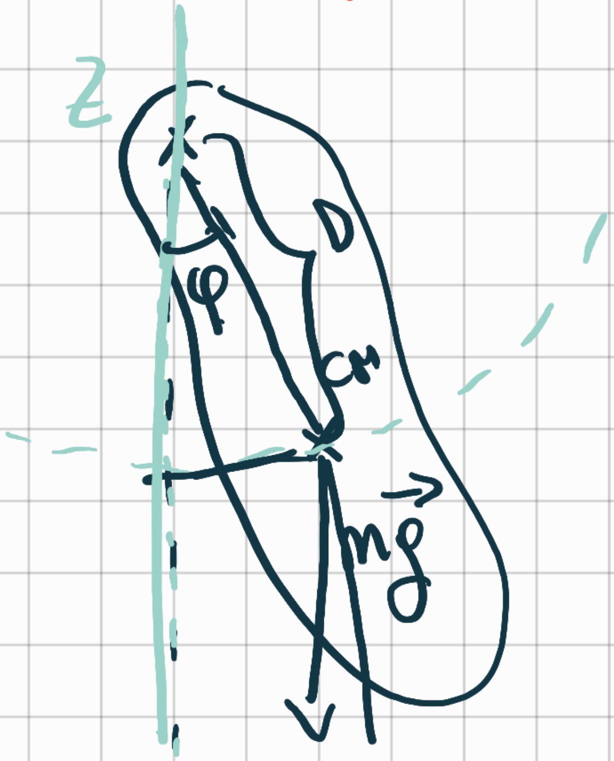
PENDOLO COMPOSTO

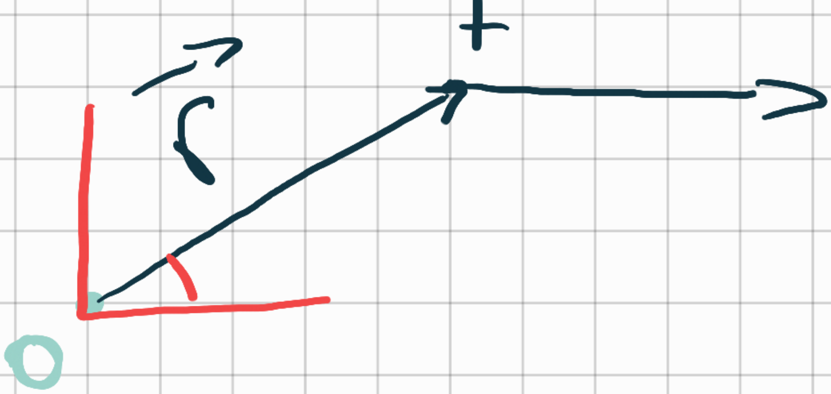


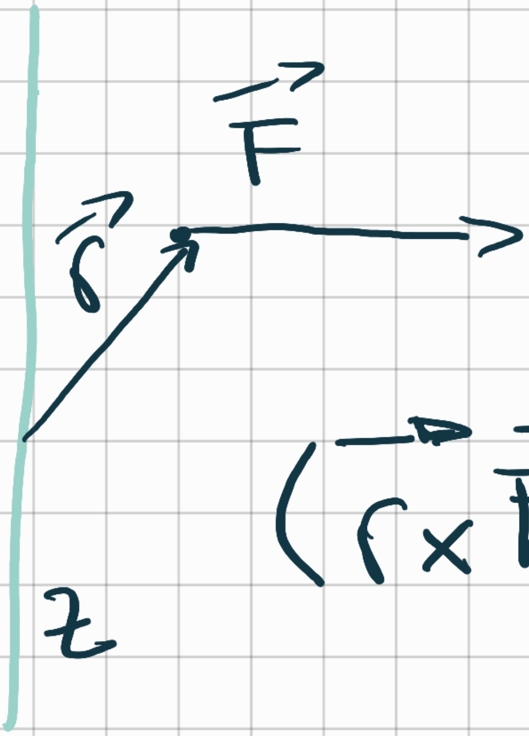
$$\sum_{i=1}^{n} \vec{F}_i^{(e)} = m\vec{a}_{cm}$$

$$\vec{R} + m\vec{g} = m\vec{a}_{cm}$$

$$\tau_z^{(e)} = \dot{L}_z^{(e)}$$

$$\vec{\tau}_o^{(e)} = \dot{\vec{L}}_o^{(e)}$$





$$(\vec{r} \times \vec{F}) \cdot \hat{k}$$

versore dell'asse z

$$\tau_{m\vec{g}} = (\vec{r} \times m\vec{g}) \cdot \hat{k}$$

$$\vec{L} = \vec{r} \times m\vec{v}$$

$$L_z = (\vec{r} \times m\vec{v}) \cdot \hat{k}$$

$$\vec{L} = \sum \vec{r} \times m\vec{v}$$

$$\downarrow$$

ωR

$$\downarrow$$

$$\dot{\varphi}$$

$$L_z = (\vec{r} \times m\dot{\vec{\omega}}\vec{R}) \cdot \hat{k}$$

RIVEDI

$$(r^2 \times m\dot{\omega}) \cdot \hat{k}$$

$$(mr^2 \times \omega) \cdot \hat{k}$$

$$mr^2 \times \dot{\vec{\omega}} \cdot \hat{k}$$

$$mr^2 \times \ddot{\varphi} \cdot \hat{k}$$

$$\sum_{i=1}^{n} m_i r_i^2 = I_z$$

$$I_z \ddot{\varphi} = \dot{L}_z$$

$$I_z \ddot{\varphi} = (2 \times mg) \cdot k$$

$$I_z \ddot{\varphi} = -D\, mg \sin\varphi$$

$$\ddot{\varphi} = -\frac{mgD}{I_z} \sin\varphi$$

$$\sin\varphi \simeq \varphi$$

$$\ddot{\varphi} = -\frac{mgD}{I_z} \varphi$$

$\to \omega^2$

$$\ddot{\varphi} = -\omega^2 \varphi$$

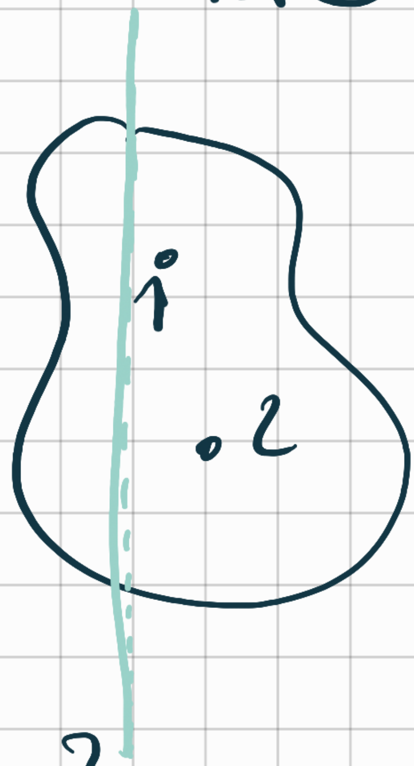
$$\varphi(t) = \varphi \cos(\omega t)$$

$\hookrightarrow \frac{2\pi}{T}$

$$\frac{2\pi}{T} = \sqrt{\frac{MGD}{I_z}}$$

$$T = 2\pi\sqrt{\frac{I_z}{MGD}}$$

MOTO CORPO RIGIDO



$$m_1 = \frac{m_1 v^2}{2} \qquad m_2 = \frac{m_2 v^2}{2}$$

$$\sum_i \frac{m_i v_i^2}{2} =$$

$$v = \omega R$$

$$\sum_i \frac{m_i \omega^2 R_i^2}{2} = \frac{1}{2}\omega^2 \underbrace{\sum m_i R_i^2}_{I_z}$$

R sono calcolati rispetto all'asse z

$$L_z = \sum_i \vec{l_i} \sin\varphi$$

$v = \omega r$

$$= \sum_i r_i \cdot m_i \cdot v_i \sin\varphi$$

$$= \sum_i r_i \cdot m_i \cdot \omega r_i = \sum r_i^2 m \omega$$

$$= \omega \underbrace{\sum_i m_i r_i^2}_{I_z}$$

TH Forze Vive

$$\vec{F} = m\vec{a}$$

$$\vec{F} = m\frac{d\vec{v}}{dt}$$

$$\longrightarrow x$$

$$F_x = m\frac{dV_x}{dt}$$

$$F_x = m\frac{dV_x}{dt}\cdot\frac{dx}{dx} \qquad \frac{dx}{dt} = V_x$$

$$\frac{}{dt} \quad \frac{}{dx}$$

$$\vec{F}_x = m \frac{dV_x}{dx} \cdot v_x$$

$$\vec{F}_x \, d\vec{x} = m \quad V_x \, dV_x$$

$$\int_{x_1}^{x_2} \vec{F}_x \cdot d\vec{x} = m \int_{V_1}^{V_2} V_x \, dV_x$$

$$\int_{x_1}^{x_2} \vec{F}_x \cdot d\vec{x} = \frac{m V_{x_2}^2}{2} - \frac{m V_{x_1}^2}{2}$$

$dx \Rightarrow$ lavoro elementare

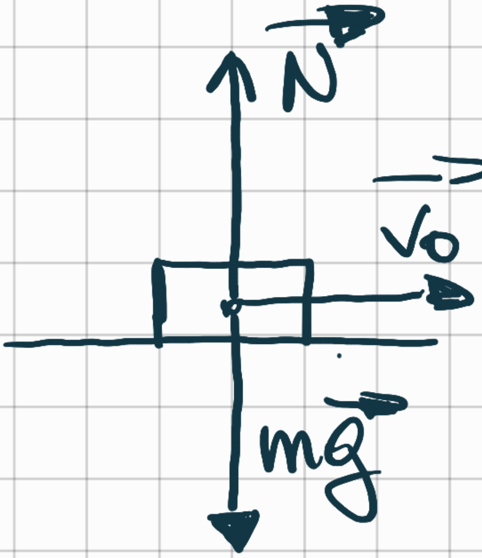
$$\vec{F} \quad \vec{} \quad F \quad |$$

$$F_x \cdot dx = F_x\, dx \cdot \cos\alpha$$

-180

$$= -F_x\, dx$$

$\alpha = 90$



$\vec{F} \cdot \vec{ds}$

$$\int_{y_1}^{y_2} \vec{F} \cdot \vec{ds} = \int_{y_1}^{y_2} m\vec{g} \cdot \vec{ds_y} = mgh \Big|_{y_1}^{y_2}$$

$$mgh_2 - mgh_1$$

$$\Delta U$$

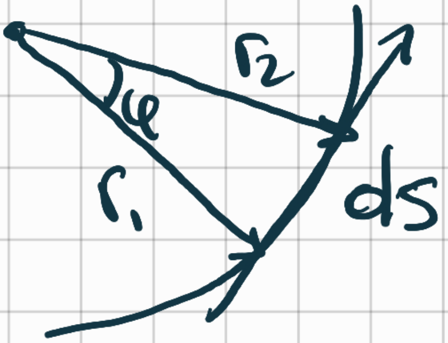
$$-\Delta U = mgh_1 - mgh_2$$

$$\frac{mv_2^2}{2} - \frac{mv_1^2}{2} = mgh_1 - mgh_2$$

$$\underbrace{mgh_1 + \frac{mv_1^2}{2}}_{E_M} = mgh_2 + \frac{mv_2^2}{2}$$

$$\Delta E_M = L_{NC}$$

MOMENTO DELLE FORZE

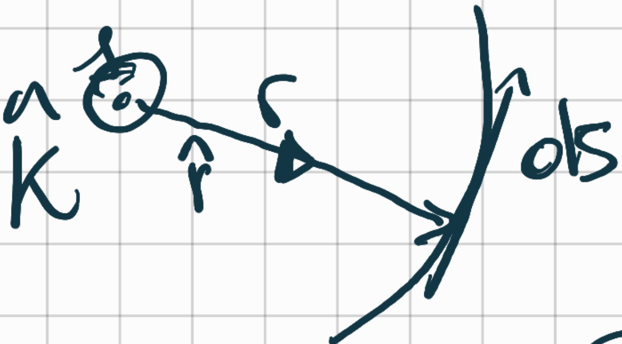


$$ds = r\,d\varphi$$

$$\int F\cdot ds = \int F\cdot r\,d\varphi$$

$$\hat{k}\times\hat{r} = \hat{s}$$

$$\uparrow U \qquad U\cdot\hat{U} = \vec{U}$$



ortogonale

$$d\vec{s} = \hat{k}\times\hat{r}\,ds$$

$$\int F\cdot r\,d\varphi = F\cdot\hat{k}\times\underbrace{\hat{r}\cdot r}_{\vec{r}}\,d\varphi$$

$$= \underline{F\cdot\hat{k}}\times\vec{r}\,d\varphi$$

$$= \hat{k}\cdot\vec{r}\times\vec{F}\,d\varphi$$

$$\underbrace{\tau_0^{(e)}}_{\tau_k^{(e)}}$$

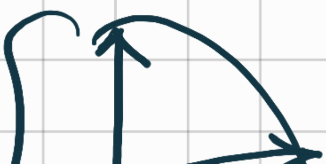
$$\vec{F} \cdot \hat{k} \times \vec{r}\, d\varphi = \vec{r} \cdot \vec{F} \times \hat{k}\, d\varphi$$
$$= \hat{k} \cdot (\vec{r} \times \vec{F})\, d\varphi$$

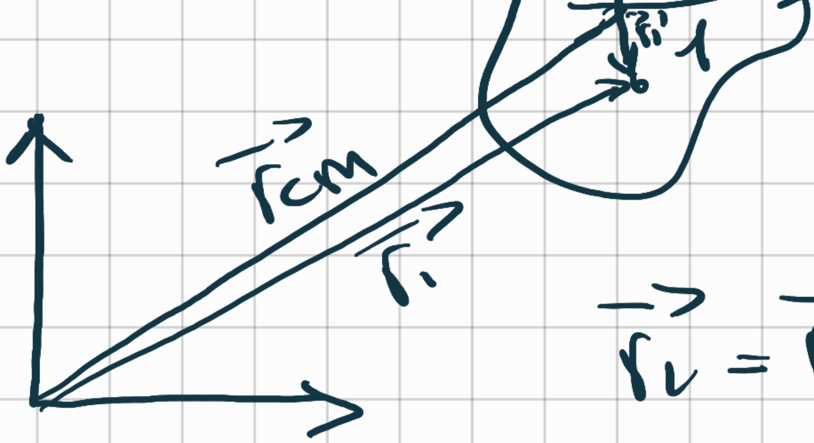
$[N \cdot m]$

I th König

$$\vec{L} = \vec{r} \times m\vec{v}$$

$$\sum_i \vec{r}_i \times m_i \vec{v}_i$$





$$\vec{r}_i = \vec{r}_{cm} + \vec{r}_i'$$

$$\vec{v}_i = \vec{v}_{cm} + \vec{v}_i'$$

$$\sum_i \vec{r}_i \times m_i \vec{v}_i =$$

$$\sum_i (\vec{r}_{cm} + \vec{r}_i') \times m_i (\vec{v}_{cm} + \vec{v}_i')$$

$$\sum \vec{r}_{cm} \times m_i \vec{v}_{cm} + \vec{r}_{cm} \times m_i \vec{v}_i' + \vec{r}_i' \times m_i \vec{v}_{cm} + \vec{r}_i' \times m_i \vec{v}_i'$$

$$\sum \vec{r}_{cm} \times m_i \vec{v}_{cm} = \vec{L}_{cm}$$

$$\sum \vec{r}_{cm} \times m_i \vec{v}_i' = \vec{r}_{cm} \times \sum m_i \vec{v}_i'$$

$$= \vec{r}_{cm} \times M \sum \frac{m_i \vec{v}_i'}{M} = 0$$

$$\sum \vec{r}_i' \times m_i \vec{v}_{cm} = M \sum \frac{\vec{r}_i' m_i}{M} \times \vec{v}_{cm}$$

$$\Downarrow$$

$$0$$

$$\sum \vec{r}_i' \times m_i \vec{v}_i' = \vec{L}'$$

$$\vec{L}_{TOT} = \vec{L}_{cm} + \vec{L}'$$

II König

$$E_c = \frac{1}{2} m v^2$$

$$\sum_i \frac{1}{2} m_i v_i^2$$

$$v_1^2 = \vec{v_1} \cdot \vec{v_1} = (\vec{v_i'} + \vec{v_{cm}})^2$$

$$= v_i'^2 + v_{cm}^2 + 2\vec{v_i'}\vec{v_{cm}}$$

$$\sum \frac{1}{2} m_i (v_i'^2 + v_{cm}^2 + 2\vec{v_i'}\vec{v_{cm}}) =$$

$$= \frac{1}{2}\sum m_i v_i'^2 + m_i v_{cm}^2 + 2 m_i \vec{v_i'}\vec{v_{cm}}$$

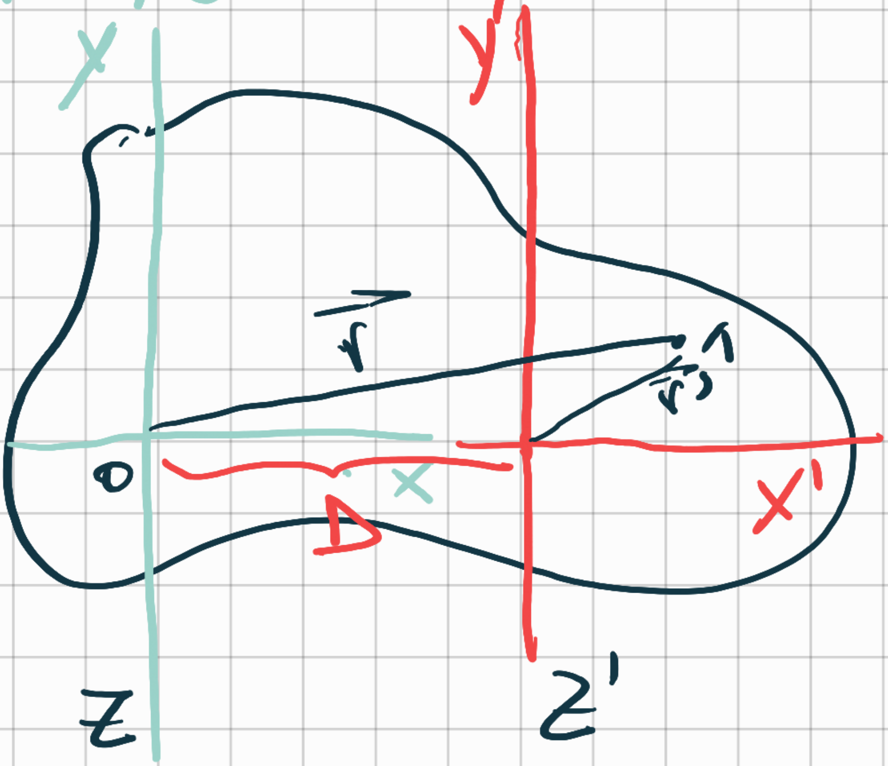
$$\frac{1}{2}\sum_i^{\infty} m_i v_i'^2 = E_c'$$

$$\frac{1}{2}\sum m_i v_{cm}^2 = E_{cm}$$

$$\cancel{\frac{1}{2}}\sum \cancel{2} m_i \vec{v_i'}\vec{v_{cm}} = M \underbrace{\sum \frac{m_i \vec{v_i'}}{M}}_{0} \cdot \vec{v_{cm}} = 0$$

$$E_{c\,TOT} = E_c' + E_{cm}$$

TH HUYGENS-STEINER



$$\vec{r} = \vec{r}\,' + D$$

$$x = x' + D$$
$$y' = y$$

$$r_1^2 = \left[(x'+D)^2 + y'^2\right] =$$

$$= \underline{x'^2} + D^2 + 2x'D + \underline{y'^2}$$

$$I_z = \sum_i m_i r_i^2$$

$$= \sum m_i \cdot \left(x'^2 + \Delta^2 + 2x'\Delta + y'^2\right) =$$

$$= \sum m_i \underbrace{\left(x'^2 + y'^2\right)}_{r_i^2} = I_z'$$

$$\sum m_i \Delta^2$$

$$\sum m\, 2x'\Delta = M \sum \frac{m_i x'}{M} 2\Delta$$

$\rightarrow 0$

$$I_z = I_z' + M\Delta^2$$

SISTEMI NON INERZIALI

ROTOTRASLAZIONE

uso sistema $O'x'y'$

$$\vec{r} = r_x + r_y = x \cdot \hat{i} + y \cdot \hat{j}$$

$$\vec{v}' = \dot{\vec{r}}' = (x \cdot \hat{i} + y \cdot \hat{j})^{\circ} = \dot{x}'\hat{i}' + x'\dot{\hat{i}}' + \dot{y}'\hat{j}' + y'\dot{\hat{j}}'$$

$$\dot{x}'\hat{i}' + \dot{y}'\hat{j}' = \vec{v}'$$

sono su un moto rotatorio
dunque ho $\dot{\hat{i}}' = \vec{\omega} \times \hat{i}'$

$\dot{\hat{j}}' = \vec{\omega} \times \hat{j}'$

$$x'\dot{\hat{i}}' + y\dot{\hat{j}} = x(\vec{\omega} \times \hat{i}) + y(\hat{\omega} \times \hat{j})$$

$$= \vec{\omega} \times (x' \cdot \hat{i}') + \vec{\omega} \times (y' \cdot \hat{j}') =$$

$$= \vec{\omega} \times (\vec{x} + \vec{y}) = \vec{\omega} \times \vec{r}$$

$$V = \vec{v}' + \vec{\omega} \times \vec{r}$$

$$\vec{a} = \dot{\vec{v}} = \dot{\vec{v}'} + (\dot{\vec{\omega} \times \vec{r}})$$

$$= \dot{\vec{v}'} + \vec{\omega} \times \dot{\vec{r}}$$

$$= \dot{\vec{v}'} + \vec{\omega} \times (\vec{v}' + \vec{\omega} \times \vec{r})$$

$$= \dot{\vec{v}'} + \vec{\omega} \times \vec{v}' + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$\vec{v}' = (x'\hat{\imath}' + y'\hat{\jmath}') = \dot{x}'\hat{\imath}' + x'\dot{\hat{\imath}}' + \dot{y}'\hat{\jmath}' + y'\dot{\hat{\jmath}}'$$

$$\dot{\vec{v}'} = \dot{\vec{v}'} + \dot{x}'\dot{\hat{\imath}}' + \dot{y}'\dot{\hat{\jmath}}' =$$

$$= \vec{a}' + \dot{x}\,(\vec{\omega} \times \hat{\imath}) + \dot{y}\,(\vec{\omega} \times \hat{\jmath}) =$$

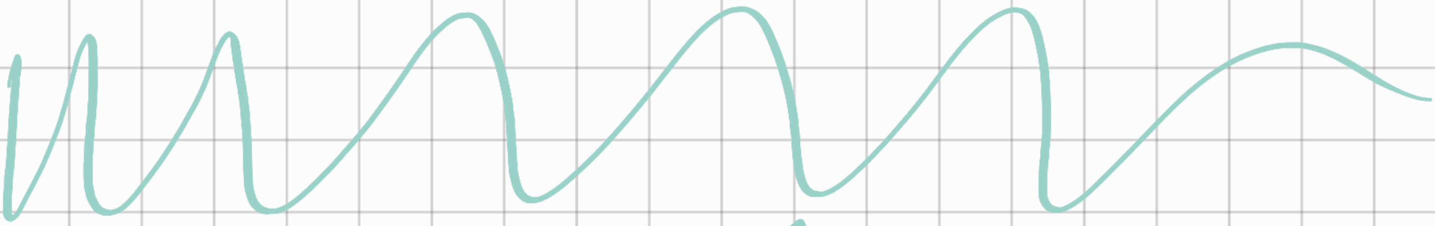
$$= \vec{a}' + \vec{\omega} \times (\dot{\vec{x}} \cdot \hat{\imath}) + \vec{\omega} \times (\dot{\vec{y}} \cdot \hat{\jmath})$$

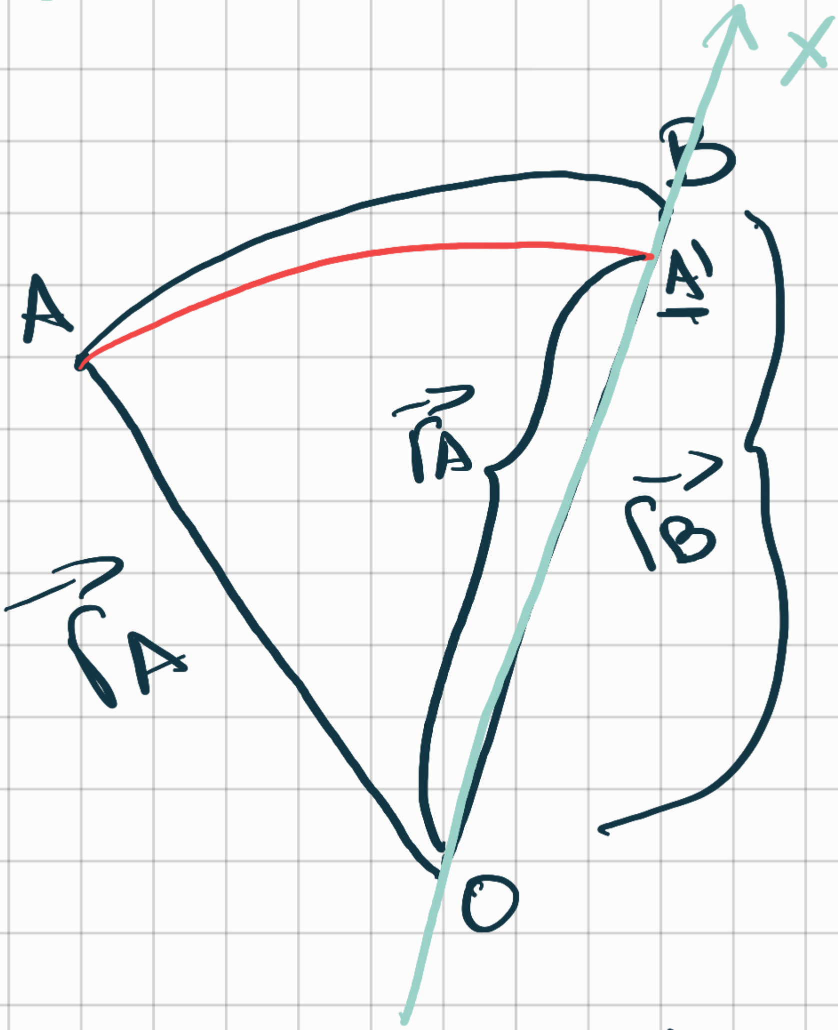
$$= \vec{a}' + \vec{\omega} \times \vec{v}'$$

$$\vec{\omega} \times \vec{v}' + \vec{\omega} \times \vec{v}' + \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \vec{a}'$$

$$\underbrace{2\vec{\omega} \times \vec{v}} + \underbrace{\vec{\omega} \times (\vec{\omega} \times \vec{r})} + \vec{a}'$$

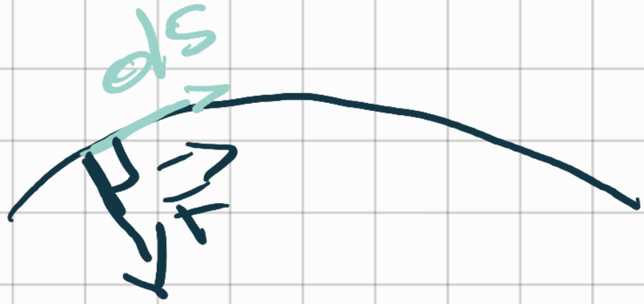
CORIOLIS CENTRIPETA





$$\vec{F} = \frac{k}{x^2} = \frac{-GmM}{r^2}$$

$$\int_A^B \vec{F}\cdot d\vec{s} = \int_A^{A'} \vec{F}\cdot d\vec{s} + \int_{A'}^B \vec{F}\cdot d\vec{s}$$

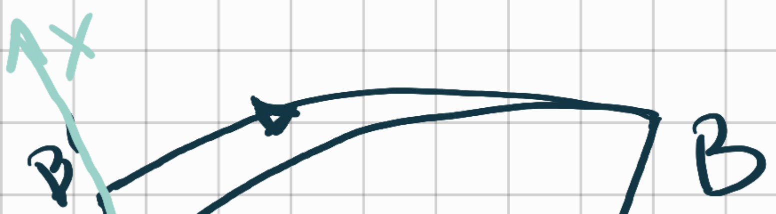


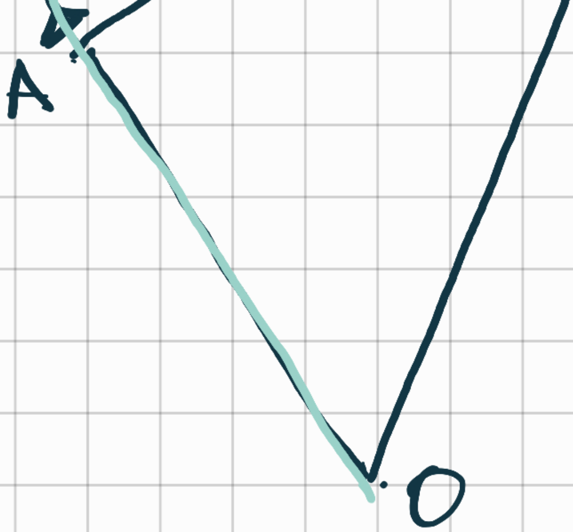
$$F \cdot ds \cdot \cos\alpha = 0$$

$$\int_A^B \vec{F} \cdot \vec{ds} = \int_{A'}^B \vec{F} \cdot \vec{ds} = \int_{A'}^B \frac{K}{x^2} dx$$

$$= \frac{K}{x}\Big|_{A'}^{B} = \frac{K}{x_B} - \frac{K}{x_{A'}}$$

$$= \frac{K}{r_B} - \frac{K}{r_A}$$





$$\int_A^B \vec{F}\cdot d\vec{s} =$$

$$\int_A^{B'} \vec{F}\cdot d\vec{s} + \int_{B'}^{B} \vec{F}\cdot d\vec{s} =$$

$$= \int_A^{B'} F\cdot ds = \left.\frac{K}{x}\right|_A^{B'}$$

$$= \frac{K}{r_B} - \frac{K}{r_A}$$

SEMPRE DA A → B

T DINAMICA

$$\Delta U = Q - L$$

$$dU = dQ - dL$$

$$dQ = dU + dL$$

$$U = \frac{3}{2} NRT = nC_V dT$$

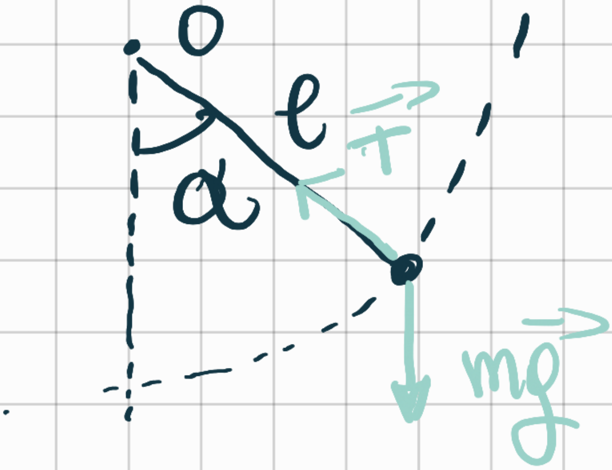
$$dL = PdV \Rightarrow NRdT$$

$$dQ = nC_V dT + nRdT$$

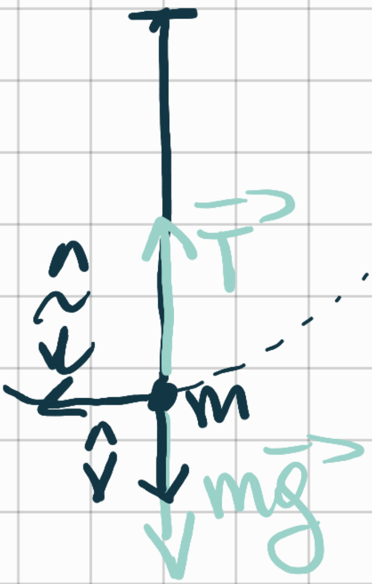
$$= n\underbrace{(C_V + R)}_{C_P}dT$$

$$C_P = C_V + R \qquad \frac{J}{mol \cdot K}$$

PENDOLO SEPLICE



$$\vec{T}+m\vec{g}=m\vec{a}$$



lungo $\hat{\tau}$

$mg\sin\alpha = 0$

lungo $\hat{\nu}$

$mg\cos\alpha = T$

lungo $\hat{\tau}$

$-mg\sin\alpha = ma_\tau$

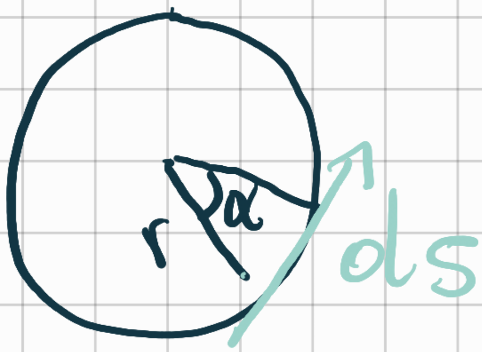
lungo $\hat{v}$

$-mg\cos\alpha + T = ma_v$

$-\cancel{m}g\sin\alpha = \cancel{m}a_\tau$

$-g\sin\alpha = \dot{v}$

$\dot{v} = \ddot{s} = r\cdot\ddot{\alpha}$



$ds = r\cdot d\alpha$

$-g\sin\alpha = r\ddot{\alpha}$

$r = \ell$

$-g\sin\alpha = \ell\ddot{\alpha}$

$$\ddot{\alpha} = -g/\ell \sin\alpha$$

$$\sin\alpha \simeq \alpha$$

$$\ddot{\alpha} = -g/\ell \, \alpha$$

$$\omega^2 = g/\ell$$

$$\ddot{\alpha} = -\omega^2 \alpha$$

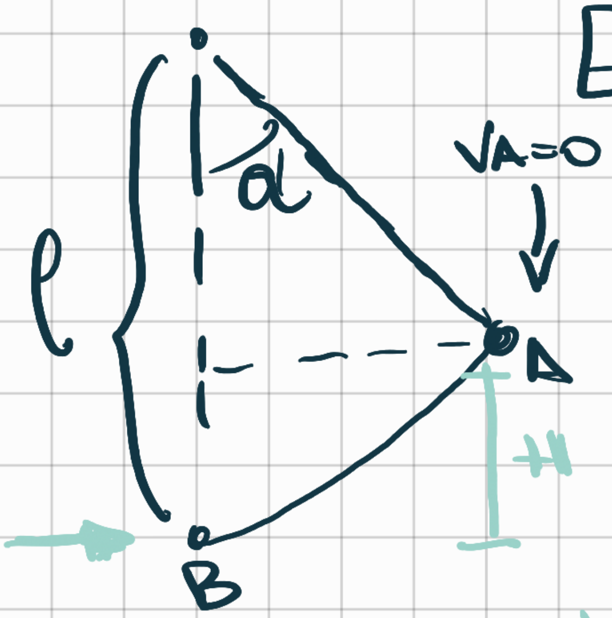
$$\alpha(t) - \alpha\cos(\omega t)$$

$$\frac{2\pi}{T} = \sqrt{g/\ell}$$

$$T = 2\pi\sqrt{\ell/g}$$

$-mg\cos\alpha + T = ma_v$

$-mg\cos\alpha + T = m\dfrac{v^2}{R}$



$E_{M_B} = \frac{1}{2} m v_B^2$

$E_{MA} = mgh$

$h = \ell - \ell\cos\alpha$
$= \ell(1-\cos\alpha)$

$h = 0 \Rightarrow mgh = 0$

$E_{MA} = E_{MB}$

$\cancel{m}g\ell(1-\cos\alpha) = \frac{1}{2}\cancel{m}v_B^2$

$v_B = \sqrt{2g\ell(1-\cos\alpha)}$

$-mg\cos\alpha + T = m\dfrac{v^2}{R}$ $\quad R = \ell$

$-mg\cos\alpha + T = m\,2g\ell(1-\cos\alpha)$

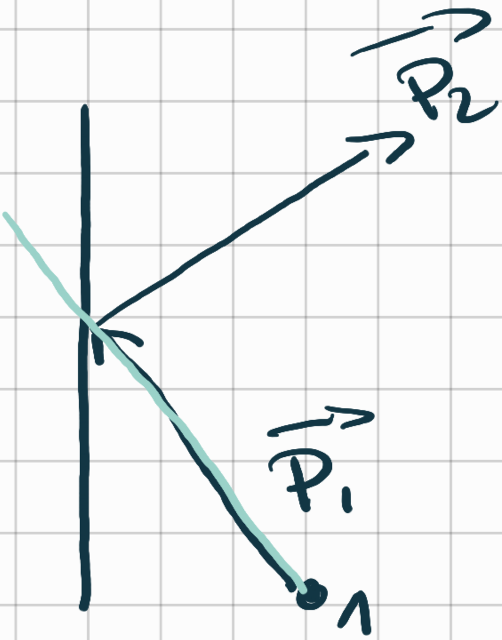
$$mg\cos\alpha + T = m\frac{2g\ell(1-\cos\alpha)}{\ell}$$

$$-mg\cos\alpha + T = 2mg(1-\cos\alpha)$$

$$T = 2mg - 2mg\cos\alpha + mg\cos\alpha$$

$$= 2mg\left(1 - \cos\alpha + \frac{\cos\alpha}{2}\right)$$

$$= 2mg\left(1 - \frac{\cos\alpha}{2}\right)$$



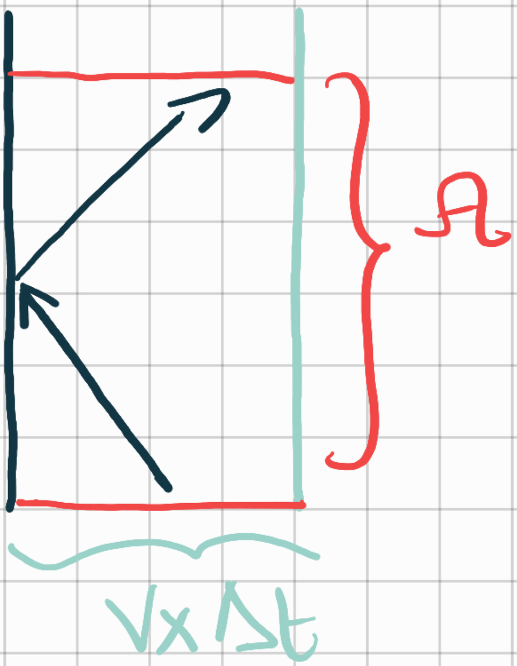
$$\vec{p} = m\vec{v}$$

$$p_1 = mv_x$$

$$p_2 = -mv_x$$

$$\vec{p} = -2mv_x$$

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$



$$\frac{N}{V}$$

$$\frac{N}{V} \quad V$$

$$N \cdot \Delta\vec{p}$$

$$V = V_x \Delta t \cdot A$$

$$\frac{N}{V} V = \frac{N}{V} V_x \Delta t$$

$$N \cdot \Delta\vec{p} = \frac{N}{V} V_x \Delta t \cdot (-2mV_x)$$

$$\Delta\vec{p}_{TOT} = \frac{N}{V} V_x \Delta t A \cdot (-2mV_x)$$

$$\frac{\Delta p}{\Delta t} = F$$

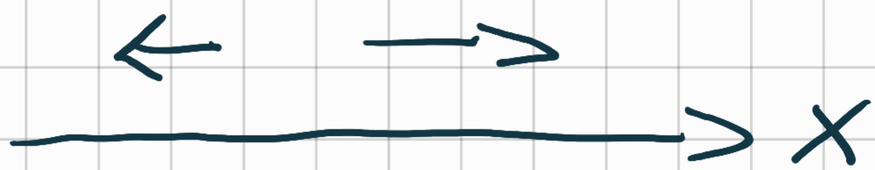
$$\vec{F} = \frac{N}{V} V_x A \cdot (-2 m V_x)$$

$$\frac{F}{A} = \frac{N}{V} V_x \cdot (-2 m V_x)$$

$$P = \frac{N}{V} V_x (-2 m V_x)$$

$$P = -\frac{N}{V} V_x^2 2m$$

$$P = \cancel{2} \frac{N}{V} V_x^2 m$$



$$\langle V_{x_S} \rangle = \langle V_{x_D} \rangle$$

$$P = \frac{N}{V} \langle v_x^2 \rangle m$$

$$P = \frac{N}{V} 2 \left\langle \frac{mv^2}{2} \right\rangle$$

$$P = 2 \frac{N}{V} \frac{\langle E_c \rangle}{3}$$

$$P = \frac{2}{3} \frac{N}{V} \langle E_c \rangle \Rightarrow N E_c = U$$

traslazionale

$$E_c = \frac{3}{2} KT$$

$$P = \frac{N}{V} KT$$

$$PV = NKT$$

$$PV = nRT$$

$$PV = \frac{2}{3} U \Rightarrow U = \frac{3}{2} PV$$

$$= \frac{3}{2} N_0 R T$$