Students:
Section 2.5 is a part of 1 assignment:
Reading Assignment 2

Requirements: PA
No due date

2.5 Set identities

The set operations intersection, union and complement are defined in terms of logical operations. All the elements are assumed to be contained in a universal set *U*.

$$x \in A \cap B \;\leftrightarrow\; (x \in A) \land (x \in B)$$

$$x \in A \cup B \;\leftrightarrow\; (x \in A) \lor (x \in B)$$

$$x \in \overline{A} \;\leftrightarrow\; \neg(x \in A)$$

The sets *U* and ∅ correspond to the constants true (T) and false (F):

$$x \in \varnothing \;\leftrightarrow\; F$$

$$x \in U \;\leftrightarrow\; T$$

The laws of propositional logic can be used to derive corresponding set identities. A ***set identity*** is an equation involving sets that is true regardless of the contents of the sets in the expression. The idea is similar to an equivalence in logic which holds regardless of the truth values of the individual variable in the expressions. The animation below shows the derivation of De Morgan's set identity using De Morgan's law for propositional logic.

PARTICIPATION ACTIVITY 2.5.1: Deriving De Morgan's set identity using logic.

1 2 3 4 5 6 2x speed

$x \in A \cap B \longleftrightarrow \neg(x \in A \cap B)$	Definition of complement
$\longleftrightarrow \neg(x \in A \land x \in B)$	Definition of intersection
$\equiv \neg(x \in A) \lor \neg(x \in B)$	De Morgan's law for proposition
$\longleftrightarrow x \in A \lor x \in B$	Definition of complement
$\longleftrightarrow x \in (A \cup B)$	Definition of Union
$A \cap B = A \cup B$	De Morgan's set identity

Therefore, De Morgan's set identity is true: the complement of is equal to the union of the complement of and the complement of .

Captions

1. The definition of set complement says that x is an element of the complement of if and only if .
2. The definition of set intersection says that if and only if .
3. De Morgan's law for propositions says that is equivalent to .
4. By the definition of set complement: if and only if is in the complement of and if and only if is in the complement of .
5. The definition of set union says that if and only if .
6. Therefore, De Morgan's set identity is true: the complement of is equal to the union of the complement of and the complement of .

Feedback?

All the set identities in the table below can be proven in a similar manner using the definitions for set operations and the laws of propositional logic.

Table 2.5.1: Set identities.

Name	Identities	
Idempotent laws	A ∪ A = A	A ∩ A = A
Associative laws	(A ∪ B) ∪ C = A ∪ (B ∪ C)	(A ∩ B) ∩ C = A ∩ (B ∩ C)
Commutative laws	A ∪ B = B ∪ A	A ∩ B = B ∩ A
Distributive laws	A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)	A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)
Identity laws	A ∪ ∅ = A	A ∩ *U* = A
Domination laws	A ∩ ∅ = ∅	A ∪ *U* = *U*
Double complement law		

Complement laws	$A \cap \overline{A} = \varnothing$ $\overline{U} = \varnothing$	$A \cup \overline{A} = U$ $\overline{\varnothing} = U$
De Morgan's laws	$\overline{A \cup B} = \overline{A} \cap \overline{B}$	$\overline{A \cap B} = \overline{A} \cup \overline{B}$
Absorption laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$

Feedback?

PARTICIPATION ACTIVITY 2.5.2: Using the set identities.

Match equivalent sets.

$\varnothing$

$\varnothing \cap (B \cup C)$

The domination law says that the intersection of $\varnothing$ and any other set is equal to $\varnothing$.

Correct

B

$B \cup \varnothing$

The identity law says that the union of $\varnothing$ and any other set B is equal to B.

Correct

U

$B \cup \overline{B}$

The complement law says that the union of a set and its complement is equal to *U*.

Correct

C

$C \cup (C \cap B)$

The absorption law says that $C \cup (C \cap B) = C$.

Correct

B ∪ C

The double complement law says that the complement of the complement of a set $B \cup C$ is

Correct

equal to B ∪ C.

Reset

Feedback?

CHALLENGE ACTIVITY | 2.5.1: Set identities.

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Jump to level 1

1

2

3

4

The following two sets are not equivalent.

and

Select the example for sets and that proves the two sets are not equivalent.

A = {1, 2, 3} and B = {4, 5}

1	2	3	**4**

Check Next **Done**. Click any level to practice more. Completion is preserved.

✔ Expected: and

When and , then . Therefore, is not equivalent to .

Feedback?

Additional exercises

EXERCISE | 2.5.1: Name the set identity.

Name the set identity that is used to justify each of the identities given below.

(a) $(B \cap C) \cup \overline{B \cap C} = U$

(b) $\overline{A \cup (A \cap B)} = \overline{A}$

(c) $A \cup (\overline{B \cap C}) = A \cup (\overline{B} \cup \overline{C})$

(d)

(e) $(B - A) \cup (B - A) = (B - A)$

(f) $((A \oplus B) - C) \cap \varnothing = \varnothing$

Feedback?

EXERCISE 2.5.2: Proving set identities.

Use the set identities given in the table to prove the following new identities. Label each step in your proof with the set identity used to establish that step.

(a) $(\overline{A} \cap C) \cup (A \cap C) = C$

(b) $(B \cup A) \cap (\overline{B} \cup A) = A$

(c)

(d) $\overline{A} \cap (A \cup B) = \overline{A} \cap B$

(e) $\overline{A} \cup (A \cap B) = \overline{A} \cup B$

(f) $A \cap (B \cap \overline{B}) = \varnothing$

(g) $A \cup (B \cup \overline{B}) = U$

Feedback?

EXERCISE 2.5.3: Showing set equations that are not identities.

A set equation is not an identity if there are examples for the variables denoting the sets that cause the equation to be false. For example $A \cup B = A \cap B$ is not an identity because if $A = \{1, 2\}$ and $B = \{1\}$, then $A \cup B = \{1, 2\}$ and $A \cap B = \{1\}$, which means that $A \cup B \neq A \cap B$.

Show that each set equation given below is not a set identity.

(a) $A \cup (B \cap A) = B$

(b) $A - (B \cap A) = A$

(c) $(A \cup B) - (A \cap B) = A - B$

(d) $(B - A) \cup A = A$

(e) $A \cup B = A \oplus B$

Feedback?

EXERCISE 2.5.4: Proving set identities with the set difference operation.

The set subtraction law states that $A - B = A \cap \overline{B}$. Use the set subtraction law as well as the other set identities given in the table to prove each of the following new identities. Label each step in your proof with the set identity used to establish that step.

(a) $A - (B \cap A) = A - B$

(b) $A \cap (B - A) = \varnothing$

(c) $A \cup (B - A) = A \cup B$

(d) $A - (B - A) = A$

(e) $(A - B) - A = \varnothing$

Feedback?

How was this section? Provide feedback

Activity summary for assignment: Reading Assignment 2 80 / 105 pts

No due date

80 / 105 pts submitted to canvas

Completion details