Students:
Section 2.1 is a part of 1 assignment:
Reading Assignment 2

Requirements: PA CA
No due date

2.1 Sets and subsets

Sets play an important role in almost every area of mathematics, including discrete math. Set theory is a well-developed branch of mathematics in its own right, most of which is beyond the scope of this material. We will start with some basic definitions, notations, and ideas related to sets, all of which will be used extensively in the rest of the topics covered.

A ***set*** is a collection of objects. Objects may be of various types, such as titles of books, names of bridges, or rational numbers. This material is mostly concerned with sets of mathematical objects like numbers. The objects in a set are called ***elements***. A set may contain elements of different varieties, e.g., a set whose elements are the number 2, a strawberry, and a monkey.

A set is defined by indicating which elements belong to it. If the number of elements in a set is small, the easiest way to describe the set is by listing its elements. The ***roster notation*** definition of a set is a list of the elements enclosed in curly braces with the individual elements separated by commas. The following definition of the set A uses roster notation:

$$A = \{ 2, 4, 6, 10 \}$$

The order in which the elements are listed is unimportant. So the set A can also be expressed as:

$$A = \{ 10, 6, 4, 2 \}$$

Repeating an element does not change the set. So the set A can also be expressed as:

$$A = \{ 2, 2, 4, 6, 10 \}$$

PARTICIPATION ACTIVITY 2.1.1: Example of sets.

1 2 3 2x speed

The set N The set F The set M

N = { 2, 4, 6, 10 }

This set has four real-number elements

F = { Watermelon, Strawberry, Banana }

This set has three fruit elements

M = { 2, Strawberry, Monkey }

Set elements may be of different varieties

The set { 2, Strawberry, Monkey } has elements of different varieties.

Captions

1. The set { 2, 4, 6, 10 } has four real-number elements.
2. The set { Watermelon, Strawberry, Banana } has three fruit elements.
3. The set { 2, Strawberry, Monkey } has elements of different varieties.

Feedback?

PARTICIPATION ACTIVITY 2.1.2: Sets in roster notation.

Select the statement that is false.

1) The set S = {1, 2, 3, 4}.

- S = {4, 3, 2, 1}
- S = {4, 4, 1, 2, 3}
- S = {1, 2, 2, 2, 3}

2) The set T = {a, A, c, d}

- T = {a, c, d, A, b}
- T = {d, A, A, c, a}
- T = {A, a, c, a, d, A}

Feedback?

More notation related to sets

The symbol ∈ is used to indicate that an element is in a set, as in 2 ∈ A. The symbol ∉ indicates that an element is not in a set, as in 5 ∉ A. Typically, capital letters will be used as variables denoting sets, and lower case letters will be used for elements in the set. Variables can be used to indicate an unspecified member of a set. For example, if a ∈ A, then a is equal to 2, 4, 6, or 10.

The set with no elements is called the ***empty set*** and is denoted by the symbol ∅. The empty set is sometimes referred to as the ***null set*** and can also be denoted by {}. Because the empty set has no elements, for any element a, a ∉ ∅ is true.

A ***finite set*** is a set that is either empty or whose elements can be numbered 1 through n for some positive integer n. An ***infinite set*** is a set that is not finite. The ***cardinality*** of a finite set A, denoted by |A|, is the number of distinct elements in A. If A = { 2, 4, 6, 10 }, then |A| = 4. The cardinality of the empty set |∅| is zero.

When there are many elements in a set, it may not be practical to provide an exhaustive list. In this case, ellipses (...) are used to denote a long (possibly infinite) sequence of numbers. However, the sequence's pattern should be clear so that the reader can infer which elements are missing. Below are some examples of the use of ellipses to describe sets:

$$B = \{ 1, 3, 5, \ldots, 99 \}$$

$$C = \{ 3, 6, 9, 12, \ldots \}$$

In the first example, the set B contains all odd integers between 0 and 100. In the second case, the set C is an infinite set containing all positive integer multiples of 3.

Two sets are equal if they have exactly the same elements. Define:

$$D = \{ 3, 4, 5 \}$$

$$E = \{ 5, 3, 4 \}$$

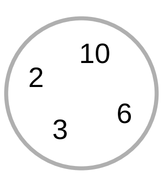
$$F = \{ 5, 3, 4, 6 \}$$

D = E because for any a, a ∈ D if and only if a ∈ E. Remember that the order in which the elements are listed in the definitions of D and E is unimportant. F ≠ E because 6 ∈ F but 6 ∉ E.

PARTICIPATION ACTIVITY | 2.1.3: Set basics.

Start 2x speed

The set A



A = { 2, 3, 6, 10 }
= { 3, 2, 10, 6 }

Order does not matter in listing elements

|A| = 4

|A| is the cardinality of A, which is the number of distinct elements in A

The cardinality is finite A is finite set

2 A indicates that an element is in a set

5 A indicates that an element is *not* in a set

The empty set

$\varnothing$ = { } The empty set has no elements and is denoted

Captions

Feedback?

PARTICIPATION ACTIVITY | 2.1.4: Set membership and cardinality.

Consider the following sets:

A = { 4, 6, 3 }
B = { 2, 4, 6, ..., 20 }
C = { 2, 4, 6, ... }
D = { 3, 4, 6 }

1) A = D
- True
- False

2) $5 \in A$
- True
- False

3) $5 \in \varnothing$
- True
- False

4) C is finite.
- True
- False

5) |B| = 10
- True
- False

6) |A| = |D|
- True

False

Feedback?

Example 2.1.1: Sets in applications.

Many pieces of software need to maintain sets of items. For example, a database is a large set of pieces of information. A university maintains a set of all the students enrolled. An airline maintains a set of all past and future flights. One of the most basic types of operations a programmer might perform on a set of data is to ask whether a particular item is an element of that set. Discrete math is primarily concerned with mathematical operations on sets, but software developers need to think about how to actually represent the set in order to satisfy queries to the set of items.

Feedback?

Some sets of numbers are used so frequently in mathematics that they have their own symbols. Here are some of the most common examples:

Table 2.1.1: Common mathematical sets.

Set	Symbol	Examples of elements
N is the set of ***natural numbers***, which includes all integers greater than or equal to 0.	**N**	0, 1, 2, 3, ...
Z is the set of all integers.	**Z**	..., -2, -1, 0, 1, 2, ...
Q is the set of ***rational numbers***, which includes all real numbers that can be expressed as a/b, where a and b are integers and b ≠ 0.	**Q**	0, 1/2, 5.23, -5/3
R is the set of real numbers.	**R**	0, 1/2, 5.23, -5/3, π,

Feedback?

The superscript + is used to indicate the positive elements of a particular set. For example, the set $\mathbf{R}^+$ is the set of all positive real numbers, and $\mathbf{Z}^+$ is the set of all positive integers. A number x is ***positive*** if x > 0. The superscript - is used to indicate the negative elements of a particular set. For example, the set $\mathbf{R}^-$ is the set of all negative real numbers, and $\mathbf{Z}^-$ is the set of all negative integers. A number x is ***negative*** if x < 0. The number 0 is neither positive nor negative, so $0 \notin \mathbf{Z}^+$ and $0 \notin \mathbf{Z}^-$.

A number x is ***non-negative*** if x ≥ 0. The natural numbers, as defined in the table above, is equal to the set of non-negative integers. Some authors define the natural numbers to be the set of positive integers (i.e., excluding 0).

PARTICIPATION ACTIVITY | 2.1.5: Membership in numerical sets.

1) $-3 \in \mathbf{Z}^+$
   - True
   - False

2) $0 \in \mathbf{Z}^+$
   - True
   - False

3) 0 is a non-negative integer.
   - True
   - False

4) $5 \in \mathbf{R}^+$
   - True
   - False

5) -5 is a member of the set of all non-negative integers.
   - True
   - False

6) $0 \in \mathbf{Q}$
   - True
   - False

Feedback?

In ***set builder notation***, a set is defined by specifying that the set includes all elements in a larger set that also satisfy certain conditions. The notation would look like:

$$A = \{ x \in S : P(x) \}$$

S is the larger set from which the elements in A are taken. P(x) is some condition for membership in A. The colon symbol ":" is read "such that". The description for A above would read: "all x in S such that P(x)". Often, the set S will be one of the standard mathematical sets from the table above. For example, the set:

$$C = \{ x \in \mathbf{Z} : 0 < x < 100 \text{ and } x \text{ is prime}\}$$

would be all prime integers between 0 and 100. The set:

$$D = \{ x \in \mathbf{R} : |x| < 1 \}$$

would be all real numbers between -1 and 1, not including 1 or -1.

PARTICIPATION ACTIVITY | 2.1.6: Set equality.

For each set, find the matching set.

{ 2, 3, 5, 7, 11, 13, 17, 19 }

$\{ x \in \mathbf{Z} : 0 < x < 20 \text{ and } x \text{ is prime} \}$

The set { 2, 3, 5, 7, 11, 13, 17, 19 } consists of exactly those integers that are between 0 and 20 and are prime.

Correct

{ -1, 0 ,1 }

$\{ x \in \mathbf{R} : |x| = x^2 \}$

The only real numbers that satisfy $|x| = x^2$ are 0, 1, and -1.

Correct

{ 0, 2, 4, 6, ... }

$\{ x \in \mathbf{N} : x \text{ is even} \}$

The set { 0, 2, 4, 6, ... } is an infinite set consisting of all natural numbers that are even.

Correct

{ 0, 2, 4, 6, ..., 18 }

$\{ x \in \mathbf{N} : x \text{ is even and } x < 20 \}$

The set { 0, 2, 4, 6, ..., 18 } is a finite set consisting of all natural

Correct

set consisting of all natural numbers that are even and less than 20.

Reset

Feedback?

CHALLENGE ACTIVITY | 2.1.1: Cardinality and equality of sets.

422102.2723990.qx3zqy7

Jump to level 1

1
2
3
4

2

1	2	3	4

Check Next **Done**. Click any level to practice more. Completion is preserved.

Expected:

has elements, so .

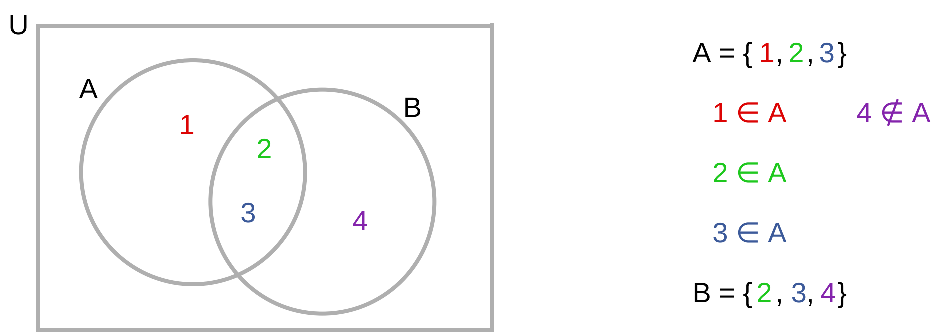
Feedback?

The ***universal set***, usually denoted by the variable *U*, is a set that contains all elements mentioned in a particular context. For example, in a discussion about certain types of real numbers, it would be understood that any element in the discussion is a real number. In this case, the universal set would be the set of real numbers. In a discussion regarding the academic standing of certain students at a school, the universal set would be the set of all students enrolled in the school.

Sets are often represented pictorially with ***Venn diagrams***. A rectangle is used to denote the universal set *U*, and oval shapes are used to denote sets within *U*. Sometimes, Venn diagrams can indicate which specific elements are inside and outside the set. An element is drawn inside the oval if it is in the set represented by the oval. The following animation illustrates the use of Venn diagrams.

PARTICIPATION ACTIVITY | 2.1.7: Venn diagrams.

1 2 2x speed



If the set B = {2, 3, 4} is added to the Venn diagram, then the circle for B contains 2, 3, and 4 but does not contain 1.

Captions

1. A Venn diagram depicts set A = {1, 2, 3} as an oval (or circle) that contains 1, 2, and 3. The element 4 is outside the circle for A because 4 ∉ A.
2. If the set B = {2, 3, 4} is added to the Venn diagram, then the circle for B contains 2, 3, and 4 but does not contain 1.

Feedback?

If every element in A is also an element of B, then A is a ***subset*** of B, denoted as A ⊆ B. If there is an element of A that is not an element of B, then A is not a subset of B, denoted as A ⊈ B. If the universal set is *U*, then for every set A:

$$\emptyset \subseteq A \subseteq U$$

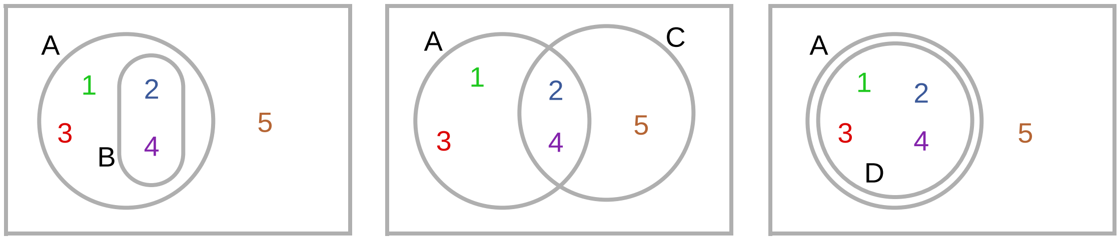
Two sets are equal if and only if each is a subset of the other:

$$A = B \text{ if and only if } A \subseteq B \text{ and } B \subseteq A$$

If A ⊆ B and there is an element of B that is not an element of A (i.e., A ≠ B), then A is a ***proper subset*** of B, denoted as A ⊂ B. Venn diagrams are particularly useful for visualizing subset relationships between sets.

PARTICIPATION ACTIVITY | 2.1.8: Venn diagrams with subsets.

1 2 3 2x speed



A = { 1, 2, 3, 4}

B = { 2, 4}

B A

3 A 3 B

B A

A = { 1, 2, 3, 4}

C = { 2, 4, 5}

5 C 5 A

C A

A = { 1, 2, 3, 4}

D = { 1, 2, 3, 4}

A D, D A A = D

If D = {1, 2, 3, 4}, D A and A D, which implies that A = D. D is not a proper subset of A (D A).

Captions

1. B = {2, 4} is a subset of A = {1, 2, 3, 4}. In a Venn diagram, the oval for B is inside the circle for A. Since 3 A and 3 B, B is a proper subset of A (B A).
2. C = {2, 4, 5} is not a subset of A (C A) because 5 C and 5 A. In the Venn diagram, the circle for C is not contained in the circle for A.
3. If D = {1, 2, 3, 4}, D A and A D, which implies that A = D. D is not a proper subset of A (D A).

Feedback?

PARTICIPATION ACTIVITY 2.1.9: Sets and subsets.

Consider the following sets:

A = { 3, 4, 5 }
B = { 4, 5, 3 }
C = { x ∈ **Z**: x is odd }
D = { 3, 5, 7, 9 }

1) $A \subset B$?

- Yes
- No

Correct

$A \subset B$ if and only if $A \subseteq B$ and $A \neq B$. Since A = B, A is not a proper subset of B.

2) $A \subseteq B$?

Yes

No

Correct

Every element in A is also an element of B. Therefore A ⊆ B.

3) C ⊂ **Z**?

Yes

No

Correct

The set of odd integers is a subset of the set of all integers. C is a proper subset of the integers because the even integers are not in C.

4) B ⊆ D?

Yes

No

Correct

4 ∈ B, but 4 ∉ D.

5) D ⊆ C?

Yes

No

Correct

Every element in D is an odd integer.

6) **N** ⊂ **Z** ⊂ **Q** ⊂ **R** ?

Yes

No

Correct

Natural numbers are a proper subset of integers. Integers are a proper subset of rational numbers. Rational numbers are a proper subset of real numbers.

7) Is the following statement true?
For any two sets, X and Y, if X ⊂ Y, then X ⊆ Y.

Yes

No

Correct

If X ⊂ Y, then every element of X is also an element of Y, so X ⊆ Y.

8) Is the following statement true?
For any two sets, X and Y, if X ⊆ Y, then X ⊂ Y.

Yes

No

Correct

It is possible that X = Y in which case X ⊆ Y, but X ⊄ Y.

Feedback?

Additional exercises

Additional exercises

EXERCISE 2.1.1: Set membership and subsets - true or false.

Use the definitions for the sets given below to determine whether each statement is true or false:

A = { x ∈ **Z**: x is an integer multiple of 3 }
B = { x ∈ **Z**: x is a perfect square }
C = { 4, 5, 9, 10 }
D = { 2, 4, 11, 14 }
E = { 3, 6, 9 }
F = { 4, 6, 16 }

An integer x is a perfect square if there is an integer y such that $x = y^2$.

(a) 27 ∈ A

(b) 27 ∈ B

(c) 100 ∈ B.

(d) E ⊆ C or C ⊆ E.

(e) E ⊆ A

(f) A ⊂ E

(g) E ∈ A

Feedback?

EXERCISE 2.1.2: Set membership and subsets - true or false, cont.

Use the definitions for the sets given below to determine whether each statement is true or false:

A = { x ∈ **Z**: x is an integer multiple of 3 }
B = { x ∈ **Z**: x is a perfect square }
C = { 4, 5, 9, 10 }
D = { 2, 4, 11, 14 }
E = { 3, 6, 9 }
F = { 4, 6, 16 }

An integer x is a perfect square if there is an integer y such that $x = y^2$.

(a) $15 \subset A$

(b) $\{15\} \subset A$

(c) $\emptyset \subset A$

(d) $A \subseteq A$

(e) $\emptyset \in B$

(f) A is an infinite set.

(g) B is a finite set.

(h) $|E| = 3$

(i) $|E| = |F|$

Feedback?

EXERCISE 2.1.3: Subset relationships between common numerical sets.

Indicate whether the statement is true or false.

(a) $\mathbf{Z} \subset \mathbf{R}$

(b) $\mathbf{Z} \subseteq \mathbf{R}$

(c) $\mathbf{Z} \subseteq \mathbf{R}^+$

(d) $\mathbf{N} \subset \mathbf{R}$

(e) $\mathbf{Z}^+ \subset \mathbf{N}$

Feedback?

EXERCISE 2.1.4: Sets, subsets, and set equality.

Define sets A, B and C as follows:

$A = \{2, 4, 6, 8\}$

$B = \{x \in \mathbf{Z}: x \text{ is even and } 0 < x < 10\}$

B = {x ∈ **Z**: x is even and 0 < x < 10}
C = {x ∈ **Z**: x is even and 0 < x ≤ 10}

Indicate whether each statement about the sets A, B and C is true or false.

(a) $A \subseteq B$

(b) $A \subset B$

(c) $A \subseteq C$

(d) $A \subset C$

(e) $C \subseteq B$

(f) $A = C$

(g) $A = B$

Feedback?

EXERCISE 2.1.5: Expressing sets in set builder notation.

Express each set using set builder notation. Then if the set is finite, give its cardinality. Otherwise, indicate that the set is infinite.

(a) {-2, -1, 0, 1, 2}

(b) { 3, 6, 9, 12, ... }

(c) { -3, -1, 1, 3, 5, 7, 9 }

(d) { 0, 10, 20, 30, ..., 1000 }

Feedback?

EXERCISE 2.1.6: Which statements are true for all sets?

Determine whether each statement is true or false for any two sets A and B. If the statement is false, explain why.

(a) If $A \subseteq B$, then $A \subset B$.

(b) If $A \subset B$, then $A \subseteq B$.

(c) If A = B, then $A \subseteq B$.

(d) If A = B, then $A \subset B$.

(e) If $A \subset B$, then $A \neq B$.

Feedback?

How was this section?

Provide feedback

Activity summary for assignment: Reading Assignment 2 30 / 105 pts

No due date

30 / 105 pts submitted to canvas

Completion details