factorial but instead of multiplying each element you will add them together.

TO DO:
1. Write this method so that when you input an integer *n* the method will return n + (n - 1) + (n - 2) + ... + 1.

The Harmonic Summation Method

The Harmonic Summation will return the sum of the harmonic series.

TO DO:
1. Set up this method so that when you input an integer *n* the method will return the sum of 1 + 1/2

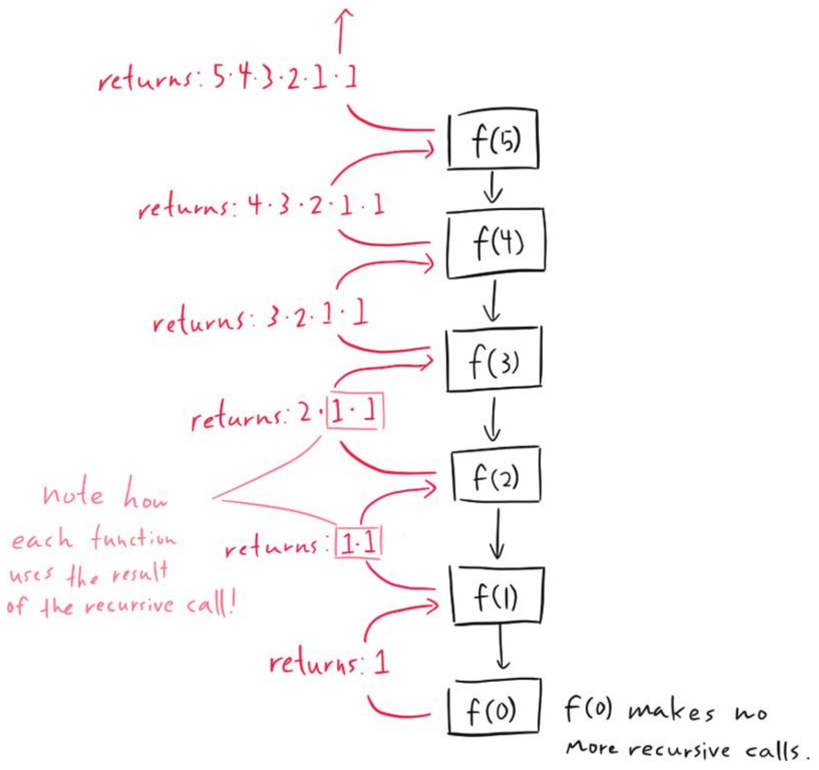
+ 1/3 + ... + 1/(n-1) + 1/n.

The Geometric Summation Method

The Geometric Summation will return the sum of the geometric series.

TO DO:
1. Set up this method so that when you input an integer *n* the method will return the sum of the sum 1 + 1/2 + 1/4 + 1/8 + ... + 1/Math.pow(2,n),

To understand how these methods are exhibiting linear recursion, draw out the stack trace for a reasonable n value like 4. Here's an example of this for factorial for n = 5:



If you've implemented everything correctly your output should look like this:

```
Testing the factorial method
factorial(4) should be 24 -> 24
factorial(6) should be 720 -> 720
factorial(0) should be 1 -> 1

Testing the summation method
sum(4) should be 10 -> 10
sum(10) should be 55 -> 55
sum(0) should be 0 -> 0

Testing out the harmonicSum method
harmonicSum(0) should be 0.0000 -> 0.0000
harmonicSum(7) should be 2.5929 -> 2.5929
harmonicSum(8) should be 2.7179 -> 2.7179
harmonicSum(13) should be 3.1801 -> 3.1801
harmonicSum(24) should be 3.7760 -> 3.7760

Testing out the geometricSum method
geometricSum(0) should be 1.0000 -> 1.0000
geometricSum(1) should be 1.5000 -> 1.5000
geometricSum(2) should be 1.7500 -> 1.7500
geometricSum(7) should be 1.9922 -> 1.9922
geometricSum(24) should be 2.0000 -> 2.0000
```

Submission

In Submit mode, select "Submit for grading" when you are ready to turn in your assignment. Each method will be tested.

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LAB ACTIVITY 3.13.1: Lab 3 - Review Linear Recursion 7 / 7

LinearRecursion.java Load default template...

```java
/**
 * Recitation created by Toby Patterson, 5/22/2020
 * For CS165 at CSU
 * With the help of https://www.cs.colostate.edu/~cs165/.Spring20/recitations/L6/doc/
 *
 * NOTE: All methods should be implemented recursively, no iteration allowed!
 */

public class LinearRecursion {

    /**
     * params: integer n
     * return: n! or n * (n - 1) * (n - 2) * ... * 1
     * TODO: implement this method
     */


```

Develop mode **Submit mode**

Run your program as often as you'd like, before submitting for grading. Below, type any needed input values in the first box, then click **Run program** and observe the program's output in the second box.

Enter program input (optional)

If your code requires input values, provide them here.

Run program

Input (from above) → LinearRecursion.java (Your program) →

Program output displayed here

Coding trail of your work What is this?

```
9/6 T--------2,0,4,0,0,0,0,0------------ R--------------6--
--7 min:53
```

3.14 Lab 4 - Branching Recursion

Module 2: Lab 4 - Branching Recursion

Recursion Practice

Welcome back! In this lab, we will be reviewing recursion by practicing with some basic recursion problems.

Objectives

- Increase familiarity with recursive logic by working through several recursive problems.
- Taking into consideration a few corner cases through analyzing the test cases.
- Using a regex expression that will remove punctuation.

Getting Started
This lab includes the following .java file:

```
L4/
└── Recursion.java
└── Main.java*
```

**Main.java is a read-only file used for testing. It is not included in the starter jar.*

Here is the starter jar if you would like to code in a different environment: L4.jar.

Please complete *ALL* functions. Make sure to read the description for each function carefully.

Do not include any for or while loops in your methods. These can all be completed in a purely recursive style, so do it recursively!

In the spirit of incremental development, implement each method one at a time, look at the test cases and take into consideration what is being tested, then uncomment the corresponding test code and see if it works. If not, try to determine what went wrong and try again.

Lab Assignment

In this lab, you will complete the following methods:
- fib
- mult
- expt
- isPalindrome
- longestWordLength
- dedupeChars

1. The Fibonacci Method

TO DO:
1. Write this method so that it returns the Fibonacci number for any input integer *n* .

The Fibonacci sequence begins with 0 and then 1 follows. All subsequent values are the sum of the previous two, for example: 0, 1, 1, 2, 3, 5, 8, 13. Complete the fib() method, which takes in an index, n, and returns the nth value in the sequence. Every number after the first two is the sum of the two preceding ones.

For each index 0, 1, 2, 3, 4, 5, 6...
the output is: 0, 1, 1, 2, 3, 5, 8...

When you run this method, your output should look like this:

```
Testing the fibonacci method
fib(0) should be 0 -> 0
fib(1) should be 1 -> 1
fib(2) should be 1 -> 1
fib(5) should be 5 -> 5
fib(10) should be 55 -> 55
fib(13) should be 233 -> 233
```

2. The Multiplication Method

TO DO:
1. Write a multiplication method recursively using repeated addition.
Do not use the multiplication operator or a loop.

When you run this method, your output should look like this:

```
Testing out the multiplication method
mult(8, 2) should be 16 -> 16
mult(2, 8) should be 16 -> 16
mult(-2, -8) should be 16 -> 16
mult(4, -3) should be -12 -> -12
mult(-3, 4) should be -12 -> -12
```

3. The Exponent Method

TO DO:
1. Write a method that computes j^k.
Do not use Math.pow() or a loop.

When you run this method, your output should look like this:

```
Testing out the exponent method
expt(2, 5) should be 32 -> 32
expt(5, 2) should be 25 -> 25
```

4. The isPalindrome Method

TO DO:
1. Write a method that checks to see if a word is a palindrome. This should be case-independent.

When you run this method, your output should look like this:

```
Testing out the palindrome method
isPalindrome("a") should be true -> true
isPalindrome("Aibohphobia") should be true -> true
isPalindrome("noon") should be true -> true
isPalindrome("Recursion") should be false -> false
```

5. The Longest Word Length Method

TO DO:
1. Write a method that returns length of the longest word in the given String using recursion (no loops).

Hint: a Scanner may be helpful for finding word boundaries. After delimiting by space, use the following method on your String to remove punctuation .replaceAll("[^a-zA-Z]", "") If you use a Scanner, you will need a helper method to do the recursion on the Scanner object.

When you run this method, your output should look like this:

```
Testing out the longestWordLength method

The longest word in the following quote:
Grit, one of the keys to success. The person who perseveres is the
one who
will surely win. Success does not come from giving up, it comes from
believing
in yourself and continuously working towards the realization of a
worthy ideal.
Do not ever give up on what you want most. You know what you truly
want.
Believe in your dreams and goals and take daily consistent action in
order to
make your dreams a reality.
should be 12 -> 12

The longest word in the following quote:
Try to be like the turtle – at ease in your own shell.
should be 6 -> 6
```

6. The Remove Duplicates Method

TO DO:
1. Write a method to remove consecutive duplicate characters from a String.

If two or more consecutive duplicate characters have different cases, then the first letter should be kept.

When you run this method, your output should look like this:

```
Testing the dedupeChars method
dedupeChars("a") should be a -> a
dedupeChars("aa") should be a -> a
dedupeChars("MiSsisSiPpi") should be MiSisiPi -> MiSisiPi
dedupeChars("swimMmMming") should be swiming -> swiming
```

Submission

In Submit mode, select "Submit for grading" when you are ready to turn in your assignment. Do **not** change or delete any of the print statements in main. Your output must match exactly.

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LAB ACTIVITY 3.14.1: Lab 4 - Branching Recursion 0 / 7

Downloadable files

Recursion.java **Download**

Current file: **Recursion.java** Load default template...

```
import java.util.Scanner;

/**
 * Recitation created by Gareth Halladay, 08/17. <br>
 * Content was gathered from two sources:
 * <ul>
 *     <li> http://www.cs.wustl.edu/~kjg/cse131/modules/recursion/lab.html
 *     <li> http://codingbat.com/prob/p273235?parent=/home/bono@usc.edu/recursionLab
 * </ul>
 *
 */
public class Recursion {

    /**
     * Every number after the first two is the sum of the two preceding ones. <br>
     * index:  0, 1, 2, 3, 4, 5, 6... <br>
     * output: 0, 1, 1, 2, 3, 5, 8...
```

Develop mode **Submit mode**

Run your program as often as you'd like, before submitting for grading. Below, type any needed input values in the first box, then click **Run program** and observe the program's output in the second box.

Enter program input (optional)

If your code requires input values, provide them here.

Run program

Input (from above) → Recursion.java (Your program) →

Program output displayed here

Coding trail of your work What is this?

```
9/13 T----------------- R- F--0,0---------------------0-----
------------------------0,0,0,0,0,0,0,0,0,0,0,0,0 min:147
```

3.15 LAB: Fibonacci sequence (recursion)

The Fibonacci sequence begins with 0 and then 1 follows. All subsequent values are the sum of the previous two, for example: 0, 1, 1, 2, 3, 5, 8, 13. Complete the fibonacci() method, which takes in an index, n, and returns the nth value in the sequence. Any negative index values should return -1.

Ex: If the input is:

```
7
```

the output is:

```
fibonacci(7) is 13
```

Note: Use recursion and **DO NOT** use any loops.

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LAB ACTIVITY 3.15.1: LAB: Fibonacci sequence (recursion) 10 / 10

LabProgram.java Load default template...

```
import java.util.Scanner;

public class LabProgram {

   public static int fibonacci(int n) {
      if ( n < 0 ) {
         return -1;
```

Develop mode Submit mode

Run your program as often as you'd like, before submitting for grading. Below, type any needed input values in the first box, then click **Run program** and observe the program's output in the second box.

Enter program input (optional)

If your code requires input values, provide them here.

Run program

Input (from above) → LabProgram.java (Your program) →

Program output displayed here

Coding trail of your work What is this?

```
9/8 R-------10----------10,10 min:19
```