Tracking Controller

H S Helson Go

March 6, 2024

This package is initially a port of mavros_controllers, specifically it src/geometric_controller.cpp to functional-styled python accelerated by jax. It was then backported to C++.

This package implements the tracking control law:

Definition 1. *A quadrotor has states*

$$\{\mathbf{p}, \mathbf{R}, \mathbf{v}\}, \tag{1}$$

respectively its absolute position, attitude (body-to-earth rotation), and velocity.

Given control references

$$\{\mathbf{p}_d, \mathbf{v}_d, \psi_d, [, \mathbf{a}_d]\} \tag{2}$$

respectively absolute *position, attitude, velocity and acceleration setpoints, the position error is defined as*

$$\mathbf{e}_p = \mathbf{p} - \mathbf{p}_d, \tag{3}$$

and the velocity error as

$$\mathbf{e}_v = \mathbf{v} - \mathbf{v}_d. \tag{4}$$

Given parameters $\mathbf{k}_p \in \mathbb{R}^3$, $\mathbf{k}_v \in \mathbb{R}^3$, $\mathbf{D} \in \mathbb{R}^{3\times3}$, *respectively the position control gain, velocity control gain, and the drag model matrix, the tracking control law computes acceleration setpoints*

$$\mathbf{a}_{sp} = -\mathbf{k}_p \circ \mathbf{e}_p - \mathbf{k}_v \circ \mathbf{e}_v - \mathbf{R}_d \mathbf{D} \mathbf{R}_d \mathbf{v}_d + g\mathbf{1}_3 + \mathbf{a}_d \tag{5}$$

where $\circ$ *is element-wise multiplication,* $\mathbf{R}_d = \texttt{accelerationVectorToRotation}(\mathbf{a}_d + g\mathbf{1}_3, \psi_d)$.

`accelerationVectorToRotation` *is a quasi-Gram-Schmidt process to compute a spatial rotation to align a quadrotor's body-z axis with a desired acceleration vector.*

and the attitude control law

Definition 2. *Given control references*

$$\{\mathbf{R}_d = \texttt{accelerationVectorToRotation}(\mathbf{a}_{sp}, \psi_d)\}, \tag{6}$$

the attitude error is

$$\mathbf{e}_r = \frac{1}{2}\left(\mathbf{R}_d^\top \mathbf{R} - \mathbf{R}^\top \mathbf{R}_d\right)^\vee \tag{7}$$

The attitude control law computes angular velocity setpoints

$$\boldsymbol{\omega}_{sp} = -\frac{2}{\tau}\mathbf{e}_r \tag{8}$$