I am using G_k to denote the event of choosing a door with a goat behind it after k doors have been revealed, and similarly C_k to denote choosing a car after k doors revealed.

---

(1) a) $P(\text{win}) = P(C_0) = 1/3$

b) $P(\text{win}) = P(C_1) = P(C_1|G_0)P(G_0) + \cancelto{0}{P(C_1|C_0)}P(C_0) = 1 \times \frac{2}{3} + 0 = 2/3$

(2) a) $P(\text{win}) = P(C_0) = 1/n$

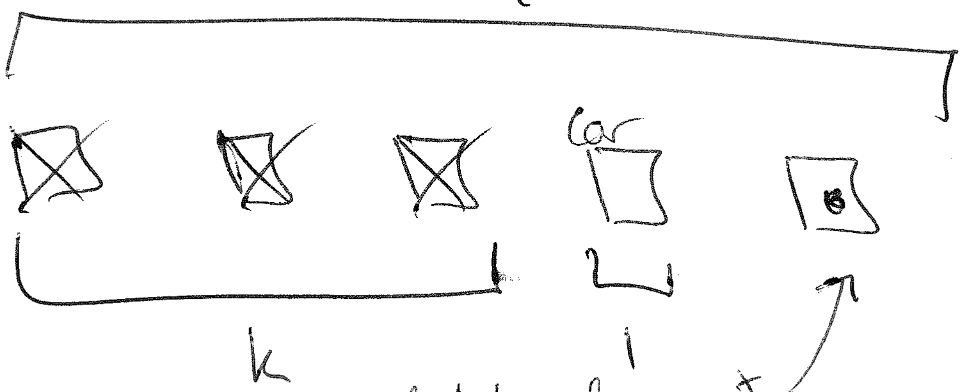
b) $P(\text{win}) = P(C_1) = P(C_1|G_0)P(G_0) + \cancelto{0}{P(C_1|C_0)}P(C_0) = \left(\frac{1}{n-2}\right)\left(\frac{n-1}{n}\right)$

(3) a) $P(\text{win}) = P(C_0) = 1/n$

b) $P(\text{win}) = \cancel{P(G_{n-2})}\ P(G_{n-2})$

which is given by the recurrence relation $P(G_k) = 1 - \frac{P(G_{k-1})}{n-k}$, $P(G_0) = \frac{n-1}{n}$

as derived on the next page



$$P(win) = P(G_k)$$

$$= P(G_k \mid G_{k-1})P(G_{k-1}) + P(G_k \mid C_{k-1})P(C_{k-1})$$

$$= \left(\frac{n-k-1}{n-k}\right)P(G_{k-1}) + \left(\cancel{\frac{n-k}{n-k}}\ 1\right)P(C_{k-1})$$

$$= \left(\frac{n-k-1}{n-k}\right)P(G_{k-1}) + \left(1 - P(G_{k-1})\right)$$

$$\cancel{= 1 - \left(\frac{n-k-1}{n-1}\right)P(G_{k-1})}$$ * algebra?

$$= 1 - \frac{P(G_{k-1})}{n-k}$$

Recurrence relation!

note: $P(G_0) = \frac{n-1}{n}$, can build up to $P(win)$ from there

200256677_1_Beerli

May 29, 2020

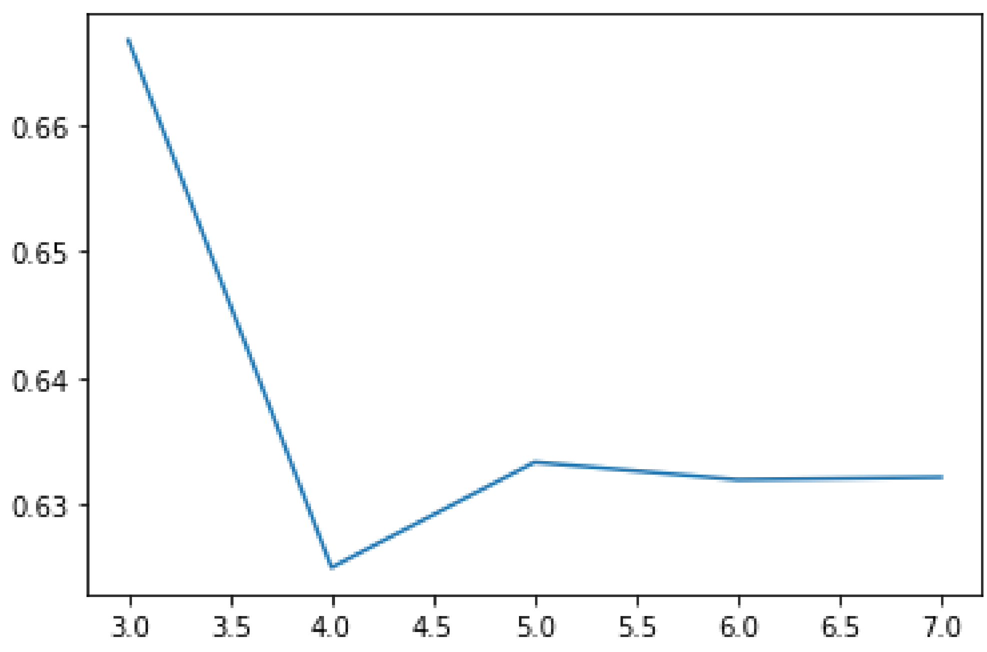
[2]:
```python
import numpy as np
from matplotlib import pyplot as plt
```

0.0.1 3B - Always Switching

My answer for 3B is on a hand-written page. Here, I am just testing my answer with a simulation.

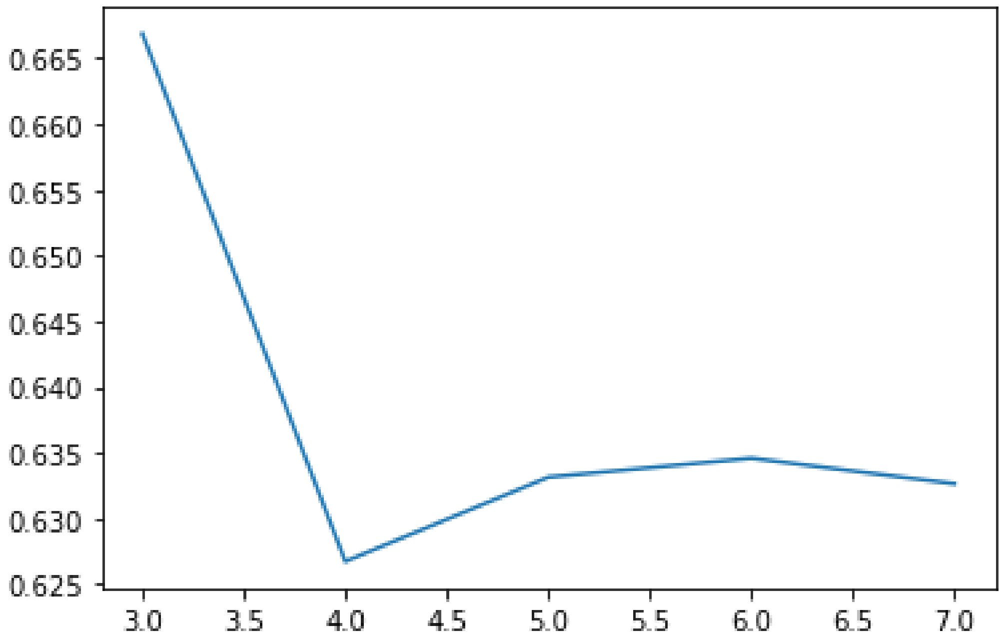
[4]:
```python
n = 3; k = n - 2
# exact probability
def prob(n):
    k = n - 2
    p = (n - 1) / n
    for i in range(1,k+1):
        p = 1 - p / (n - i)
    return p

plt.plot(range(3,8), [prob(ni) for ni in range(3,8)])
```

[4]: 0.6333333333333333



[277]:
```
probs = []; ns = list(range(3, 8))
for n in ns:

    k = n-2

    # simulation of many games, n doors, k doors opened, switch every time
    reps = 99999; wins = 0
    for game in range(reps):
        car = np.random.randint(0, n)
        doors = np.zeros(n); doors[car] = 1
        goatlist = []

        # choose first door
        choice = np.random.choice(list(set(range(n))))
        for i in range(k):
            # open random goat door
            goat = np.random.choice(list(set(range(n)) - set(goatlist) -␣
↪set([car, choice])))
            goatlist.append(goat)
            # switch
            choice = np.random.choice(list(set(range(n)) - set(goatlist) -␣
↪set([choice])))

        if(choice == car):
            wins = wins + 1

    # experimental probability
    probs.append(wins / reps)

plt.plot(ns, probs)
```

[277]: [<matplotlib.lines.Line2D at 0x7f7e790fced0>]



0.0.2 3C - Switching With Probability p

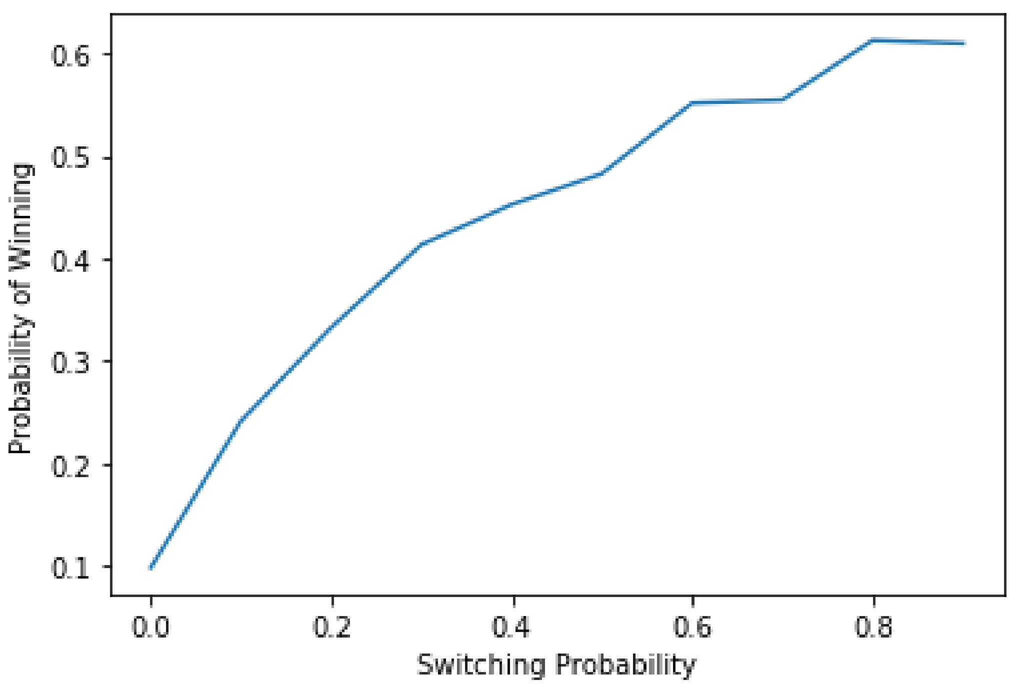
[13]:
```
# over a range of switching probabilities
win_chance = []
for p in np.arange(0, 1, 0.1):
    print(p)

    # simulation of many games, n doors, k doors opened, switch with␣
↪probability p
    n = 10; k = n - 2; reps = 999; wins = 0
    for game in range(reps):
        car = np.random.randint(0, n)
        doors = np.zeros(n); doors[car] = 1
        goatlist = []

        # choose first door
        choice = np.random.choice(list(set(range(n)) - set(goatlist)))
        for i in range(k):
            # open random goat door
            goat = np.random.choice(list(set(range(n)) - set(goatlist) -␣
↪set([car, choice])))
            goatlist.append(goat)
            # switch with probability p
            if(np.random.rand() < p):
```

```
                choice = np.random.choice(list(set(range(n)) - set(goatlist) -␣
↪set([choice])))

        if(choice == car):
            wins = wins + 1

    # experimental probability to win, for this particular switching probability
    win_chance.append(wins / reps)
```

```
0.0
0.1
0.2
0.30000000000000004
0.4
0.5
0.6000000000000001
0.7000000000000001
0.8
0.9
```

[14]:
```
plt.plot(np.arange(0, 1, 0.1), win_chance)
plt.xlabel("Switching Probability")
plt.ylabel("Probability of Winning")
```

[14]:
```
Text(0, 0.5, 'Probability of Winning')
```



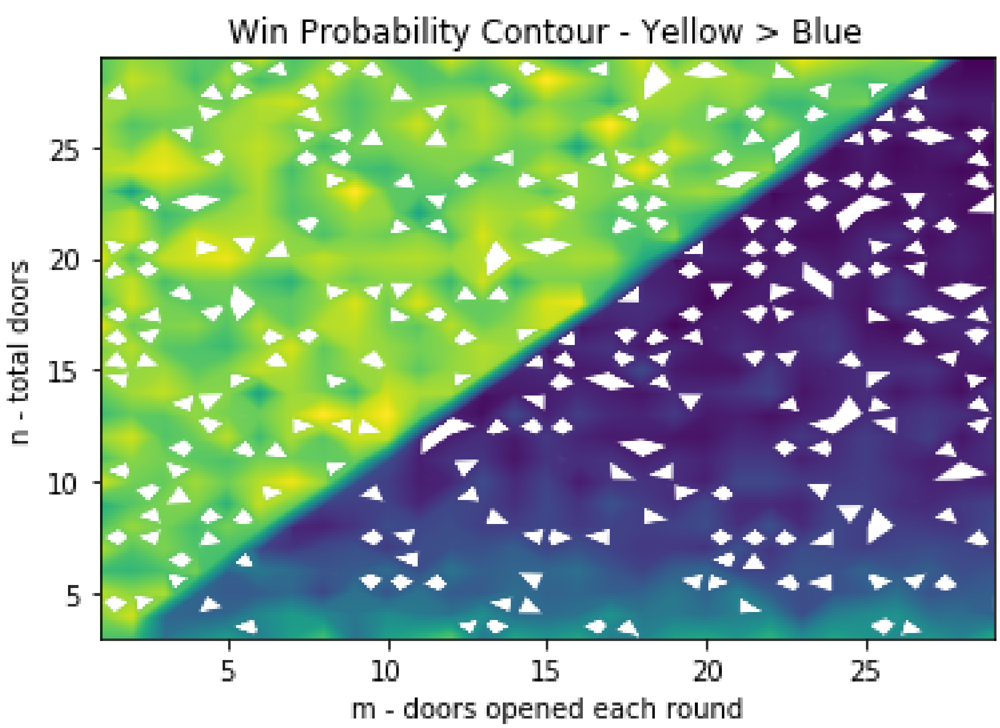
0.1 4 - Extended Monty Hall

[86]:
```
# in dire need of more performance, so let's revisit the problem even though␣
↪it's 5:30am friday morning ok sure good idea
def game(n, k, m, p) :

    # integer in [0,n-1] that represents a car
    car = np.random.randint(0, n)
    # the rest of the integers in [0,n-1]
    goats = np.array([i for i in np.arange(n) if i != car])
    # shuffle the goat doors so that we can avoid appends by drawing␣
↪sequentially from randomized list
    np.random.shuffle(goats)

    # choose first door
    choice = np.random.randint(0, n)
    i = 0
    while(i + m <= k):
        # monty opens m doors
        i = i + m
        # choose a new door with probability p
        if(np.random.rand() < p):
            choice = np.random.choice(list(set(np.arange(n)) - set(goats[:i])))

        # if monty can't open m doors, he opens the remaining doors so k are␣
↪open for last round of switching
        # this is what makes the hard line y=x in the contour plot
        if(i < k):
            # monty opens remaining k - i doors, leaving 2 closed for last round
            i = k
            # choose a new door with probability p
            if(np.random.rand() < p):
                choice = np.random.choice(list(set(np.arange(n)) - set(goats[:
↪i])))

    if(choice == car):
        return 1
    else:
        return 0
```

[87]:
```
p = 1
reps = 100
ns = range(3,30) # range of n total doors
ms = range(1,30) # range of m doors to be opened at once

win_chance = np.zeros(shape=(len(ns),len(ms)))
```

```python
# for a range of n total doors
for n in ns:

    k = n - 2

    # for a range of m doors opened at once
    for m in ms:

        # for multiple reps per parameter set
        wins = 0
        for rep in range(reps):
            wins = wins + game(n, k, m, p)

        # enter average win probabilty for parameters n,m
        win_chance[n-ns[0]][m-ms[0]] = (wins / reps)

plt.contour(ms, ns, win_chance,len(ns) * len(ms))
plt.ylabel("n - total doors")
plt.xlabel("m - doors opened each round")
plt.title("Win Probability Contour - Yellow > Blue")
```

[87]: Text(0.5, 1.0, 'Win Probability Contour - Yellow > Blue')

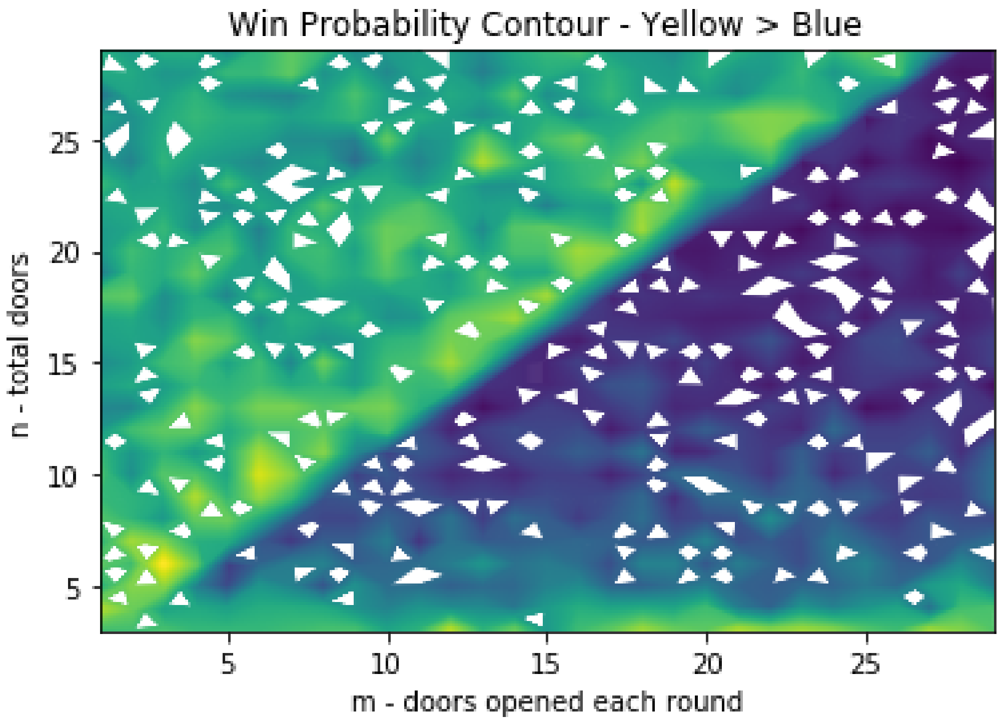


[88]:

```python
p = 0.5
reps = 100
ns = range(3,30) # range of n total doors
ms = range(1,30) # range of m doors to be opened at once

win_chance = np.zeros(shape=(len(ns),len(ms)))

# for a range of n total doors
for n in ns:

    k = n - 2

    # for a range of m doors opened at once
    for m in ms:

        # for multiple reps per parameter set
        wins = 0
        for rep in range(reps):
            wins = wins + game(n, k, m, p)

        # enter average win probabilty for parameters n,m
        win_chance[n-ns[0]][m-ms[0]] = (wins / reps)

plt.contour(ms, ns, win_chance,len(ns) * len(ms))
plt.ylabel("n - total doors")
plt.xlabel("m - doors opened each round")
plt.title("Win Probability Contour - Yellow > Blue")
```

[88]:

```
Text(0.5, 1.0, 'Win Probability Contour - Yellow > Blue')
```


Win Probability Contour - Yellow > Blue
n - total doors
m - doors opened each round
5
10
15
20
25

[ ]: