CS 220

Written Assignment 1

1. Let a and b be rational numbers such that $a > b$. Then there exists a rational number c such that $a > c > b$.

2. Let a, b, and c be integers that are not all odd. Prove that $a \cdot b \cdot c$ is even.

3. Show that the product of three odd integers is odd.

4. Prove that if n is a positive integer, then n is even if and only if $7n + 4$ is even.

   Note that an "if and only if" proof involves 2 proofs:
   n is even implies $7n + 4$ is even,
   $7n + 4$ is even implies n is even.
   One of these is best done as a direct proof, the other as a proof by contrapositive.

5. Prove that if $x + y \geq 4$, where x and y are real numbers, then $x \geq 2$ or $y \geq 2$.

6. Prove: There is no largest odd number.

7. Consider the following statement: Let a and b be rational numbers, then a^b is rational. Is it true? If yes, prove it; otherwise provide a counterexample.

8. Let $\min(x, y)$ be the function whose value is the minimum of the two arguments. Use a proof by cases to show that:

   $$\min(x, \min(y, z)) = \min(\min(x, y), z),$$

   whenever x, y, z are real numbers.

   Hint: You will need 3 cases: one for each of x, y, z assumed to be the minimum.

9. Let x, y be integers, one of which is odd, and one of which is even. Show that $7x + 7y$ is odd.