Q4. Monte Carlo: (Dr. Shanbhag)
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Method

For Ellipse E_1 we have:

$$E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

We know that Ellipse E_2 has the same shape as E_1 but it is rotated counter clockwise for $\frac{\pi}{3}$, then we have:

$$\begin{bmatrix} x_{new} \\ y_{new} \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} xcos\theta + ysin\theta \\ xsin\theta - ycos\theta \end{bmatrix}$$

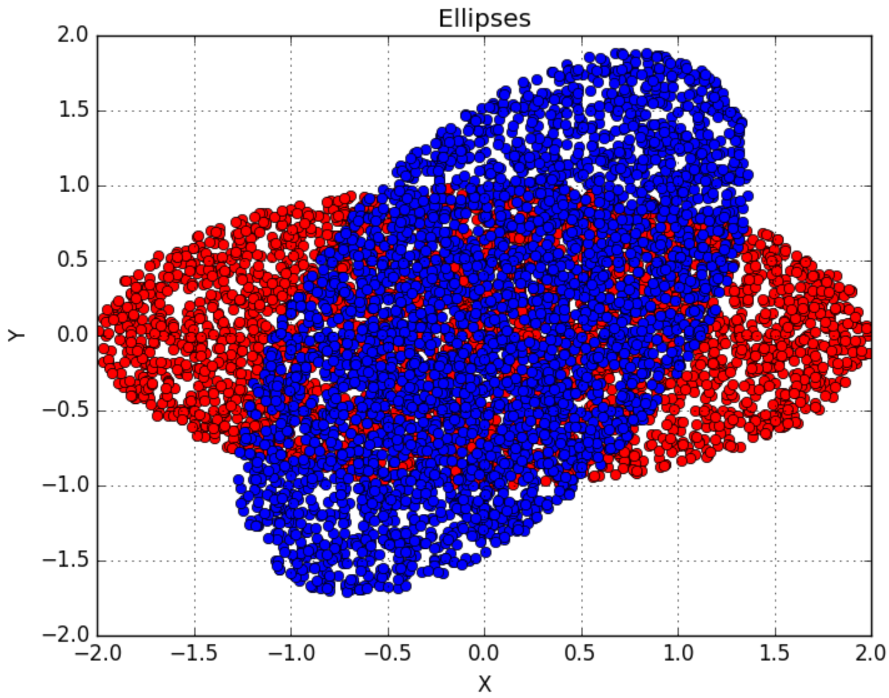
Also, we know that the center of E_2 , O' is not $O(0,0)$, but we know $OO' = \delta = 0.1$. Using the right-angle triangle we have with legs of x_{new} and y_{new} which leads us to:

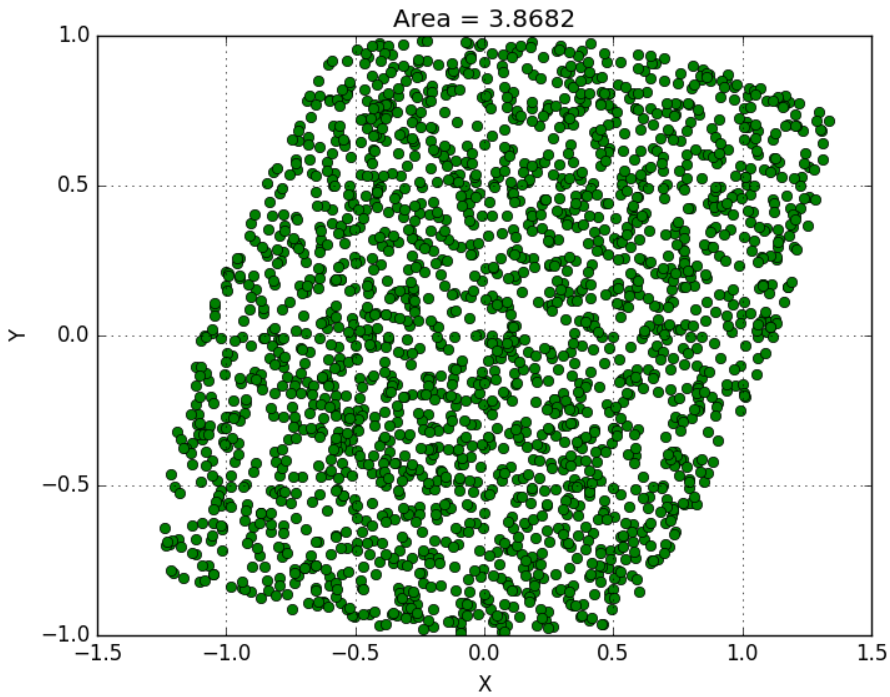
$$O' = (x_{new}, y_{new}) = (\delta cos(\frac{\pi}{3}), \delta sin(\frac{\pi}{3}))$$

$$E_2 : \frac{[(x - x_{new})cos\theta + (y - y_{new})sin\theta]^2}{a^2} + \frac{[(x - x_{new})sin\theta - (y - y_{new})cos\theta]^2}{b^2} = 1$$

For all the equations, $a = 2$ and $b = 1$.
With using simple monte carlo with considering the criteria that each point should be true for: $E1 <= 1$ AND $E2 <= 1$ For The Area we will have:

$$Area = 3.8682$$


Ellipses
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
Y
−2.0 −1.5 −1.0 −0.5 0.0 0.5 1.0 1.5 2.0
X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