A2

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1 Sheet 02, Exercise 2

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[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib
matplotlib.rcParams["figure.figsize"] = (15,12)
%matplotlib inline
```

$$
\begin{aligned}
f_1(x) &= \sin(x) && (1)\\
f_1'(x) &= \cos(x) && (2)\\
f_2(x) &= 2\left\lfloor \frac{x}{\pi} \right\rfloor - \cos(x \mod \pi) + 1 && (3)\\
f_2'(x) &= \sin(x \mod \pi) \quad \text{for} \sin(x) \neq 0 && (4)
\end{aligned}
$$

[ ]:
```
def f1(x):
    return np.sin(x)


def f2(x):
    return 2 * np.floor(x / np.pi) - np.cos(x % np.pi) + 1


def f1Prime(x):
    return np.cos(x)


def f2Prime(x):
    return np.sin(x % np.pi)
```

[ ]:
```
def zweipunkt(f, x, h):
    return (f(x + h) - f(x - h)) / (2 * h)


def vierpunkt(f, x, h):
```

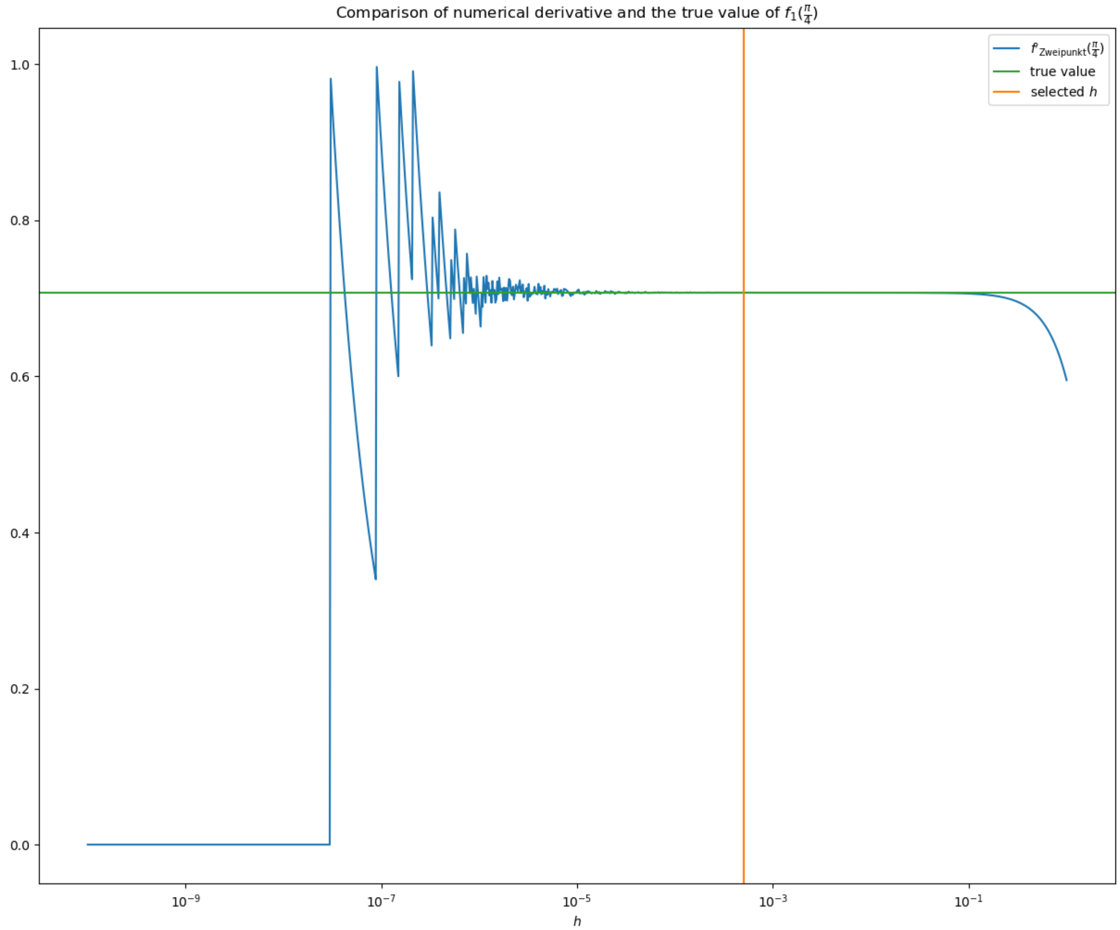
```
    return (f(x - 2 * h) + 8 * f(x + h) - f(x + 2 * h) - 8 * f(x - h)) / (12 *␣
↪h)
```

1.1 a)

[ ]:
```
x = np.float32(np.pi / 4)  # The x value at wich we look for a good h
h_arr = np.logspace(-10, 0, 1000, dtype=np.float32)
h1 = 5e-4  # result of plot below
```

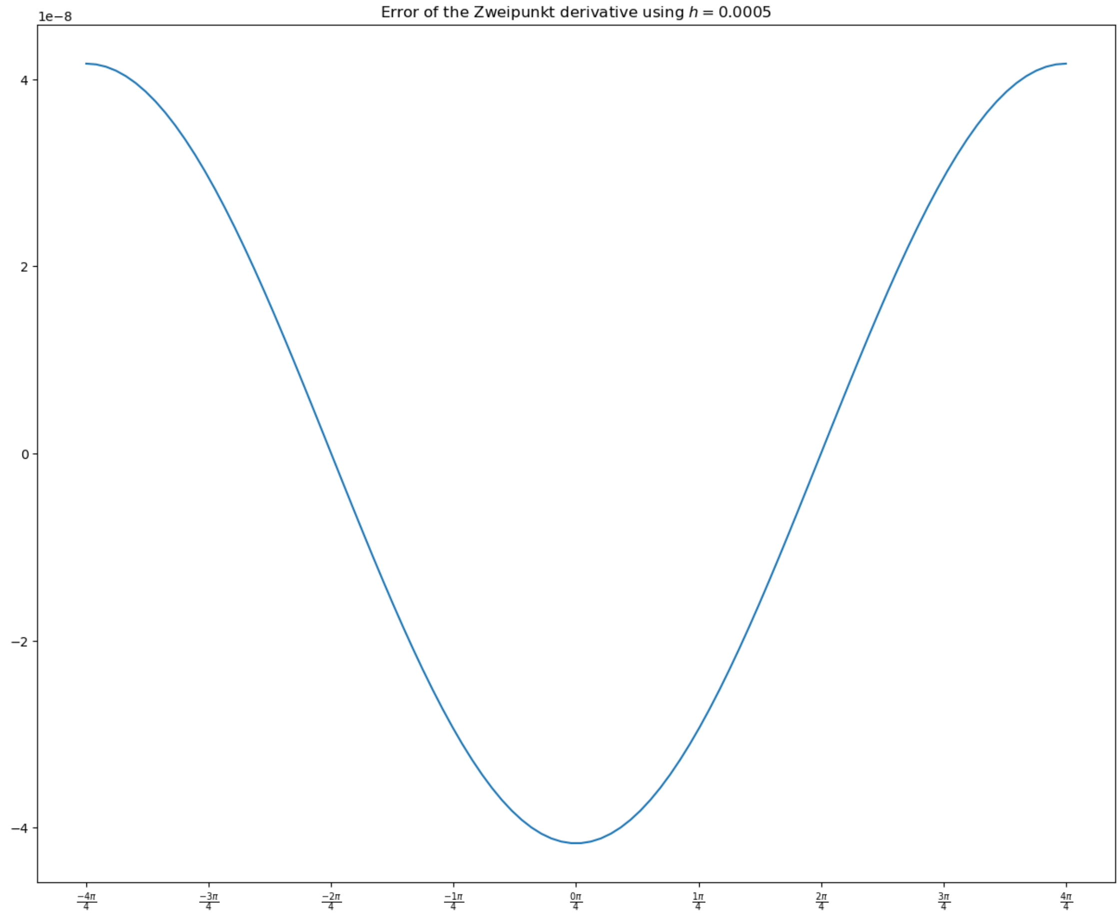
[ ]:
```
plt.plot(
    h_arr, zweipunkt(f1, x, h_arr),␣
↪label=r"$f'_\mathrm{Zweipunkt}(\frac{\pi}{4})$"
)
plt.xscale("log")
plt.axhline(y=f1Prime(x), color="tab:green", label="true value")
plt.axvline(x=h1, color="tab:orange", label="selected $h$")
plt.legend()
plt.title(
    r"Comparison of numerical derivative and the true value of␣
↪$f_1(\frac{\pi}{4})$"
)
plt.xlabel("$h$")
```

[ ]:
```
Text(0.5, 0, '$h$')
```



[ ]:
```
x_arr = np.linspace(-np.pi, np.pi, 100)
plt.plot(x_arr, (zweipunkt(f1, x_arr, h1) - f1Prime(x_arr)))
plt.xticks(
    np.linspace(-np.pi, np.pi, 9),
    [rf"$\frac{{{x}\pi}}{{4}}$" for x in np.linspace(-4, 4, 9, dtype=int)],
)  # This should be a build in thing
plt.title(f"Error of the Zweipunkt derivative using $h = {h1}$")
```

[ ]:
```
Text(0.5, 1.0, 'Error of the Zweipunkt derivative using $h = 0.0005$')
```



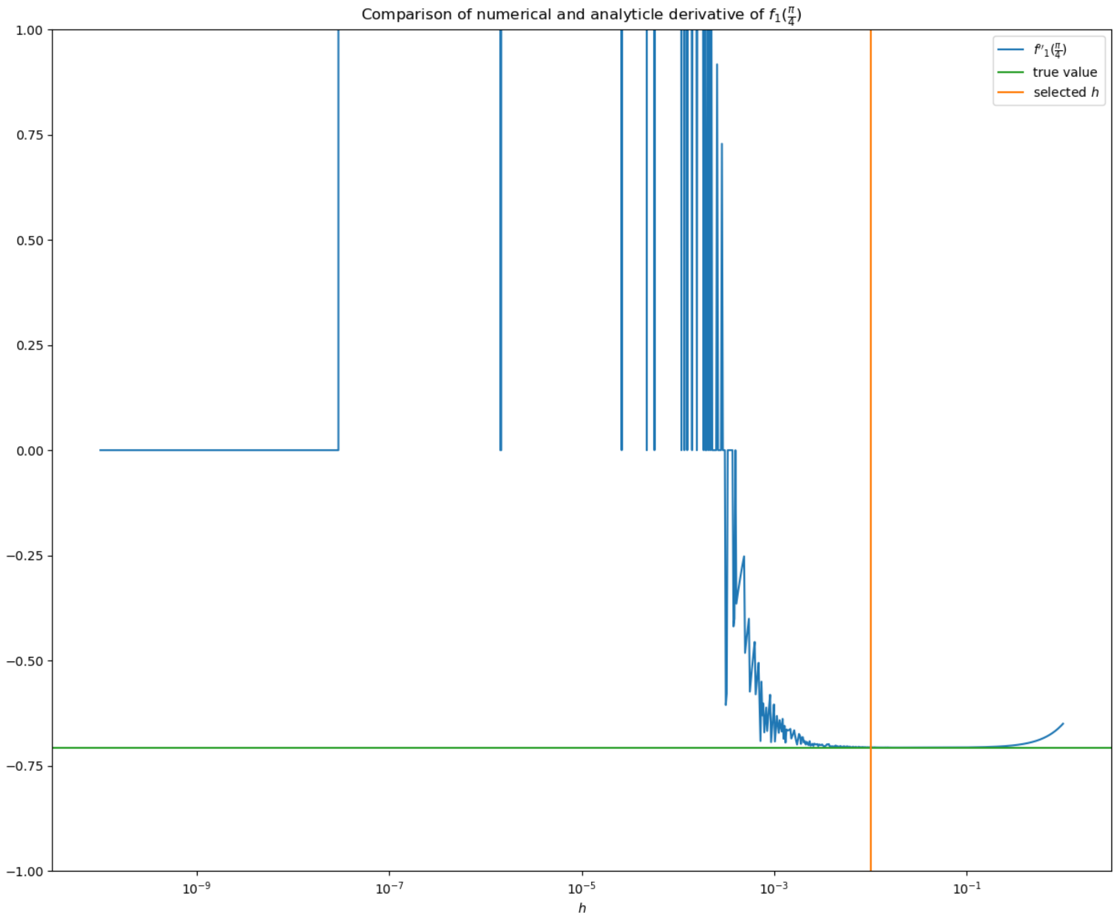
1.2 b)

[ ]:
```
def twoPrime(f, x, h):
    return (f(x + h) - 2 * f(x) + f(x - h)) / h**2


def f1TwoPrime(x):
    return -np.sin(x)
```

[ ]:
```
h2 = 1e-2

plt.plot(h_arr, twoPrime(f1, x, h_arr), label=r"$f''_1(\frac{\pi}{4})$")
plt.xscale("log")
plt.ylim(-1, 1)
plt.axhline(y=f1TwoPrime(x), color="tab:green", label="true value")
plt.axvline(x=h2, color="tab:orange", label="selected $h$")
plt.legend()
```

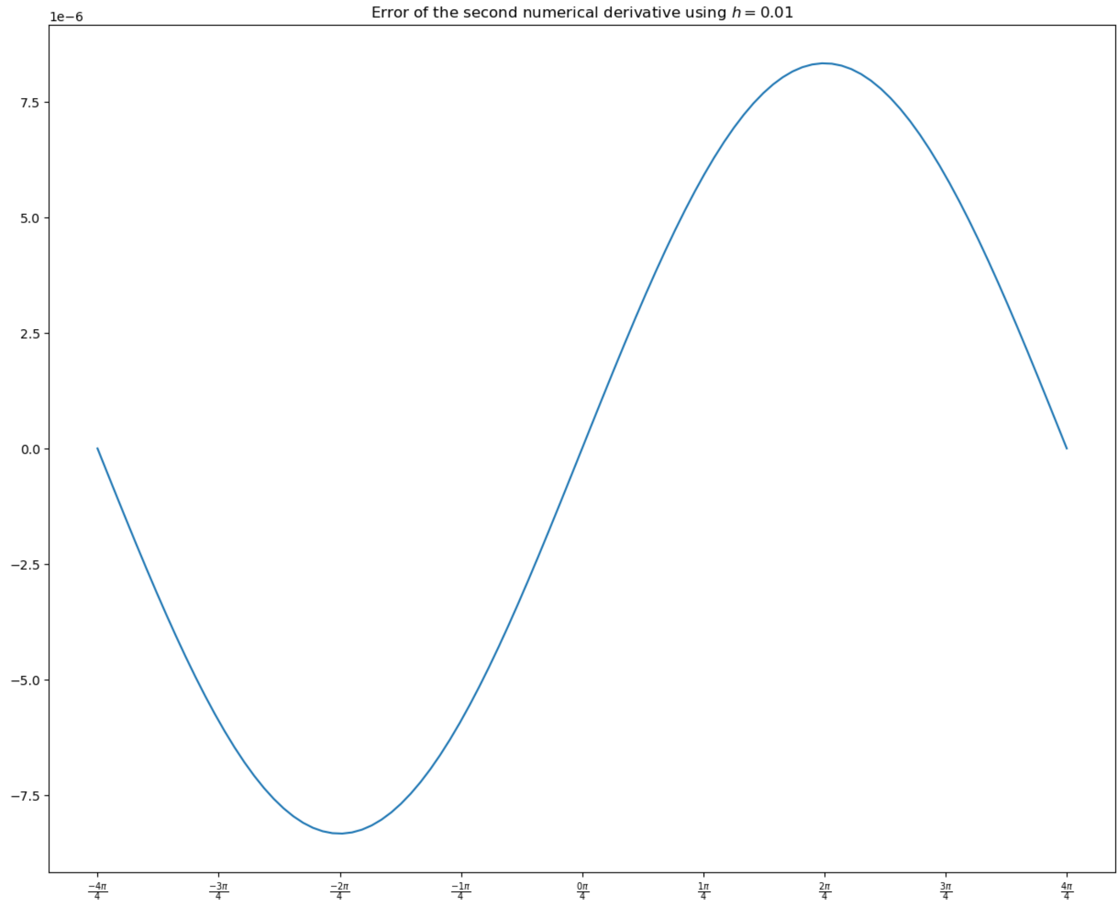
```
plt.title(r"Comparison of numerical and analyticle derivative of␣
↪$f_1(\frac{\pi}{4})$")
plt.xlabel("$h$")
```

[ ]: Text(0.5, 0, 'h')



[ ]:
```
plt.plot(x_arr, (twoPrime(f1, x_arr, h2) - f1TwoPrime(x_arr)))
plt.xticks(
    np.linspace(-np.pi, np.pi, 9),
    [rf"$\frac{{{x}\pi}}{{4}}$" for x in np.linspace(-4, 4, 9, dtype=int)],
)  # This should be a build in thing
plt.title(f"Error of the second numerical derivative using $h = {h2}$")
```

[ ]: Text(0.5, 1.0, 'Error of the second numerical derivative using $h = 0.01$')

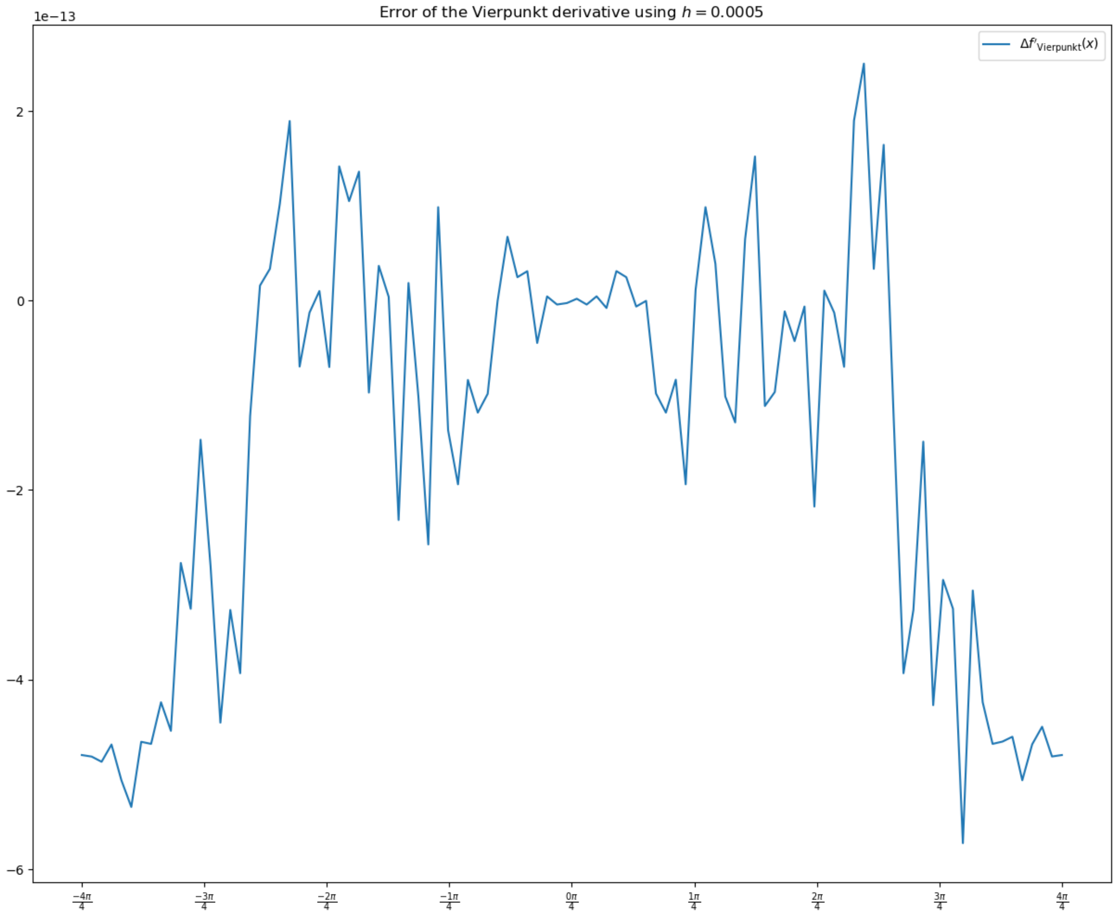


The errors of the second derivative are 2 orders of magnitude bigger. This may be, because we must choose a bigger h to be in numerical stable.

1.3 c)

[ ]:
```
plt.plot(
    x_arr,
    (vierpunkt(f1, x_arr, h1) - f1Prime(x_arr)),
    label="$\Delta f'_\mathrm{Vierpunkt}(x)$",
)
plt.xticks(
    np.linspace(-np.pi, np.pi, 9),
    [rf"$\frac{{{x}\pi}}{{4}}$" for x in np.linspace(-4, 4, 9, dtype=int)],
)  # This should be a build in thing
plt.title(f"Error of the Vierpunkt derivative using $h = {h1}$")
plt.legend()
```

[ ]:
```
<matplotlib.legend.Legend at 0x7f6317dee190>
```



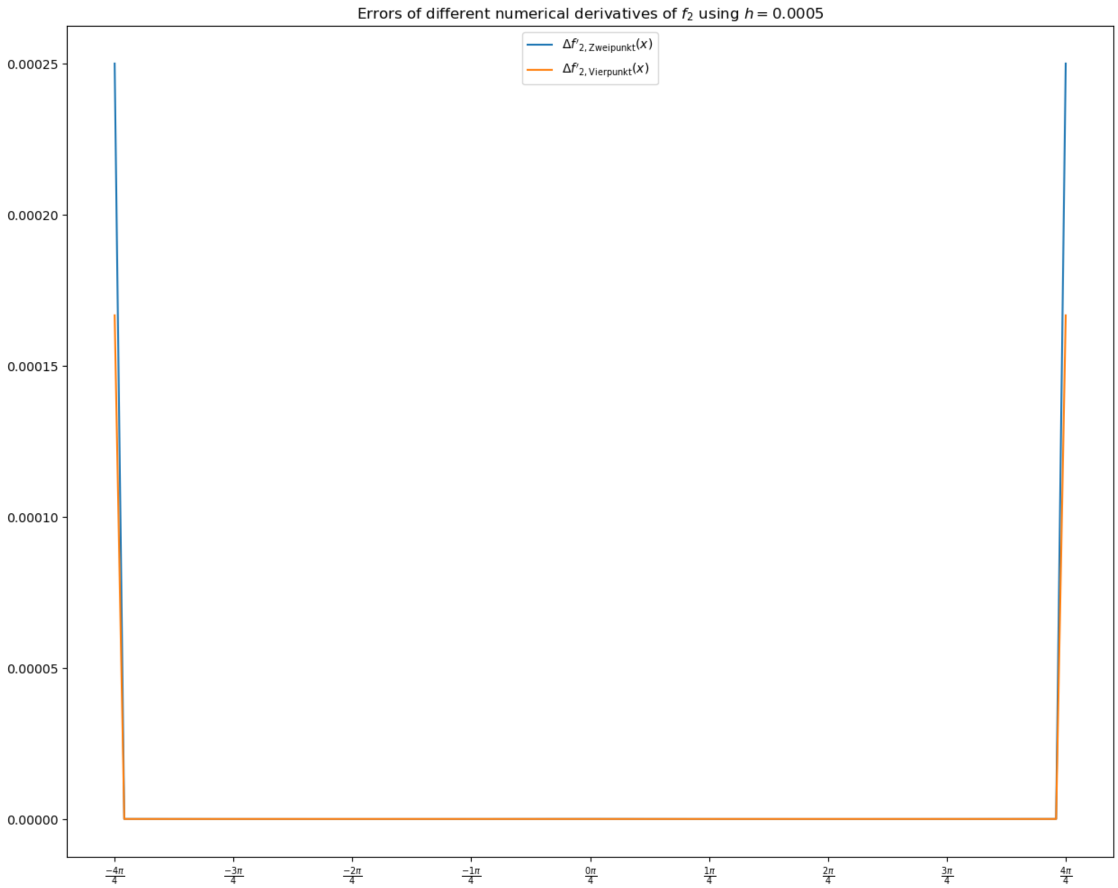
The errors of the Vierpunktregel are about five orders of magnitude smaller.

1.4 d)

[ ]:
```
plt.plot(
    x_arr,
    zweipunkt(f2, x_arr, h1) - f2Prime(x_arr),
    label="$\Delta f'_{2,\mathrm{Zweipunkt}}(x)$",
)
plt.plot(
    x_arr,
    vierpunkt(f2, x_arr, h1) - f2Prime(x_arr),
    label="$\Delta f'_{2,\mathrm{Vierpunkt}}(x)$",
)
plt.xticks(
    np.linspace(-np.pi, np.pi, 9),
    [rf"$\frac{{{x}\pi}}{{4}}$" for x in np.linspace(-4, 4, 9, dtype=int)],
)  # This should be a build in thing
```

```python
plt.legend()
plt.title(f"Errors of different numerical derivatives of $f_2$ using $h={h1}$")
```

[ ]: Text(0.5, 1.0, 'Errors of different numerical derivatives of f_2 using $h=0.0005$')



For f_2 the errors of both methods are very similar.