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Q2 Fourier Analysis

(Dr. Meyer-Baese; Spring **2014**)

Part 2

Show that the Fourier Transform of the function $\boldsymbol{g(t) = sinc(\pi t)\ e^{i10\omega}}$ is $\boldsymbol{G(\omega) = rect\left(\frac{\omega-10}{2\pi}\right)}$.

$$\boldsymbol{g(t) = (\tfrac{1}{2\pi}) \int_{-\infty}^{\infty} G(\omega)\ e^{iwt} d\omega}$$, where ***i*** represents the imaginary number.

From the definition of the function $\boldsymbol{sinc}$ we have that $\boldsymbol{sinc(\pi t) = sin(\pi t)/(\pi t)}$.

Starting with the equation $\boldsymbol{g(t\,) = (\tfrac{1}{2\pi}) \int_{-\infty}^{\infty} G(\omega)\ e^{iwt} d\omega}$

Insert the given $\boldsymbol{G(\omega)}$ to obtain $\boldsymbol{g(t) = (\tfrac{1}{2\pi}) \int_{-\infty}^{\infty} rect\left(\frac{\omega-10}{2\pi}\right)\ e^{iwt} d\omega}$

<u>Consider the values of of ω from $\omega = -\pi + 10$ to $\omega = \pi + 10$</u>:

The value of the $\boldsymbol{rect\left(\frac{\omega-10}{2\pi}\right)}$ function equals **1** from $\boldsymbol{\omega = -\pi + 10}$ **to** $\boldsymbol{\omega = \pi + 10}$:

$$\boldsymbol{g(t) = (\tfrac{1}{2\pi}) \int_{-\pi+10}^{\pi+10} 1\ e^{iwt} d\omega}$$

Simplifying:

$$\boldsymbol{g(t) = (\tfrac{1}{2\pi}) \int_{-\pi+10}^{\pi+10} e^{w(it)} d\omega}$$

Using standard integral formula:

$$\boldsymbol{g(t) = \left(\frac{1}{2\pi}\right)\left(\frac{1}{it}\right) e^{w(it)} \Bigg|_{-\pi+10}^{\pi+10}}$$

Completing integration process:

$$\boldsymbol{g(t) = \left(\frac{1}{2\pi}\right)\left(\frac{1}{it}\right) \left(e^{(\pi+10)(it)} - e^{(-\pi+10)(it)}\right)}$$

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Part 2 continued

From previous page:

$$\boldsymbol{g(t)} = \left(\frac{\mathbf{1}}{\mathbf{2\pi}}\right)\left(\frac{\mathbf{1}}{\boldsymbol{it}}\right)\ (\ \boldsymbol{e^{(\pi+10)(it)}} - \ \boldsymbol{e^{(-\pi+10)(it)}})$$

Rearranging terms:

$$\boldsymbol{g(t)} = \left(\frac{\mathbf{1}}{\mathbf{2\pi}}\right)\left(\frac{\mathbf{1}}{\boldsymbol{it}}\right)\ (\ \boldsymbol{e^{i(\pi t)+i(10t)}} - \ \boldsymbol{e^{-i(\pi t)+i(10t)}})$$

Factoring out an exponential:

$$\boldsymbol{g(t)} = \left(\frac{\mathbf{1}}{\boldsymbol{\pi t}}\right)\left(\frac{\mathbf{1}}{\boldsymbol{2i}}\right)\ (\ \boldsymbol{e^{i(10t)}})\ (\ \boldsymbol{e^{i(\pi t)}} - \ \boldsymbol{e^{-i(\pi t)}}\ )$$

Rearranging terms:

$$\boldsymbol{g(t)} = \ (\ \boldsymbol{e^{i(10t)}})\left(\frac{\mathbf{1}}{\boldsymbol{\pi t}}\right)\left(\frac{\mathbf{1}}{\boldsymbol{2i}}\right)\ (\ \boldsymbol{e^{i(\pi t)}} - \ \boldsymbol{e^{-i(\pi t)}}\ )$$

Exponential form of $\boldsymbol{sin(\Theta)}$ is: $\boldsymbol{sin(x)} = \left(\frac{\mathbf{1}}{\boldsymbol{2i}}\right)(\ \boldsymbol{e^{i\Theta}} - \ \boldsymbol{e^{-i\Theta}})$

$$\boldsymbol{sin(\pi t)} = \left(\frac{\mathbf{1}}{\boldsymbol{2i}}\right)\ (\ \boldsymbol{e^{i(\pi t)}} - \ \boldsymbol{e^{-i(\pi t)}}\ )$$

Substitution of exponential form of $\boldsymbol{sin(\pi t)}$:

$$\boldsymbol{g(t)} = \ (\ \boldsymbol{e^{i(10t)}})\left(\frac{\mathbf{1}}{\boldsymbol{\pi t}}\right)\boldsymbol{sin(\pi t)}$$

Rearranging terms:

$$\boldsymbol{g(t)} = \ \left(\frac{\mathbf{1}}{\boldsymbol{\pi t}}\right)(\boldsymbol{sin(\pi t)})\ (\ \boldsymbol{e^{i(10t)}})$$

Substitution of previous formula for $\boldsymbol{sinc(\pi t)} = \boldsymbol{sin(\pi t)}/(\boldsymbol{\pi t})$:

$$\boldsymbol{g(t)} = (\boldsymbol{sinc(\pi t)})\ \ (\ \boldsymbol{e^{i(10t)}})$$

The preceding shows that the Fourier Transform of the function $\boldsymbol{g(t)} = (\boldsymbol{sinc(\pi t)})\ \ (\ \boldsymbol{e^{i(10t)}})$

is $\boldsymbol{G(\omega)} = \boldsymbol{rect}\left(\frac{\boldsymbol{\omega - 10}}{\mathbf{2\pi}}\right)$ **for values of** $\boldsymbol{\omega}$ **from** $\boldsymbol{\omega = -\pi + 10}$ **to** $\boldsymbol{\omega = \pi + 10}$**.**