ECSE 543A NUMERICAL METHODS IN ELECTRICAL ENGINEERING

Assignment 3

Set: 09-Nov-2022 Due: 05-Dec-2022

NOTE: QUESTION 1 IS WORTH 20 MARKS

1. You are given a list of measured BH points for M19 steel (Table 1), with which to construct a continuous graph of B versus H.

(a) Interpolate the first 6 points using full-domain Lagrange polynomials. Is the result plausible, i.e. do you think it lies close to the true B versus H graph over this range?

(b) Now use the same type of interpolation for the 6 points at B = 0, 1.3, 1.4, 1.7, 1.8, 1.9. Is this result plausible?

(c) An alternative to full-domain Lagrange polynomials is to interpolate using cubic Hermite polynomials in each of the 5 subdomains between the 6 points given in (b). With this approach, there remain 6 degrees of freedom - the slopes at the 6 points. Suggest ways of fixing the 6 slopes to get a good interpolation of the points. Test your suggestion and comment on the results.

(d) The magnetic circuit of Figure 1 has a core made of Ml9 steel, with a cross-sectional area of 1 cm^2. L_c = 30 cm and L_a = 0.5 cm. The coil has N = 800 turns and carries a current I = 10 A. Derive a (nonlinear) equation for the flux ψ in the core, of the form $f(\psi) = 0$.

(e) Solve the nonlinear equation using Newton-Raphson. Use a piecewise-linear interpolation of the data in Table 1. Start with zero flux and finish when $| f(\psi) / f(0) | < 10^{-6}$. Record the final flux, and the number of steps taken.

(f) Try solving the same problem with successive substitution. If the method does not converge, suggest and test a modification of the method that *does* converge.

B (T)	H (A/m)
0.0	0.0
0.2	14.7
0.4	36.5
0.6	71.7
0.8	121.4
1.0	197.4
1.1	256.2
1.2	348.7
1.3	540.6
1.4	1062.8
1.5	2318.0
1.6	4781.9
1.7	8687.4
1.8	13924.3
1.9	22650.2

Table 1: BH Data for M19 Steel

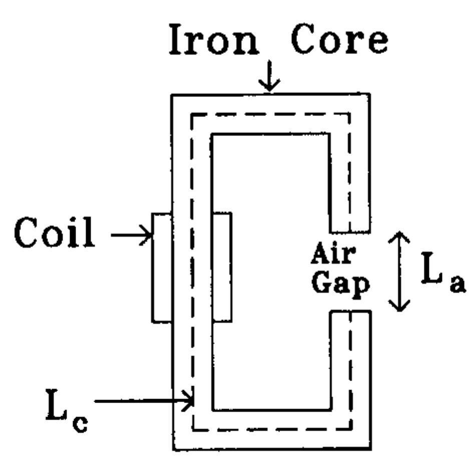

Figure 1

<u>NOTE</u>: ANSWER ONLY <u>ONE</u> OF THE TWO FOLLOWING QUESTIONS (EACH IS WORTH 10 MARKS)

2. For the circuit shown in Figure 2 below, the DC voltage E is 200 mV, the resistance R is 512 Ω, the reverse saturation current for diode A is I_{sA} = 0.8 μA, the reverse saturation current for diode B is I_{sB} = 1.1 μA, and assume kT/q = 25 mV.

   (a) Derive nonlinear equations for a vector of nodal voltages, $\mathbf{v}_n$, in the form $\mathbf{f}(\mathbf{v}_n) = 0$. Give $\mathbf{f}$ explicitly in terms of the variables I_{sA}, I_{sB}, E, R and $\mathbf{v}_n$.

   (b) Solve the equation $\mathbf{f} = 0$ by the Newton-Raphson method. At each step, record $\mathbf{f}$ and the voltage across each diode. Is the convergence quadratic? [Hint: define a suitable error measure ε_k].

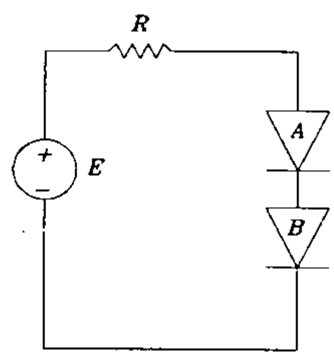

Figure 2

3. Write a program that accepts as input the values for the parameters x_0, x_N, and N and integrates a function $f(x)$ on the interval $x = x_0$ to $x = x_N$ by dividing the interval into N equal segments and using one-point Gauss-Legendre integration for each segment.

   (a) Use your program to integrate the function $f(x) = sin(x)$ on the interval $x_0 = 0$ to $x_N = 1$ for N = 1, 2,…, 20. Plot $log_{10}(E)$ versus $log_{10}(N)$ for N=1,2,…,20, where E is the absolute error in the computed integral. Comment on the result.

   (b) Repeat part (a) for the function $f(x) = ln(x)$, only this time for N = 10, 20,…, 200. Comment on the result.

   (c) Repeat part (b) for the function $f(x) = ln(0.2|\sin(x)|)$. Comment on the result.

   (d) An alternative to dividing the interval into equal segments is to use smaller segments in more difficult parts of the interval. Experiment with a scheme of this kind, and see how accurately you can integrate $f(x)$ in part (b) and (c) using only 10 segments. Comment on the results.