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THE RELATIVE IMPORTANCE OF MODEL PARAMETERS IN PREDICTIVE TRANSIENT MODELS

Filip Sund[1,2]

[1]Norwegian University of Science and Technology
Department of Energy and Process Engineering
7491 Trondheim, Norway

[2]Uni Research Polytec
5527 Haugesund, Norway
e-mail: filip.sund@polytec.no

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Abstract. *A sensitivity study on a numerical transient flow model for compressible gas was performed to determine the most important parameters when simulating long offshore gas pipelines. A simplified pipeline was simulated with synthetic transient boundary conditions, while systematically modifying different model parameters and correlations. It was found that, for the mass flow and pressure, the most important parameters by a large margin, are the friction factor and the compressibility factor. For the temperature, the parameter with the highest impact was found to be the derivative of the compressibility factor with respect to temperature (at constant density), closely followed by the isochoric gas heat capacity and the friction factor.*

1 INTRODUCTION

Natural gas exported from Norway to Europe accounts for around 25 percent of the yearly gas consumption in the European Union. The gas is transported from Norway through pipelines that are up to 1166 km long. To ensure that the pipelines stay within their operating limits, to monitor the pipelines for leaks, and to track changes in gas quality, it is important to know the state of the gas in the pipelines. But measurements of the state of the gas are usually only available at the inlet and outlet, which means that numerical models are necessary to know the state of the gas between the endpoints.

Simulating compressible gas flow is a highly complex issue, so to reduce the problem to a tractable one, several empirical correlations and simplifications like the Colebrook-White equation [1] and the Dittus-Boelter equation [2, 3] are typically used to model different aspects of the system. When doing this, inaccuracies are introduced into the simulations, the total effect of which can be hard to calculate *a priori*, and which will depend on the state and system being simulated.

The objective of this study is to investigate which parameters and correlations in the gas models that have greatest impact on the modelled results, especially during transient conditions, to know where to apply effort when trying to improve the models. A similar study limited to steady state models was done by Langelandsvik [4], and some work using transient models was by Helgaker [5]. The present work deals with transient one-dimensional non-isothermal models for compressible natural gas mixtures, and a simplified pipeline is modelled using synthetic but representative flow transients as boundary conditions.

This article is structured as follows: The theoretical foundation and underlying equations are presented in section 2, followed by a presentation of the studied pipeline system in section 4. Results are presented and discussed in section 5 while concluding remarks are drawn in section 6.

2 THEORY

The description of the theoretical foundation closely follows the description in [6].

2.1 Conservation laws

The governing equations for compressible, non-isothermal, transient pipeline gas flow are derived by averaging over the cross-section of Reynolds time-averaged conservation laws for viscous flow, resulting in:

the **continuity equation**

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = 0, \tag{1}$$

the **momentum equation** [7]

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) + \frac{\partial p}{\partial x} = -\frac{f \rho \left| u \right| u}{2D} - \rho g \sin \theta, \tag{2}$$

and the **energy equation** [8]

$$\rho \left(\frac{\partial e}{\partial t} + u \frac{\partial e}{\partial x} \right) + p \frac{\partial u}{\partial x} = \frac{f \rho u^3}{2D} + \frac{\Omega}{A_h}, \tag{3}$$

where ρ is gas density, e is internal energy, f is the friction factor, Ω is heat transfer through the pipe wall, and A_h is the area through which the heat is transferred.

The two terms containing the friction factor f in eqs. (2) and (3) model viscous dissipation – the transfer of mechanical energy to thermal energy via viscous stresses, and should account for dissipation at all length scales. It has been shown that a correction factor of up to 10 % should be applied to the dissipation term in eq. (3), depending on the Reynolds number and pipe roughness [9]. Additionally, the friction factor is calculated using empirical equations and the *sand grain equivalent roughness*, which is not directly correlated to the physical roughness of the pipe wall [10], so there are some uncertainties regarding the accuracy of these terms and the friction factor itself.

Using a real gas equation of state

$$\frac{p}{\rho} = ZRT, \tag{4}$$

where Z is the compressibility factor, and introducing the mass flow rate $\dot{m} = \rho u A$, the governing equations are developed into partial differential equations for mass flow $\dot{m}$, pressure p, and temperature T

$$\frac{\partial p}{\partial t} = \left(\frac{1}{p} - \frac{1}{Z}\left.\frac{\partial Z}{\partial p}\right|_T\right)^{-1}\left[\left(\frac{1}{T} + \frac{1}{Z}\left.\frac{\partial Z}{\partial T}\right|_p\right)\frac{\partial T}{\partial t} - \frac{ZRT}{pA}\frac{\partial \dot{m}}{\partial x}\right] \tag{5}$$

$$\begin{aligned}\frac{\partial \dot{m}}{\partial t} = {} & \frac{\dot{m}ZRT}{pA}\left[-2\frac{\partial \dot{m}}{\partial x} + \dot{m}\left(\frac{1}{p} - \frac{1}{Z}\left.\frac{\partial Z}{\partial p}\right|_T\right)\frac{\partial p}{\partial x} - \dot{m}\left(\frac{1}{T} + \frac{1}{Z}\left.\frac{\partial Z}{\partial T}\right|_p\right)\frac{\partial T}{\partial x}\right] \\ & - A\frac{\partial p}{\partial x} - \frac{fZRT\dot{m}\left|\dot{m}\right|}{2DAp} - \frac{pA}{ZRT}g\sin\theta\end{aligned} \tag{6}$$

$$\begin{aligned}\frac{\partial T}{\partial t} = {} & -\frac{\dot{m}ZRT}{pA}\frac{\partial T}{\partial x} - \frac{\dot{m}\left(ZRT\right)^2}{pAc_v}T\left(\frac{1}{T} + \frac{1}{Z}\left.\frac{\partial Z}{\partial T}\right|_\rho\right) \\ & \cdot\left[\frac{1}{\dot{m}}\frac{\partial \dot{m}}{\partial x} + \left(\frac{1}{T} + \frac{1}{Z}\left.\frac{\partial Z}{\partial T}\right|_p\right)\frac{\partial T}{\partial x} - \left(\frac{1}{p} - \frac{1}{Z}\left.\frac{\partial Z}{\partial p}\right|_T\right)\frac{\partial p}{\partial x}\right] \\ & + \frac{f}{2c_vD}\left(\frac{ZRT|\dot{m}|}{pA}\right)^3 + \frac{ZRT}{pc_v}\frac{\Omega}{A_h}.\end{aligned} \tag{7}$$

The resulting non-linear partial differential equations are discretized using the cell-centered backward-time centered-space (BTCS) implicit finite difference method [11, 12], and solved using matrix inversion and the Jacobi iterative method [13], as described in further detail in section 3.1.

2.2 Closure relations

2.2.1 Heat transfer

To calculate the heat transfer Ω between the gas and the surroundings, a transient one-dimensional radial model [14] is used. This model includes heat storage in the pipeline wall and surrounding medium, and has been shown to give accurate results for the temperature development in long off-shore pipelines [15–17], given accurate ambient temperatures [18].

When calculating the heat transfer Ω, the inner and outer heat transfer coefficients are used to calculate respectively the heat transfer between the gas and the pipeline wall, and the heat transfer between the pipeline wall and the ambient. The heat transfer coefficient h can be determined from the Nusselt number for pipe flow

$$\mathrm{Nu_D} = \frac{hD}{k}, \tag{8}$$

where D is the (inner or outer) diameter and k is the thermal conductivity of the fluid (the gas or the ambient fluid).

The *inner* film heat transfer coefficient can be determined from the Dittus-Boelter relation [2, 3], which is valid for forced convection in turbulent pipe flow with Reynolds numbers larger than 10^4 [19]. The Dittus-Boelter relation is

$$\mathrm{Nu_D} = 0.023 \cdot \mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.4}, \tag{9}$$

where Re and Pr is respectively the Reynolds number and the Prandtl number of the gas.

The outer film heat transfer coefficient can be determined from a similar equation, valid for circular cylinders in cross flow with Reynolds numbers between 10^3 and $2 \cdot 10^5$ [19]

$$\mathrm{Nu_D} = 0.26 \cdot \mathrm{Re}^{0.6}\,\mathrm{Pr}^{0.3}, \tag{10}$$

where Re and Pr is respectively the Reynolds number and the Prandtl number of the ambient medium.

2.2.2 Equation of state

For high pressures, such as in the Norwegian export network, the selection of equation of state can have a significant impact on the simulation results [15, 20]. In this study the BWRS (Benedict–Webb–Rubin-Starling) equation of state [21] is used, to determine the gas density, and the compressibility factor Z and its derivatives. The BWRS equation is the following function of molar density ρ_m and temperature

$$\begin{aligned} P = \rho_m RT &+ \left(B_0 RT - A_0 - \frac{C_0}{T^2} + \frac{D_0}{T^3} - \frac{E_0}{T^4}\right)\rho_m^2 + \left(bRT - a - \frac{d}{T}\right)\rho_m^3 \\ &+ \alpha\left(a + \frac{d}{T}\right)\rho_m^6 + \frac{c\rho_m^3}{T^2}\left(1 + \gamma\rho_m^2\right)\exp\left(-\gamma\rho_m^2\right). \end{aligned} \tag{11}$$

The parameters A_0, B_0, etc. are 11 mixture parameters specific to BWRS, and are calculated using mixing rules and pure component properties given in [21], and a set of parameters A_i and B_i. The set of parameters A_i and B_i used in this study has been especially tuned for the Norwegian gas transport network [22].

2.2.3 Friction factor and viscosity

The Colebrook-White equation [1] is used to calculate the friction factor f

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\epsilon}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right), \tag{12}$$

where ϵ is the sand grain equivalent roughness of the inner pipeline wall. Here a value of 3 micrometer was used for the roughness. The Colebrook-White equation is an implicit equation, which is solved using the Newton-Rhapson method.

The Lee-Gonzales-Eakin correlation [23] is used to calculate the viscosity of the gas μ

$$\mu = K\exp\left(X\rho^{Y}\right), \tag{13}$$

where

$$K = \frac{(9.4 + 0.02M)\,T^{1.5}}{209 + 19M + T}, \tag{14}$$

$$X = 3.5 + \frac{986}{T} + 0.01M, \tag{15}$$

$$Y = 2.4 - 0.2X, \tag{16}$$

and M is the molecular weight of the gas.

3 NUMERICAL SCHEME

3.1 Governing equations

To solve the non-linear partial differential equations for the three *state variables* mass flow $\dot{m}$, pressure p, and temperature T, eqs. (5) to (7) are first discretized using a scheme similar to the BTCS (backward time, centered space) finite difference scheme, using cell averages [11, 12]. The pipeline is divided into N grid points, and the different variables are approximated at each section between the grid points by

$$y \approx \frac{y_{i+1}^{n+1} + y_i^{n+1}}{2}, \tag{17}$$

where y represents a general variable, superscripts n and $n+1$ denote time level, and subscripts i and $i+1$ denote grid points. Time derivatives are approximated by

$$\frac{\partial y}{\partial t} \approx \frac{y_{i+1}^{n+1} + y_i^{n+1} - \left(y_{i+1}^{n} + y_i^{n}\right)}{2\Delta t}, \tag{18}$$

and spatial derivatives by

$$\frac{\partial y}{\partial x} \approx \frac{y_{i+1}^{n+1} - y_i^{n}}{\Delta x}. \tag{19}$$

This scheme is first order accurate in time, and second order accurate in space, and have been shown to give accurate results [15, 24].

Central difference schemes are known to be very prone to oscillations, but this problem is avoided by choosing an appropriate time step and grid spacing, and by using smooth transients, and avoiding discontinous changes in boundary conditions [24]. A different approach could be using an upwind (backward difference) scheme for the spatial derivatives, but upwind schemes does not take into account acoustic information traveling from points which are downstream [25], so central differences are preferred.

When replacing eqs. (17) to (19) in eqs. (5) to (7) non-linear equations in $\dot{m}$, p and T are aquired. These equations are linearized by "lagging" behind parts of the non-linear terms [26]

$$y^{n+1} \to y^{n}. \tag{20}$$

The result is a set of linear equations, with three equations for each pipe section, and $N-1$ total pipe sections, giving a total of $3(N-1)$ equations. The number of unknowns at time level $n+1$ is $3N$ (N for each state variable), so three boundary conditions are needed. Here the inlet mass flow $\dot{m}_1$, outlet pressure p_N, and inlet temperature T_1 are chosen. The linear equations with boundary conditions are written on matrix form

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}, \tag{21}$$

where the vector $\boldsymbol{x}$ has length $3(N-1)$ and contains the unknowns

$$\boldsymbol{x} = \left[\dot{m}_2^{n+1}, \ldots, \dot{m}_N^{n+1}, p_1^{n+1}, \ldots, p_{N-1}^{n+1}, T_2^{n+1}, \ldots, T_N^{n+1}\right]^{-1}, \tag{22}$$

the matrix $\boldsymbol{A}$ has shape $3(N-1) \times 3(N-1)$ and contains the coefficients in front of the unknowns, and the vector $\boldsymbol{b}$ contains the known terms including the boundary conditions. Equation (21) is solved using matrix inversion and the Jacobi iterative method [13]. This entails finding $\boldsymbol{x}$ using matrix inversion

$$\boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}, \tag{23}$$

where the inverse $\boldsymbol{A}^{-1}$ is found using a linear algebra library. In the Jacobi iterative method the unknows at time level $n+1$ are given the values from $\boldsymbol{x}$, and terms like the friction factor, compressibility factor etc., are updated using the new mass flow, pressure and temperature. This gives a new set of coefficients $\boldsymbol{A}$ and known terms $\boldsymbol{b}$, and the procedure is repeated until the unknowns converge.

4 SIMULATIONS

4.1 Pipeline description

A simplified pipeline was modelled, based on typical off-shore pipelines that transport gas from Norway to Europe. The simplified pipeline profile is illustrated in fig. 1, and the pipe wall composition in fig. 2.

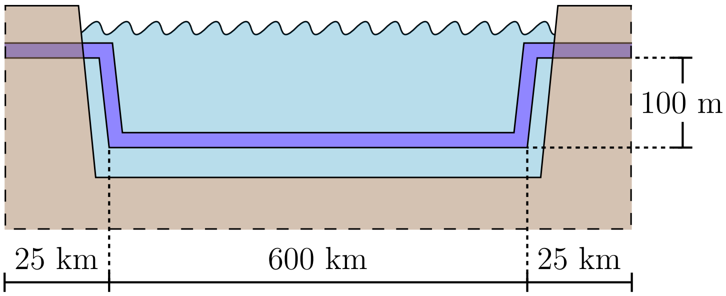

Figure 1: Illustration of the simplified pipeline. The pipe is on-shore and buried 2 m underground for the first and last 25 km, and 100 m below sea level and exposed to sea water for 600 km between the on-shore sections. Figure created freely after figure in [5].

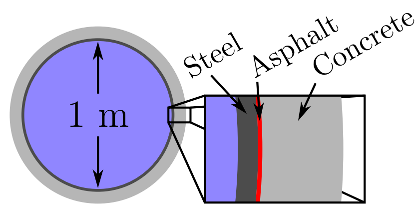

Figure 2: Illustration of pipe and pipe wall materials. The model pipeline has an inner diameter of 1 m, and the pipe wall consists of 24 mm of steel, coated with 7 mm of a protective asphalt coating, and finally 80 mm of concrete.

The pipeline has an inner diameter of 1 m, and consists of a steel pipeline, coated with a protective asphalt coating, and an outer concrete shell. The pipeline is on-shore and buried underground for the first 25 km. It is then 100 m below sea level and exposed to sea water for 600 km, before it is on-shore and buried for the final 25 km.

4.1.1 Boundary conditions

The boundary conditions for the pipeline was constant inlet temperature of 33 °C, constant outlet pressure of 10 MPa, and constant air and sea water temperatures of respectively 6 °C and 4 °C. The system was thermalized with constant inlet mass flow of 600 kg/s, and then the mass flow rate was gradually decreased from 600 kg/s to 200 kg/s in a span of 4 minutes to emulate a transient. The mass flow transient is shown in fig. 3. These conditions correspond to a Reynolds number of 40 to 50 million.

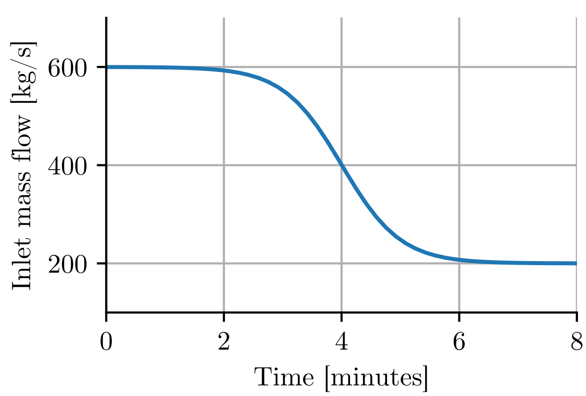

Figure 3: Plot of the inlet mass flow boundary condition, which simulates a transient occuring in a time span of approx 4 minutes.

The gas composition was kept fixed at the values shown in table 1.

4.2 Sensitivity study

Which parameters to include in the sensitivity study were determined by looking at which variables appear in the governing equations (eqs. (5) to (7)), in addition to other correlations that are used in the simulations. The focus was on simplifications and empirical correlations in the models, not on input parameters like pipe diameter, ambient temperature, etc. The following nine parameters are included in the study:

- the Colebrook-White correlation for the friction factor f, eq. (12)
- the compressibility factor Z and three derivatives: $\left.\frac{\partial Z}{\partial T}\right|_p$, $\left.\frac{\partial Z}{\partial p}\right|_T$ and $\left.\frac{\partial Z}{\partial T}\right|_\rho$, which are all calculated from the equation of state
- Nusselt number relations (the Dittus-Boelter equation and eq. (10)) for inner and outer film heat transfer coefficients, which go into the calculation of the heat transfer between the gas and the surroundings Ω
- the correlation for heat capacity of the gas at constant volume c_v
- the Lee-Gonzales-Eakin correlation for the viscosity of the gas μ, eq. (13), which mainly enters the simulations via the Reynolds number, $\mathrm{Re} = \frac{\rho u D}{\mu}$

To investigate the sensitivity of the model a *base case* was first established using standard model parameters and the boundary conditions described in section 4.1. The inlet flow rate transient was initiated at around 2 hours, and the pipeline was simulated for 104 hours after the transient. The simulation was then repeated several times, with a different parameter modified by a constant factor of 1.2 each time. The corresponding response in the modelled flow, pressure, and temperature were recorded for each case.

Table 1: The gas. composition used for the simulations.

Component	Mole fraction
CH_4	0.8916
C_2H_6	0.073513
C_3H_8	0.005104
iC_4H_{10}	0.000251
nC_4H_{10}	0.000311
iC_5H_{12}	0.000009
nC_5H_{12}	0.000024
N_2	0.006980
CO_2	0.022208

5 RESULTS AND DISCUSSION

5.1 Simulation results

In fig. 4 is a plot of the boundary conditions for the base case, and the results from a simulation where the compressibility factor Z was increased by 20 %. From the figure

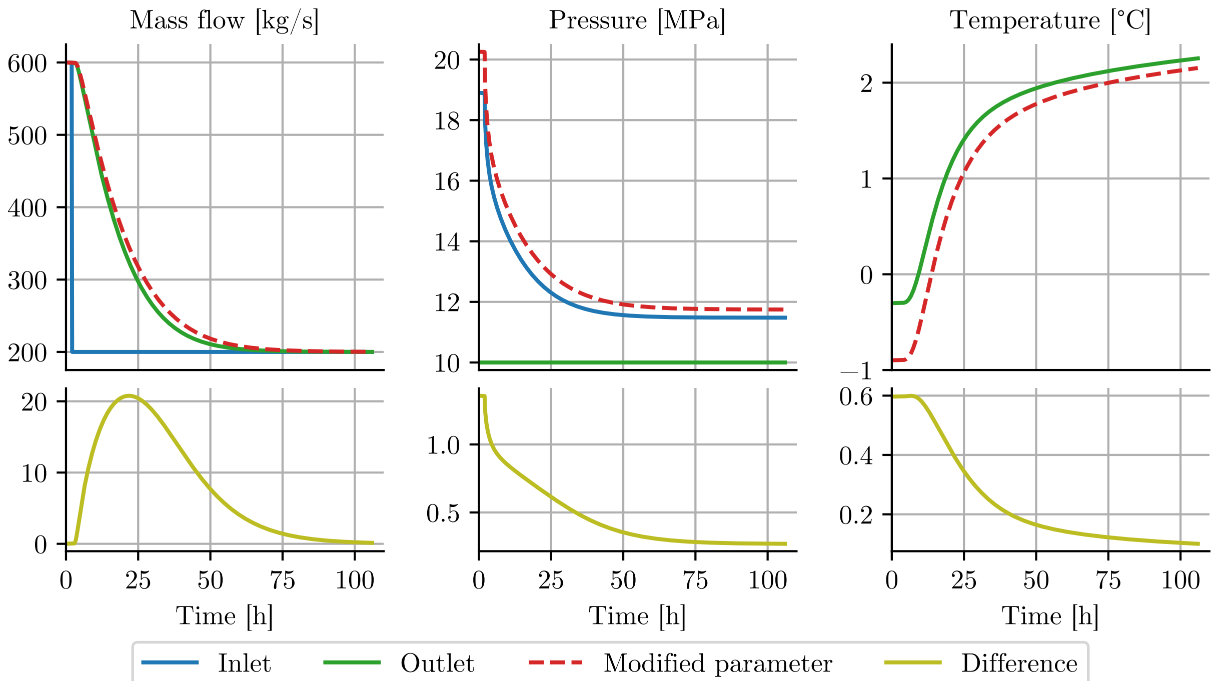

Figure 4: A plot of the results from the base case, and from a simulation with a modified parameter (the compressibility factor Z). The inlet flow rate transient occurs at 2 hours, and the boundary conditions are then kept constant for 104 hours. The three bottom plots show the difference between the base case and the modified parameter simulation, at respectively the outlet, the inlet and the outlet. The constant inlet temperature of 33 °C is not shown.

it is clear that the difference between the base case (y^0) and the cases with a modified parameter (y) vary with time during the transient, so to more easily analyze the results, both the time average relative difference

$$\overline{\Delta y_{\mathrm{rel}}} = \frac{1}{n}\sum_{i=1}^{n} \Delta y_{\mathrm{rel},i} = \frac{1}{n}\sum_{i=1}^{n} \frac{y_i - y_i^0}{y_i^0}, \qquad \text{where } n \text{ is the number of time steps,} \tag{24}$$

and the maximum relative difference

$$\max\left(\Delta y_{\mathrm{rel}}\right) = \max\left(\frac{y_i - y_i^0}{y_i^0}\right) \tag{25}$$

was calculated for mass flow, pressure and temperature. This was calculated for each parameter in the sensitivity study, at every grid point in the simulations. Since a transient case is simulated, and both the temperature and the pressure vary between the inlet and the outlet, relative differences are used. A plot of the maximum and average differences as function of position, for all parameters in the study, are shown in fig. 5.

From fig. 5 a) and b) it can be seen that the parameters with the highest impact on both mass flow and pressure are the friction factor f and the compressibility factor Z. For the mass flow the greatest impact is at the outlet, with a gradual decrease from the inlet to the outlet. For the pressure the situation is reversed. This behaviour is caused by how the simulations are set up, with the mass flow as boundary condition at the inlet and the pressure as boundary condition at the outlet.

The average impact for the whole pipeline was calculated by averaging over the time averaged difference for each grid point. A list of the average impact of all parameters is listed in table 2. The average impact on mass flow is found to be 1.43 % for the friction factor and 0.90 % for the compressibility factor, which is respectively 6.6 and 4.1 times higher than the third most important factor, $\partial Z/\partial p|_T$, which has an average impact of 0.22 %. The average impact on pressure is 3.09 % for the friction factor and 2.07 % for the compressibility factor, which is respectively 14.3 and 9.6 times higher than the third most important factor, the viscosity μ, which has an average impact of 0.22 %.

Table 2: Table of the average relative difference in mass flow, pressure and temperature, for all parameters in the sensitivity study.

	Average relative difference		
Parameter	Mass flow [%]	Pressure [%]	Temperature [%]
Z	0.90	2.1	0.029
$\partial Z/\partial p\|_T$	0.22	0.053	0.0084
$\partial Z/\partial T\|_p$	0.19	0.061	0.016
$\partial Z/\partial T\|_\rho$	0.10	0.057	0.045
h_{outer}	0.017	0.0073	0.0061
h_{inner}	0.0033	0.0015	0.0015
c_v (gas)	0.088	0.023	0.032
μ (gas)	0.078	0.22	0.0027
f	1.4	3.1	0.031

For the temperature it is seen from fig. 5 c) that the situation is less clear-cut. From table 2 it is seen that the parameter which gives the highest average difference along the whole pipeline is the derivative of the compressibility factor $\partial Z/\partial T|_\rho$, with an average difference of 0.045 %, 1.41 times the impact of the next parameter (c_v) and 1.48 times the impact of the third parameter (f). The effects on temperature are in general seen to be much smaller than for mass flow and pressure, and there are no parameters that stand out in the same way they for mass flow and pressure

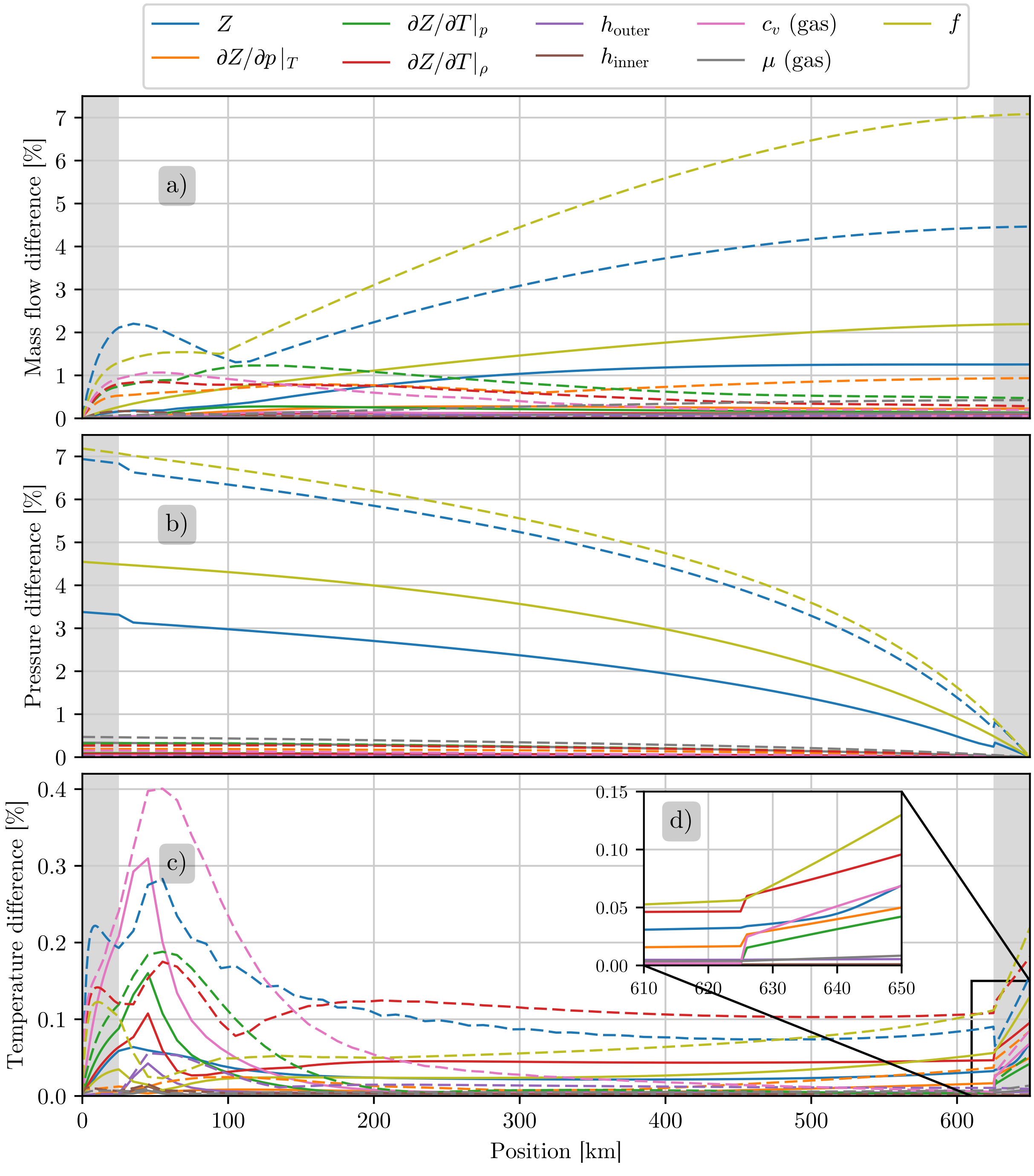

Figure 5: Plots of the maximum (dashed lines) and time average (solid lines) difference between the base case and the cases with a parameter increased by 20 %, for each grid point used in the simulations. The gray shaded areas on the left and right side illustrate the on-shore parts of the pipeline.

In fig. 5 c), peaks in the temperature responses of up to 0.31 % are observed around 20 km off-shore, before the impact of all parameters steadily decrease until they stabilize around 100 km to 200 km off-shore. Also, between 125 km (100 km off-shore) and landfall at 625 km, all parameters have much lower impact than closer to the start of the pipeline; no parameter have a higher maximum impact than 0.15 % or an average impact of more than 0.06 %. The stable behaviour in this area is caused by the fixed sea temperature, which acts as a thermal reservoir, so after a long off-shore section the gas comes to a thermal equilibrium with the sea water, and the gas temperature is governed by the ambient temperature.

Finally, there is a steady increase in the impact on temperature between landfall and the outlet for most parameters. This is because the boundary condition for the thermal exchange between the gas and the ambient changes at landfall (the pipeline goes from being exposed to sea water to buried under ground), and the thermal equilibrium between the gas and the ambient is disturbed. Some details on the temperature response near the outlet can be seen in the inset in fig. 5 d). It is clear that some of the parameters that have very little effect on the temperature while off-shore, like the heat capacity c_v and the derivative of the compressibility factor $\partial Z/\partial T|_p$, have a much higher effect after going on-shore, and on the final outlet temperature, even though the final effect is still small. The impact of the derivative of the compressibility increases from 0.0012 % at 625 km to 0.042 % at the outlet, a factor of 12.9, and the impact of the heat capacity c_v increases from 0.0018 % at 625 km to 0.069 % at the outlet, a factor of 13.8. The trend in the plot indicates that the impact would be even bigger with a longer on-shore section. In general it is observed that the exposure to the sea seem to reduce the impact of changes in the model parameters on the temperature.

To further analyze the temperature responses, the maximum and average temperature differences at certain points of interest along the pipeline are shown in a bar chart in fig. 6. It is seen that the heat capacity c_v is the parameter with the highest impact between all the selected points on the pipeline, with the two highest *average* temperature differences (respectively at 20 km off-shore, and at the end of the on-shore section), and the highest and third highest *maximum* differences (respectively at 20 km off-shore and at the end of the on-shore section). The parameter with the highest impact at 300 km off-shore is the derivative of the compressibility factor $\partial Z/\partial T|_\rho$, while at the end of the off-shore section and at the outlet it is the friction factor f.

5.2 Comparisons with literature

The sensitivity study by Langelandsvik [4] was performed using steady state models, and most of the parameters are hard to relate to the present study. One parameter that is common for both studies is the viscosity. The viscosity was increased by 1 % by Langelandsvik, which lead to a change in volume flow of 0.03 % to 0.05 % and no observable change in outlet temperature (less than 0.01 °C or 0.004 %). Further, the density was modified by 1 %, which is equivalent to changing the compressibility factor

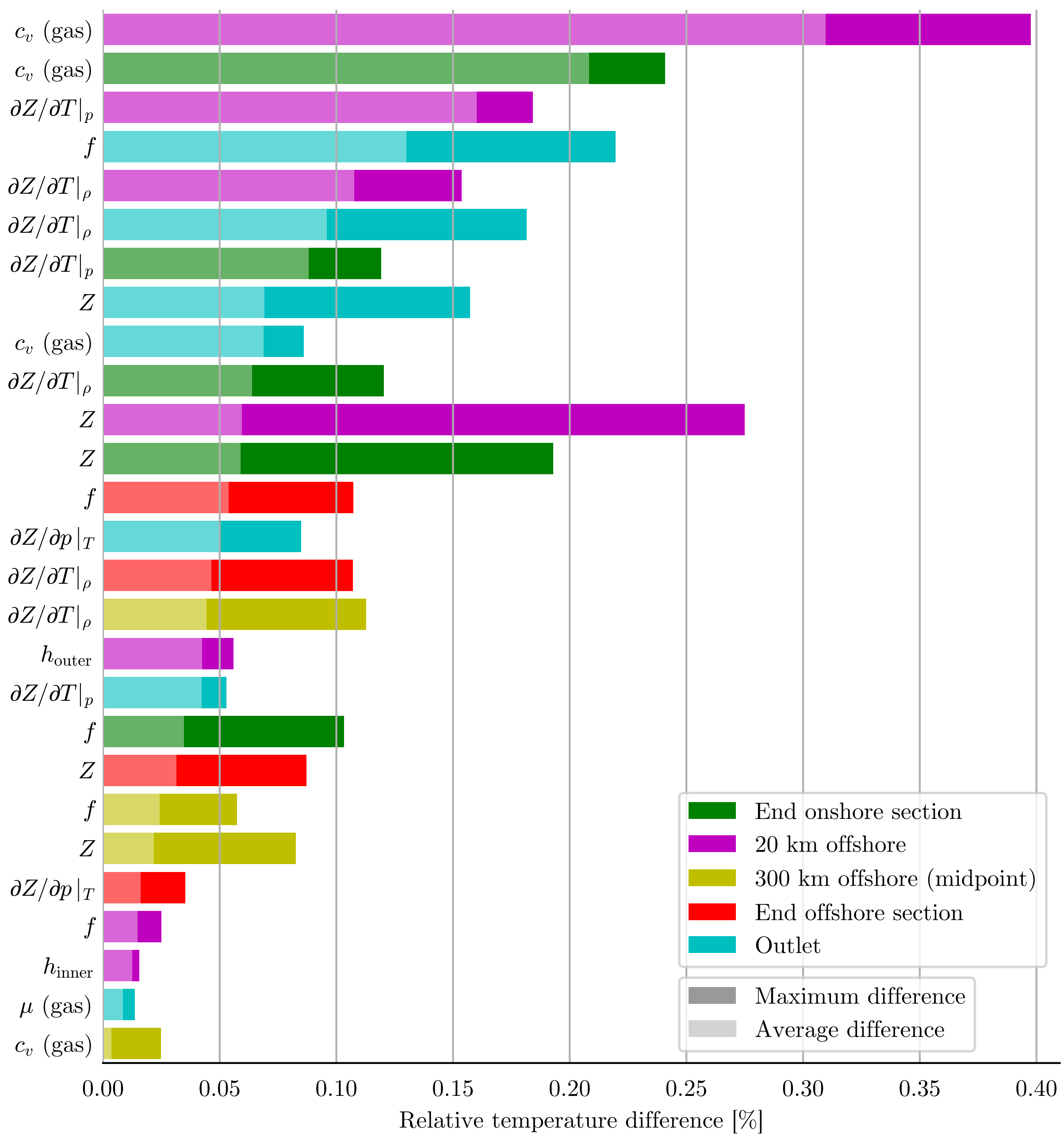

Figure 6: Chart of the average relative difference (the hatched bars) and maximum relative difference (the solid bars) in temperature between the base case and the cases with modified parameters, for 5 different points on the pipeline (the end of the on-shore section, 20 km off shore, 300 km off shore, and the end of the off-shore section). The bars are sorted by descending average error. Only parameters/points with average relative difference above 0.0036 % are shown.

by 1 %, since these values are linked via the equation of state

$$\frac{p}{\rho} = ZRT. \tag{4}$$

The modified density lead to a change in volume flow of 0.50 % and a change in the outlet temperature of around 0.01 °C (0.004 %).

These results does not agree that well with the present results. Here a change in the viscosity of 20 % leads to an average change in mass flow of 0.078 % and an average change in the outlet temperature of 0.84 %. Further, a change in the compressibility factor of 20 % leads to an average change in mass flow of 0.90 % and an average change in the outlet temperature of 0.84 %. The effects on temperature are somewhat in agreement, but the effect on the flow rates are not proportional to the changes in the two parameters. This is most likely caused by how the simulations performed by Langelandsvik are set up: the boundary conditions are inlet temperature, inlet pressure and outlet pressure. This means that the pressure drop is fixed, and any impact on pressure observed in the present work would appear as impacts on flow rate in the work by Langelandsvik.

The sensitivity study by Helgaker [5] used a similar simplified pipeline as in the present study, but a different methodology, so the results are not directly comparable. Helgaker used real boundary conditions based on measurement data, but with flow rates, pressures and temperatures in the same range as synthetic transient used in the present study. In the present study, different correlations were modified directly by a constant factor, while Helgaker modified the friction factor by changing the equivalent sand grain pipeline roughness; tested different equations of state; and modified different parameters used by the heat transfer model, like the thermal conductivity and heat capacity of the pipe wall and burial medium, the burial depth, the length of the buried sections, and the ambient temperature.

Helgaker used sand grain equivalent roughnesses (ϵ in eq. (12)) of 2 µm, 3 µm and 4 µm, and found no observable difference in outlet mass flow or outlet temperature between the three roughnesses, but a difference in inlet pressure of 0.15 MPa (approximately 0.8 % to 1.0 %). This contradicts the results from the present study, which shows that the friction factor is important for both mass flow, pressure and temperature. A rough estimate using the Colebrook-White equation shows that a change in the roughness of 1 µm leads to a change in the friction factor in the order of 2 %, compared to a change in the friction factor of 20 % used in the present study. This relatively small change in the friction factor used by Helgaker might explain why no difference was observed in the mass flow or temperature. This estimate also shows that the relative effect on pressure when changing the friction factor seems to agree between the two studies, with a change in the friction factor of 0.8 % to 1.0 % leading to a change in the inlet pressure of approximately 1 % in the work by Helgaker, and a change in the friction factor of 20 % leading to a change in the inlet pressure of approximately 4.5 % to 7.2 % in the present work (see fig. 5).

The results of Helgaker for the heat transfer model are hard to compare to the present results, since the physical properties of the pipeline and surroundings were modified, while in the present study the correlations for the inner and outer heat transfer coefficients are modified. This is by design, since the present study sets out to determine where to apply effort when trying to improve the models themselves, not to determine which input to the models that are most important.

Helgaker also looked at the effect of the partial derivatives of Z, by setting them all to zero. He found that this had a small impact on the mass flow, but a considerable effect on the pressure during transients. It also had a large impact on the temperature, since setting $\frac{\partial Z}{\partial T}|_p$ to zero effectively removes the Joule-Thompson effect of cooling upon expansion. It is hard to compare these results to the present study, since setting the derivatives to zero is very different from modifying them by a constant factor of 20 %. But the maximum difference caused by modifying the different derivatives observed in fig. 5 indicate that these parameters are important during transients, which agree with the results of Helgaker.

6 CONCLUSIONS

To determine where to apply effort when trying to improve compressible gas flow models for long off-shore pipelines, the relative importance of a selection of model parameters were determined, by modifying nine different model parameters by 20 % one at the time, and investigating the response in mass flow, pressure and temperature.

It was found that, for the mass flow and pressure, the most important parameters by a factor of between 4 and 14 are the friction factor f and the compressibility factor Z, with an average impact on the modelled mass flow of 1.43 % for the friction factor and 0.90 % for the compressibility factor, and an average impact on the modelled pressure of 3.09 % for the friction factor and 2.07 % for the compressibility factor.

For the temperature, none of the parameters stood out like they did for mass flow and pressure, but it was found that the parameter with the highest average impact on the modelled temperature is the derivative of the compressibility factor with respect to temperature (at constant density) $\partial Z/\partial T|_\rho$, with an average difference of 0.045 %, 1.4 times the impact of the next parameter (the gas heat capacity c_v) and 1.5 times the impact of the third parameter (the friction factor f). Further, a peak in the temperature response for most of the parameters was observed around 20 km after going off-shore, with the highest peak attributed to the gas heat capacity c_v. The response of all parameters are greatly diminished after a long off-shore section, and the highest impact at 300 km off-shore is attributed to $\partial Z/\partial T|_\rho$. Finally, at the end of the off-shore section, and at the outlet, the most important parameter is the friction factor f.

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