Seth Boren

Q2 Fourier Analysis

(Dr. Meyer-Baese; Spring **2013**)

Part 2

Show that the Fourier Transform of the function $\boldsymbol{g(t) = sinc(t-5)}$ is $\boldsymbol{G(\omega) = rect\left(\frac{\omega}{2}\right)\ e^{-i5\omega}}$.

The given equation $\boldsymbol{G(\omega) = \int_{-\infty}^{\infty} g(t)\, e^{-iwt} dt}$

has a companion equation $\boldsymbol{g(t) = (\frac{1}{2\pi}) \int_{-\infty}^{\infty} G(\omega)\, e^{iwt} d\omega}$, where ***i*** represents the imaginary number.

From the definition of the function $\boldsymbol{sinc}$ we have that $\boldsymbol{sinc(t-5) = sin(t-5)/(t-5)}$.

Starting with the equation $\boldsymbol{g(t\,) = (\frac{1}{2\pi}) \int_{-\infty}^{\infty} G(\omega)\, e^{iwt} d\omega}$

Insert the given $\boldsymbol{G(\omega)}$ to obtain $\boldsymbol{g(t) = (\frac{1}{2\pi}) \int_{-\infty}^{\infty} rect\left(\frac{\omega}{2}\right)\ e^{-i5\omega}\, e^{iwt} d\omega}$

Consider the values of of $\boldsymbol{\omega}$ from $\boldsymbol{\omega = -1}$ to $\boldsymbol{\omega = 1}$:

The value of the $\boldsymbol{rect\left(\frac{\omega}{2}\right)}$ function equals **1** from $\boldsymbol{\omega = -1}$ **to** $\boldsymbol{\omega = 1}$:

$$\boldsymbol{g(t) = (\frac{1}{2\pi}) \int_{-1}^{1} 1\ e^{-i5\omega}\, e^{iwt} d\omega}$$

Combining exponential terms:

$$\boldsymbol{g(t) = (\frac{1}{2\pi}) \int_{-1}^{1} 1\ e^{w(i(t-5))} d\omega}$$

Using standard integral formula:

$$\boldsymbol{g(t) = \left(\frac{1}{2\pi}\right)(1/i(t-5))\, e^{w(i(t-5))}} \Bigg|_{-1}^{1}$$

Completing integration process:

$$\boldsymbol{g(t) = \left(\frac{1}{2\pi}\right)(1/(t-5))\left(\frac{1}{i}\right)(\,e^{i(t-5)} - \ e^{-i(t-5)})}$$

Part 2 continued

From previous page:

$$\boldsymbol{g(t) = \left(\frac{1}{2\pi}\right)(1/(t-5))\left(\frac{1}{i}\right)\left(e^{i(t-5)} - e^{-i(t-5)}\right)}$$

Rearranging terms:

$$\boldsymbol{g(t) = \left(\frac{1}{\pi}\right)(1/(t-5))\left(\frac{1}{2i}\right)\left(e^{i(t-5)} - e^{-i(t-5)}\right)}$$

Exponential form of $\boldsymbol{sin(x)}$ is $\boldsymbol{sin(x) = \left(\frac{1}{2i}\right)\left(e^{i\Theta} - e^{-i\Theta}\right)}$

Substitution of exponential form of $\boldsymbol{sin(x)}$:

$$\boldsymbol{g(t) = \left(\frac{1}{\pi}\right)\left(\frac{1}{t-5}\right) sin(t-5)}$$

Substitution of previous formula for $\boldsymbol{sinc(t-5) = sin(t-5)/(t-5)}$:

$$\boldsymbol{g(t) = \left(\frac{1}{\pi}\right) sinc(t-5)}$$

The preceding shows that the Fourier Transform of the function $\boldsymbol{g(t) = sinc(t-5)}$

is $\boldsymbol{G(\omega) = rect\left(\frac{\omega}{2}\right)\ e^{-i5\omega}}$ **for values of $\boldsymbol{\omega}$ from** $\boldsymbol{\omega = -1}$ to $\boldsymbol{\omega = 1.}$