I KONIG (MOMENTO ANGOLARE)

$$\mathcal{L} = \vec{r} \times m\vec{v}$$

ricordando che $\vec{r} = \vec{r}_{cm} + \vec{r}'$
e $\vec{v} = \vec{v}_{cm} + \vec{v}'$

$$\mathcal{L} = (\vec{r}_{cm} + \vec{r}') \times m(\vec{v}' + \vec{v}_{cm})$$

Se mi trovo in un sisteme di pti ho:

$$\mathcal{L} = \sum_i (\vec{r}_{cm} + \vec{r}_i') \times m_i(\vec{v}_i' + \vec{v}_{cm})$$

$$= \sum \vec{r}_{cm} \times m\vec{v}_i + \vec{r}_{cm} \times m_i\vec{v}_{cm} + \vec{r}_i' \times m_i \vec{v}_i + \vec{r}_i' \times m_i \vec{v}_{cm}$$

①

$$= \sum \vec{r}_{cm} \times m_i \vec{v}_i = \vec{r}_{cm} \times \sum m_i \vec{v}_i$$

$$= \vec{r}_{cm} \times M \sum \frac{m_i \vec{v}_i}{M} = 0$$

(2) $$\sum \vec{r}_{cm} \times m_i \vec{v}_{cm} = L$$ come se tutto fosse concentrato in cm

(3) $$\sum r_i' \times m \vec{v}_i' = L_{cm}$$

(4) $$\sum \vec{r}_i' \times m_i \vec{v}_{cm} = \sum \vec{r}_i m_i \times v_{cm}$$

$$= M \sum \frac{\vec{r}_i m_i}{M} \times \vec{v}_{cm} = 0$$

$$L_{TOT} = L_{CM} + \vec{r}_{cm} \times m_i \vec{v}_{cm}$$

II KONIG (E cinetica)

$$E_c = \frac{1}{2} m v^2$$

ricordo che $\vec{v} = \vec{v}_{cm} + \vec{v}'$

quindi $v^2 = v_{cm}^2 + v'^2 + 2\vec{v}'\vec{v}_{cm}$

$$E_c = \frac{1}{2} m (v_{cm}^2 + v'^2 + 2\vec{v}'\vec{v}_{cm})$$

in un sisteme di punti

$$E_c = \sum \frac{1}{2} m_i (\vec{v}_{cm} + v_i'^2 + 2\vec{v}_i'\vec{v}_{cm})$$

1. $\sum \frac{1}{2} m_i v_{cm}^2 = E_c$ come se tutto fosse in cm

2. $\sum \frac{1}{2} m_i v_i^2 = E_{c\,cm}$

3. $\sum \frac{1}{\cancel{2}} m_i (\cancel{2}\vec{v}_i'\vec{v}_{cm}) = \sum m_i \vec{v}_i'\vec{v}_{cm}$

$$= \vec{v}_{cm} \sum m_i \vec{v}_i = \vec{v}_{cm} M \sum \frac{m_i v_i}{M}$$

$= 0$

$$E_{c\,tot} = E_{c\,cm} + \frac{1}{2} m v_{cm}^2$$

$$\vec{\tau} = \dot{\vec{L}}$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = (\vec{r}\,' + \vec{r}_{cm}) \times \vec{F} = \vec{r}\,' \times \vec{F} + \vec{r}_{cm} \times \vec{F}$$

$$\dot{\vec{L}} = \dot{L}_{cm} + \vec{r}_{cm} \times m \dot{\vec{v}}_{cm} = \dot{\vec{L}}_{cm} + \vec{r}_{cm} \times m \vec{a}_{cm}$$

$$\vec{\tau} = \dot{\vec{L}} \Rightarrow \vec{\tau}_{cm} + \cancel{\vec{r}_{cm} \times \vec{F}} = \dot{\vec{L}}_{cm} + \cancel{\vec{r}_{cm} \times \vec{F}}$$

$$\vec{\tau}_{cm} = \dot{\vec{L}}_{cm}$$

PENDOLO COMPOSTO

$(\vec{F}^{(e)}, \vec{M}$

$$\begin{cases} \quad = M a_{cm} \\ \vec{\tau} = \dot{\vec{L}} \end{cases}$$

z orizzontale

$\vec{\omega} \otimes$

R costante

$$\tau_z = \dot{L}_z$$

$$\vec{r} \times \vec{F} = \vec{r} \times m \vec{v}$$

poiché mi trovo in un moto rotatorio $\vec{v} = \vec{\omega} R$

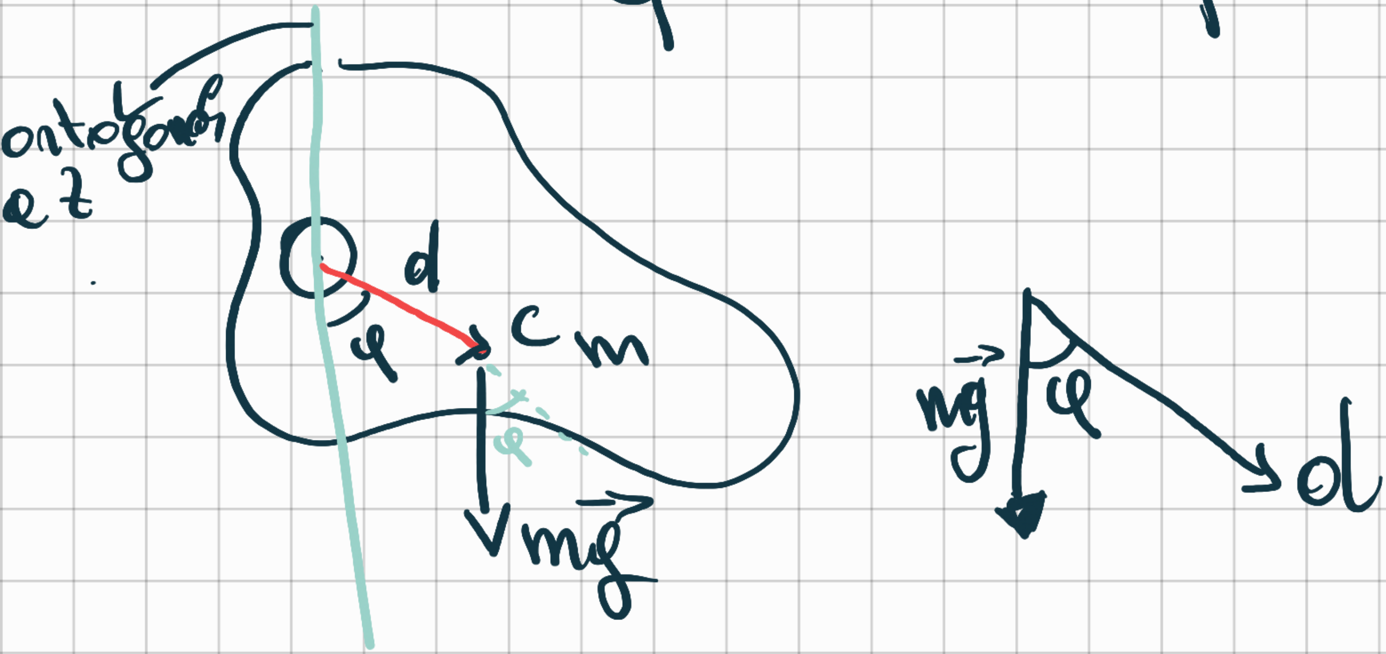
$$L = r \times m \vec{\omega} r$$
$$= r^2 \times m \vec{\omega}$$

$$= \underbrace{m r^2} \times \vec{\omega} \Rightarrow I_z \vec{\omega}$$

I_z

$$\dot{\vec{L}} = (I_z \dot{\vec{\omega}}) = I_z \dot{\vec{\omega}}$$

ma $\vec{\omega} = \dot{\varphi} \Rightarrow \dot{\vec{\omega}} = \ddot{\varphi}$



$$\tau_{mg} = -mgd \sin\varphi$$

$$-mgd\sin\varphi = I_z \ddot{\varphi}$$

$$\ddot{\varphi} = -\frac{mgd}{I_z} \sin\varphi$$

$$\frac{mgd}{I_z} = \omega^2$$

I_z

se ho piccole oscillazioni

$\sin\varphi \simeq \varphi$

$-\omega^2 \varphi = I_z \ddot{\varphi}$

se ricordo che $\omega = \frac{2\pi}{T}$

$$\omega = \sqrt{\frac{Mgd}{I_z}} = \frac{2\pi}{T}$$

$$\sqrt{\frac{Mgd}{I_z}} = \frac{2\pi}{T}$$

$$\frac{T}{2\pi} = \sqrt{\frac{I_z}{MgD}} \Rightarrow T = 2\pi \sqrt{\frac{I_z}{Mgd}}$$

PENDOLO SEMPLICE

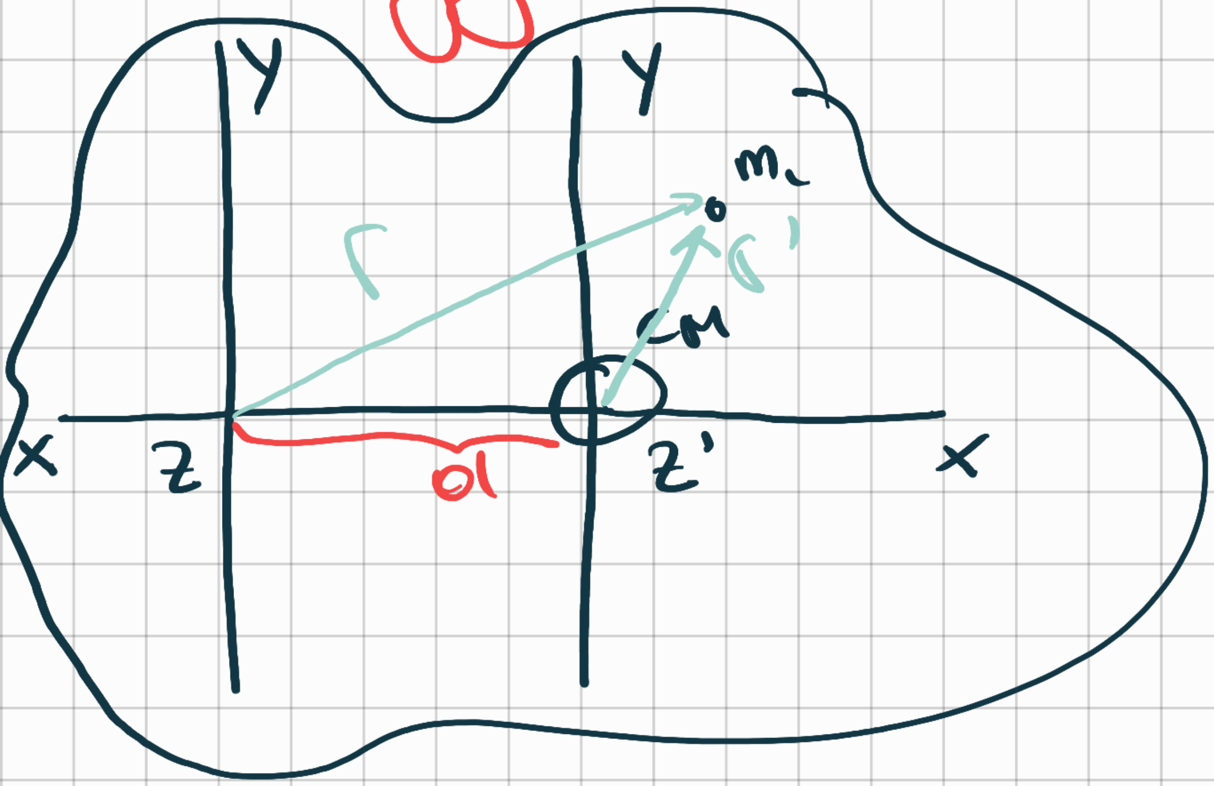
$I_z = mr^2$

$$= ml^2$$

$$Mgd = mgl$$

$$\omega = \sqrt{\frac{\cancel{m}g\cancel{l}}{\cancel{m}l^{\cancel{2}}}} = \sqrt{g/l}$$

Th Huygens Steiner (I_z)



z' è un'asse passante per cm sistema e parallelo a z. avente l'asse x comune col primo sistema

$\vec{r} = \vec{r}' + d$

$I_z = m r^2$

$= m(\vec{r}' + d)^2$

$= m r'^2 + m d^2 + m 2 r' d$

$\sum_i \underbrace{m_i r_i^2}_{I \text{ del cm}} + m d^2 + \underbrace{m_i 2 r_i' d}$

$\sum 2 m_i r_i' d$

$2d \sum m_i r_i'$

$2d\, M \sum \frac{m_i r_i'}{M}$

$= 0$