Q2. Linear Algebra: (Dr. Wang) Summer 2015

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Consider the following matrix:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Part-a):
Using mathematical induction we have:

$$n = 3 \longrightarrow A^3 = A + A^2 - I$$
$$p(3) = A^3 = A + A^2 - I$$

our formula is true for n=3

$$p(k) = A^k = A^{k-2} + A^2 - I$$

we assume that is true for n = k

$$p(k+1) = A^{k+1} = A^{k-1} + A^2 - I$$

we are going to show that it is true for n = k+1.

$$Ap(k) = A(A^k) = A(A^{k-2} + A^2 - I) = A^{k-1} + A^3 - A$$

Now we use our $p(3)$ here:

$$\Longrightarrow Ap(k) = A^{k-1} + A + A^2 - I - A = A^{k-1} + A^2 - I = p(k+1)$$

Part-b):

For calculating A^{100} we have:

Using formula: $A^n = A^{n-2} + A^2 - I$ we have:

$$n = 4 \longrightarrow A^4 = A^2 + A^2 - I = 2A^2 - I$$
$$n = 6 \longrightarrow A^6 = A^4 + A^2 - I = 2A^2 - I + A^2 - I = 3A^2 - 2I$$

$$n = 8 \longrightarrow A^8 = A^6 + A^2 - I = 3A^2 - 2I + A^2 - I = 4A^2 - 3I$$

$$\vdots$$

$$\vdots$$

$$A^N = \frac{N}{2}A^2 - (\frac{N}{2} - 1)I$$

Here for $N = 100$ we have:

$$A^{100} = 50A^2 - 49I$$

We have:

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

So, we have to just multiply A^2 with 50 and subtract from $49I$, then we will have:

$$A^{100} = \begin{bmatrix} 1 & 0 & 0 \\ 50 & 1 & 0 \\ 50 & 0 & 1 \end{bmatrix}$$