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Q2 Fourier Analysis

(Dr. Meyer-Baese; Spring **2014**)

Part 1a and 1b combined

$$g(t) = 14sin(8t) + 6\,sin\left(7t - \frac{\pi}{2}\right) + 5cos(10t - \frac{\pi}{10})$$

Convert equation using trigonometric rules (aided by online symbolab trig caculator).

$$g(t) = 14sin(8t) - 6\,cos(7t) - 0.309\cos(10t) + 0.951\sin(10t)$$

Standard Euler Equations:

$$\cos(\Theta) = \left(\frac{1}{2}\right)\left(e^{i\Theta}\right) + \left(\frac{1}{2}\right)(e^{-i\Theta}) \quad \textit{and} \quad \sin(\Theta) = \left(\frac{1}{2i}\right)\left(e^{i\Theta}\right) - \left(\frac{1}{2i}\right)(e^{-i\Theta})$$

Replacing Trig terms with exponential Euler terms:

$$g(t) = 14\left(\left(\frac{1}{2i}\right)\left(e^{i8t}\right) - \left(\frac{1}{2i}\right)(e^{-i8t})\right) - 6\left(\left(\frac{1}{2}\right)\left(e^{i7t}\right) + \left(\frac{1}{2}\right)(e^{-i7t})\right) - 0.309\left(\left(\frac{1}{2}\right)\left(e^{i10t}\right) + \left(\frac{1}{2}\right)(e^{-i10t})\right) + \cdots$$

$$\ldots + 0.951\left(\left(\frac{1}{2i}\right)\left(e^{i10t}\right) - \left(\frac{1}{2i}\right)(e^{-i10t})\right)$$

Simplifying terms:

$$g(t) = (-7i)\left(e^{i8t}\right) + (7i)(e^{-i8t}) - 3\left(e^{i7t}\right) - 3(e^{-i7t}) - (0.154)\left(e^{i10t}\right) - (0.154)(e^{-i107t}) + \cdots$$

$$\ldots + (-0.475i)\left(e^{i10t}\right) - (-0.475i)(e^{-i10t})$$

Combining like terms to arrive at **exponential form of Fourier Series** for function g(t) :

$$g(t) = (-7i)\left(e^{i8t}\right) + (7i)(e^{-i8t}) - 3\left(e^{i7t}\right) - 3(e^{-i7t}) - (0.154 + 0.475i)\left(e^{i10t}\right) - (0.154 - 0.475i)(e^{-i10t})$$

$$g(t) = (-7i)\left(e^{i8t}\right) + (7i)\left(e^{-i8t}\right) - 3\left(e^{i7t}\right) - 3\left(e^{-i7t}\right) - (0.154 + 0.475i)\left(e^{i10t}\right) - (0.154 - 0.475i)\left(e^{-i10t}\right)$$

Extracting Coefficients $\boldsymbol{a_n}$ from the exponential Fourier Series equation (for periodic interval 2π):

n = 7 $a_7 = -3$

n = -7 $a_{-7} = -3$

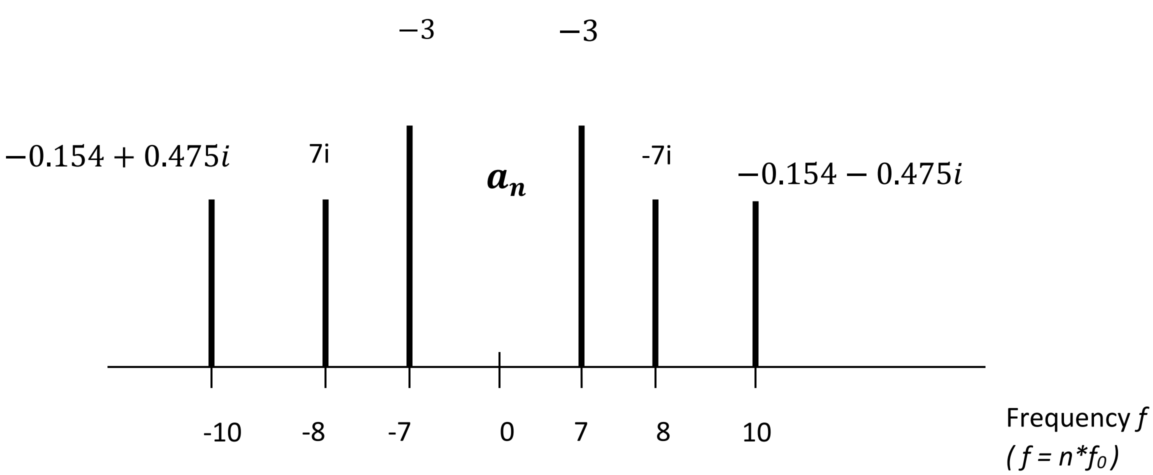
n = 8 $a_8 = -7i$

n = 8 $a_{-8} = 7i$

n = 10 $a_{10} = -0.154 - 0.475i$

n = -10 $a_{-10} = -0.154 + 0.475i$

Using the coefficients from the exponential Fourier Series equation to **plot the Spectra** (for periodic interval 2π):



Fourier Series Spectra