

Part 1

$$\pi(y=0) = \frac{1}{2} \quad \pi(y=1) = \frac{1}{2}$$

$$\pi(x|y) = \begin{cases} N(\mu=1, \sigma=2), & \text{if } y=0 \\ N(\mu=2, \sigma=2), & \text{if } y=1 \end{cases}$$

$$\pi(x) = \sum_y \pi(x|y) \cdot \pi(y)$$

$$\pi(x) = \pi(x|y=0) \cdot \pi(y=0) + \pi(x|y=1) \cdot \pi(y=1)$$

$$\pi(x) = \frac{1}{2} [\pi(x|y=0)] + \frac{1}{2} [\pi(x|y=1)]$$

$$\pi(x) = \frac{1}{2} N(\mu=1, \sigma=2) + \frac{1}{2} N(\mu=2, \sigma=2)$$

use interval of 3σ for plotting

$$1 - 3\sigma = -5$$

$$2 + 3\sigma = 8 \quad \rightarrow \text{interval } [-5, 8]$$

See attached Matlab code

See attached Plot of $\pi(x)$