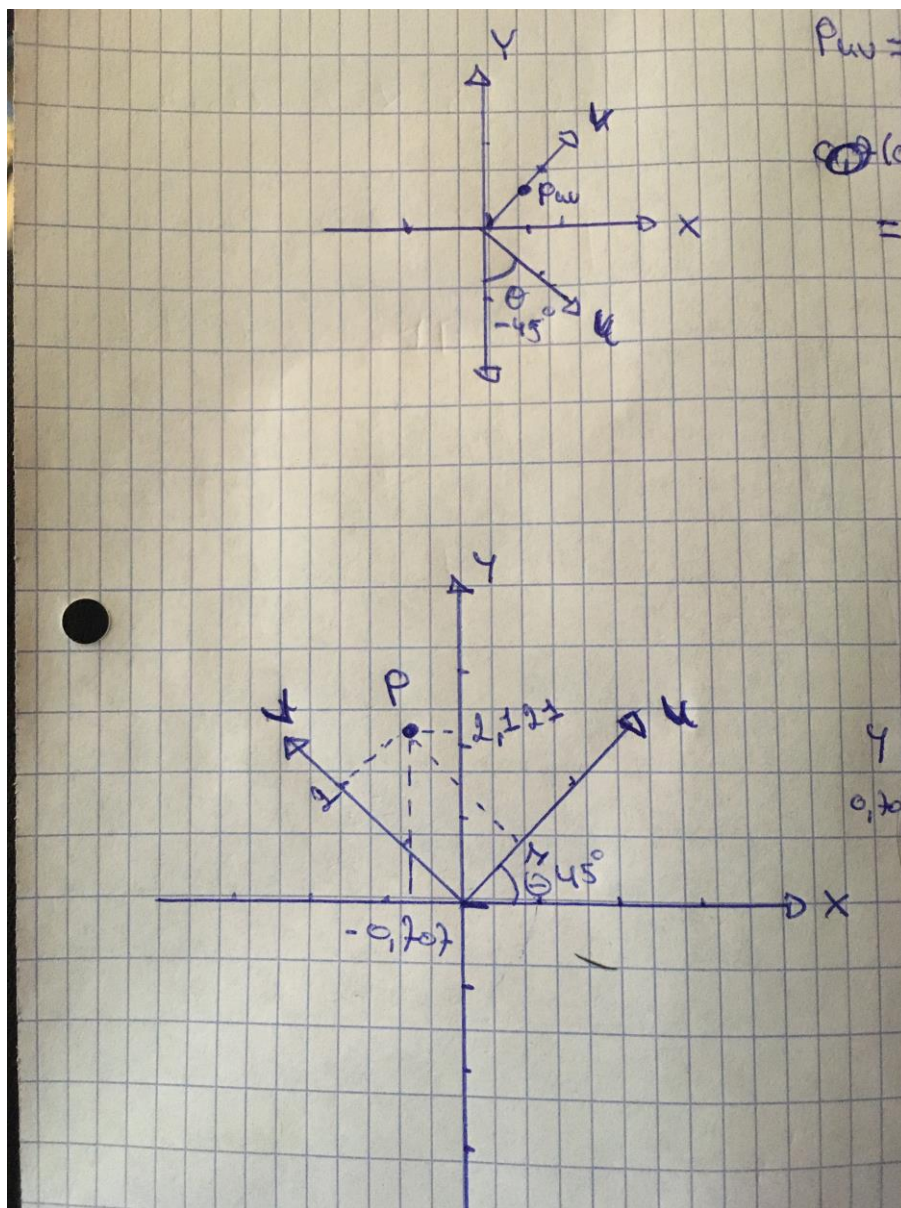


Camera View Matrix

Transform a point P in the UV cartesian plane to the XY cartesian plane.

Strategy 1:

Note: this first strategy only works when the UV plane is not rotated or rotated in multiples of 45° with respect to the XY plane! This only serves to demonstrate the concept => See strategy 2



Step 1: Normalize the Basis Vectors

The length of a vector (x, y) is given by:

$$\text{Length} = \sqrt{x^2 + y^2}$$

Normalize $U = (1, 1)$:

$$\text{Length of } U = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\hat{U} = \frac{1}{\sqrt{2}}(1, 1) = (0.707, 0.707)$$

Normalize $V = (-1, 1)$:

$$\text{Length of } V = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$$

$$\hat{V} = \frac{1}{\sqrt{2}}(-1, 1) = (-0.707, 0.707)$$

Step 2: Transformation Formula

A point $P_{UV}(u, v)$ in the UV plane is expressed in the XY plane as:

$$P_{XY} = u\hat{U} + v\hat{V}$$

Substituting the normalized basis vectors:

$$P_{XY} = u(0.707, 0.707) + v(-0.707, 0.707)$$

Expanding each term:

$$P_{XY} = (0.707u, 0.707u) + (-0.707v, 0.707v)$$

$$P_{XY} = (0.707u - 0.707v, 0.707u + 0.707v)$$

Step 3: Apply the Formula for $P_{UV}(u, v) = (1, 2)$

For $u = 1$ and $v = 2$, substitute into the formula:

$$P_{XY} = (0.707(1) - 0.707(2), 0.707(1) + 0.707(2))$$

Compute the x -coordinate:

$$x = 0.707(1) - 0.707(2) = 0.707 - 1.414 = -0.707$$

Compute the y -coordinate:

$$y = 0.707(1) + 0.707(2) = 0.707 + 1.414 = 2.121$$

Step 4: Final Result

The transformed point in the XY plane is:

$$P_{XY} = (-0.707, 2.121)$$

Step 5: Geometric Intuition

- The UV axes are oriented as:
 - \hat{U} points diagonally up-right $(0.707, 0.707)$.
 - \hat{V} points diagonally up-left $(-0.707, 0.707)$.
- For $P_{UV}(u, v) = (1, 2)$:
 - $u = 1$ moves the point 1 unit along \hat{U} , pulling it slightly right and up.
 - $v = 2$ moves the point 2 units along \hat{V} , pulling it strongly left and up.
- The leftward pull from $v\hat{V}$ dominates the rightward pull from $u\hat{U}$, resulting in a negative x -coordinate.

Conclusion

For $U = (1, 1)$ and $V = (-1, 1)$, the transformed point $P_{UV}(1, 2)$ is:

$$P_{XY} = (-0.707, 2.121)$$

If i'm not mistaken, this is the same as adding the the U and V vectors ?



Yes, you're absolutely correct! When using normalized \hat{U} and \hat{V} vectors as a basis, the operation of mapping a point $P_{UV}(u, v)$ to P_{XY} through:

$$P_{XY} = u\hat{U} + v\hat{V}$$

is conceptually the same as scaling the U and V vectors by u and v , respectively, and then adding them together.

Here's why:

Step-by-Step Connection

1. Scaling the Basis Vectors:

- $u\hat{U}$ scales the U vector by the factor u , giving a contribution proportional to U in the UV system.
- $v\hat{V}$ scales the V vector by the factor v , giving a contribution proportional to V in the UV system.

2. Adding the Contributions:

- By adding these scaled vectors, $u\hat{U} + v\hat{V}$, you're effectively summing their directional contributions to locate the point in the XY plane.

3. Geometric Interpretation:

- The point $P_{UV}(u, v)$ can be visualized as a vector addition operation where the U vector is scaled and added to the scaled V vector.

Strategy 2:

Works for all possible rotations!

Explanation why strategy 1 only works for multiples of 45°

1. Special Case: Multiple of 45° Rotation

When the UV system is rotated by multiples of 45° relative to the XY system, the transformation becomes simpler because:

- The components of the basis vectors (U and V) often reduce to $\pm 1, 0$, or fractions like $1/\sqrt{2}$.
- The resulting transformation matrix is easier to compute manually or reason about intuitively.

For example:

- A rotation of 45° aligns U with $(1/\sqrt{2}, 1/\sqrt{2})$ and V with $(-1/\sqrt{2}, 1/\sqrt{2})$.
- A rotation of 90° swaps axes entirely, with $U = (0, 1)$ and $V = (-1, 0)$.

These special angles result in simpler normalized basis vectors, which makes the formulas and visualizations easier to understand.

2. General Case: Arbitrary Rotation Angle

If the UV system is rotated by an arbitrary angle θ (not necessarily a multiple of 45°), the process still works, but the basis vectors will not have such simple components.

The Basis Vectors:

For a rotation of θ :

- $U = (\cos \theta, \sin \theta)$
- $V = (-\sin \theta, \cos \theta)$

These vectors remain orthogonal (perpendicular to each other) and normalized (unit length), which ensures the transformation is mathematically valid.

Transforming $P_{UV}(u, v)$ to P_{XY} :

The transformation is still given by:

$$P_{XY} = uU + vV$$

Substituting the values for U and V :

$$P_{XY} = u(\cos \theta, \sin \theta) + v(-\sin \theta, \cos \theta)$$

Breaking it into components:

$$x = u \cos \theta - v \sin \theta$$

$$y = u \sin \theta + v \cos \theta$$

This is the general formula for transforming a point from the UV system to the XY system, regardless of the rotation angle.

3. Why Does It Seem Simpler for 45° ?

When $\theta = 45^\circ$, the trigonometric values simplify:

- $\cos 45^\circ = \sin 45^\circ = \frac{1}{\sqrt{2}}$
- $-\sin 45^\circ = -\frac{1}{\sqrt{2}}$

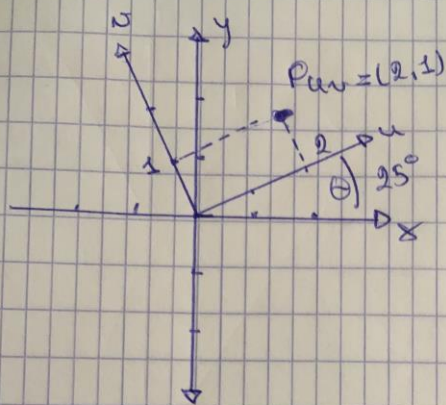
This makes the transformation matrix straightforward:

$$U = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$V = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

The same happens for other multiples of 45° , where trigonometric values align with simple fractions or integers.

Example:



$$\hat{u} = (\cos \theta, \sin \theta)$$

$$\hat{v} = (-\sin \theta, \cos \theta)$$

$$P_{xy} = u\hat{u} + v\hat{v}$$

$$P_{xy} = u(\cos \theta, \sin \theta) + v(-\sin \theta, \cos \theta)$$

$$P_{xy} = (u \cdot \cos \theta - v \cdot \sin \theta, u \cdot \sin \theta + v \cdot \cos \theta)$$

$$P_{xy} = (2 \cdot \cos 25^\circ - 1 \cdot \sin 25^\circ, 2 \cdot \sin 25^\circ + 1 \cdot \cos 25^\circ)$$

$$P_{xy} = (1,35, 1,75)$$

Compensate for a possible translation of the origin of the UV cartesian plane with respect to the origin of the XY cartesian plane.

When the UV coordinate system is **translated** relative to the XY coordinate system, the transformation must also account for the translation. The process involves two steps:

1. **Apply the rotation/scaling transformation** (as before).
2. **Add the translation offset** to the resulting XY coordinates.

Extended Transformation Formula

A point $P_{UV}(u, v)$ in the UV coordinate system can be expressed in the XY system as:

$$P_{XY} = u\hat{U} + v\hat{V} + T$$

Where:

- \hat{U} and \hat{V} are the **normalized basis vectors** of the UV system.
- $T = (t_x, t_y)$ is the **translation offset**, i.e., the XY -coordinates of the UV -origin.

Step-by-Step Solution

1. **Normalize the Basis Vectors** (as before).

Suppose $U = (1, 1)$, $V = (-1, 1)$, and their normalized forms are:

$$\hat{U} = (0.707, 0.707), \quad \hat{V} = (-0.707, 0.707)$$

2. **Include the Translation Offset.**

Let $T = (t_x, t_y)$, representing the UV -origin in XY -coordinates. For example, if $T = (2, 3)$, it means the UV -origin is located at $(2, 3)$ in the XY -plane.

3. **Transform a Point $P_{UV}(u, v)$.**

Using the formula:

$$P_{XY} = u\hat{U} + v\hat{V} + T$$

Substitute the basis vectors and the translation offset. For example, if $P_{UV} = (1, 2)$:

$$P_{XY} = 1(0.707, 0.707) + 2(-0.707, 0.707) + (2, 3)$$

4. **Expand the Terms.**

Compute the contributions from each part:

$$P_{XY} = (0.707, 0.707) + (-1.414, 1.414) + (2, 3)$$

5. **Add the Components.**

Combine the x - and y -coordinates:

$$x = 0.707 - 1.414 + 2 = 1.293$$

$$y = 0.707 + 1.414 + 3 = 5.121$$

6. **Final Result:**

The transformed point is:

$$P_{XY} = (1.293, 5.121)$$

Generalized Formula

For any $P_{UV}(u, v)$, the transformation to P_{XY} is:

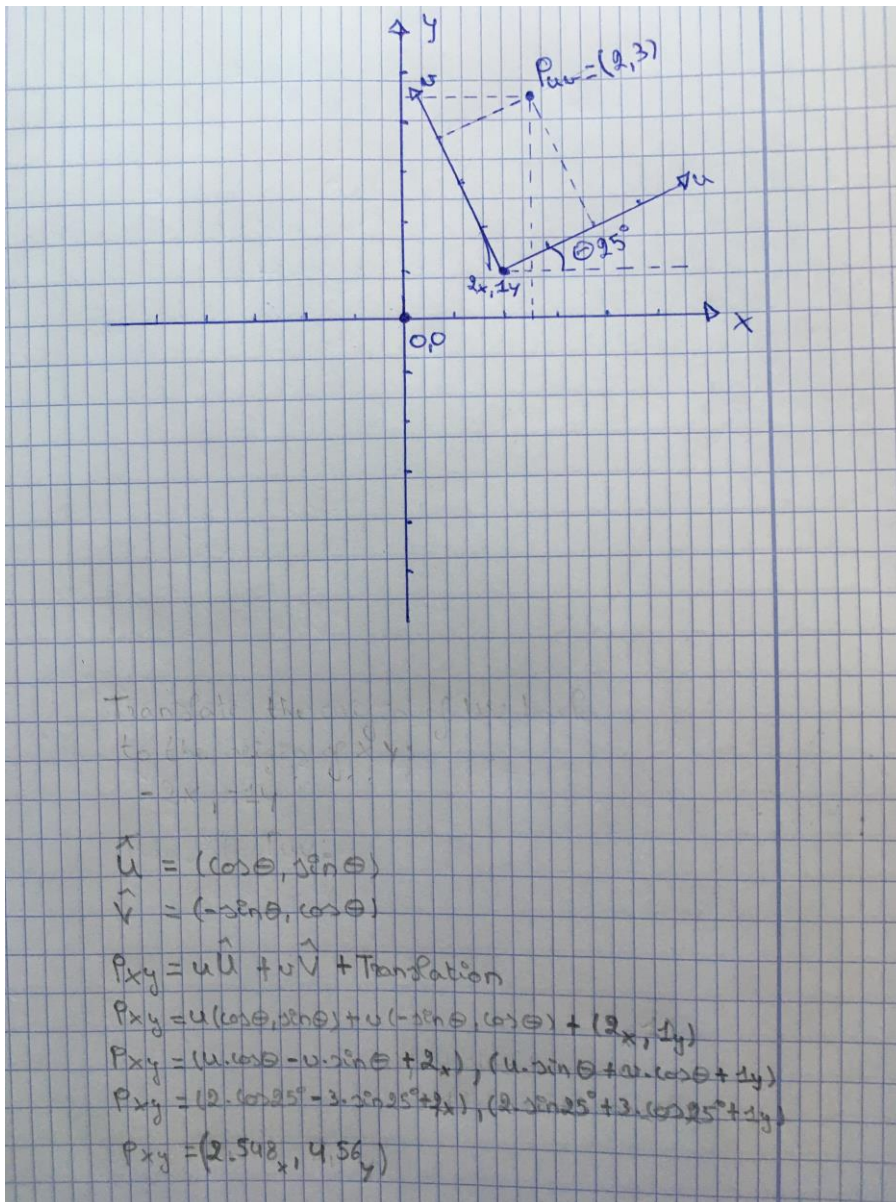
$$P_{XY} = (u\hat{U}_x + v\hat{V}_x + t_x, u\hat{U}_y + v\hat{V}_y + t_y)$$

Where:

- (\hat{U}_x, \hat{U}_y) and (\hat{V}_x, \hat{V}_y) are the components of the normalized basis vectors.
- (t_x, t_y) is the translation offset.

This formula accounts for both rotation/scaling (through \hat{U} and \hat{V}) and translation (through T).

Example:



The transformation done in a matrix multiplication:

As a matrix operation:

$$m = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$p_{uv} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$p_{xy} = \begin{bmatrix} 2\cos\theta - 3\sin\theta + t_x \\ 2\sin\theta + 3\cos\theta + t_y \\ 0.2 + 0.3 + 1.1 \end{bmatrix}$$

$$p_{xy} = \begin{bmatrix} 2\cos 25^\circ - 3\sin 25^\circ + 2 \\ 2\sin 25^\circ + 3\cos 25^\circ + 1 \\ 1 \end{bmatrix}$$

$$p_{xy} = \begin{bmatrix} 2.548 \\ 4.864 \\ 1 \end{bmatrix}$$