Part 4

For $x_j^{\dagger} = \dfrac{2\pi j}{N}$

$$\sum_{j=1}^{N} \cos(k \cdot x_j^{\dagger}) = \sum_{j=1}^{N} \cos\left(\dfrac{2\pi k j}{N}\right) = \begin{cases} N & \text{if } k = mN \\ 0 & \text{otherwise} \end{cases}$$

where $\boxed{m}$ is an integer; $\boxed{k}$ is a multiple of $\boxed{N}$.

From Part 3

$$IN - I = \dfrac{2\pi}{N} \sum_{k=1}^{\infty} a_k^{\dagger} \sum_{j=1}^{N} \cos\left(\dfrac{2\pi k j}{N}\right)$$

Let $k = m \cdot N$, so $\boxed{k}$ is a multiple of $\boxed{N}$.

$$IN - I = \dfrac{2\pi}{N} \sum_{m=1}^{\infty} a_{mN}^{\dagger\dagger} \cdot N$$

$$IN - I = 2\pi \sum_{m=1}^{\infty} a_{mN}^{\dagger\dagger}$$

$\underleftarrow{\qquad}$ multiples of $\boxed{N}$