8 #2

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Newton's Method: $\nabla^2 f(x_k) p = -\nabla f(x_k)$

on $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$

with starting point $x_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

and update step $x_{k+1} = x_k - [\nabla^2 f(x_k)]^{-1} \nabla f(x_k)$

$$\nabla f(x_k) = \begin{bmatrix} 1 + 4x_1 + 2x_2 \\ -1 + 2x_1 + 4x_2 \end{bmatrix}, \quad \nabla^2 f(x_k) = H = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}, \quad H^{-1} = \begin{bmatrix} 1/3 & -1/6 \\ -1/6 & 1/3 \end{bmatrix}$$

note: H constant, independent of x_k

take one step using Newton's method:

$$x_2 = x_1 - H^{-1} \nabla f(x_1)$$

$$= - \begin{bmatrix} 1/3 & -1/6 \\ -1/6 & 1/3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 \\ 1/2 \end{bmatrix}$$

$$g_2 = \nabla f(x_2) = \begin{bmatrix} 1 + 4(-1/2) + 2(1/2) \\ -1 + 2(-1/2) + 4(1/2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\nabla f(x_2) = \vec{0}$, so the method has indeed converged in one iteration