Jump process

Simon Frost (@sdwfrost), 2020-04-27

Introduction

This implementation defines the model as a combination of two jump processes, infection and recovery, simulated using the Doob-Gillespie algorithm.

Libraries

```
using DifferentialEquations
using SimpleDiffEq
using Random
using DataFrames
using StatsPlots
using BenchmarkTools
```

Transitions

For each process, we define the rate at which it occurs, and how the state variables change at each jump. Note that these are total rates, not *per capita*, and that the change in state variables occurs in-place.

```
function infection_rate(u,p,t)
    (S,I,R) = u
    (β,c,γ) = p
    N = S+I+R
    β*c*I/N*S
end
function infection!(integrator)
  integrator.u[1] -= 1
  integrator.u[2] += 1
end
infection_jump = ConstantRateJump(infection_rate,infection!);

function recovery_rate(u,p,t)
    (S,I,R) = u
    (β,c,γ) = p
    γ*I
end
function recovery!(integrator)
  integrator.u[2] -= 1
  integrator.u[3] += 1
end
recovery_jump = ConstantRateJump(recovery_rate,recovery!);
```

Time domain

```
tmax = 40.0
tspan = (0.0,tmax);
```

For plotting, we can also define a separate time series.

```
δt = 0.1
t = 0:δt:tmax;
```

Initial conditions

```
u0 = [990,10,0]; # S,I,R
```

Parameter values

```
p = [0.05,10.0,0.25]; # β,c,γ
```

Random number seed

We set a random number seed for reproducibility.

```
Random.seed!(1234);
```

Running the model

Running this model involves:

- Setting up the problem as a `DiscreteProblem`;
- Adding the jumps and setting the algorithm using `JumpProblem`; and
- Running the model, specifying `SSAStepper()`

```
prob = DiscreteProblem(u0,tspan,p)
```

```
DiscreteProblem with uType Array{Int64,1} and tType Float64. In-place: true
timespan: (0.0, 40.0)
u0: [990, 10, 0]
```

```
prob_jump = JumpProblem(prob,Direct(),infection_jump,recovery_jump)
```

```
DiffEqJump.JumpProblem with problem DiffEqBase.DiscreteProblem and aggregat
or DiffEqJump.Direct
Number of constant rate jumps: 2
Number of variable rate jumps: 0
```

```
sol_jump = solve(prob_jump,SSAStepper());
```

Post-processing

In order to get output comparable across implementations, we output the model at a fixed set of times.

```
out_jump = sol_jump(t);
```

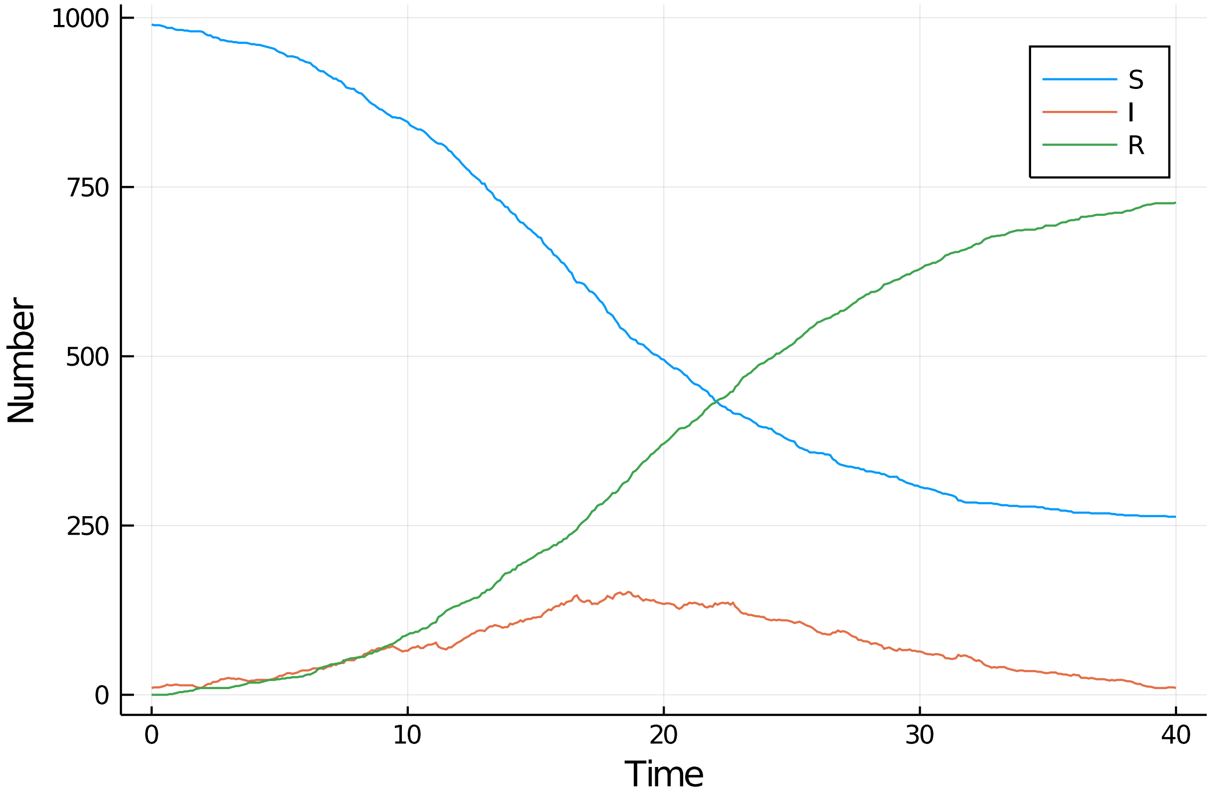
We can convert to a dataframe for convenience.

```
df_jump = DataFrame(out_jump')
df_jump[!,:t] = out_jump.t;
```

Plotting

We can now plot the results.

```
@df df_jump plot(:t,
    [:x1 :x2 :x3],
    label=["S" "I" "R"],
    xlabel="Time",
    ylabel="Number")
```



Benchmarking

```
@benchmark solve(prob_jump,FunctionMap())
```

```
BenchmarkTools.Trial:
  memory estimate:  14.44 KiB
  allocs estimate:  125
```

```
--------------
minimum time:     65.600 μs (0.00% GC)
median time:      781.750 μs (0.00% GC)
mean time:        906.109 μs (5.32% GC)
maximum time:     18.463 ms (92.48% GC)
--------------
samples:          5478
evals/sample:     1
```

Appendix

Computer Information

```
Julia Version 1.4.0
Commit b8e9a9ecc6 (2020-03-21 16:36 UTC)
Platform Info:
  OS: Windows (x86_64-w64-mingw32)
  CPU: Intel(R) Core(TM) i7-8550U CPU @ 1.80GHz
  WORD_SIZE: 64
  LIBM: libopenlibm
  LLVM: libLLVM-8.0.1 (ORCJIT, skylake)
Environment:
  JULIA_NUM_THREADS = 4
```

Package Information

```
Status `~\.julia\environments\v1.4\Project.toml`
[80f14c24-f653-4e6a-9b94-39d6b0f70001] AbstractMCMC 1.0.1
[46ada45e-f475-11e8-01d0-f70cc89e6671] Agents 3.1.0
[b19378d9-d87a-599a-927f-45f220a2c452] ArrayFire 1.0.6
[c52e3926-4ff0-5f6e-af25-54175e0327b1] Atom 0.12.10
[6e4b80f9-dd63-53aa-95a3-0cdb28fa8baf] BenchmarkTools 0.5.0
[a134a8b2-14d6-55f6-9291-3336d3ab0209] BlackBoxOptim 0.5.0
[336ed68f-0bac-5ca0-87d4-7b16caf5d00b] CSV 0.6.2
[be33ccc6-a3ff-5ff2-a52e-74243cff1e17] CUDAnative 3.1.0
[3a865a2d-5b23-5a0f-bc46-62713ec82fae] CuArrays 2.2.0
[717857b8-e6f2-59f4-9121-6e50c889abd2] DSP 0.6.6
[2445eb08-9709-466a-b3fc-47e12bd697a2] DataDrivenDiffEq 0.3.1
[a93c6f00-e57d-5684-b7b6-d8193f3e46c0] DataFrames 0.21.0
[1313f7d8-7da2-5740-9ea0-a2ca25f37964] DataFramesMeta 0.5.1
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[eb300fae-53e8-50a0-950c-e21f52c2b7e0] DiffEqBiological 4.3.0
[459566f4-90b8-5000-8ac3-15dfb0a30def] DiffEqCallbacks 2.13.1
[aae7a2af-3d4f-5e19-a356-7da93b79d9d0] DiffEqFlux 1.10.2
[c894b116-72e5-5b58-be3c-e6d8d4ac2b12] DiffEqJump 6.7.5
[1130ab10-4a5a-5621-a13d-e4788d82bd4c] DiffEqParamEstim 1.14.1
[41bf760c-e81c-5289-8e54-58b1f1f8abe2] DiffEqSensitivity 6.14.1
[6d1b261a-3be8-11e9-3f2f-0b112a9a8436] DiffEqTutorials 0.1.0
```

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[0c46a032-eb83-5123-abaf-570d42b7fbaa] DifferentialEquations 6.14.0
[31c24e10-a181-5473-b8eb-7969acd0382f] Distributions 0.23.2
[634d3b9d-ee7a-5ddf-bec9-22491ea816e1] DrWatson 1.11.0
[587475ba-b771-5e3f-ad9e-33799f191a9c] Flux 0.10.4
[0c68f7d7-f131-5f86-a1c3-88cf8149b2d7] GPUArrays 3.3.0
[28b8d3ca-fb5f-59d9-8090-bfdbd6d07a71] GR 0.49.1
[523d8e89-b243-5607-941c-87d699ea6713] Gillespie 0.1.0
[7073ff75-c697-5162-941a-fcdaad2a7d2a] IJulia 1.21.2
[e5e0dc1b-0480-54bc-9374-aad01c23163d] Juno 0.8.1
[23fbe1c1-3f47-55db-b15f-69d7ec21a316] Latexify 0.13.2
[961ee093-0014-501f-94e3-6117800e7a78] ModelingToolkit 3.4.0
[d41bc354-129a-5804-8e4c-c37616107c6c] NLSolversBase 7.6.1
[76087f3c-5699-56af-9a33-bf431cd00edd] NLopt 0.6.0
[d9ec5142-1e00-5aa0-9d6a-321866360f50] NamedTupleTools 0.13.2
[73a701b4-84e1-5df0-88ff-1968ee2ee8dc] NamedTuples 5.0.0
[429524aa-4258-5aef-a3af-852621145aeb] Optim 0.20.1
[1dea7af3-3e70-54e6-95c3-0bf5283fa5ed] OrdinaryDiffEq 5.38.1
[91a5bcdd-55d7-5caf-9e0b-520d859cae80] Plots 1.2.6
[e6cf234a-135c-5ec9-84dd-332b85af5143] RandomNumbers 1.4.0
[c5292f4c-5179-55e1-98c5-05642aab7184] ResumableFunctions 0.5.1
[428bdadb-6287-5aa5-874b-9969638295fd] SimJulia 0.8.0
[05bca326-078c-5bf0-a5bf-ce7c7982d7fd] SimpleDiffEq 1.1.0
[8ce77f84-9b61-11e8-39ff-d17a774bf41c] Soss 0.12.0
[2913bbd2-ae8a-5f71-8c99-4fb6c76f3a91] StatsBase 0.32.2
[4c63d2b9-4356-54db-8cca-17b64c39e42c] StatsFuns 0.9.4
[f3b207a7-027a-5e70-b257-86293d7955fd] StatsPlots 0.14.6
[789caeaf-c7a9-5a7d-9973-96adeb23e2a0] StochasticDiffEq 6.22.0
[a759f4b9-e2f1-59dc-863e-4aeb61b1ea8f] TimerOutputs 0.5.5
[fce5fe82-541a-59a6-adf8-730c64b5f9a0] Turing 0.7.1
[44d3d7a6-8a23-5bf8-98c5-b353f8df5ec9] Weave 0.9.2
[37e2e46d-f89d-539d-b4ee-838fcccc9c8e] LinearAlgebra
[cf7118a7-6976-5b1a-9a39-7adc72f591a4] UUIDs
```