Multiplayer Target Defense Game Between Quadrotors: Use Cases Studies

ESC499 Thesis Presentation
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Introduction

- **Rise of UAVs**: Increasing use of unmanned aerial vehicles in surveillance, delivery, and defense.
- **Security Challenges**: Growing need for efficient interception strategies due to UAVs proliferation.
- **Strategic Gap**: Limited current strategies for addressing faster-moving targets.
- **Research Aim**: To develop and evaluate advanced interception methods and conduct parameter analysis

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Problem Formulation

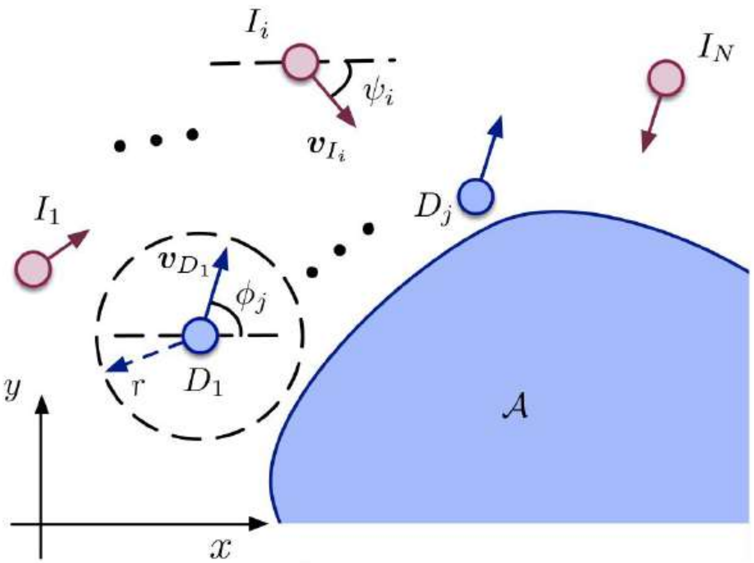

Figure 1: Schematic of the defense scenario illustrating defenders, invaders, and the target area

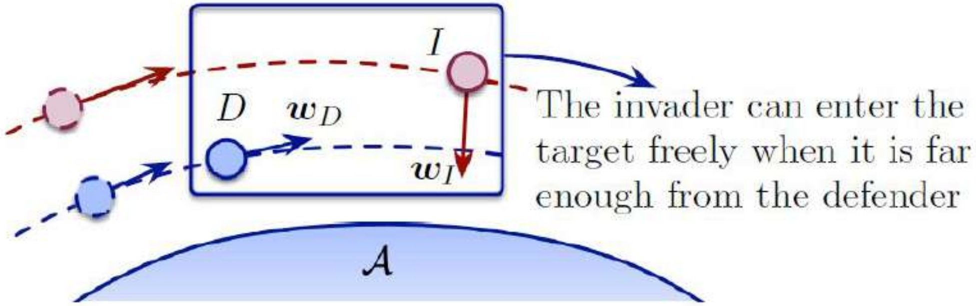

Figure 2: Detailed interaction dynamics between a defender and an invader

Challenge: How can a slower defender effectively intercept a fast-moving intruder?

[1] Fu, H., & Liu, H. H. T. (2021). Optimal solution of a target defense game with two defenders and a faster intrude. *Unmanned Systems*, *9*(03), 247-262.

Dynamics of the Two Defenders and One Invader

The kinematics of the 2DSI game is

$$\dot{x}_I = v_I \cos\psi_I, \quad \dot{y}_I = v_I \sin\psi_I,$$
$$\dot{x}_i = v_D \cos\psi_i, \quad \dot{y}_i = v_D \sin\psi_i, \; i \in \{1,2\},$$

The concept of capture is defined as follows:

$$\begin{cases} \|ID_i\| < r, \quad \exists i \in \{1,2\}, & \text{or} \\ \|ID_i\| = r, \quad \text{and} \quad \dfrac{d\|ID_i\|}{dt} < 0, & \exists i \in \{1,2\}, \end{cases}$$

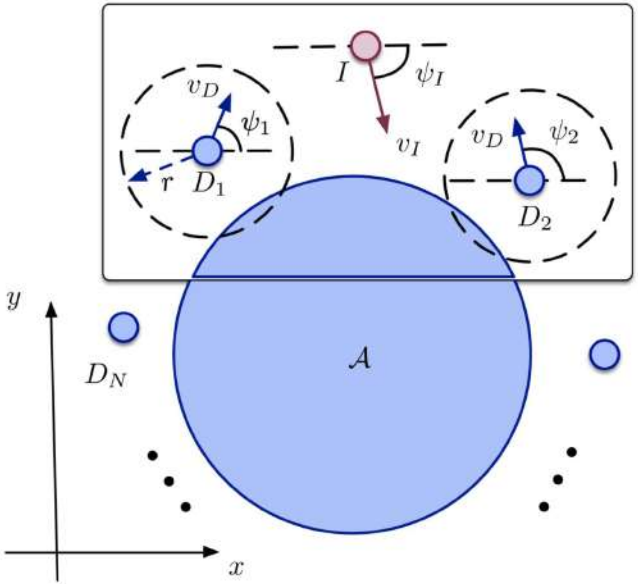

Figure 3: Definition of the 2DSI game

[1] Fu, H., & Liu, H. H. T. (2021). Optimal solution of a target defense game with two defenders and a faster intrude. *Unmanned Systems*, *9*(03), 247-262.

Related Work

In Polar Coordinate, (ρ_D, ρ_I) state space

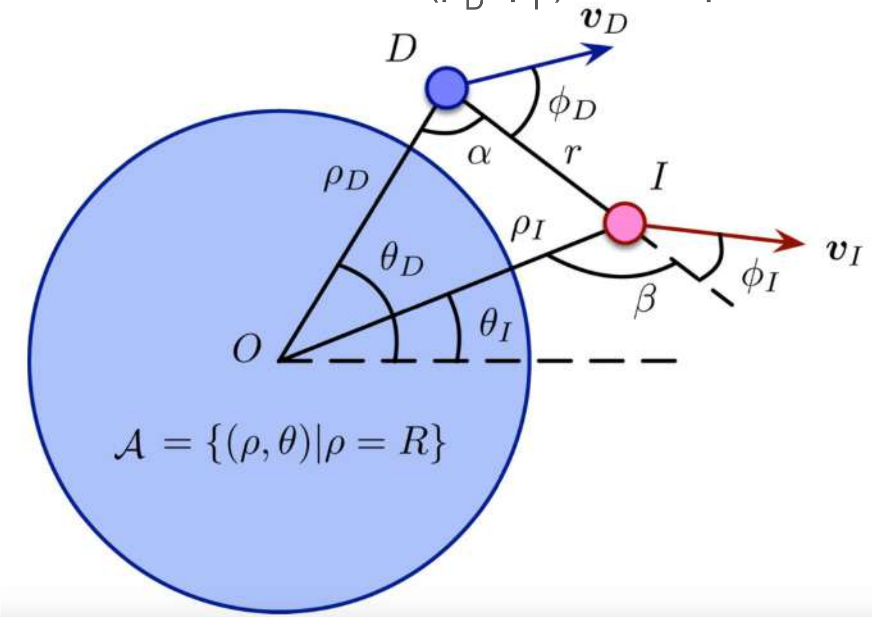

Figure 4: Representation of defender (D) and invader (I) in polar coordinates with radius (ρ) and angle (θ) variables in the target area (A)

Defender wins by maintaining a strategic distance

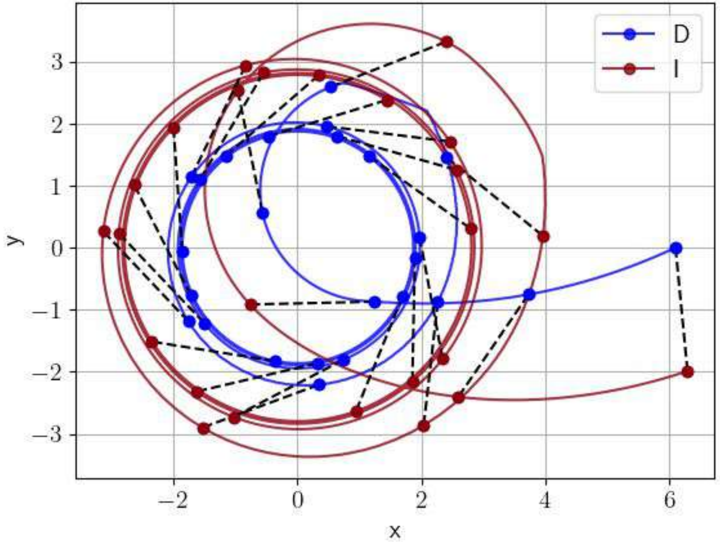

Figure 5: Optimal trajectories for a circular target with parameters r = 2 m, vD = 1 m/s, vI = 1.5 m/s, and initial state (ρ_D,ρ_I) = (6.1m,6.6m)

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Related Work

In Polar Coordinate, (ρ_D, ρ_I) state space

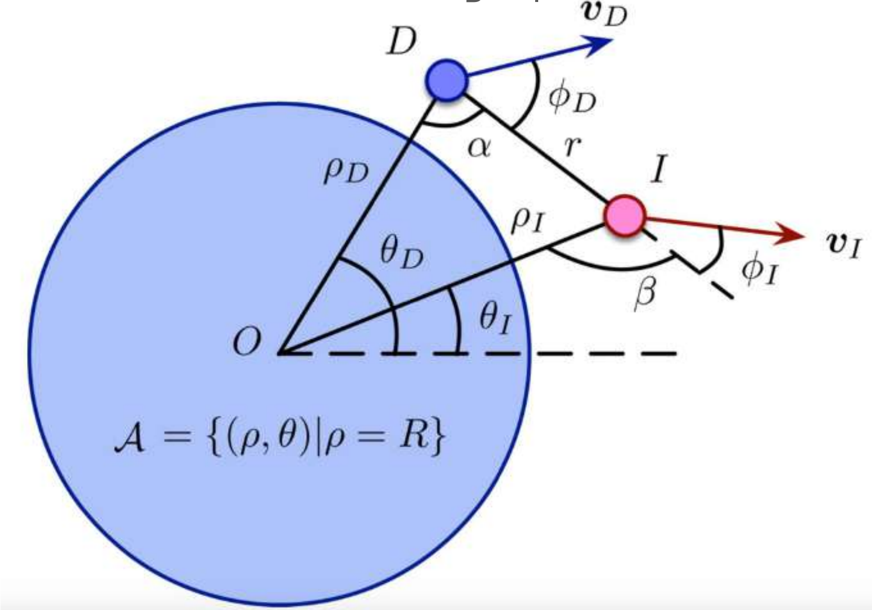

Figure 4: Representation of defender (D) and invader (I) in polar coordinates with radius (ρ) and angle (θ) variables in the target area (A)

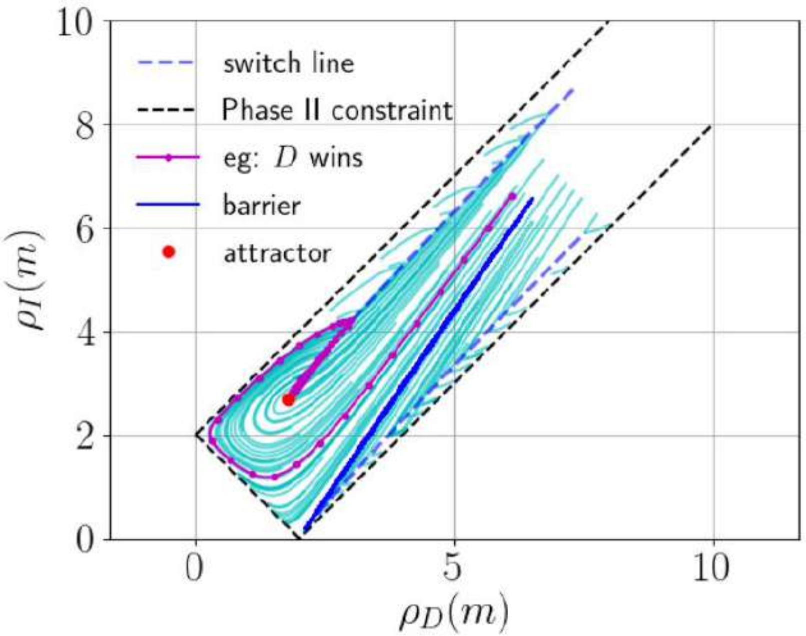

Figure 6: Optimal trajectories in the (ρ_D, ρ_I) state space

[1] Fu, H., & Liu, H. H. T. (2021). Optimal solution of a target defense game with two defenders and a faster intrude. *Unmanned Systems*, *9*(03), 247-262.

My Contributions

Dr. Fu's work:

- Based on differential game theory
- The solution is customized for circular targets
- Not designed for real-time deployment on UAVs
- Deployment strategies for defender drones were not addressed

My work:

- Individualized treatment for defender and intruder strategies
- Enhanced flexibility in drone strategy development
- Capable of real-time execution on drone systems
- Allows easy parameter modification to observe changes in game outcomes

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Base Model: MATLAB Simulation

Input parameters

- Velocity of the intruder: v_{intruder}
- Velocity of the defender 1: v_{defender_1}
- Velocity of the defender 2: v_{defender_2}
- Heading angle of the intruder: ψ_{intruder}
- Heading angle of the defender 1: ψ_{defender_1}
- Heading angle of the defender 2: ψ_{defender_2}
- Initial positions of the intruder: $(x_{0,\text{intruder}}, y_{0,\text{intruder}})$
- Initial positions of defender 1: $(x_{0,\text{defender}_1}, y_{0,\text{defender}_1})$
- Initial positions of defender 2: $(x_{0,\text{defender}_2}, y_{0,\text{defender}_2})$

$V_{\text{defender1}} = 1$ m/s
$\Psi_{\text{defender1}} = 5*\pi/4$
$x_{0,\text{defender1}} = 20$
$y_{0,\text{defender1}} = 28$

$V_{\text{intruder}} = 1.5$ m/s
$\Psi_{\text{intruder}} = 5*\pi/4$
$x_{0,\text{intruder}} = 30$
$y_{0,\text{intruder}} = 30$

$V_{\text{defender2}} = 1$ m/s
$\Psi_{\text{defender2}} = 5*\pi/4$
$x_{0,\text{defender2}} = 35$
$y_{0,\text{defender2}} = 27$

Target area with r = 2m

Intruder wins

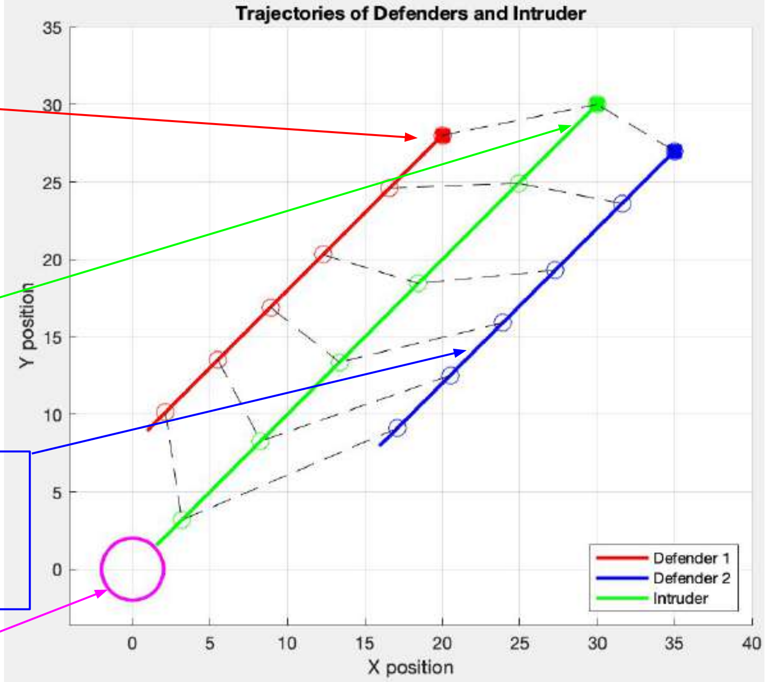

Figure 7: Trajectories of Defender 1 (in red), Defender 2 (in blue), and the Intruder (in green) without any strategy

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Methodology: Implementation of PNG on Defenders

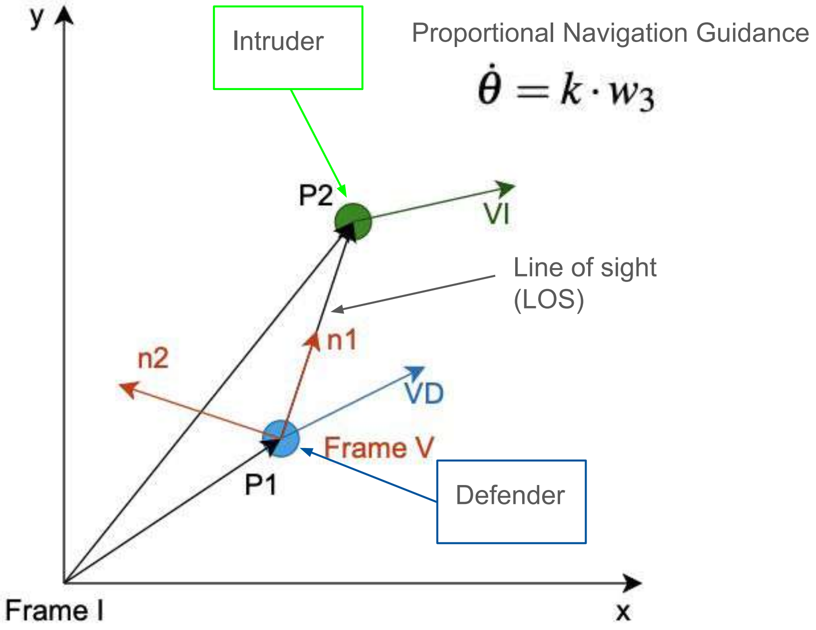

Figure 8: Illustration of the engagement geometry between the defender (P1) and the intruder (P2).

Defender wins

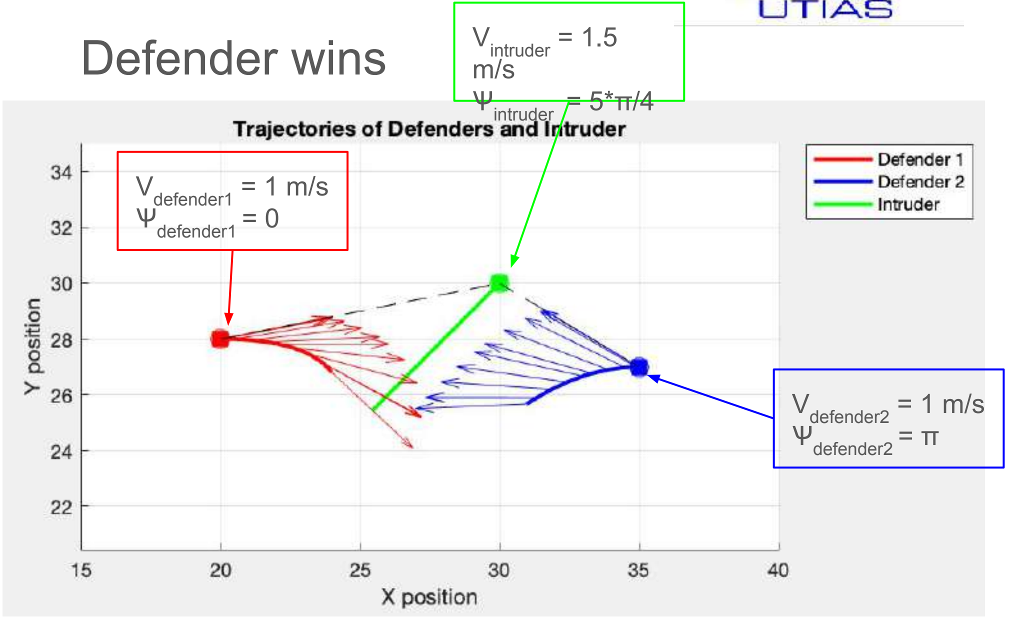

Figure 9: Trajectories of Defender 1 (in red), Defender 2 (in blue), and the Intruder (in green) with PNG law applied on defenders

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Methodology: Monte Carlo Simulation

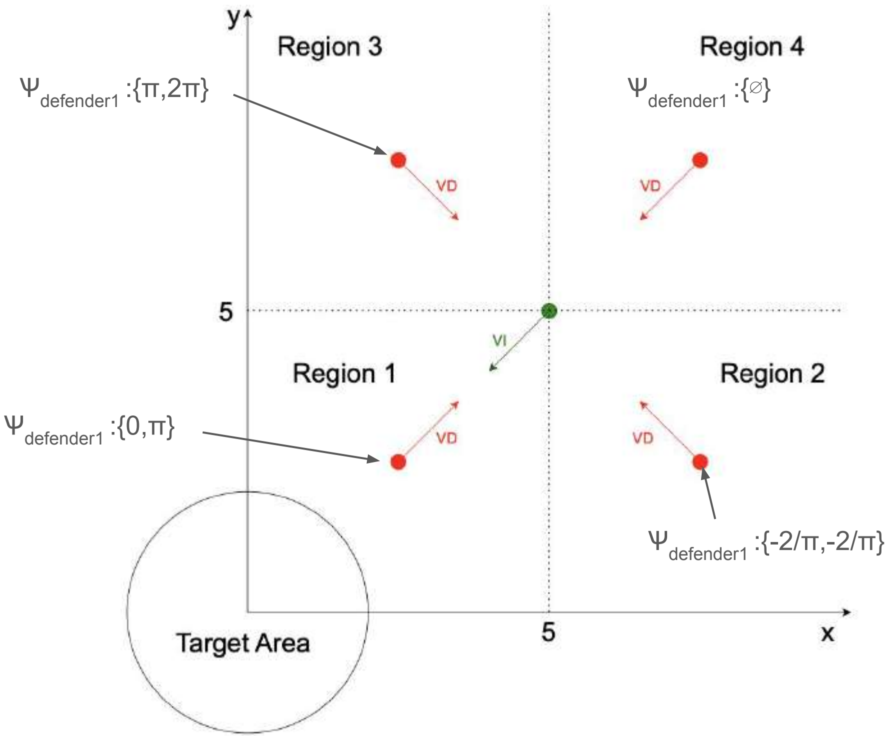

Figure 10: The 2D plane divided into four regions with different strategic significance

$V_{defender1} = 1$ m/s
$V_{intruder} = 1.5$ m/s
$\Psi_{intruder} = 5*\pi/4$
$x_{0,intruder} = 5$
$y_{0,intruder} = 5$

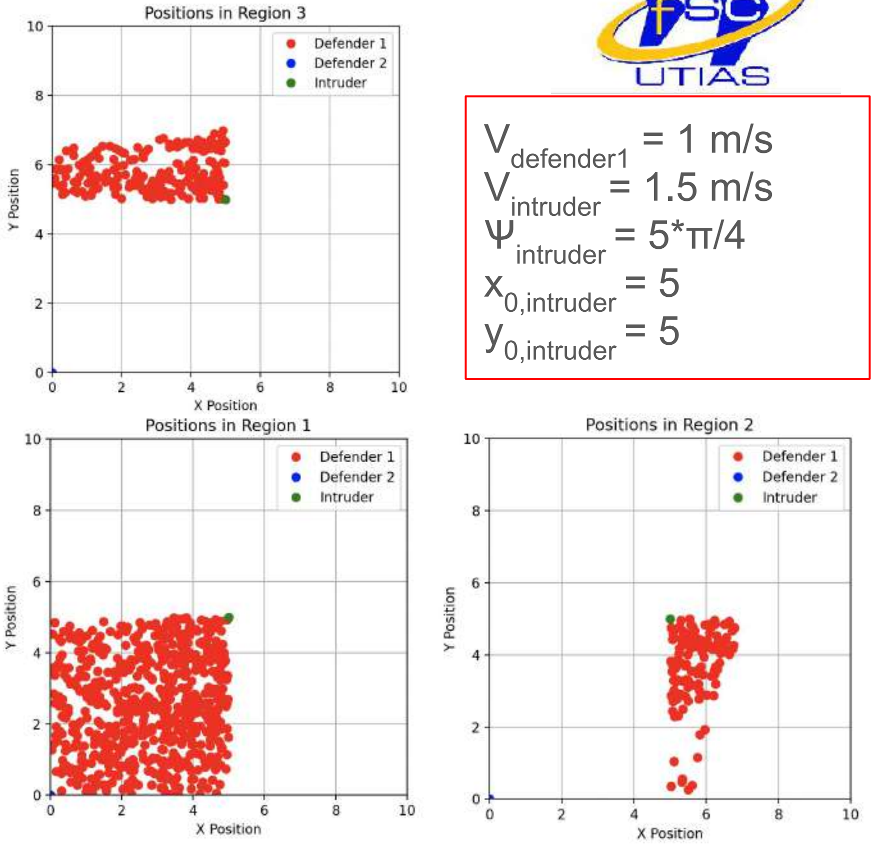

Figure 11: Initial positions of successful interceptions by Defender 1 in Regions 1, 2, and 3

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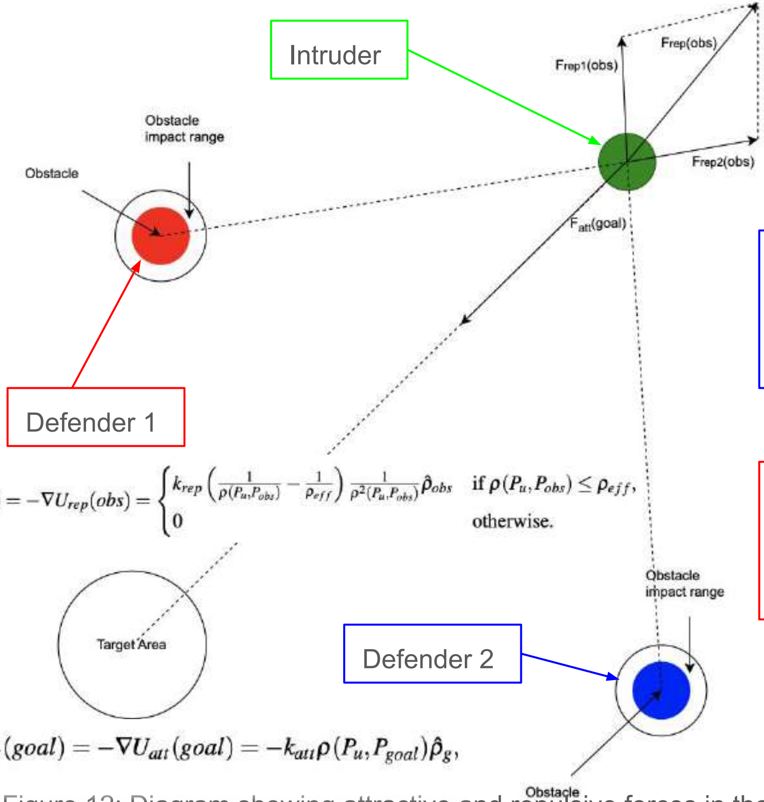
Methodology: Implementation of APF on Intruder



$$F_{rep}(obs) = -\nabla U_{rep}(obs) = \begin{cases} k_{rep}\left(\frac{1}{\rho(P_u,P_{obs})} - \frac{1}{\rho_{eff}}\right)\frac{1}{\rho^2(P_u,P_{obs})}\hat{\rho}_{obs} & \text{if } \rho(P_u,P_{obs}) \leq \rho_{eff}, \\ 0 & \text{otherwise.} \end{cases}$$

$$F_{att}(goal) = -\nabla U_{att}(goal) = -k_{att}\rho(P_u,P_{goal})\hat{\rho}_g,$$

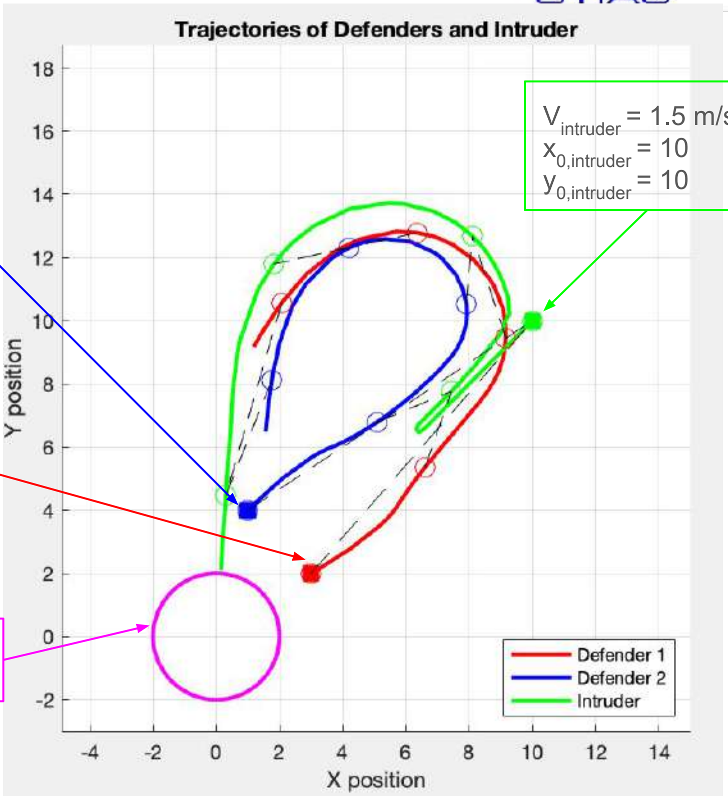
Figure 12: Diagram showing attractive and repulsive forces in the Artificial Potential Field (APF)

Coefficients:
$k_{att} = 1.5$
$k_{rep} = 1000$
$\rho_{eff} = 3$

$V_{defender2} = 1$ m/s
$\Psi_{defender2} = \pi/4$
$x_{0,defender2} = 1$
$y_{0,defender2} = 4$

$V_{defender1} = 1$ m/s
$\Psi_{defender1} = \pi/6$
$x_{0,defender1} = 3$
$y_{0,defender1} = 2$

Target area with r = 2m



$V_{intruder} = 1.5$ m/s
$x_{0,intruder} = 10$
$y_{0,intruder} = 10$

Figure 13: Trajectories of Defender 1 (in red), Defender 2 (in blue), and the Intruder (in green) with PNG law applied on defenders and APF on intruder

Methodology: Predictive Interception Strategy on Defenders

Future Intruder Position Prediction:

$$P_{I_{future}} = P_{I_{current}} + V_I \times \Delta t \quad (1)$$

Relative Position to Defender:

$$P_{relative} = P_{I_{future}} - P_D \quad (2)$$

Heading Angle for Defender:

$$\psi_D = \text{atan2}(P_{relative_y}, P_{relative_x}) \quad (3)$$

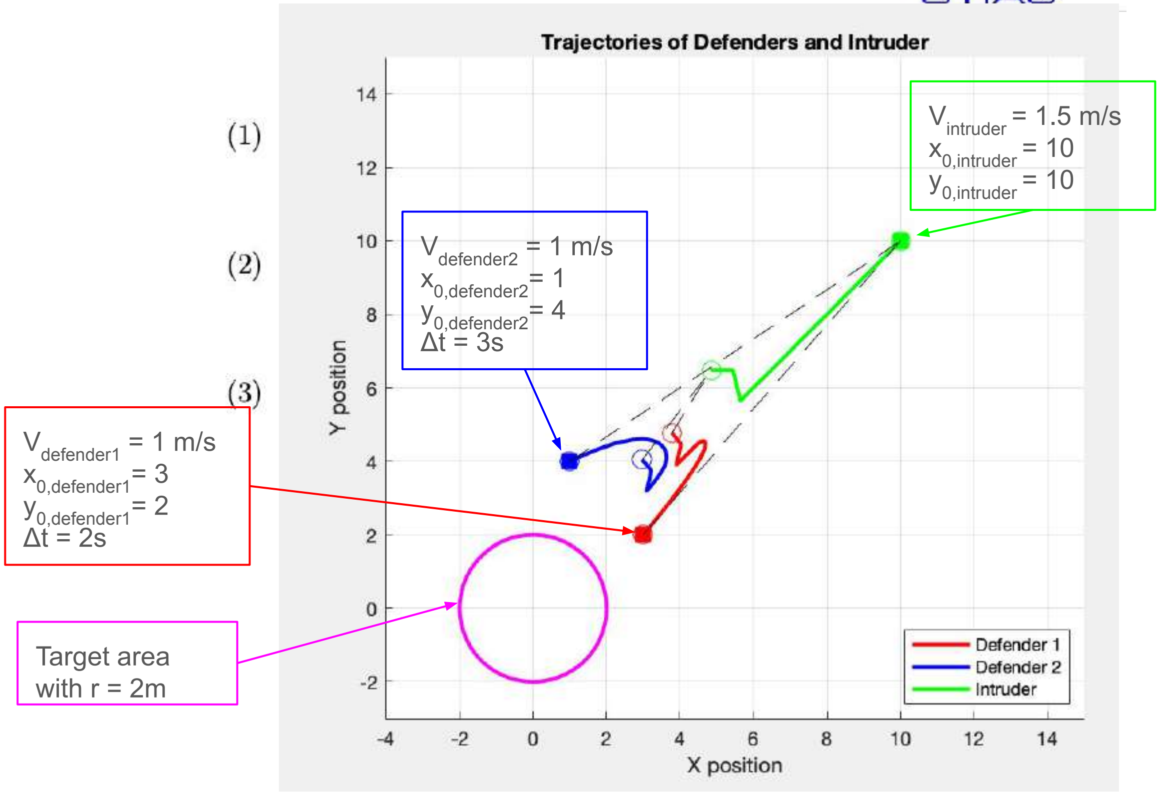


Figure 14: Trajectories of Defender 1 (in red), Defender 2 (in blue), and the Intruder (in green) with Predictive applied on defenders and APF on intruder

Parameter Analysis

Intruder Velocity	Defender 1 Velocity	Defender 2 Velocity	Outcome
1.5	0.8	0.8	Intruder Win
1.5	1.0	1.0	Defender Win
1.5	1.2	1.2	Defender Win
2.0	0.8	0.8	Intruder Win
2.0	1.0	1.0	Defender Win
2.0	1.2	1.2	Defender Win
2.5	0.8	0.8	Intruder Win
2.5	1.0	1.0	Intruder Win
2.5	1.2	1.2	Defender Win

Table 1: Simulation results for varying drone velocities

k_{att}	η	ρ_{eff} (units)	Outcome
1.0	500	3	Defender Win
1.0	500	4	Defender Win
1.0	500	5	Defender Win
1.0	1000	3	Defender Win
1.0	1000	4	Defender Win
1.0	1000	5	Defender Win
1.0	1500	3	Intruder Win
1.0	1500	4	Defender Win
1.0	1500	5	Defender Win
1.5	500	3	Defender Win
1.5	500	4	Defender Win
1.5	500	5	Defender Win
1.5	1000	3	Defender Win
1.5	1000	4	Defender Win
1.5	1000	5	Defender Win
1.5	1500	3	Intruder Win
1.5	1500	4	Defender Win
1.5	1500	5	Defender Win
2.0	500	3	Defender Win
2.0	500	4	Defender Win
2.0	500	5	Defender Win
2.0	1000	3	Defender Win
2.0	1000	4	Defender Win
2.0	1000	5	Defender Win
2.0	1500	3	Defender Win
2.0	1500	4	Defender Win
2.0	1500	5	Defender Win

Table 2: Simulation results for varying APF coefficients

Defender 1 Time Step (Δt_{D1})	Defender 2 Time Step (Δt_{D2})	Outcome
0.5	0.5	Intruder Win
0.5	1	Intruder Win
0.5	1.5	Intruder Win
1	1	Intruder Win
1	1.5	Intruder Win
1	2	Intruder Win
1.5	1.5	Intruder Win
1.5	2	Intruder Win
1.5	2.5	Intruder Win
2	2	Intruder Win
2	2.5	Defender Win
2	3	Defender Win
2.5	2.5	Defender Win
2.5	3	Defender Win
2.5	3.5	Defender Win
3	3	Defender Win
3	3.5	Execution Too Long
3	4	Execution Too Long

Table 3: Simulation results for varying prediction time steps

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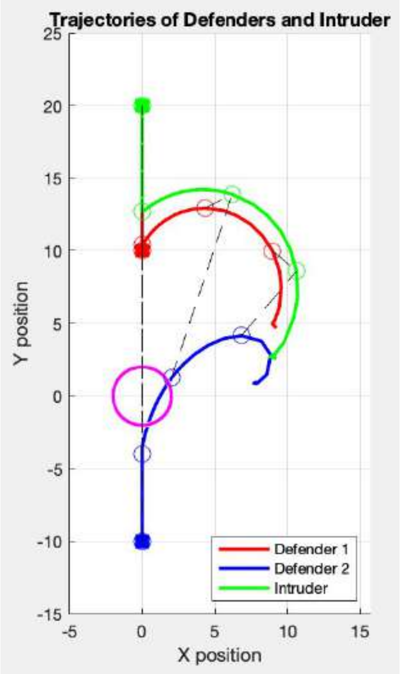
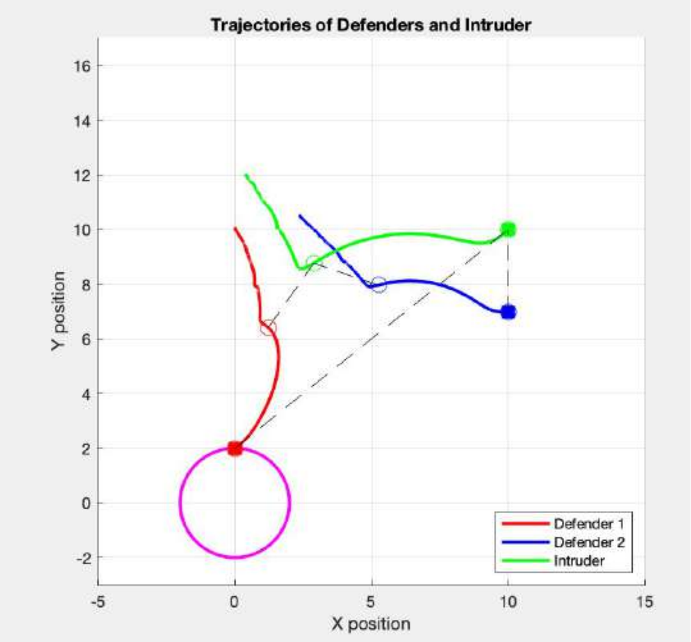
Influence of Initial Drone Positions

Case 1: Head-to-Head Confrontation



Case 2: Target Protection



Case 3: Flanking Maneuver

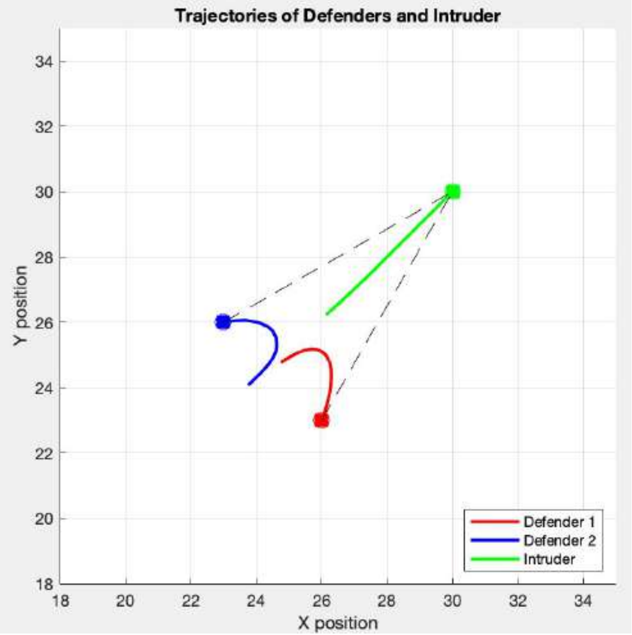


Figure 15: Trajectories of Defender 1 (in red), Defender 2 (in blue), and the Intruder (in green) with Predictive applied on defenders and APF on intruder

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Similar Distance Maintaining Behavior

$V_{intruder}$ = 1.5 m/s
$x_{0,intruder}$ = 6.6
$y_{0,intruder}$ = -2

$V_{defender1}$ = 1 m/s
$x_{0,defender1}$ = 7.1
$y_{0,defender1}$ = 2
Δt = 2s

$V_{defender2}$ = 1 m/s
$x_{0,defender2}$ = 6.1
$y_{0,defender2}$ = 0
Δt = 3s

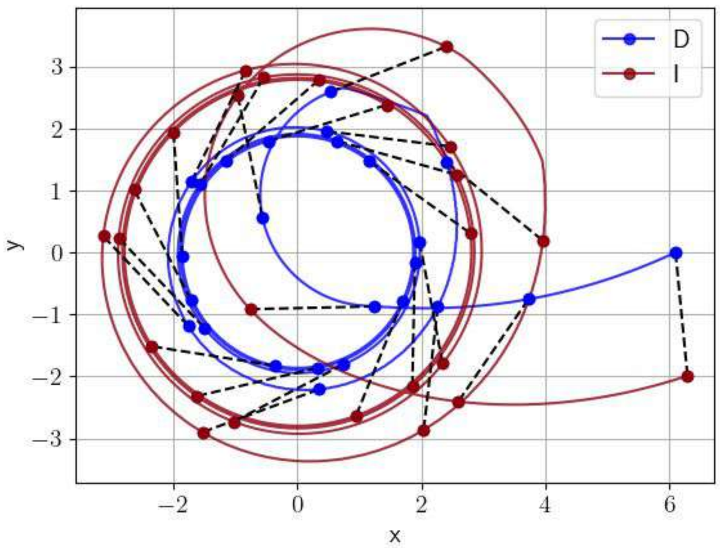


Figure 5: Optimal trajectories for a circular target with parameters r = 2 m, vD = 1 m/s, vI = 1.5 m/s, and initial state (ρ_D, ρ_I) = (6.1m,6.6m)

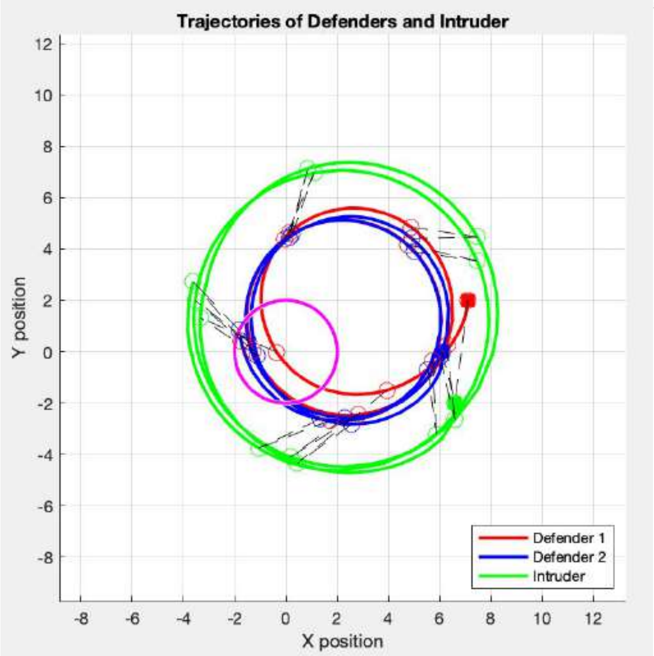


Figure 16: Trajectories shows similar distance maintaining behavior

[1] Fu, H., & Liu, H. H. T. (2021). Optimal solution of a target defense game with two defenders and a faster intrude. *Unmanned Systems*, *9*(03), 247-262.

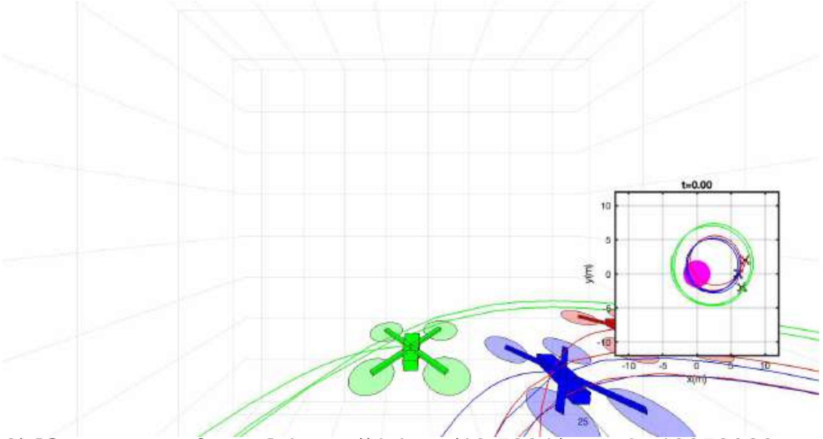
Conclusion and Future Work

Conclusions:

- **Use different strategies to achieve similar result as Dr. Fu's work**
- **Investigate different parameters effect on game**

Future Work:

- **Real-World Testing**
- **Collaborative Defense**
- **3D scenarios**



[2]Qian, L. (2023). Matlab Drone Simulation Visualization Package (Version 1.0.0) [Computer software]. https://doi.org/10.5281/zenodo.10359983

Acknowledgment

I would like to express my sincere gratitude to my supervisor, Professor Hugh H.T. Liu, and Dr. Longhao Qian for their invaluable contributions to this thesis project. Their mentorship and expertise were crucial in making this work possible.

Thank you!