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Q11 Stability and Convergence of Numerical PDE's

(Dr. Plewa; Summer 2017)

Part 1, Spatial Convergence Study :

A Spatial Convergence Study is conducted using the matlab programs for a one-dimensional heat diffusion problem from Dr. Burkardt's website. The approximate data $u(x,t)$ are stored in matrix hmat.

An L-1 norm is found for the approximate solution by adding the code "L-1 norm = norm(hmat , 1) ."
Each L-1 norm is scaled back to the coarse grid by dividing by 2, 4, and 8 respectively.
An L-1 norm is also found for a **very Fine grid** of t_num = 10,241 and x_num of 161.
The **very Fine grid** solution is used to approximate the exact solution.
The "L1 norm **error"** is assumed to be the difference between the Coarse grid L-1 norm and the very Fine grid L-1 norm.

Holding the step size for time to be dt = Δt = 0.0039047 the following data are generated for four different step spatial step sizes of dx = Δx:

Spatial Convergence Study							
tnum	**xnum**	**L1_norm**	**dx**	**LN(dx)**	**Coarse L1_norm**	**L1 norm error**	**LN(L1 norm error)**
2561	11	830.0736	0.1000	-2.30259	830.0736	75.1986	4.32013
2561	21	1579.9000	0.0500	-2.99573	789.9500	35.0750	3.55749
2561	41	3079.6000	0.0250	-3.68888	769.9000	15.025	2.70972
2561	81	6079.2000	0.0125	-4.38203	759.9000	5.025	1.61443

dx = x step size = 1/(xnum-1)

The L-1 error norm **model** for varying step sizes in the spatial dimension x is written as:

$$\boldsymbol{L_1 = A(\Delta x)^{\alpha}}$$

$$Ln(L_1) = Ln(A(\Delta x)^{\alpha})$$

$$Ln(L_1) = Ln\,\mathrm{A} + Ln\,(\Delta x)^{\alpha}$$

$$Ln(L_1) = Ln\,\mathrm{A} + \alpha\, Ln\,(\Delta x)$$

$Ln\,\mathrm{A} + \alpha\, Ln\,(\Delta x) = Ln(L_1)$ letting $a = Ln\,\mathrm{A}$ and $b = \alpha$

$$\boldsymbol{a + b\, Ln\,(\Delta x) = Ln(L_1)}$$

Create a system of equation using the data from above:

$$\boldsymbol{a + b(-2.30259) = 4.32013}$$

$$\boldsymbol{a + b(-2.99573) = 3.55749}$$

$$\boldsymbol{a + b(-3.68888) = 2.70972}$$

$$\boldsymbol{a + b(-4.38203) = 1.61443}$$

Putting the system of equations into matrix form:

$$\begin{bmatrix} 1 & -\mathbf{2.30259} \\ 1 & -\mathbf{2.99573} \\ 1 & -\mathbf{3.68888} \\ 1 & -\mathbf{4.38203} \end{bmatrix} \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{bmatrix} = \begin{bmatrix} \mathbf{4.23013} \\ \mathbf{3.55749} \\ \mathbf{2.70972} \\ \mathbf{1.61443} \end{bmatrix}$$

Letting A = $\begin{bmatrix} 1 & -\mathbf{2.30259} \\ 1 & -\mathbf{2.99573} \\ 1 & -\mathbf{3.68888} \\ 1 & -\mathbf{4.38203} \end{bmatrix}$, **x =** $\begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{bmatrix}$, and b = $\begin{bmatrix} \mathbf{4.23013} \\ \mathbf{3.55749} \\ \mathbf{2.70972} \\ \mathbf{1.61443} \end{bmatrix}$

Solve for $\boldsymbol{a}$ and $\boldsymbol{b}$ using matrix **method of least squares**:

A x = b

$A^T A x = A^T b$

From Matlab

$x = A^T A \setminus A^T b$

x = $\begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{bmatrix}$

Plugging $\boldsymbol{a}$ and $\boldsymbol{b}$ into the previous equations:

$\boldsymbol{a} = 7.22056435$

$a = Ln$ A

Ln A = 7.22056435

A = $e^{7.22056435}$

A = 1,367.26

$\boldsymbol{b} = 1.25441031$

$b = \alpha$

α = 1.25441031

$\boldsymbol{L_1} = \boldsymbol{A(\Delta x)^{\alpha}}$

$\boldsymbol{L_1} = \mathbf{1,367.26}(\boldsymbol{\Delta x})^{\mathbf{1.25441031}}$ **Error Norm for Spatial Study**

Part 2, Temporal Convergence Study :

A Temporal Convergence Study is conducted using the matlab programs for a one-dimensional heat diffusion problem from Dr. Burkardt's website. The approximate data $u(x, t)$ are stored in matrix hmat.

An L-1 norm is found for the approximate solution by adding the code "L-1 norm = norm(hmat , 1) ."
An L-1 norm is also found for a **very Fine grid** of t_num = 10,241 and x_num of 161.
The **very Fine grid** solution is used to approximate the exact solution.
The "L1 norm **error"** is assumed to be the difference between the Coarse grid L-1 norm and the very Fine grid L-1 norm.

Holding the spatial x step size to be dx = Δx = 0.05 the following data are generated for four different step temporal step sizes of dt = Δt:

Temporal Convergence Study							
tnum	**xnum**	**L1_norm**	**dt**	**LN(dt)**		**L1 norm error**	**LN (L1 norm error)**
41	11	831.5980	0.25000	-1.3863		76.7230	4.34020
81	11	830.8196	0.12500	-2.0794		75.9446	4.33000
161	11	830.4336	0.06250	-2.7726		75.5586	4.32491
321	11	830.2414	0.03125	-3.4657		75.3664	4.32236

dt = t step size = 10/(tnum -1)

The L-1 error norm **model** for varying step sizes in the spatial dimension x is written as:

$$\boldsymbol{L_1 = B(\Delta t)^{\beta}}$$

$$Ln(L_1) = Ln\left(B(\Delta t)^{\beta}\right)$$

$$Ln(L_1) = Ln\,\mathrm{B} + Ln\,(\Delta t)^{\beta}$$

$$Ln(L_1) = Ln\,\mathrm{B} + \beta\,Ln\,(\Delta t)$$

$Ln\,\mathrm{B} + \beta\,Ln\,(\Delta t) = Ln(L_1)$ letting $a = Ln\,\mathrm{B}$ and $b = \beta$

$$\boldsymbol{a + b\,Ln\,(\Delta t) = Ln(L_1)}$$

Create a system of equation using the data from above:

$$\boldsymbol{a + b(-1.3863) = 4.34020}$$

$$\boldsymbol{a + b(-2.0794) = 4.33000}$$

$$\boldsymbol{a + b(-2.7726) = 4.32491}$$

$$\boldsymbol{a + b(-3.4657) = 4.32236}$$

Putting the system of equations into matrix form:

$$\begin{bmatrix} 1 & \mathbf{-1.3863} \\ 1 & \mathbf{-2.0794} \\ 1 & \mathbf{-2.7726} \\ 1 & \mathbf{-3.4657} \end{bmatrix} \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{bmatrix} = \begin{bmatrix} \mathbf{4.34020} \\ \mathbf{4.33000} \\ \mathbf{4.32491} \\ \mathbf{4.32236} \end{bmatrix}$$

Letting A = $\begin{bmatrix} 1 & \mathbf{-1.3863} \\ 1 & \mathbf{-2.0794} \\ 1 & \mathbf{-2.7726} \\ 1 & \mathbf{-3.4657} \end{bmatrix}$, **x =** $\begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{bmatrix}$, and b = $\begin{bmatrix} \mathbf{4.34020} \\ \mathbf{4.33000} \\ \mathbf{4.32491} \\ \mathbf{4.32236} \end{bmatrix}$.

Solve for $\boldsymbol{a}$ and $\boldsymbol{b}$ using matrix **method of least squares**:

A x = b

$A^T A$ x= A^T b

From Matlab

x = $A^T A \setminus A^T$ b

x = $\begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{bmatrix}$

Plugging $\boldsymbol{a}$ and $\boldsymbol{b}$ into the previous equations:

$a =$ 4.34636825

$a = Ln$ B

Ln B = 4.34636825

B = $e^{4.34636825}$

B = 77.1976

$b =$ 0.00754683

$b = \beta$

β = 0.00754683

$\boldsymbol{L_1} = \boldsymbol{B}(\boldsymbol{\Delta t})^{\boldsymbol{\beta}}$

$\boldsymbol{L_1} = \mathbf{77.1976}(\boldsymbol{\Delta t})^{\mathbf{0.00754683}}$ **Error Norm for Temporal Study**

Part 2, Spatial-Temporal Convergence Study :

A Spatial Temporal Convergence Study is conducted using the matlab programs for a one-dimensional heat diffusion problem from Dr. Burkardt's website. The approximate data $u(x,t)$ are stored in matrix hmat.

An L-1 norm is found for the approximate solution by adding the code "L-1 norm = norm(hmat , 1) ."
Each L-1 norm is scaled back to the coarse grid by dividing by 2, 4, and 8 respectively.
An L-1 norm is also found for a **very Fine grid** of t_num = 10,241 and x_num of 161.
The **very Fine grid** solution is used to approximate the exact solution.
The "L1 norm **error"** is assumed to be the difference between the Coarse grid L-1 norm and the very Fine grid L-1 norm.

The following L1_norm data are generated for seven different combinations of temporal step sizes dt = Δt and spatial step sizes of dx = Δx :

Spatial/Temporal Convergence Study						
tnum	**dt**	**xnum**	**dx**	**L1_norm**	**Coarse L1_norm**	**L1 norm error**
161	0.0625	21	0.05	1580.6000	790.3	35.425
321	0.03125	21	0.05	1580.2000	790.1	35.225
641	0.015625	21	0.05	1580.0000	790	35.125
641	0.015625	41	0.025	3079.9000	769.975	15.1
1281	0.007813	11	0.1	830.0976	830.0976	75.2226
1281	0.007813	21	0.05	1579.9000	789.95	35.075
1281	0.007813	41	0.025	3079.7000	769.925	15.05

The L-1 error norm **model** for varying step sizes in the spatial dimension x and the temporal dimension t is written as:

$$\boldsymbol{L_1} = \boldsymbol{A}(\boldsymbol{\Delta x})^{\alpha} + \boldsymbol{B}(\boldsymbol{\Delta t})^{\beta} + \boldsymbol{C}(\boldsymbol{\Delta x \Delta t})^{\gamma}$$

Using the formulas derived from the previous two studies:

$$\boldsymbol{A}(\boldsymbol{\Delta x})^{\alpha} = \mathbf{1,367.26}(\boldsymbol{\Delta x})^{\mathbf{1.25441031}}$$

and

$$\boldsymbol{B}(\boldsymbol{\Delta t})^{\beta} = \mathbf{77.1976}(\boldsymbol{\Delta t})^{\mathbf{0.00754683}}$$

The combined L-1 error norm equation for the Spatial-Temporal Convergence study is found to be:

$$\boldsymbol{L_1} \textbf{ norm error} = \mathbf{1,367.26}(\boldsymbol{\Delta x})^{\mathbf{1.25441031}} + \mathbf{77.1976}(\boldsymbol{\Delta t})^{\mathbf{0.00754683}} + \boldsymbol{C}(\boldsymbol{\Delta x \Delta t})^{\gamma}$$

Spatial/Temporal Convergence Study							
tnum	dt	xnum	dx	L1_norm	Coarse L1_norm	L1 Spatial	L1 Temporal
161	0.0625	21	0.05	1580.6000	790.3	31.9026	75.59907992
321	0.03125	21	0.05	1580.2000	790.1	31.9026	75.20464884
641	0.015625	21	0.05	1580.0000	790	31.9026	74.81227568
641	0.015625	41	0.025	3079.9000	769.975	13.3724	74.81227568
1281	0.007813	11	0.1	830.0976	830.0976	76.1098	74.42194968
1281	0.007813	21	0.05	1579.9000	789.95	31.9026	74.42194968
1281	0.007813	41	0.025	3079.7000	769.925	13.3724	74.42194968

$\boldsymbol{L_1}$ **error norm =** $\boldsymbol{1,367.26(\Delta x)^{1.25441031} + 77.1976(\Delta t)^{0.00754683} + C(\Delta x \Delta t)^{\gamma}}$

$\mathbf{L_1}$ **error norm =** $\mathbf{L1\ Spatial\ error + L1\ Temporal\ error} + \boldsymbol{C(\Delta x \Delta t)^{\gamma}}$

Creating an equation for each combination from the combined spatial-temporal charts above:

35.4250 = 31.9026 + 75.59907992 $\boldsymbol{+ C(0.003125)^{\gamma}}$

35.2250 = 31.9026 + 75.20464884 $\boldsymbol{+ C(0.0015625)^{\gamma}}$

35.1250 = 31.9026 + 74.81227568 $\boldsymbol{+ C(0.0007813)^{\gamma}}$

15.1000 = 13.3724 + 74.81227568 $\boldsymbol{+ C(0.0003906)^{\gamma}}$

75.2226 = 76.1098 + 74.42194968 $\boldsymbol{+ C(0.0007813)^{\gamma}}$

35.0750 = 31.9026 + 74.42194968 $\boldsymbol{+ C(0.0003907)^{\gamma}}$

15.0500 = 13.3724 + 74.42194968 $\boldsymbol{+ C(0.0001953)^{\gamma}}$

Combining numerical terms:

-72.07668 = $\boldsymbol{C(0.003125)^{\gamma}}$

-71.88225 = $\boldsymbol{C(0.0015625)^{\gamma}}$

-71.58987 = $\boldsymbol{C(0.0007813)^{\gamma}}$

-73.08468 = $\boldsymbol{C(0.0003906)^{\gamma}}$

-75.30915 = $\boldsymbol{C(0.0007813)^{\gamma}}$

-71.24955 = $\boldsymbol{C(0.0003907)^{\gamma}}$

-72.74435 = $\boldsymbol{C(0.0001953)^{\gamma}}$

Multiplying both sides times -1 :

72.07668 = $-\boldsymbol{C}(\mathbf{0.003125})^{\boldsymbol{\gamma}}$

71.88225 = $-\boldsymbol{C}(\mathbf{0.0015625})^{\boldsymbol{\gamma}}$

71.58987 = $-\boldsymbol{C}(\mathbf{0.0007813})^{\boldsymbol{\gamma}}$

73.08468 = $-\boldsymbol{C}(\mathbf{0.0003906})^{\boldsymbol{\gamma}}$

75.30915 = $-\boldsymbol{C}(\mathbf{0.0007813})^{\boldsymbol{\gamma}}$

71.24955 = $-\boldsymbol{C}(\mathbf{0.0003907})^{\boldsymbol{\gamma}}$

72.74435 = $-\boldsymbol{C}(\mathbf{0.0001953})^{\boldsymbol{\gamma}}$

Taking the natural log of both sides of the equation:

$\boldsymbol{Ln}$ **(72.07668)** = $\boldsymbol{Ln}(-\boldsymbol{C}) + \boldsymbol{Ln}(\mathbf{0.003125})^{\boldsymbol{\gamma}}$

$\boldsymbol{Ln}$ **(71.88225)** = $\boldsymbol{Ln}(-\boldsymbol{C}) + \boldsymbol{Ln}(\mathbf{0.0015625})^{\boldsymbol{\gamma}}$

$\boldsymbol{Ln}$ **(71.58987)** = $\boldsymbol{Ln}(-\boldsymbol{C}) + \boldsymbol{Ln}(\mathbf{0.0007813})^{\boldsymbol{\gamma}}$

$\boldsymbol{Ln}$ **(73.08468)** = $\boldsymbol{Ln}(-\boldsymbol{C}) + \boldsymbol{Ln}(\mathbf{0.0003906})^{\boldsymbol{\gamma}}$

$\boldsymbol{Ln}$(**75.30915)** = $\boldsymbol{Ln}(-\boldsymbol{C}) + \boldsymbol{Ln}(\mathbf{0.0007813})^{\boldsymbol{\gamma}}$

$\boldsymbol{Ln}$ **(71.24955)** = $\boldsymbol{Ln}(-\boldsymbol{C}) + \boldsymbol{Ln}(\mathbf{0.0003907})^{\boldsymbol{\gamma}}$

$\boldsymbol{Ln}$ **(72.74435)** = $\boldsymbol{Ln}(-\boldsymbol{C}) + \boldsymbol{Ln}(\mathbf{0.0001953})^{\boldsymbol{\gamma}}$

Letting $\boldsymbol{a} = \boldsymbol{Ln}(-\boldsymbol{C})$ **and** $\boldsymbol{b} = \boldsymbol{\gamma}$ **:**

$\boldsymbol{a}$ **+** $\boldsymbol{b}\ \boldsymbol{Ln}\ (\boldsymbol{\Delta t})$ **=** $\boldsymbol{Ln}(\boldsymbol{L_1})$

$\boldsymbol{a} + \boldsymbol{b}\ \boldsymbol{Ln}(\mathbf{0.003125}) =$ $\boldsymbol{Ln}$ **(72.07668)**

$\boldsymbol{a} + \boldsymbol{b}\ \boldsymbol{Ln}(\mathbf{0.0015625}) =$ $\boldsymbol{Ln}$ **(71.88225)**

$\boldsymbol{a} + \boldsymbol{b}\ \boldsymbol{Ln}(\mathbf{0.0007813}) =$ $\boldsymbol{Ln}$ **(71.58987)**

$\boldsymbol{a} + \boldsymbol{b}\ \boldsymbol{Ln}(\mathbf{0.0003906}) =$ $\boldsymbol{Ln}$ **(73.08468)**

$\boldsymbol{a} + \boldsymbol{b}\ \boldsymbol{Ln}(\mathbf{0.0007813}) =$ $\boldsymbol{Ln}$(**75.30915)**

$\boldsymbol{a} + \boldsymbol{b}\ \boldsymbol{Ln}(\mathbf{0.0003907}) =$ $\boldsymbol{Ln}$ **(71.24955)**

$\boldsymbol{a} + \boldsymbol{b}\,\boldsymbol{Ln}(\mathbf{0.0001953}) = \boldsymbol{Ln}$ **(72.74435)**

Create a system of equation using the data from above:

$$\boldsymbol{a} + \boldsymbol{b}(\mathbf{-5.76832}) = \mathbf{4.27773}$$

$$\boldsymbol{a} + \boldsymbol{b}(\mathbf{-6.46147}) = \mathbf{4.27503}$$

$$\boldsymbol{a} + \boldsymbol{b}(\mathbf{-7.15455}) = \mathbf{4.27095}$$

$$\boldsymbol{a} + \boldsymbol{b}(\mathbf{-5.54524}) = \mathbf{4.29162}$$

$$\boldsymbol{a} + \boldsymbol{b}(\mathbf{-7.15455}) = \mathbf{4.32160}$$

$$\boldsymbol{a} + \boldsymbol{b}(\mathbf{-5.54499}) = \mathbf{4.26619}$$

$$\boldsymbol{a} + \boldsymbol{b}(\mathbf{-8.54097}) = \mathbf{4.28695}$$

Putting the system of equations into matrix form:

$$\begin{bmatrix} 1 & \mathbf{-5.76832} \\ 1 & \mathbf{-6.46147} \\ 1 & \mathbf{-7.15455} \\ 1 & \mathbf{-5.54524} \\ \mathbf{1} & \mathbf{-7.15455} \\ \mathbf{1} & \mathbf{-5.54499} \\ \mathbf{1} & \mathbf{-8.54097} \end{bmatrix} \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{bmatrix} = \begin{bmatrix} \mathbf{4.27773} \\ \mathbf{4.27503} \\ \mathbf{4.27095} \\ \mathbf{4.29162} \\ \mathbf{4.32160} \\ \mathbf{4.26619} \\ \mathbf{4.28695} \end{bmatrix}$$

Letting $A = \begin{bmatrix} 1 & \mathbf{-5.76832} \\ 1 & \mathbf{-6.46147} \\ 1 & \mathbf{-7.15455} \\ 1 & \mathbf{-5.54524} \\ \mathbf{1} & \mathbf{-7.15455} \\ \mathbf{1} & \mathbf{-5.54499} \\ \mathbf{1} & \mathbf{-8.54097} \end{bmatrix}$, **x** = $\begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{bmatrix}$, and $b = \begin{bmatrix} \mathbf{4.27773} \\ \mathbf{4.27503} \\ \mathbf{4.27095} \\ \mathbf{4.29162} \\ \mathbf{4.32160} \\ \mathbf{4.26619} \\ \mathbf{4.28695} \end{bmatrix}$

.

Solve for $\boldsymbol{a}$ and $\boldsymbol{b}$ using matrix **method of least squares**:

$A\,x = b$

$A^T A\,x = A^T b$

From Matlab

$x = A^T A \setminus A^T b$

$x = \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{bmatrix}$

Plugging $\boldsymbol{a}$ and $\boldsymbol{b}$ into the previous equations:

$\boldsymbol{a} =$ 4.25134868

$a = Ln$ (-C)

Ln (-C) = 4.25134868

- C = $e^{4.25134868}$

C = -$e^{4.25134868}$

C = -70.2000

$\boldsymbol{b} =$ -0.00499521

$b = \gamma$

γ = -0.00499521

Error Norm for Combined Spatial-Temporal Convergence Study:

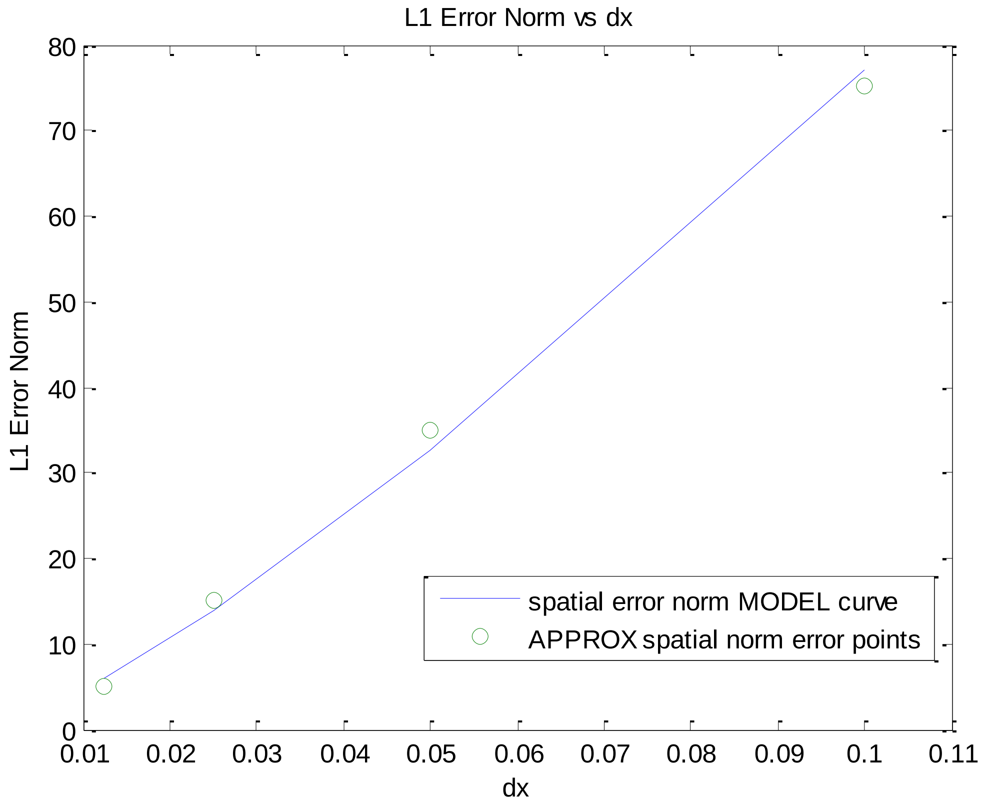
$\boldsymbol{L_1} = \boldsymbol{A}(\boldsymbol{\Delta x})^{\alpha} + \boldsymbol{B}(\boldsymbol{\Delta t})^{\beta} + \boldsymbol{C}(\boldsymbol{\Delta x \Delta t})^{\gamma}$

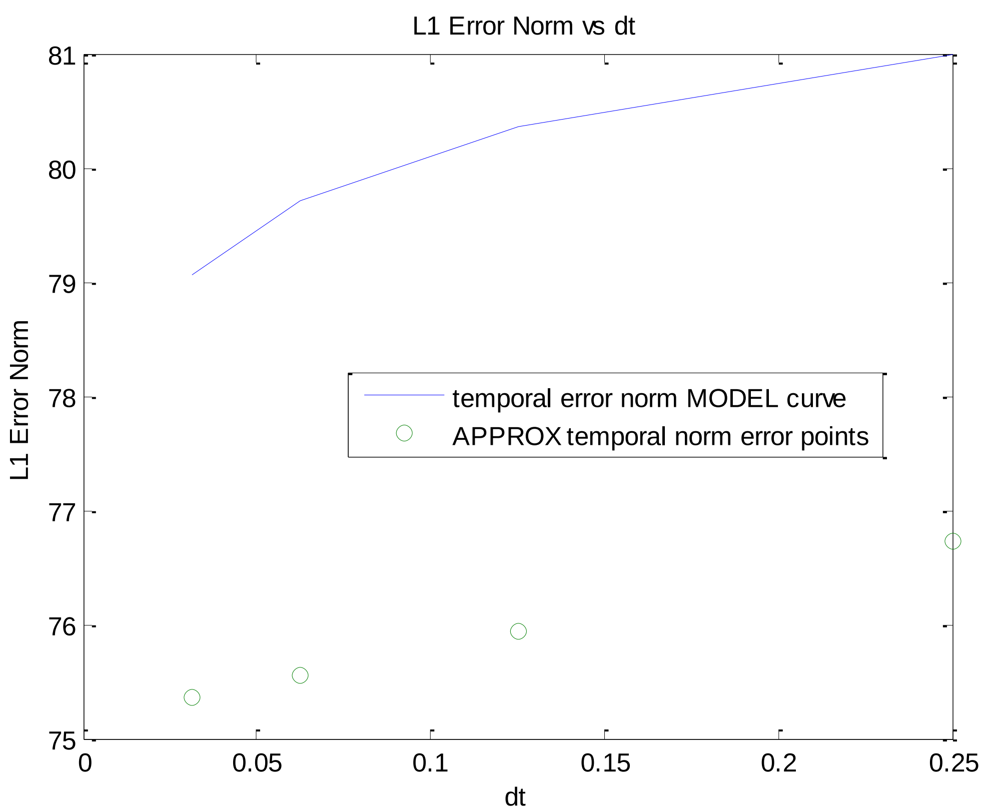
L1 error norm = $\mathbf{1,367.26}(\boldsymbol{\Delta x})^{\mathbf{1.25441031}} + \mathbf{77.1976}(\boldsymbol{\Delta t})^{\mathbf{0.00754683}} + \boldsymbol{C}(\boldsymbol{\Delta x \Delta t})^{\gamma}$

L1 error norm = $\mathbf{1,367.26}(\boldsymbol{\Delta x})^{\mathbf{1.25441031}} + \mathbf{77.1976}(\boldsymbol{\Delta t})^{\mathbf{0.00754683}} - \mathbf{70.20}(\boldsymbol{\Delta x \Delta t})^{\mathbf{-0.00499521}}$

The L1 error norm rate of convergence for the Δx term is much larger than the rate of convergence term for the Δt term. The Δx term should be higher because the PDE equation is a 2nd order derivative with respect to the spatial dimension x. The PDE is only a 1st order derivative with respect to temporal variable t for time.

The graphs on the next page show model curve for the L1 error norm versus actual data points found using the approximation solution code. The spatial data fits the model curve very well. The temporal data follows the outline of the model curve but is offset slightly downward.


L1 Error Norm vs dx
L1 Error Norm
dx
0
10
20
30
40
50
60
70
80
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
0.11
spatial error norm MODEL curve
APPROX spatial norm error points


L1 Error Norm vs dt
L1 Error Norm
dt
75
76
77
78
79
80
81
0
0.05
0.1
0.15
0.2
0.25
temporal error norm MODEL curve
APPROX temporal norm error points

MATLAB CODES

```
%Spatial Convergence

A1 = [1 -2.30259 ; 1 -2.99573 ; 1 -3.68888 ; 1 -4.38202 ];

b1 = [4.23013 ; 3.55749 ; 2.70972 ; 1.61443];

%solve for a and b using matrix method of least squares
% Ax = b
% ATAx = ATb
% x = ATA\ATb

AT_1 = transpose(A1);
x1 = ( (AT_1)*(A1) ) \ ( (AT_1)*(b1) );

x1

%Temporal Convergence

A2 = [1 -1.3863 ; 1 -2.0794 ; 1 -2.0794 ; 1 -3.4657];

b2 = [4.34020 ; 4.33000 ; 4.32491 ; 4.32236];

%solve for a and b using matrix method of least squares
% Ax = b
% ATAx = ATb
% x = ATA\ATb

AT_2 = transpose(A2);
x2 = ( (AT_2)*(A2) ) \ ( (AT_2)*(b2) );

x2

%Spatial/Temporal Convergence

A3 = [1 -5.76832 ; 1 -6.46147 ;1 -7.15455 ; 1 -5.54524 ; 1 -7.15455 ; 1 -5.54499 ; 1 -
8.54097 ];

b3 = [4.27773 ; 4.27503 ; 4.27095 ; 4.29162 ; 4.32160 ; 4.26619 ; 4.28695];

%solve for a and b using matrix method of least squares
% Ax = b
% ATAx = ATb
% x = ATA\ATb

AT_3 = transpose(A3);
x3 = ( (AT_3)*(A3) ) \ ( (AT_3)*(b3) );

x3
```

```
%graph 1
dx1 = [.1 ; .05 ; .025 ; .0125];
dt1 = 0.0039047;
spatial_norm_error = [75.1986 ; 35.0750 ; 15.025 ; 5.025];

%initializing Error Norm
spatial_norm_curve = [0 ; 0 ; 0 ; 0];

spatial_norm_curve = 1367.26 * ((dx1).^(1.25441031)) + 77.1976 * ((dt1).^(0.00754683)) -
70.20*((dx1*dt1).^(-0.00499521));

figure ( 1 )
plot(dx1,spatial_norm_curve)
hold on
scatter(dx1,spatial_norm_error)
hold on
%plot(dx1,spatial_norm_error)
legend('spatial error norm MODEL curve','APPROX spatial norm error points')
title ( ' L1 Error Norm vs dx' );
xlabel ( 'dx' );
ylabel ( 'L1 Error Norm' );

hold on




%graph 2
dx2 = 0.10;
dt2 = [0.25 ; 0.125 ; 0.0625 ; 0.03125];

temporal_norm_error = [76.7230 ; 75.9446 ; 75.5586 ; 75.3664];


%Initializing Error Norm
temporal_norm_curve = [0 ; 0 ; 0 ; 0];

temporal_norm_curve = 1367.26 * ((dx2).^(1.25441031)) + 77.1976 * ((dt2).^(0.00754683)) -
70.20*((dx2*dt2).^(-0.00499521));

figure ( 2 )
plot( dt2, temporal_norm_curve)
hold on
scatter(dt2, temporal_norm_error)
hold on
%plot(dt2, temporal_norm_error)
legend('temporal error norm MODEL curve', 'APPROX temporal norm error points'  )
title ( '  L1 Error Norm vs dt' );
xlabel ( 'dt' );
ylabel ( 'L1 Error Norm' );
```