Prelim Summer 2017, Q4 5894371007489034

Part 3

$$\pi(y|x) = \frac{\pi(y)\,\pi(x|y)}{\pi(x)}$$ Bayes theorem

$\boxed{y}$ is a discrete random variable with only two possible outcomes.

$$\pi(y=0|x) = \frac{\pi(y=0)\,\pi(x|y=0)}{\pi(x)}$$

$\pi(y=0) = 1/2$

$\pi(x|y=0) = N(\mu=1, \sigma)$

$\pi(x) = \frac{1}{2}(N(\mu=1,\sigma)) + \frac{1}{2}(N(\mu=2,\sigma))$

$y=0$ $\rightarrow$ $$\pi(y=0|x) = \frac{\frac{1}{2}[N(\mu=1,\sigma)]}{\frac{1}{2}(N(\mu=1,\sigma)) + \frac{1}{2}(N(\mu=2,\sigma))}$$

$$\pi(y=1|x) = \frac{\pi(y=1)\,\pi(x|y=1)}{\pi(x)}$$

$\pi(y=1) = 1/2$

$\pi(x|y=1) = N(\mu=2, \sigma)$

$\pi(x) = \frac{1}{2}(N(\mu=1,\sigma)) + \frac{1}{2}(N(\mu=2,\sigma))$

$y=1$ $\rightarrow$ $$\pi(y=1|x) = \frac{\frac{1}{2}[N(\mu=2,\sigma)]}{\frac{1}{2}(N(\mu=1,\sigma)) + \frac{1}{2}(N(\mu=2,\sigma))}$$

→ see attached matlab code and plots

As the standard deviation increases, the two different distributions $N(\mu=1,\sigma)$ and $N(\mu=2,\sigma)$ start to overlap and become one distribution.