MOTO ESPONENZIALE

$$F = m\vec{a}$$

$$m\frac{dv}{dt} = -b\vec{v}$$

$$\frac{dv}{dt} = -\frac{bv}{m}$$

$$\frac{dv}{v} = -\frac{b}{m}\,dt$$

$$\int_{v_i}^{v} \frac{1}{v}\,dv = -\frac{b}{m}\int_{t_i}^{t} dt \qquad t_i = 0$$

$$\ln\frac{v}{v_0} = -\frac{b}{m}t + \frac{b}{m}t_i$$

$$\frac{V}{V_0} = e^{-\frac{b}{m}t} \qquad \frac{m}{b} = \tau$$

$$\frac{V}{V_0} = e^{-\frac{t}{\tau}}$$

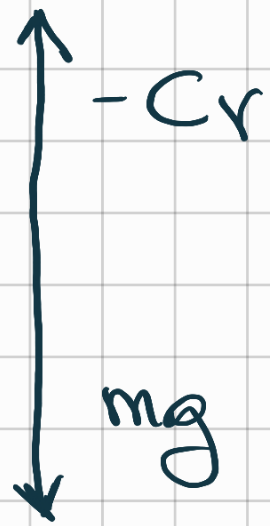
$$V_0 \neq 0$$

$$x(t) = \int v$$

$$= V_0 \int_{t_0}^{t} e^{-\frac{t}{\tau}}\, dt = V_0\left(-\tau e^{-t/\tau}\right)$$

$$= V_0\left(\tau e^{-\frac{t}{\tau}} + \tau\right) = V_0 \tau\left(1 - e^{t/\tau}\right)$$

CADUTA DI UN GRAVE CON ATTRITO



$$\vec{F} = m\vec{g} - b\vec{v}$$

$$m\vec{a} = m\vec{g} - b\vec{v}$$

$$m\frac{dv}{dt} = m\vec{g} - b\vec{v}$$

esprimo l'accelerazione come derivata di v

$$\frac{dv}{dt} = g - \frac{b}{m}v \Rightarrow \frac{dv}{dt} = g - \frac{v}{\tau} \qquad \tau = \frac{m}{b}$$

moltiplico per dt

$$dv = \left(g - \frac{b}{m} v\right) dt$$

$$\frac{dv}{g - \frac{b}{m} v} = dt$$

$$\int_{v_0}^{v} \frac{dv}{g - \frac{v}{\tau}} = \int_0^t dt$$

$$g - \frac{v}{\tau} = k$$

$$d\left(g - \frac{v}{\tau}\right) = dk$$

$$\downarrow$$

$$-\frac{1}{\tau} dv = dk$$

$$dv = -\tau dk$$

$$\int^{k} -\tau dk = \int^{t} dt$$

$$\int_{K_0} \frac{-\tau\, dK}{K} = \int_{t_0} dt$$

$$K = g - \frac{V}{\tau}$$

estremo sup è V_f

$$K = g - \frac{V_f}{g}$$

estremo inf $\Rightarrow V = 0$

$$K = g - 0$$

$$\int_{g}^{g-\frac{V}{\tau}} \frac{-\tau\, dK}{K} = \int_{t_0}^{t} dt$$

$$-\tau \int_{g}^{g-V/\tau} \frac{dK}{K} = \int_{t_0}^{t} dt$$

$$\Downarrow$$

$$-\tau \quad \ln\left(\frac{g-v/\tau}{g}\right) = \frac{1}{t}$$

$$\hookrightarrow \ln\left(\frac{g-v/\tau}{g}\right) = -\tau/$$

$$\downarrow$$

esponenziali

$$\frac{g-v/\tau}{g} = e^{-t/\tau}$$

$$g - v/\tau = g e^{-t/\tau}$$

$$-v/\tau = g e^{-t/\tau} - g$$

$$v = \left(g e^{-t/\tau} - g\right)(-\tau)$$

$$= -\tau g e^{-t/\tau} + g\tau$$

$$= g\tau\left(1 - e^{-t/\tau}\right)$$

$$x(t) = \int_0^t g\tau \, dt - \int_0^t g\tau e^{-t/\tau} \, dt$$

$$= g\tau \cdot t - (g\tau)(-\tau e^{-t/\tau}) + g\tau(-\tau)$$

$$= g\tau t + \underbrace{g\tau^2 e^{-t/\tau} - g\tau^2}_{}$$

$$g\tau$$

$$t - \left(1 - e^{-t/\tau}\right)$$

In 2 dimensioni

$$\begin{cases} m\frac{dv_x}{dt} = -b v_x \end{cases} \qquad v_{x0} = 0$$

$$\frac{d}{dt}$$ $V_{0y} = 0$

$V_{0x} = 0$

$$m\frac{dV_y}{dt} = -bV_y + mg$$

$$\frac{dV_x}{dt} = -\frac{b}{m}V_x \Rightarrow \frac{dV_x}{V_x} = -\frac{b}{m}dt$$

$$\Rightarrow \ln\left(\frac{V_2}{V_1}\right) = -\frac{1}{\tau}t_2$$

$$\frac{V_2}{V_1} = e^{\frac{t_2}{\tau}}$$

$$V_2 = V_1 e^{-t/\tau}$$

lungo y

$$m\frac{dV_y}{dt} = -bV_y + mg$$

$$\frac{dV_y}{dt} = -\frac{b}{m} + g \Rightarrow$$

$$= -\tau \ln\left(\frac{g - v/\tau}{g}\right) = \frac{1}{t}$$

$$= \ln\left(\frac{g - v/\tau}{g}\right) = -\frac{t}{\tau}$$

$$= \frac{g - v/\tau}{g} = e^{-t/\tau}$$

$$g - v/\tau = g e^{-t/\tau}$$

$$-v/\tau = g e^{-t/\tau} - g$$

$$-v/\tau = g\left(e^{-t/\tau} - 1\right)$$

$$v/\tau = g\left(1 - e^{-t/\tau}\right)$$

$$v = g\tau\left(1 - e^{-t/\tau}\right)$$

$$v_y = g\tau(1-e^{-t/\tau})$$

$$y = \int_{t_1}^{t_2} g\tau(1-e^{-t/\tau})$$

$$= \int g\tau - \int g\tau e^{-t/\tau}$$

$$g\tau t - g\tau \int 1 e^{-t/\tau}$$

$$-\frac{t}{\tau} = k$$

$$\frac{dt}{-\tau} = dk$$

$$dt = -\tau dk$$

$$g\tau t - g\tau \int_0^{-t/\tau} e^{k} - \tau dk$$

$$g\tau^2 \cdot e^{k}\Big|_0^{-t/\tau} = g\tau^2\left(e^{-t/\tau}-1\right)$$

RIVEDI INTEGRALI