Students:
Section 3.5 is a part of 1 assignment:
Reading Assignment 3

Requirements: PA CA
No due date

3.5 Composition of functions

Let f be a function that assigns employees to offices in a company. Let g be the function that maps each office to the telephone number for the phone in that office. An employee, Rajiv, is assigned the office f(Rajiv). Rajiv's work phone number is g(f(Rajiv)). The process of applying a function to the result of another function is called ***composition***.

Definition 3.5.1: Composition of functions.

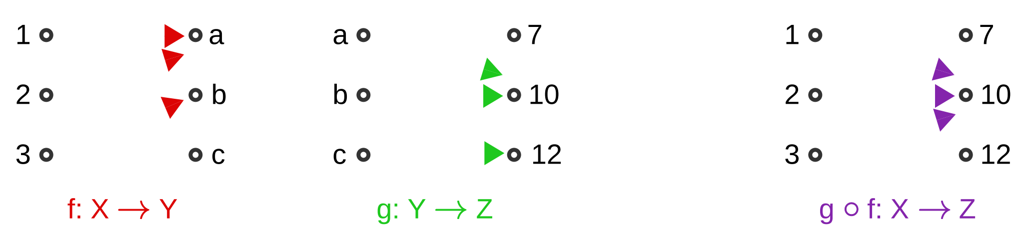
f and g are two functions, where f: X → Y and g: Y → Z. The composition of g with f, denoted g o f, is the function (g o f): X → Z, such that for all x ∈ X, (g o f)(x) = g(f(x)).

Feedback?

PARTICIPATION ACTIVITY 3.5.1: Composition of functions.

Start 2x speed

X = { 1, 2, 3 }
Y = { a, b, c }
Z = { 7, 10, 12 }



Captions

Feedback?

Generally, the order in which the functions are applied is important, so f o g is not the same as g o f. Define:

$$f: \mathbf{R}^+ \rightarrow \mathbf{R}^+, f(x) = x^3$$

$$g: \mathbf{R}^+ \rightarrow \mathbf{R}^+, g(x) = x + 2$$

Then

$$(f \circ g)(x) = f(g(x)) = (x + 2)^3$$

$$(g \circ f)(x) = g(f(x)) = x^3 + 2$$

It is possible to compose more than two functions. Composition is associative, so the order in which one composes the functions does not matter:

$$f \circ g \circ h = (f \circ g) \circ h = f \circ (g \circ h) = f(g(h(x)))$$

The ***identity function*** always maps a set onto itself and maps every element onto itself.

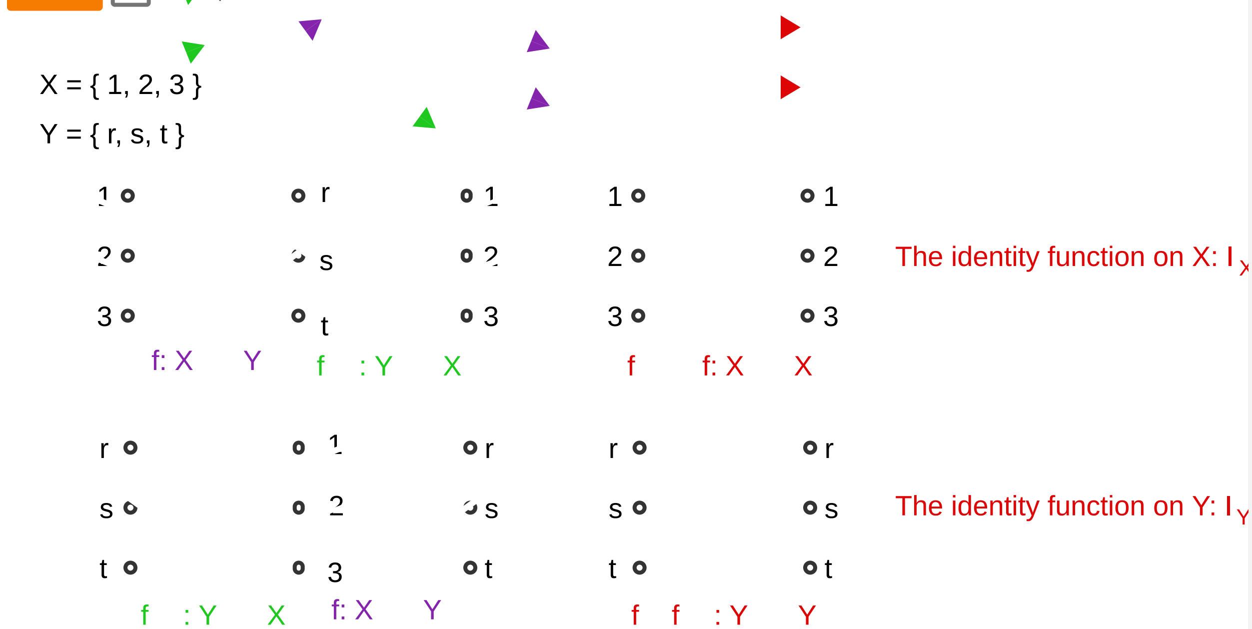
The identity function on A, denoted I_A: A → A, is defined as $I_A(a) = a$, for all a ∈ A.

If a function f from A to B has an inverse, then f composed with its inverse is the identity function. If f(a) = b, then $f^{-1}(b) = a$, and $(f^{-1} \circ f)(a) = f^{-1}(f(a)) = f^{-1}(b) = a$.

Let f: A → B be a bijection. Then $f^{-1} \circ f = I_A$ and $f \circ f^{-1} = I_B$.

PARTICIPATION ACTIVITY 3.5.2: Composition of a function and its inverse.

Start 2x speed



Captions

Feedback?

PARTICIPATION ACTIVITY

3.5.3: Composition of functions.

Define functions f, g, h, all of which have **R** as their domain and **R** as their target.

$f(x) = 3x + 1$
$g(x) = x^2$
$h(x) = 2^x$

1) What is (f o g)(2)?

Check Show answer

2) What is (g o h)(3)?

Check Show answer

3) What is (f o g o h)(0)?

Check Show answer

4) What is (f o f^{-1})(17)?

Check Show answer

Feedback?

CHALLENGE ACTIVITY

3.5.1: Composition of functions.

422102.2723990.qx3zqy7

Start

Given:

Ex: 15

1 2 3 4 5

1 2 3 4 5

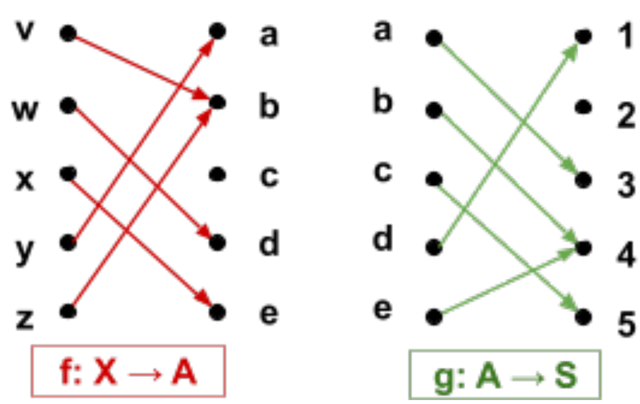
Check Next

Feedback?

Additional exercises

EXERCISE 3.5.1: Function composition from arrow diagrams.

The drawing below gives arrow diagrams for two functions, f and g.



(a) What is the domain of g o f?

(b) What is the target of g o f?

(c) Give the arrow diagram for g o f.

(d) What is the range of g o f?

Feedback?

EXERCISE 3.5.2: Composition of functions on integers.

Consider three functions f, g, and h, whose domain and target are **Z**.
Let

(a) Evaluate (f o g)(0)

(b) Evaluate (f o h)(52)

(c) Evaluate (g o h o f)(4)

(d) Give a mathematical expression for h o f.

(e) Give a mathematical expression for f o g.

Feedback?

EXERCISE 3.5.3: Function composition and the identity function.

Define f to be a function whose domain is X and whose target is Y such that X ∩ Y = ∅. For each of the following functions, indicate whether the function is well-defined. If your answer is "well-defined", indicate how the function relates to f.

(a) f o I_X

(b) f o I_Y

(c) I_X o f

(d) I_Y o f

Feedback?

EXERCISE 3.5.4: Composition of onto and one-to-one functions.

Let f: X → Y and g: Y → Z be two functions.

(a) Is it possible that f is not onto and g o f is onto? Justify your answer. If the answer is "yes", give a specific example for f and g.

(b) Is it possible that g is not onto and g o f is onto? Justify your answer. If the answer is "yes", give a specific example for f and g.

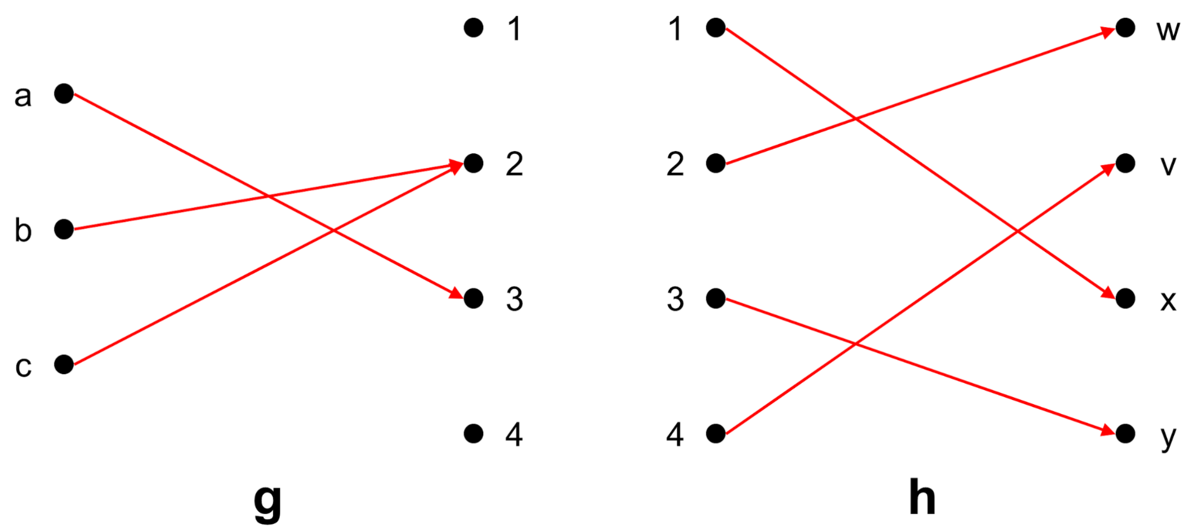
(c) Is it possible that f is not one-to-one and g o f is one-to-one? Justify your answer. If the answer is "yes", give a specific example for f and g.

(d) Is it possible that g is not one-to-one and g o f is one-to-one? Justify your answer. If the answer is "yes", give a specific example for f and g.

Feedback?

EXERCISE | 3.5.5: Composition of functions defined by arrow diagrams.

Define two functions: g: {a, b, c} → {1, 2, 3, 4} and h: {1, 2, 3, 4} → {w, v, x, y}. The functions are shown in the arrow diagrams below.



(a) What is the range of g?

(b) What is the domain of h o g?

(c) What is $h^{-1}(y)$?

(d) What is the domain of h^{-1} o h?

(e) What is (h o g)(b)?

(f) Is g one-to-one or onto?

(g) Are either g or h a bijection?

Feedback?

EXERCISE 3.5.6: Composition of functions on sets of strings.

Define the following functions f, g, and h:

- f: $\{0, 1\}^3 \rightarrow \{0, 1\}^3$. The output of f is obtained by taking the input string and replacing the first bit by 1, regardless of whether the first bit is a 0 or 1. For example, f(001) = 101 and f(110) = 110.
- g: $\{0, 1\}^3 \rightarrow \{0, 1\}^3$. The output of g is obtained by taking the input string and reversing the bits. For example, g(011) = 110.
- h: $\{0, 1\}^3 \rightarrow \{0, 1\}^3$. The output of h is obtained by taking the input string x, and replacing the last bit with a copy of the first bit. For example, h(011) = 010.

(a) What is (g o f)(010)?

(b) What is (g o h)(010)?

(c) What is (h o f)(010)?

(d) What is the range of h o f?

(e) What is the range of g o f?

Feedback?

EXERCISE 3.5.7: Composition of functions on sets of strings, part 2.

Let d, f, and g be defined as follows.

- d: $\{0, 1\}^4 \rightarrow \{0, 1\}^4$. d(x) is obtained from x by removing the second bit and placing it at the end. For example, d(1011) = 1110.
- f: $\{0, 1\}^4 \rightarrow \{0, 1\}^4$. f(x) is obtained from x by replacing the last bit with 1. For example, f(1000) = 1001.
- g: $\{0, 1\}^4 \rightarrow \{0, 1\}^3$. g(x) is obtained from x by removing the first bit. For example, g(1000) = 000.

(a) What is $d^{-1}(1001)$?

(b) Which of the following functions is not well defined, f o g or g o f?

(c) What is the range of g o f?

(d) What is (f o d)(1011)?

Feedback?

EXERCISE | 3.5.8: Explicit formulas for compositions of functions.

The domain and target set of functions f, g, and h are **Z**. The functions are defined as:

- $f(x) = 2x + 3$
- $g(x) = 5x + 7$
- $h(x) = x^2 + 1$

Give an explicit formula for each function given below.

(a) f o g

(b) g o f

(c) f o h

(d) h o f

Feedback?

How was this section? Provide feedback

Activity summary for assignment: Reading Assignment 3 57 / 90 pts

No due date 57 / 90 pts submitted to canvas

Completion details