ISC5907 - Prelim Preparation Class
Spring 2013 Prelim Exam Question 5

August 5, 2016

1 Question 5: Linear Algebra Prelim Question

1.1 Question 1a: Deriving Equations for L and D

$$\mathbf{A} = \mathbf{LDL}^{\mathrm{T}} = \begin{pmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{pmatrix} \begin{pmatrix} D_1 & 0 & 0 \\ 0 & D_2 & 0 \\ 0 & 0 & D_3 \end{pmatrix} \begin{pmatrix} 1 & L_{21} & L_{31} \\ 0 & 1 & L_{32} \\ 0 & 0 & 1 \end{pmatrix} \tag{1}$$

$$= \begin{pmatrix} D_1 & & \text{(symmetric)} \\ L_{21}D_1 & L_{21}^2 D_1 + D_2 & \\ L_{31}D_1 & L_{31}L_{21}D_1 + L_{32}D_2 & L_{31}^2 D_1 + L_{32}^2 D_2 + D_3. \end{pmatrix} \tag{2}$$

$$D_j = A_{jj} - \sum_{k=1}^{j-1} L_{jk}^2 D_k \tag{3}$$

$$L_{ij} = \frac{1}{D_j}\left(A_{ij} - \sum_{k=1}^{j-1} L_{ik}L_{jk}D_k\right), \qquad \text{for } i > j \tag{4}$$

Source: https://en.wikipedia.org/wiki/Cholesky_decomposition

1.2 Question 1b: Pseudo-Code for LDL^T Decomposition

Algorithm 1 LDL^T Decomposition

```
1: procedure COMPUTE
2:     D_1 ← A_11
3:     L_21 ← A_21 / D_1
4:     for (i in 2...n) do:
5:         D_i ← A_ii Σ_{k=1}^{i-1} L_ik^2 D_j
6:         for (j in i...n) do:
7:             L_ij ← (1/D_j)(A_ij − Σ_{k=1}^{i-1} L_ik L_jk D_k)
```

1.3 Question 1c: Operation Count for LDL^T Decomposition

This LDL^T Decomposition requires $\frac{n^3}{3}$ operations. This makes it only about half as expensive as the LU decomposition, which is $\frac{2n^3}{3}$ operations (see Trefethen and Bau 1997).

Source: https://en.wikipedia.org/wiki/Cholesky_decomposition#Computation
Wikipedia gives the number of operation counts explicitly and states, with reference, that the LDLT decomposition uses the same number of operations.

1.4 Question 1d: Advantages over Cholesky Decomposition

LDL^T Decomposition is very advantageous because it eliminates the need to compute square roots, which can be expensive to compute and can slow down the factorization.

Source: https://en.wikipedia.org/wiki/Cholesky_decomposition#Computation