

Q2 Fourier Analysis

(Dr. Meyer-Baese; Spring 2013)

Part 1a and 1b combined

$$g(t) = 5\sin(2t) + 4\sin\left(4t - \frac{\pi}{2}\right) + 5\cos\left(7t - \frac{\pi}{5}\right)$$

Convert equation using trigonometric rules (aided by online symbolab trig caculator).

$$g(t) = 5\sin(2t) - 4\cos(4t) + 4.045\cos(7t) + 2.939\sin(7t)$$

Standard Euler Equations:

$$\cos(\theta) = \left(\frac{1}{2}\right)(e^{i\theta}) + \left(\frac{1}{2}\right)(e^{-i\theta}) \quad \text{and} \quad \sin(\theta) = \left(\frac{1}{2i}\right)(e^{i\theta}) - \left(\frac{1}{2i}\right)(e^{-i\theta})$$

Replacing Trig terms with exponential Euler terms:

$$g(t) = 5\left(\left(\frac{1}{2i}\right)(e^{i2t}) - \left(\frac{1}{2i}\right)(e^{-i2t})\right) - 4\left(\left(\frac{1}{2}\right)(e^{i4t}) + \left(\frac{1}{2}\right)(e^{-i4t})\right) + 4.045\left(\left(\frac{1}{2}\right)(e^{i7t}) + \left(\frac{1}{2}\right)(e^{-i7t})\right) + \dots$$

$$\dots + 2.939\left(\left(\frac{1}{2i}\right)(e^{i7t}) - \left(\frac{1}{2i}\right)(e^{-i7t})\right)$$

Simplifying terms:

$$g(t) = \left(\frac{-5i}{2}\right)(e^{i2t}) + \left(\frac{5i}{2}\right)(e^{-i2t}) - 2(e^{i4t}) - 2(e^{-i4t}) + (2.023)(e^{i7t}) + (2.023)(e^{-i7t}) + \dots$$

$$\dots + (-1.047i)(e^{i7t}) + (-1.047i)(e^{-i7t})$$

Combining like terms to arrive at **exponential form of Fourier Series** for function g(t) :

$$g(t) = \left(\frac{-5i}{2}\right)(e^{i2t}) + \left(\frac{5i}{2}\right)(e^{-i2t}) - 2(e^{i4t}) - 2(e^{-i4t}) + (2.023 - 1.047i)(e^{i7t}) + (2.023 + 1.047i)(e^{-i7t})$$

$$g(t) = \left(\frac{-5i}{2}\right)(e^{i2t}) + \left(\frac{5i}{2}\right)(e^{-i2t}) - 2(e^{i4t}) - 2(e^{-i4t}) + (2.023 - 1.047i)(e^{i7t}) + (2.023 + 1.047i)(e^{-i7t})$$

Extracting Coefficients a_n from the exponential Fourier Series equation (for periodic interval 2π):

$$n = 2 \quad a_2 = \frac{-5i}{2}$$

$$n = -2 \quad a_{-2} = \frac{5i}{2}$$

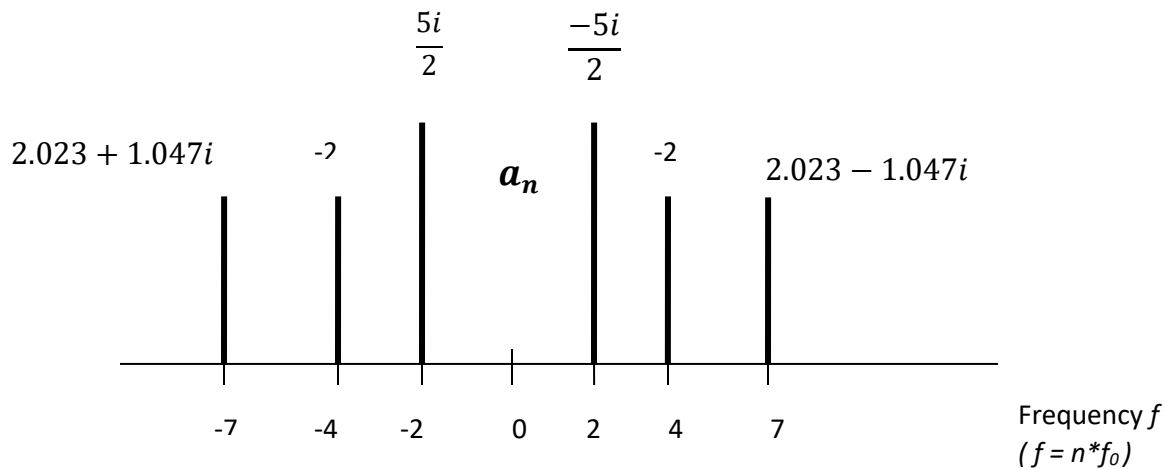
$$n = 4 \quad a_4 = -2$$

$$n = -4 \quad a_{-4} = -2$$

$$n = 7 \quad a_7 = 2.023 - 1.047i$$

$$n = -7 \quad a_{-7} = 2.023 + 1.047i$$

Using the coefficients from the exponential Fourier Series equation to **plot the Spectra** (for periodic interval 2π):



Fourier Series Spectra