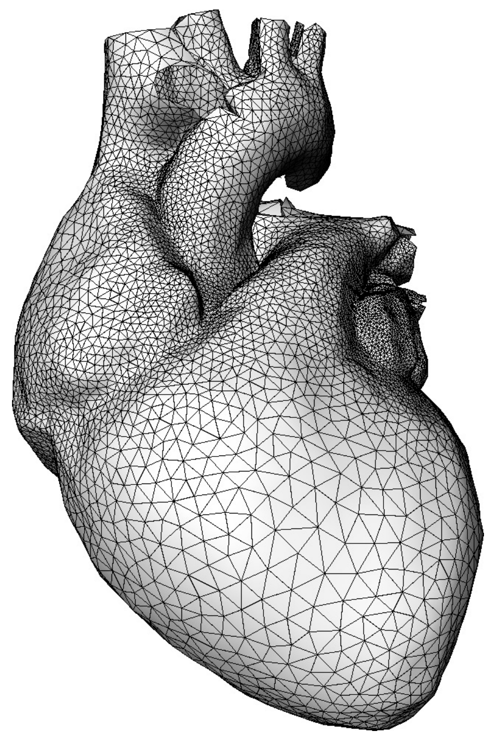
02 - Shape Representations

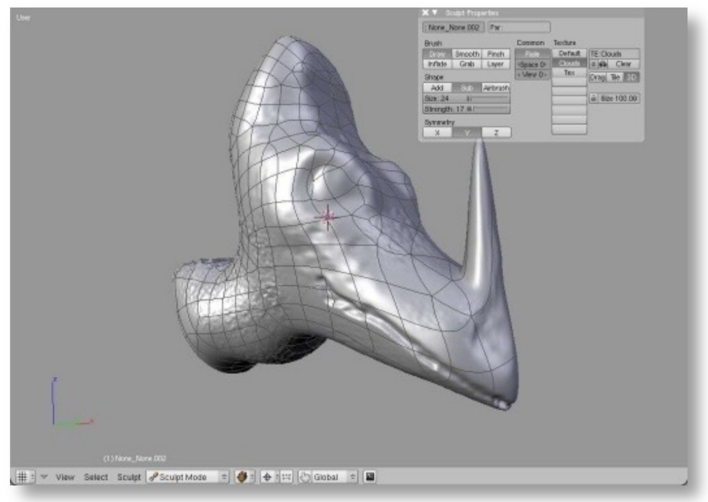
Acknowledgements: Daniele Panozzo

Shape Representation:
Origin- and Application-Dependent

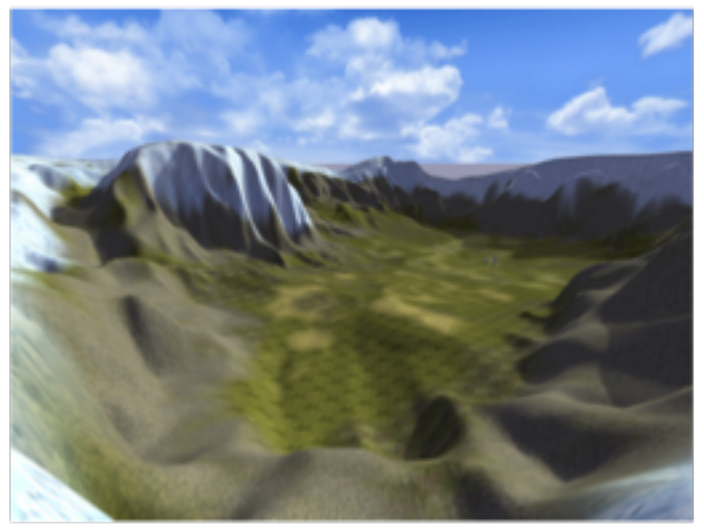
- Acquired real-world objects:
  - Discrete sampling
  - Points, meshes

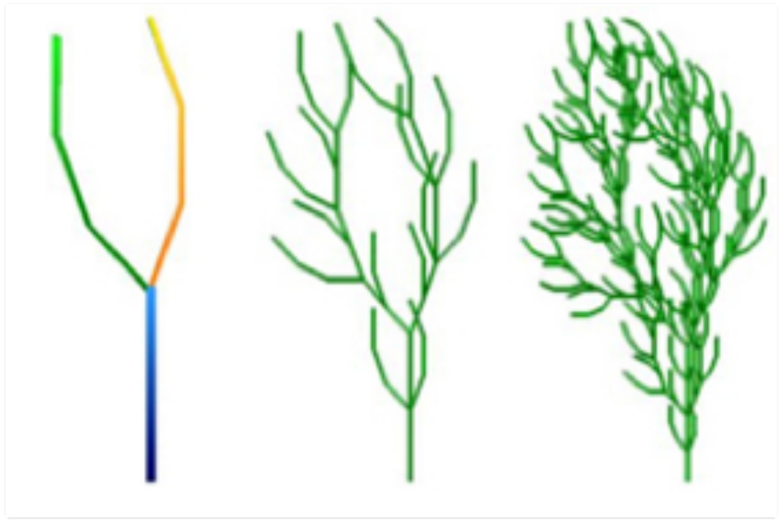


- Modeling “by hand”:
  - Higher-level representations, amendable to modification, control
  - Parametric surfaces, subdivision surfaces, implicits



- Procedural modeling
  - Algorithms, grammars

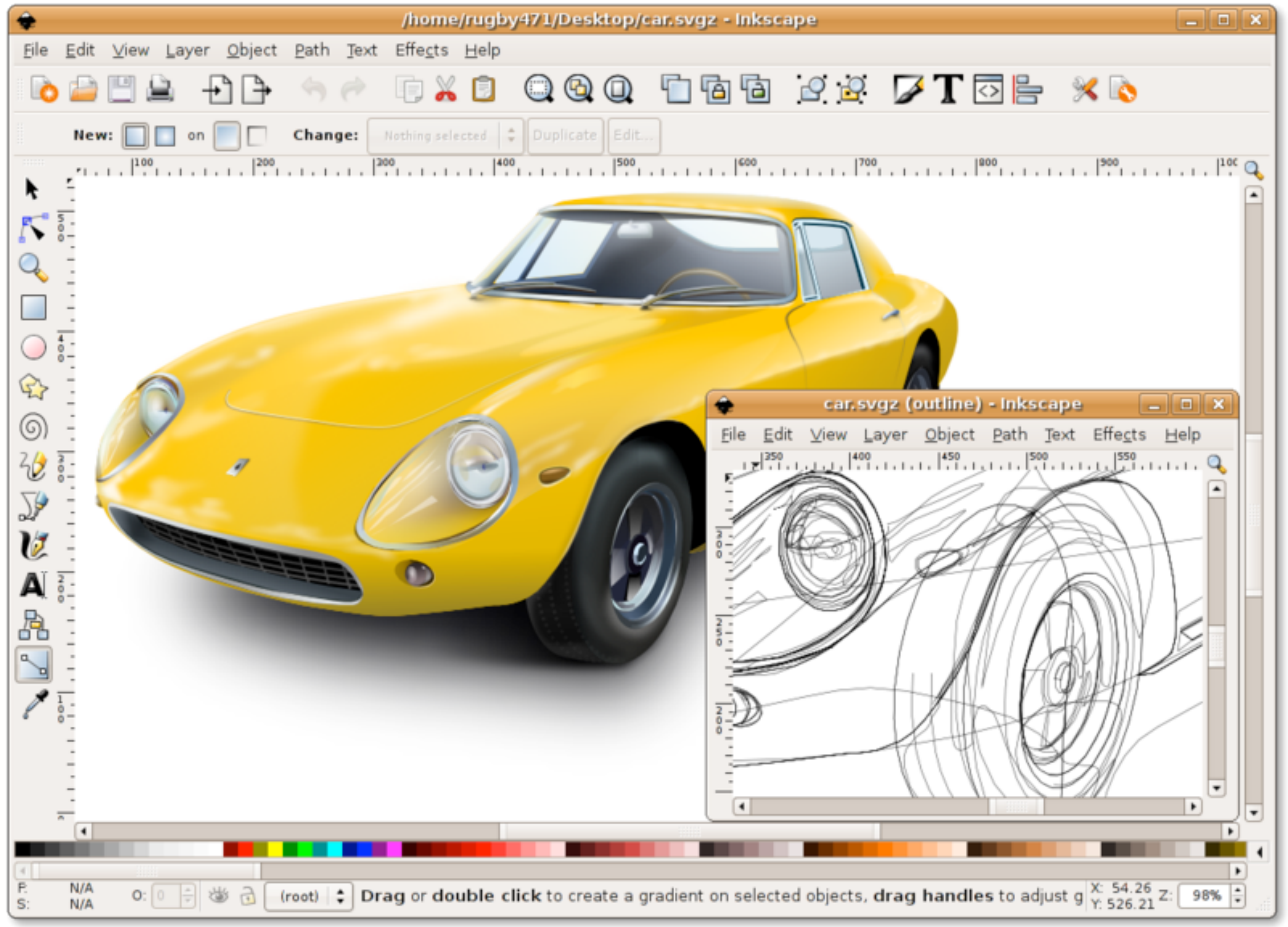




Similar to the 2D Image Domain

- Acquired digital images:
  - Discrete sampling
  - Pixels on a grid

- Painting "by hand":
  - Strokes + color/shading
  - Vector graphics
  - Controls for editing



Similar to the 2D Image Domain

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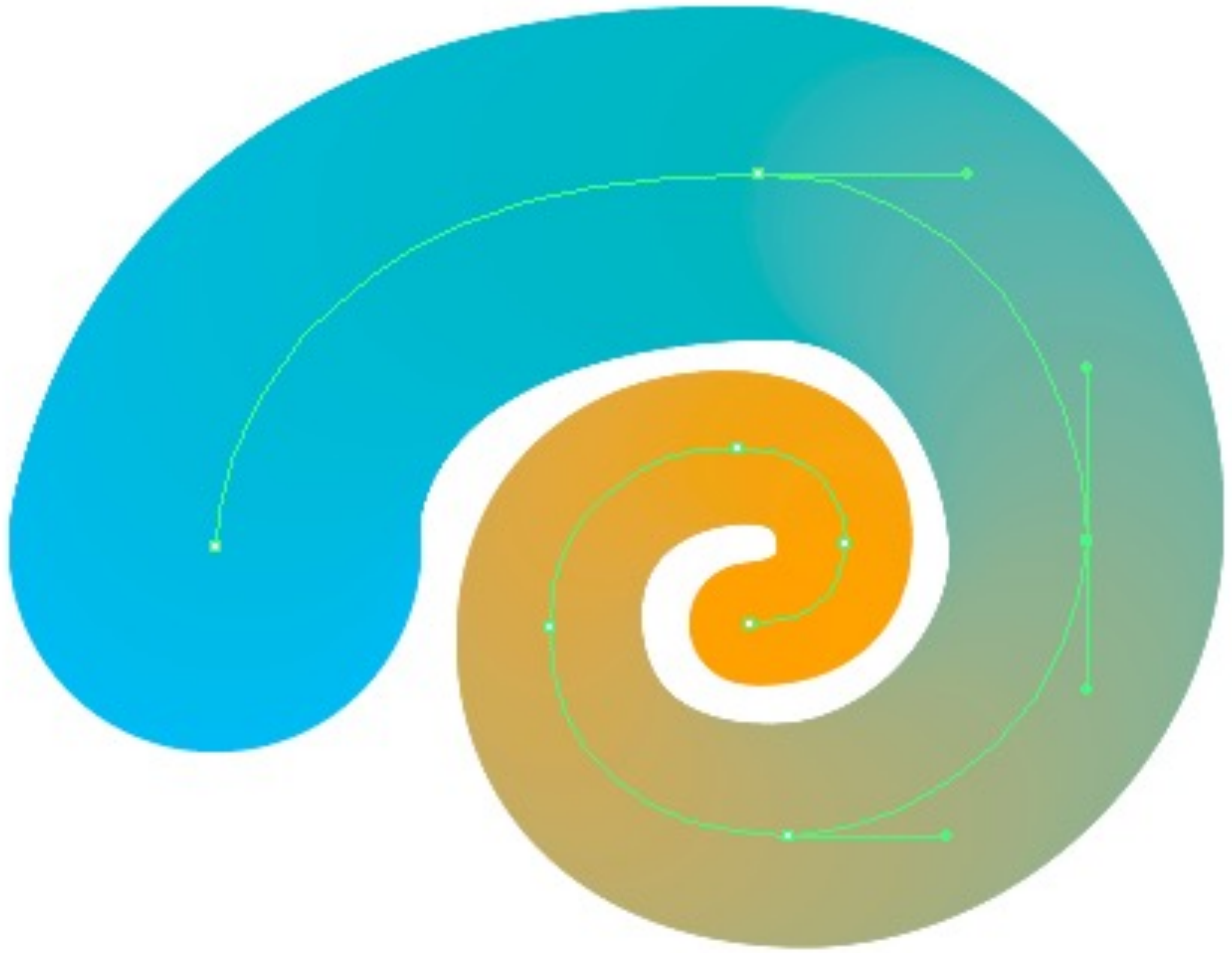
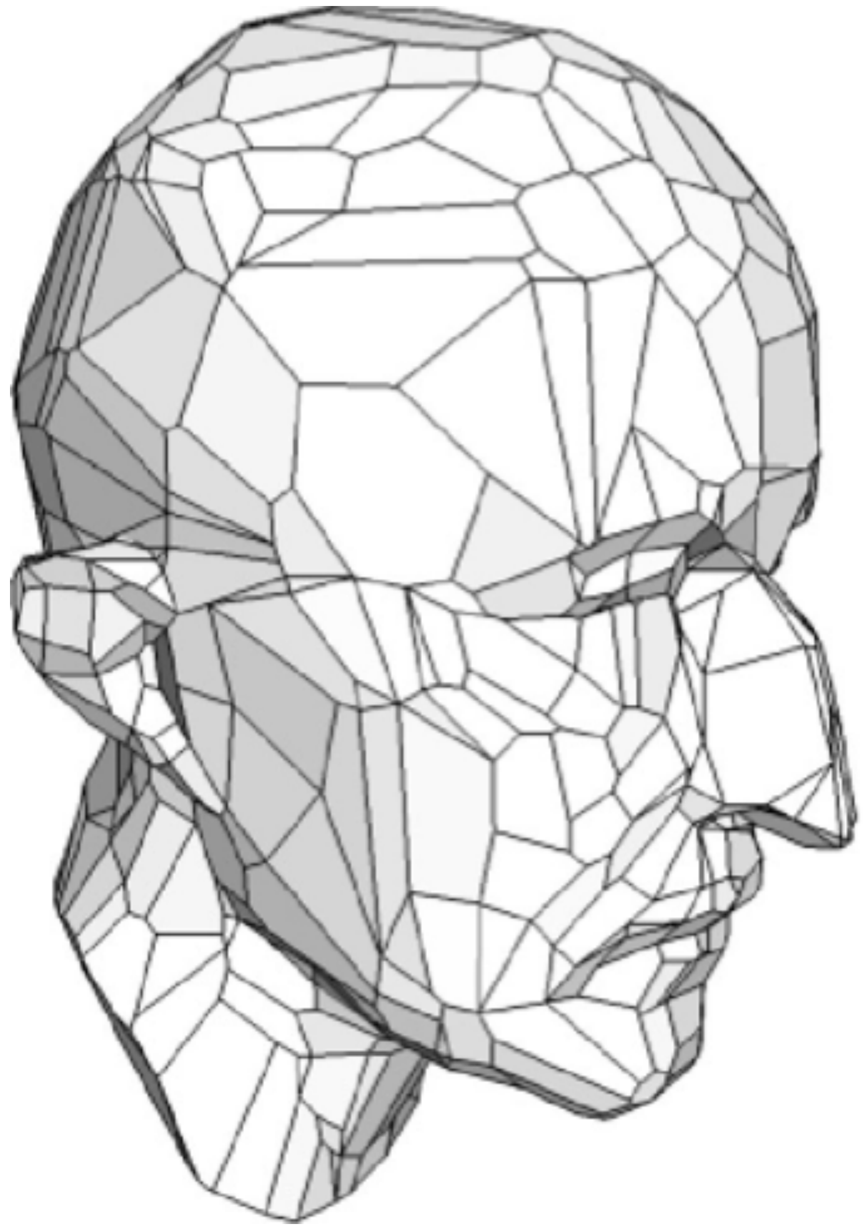


Representation Considerations

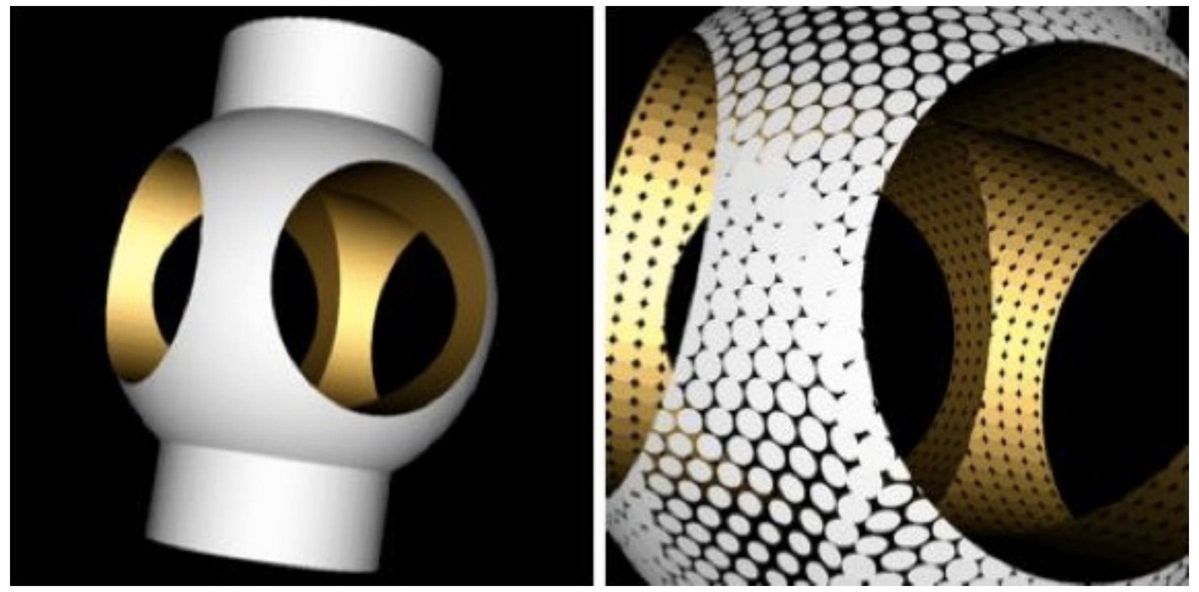
- How should we represent geometry?
  - Needs to be stored in the computer
  - Creation of new shapes
    - Input metaphors, interfaces…
  - What operations do we apply?
    - Editing, simplification, smoothing, filtering, repair…
  - How to render it?
    - Rasterization, raytracing…



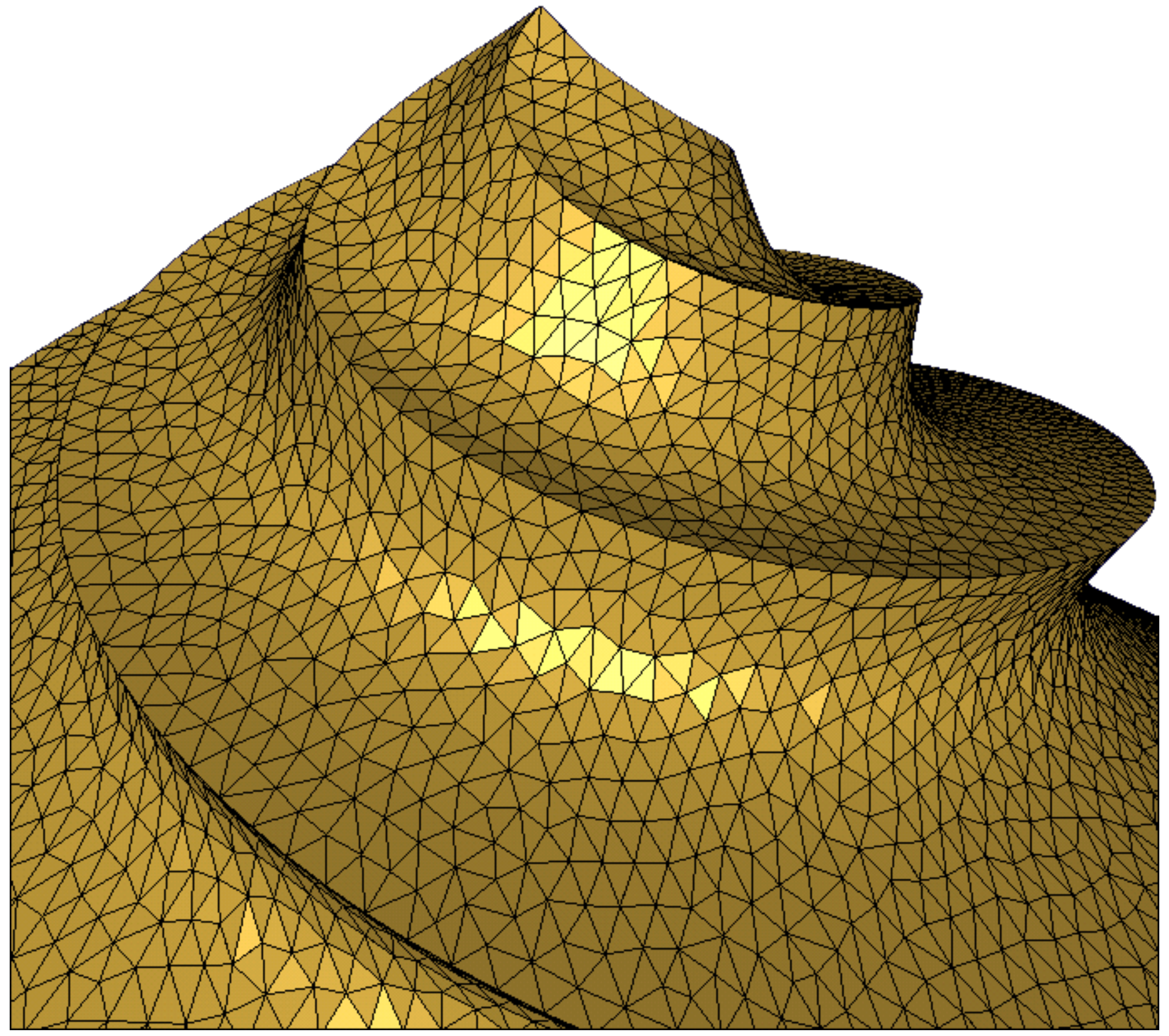
Variational Shape Approximation

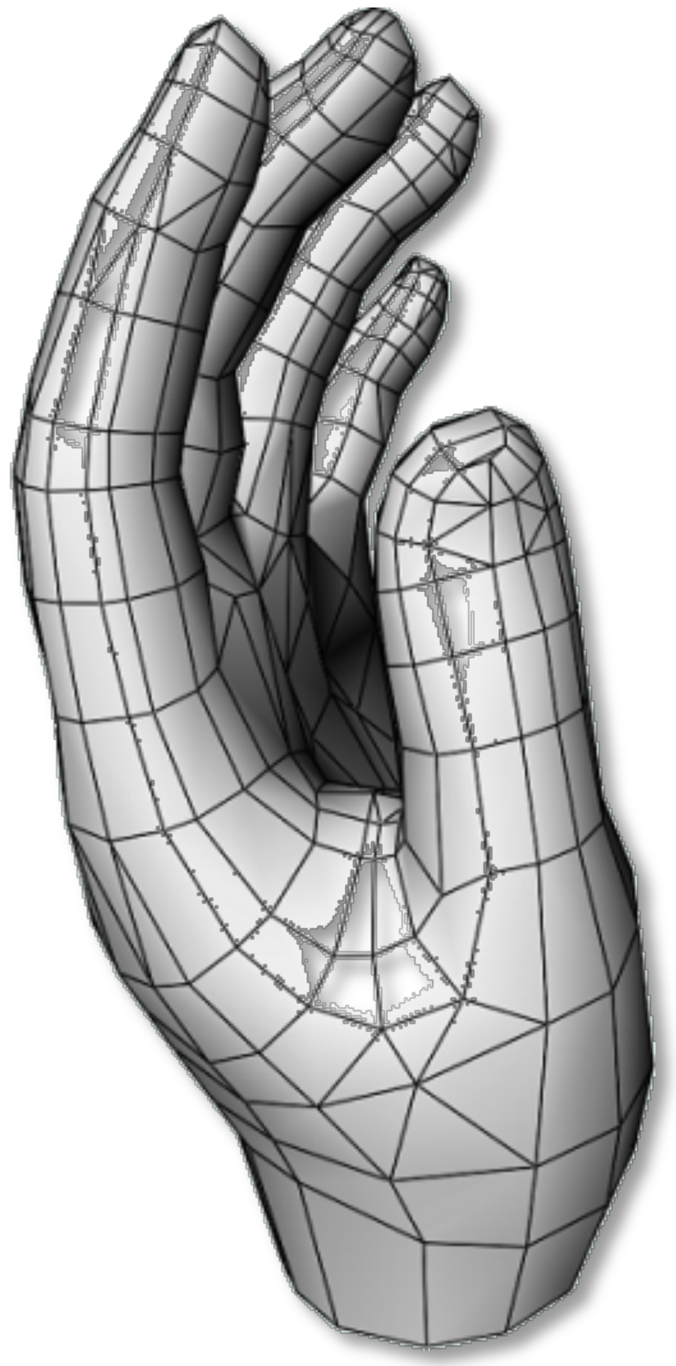
Shape Representations

- Points



- Polygonal meshes

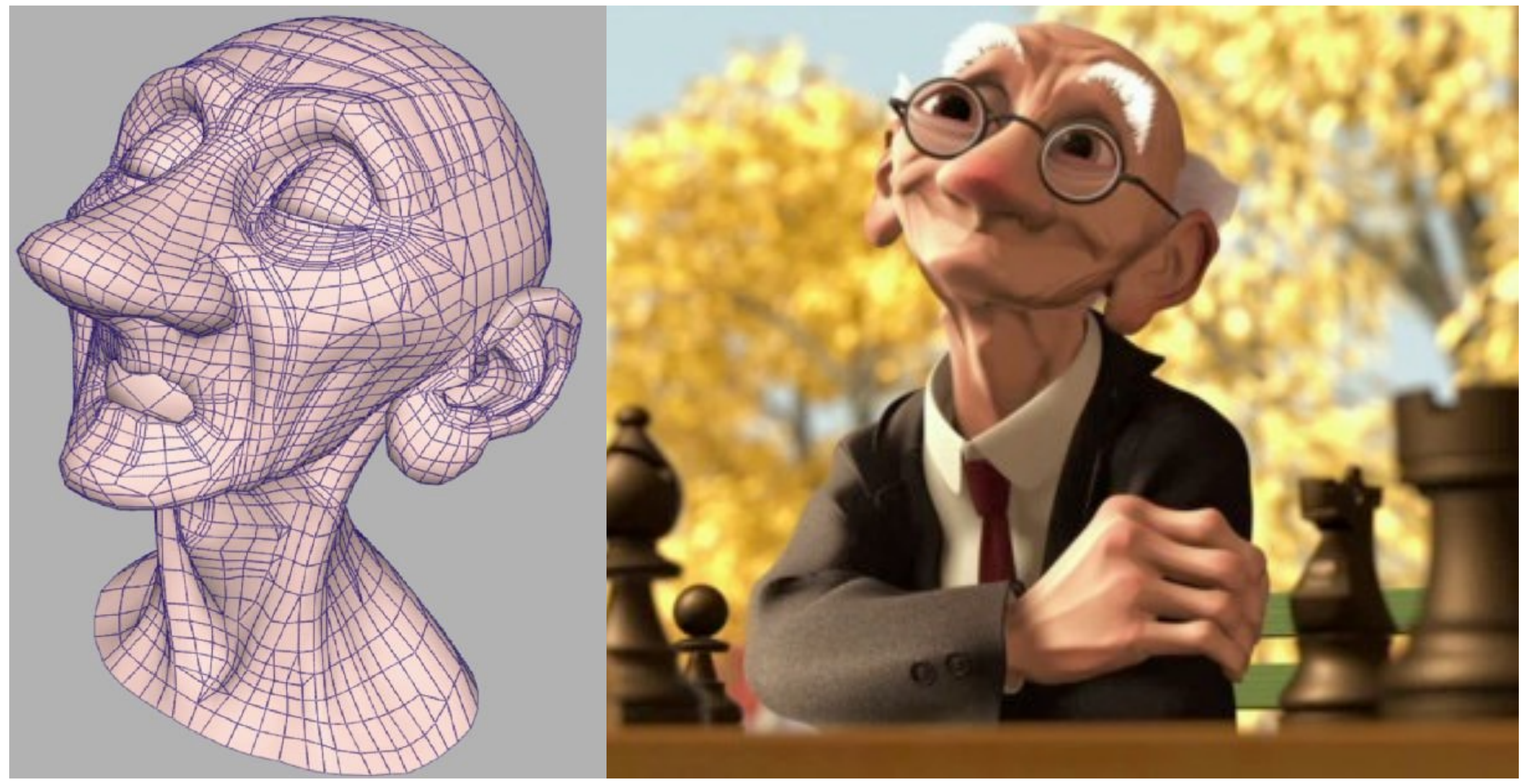




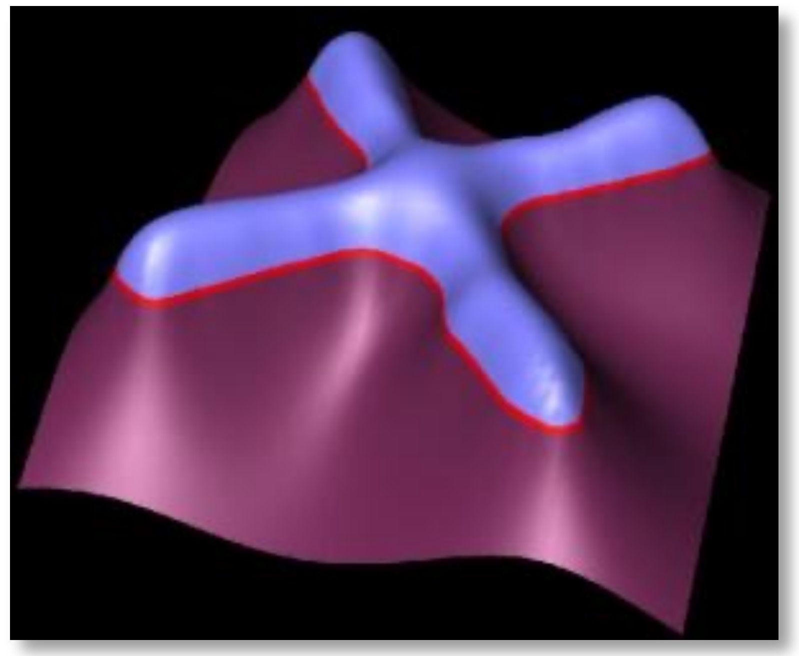
Anisotropic Polygonal Remeshing

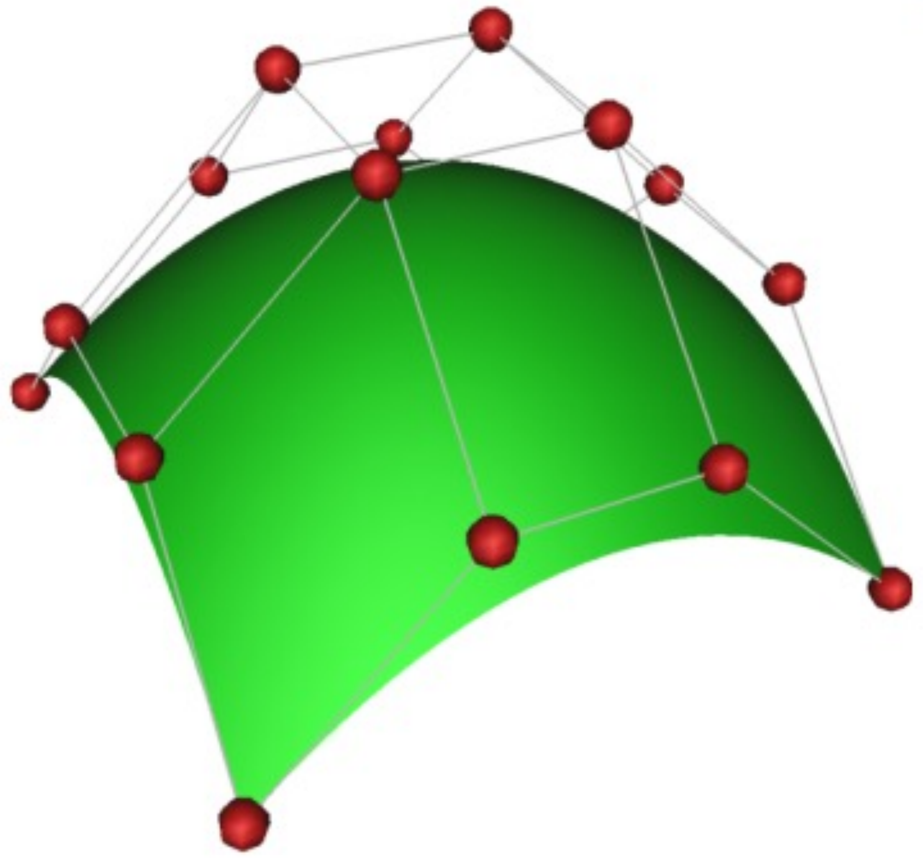
Shape Representations

- Parametric surfaces
- Subdivision surfaces
- Implicit functions



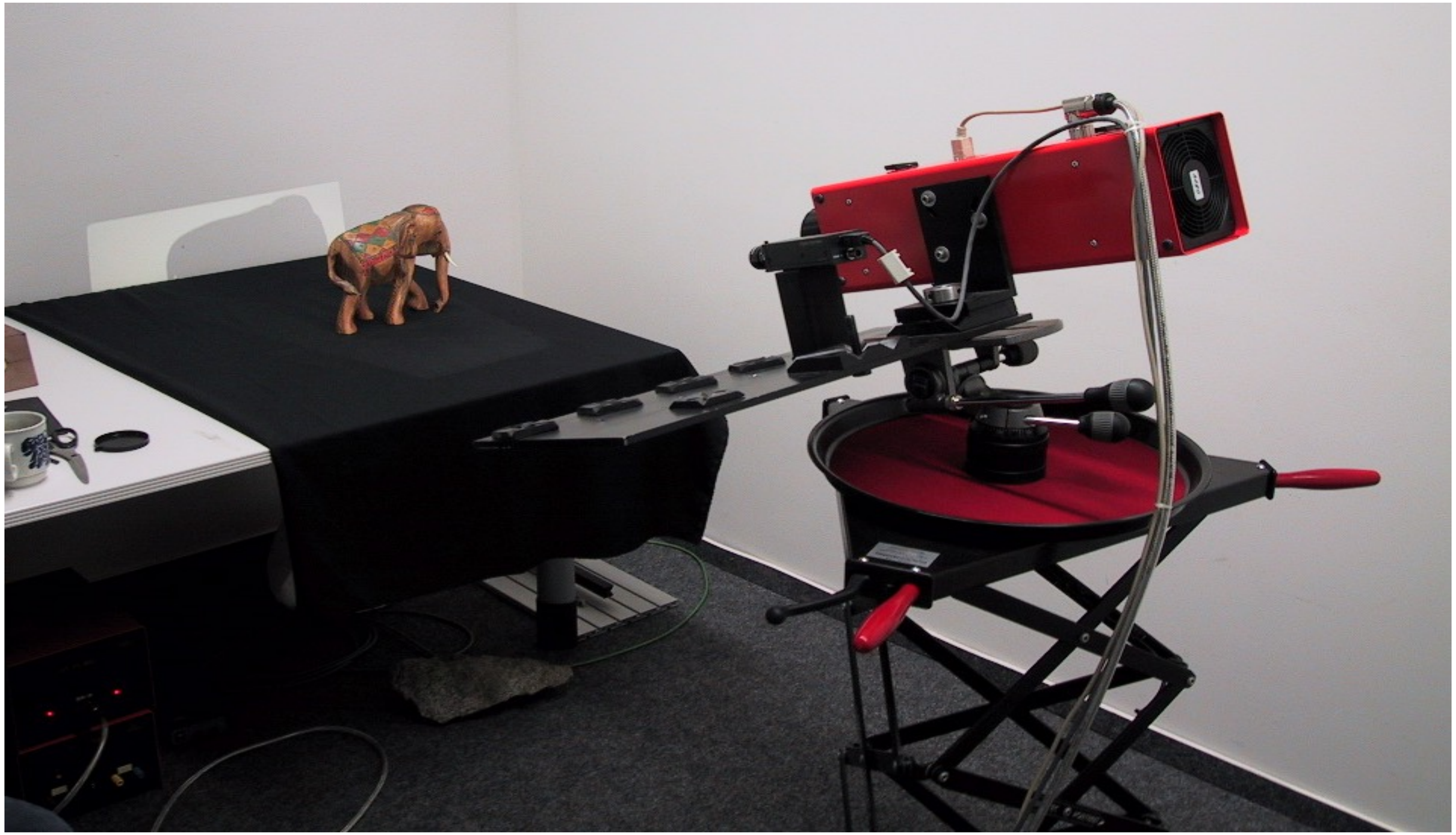
Pixar





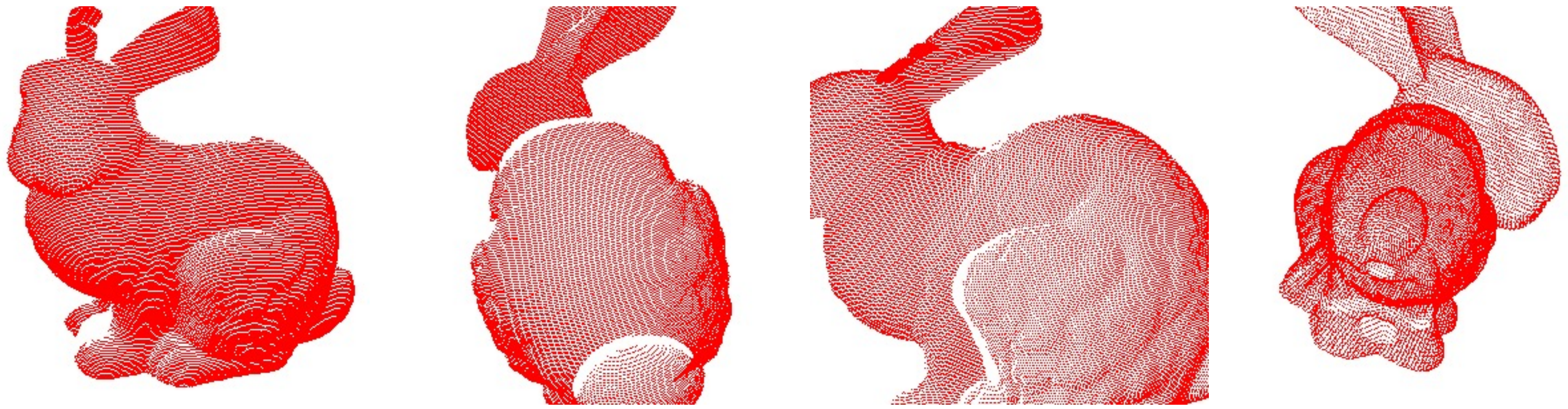
Points

Output of Acquisition

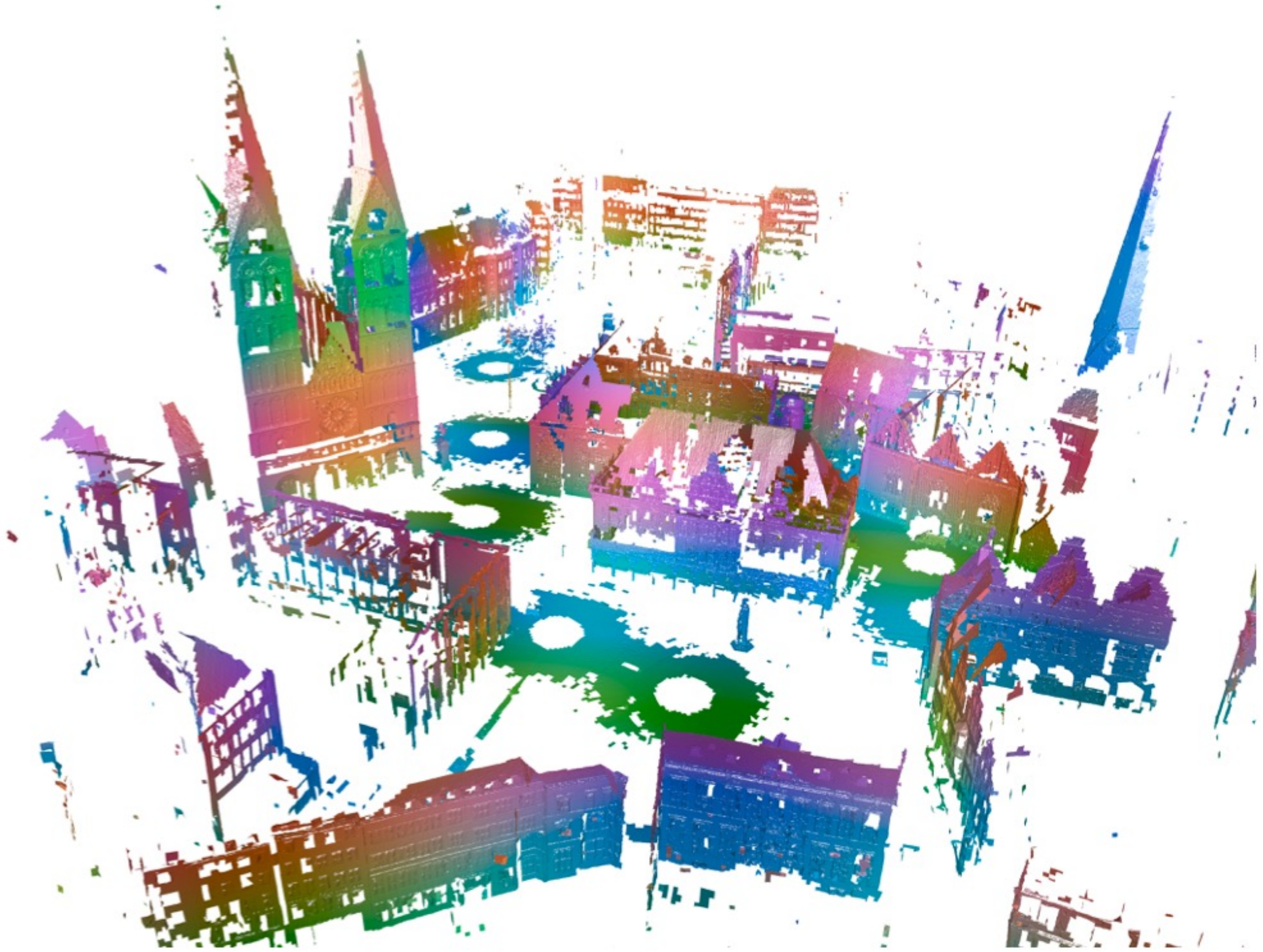


Points

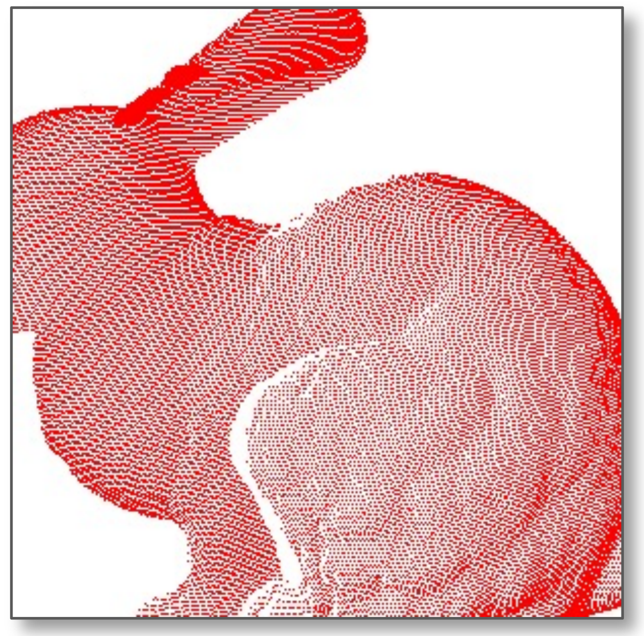
- Standard 3D data from a variety of sources
  - Often results from scanners
  - Potentially noisy



  - Depth imaging
  - Registration of multiple images



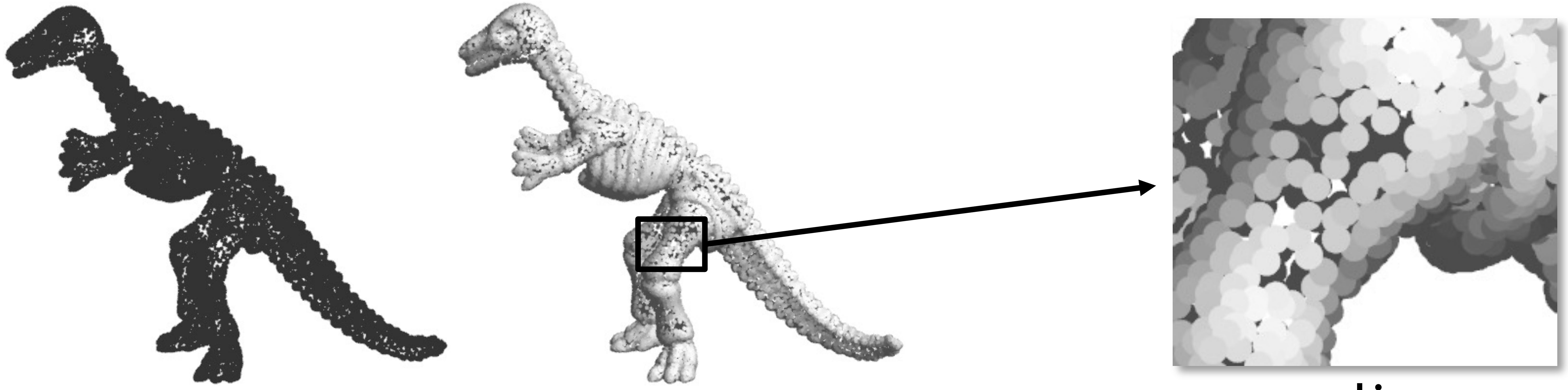
Points



- points = unordered set of 3-tuples
- Often converted to other reps
  - Meshes, implicits, parametric surfaces
  - Easier to process, edit and/or render
- Efficient point processing and modeling requires a spatial partitioning data structure
  - To figure out neighborhoods

Points: Neighborhood information

- Why do we need neighbors?



need normals (for shading)

upsampling –
need to count density

- Need sub-linear-time implementations of
  - k-nearest neighbors to point $\mathbf{x}$
  - In-radius search $\|\mathbf{p}_i - \mathbf{x}\| < \varepsilon$

Parametric Curves and Surfaces

Parametric Representation

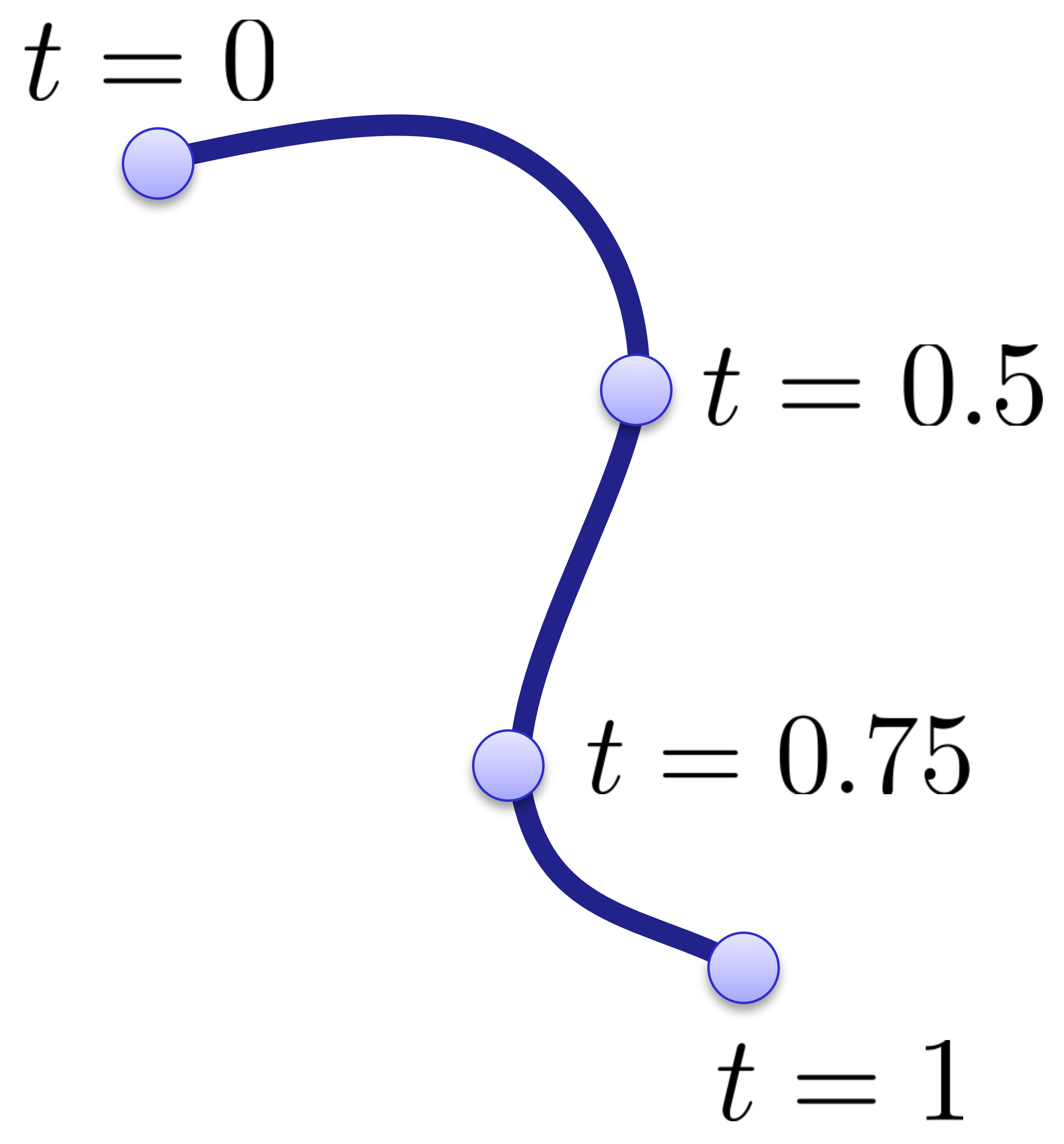
$$f : X \to Y, X \subseteq \mathbb{R}^m, Y \subseteq \mathbb{R}^n$$

- Range of a function
  - Planar curve: $m = 1, n = 2$

    $$s(t) = (x(t), y(t))$$

  - Space curve: $m = 1, n = 3$

    $$s(t) = (x(t), y(t), z(t))$$



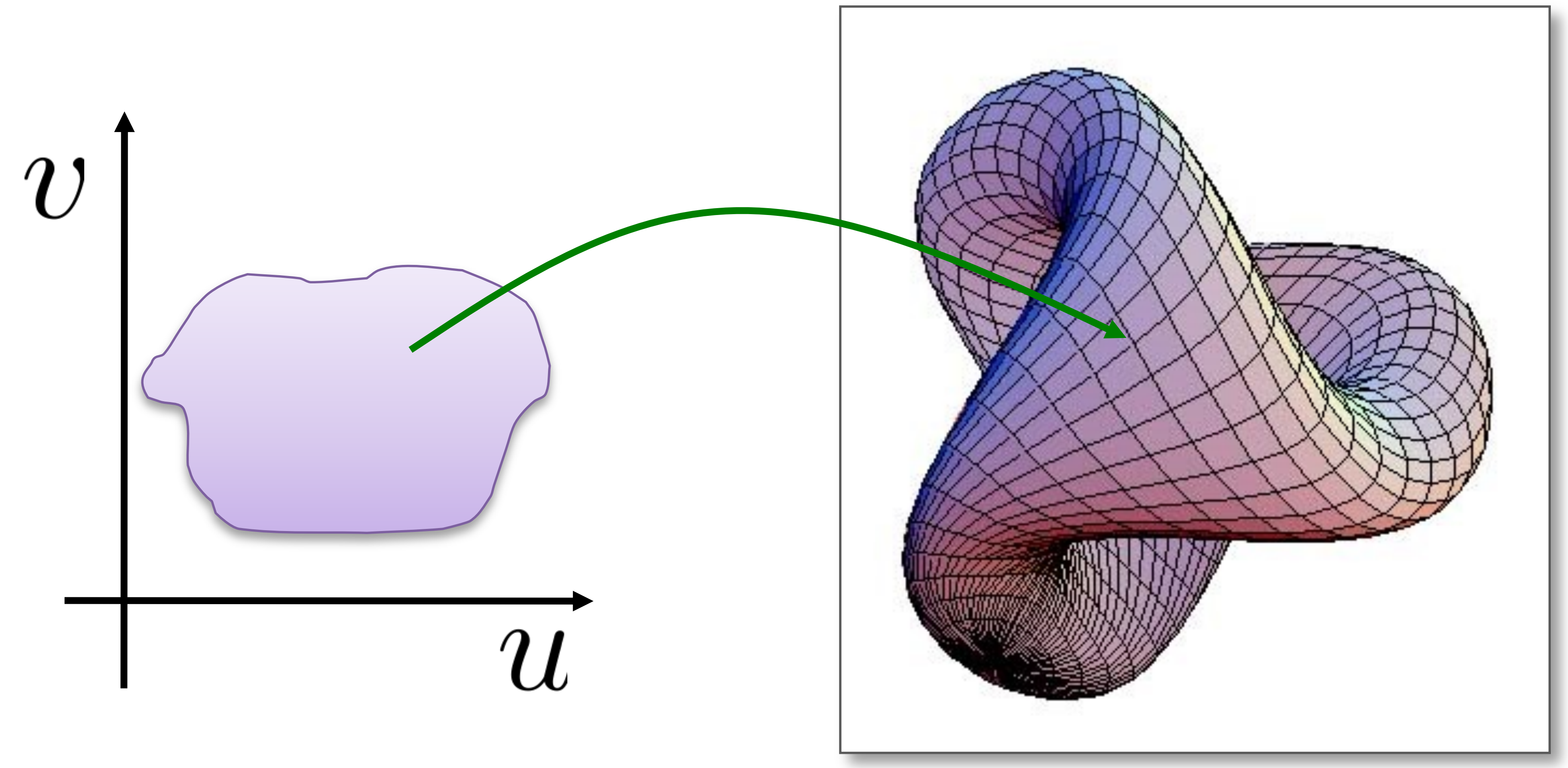
Parametric Representation

$$f : X \to Y, X \subseteq \mathbb{R}^m, Y \subseteq \mathbb{R}^n$$

- Range of a function

  - Surface in 3D:

$$m = 2, n = 3$$



$$s(u,v) = (x(u,v), y(u,v), z(u,v))$$

Parametric Curves

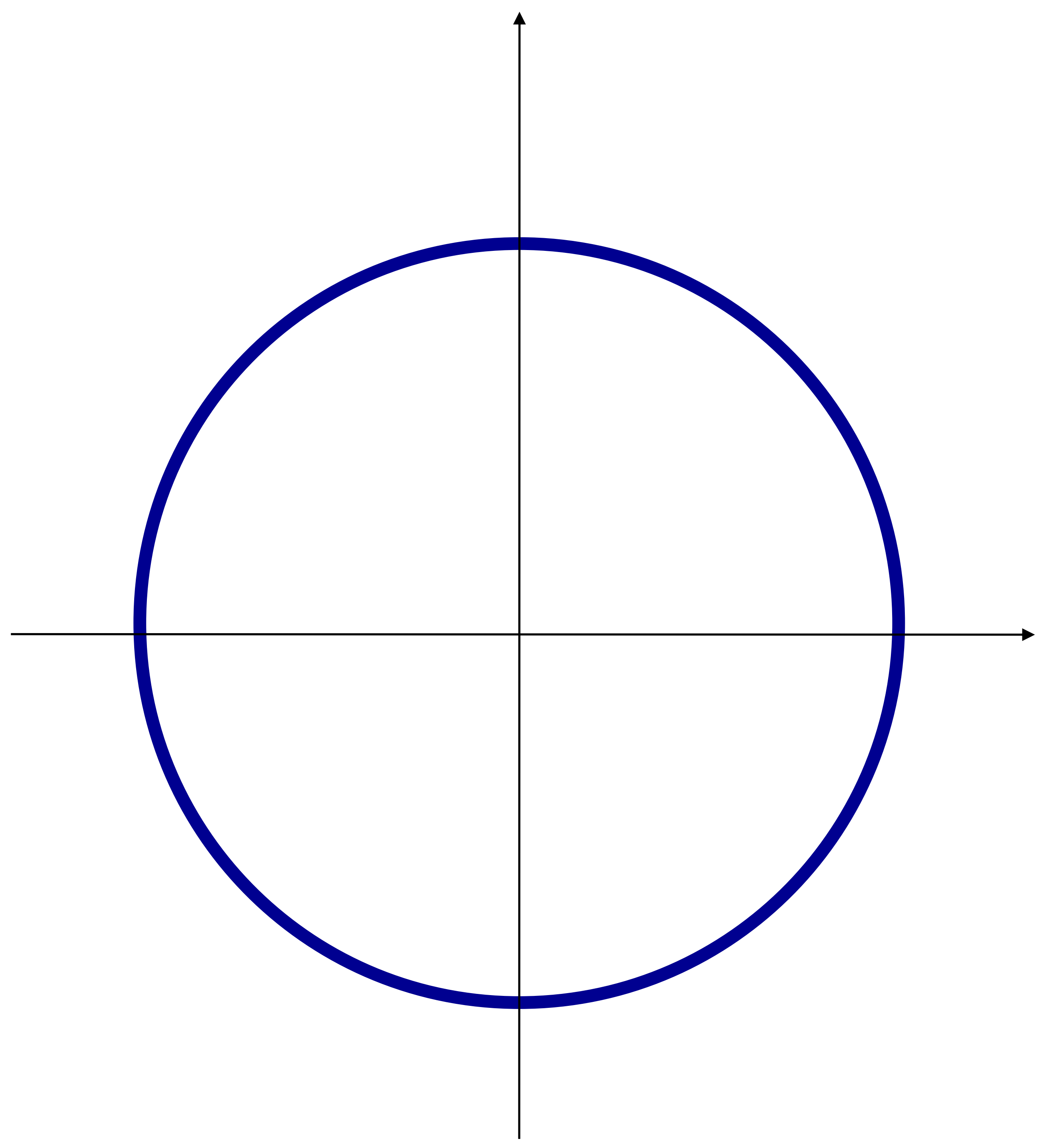
- Explicit curve/circle in 2D

$$\mathbf{p} : \mathbb{R} \to \mathbb{R}^2$$

$$t \mapsto \mathbf{p}(t) = (x(t), y(t))$$

$$\mathbf{p}(t) = r\,(\cos(t), \sin(t))$$

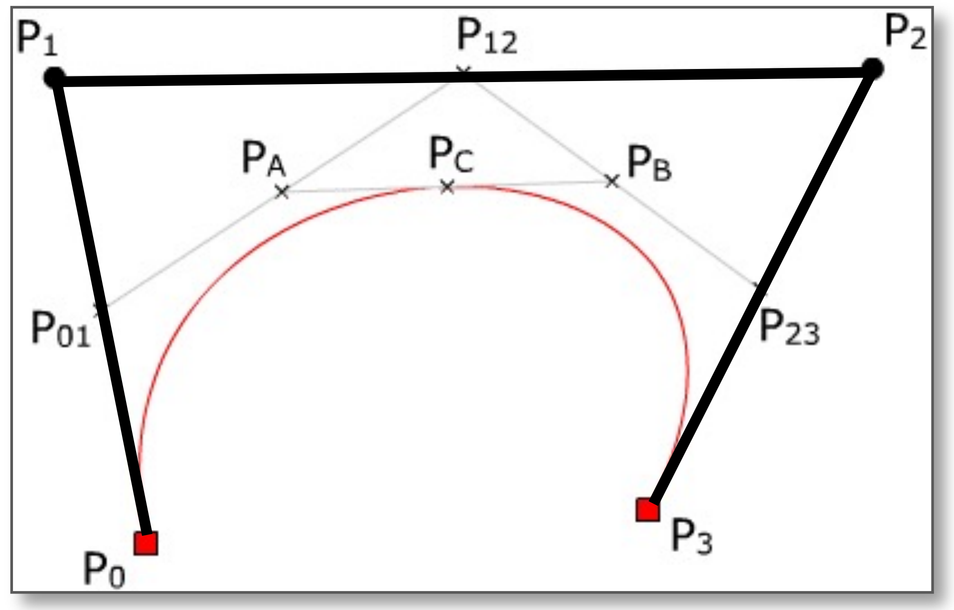
$$t \in [0, 2\pi)$$



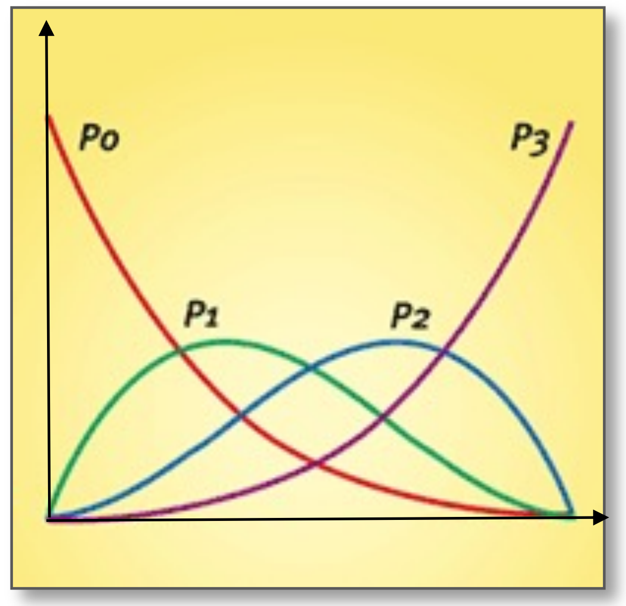
Parametric Curves

- Bezier curves, splines

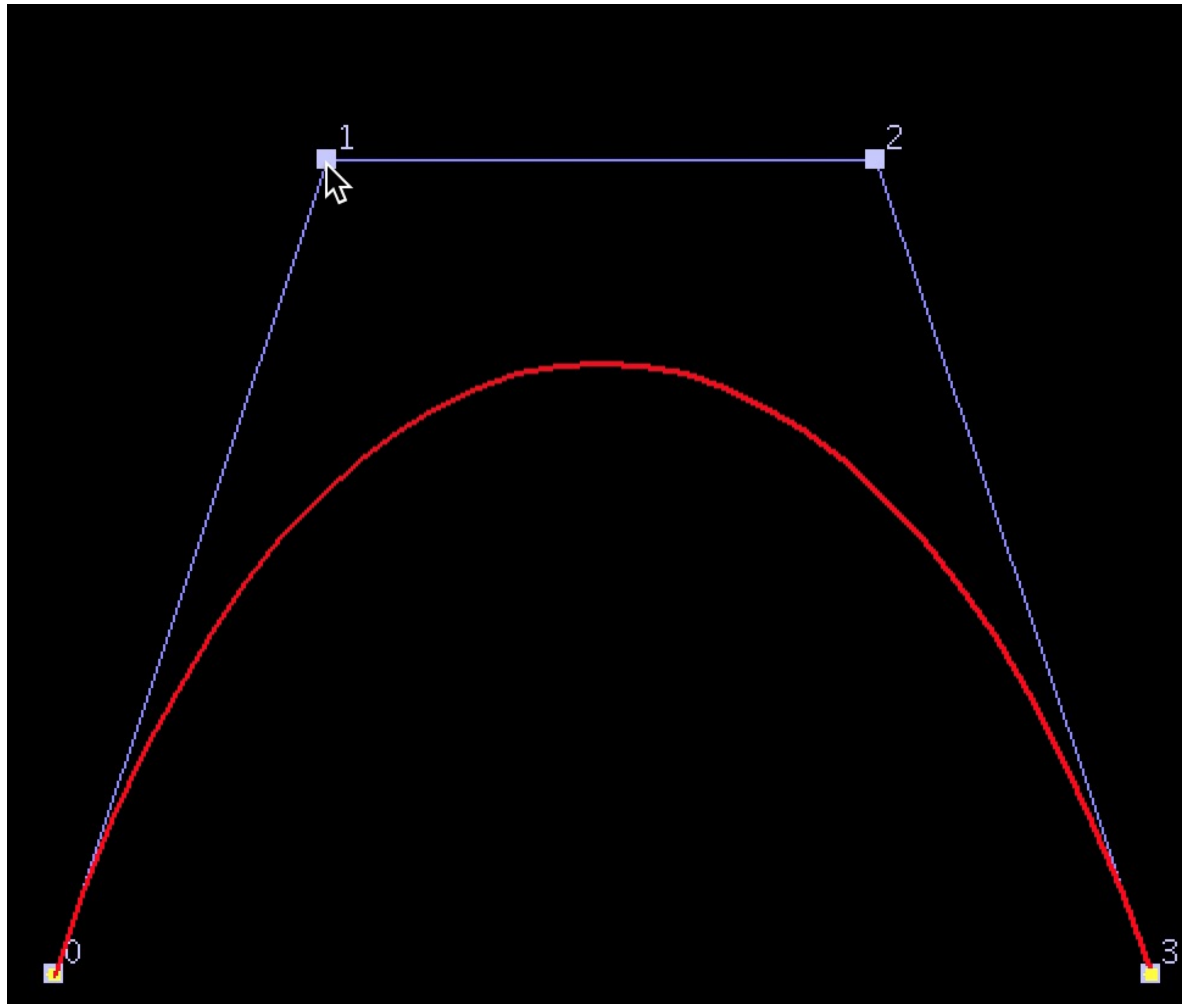
$$s(t) = \sum_{i=0}^{n} \mathbf{p}_i B_i^n(t) \qquad B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i}$$



Curve and control polygon



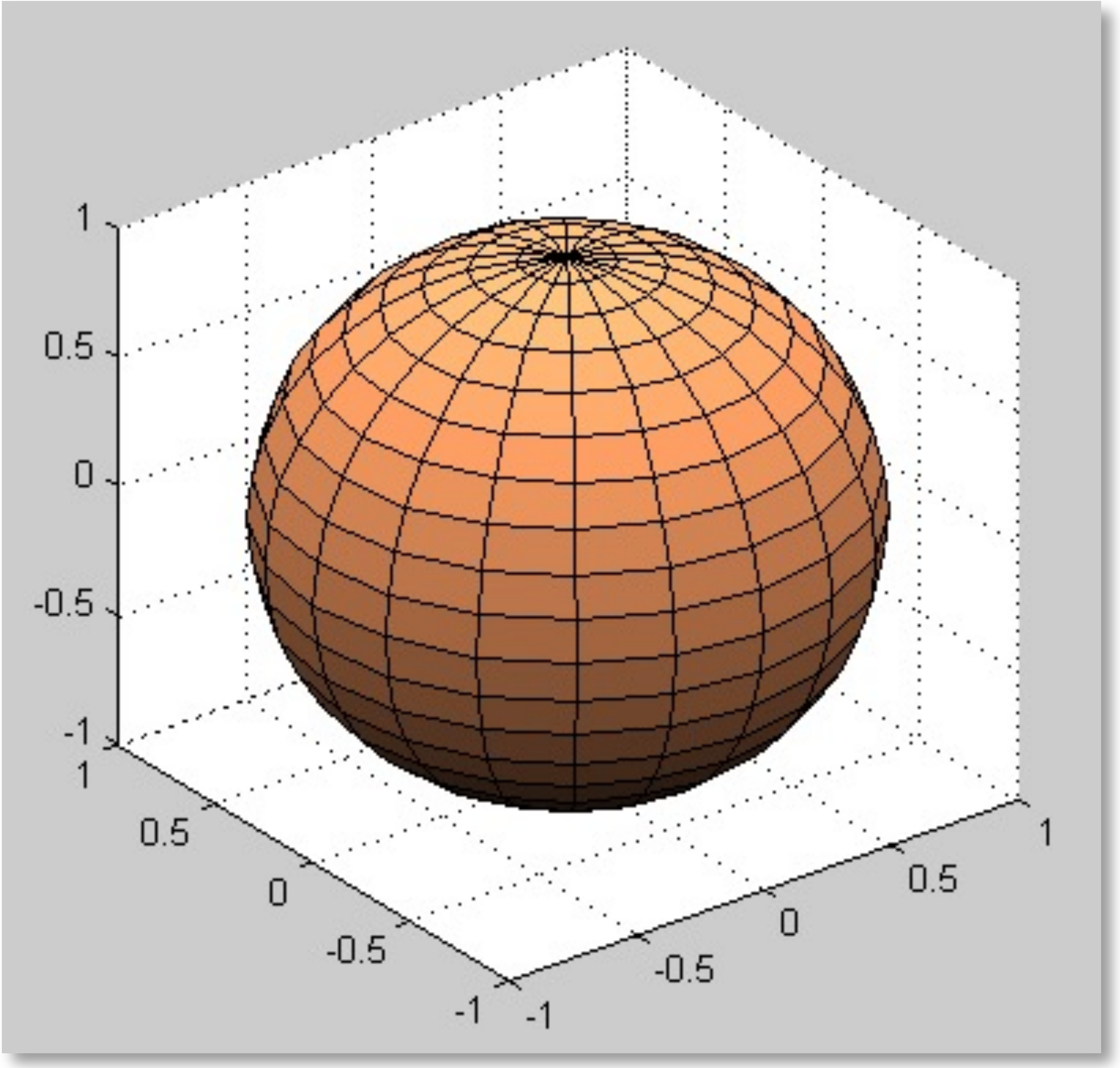
Basis functions



Parametric Surfaces

- Sphere in 3D

$$s : \mathbb{R}^2 \to \mathbb{R}^3$$



$$s(u,v) = r\left(\cos(u)\cos(v), \sin(u)\cos(v), \sin(v)\right)$$

$$(u,v) \in [0, 2\pi) \times [-\pi/2, \pi/2]$$

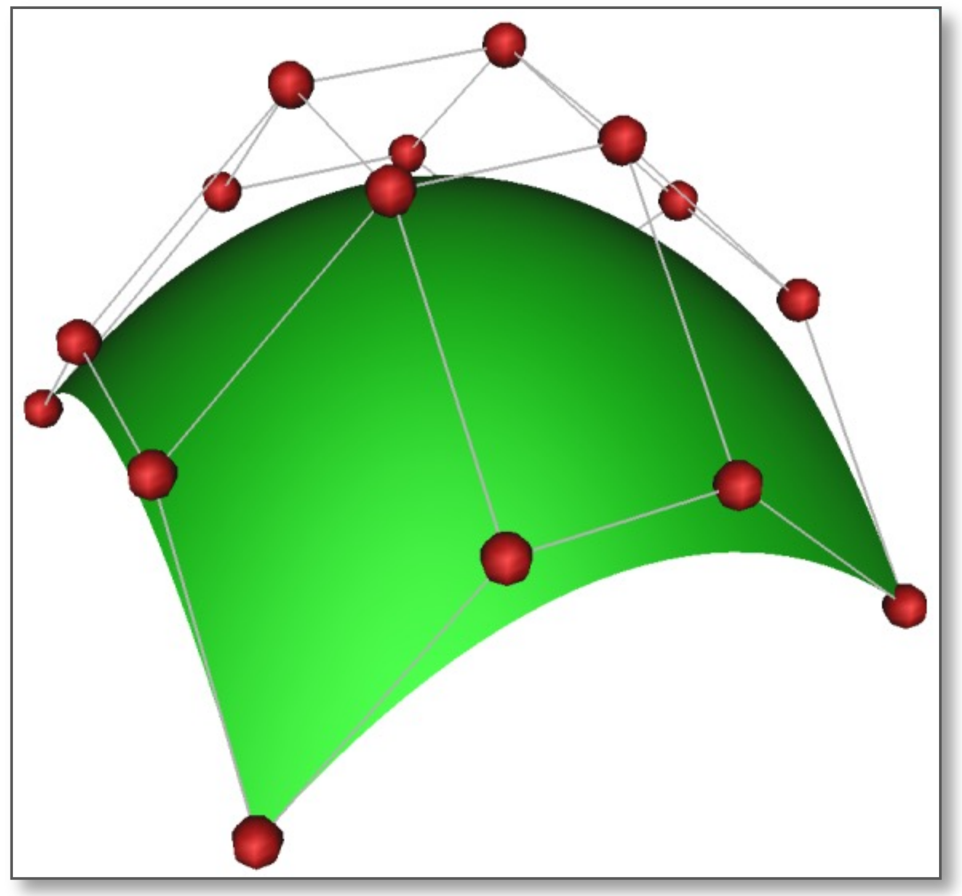
Parametric Surfaces

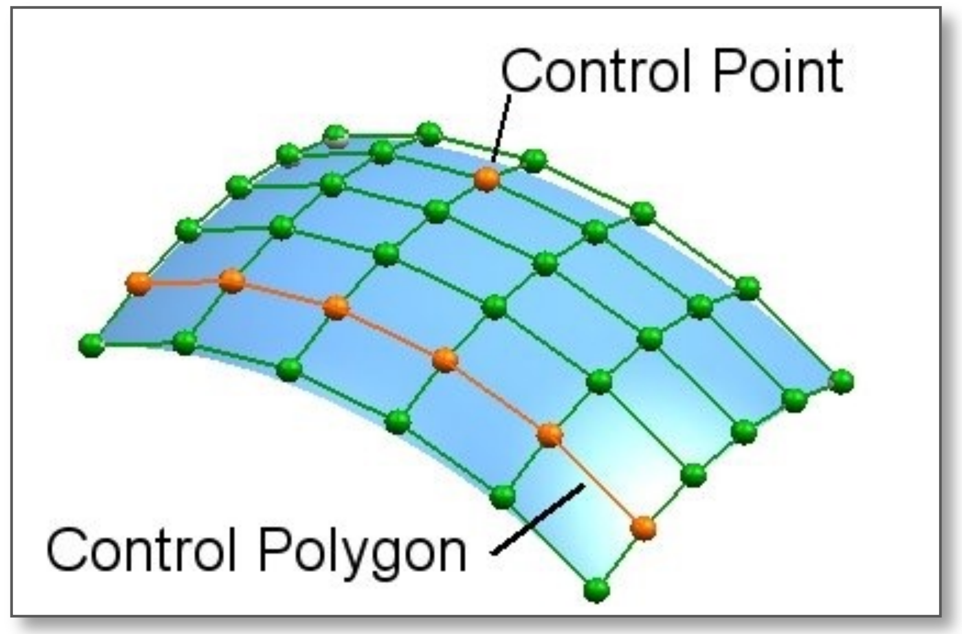
- Curve swept by another curve

$$s(u,v) = \sum_{i,j} \mathbf{p}_{i,j} B_i(u) B_j(v)$$

- Bezier surface:

$$s(u,v) = \sum_{i=0}^{m} \sum_{j=0}^{n} \mathbf{p}_{i,j} B_i^m(u) B_j^n(v)$$



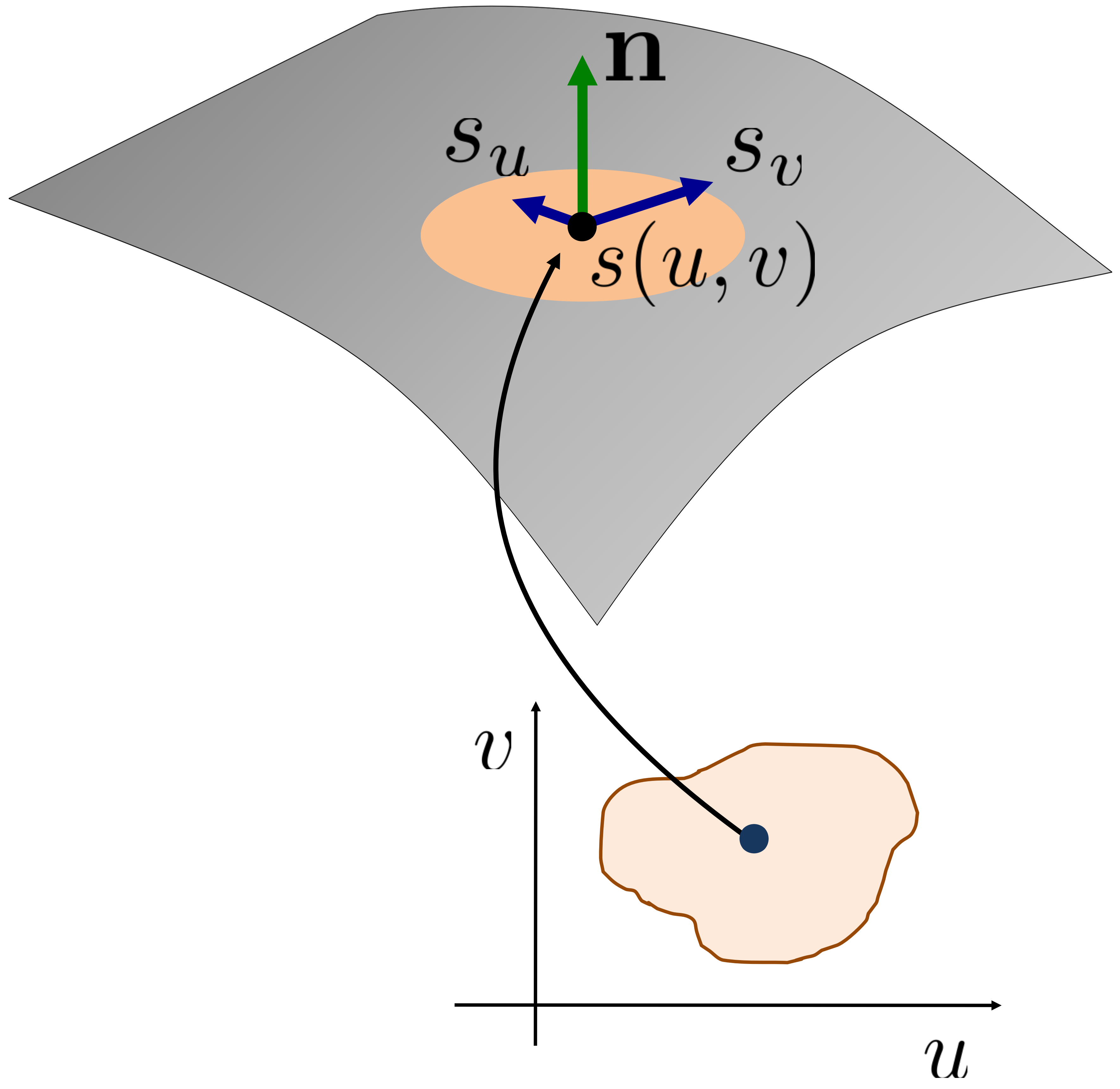


Tangents and Normal

$$s_u = \frac{\partial s(u,v)}{\partial u}$$

$$s_v = \frac{\partial s(u,v)}{\partial v}$$

$$\mathbf{n} = \frac{s_u \times s_v}{\|s_u \times s_v\|}$$



Tangent plane is normal to $\mathbf{n}$

Parametric Curves and Surfaces

- Advantages
  - Easy to generate points on the curve/surface
  - Separates x/y/z components

- Disadvantages
  - Hard to determine inside/outside
  - Hard to determine if a point is **on** the curve/surface

Implicit Curves and Surfaces

Implicit Curves and Surfaces

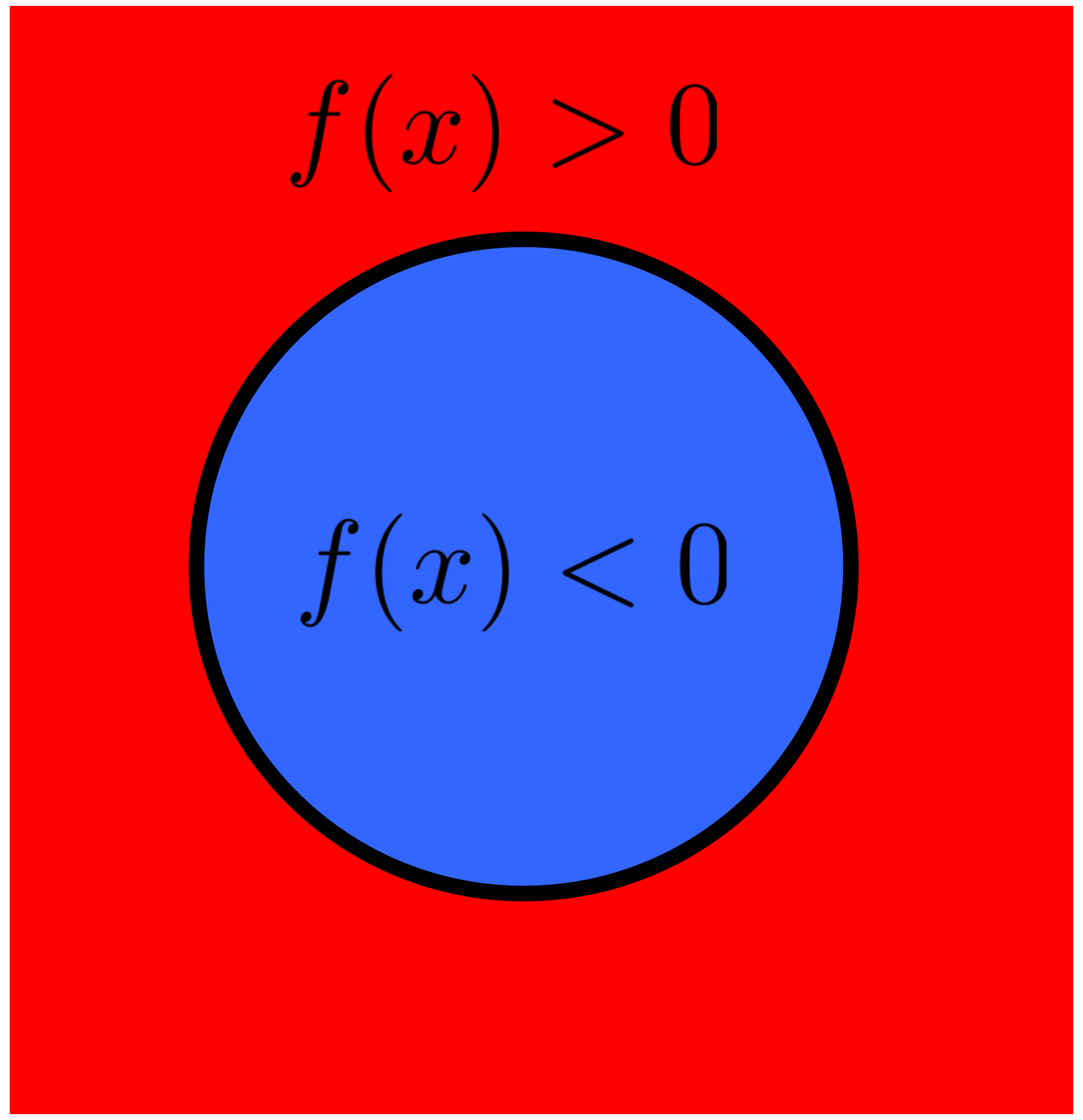
- Kernel of a scalar function $f : \mathbb{R}^m \to \mathbb{R}$
  - Curve in 2D: $S = \{x \in \mathbb{R}^2 | f(x) = 0\}$
  - Surface in 3D: $S = \{x \in \mathbb{R}^3 | f(x) = 0\}$

- Space partitioning

$\{x \in \mathbb{R}^m | f(x) > 0\}$ Outside

$\{x \in \mathbb{R}^m | f(x) = 0\}$ Curve/Surface

$\{x \in \mathbb{R}^m | f(x) < 0\}$ Inside

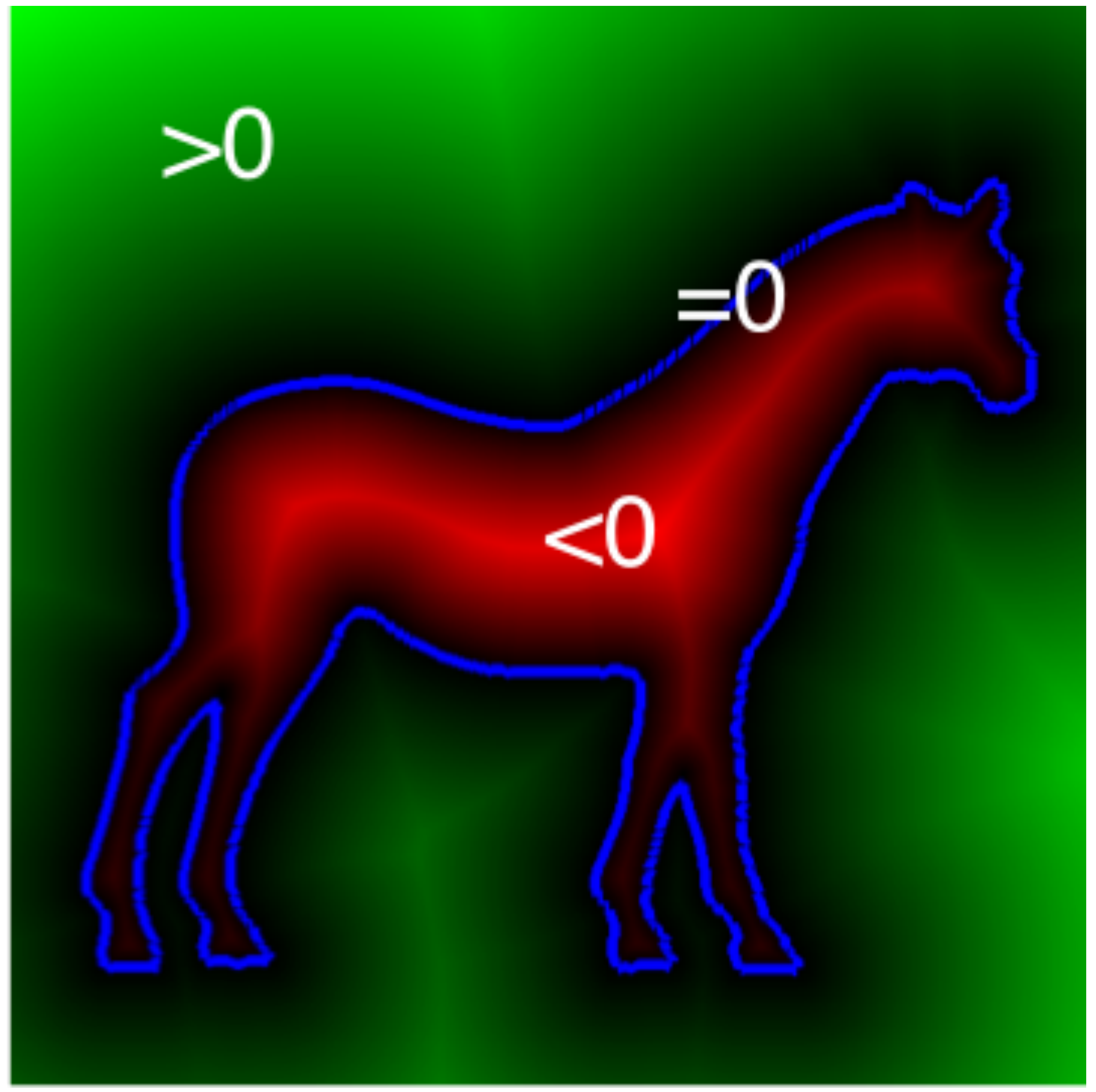


Implicit Curves and Surfaces

$$f : \mathbb{R}^m \to \mathbb{R}$$

- Kernel of a scalar function
  - Curve in 2D: $S = \{x \in \mathbb{R}^2 | f(x) = 0\}$
  - Surface in 3D: $S = \{x \in \mathbb{R}^3 | f(x) = 0\}$

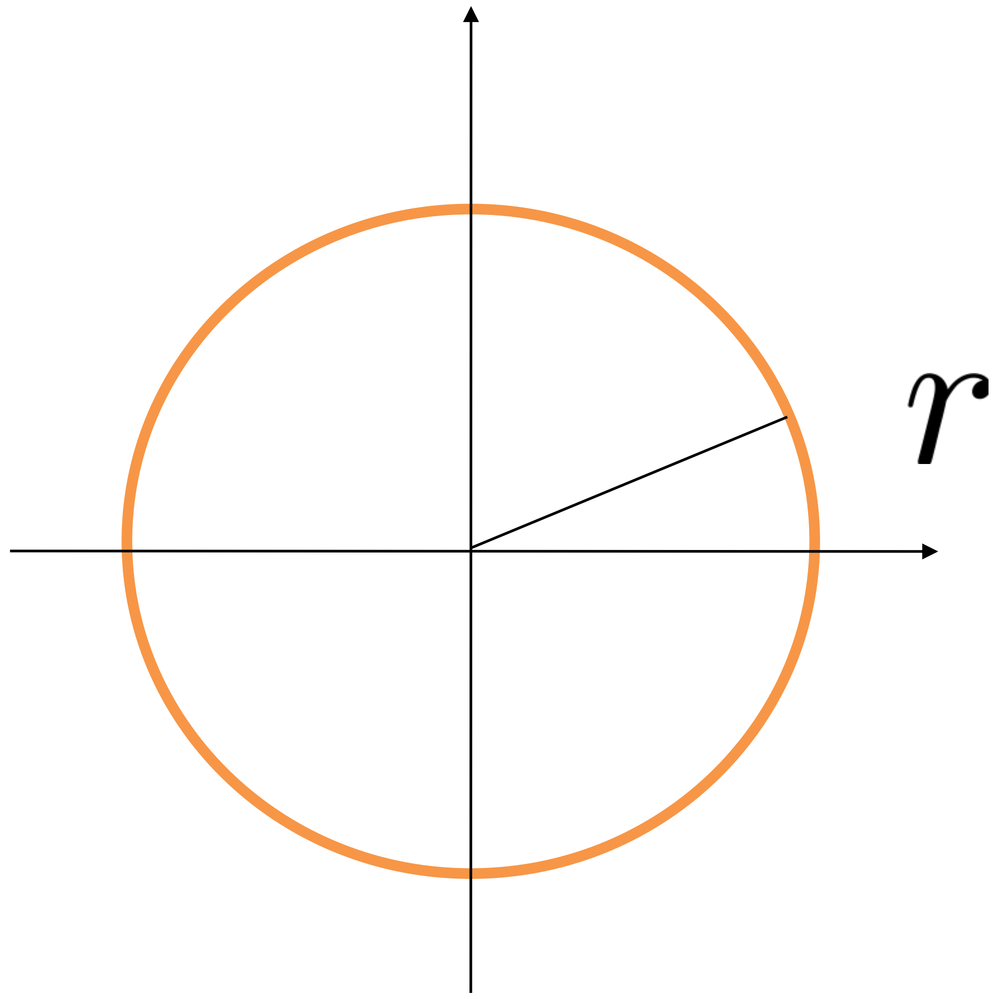
- Zero level set of signed distance function

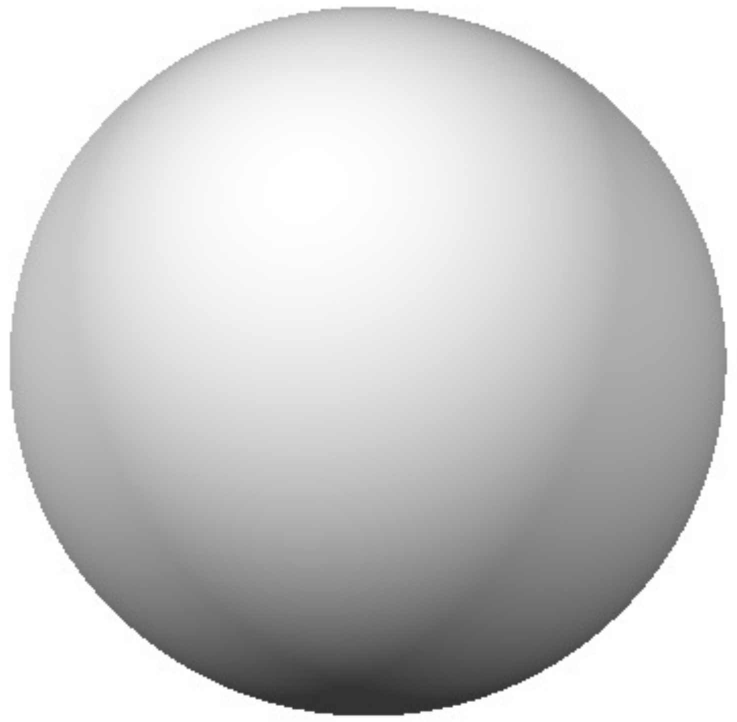


Implicit Curves and Surfaces

- Implicit circle and sphere

$$f(x,y) = x^2 + y^2 - r^2 \qquad f(x,y,z) = x^2 + y^2 + z^2 - r^2$$





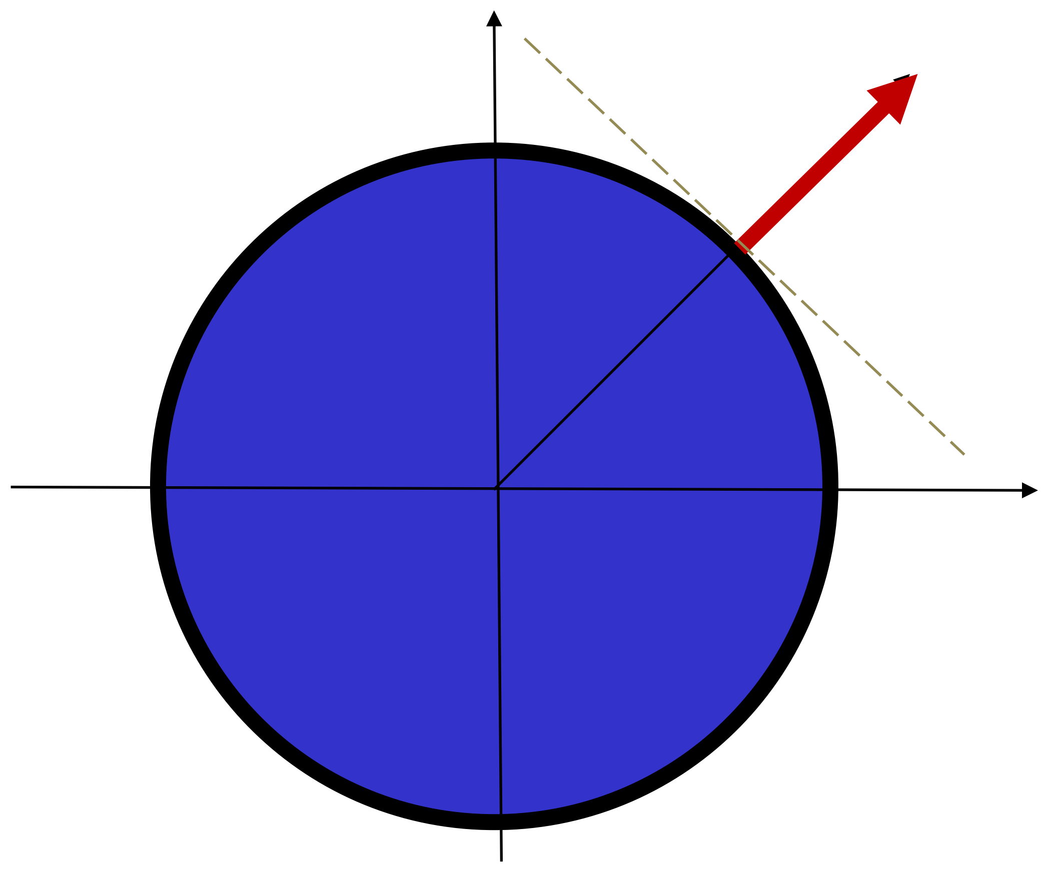
Implicit Curves and Surfaces

- The normal direction to the surface (curve) is given by the gradient of the implicit function

$$\nabla f(x,y,z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)^T$$

- Example

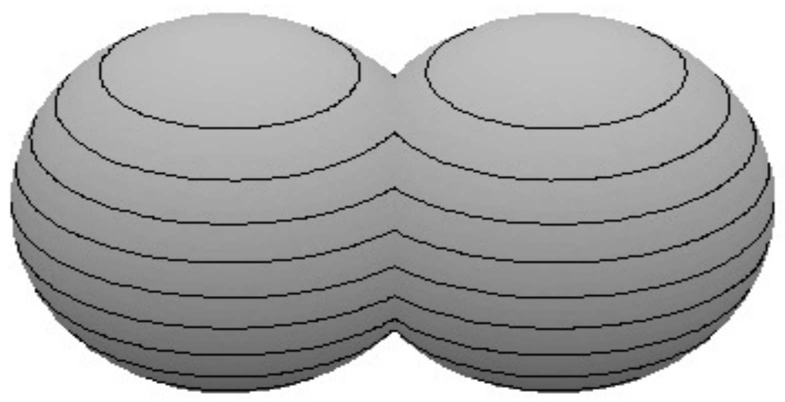
$$f(x,y,z) = x^2 + y^2 + z^2 - r^2$$
$$\nabla f(x,y,z) = (2x, 2y, 2z)^T$$

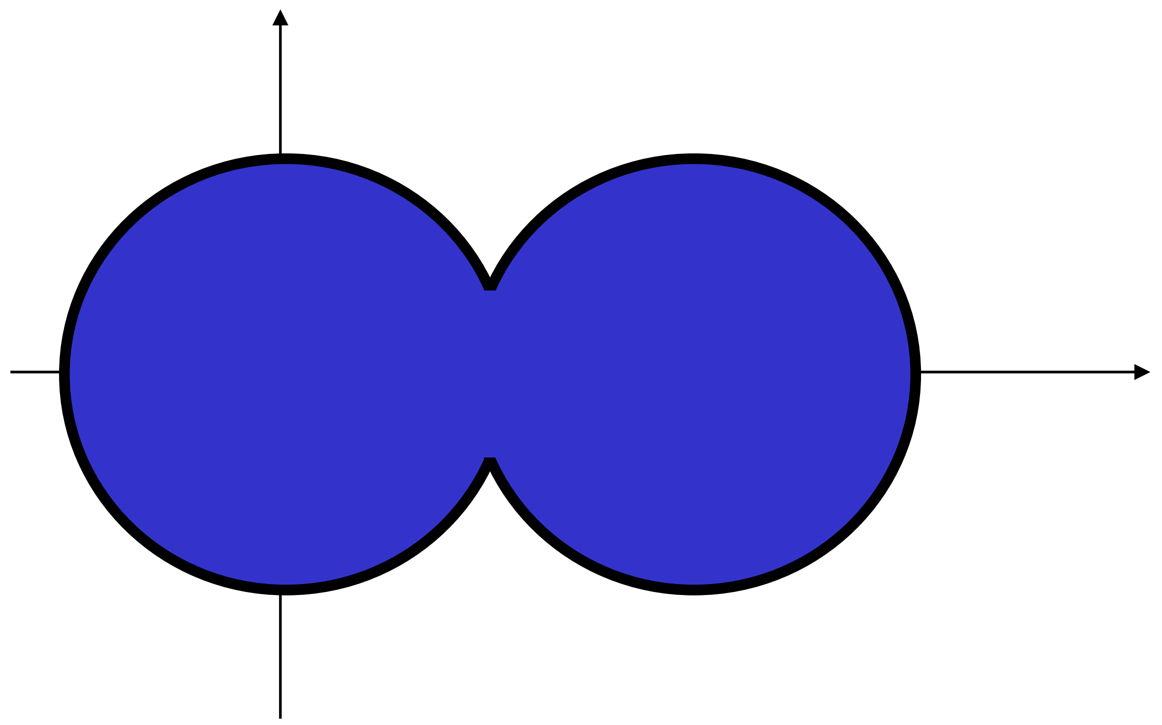


Boolean Set Operations

- Union:

$$\bigcup_i f_i(x) = \min f_i(x)$$





- Intersection:

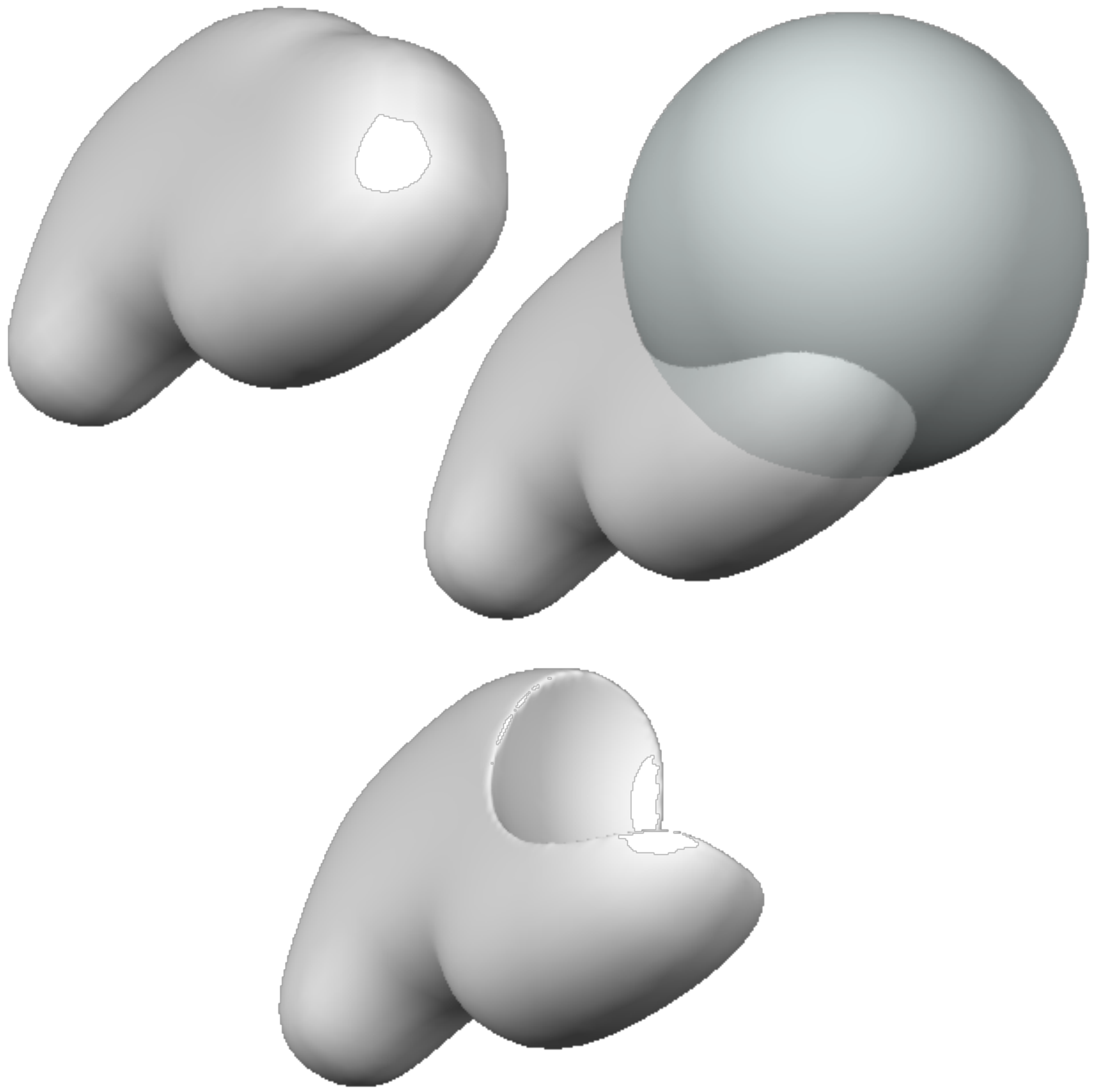
$$\bigcap_i f_i(x) = \max f_i(x)$$

Boolean Set Operations

- Positive = outside, negative = inside
- Boolean subtraction:

$$h = max(f, -g)$$

	$f > 0$	$f < 0$
$g > 0$	$h > 0$	$h < 0$
$g < 0$	$h > 0$	$h > 0$



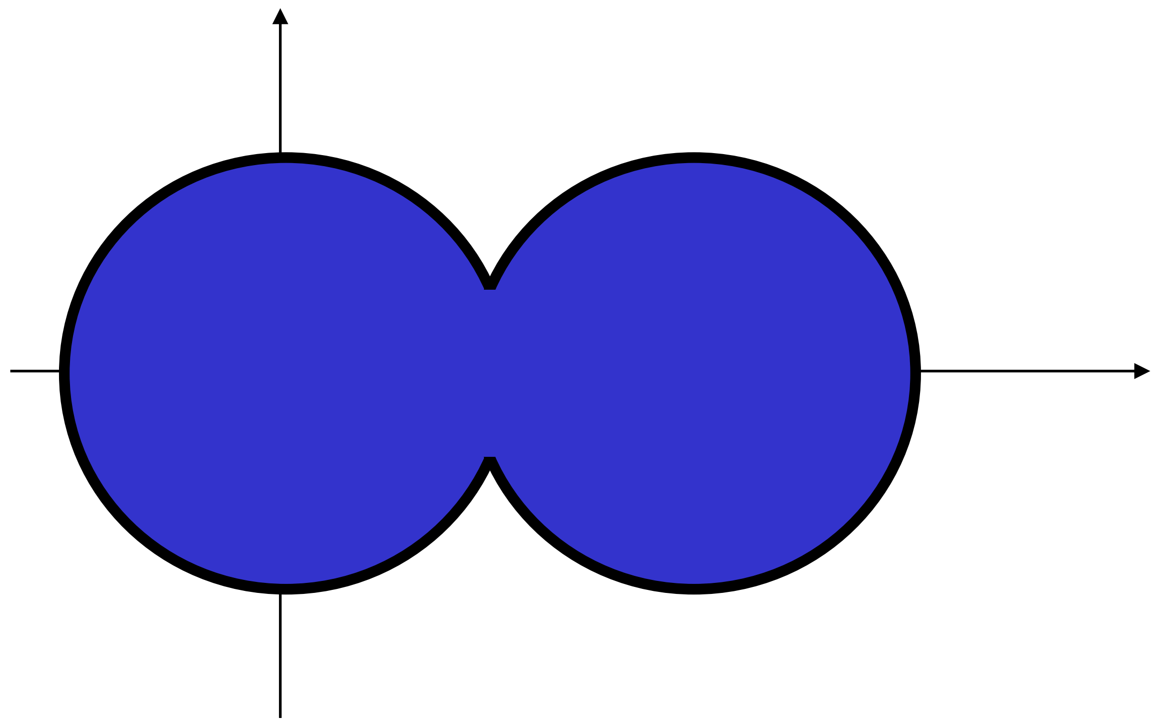
- Much easier than for parametric surfaces!

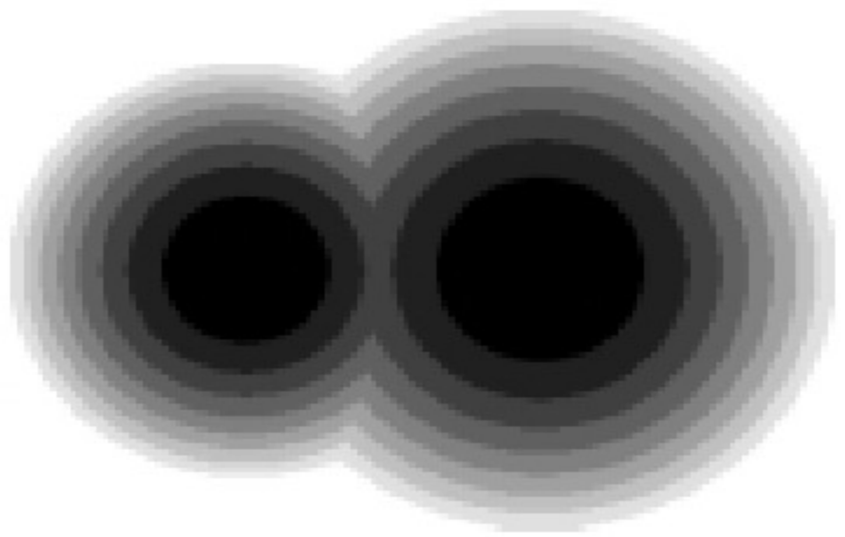
Smooth Set Operations

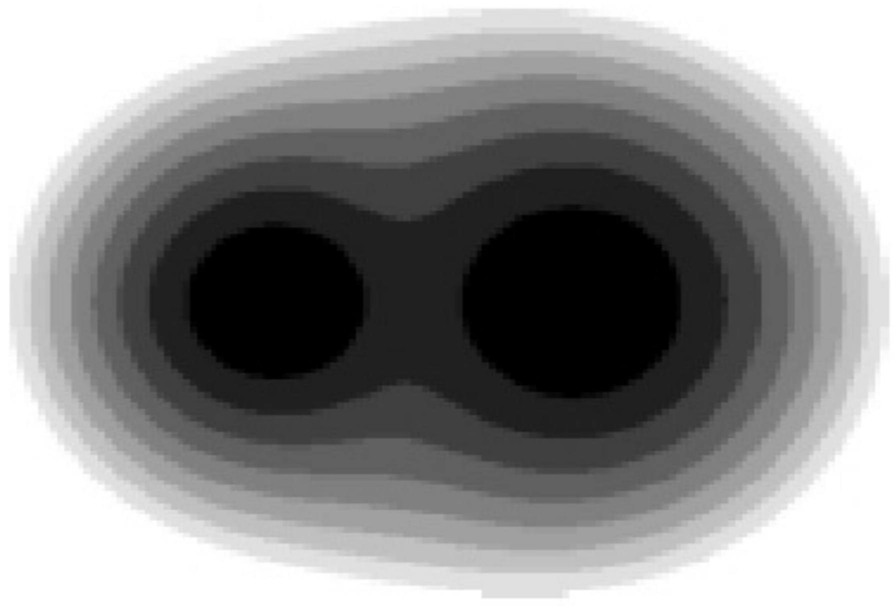
- In many cases, smooth blending is desired
  - Pasko and Savchenko, Blending operations for the functionally based constructive geometry [1994]

$$f \cup g = \tfrac{1}{1+\alpha}\left(f + g - \sqrt{f^2 + g^2 - 2\alpha f g}\right)$$

$$f \cap g = \tfrac{1}{1+\alpha}\left(f + g + \sqrt{f^2 + g^2 - 2\alpha f g}\right)$$

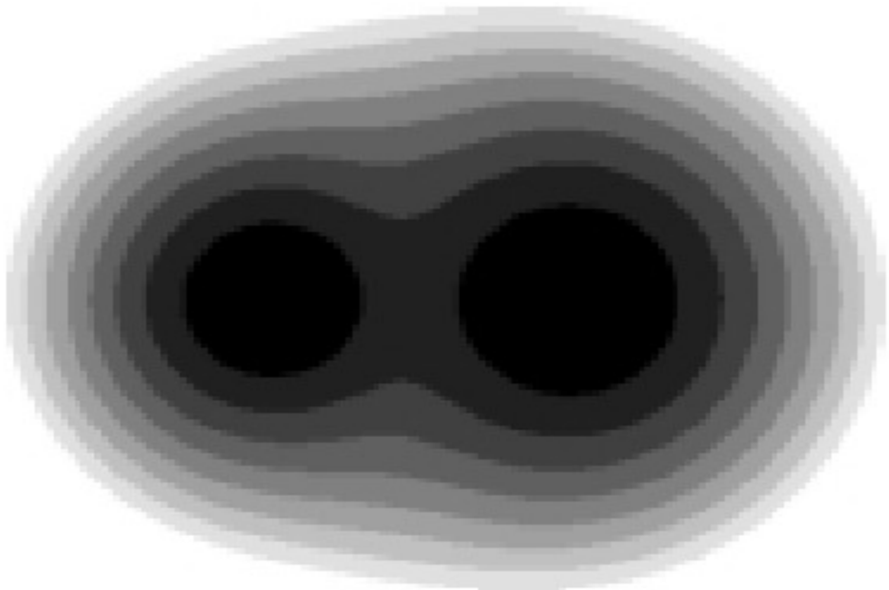




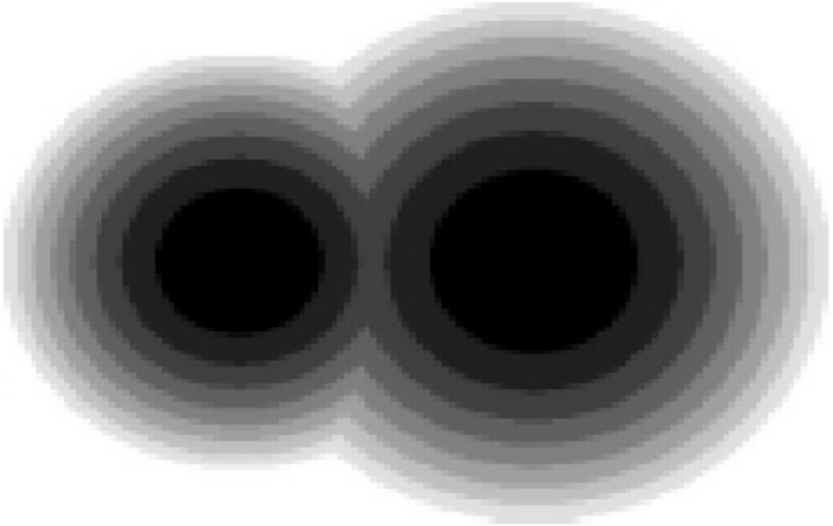


Smooth Set Operations

- Examples



$\alpha = 0$

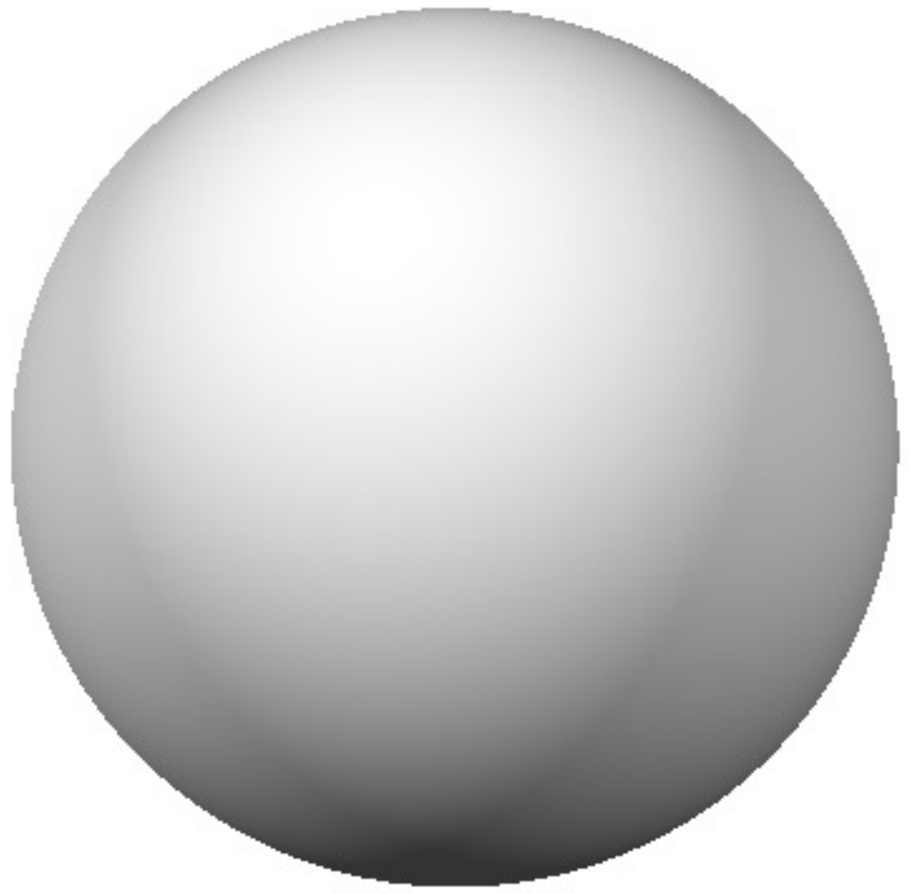


$\alpha = 1$

Designing with Implicit Surfaces

- Zero set (or level set) of a function:

$$f(\mathbf{p}) = \|\mathbf{p}\|^2 - r^2$$



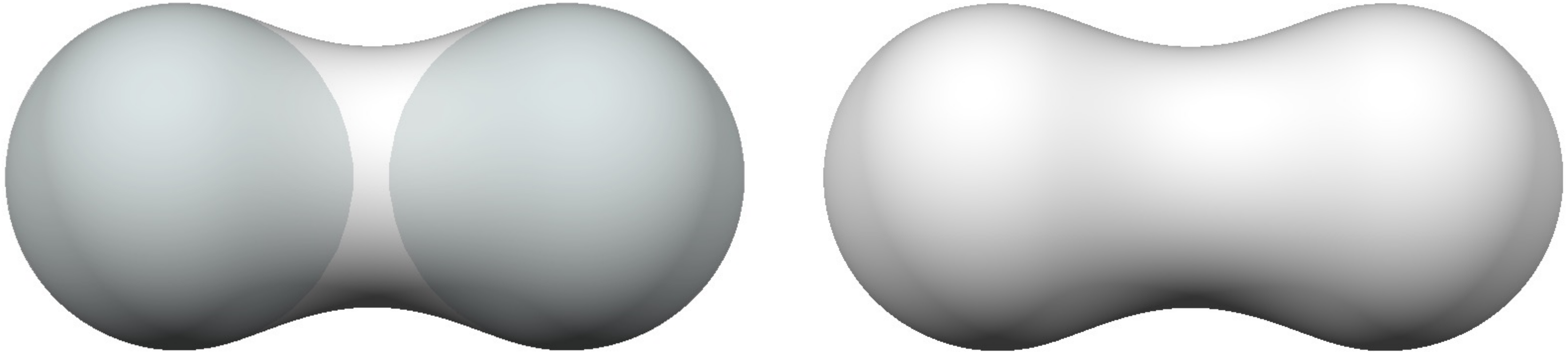
- But also a level set at value $\boldsymbol{e}^{-1}$ of this function:

$$f(\mathbf{p}) = e^{-\|\mathbf{p}\|^2/r^2} \text{ at } e^{-1}$$

Designing with Implicit Surfaces

- With smooth falloff functions, adding implicit functions generates a blend:

$$f(\mathbf{p}) = e^{-\|\mathbf{p}-\mathbf{p}_1\|^2} + e^{-\|\mathbf{p}-\mathbf{p}_2\|^2}$$



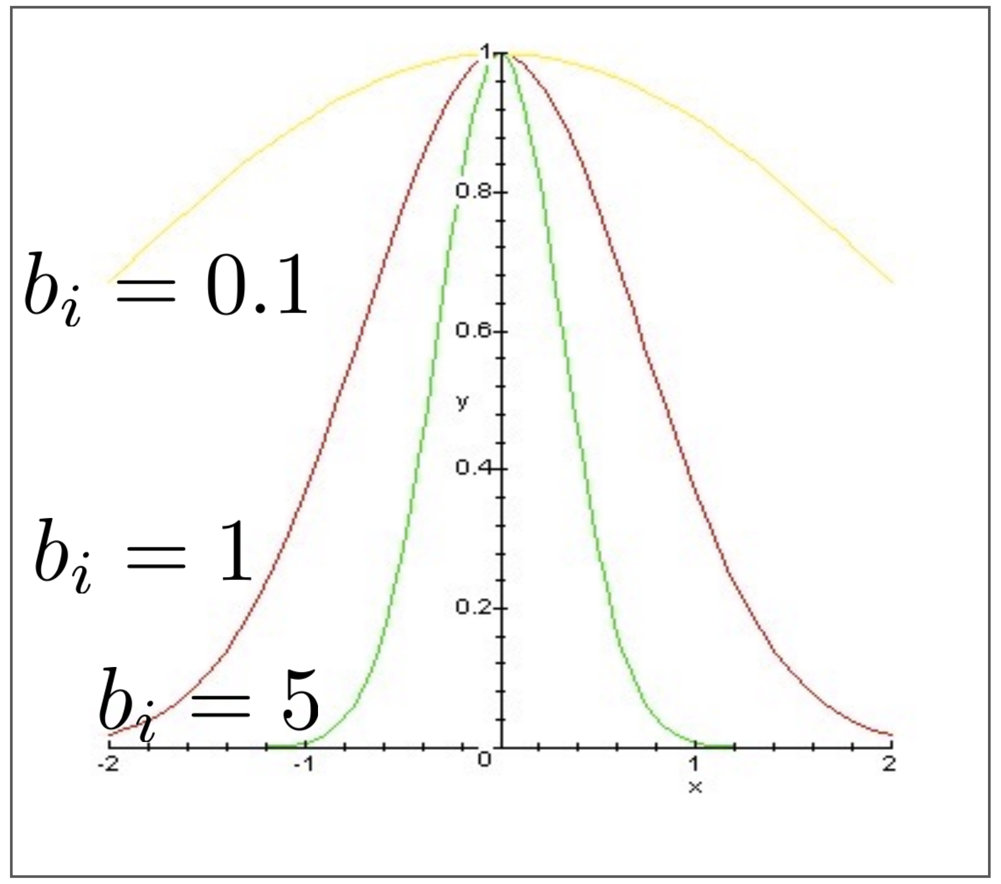
- Called "Metaballs" or "Blobs"

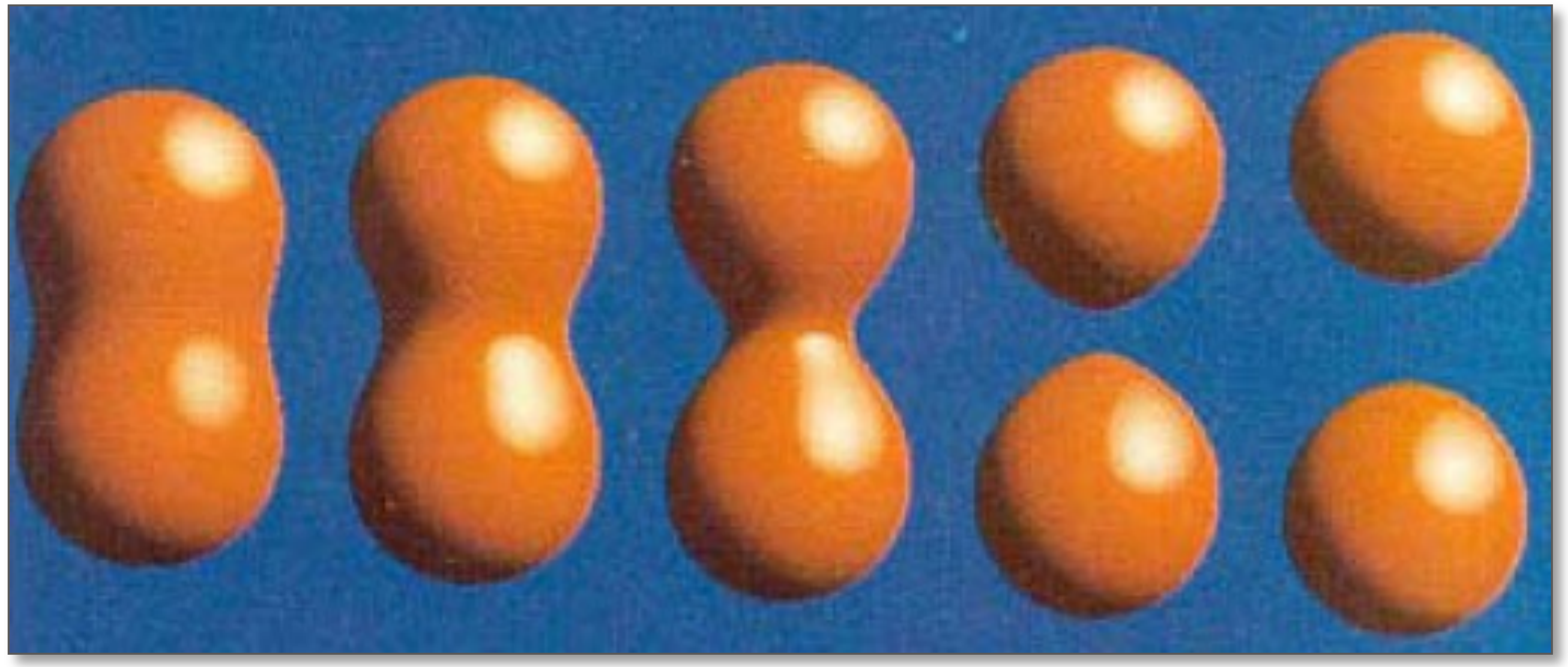
Blobs

- Suggested by Blinn [1982]
  - Defined implicitly by a potential function around a point $\mathbf{p}_i$:

$$f(\mathbf{p}) = a_i e^{-b_i \|\mathbf{p}-\mathbf{p}_i\|^2}$$

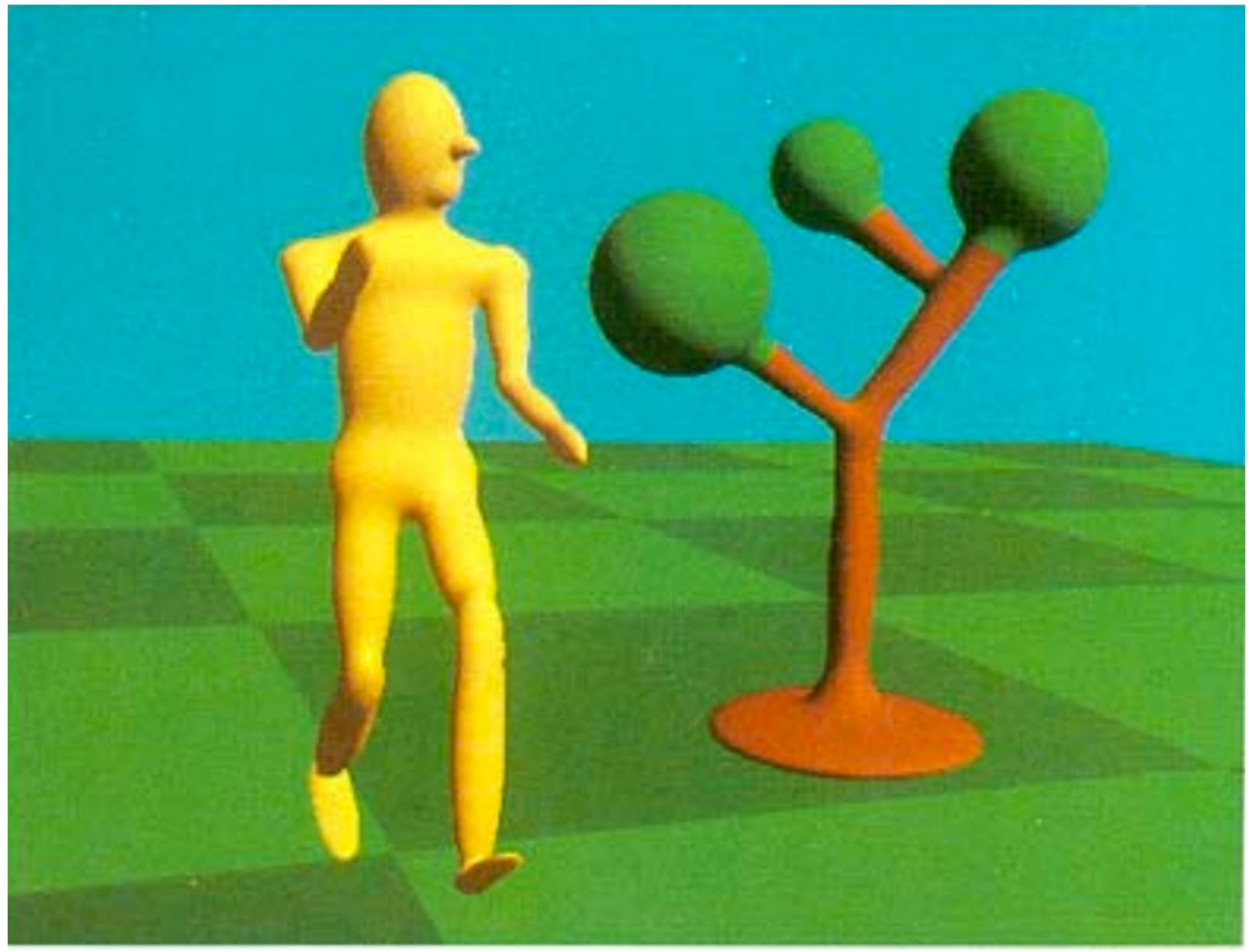
  - Set operations by simple addition/subtraction





J. Blinn, "A Generalization of Algebraic Surface Drawing", ACM Transactions on Graphics, Vol. 1, No. 3, pp. 235-256, July, 1982.

Blobs



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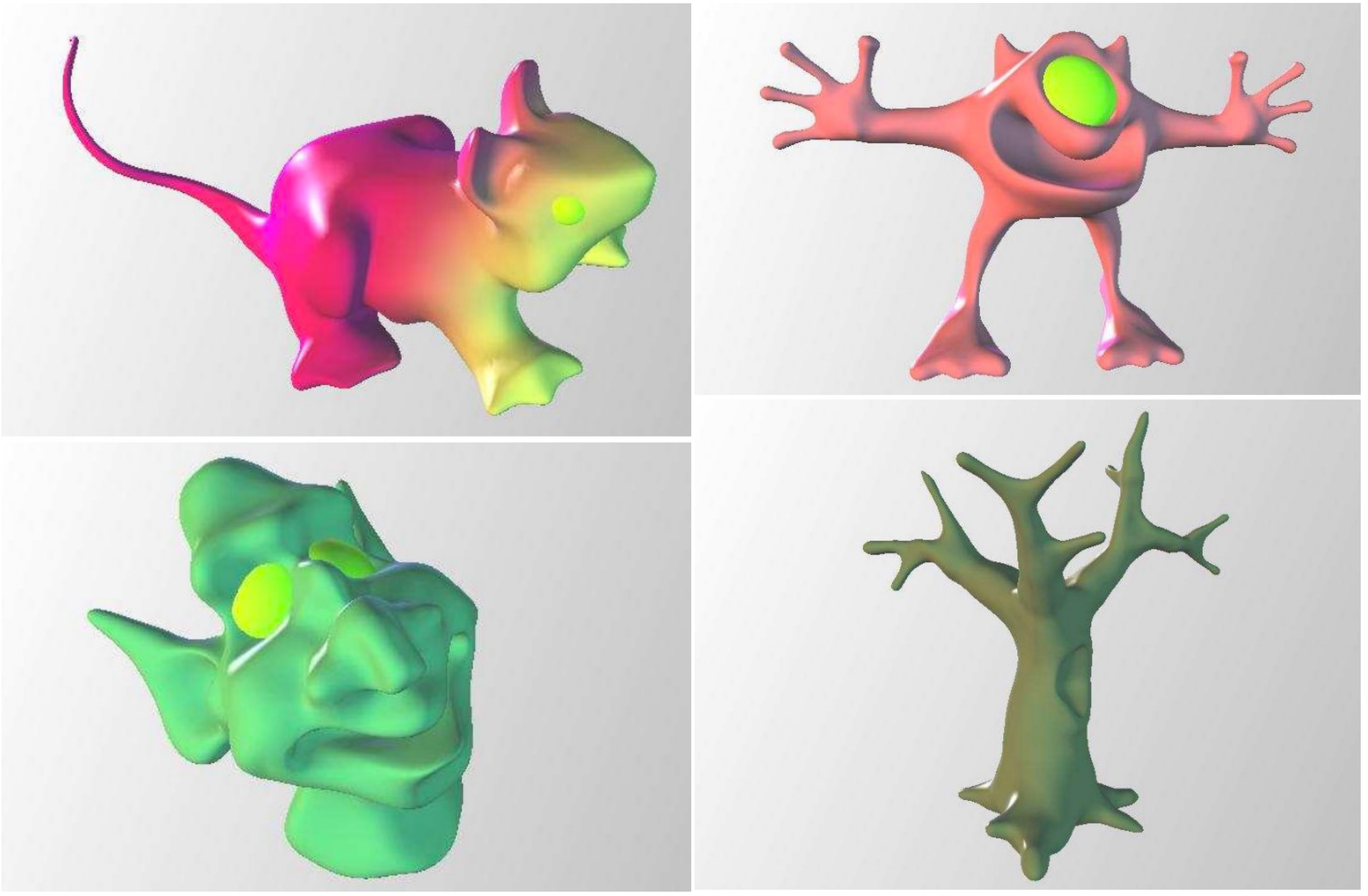
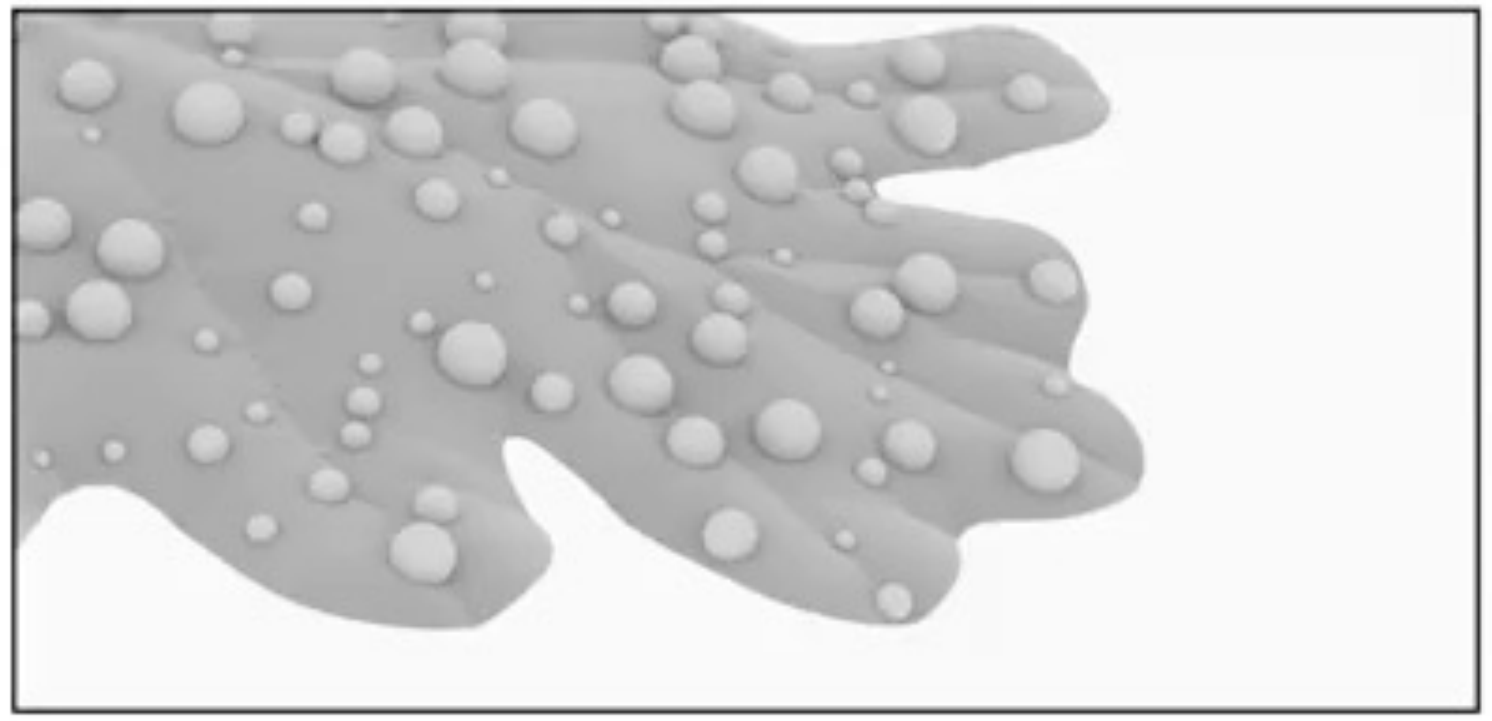
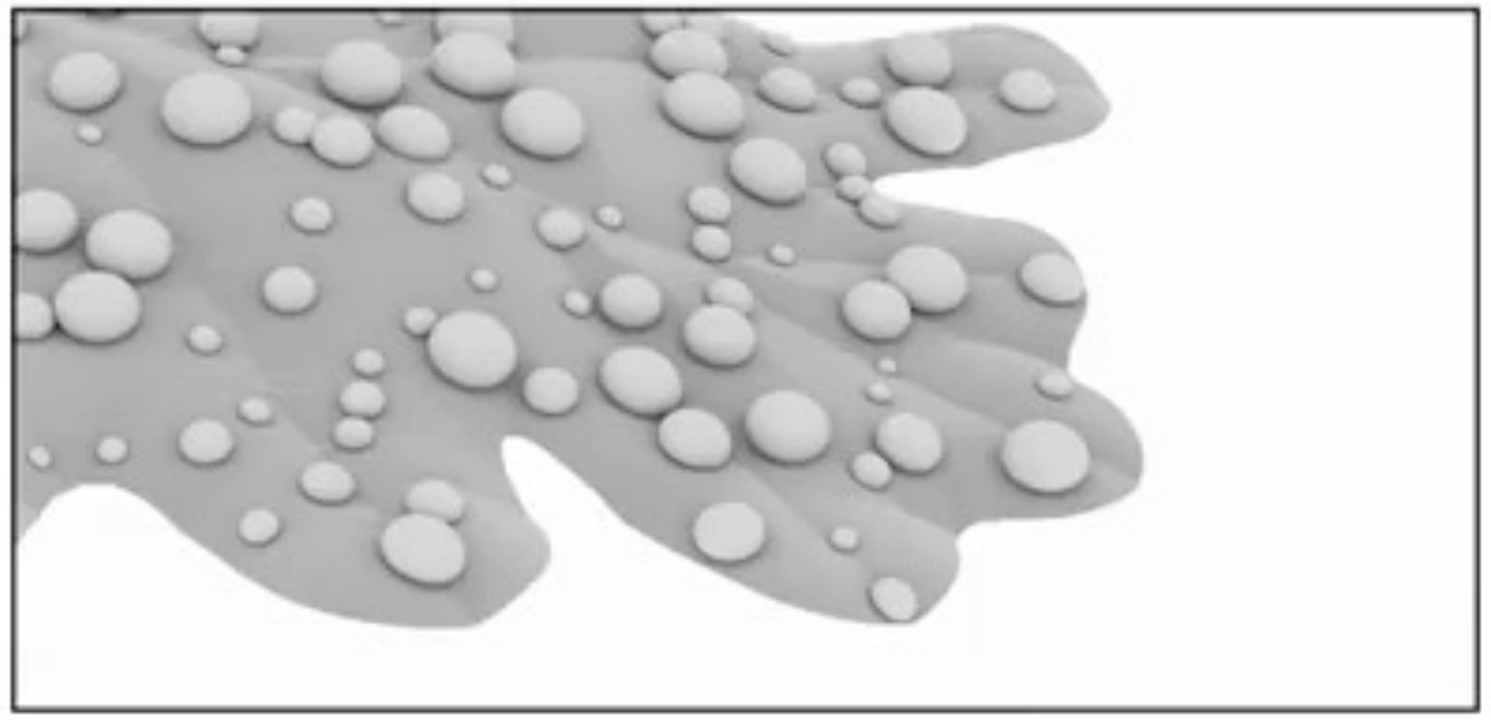


Angelidis et al., “Swirling-Sweepers: Constant-Volume Modeling”, Pacific Graphics 2004

Sketch-Based Implicit Blending

Baptiste Angles[1,2], Marco Tarini[3], Brian Wyvill[1], Loïc Barthe[2], Andrea Tagliasacchi[1]





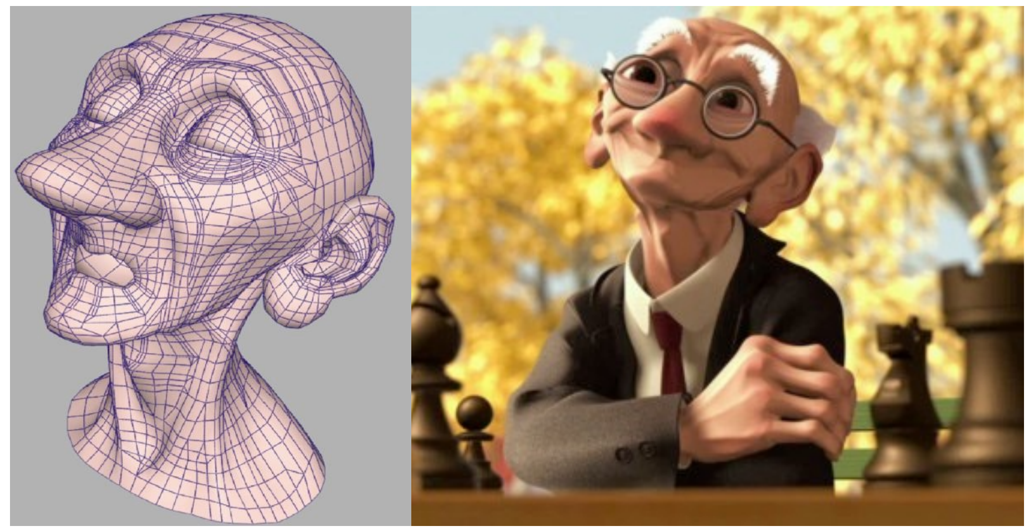
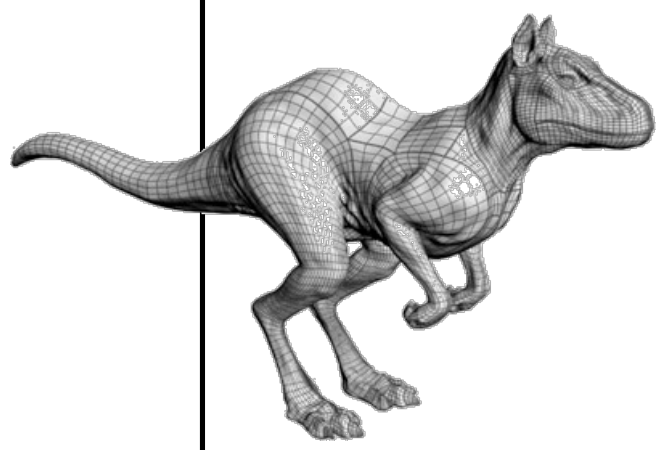
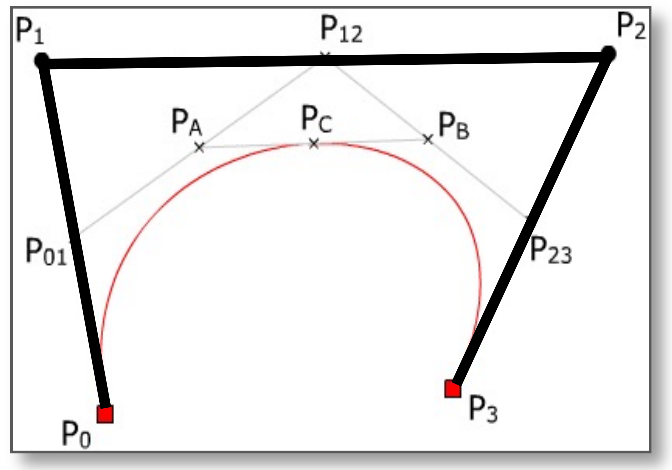
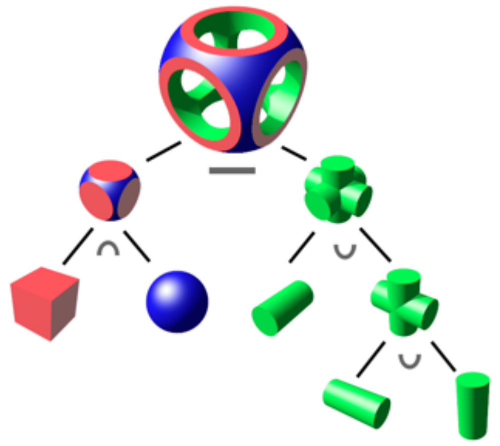
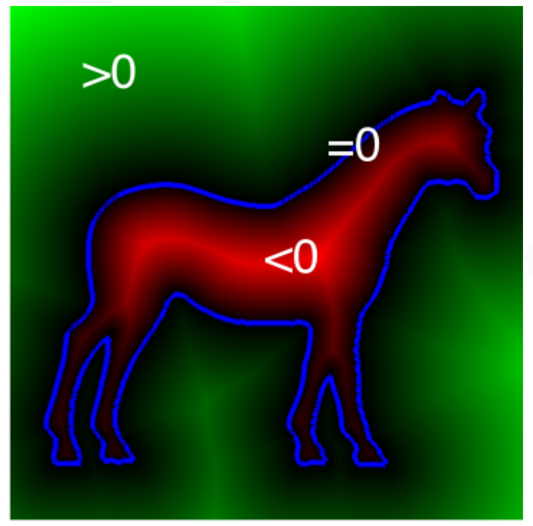
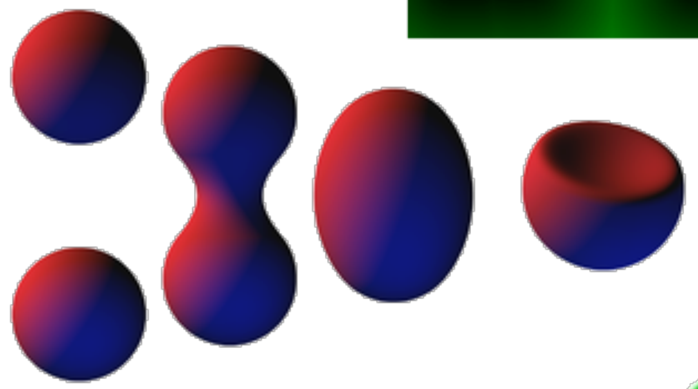
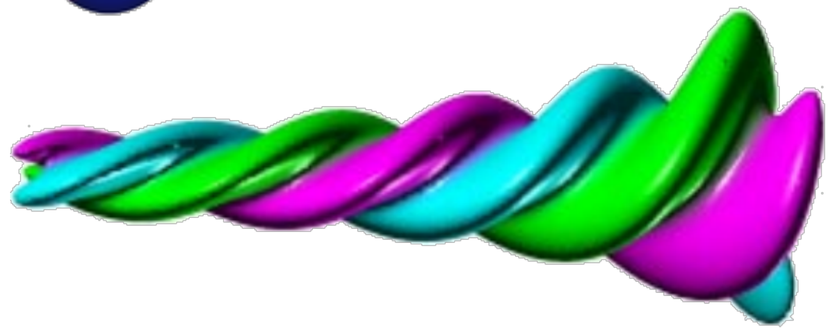
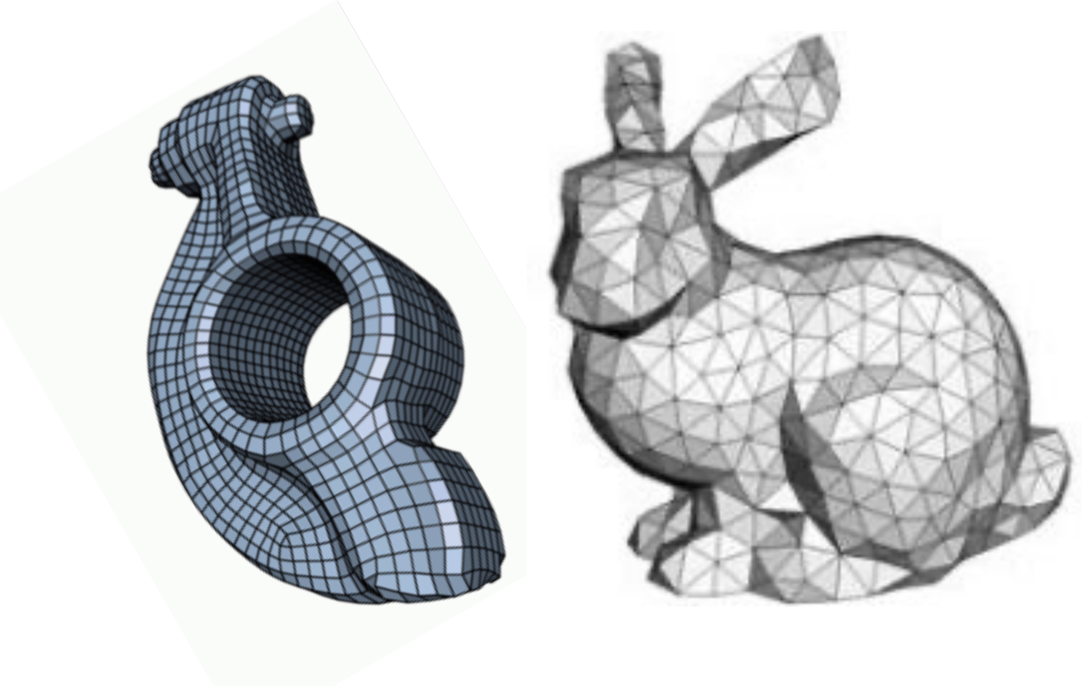
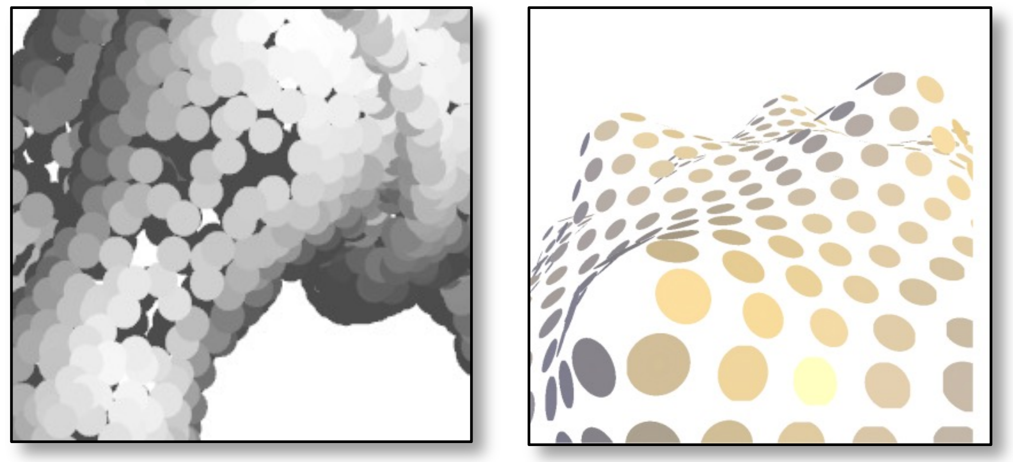
1. University of Victoria
2. Université de Toulouse, IRIT/CNRS
3. Università dell'Insubria, ISTI / CNR

Implicit Curves and Surfaces

- Advantages
  - Easy to determine inside/outside
  - Easy to determine if a point is **on** the curve/surface

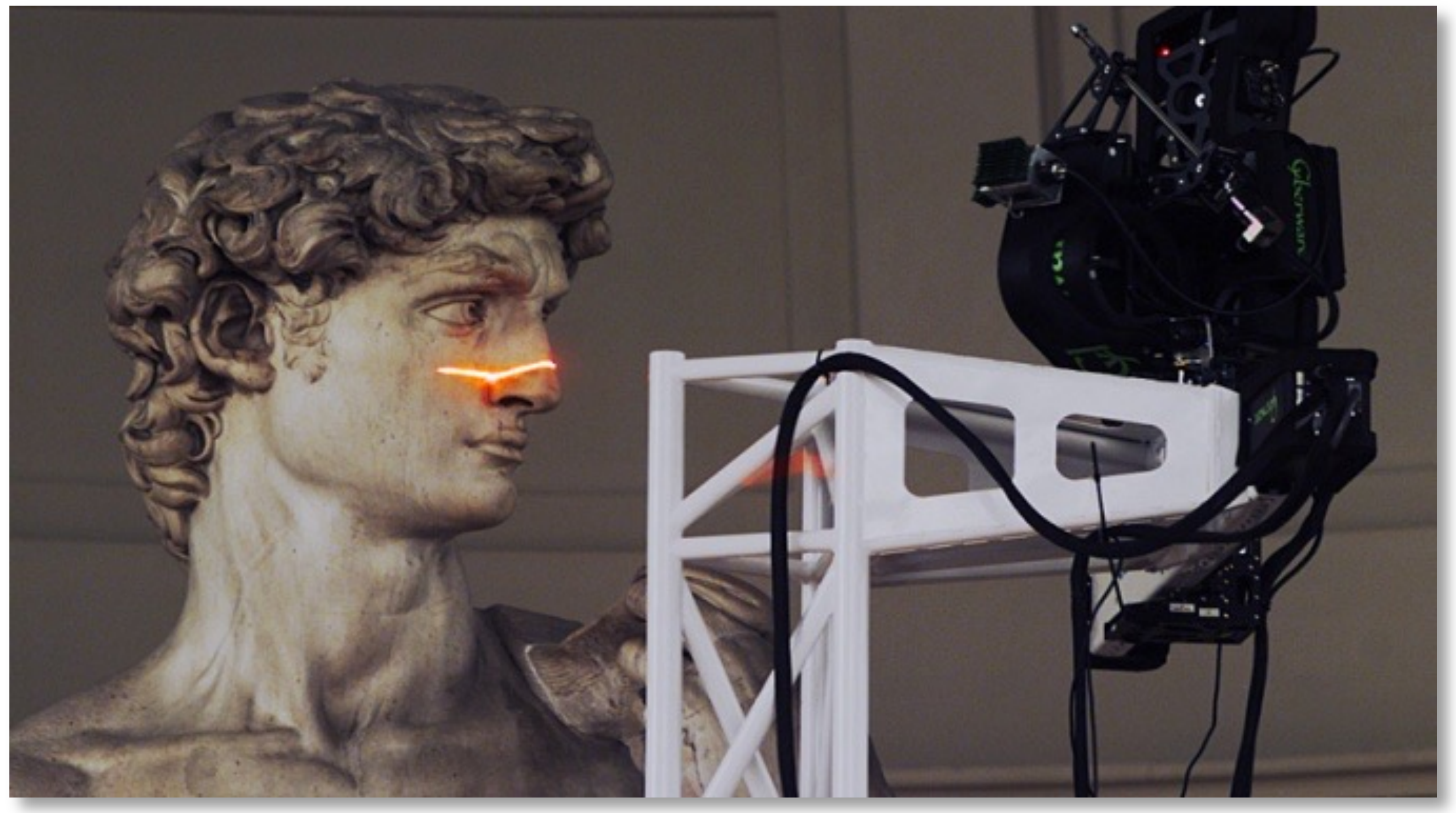
- Disadvantages
  - Hard to generate points on the curve/surface
  - Does not lend itself to (real-time) rendering

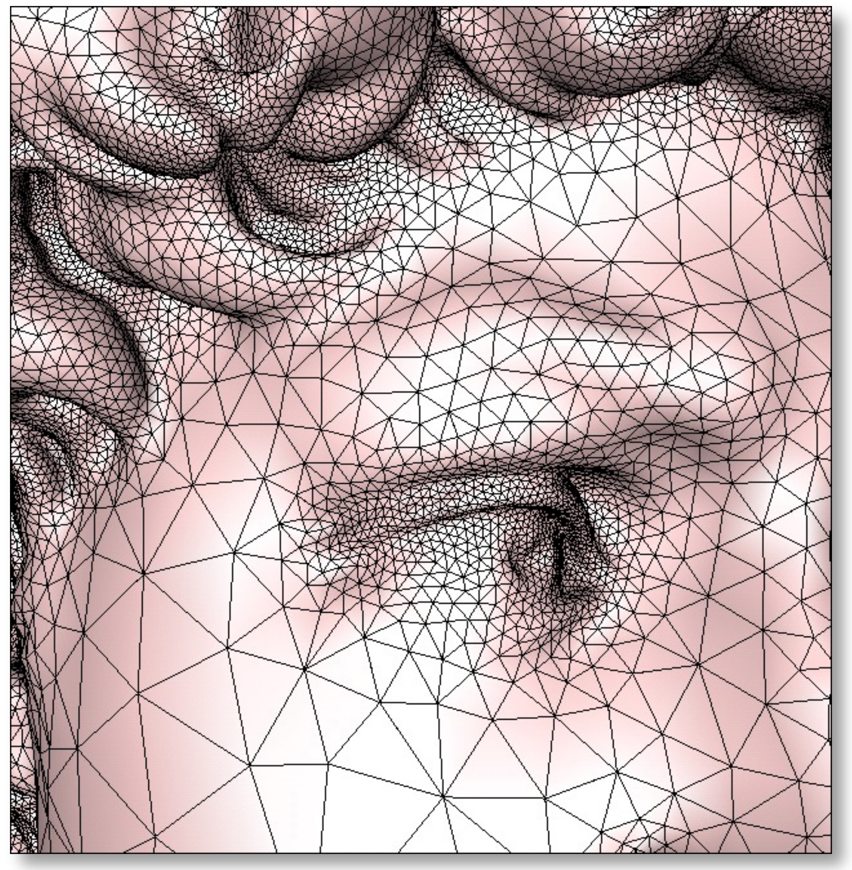
Summary

Parametric	**Implicit**	**Discrete/Sampled**
   	   	 
• Splines, tensor-product surfaces • Subdivision surfaces	• Metaballs/blobs • Distance fields	• Meshes • Point set surfaces

Next Lecture

- A bit about geometry acquisition
- All About Meshes





Thank you