Q10. Numerical Integration: (Dr. Beerli) Spring 2014

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Calculate this Integral

$$I = \frac{1}{2\pi}\int_{-1}^{1}\int_{-1}^{1} e^{-(x^2+y^2)}dxdy$$

Method

First of all, for more convenience, I tried to make the integral more easier in 1D due to the Symmetry of the problem. We have:

$$I = [\frac{1}{\sqrt{2\pi}}\int_{-1}^{1} e^{-(x^2)}dx]^2$$

Gauss Quadrature

For this part, I have used John's Burkardt Gauss-Kronrod Package. It has both Gauss Rule and Gauss-Kronrod Weights and abscissas.

Clenshaw-Curtis Quadrature

$$I = \int_{-1}^{1} f(x)dx = \int_{0}^{\pi} f(cos\theta)sin\theta d\theta$$

We can say:

$$f(cos\theta) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k cos(k\theta)$$

So, we will have:

$$I = \int_{-1}^{1} f(x)dx = \int_{0}^{\pi} f(cos\theta)sin\theta d\theta = a_0 + \sum_{k=1}^{\infty} \frac{2a_{2k}}{1-(2k)^2}$$

With :

$$a_k = \frac{2}{\pi}\int_{0}^{\pi} f(cos\theta)cos(k\theta)d\theta$$

Romberg

$$I = I_{2n} + \frac{I_{2n} - I_n}{3}$$

Which I used trapezoidal rule for calculating each part of the fractions.

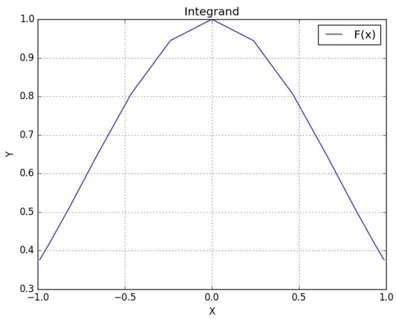
MonteCarlo

For this part, we just need to produce N sample darts between our intervals which is -1 and 1. I have used uniform distribution. Then we find the value of each sample dart at our function, multiply with the length of our interval which is 2.

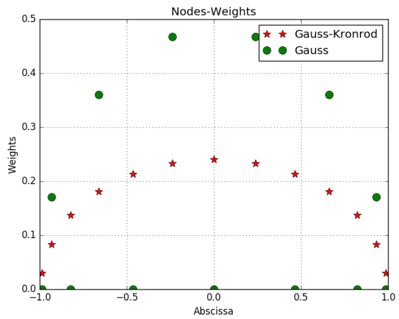
Results

Gauss Quadrature

The Integral Using Gauss rule :: I = 0.355072003479
The Integral Using Gauss-Kronrod (N=6) :: I = 0.355072313218
Relative Error of Integrals = 4.36163263248e-07



(a) Integrand using Gauss Rule



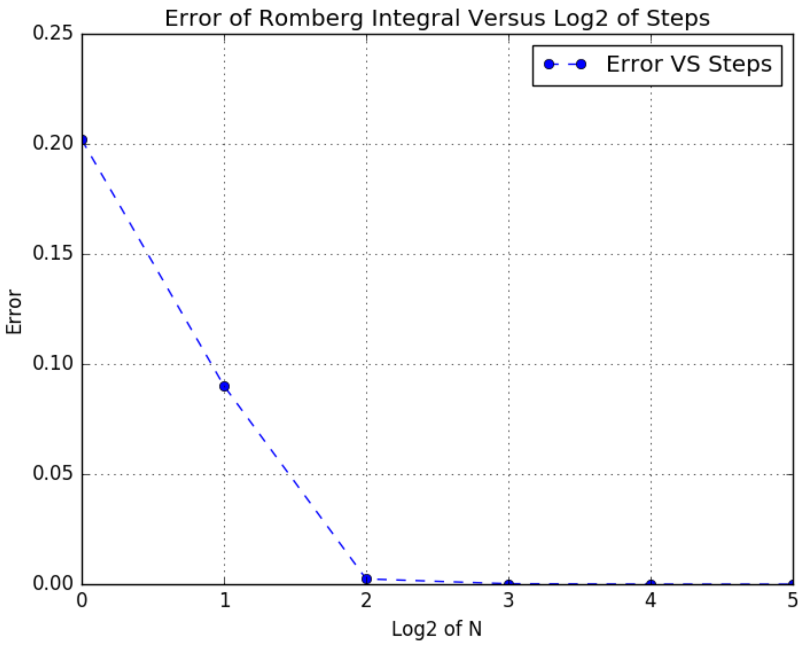
(b) Gauss and Gauss-Kronrod Rule

Clenshaw-Curtis Quadrature

The Result using ClenShaw-Curtis Integration = 0.355072313219

Romberg's Method

Step = 1 , Romberg = 0.153168208369 , Norm = 0.201901791631
Step = 2 , Romberg = 0.445237854316 , Norm = 0.0901678543157
Step = 4 , Romberg = 0.35747361225 , Norm = 0.00240361224977
Step = 8 , Romberg = 0.355280298733 , Norm = 0.000210298733121
Step = 16 , Romberg = 0.355094403579 , Norm = 2.44035792558e-05
Step = 32 , Romberg = 0.355074869388 , Norm = 4.86938789629e-06



MonteCarlo

The Result with 100000 darts = 0.355103269763
Standard Deviation = 0.00632455532034