Q5. Numerical Quadrature: (Dr. Shanbhag)

Amirhessam Tahmassebi

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1 Method

For this problem, I have used Green's Theorem:
Let C be a positively oriented, piece-wise smooth, simple closed curve in a plane, and let D be the region bounded by C. If L and M are functions of (x, y) defined on an open region containing D and have continuous partial derivatives there, then:

$$\oint_C (L\,dx + M\,dy) = \iint_S (\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y})\,dx\,dy$$

where the path of integration along C is counterclockwise.

So, easily we can implement our integrals for area, central positions and gyration radius.

Area:

$$Area = \iint_S dx\,dy$$

$$\Longrightarrow (\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}) = 1 \Longrightarrow (M = x \quad \text{and} \quad L = 0)$$

$$\Longrightarrow Area = \oint_C x\,dy$$

X_{cm}:

$$X_{cm} = \frac{1}{Area} \iint_S x\,dx\,dy$$

$$\Longrightarrow (\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}) = x \Longrightarrow (M = \frac{x^2}{2} \quad \text{and} \quad L = 0)$$

$$\Longrightarrow X_{cm} = \frac{1}{Area} \oint_C \frac{x^2}{2}\,dy$$

Y_{cm}:

$$Y_{cm} = \frac{1}{Area} \iint_S y\,dx\,dy$$

$$\Longrightarrow (\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}) = y \Longrightarrow (M = 0 \quad \text{and} \quad L = -\frac{y^2}{2})$$

$$\Longrightarrow Y_{cm} = \frac{-1}{Area} \oint_C \frac{y^2}{2}\, dy$$

But, here we need to be careful about path C which is always counter clockwise. At the end, we will come up with another negative sign due to the path.

$$\Longrightarrow Y_{cm} = \frac{1}{Area} \oint_C \frac{y^2}{2}\, dy$$

X_g^2:

$$X_g^2 = \frac{1}{Area} \iint_S (x - X_{cm})^2\, dx\, dy$$

$$\Longrightarrow (\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}) = (x - X_{cm})^2 \Longrightarrow (M = \frac{(x - X_{cm})^3}{3} \quad \text{and} \quad L = 0)$$

$$\text{Due to the path} \Longrightarrow X_g^2 = \frac{-1}{Area} \oint_C \frac{(x - X_{cm})^3}{3}\, dy$$

Y_g^2:

$$Y_g^2 = \frac{1}{Area} \iint_S (y - Y_{cm})^2\, dx\, dy$$

$$\Longrightarrow (\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}) = (y - Y_{cm})^2 \Longrightarrow (M = 0 \quad \text{and} \quad L = -\frac{(y - Y_{cm})^3}{3})$$

$$\text{Due to the path} \Longrightarrow Y_g^2 = \frac{1}{Area} \oint_C \frac{(y - Y_{cm})^3}{3}\, dy$$

$$R_{Gyration}^2 = X_g^2 + Y_g^2$$

2 Results

The Area = 565.486677646
The X_{cm} = 2.63488313796e-16
The Y_{cm} = 0.833333333333
The R_G = 101.433333333