Prelim Summer 2017, QZ 5894 3710 0798 9034

Part 3

$$f(x) = \frac{a_0^\dagger}{2} + \sum_{K=1}^{\infty} a_K^\dagger \cos(K \cdot x)$$

$$I = \pi a_0^\dagger$$

$$I\_N = IN = \frac{2\pi}{N} \sum_{j=1}^{N} f(x_j^\dagger) \quad \text{where} \quad x_j^\dagger = \frac{2\pi j}{N}$$

$$IN = \frac{2\pi}{N} \sum_{j=1}^{N} \left[\frac{a_0^\dagger}{2} + \sum_{K=1}^{\infty} a_K^\dagger \cos(K \cdot x_j^\dagger) \right]$$

$$IN = \frac{2\pi}{N} \sum_{j=1}^{N} \frac{a_0^\dagger}{2} + \frac{2\pi}{N} \sum_{j=1}^{N} \sum_{K=1}^{\infty} a_K^\dagger \cos(K \cdot x_j^\dagger)$$

$$IN = \left(\frac{2\pi}{N}\right)\left(\frac{N a_0^\dagger}{2}\right) + \frac{2\pi}{N} \sum_{K=1}^{\infty} a_K^\dagger \sum_{j=1}^{N} \cos(K \cdot x_j^\dagger)$$

$$IN = \pi a_0^\dagger + \frac{2\pi}{N} \sum_{K=1}^{\infty} a_K^\dagger \sum_{j=1}^{N} \cos\left(\frac{K \cdot 2\pi x_j^\dagger}{N}\right)$$

$$IN - I = \left(\pi a_0^\dagger + \frac{2\pi}{N} \sum_{K=1}^{\infty} a_K^\dagger \sum_{j=1}^{N} \cos\left(\frac{2\pi K j}{N}\right)\right) - \left(\pi a_0^\dagger\right)$$

(IN and I)

$$IN - I = \frac{2\pi}{N} \sum_{K=1}^{\infty} a_K^\dagger \sum_{j=1}^{N} \cos\left(\frac{2\pi K j}{N}\right)$$