

Rough-set Based Color Channel Selection

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Abstract—Color channel selection is essential for accurate segmentation of sky and clouds in images obtained from ground-based sky cameras. Most prior works in cloud segmentation use threshold based methods on color channels selected in an ad-hoc manner. In this letter, we propose the use of rough sets for color channel selection in visible-light images. Our proposed approach assesses color channels with respect to their contribution for segmentation, and identifies the most effective ones.

Index Terms—Color channel, rough set, Whole Sky Imagers (WSIs), cloud analysis.

I. INTRODUCTION

GROUND-BASED whole sky imagers (WSIs) are getting popular among the remote sensing community. They provide instantaneous data of the cloud formations and are thus useful in a variety of applications [1]. They complement satellite images with localized data of higher temporal and spatial resolution. These devices capture images of the sky at regular intervals and archive them for processing. Most WSIs use traditional cameras in the visible-light spectrum [2], while a few models capture the near-infrared range as well [3], [4].

Typical post-processing algorithms include the computation of cloud coverage percentage [5], cloud-type recognition [6], or cloud base height estimation [7]. The fundamental task for these applications is the segmentation of clouds from the sky, with each pixel of the image classified as either *sky* or *cloud*. As the sky is predominantly blue because of Rayleigh scattering, most existing approaches use thresholding on an ad-hoc combination of red and blue color channels [8], [9]. Kreuter et al. [8] used a fixed threshold for the blue/red ratio. Calbó and Sabburg [10] use various statistical features (mean, standard deviation, entropy etc.) obtained from red and blue channels for successful detection and subsequent labeling of pixels. The difference of red and blue channels is exploited in [11], [12]. The saturation channel of the HSV color model is used in [13]. Recently, Li et al. [9] proposed the use of a normalized red/blue ratio for cloud detection.

We believe only a structured review of color channels allows us to systematically select the one(s) with the most *discriminative* cues. This helps to efficiently represent the images in a lower-dimensional subspace. Such systematic analysis of color channels for sky/cloud images is important for subsequent tasks, such as cloud type recognition or feature

matching in cloud base height estimation. An analysis of several color channels for sky/cloud segmentation is provided in our previous work [14], where we used principal component analysis and bimodality to determine favorable channels. However, these techniques had certain shortcomings with regards to ranking color channels according to their relevance for cloud segmentation, as their correlation with cloud segmentation accuracy is relatively weak. Several other techniques have been used for selecting the most discriminative features in a classification problem. Serrano et al. [15] used area under the Receiver Operator Characteristic (ROC) curves for feature selection in a synthetic dataset. ROC curves are now routinely used in the field of remote sensing [16]. Information-theoretic measures like Kullback-Leibler (KL) distance or divergence were exploited for band selection in hyperspectral imaging [17].

Our main contribution in this letter is to determine those color channels that are most discriminative in identifying cloud pixels in traditional visible-light images. We extend the benchmarking done in [14] using a rough-set based approach that can accurately assess the efficiency of different color channels for cloud segmentation.¹ Rough set theory is useful for representing uncertain data with a level of approximation, and selecting the most discriminative features from the feature space. Recently, it has been successfully applied to hyperspectral band selection [18]. To the best of our knowledge, our proposed approach is the first that uses rough set theory for color channel selection in visible-light images.

The remaining letter is organized as follows. In Section II, we describe the fundamental concepts behind rough set theory and present our proposed color channel selection algorithm. Experimental results are presented in Section III. Finally, Section IV concludes the letter.

II. ROUGH-SET BASED COLOR CHANNEL SELECTION

Classical rough sets, originally defined by Pawlak [19], are an approximation of conventional sets in set theory. In the scenario where it is difficult to define the boundaries of a conventional set, rough-set theory provides a set of mathematical tools to approximately define them. It facilitates an objective analysis in a data-driven system which is vague, uncertain and incomplete. Here we first explain the related terminologies of rough sets, and subsequently define the use of rough sets for visible-light images.

A. Rough Set Theory

In rough sets, information is expressed in the form of a *decision table*. We define a decision table \mathcal{L} such that each row

¹ The source code of all simulations in this paper is available online at <https://github.com/Soumyabrata/rough-sets>.

Manuscript received 01-Jul-2016; revised 03-Oct-2016; accepted 28-Oct-2016.

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represents an observation, and each column is an attribute from attribute set \mathcal{A} . This non-empty set of observations is usually referred to as the *universe* \mathcal{U} . Formally, for each entry in the decision table, we define the function f that maps attribute \mathcal{A} to value domain \mathcal{V} , $f: \mathcal{U} \times \mathcal{A} \rightarrow \mathcal{V}$.

Any reduct \mathcal{P} from the set of attributes \mathcal{A} satisfies the *equivalence relation* $\text{IND}(\mathcal{P})$. For any reduct $\mathcal{P} \in \mathcal{A}$, the \mathcal{P} -indiscernibility relation is defined as:

$$\text{IND}(\mathcal{P}) = \{(x_m, x_n) \in \mathcal{U}^2 | \forall a \in \mathcal{P}, f(x_m, a) = f(x_n, a)\} \quad (1)$$

where x_m and x_n are two observations from the universe \mathcal{U} , and a is an element from set \mathcal{P} . This indicates that x_m and x_n are indiscernible based on the attribute \mathcal{P} , as the value function f assigns both x_m and x_n to the same value. This partition of \mathcal{U} generated by $\text{IND}(\mathcal{P})$ is denoted as:

$$\mathcal{U}/\text{IND}(\mathcal{P}) = \{[x_m]_{\mathcal{P}} | x_m \in \mathcal{U}\}. \quad (2)$$

Let \mathcal{X} be a set of observations from the universe \mathcal{U} . Rough set theory asks the question: how can we express this conventional set \mathcal{X} , using only the information in attribute set \mathcal{P} ? In general, this cannot be precisely described, and therefore approximations are generated. They are defined by their corresponding \mathcal{P} -lower- and \mathcal{P} -upper-approximations as:

$$\underline{\mathcal{P}}(\mathcal{X}) = \cup\{[x_m]_{\mathcal{P}} | [x_m]_{\mathcal{P}} \subseteq \mathcal{X}\} \quad (3a)$$

$$\overline{\mathcal{P}}(\mathcal{X}) = \cup\{[x_m]_{\mathcal{P}} | [x_m]_{\mathcal{P}} \cap \mathcal{X} \neq \emptyset\} \quad (3b)$$

The observations in the lower approximation set $\underline{\mathcal{P}}(\mathcal{X})$ are the definite members of \mathcal{X} , also called the *positive region* $\text{POS}(\mathcal{X})$. On the other hand, $\overline{\mathcal{P}}(\mathcal{X})$ represents the upper approximation of the set. It denotes the possible members of \mathcal{X} , based on the knowledge in the decision table. We illustrate this in Fig.1.

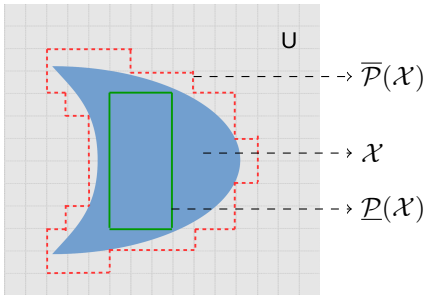


Fig. 1: Illustration of a typical rough set, which approximates a conventional set \mathcal{X} (depicted in blue). Each individual grid depicts a partition of the universe generated by an equivalence relation. The union of all such partitions indicated with solid green borders (definite members) represents the *lower approximation* $\underline{\mathcal{P}}(\mathcal{X})$; while the dotted red borders (possible members) represent the *upper approximation* $\overline{\mathcal{P}}(\mathcal{X})$.

Let us assume that the attribute set \mathcal{A} consists of both condition and decision attributes \mathcal{C} and \mathcal{D} respectively, such that $\mathcal{A} = \mathcal{C} \cup \mathcal{D}$. The *relevance* criterion is defined as the

dependence between \mathcal{C} and \mathcal{D} , and can be expressed as:

$$\gamma_{\mathcal{C}}(\mathcal{D}) = \frac{|\text{POS}_{\mathcal{C}}(\mathcal{D})|}{|\mathcal{U}|}, \quad (4)$$

where $|\cdot|$ denotes the cardinality of the set. This dependence value is an important measure to identify the most discriminate attribute from set \mathcal{A} . The value of $\gamma_{\mathcal{C}}(\mathcal{D})$ ranges between 0 and 1, where 0 indicates no dependence and 1 indicates \mathcal{D} fully depends on \mathcal{C} .

B. Color Channel Selection Using Rough Sets

We now describe the problem of color channel selection using rough-set terminology and propose our algorithm for this purpose. The main benefit of using rough set theory is that it provides a systematic method to approximate the segmentation ground truth, with the highest degree of approximation. Moreover, no prior information about the data is needed for the analysis.

We consider 16 color channels in our analysis, as shown in Table I and illustrated in Fig. 2. They include *RGB*, *HSV*, *YIQ*, *CIE L*a*b** color models, various red-blue combinations, and chroma $C = \max(R, G, B) - \min(R, G, B)$. Several of these color channels are commonly used for thresholding in sky/cloud images [9], [13], [14]. We utilize rough set theory to identify the most *informative* color channel(s) from these 16, also known as *reducts* in rough set terminology.

c_1	R	c_4	H	c_7	Y	c_{10}	L^*	c_{13}	R/B	c_{16}	C
c_2	G	c_5	S	c_8	I	c_{11}	a^*	c_{14}	$R - B$		
c_3	B	c_6	V	c_9	Q	c_{12}	b^*	c_{15}	$\frac{B-R}{B+R}$		

TABLE I: Color spaces and components used for analysis.

Suppose that \mathcal{U}_i is a non-empty finite universe of pixel observations for a single sky/cloud image \mathbf{I}_i from the image dataset $\mathcal{T} = \{\mathbf{I}_1, \mathbf{I}_2, \dots, \mathbf{I}_N\}$. The set of pixels corresponding to the ground-truth observation for an image \mathbf{I}_i is \mathcal{G}_i . We define \mathcal{A}_i as the set of its corresponding vectorized color channels $\{c_1, c_2, \dots, c_{16}\}$ for a particular image \mathbf{I}_i , along with the vectorized decision attribute \mathbf{g}_i . These 16 color channels are called condition attribute sets, and 1 ground truth vectorized image is the decision attribute set from the family of attributes \mathcal{A}_i . We define a decision table \mathcal{L}_i such that each row represents a pixel, and each column represents an attribute.

Thus, for a single image \mathbf{I}_i , the function f assigns a value v_i^{kj} in the value domain to each variable-attribute pair (q_k, a_j) , where q_k is the k -th pixel value of the image and a_j is the j -th attribute from the set \mathcal{A}_i . For the sake of brevity, we will drop the index i in our subsequent discussions.

Our objective is to characterize the ground-truth image \mathbf{g} from the knowledge of the reduct \mathcal{P} . We define reduct \mathcal{P} as the subset of condition attribute sets \mathcal{C} .

We are interested in identifying the most discriminative color channel that is strongly dependent on the sky/cloud decision attribute \mathbf{g} . This dependence of a particular color channel is measured by the corresponding *relevance* criterion of the color channels. The color channels having a high relevance value are better candidates for sky/cloud segmentation. In analogy to Eq. (4), we define the relevance criterion γ_j

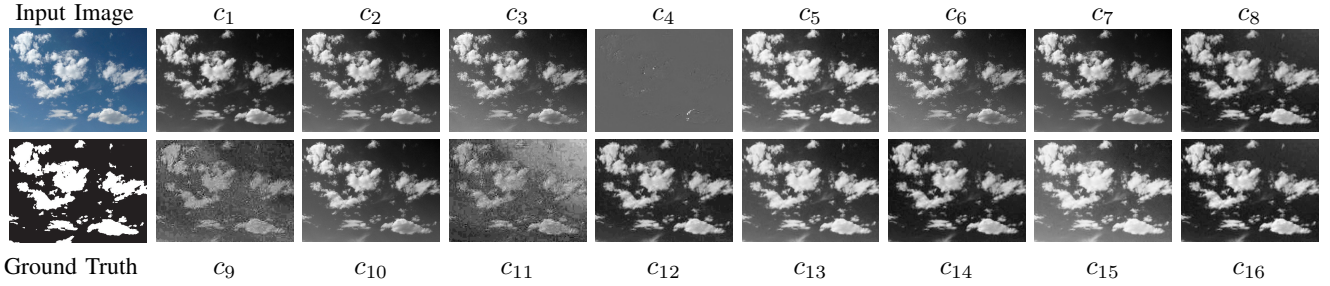


Fig. 2: Sample image and segmentation mask from the HYTA database [9], together with the 16 color channels from Table I (the color maps for c_4 , c_5 , c_9 , c_{15} , and c_{16} are inverted such that cloud pixels always have a lighter shade than sky pixels).

for each color channel c_j , which indicates the dependence between ground truth g and color channels:

$$\gamma_j = \frac{|\text{POS}(g)|}{|\mathcal{U}|}. \quad (5)$$

For image $\mathbf{I}_i \in \mathbb{R}^{r \times s}$, we generate the decision table $\mathcal{L}_i \in \mathbb{R}^{rs \times 17}$. Each row of decision table \mathcal{L}_i corresponds to a pixel. The first 16 columns represent the color channels, and the 17th column corresponds to the ground-truth label. We generate 16 distinct partitions from this decision table using the knowledge in the ground truth labels. Subsequently, the corresponding lower approximations of 16 color channels are generated using Eq. (3). In fact, this lower-approximation set is the union of all partitions (generated by the equivalence relation) that are possible members of the *cloud* label. Next, we compute the relevance value γ_j of all color channels for the image \mathbf{I}_i using Eq. (5).

We perform this for all images of dataset \mathcal{T} . Finally, we compute the average relevance $\bar{\gamma}_j$ of each color channel across all the images. The color channel with the maximum average relevance value $\bar{\gamma}_j$ is the best amongst all color channels under consideration.

III. EXPERIMENTAL RESULTS

A. Dataset

In order to check the efficacy of our proposed color channel selection algorithm, we conduct experiments on a publicly available sky/cloud image database called HYTA [9]. It consists of 32 sky/cloud images with varying sky conditions, along with their corresponding segmentation masks (cf. Fig. 2). The images were collected by sky cameras located in Beijing and Conghua, China.

B. Cloud Classification Performance

In this section, we evaluate the discriminative power of each of the 16 color channels with respect to the cloud classification task. We target a single color channel for sky/cloud image segmentation, because in our previous work [14] we observed that there is no significant improvement when using a combination of multiple color channels.

We follow a supervised learning approach and train a Support Vector Machine (SVM) to validate our findings. We use each of the 16 color channels separately as candidate feature vectors and train 16 different SVMs. The trained SVMs are

then used for sky/cloud pixel classification in order to check the efficacy of respective color channels in the classification task. We randomly segregate the HYTA dataset into 15 training images and 17 test images.

For an objective evaluation of our algorithm, we report the F-score and Accuracy. Suppose TP , FP , TN and FN denotes the true positive, false positive, true negative and false negative respectively in this binary classification task. Accuracy is defined as the ratio of pixels that are correctly classified, $(TP + TN)/(TP + TN + FP + FN)$. The F-score, a popular metric in a binary classification problems, is defined as the harmonic mean of Precision = $TP/(TP + FP)$ and Recall = $TP/(TP + FN)$.

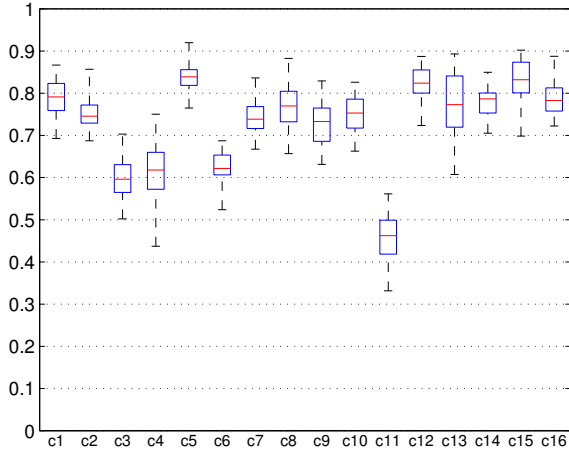
Figure 3 shows the binary classification results for each of the 16 color channels, computed over 50 different random selections of training and test sets.

We observe from Fig. 3 that color channels c_{15} ($\frac{B-R}{B+R}$) and c_5 (S) have relatively higher accuracy values and smaller spread as compared to the other color channels. Conversely, color channels c_{11} (a^*) and c_4 (H) score poorly in terms of accuracy values. Therefore, each color channel has different discriminative power to classify sky and cloud pixels from ground-based images. This ranking of color channels in terms of their classification performances serves as the ground-truth to verify the efficacy of different color channel selection algorithms.

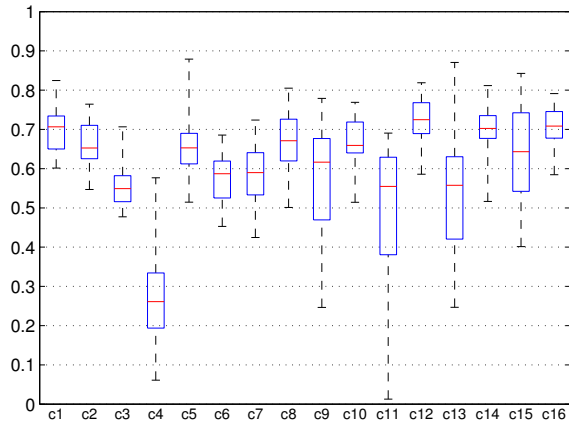
C. Benchmarking

In our proposed rough-set based method, we measure the dependency of each color channel and report its corresponding average relevance value $\bar{\gamma}$ across all images of HYTA. Table II summarizes the results. We observe that certain color channels viz. c_{15} ($\frac{B-R}{B+R}$), c_{13} ($\frac{R}{B}$), c_5 (S) have higher relevance scores as compared to others, indicating that these color channels are favorable candidates for sky/cloud image segmentation. On the other hand, color channels c_{11} (a^*) and c_4 (H) have low relevance scores. This indicates that these color channels contribute less to the decision attribute for (non-) cloud pixels. Therefore, these color channels are not conducive for sky/cloud segmentation. We verify this by obtaining the correlation of the individual relevance values of color channels with the sky/cloud segmentation results.

We benchmark our proposed rough-set based algorithm with prior works on color channel selection [14]. As we are



(a) Accuracy



(b) F-score

Fig. 3: (a) Accuracy and (b) F-scores for 16 color channels for all images in the HYTA database. For each box, the central mark indicates the median, the edges correspond to 25th and 75th percentiles, and the whiskers represent the extreme data points.

c_1	0.70	c_4	0.46	c_7	0.72	c_{10}	0.66	c_{13}	0.84	c_{16}	0.66
c_2	0.66	c_5	0.82	c_8	0.78	c_{11}	0.33	c_{14}	0.69		
c_3	0.58	c_6	0.58	c_9	0.69	c_{12}	0.61	c_{15}	0.84		

TABLE II: Average relevance value across all images of the HYTA database. The most relevant color channels are highlighted in bold.

interested in a binary segmentation, color channels exhibiting higher bimodal behavior are favorable. Pearson's Bimodality Index (PBI) [20] for all 16 color channels is computed. A PBI value close to unity indicates highest bimodality, and values higher than 1 indicate departure from bimodal behavior. Principal Component Analysis (PCA) is used to determine the color channels that capture the highest variance. We compute the absolute value of the loading factors, defined as the re-

projections of the data point on the principal component axes, for the 16 color channels, and consider the component on the first principal component axis. Those color channels with high loading factors on the first eigenvector are considered favorable color channels. Furthermore, we also compute the area under ROC curve individually for the 16 color channels. A higher area under ROC curve for a particular color channel indicates that its classifier performance is better than the random classifier. Lastly, the KL-divergence of the color channels from the binary ground truth images is computed. This distance can be interpreted as the amount of *dissimilarity* of the color channel from its ground truth image. In other words, a higher distance indicates an unfavorable color channel for cloud segmentation.

We check the correlation of the average accuracy scores for all the color channels with the normalized scores obtained from different approaches. Table III shows the correlation results of these approaches with the average classification accuracy of the color channels. Our proposed method using rough sets achieves the highest correlation.

Methods	Correlation (r)
Proposed approach	0.84 (\uparrow)
Bimodality	-0.58 (\downarrow)
Loading factors	0.57 (\uparrow)
ROC curve	0.78 (\uparrow)
Kullback-Leibler divergence	0.12 (\downarrow)

TABLE III: Correlation of cloud classification accuracy with ranking scores obtained using different methods. The \uparrow (or \downarrow) indicates if higher (or lower) magnitude signifies better performance.

Figure 4 shows the respective scatter plots. The correlation coefficient is highest ($r=0.84$) for our proposed algorithm. From Fig. 4(a), we can clearly see that color channels c_5 (S) and c_{15} ($\frac{B-R}{B+R}$) are better candidates for sky/cloud classification. Similarly, color channels c_{11} , c_4 , having lower relevance scores are poor candidates. Figure 4(b) also identifies that c_5 and c_{15} are conducive color channels as their PBI values are closer to 1. Similar results can be drawn from Fig. 4(c), where favorable color channels c_5 and c_{15} have comparatively lower KL-divergence values, and color channel c_4 has the highest KL-distance. However, the order of ranking for the other color channels is poor. We obtain a poor correlation value ($r = 0.12$).

From these results, we observe that certain color channels viz. c_5 and c_{15} always perform best in all three benchmarking methods. Conversely, color channels c_4 and c_{11} rank lower than others. The visual interpretation of c_4 and c_{11} color channels also does not demonstrate a clear contrast between sky and cloud pixels (cf. Fig. 2). We note that the relevance criterion has the highest correlation with the cloud segmentation accuracy among all methods. Therefore we conclude that our proposed rough-set based color channel selection algorithm is useful to rank color channels and identify suitable ones for image segmentation.

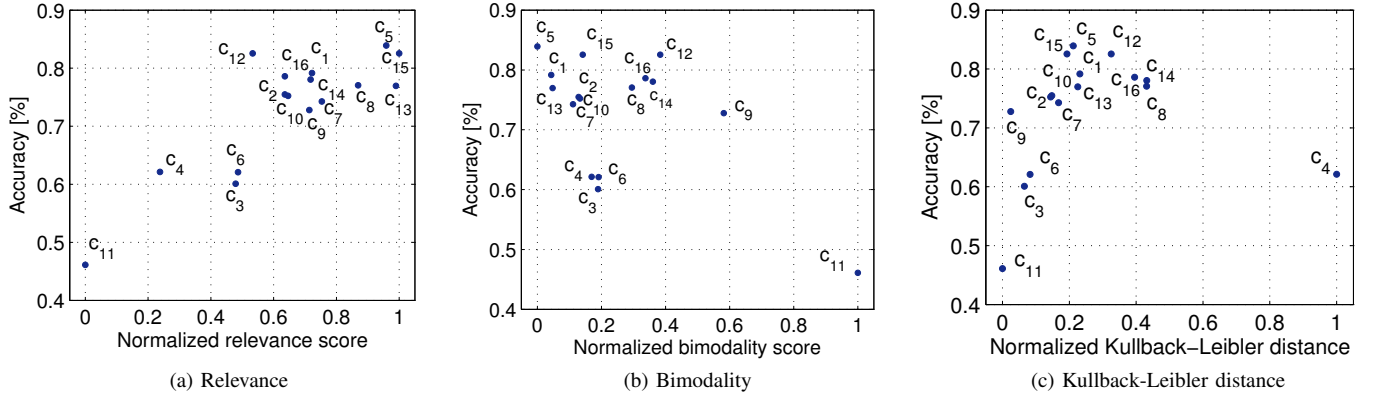


Fig. 4: Scatter plot between average accuracy scores and (a) relevance ($r=0.84$), (b) bimodality ($r=-0.57$), and (c) Kullback-Leibler distance ($r=0.12$) for all 16 color channels. Please refer to Table III for more details. Our proposed approach using relevance scores can rank the color channels w.r.t. their segmentation performance with the highest correlation.

IV. CONCLUSIONS

Sensing the earth's atmosphere using ground-based visible-light images is popular because of its low cost and the high temporal and spatial resolution of the captured images, as compared to traditional satellite images. In this letter, we have proposed a color channel selection algorithm based on rough-set theory. Extensive experimental results show the efficacy of our approach in identifying favorable color channels for image segmentation. Our proposed approach performs best as compared to other feature selection algorithms. Future work involves the extension of such rough-set based approaches to other applications.

ACKNOWLEDGMENT

This work is supported by a grant from Singapore's Defence Science & Technology Agency (DSTA).

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