9. National Conference on Computational Mechanics
MekIT'15
B. Skallerud and H I Andersson (Eds)

THE RELATIVE IMPORTANCE OF MODEL PARAMETERS IN PREDICTIVE TRANSIENT MODELS

Filip Sund[1,2]

[1]Norwegian University of Science and Technology
Department of Energy and Process Engineering
7491 Trondheim, Norway

[2]Uni Research Polytec
5527 Haugesund, Norway
e-mail: filip.sund@polytec.no

Key words: Computational Methods

Abstract. *A sensitivity study on numerical transient flow models for compressible gas was performed to determine the most important parameters when simulating long offshore gas pipelines. A simplified pipeline was simulated with synthetic transient boundary conditions, while systematically modifying different model parameters and correlations. It was found that, for the mass flow and pressure, the most important parameters are the friction factor and the compressibility factor. Further, after a long offshore section, none of the parameters has much impact on the temperature. In addition it was found that on shore, buried under ground, and initially after going offshore, the most important parameters for the temperature are the gas heat capacity, the compressibility factor and its derivatives, and the friction factor.*

209.99753 mm 159.99811 mm
8.26933 in 6.30045 in

1 INTRODUCTION

Natural gas exported from Norway to Europe accounts for around 25 percent of the yearly gas consumption in the European Union. The gas is transported from Norway through pipelines that are up to 1166 km long. To ensure that the pipelines stay within their operating limits, to monitor the pipelines for leaks, and to track changes in gas quality, it is important to know the state of the gas in the pipelines. But measurements of the state of the gas are only available at the inlet and outlet, which means that numerical models are necessary to know the state of the gas between the endpoints.

source?

Simulating compressible gas flow is a highly complex issue, so to reduce the problem to a tractable one, several empirical relations and correlations like the Colebrook-White equation and the Dittus-Boelter equation are typically used to describe different aspects of the system. When doing this errors are introduced into the simulations, the total effect of which can be hard to calculate a priori, and which will depend on the state and system being simulated.

The objective of this study is to investigate which parameters and correlations in the gas models that have greatest impact on the modelled results, especially during transient conditions, to know where to apply effort when trying to improve the models. A similar study limited to steady state models was done by Langelandsvik [11], and some work using transient models was by Helgaker [8]. The present work deals with transient one-dimensional non-isothermal models for compressible natural gas mixtures, and a simplified pipeline is modelled using synthetic but representative flow transients as boundary conditions.

This article is structured as follows: The theoretical foundation and underlying equations are presented in Section 2 followed by the presentation of the studied pipeline system in section 3. Results are presented and discussed in section 4 while concluding remarks are drawn in section 5.

2 THEORY

2.1 Conservation laws

The governing equations for one-dimensional, non-isothermal, transient pipeline gas flow are:

the **continuity equation**

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = 0, \tag{1}$$

the **momentum equation** [5]

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) + \frac{\partial p}{\partial x} = -\frac{f \rho \left| u \right| u}{2D} - \rho g \sin \theta, \tag{2}$$

and the **energy equation** [16]

$$\rho \left(\frac{\partial e}{\partial t} + u \frac{\partial e}{\partial x} \right) + p \frac{\partial u}{\partial x} = \frac{f \rho u^3}{2D} + \frac{\Omega}{A_h}, \tag{3}$$

Using a real gas equation of state

$$\frac{p}{\rho} = ZRT \tag{4}$$

and introducing the mass flow rate $\dot{m} = \rho u A$, the governing equations are developed into partial differential equations for p, $\dot{m}$ and T

$$\frac{\partial p}{\partial t} = \left(\frac{1}{p} - \frac{1}{Z}\left.\frac{\partial Z}{\partial p}\right|_T\right)^{-1}\left[\left(\frac{1}{T} + \frac{1}{Z}\left.\frac{\partial Z}{\partial T}\right|_p\right)\frac{\partial T}{\partial t} - \frac{ZRT}{pA}\frac{\partial \dot{m}}{\partial x}\right] \tag{5}$$

$$\begin{aligned}\frac{\partial \dot{m}}{\partial t} = {} & \frac{\dot{m}ZRT}{pA}\left[-2\frac{\partial \dot{m}}{\partial x} + \dot{m}\left(\frac{1}{p} - \frac{1}{Z}\left.\frac{\partial Z}{\partial p}\right|_T\right)\frac{\partial p}{\partial x} - \dot{m}\left(\frac{1}{T} + \frac{1}{Z}\left.\frac{\partial Z}{\partial T}\right|_p\right)\frac{\partial T}{\partial x}\right] \\ & - A\frac{\partial p}{\partial x} - \frac{fZRT\dot{m}\left|\dot{m}\right|}{2DAp} - \frac{pA}{ZRT}g\sin\theta\end{aligned} \tag{6}$$

$$\begin{aligned}\frac{\partial T}{\partial t} = {} & -\frac{\dot{m}ZRT}{pA}\frac{\partial T}{\partial x} - \frac{\dot{m}\left(ZRT\right)^2}{pAc_v}T\left(\frac{1}{T} + \frac{1}{Z}\left.\frac{\partial Z}{\partial T}\right|_\rho\right) \\ & \cdot\left[\frac{1}{\dot{m}}\frac{\partial \dot{m}}{\partial x} + \left(\frac{1}{T} + \frac{1}{Z}\left.\frac{\partial Z}{\partial T}\right|_p\right)\frac{\partial T}{\partial x} - \left(\frac{1}{p} - \frac{1}{Z}\left.\frac{\partial Z}{\partial p}\right|_T\right)\frac{\partial p}{\partial x}\right] \\ & + \frac{f}{2c_vD}\left(\frac{ZRT|\dot{m}|}{pA}\right)^3 + \frac{ZRT}{pc_v}\frac{\Omega}{A_h}.\end{aligned} \tag{7}$$

The resulting non-linear partial differential equations are discretized using the backward-time central space (BTCS) implicit finite difference method, and solved using matrix inversion and the Jacobi iterative method [7].

2.2 Closure relations

2.2.1 Heat transfer

To calculate the heat transfer Ω between the gas and the surroundings a transient one-dimensional radial model [3] is used. This model includes heat storage in the pipeline wall and surrounding medium, and has been shown to give accurate results for the temperature development in long offshore pipelines [10, 12, 13], given accurate ambient temperatures [15].

When calculating the heat transfer Ω, the inner and outer heat transfer coefficients are used to calculate respectively the heat transfer between the gas and the pipeline wall, and the heat transfer between the pipeline wall and the ambient.

The *inner* film heat transfer coefficient can be determined from the Dittus-Boelter relation [6, 17], which is valid for forced convection in turbulent pipe flow with Reynolds numbers ≥ 10000 [1]

$$\mathrm{Nu} = \frac{hD}{k} = 0.023 \cdot \mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.4}, \tag{8}$$

$$\text{where } \mathrm{Pr} = \frac{c_p \nu}{k} \text{ and } \mathrm{Re} = \frac{\rho u D}{\nu}, \tag{9}$$

Nu is the Nusselt number, h is the film heat transfer coefficient, D is the inner diameter of the pipe, k is the thermal conductivity of the gas, Re is the Reynolds number, and Pr is the Prandtl number.

The outer film heat transfer coefficient can be determined from a similar equation, valid for circular cylinder in cross flow with Reynolds numbers between 10^3 and $2 \cdot 10^5$ [1]

consider changing to eq. 7.52 in Incropera

$$\mathrm{Nu} = \frac{hD}{k} = 0.26 \cdot \mathrm{Re}^{0.6}\,\mathrm{Pr}^{0.3}, \tag{10}$$

where D is the outer diameter of the pipe and k is the thermal conductivity of the ambient medium.

Consider re-running simulations, since h_outer calculation has changed – used constant before.

2.2.2 Equation of state

The BWRS equation of state [14] is used to determine the gas density, the compressibility factor Z and its derivatives, and is also used to calculate properties like the heat capacity c_v. The BWRS equation is the following function of molar density ρ_m and temperature

$$\begin{aligned} P = \rho_m RT &+ \left(B_0 RT - A_0 - \frac{C_0}{T^2} + \frac{D_0}{T^3} - \frac{E_0}{T^4}\right)\rho_m^2 + \left(bRT - a - \frac{d}{T}\right)\rho_m^3 \\ &+ \alpha\left(a + \frac{d}{T}\right)\rho_m^6 + \frac{c\rho_m^3}{T^2}\left(1 + \gamma\rho_m^2\right)\exp\left(-\gamma\rho_m^2\right). \end{aligned} \tag{11}$$

The parameters A_0, B_0, etc. are 11 mixture parameters specific to BWRS, and are calculated using mixing rules from pure component parameters given in [14]. For high pressures, such as in the Norwegian export network, the selection of equation of state can have a significant impact on the simulation results [2, 9].

we use Gassco tuned parameters...

2.2.3 Other

The Colebrook-White equation [4] is used to calculate the friction factor f

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\epsilon}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right), \tag{12}$$

where ϵ is the sand grain equivalent roughness of the inner pipeline wall. Here a value of $3\,\mu\mathrm{m}$ was used.

3 SIMULATIONS(?)

3.1 Pipeline description (?)

A simplified pipeline was modelled based on typical offshore pipelines that transport gas from Norway to the EU. The simplified pipeline profile is illustrated in fig. 1, and the pipe wall composition is shown in fig. 2.

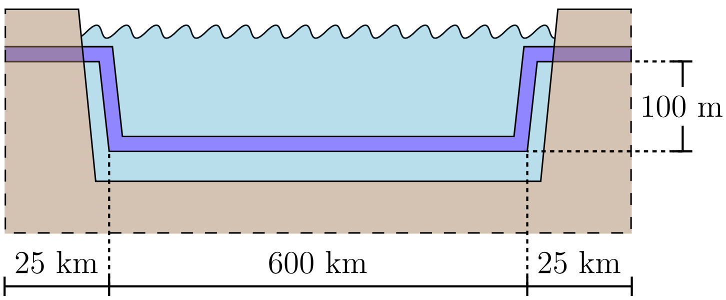

Figure 1: Illustration of the simplified pipeline. The pipe is onshore and buried 2 m underground for the first and last 25 km, and 100 m below sea level and exposed to sea water for 600 km between the onshore sections. Figure created freely after figure in [8].

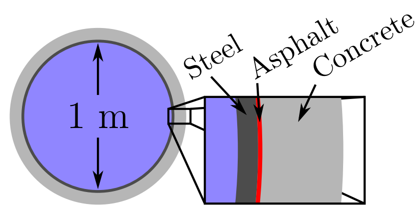

Figure 2: Illustration of pipe and pipe wall materials. The model pipeline has an inner diameter of 1 m, and the pipe wall consists of 24 mm of steel, coated with 7 mm of a protective asphalt coating, and finally 80 mm of concrete.

The boundary conditions for the pipeline was constant inlet temperature of 33 °C, constant outlet pressure of 10 MPa, and constant air and sea water temperatures of respectively 6 °C and 4 °C. The system was thermalized with constant inlet mass flow of 600 kg/s, and then the mass flow rate was gradually decreased from 600 kg/s to 200 kg/s in a span of 4 minutes to emulate a transient. The mass flow transient is shown in fig. 3. These conditions correspond to a Reynolds number of 40 to 50 million.

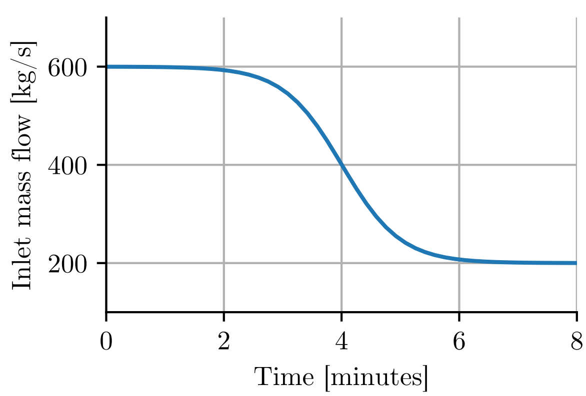

Figure 3: Plot of the inlet mass flow boundary condition, which simulates a transient occuring in a time span of approx 4 minutes.

The gas composition was kept fixed at the values shown in table 1.

3.2 Sensitivity study

Which parameters to include in the sensitivity study was decided by looking at which variables that appeared in the governing equations (eqs. (5) to (7)), in addition to other correlations that are used in the simulations. The following 9 parameters were included:

- the Colebrook-White correlation for the friction factor, f, eq. (12)
- the compressibility factor Z and three derivatives: $\left.\frac{\partial Z}{\partial T}\right|_p$, $\left.\frac{\partial Z}{\partial p}\right|_T$ and $\left.\frac{\partial Z}{\partial T}\right|_\rho$, which are all calculated from the equation of state
- Nusselt number relations (the Dittus-Boelter equation) for inner and outer film heat transfer coefficients (h_{inner} and h_{outer}), which go into the calculation of the heat transfer between the gas and the surroundings, Ω
- the correlation for heat capacity of the gas at constant volume, c_v
- the correlation for viscosity of the gas μ (Lee-Gonzales-Eakin [12]), which mainly enters the simulations via the Reynolds number, $\mathrm{Re} = \frac{\rho u D}{\mu}$

To investigate the sensitivity of the model a *base case* was first established using standard model parameters and the boundary conditions described in section 3.1. The inlet flow rate transient occurs at 2 hours, and the pipeline was simulated for 104 hours after the transient. The simulation was then repeated several times, with a different parameter modified by a constant parameter factor of 1.2 each time. The corresponding changes in the modelled flow, pressure, and temperature (compared to the base case) were recorded for each case.

Table 1: The gas. composition used for the simulations.

Component	Mole fraction
CH_4	0.8916
C_2H_6	0.073513
C_3H_8	0.005104
iC_4H_{10}	0.000251
nC_4H_{10}	0.000311
iC_5H_{12}	0.000009
nC_5H_{12}	0.000024
N_2	0.006980
CO_2	0.022208

4 RESULTS AND DISCUSSION

In fig. 4 is a plot of the boundary conditions for the base case, and the results for a simulation where the compressibility factor Z has been increased by 20 %.

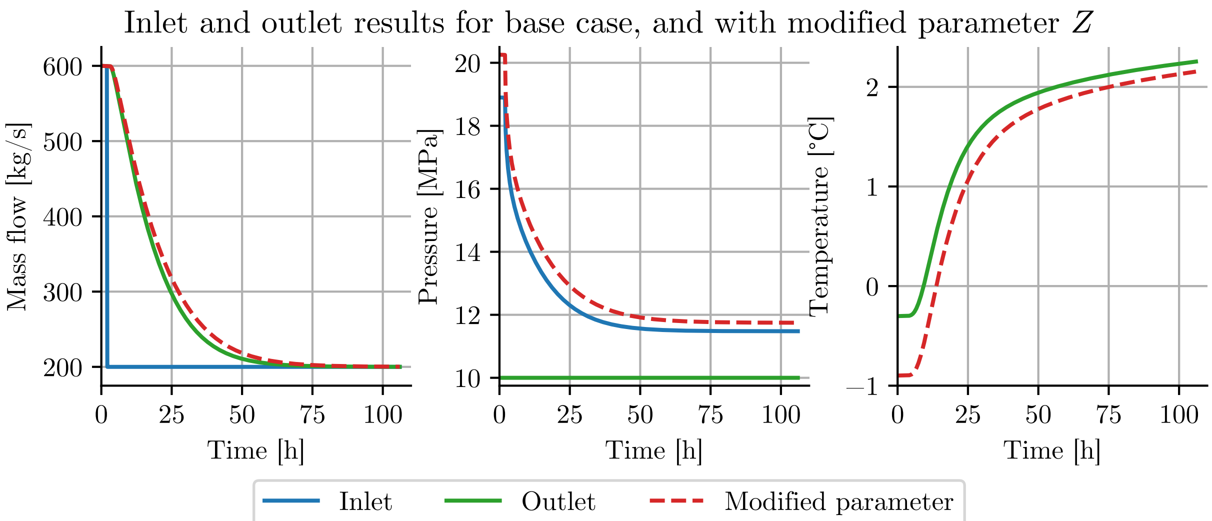

Figure 4: A plot of the results from the base case, and from a simulation with a modified parameter (the compressibility factor Z). The inlet flow rate transient occurs at 2 hours, and the boundary conditions are then kept constant for 104 hours. The constant inlet temperature of 33 °C is not shown.

From fig. 4 it is clear that the differences vary with time during the transient, so to more easily analyze the results, both the time average relative difference

$$\overline{\Delta y_{\mathrm{rel}}} = \frac{1}{n}\sum_{i=1}^{n} \Delta y_{\mathrm{rel}} = \frac{1}{n}\sum_{i=1}^{n} \frac{y_i - y_i^0}{y_i^0} \tag{13}$$

and the maximum relative difference

$$\max\left(\Delta y_{\mathrm{rel}}\right) = \max\left(\frac{y_i - y_i^0}{y_i^0}\right) \tag{14}$$

between the base case (y^0) and the simulations with modified parameters (y) was calculated, for mass flow, pressure and temperature. This was calculated for each parameter in the sensitivity study, at every grid point of the pipeline. Since a transient case is simulated, the relative differences are used, to normalize the differences. A plot of the maximum and average relative differences as function of position, for all parameters in the study, are shown in fig. 5.

From fig. 6 a) and b) it can be seen that the parameters with the highest impact on both mass flow and pressure are the friction factor f and the compressibility factor Z. For the mass flow the greatest impact is at the outlet, with a gradual decrease from the inlet to the outlet. For the pressure the greatest impact is at the inlet, with a gradual decrease from the inlet to the outlet.

A list of the average impact of all parameters is listed in table 2. The average impact on mass flow is found to be 1.32 % for the friction factor and 0.81 % for the compressibility factor, which is respectively 7.5 and 4.6 times higher than the next most important factor, $\partial Z/\partial p|_T$, which has an average impact of 0.18 %. The average impact on pressure is 2.80 % for the friction factor and 1.94 % for the compressibility factor, which is respectively 14.1 and 9.8 times higher than the next most important factor, μ, which has an average impact of 0.20 %.

Table 2: caption

Parameter	Average relative impact on mass flow [%]	Average relative impact on pressure [%]	Average impact on temperature [%]
Z	0.9007	2.0722	0.0289
$\partial Z/\partial p\|_T$	0.2179	0.0527	0.0084
$\partial Z/\partial T\|_p$	0.1859	0.0614	0.0157
$\partial Z/\partial T\|_\rho$	0.1021	0.0567	0.0454
h_{outer}	0.0171	0.0073	0.0061
h_{inner}	0.0033	0.0015	0.0015
c_v (gas)	0.0881	0.0225	0.0321
μ (gas)	0.0782	0.2155	0.0027
f	1.4316	3.0885	0.0306

For the temperature the situation is a bit more complicated. From fig. 5 c) it can be seen that between 125 km (100 km off-shore) and landfall at 625 km, all parameters have very little impact (compared closer to the start of the pipeline), and no parameter have a higher maximum impact than 0.15 % or an average impact of more than 0.06 %. This is because the gas comes to a thermal equilibrium with the ambient sea water after being transported offshore for some time. The sea has fixed temperature, and acts as a thermal reservoir, so after a long off-shore section the gas temperature will be governed by the ambient temperature, and will be independent of any parameters that affect the heat exchange rate between the gas and the ambient (as long as the heat exchange rate is different from zero).

what about things like Joule Thompson?, are there other effects??

Further, a peak is observed around 20 km off-shore in the temperature responses for most of the parameters, before the impact of all parameters steadily decrease until they stabilize around 100 km to 200 km off-shore. Finally, there is a steady increase in the impact on temperature between landfall and the outlet for most parameters. This is because at landfall the boundary conditions for the thermal exchange between the gas and the ambient changes (the pipeline goes from being exposed to sea water to buried

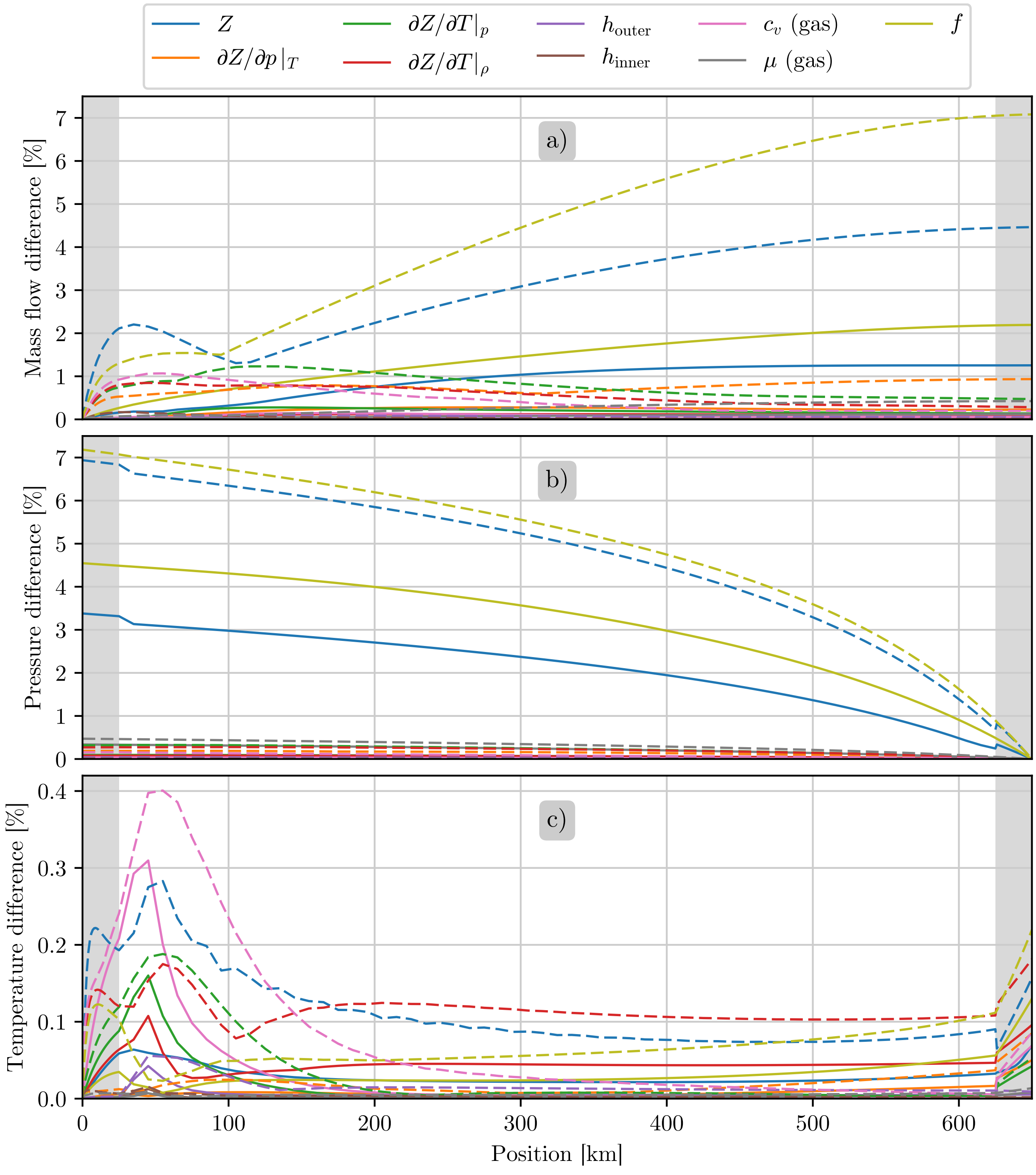

Figure 5: Plots of the maximum (dashed lines) and time average (solid lines) difference between the base case and the cases with a parameter increased by 20 %. The gray shaded areas illustrate the on-shore parts of the pipeline.

under ground), and the thermal equilibrium between the gas and the ambient is disturbed.

To further analyze the temperature responses, the maximum and average relative temperature differences at certain points of interest along the pipeline, are shown in a bar chart in fig. 6. It is seen that the heat capacity c_v is the parameter with the highest impact between all the selected points on the pipeline, with the two highest average temperature differences (respectively at 20 km off-shore, and at the end of the on-shore section), and the highest and third highest maximum differences (respectively at 20 km off-shore and at the end of the on-shore section).

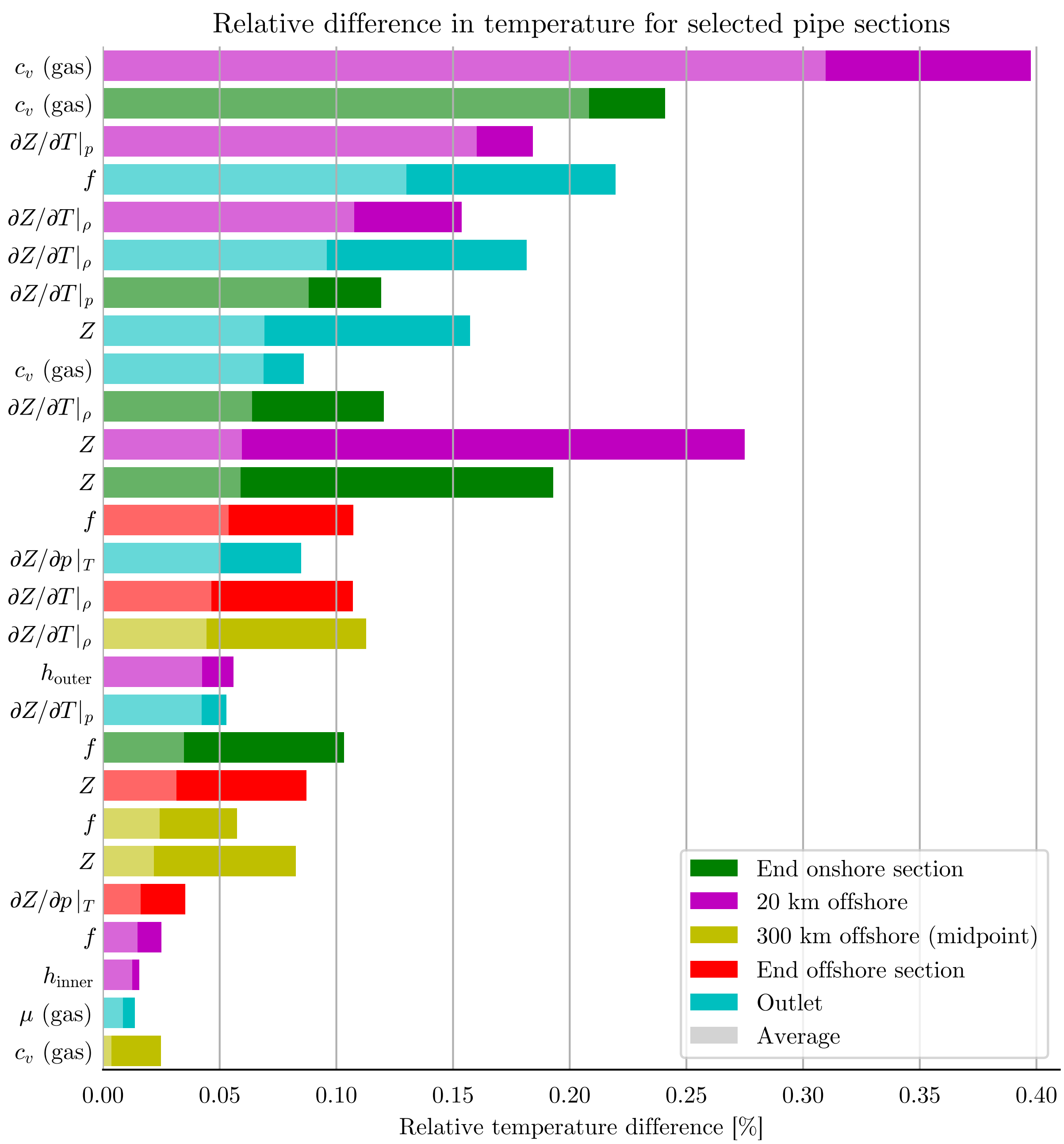

Figure 6: Chart of the average and maximum relative difference in temperature between the base case and the cases with modified parameters, for 5 different points on the pipeline (the end of the onshore section, 20 km off shore, 300 km off shore, and the end of the offshore section). The bars are sorted by descending average error. Only parameters/points with average relative difference above 0.0036 % are shown.

5 CONCLUSIONS

Table 3: caption

Parameter	Average relative impact on mass flow [%]	Average relative impact on pressure [%]	Average impact on temperature [%]
Z	0.8058	1.9431	0.0329
$\partial Z/\partial p\|_T$	0.1769	0.0483	0.0134

Acknowledgements

This work has been funded by the Norwegian gas operating company Gassco as part of a project to improve flow modeling in offshore natural gas transmission pipelines.

References

[1] T. L. Bergman et al. *Fundamentals of heat and mass transfer*. Seventh Ed. John Wiley & Sons Inc., 2011, p. 1076.

[2] M. Chaczykowski. "Sensitivity of pipeline gas flow model to the selection of the equation of state". In: *Chemical Engineering Research and Design* 87.12 (2009).

[3] M. Chaczykowski. "Transient flow in natural gas pipeline - The effect of pipeline thermal model". In: *Applied Mathematical Modelling* 34.4 (2010).

[4] C. F. Colebrook. "Turbulent flow in pipes, with particular reference to the transition region between the smooth and rough pipe laws". In: *Journal of the Institution of Civil Engineers* 11.4 (1939).

[5] H. Daneshyar. *One-Dimensional Compressible Flow*. Pergamon Press, 1976.

[6] F. W. Dittus and L. M. K. Boelter. "Heat transfer in automobile radiators of the tubular type". In: *International Communications in Heat and Mass Transfer* 12 (1985).

[7] J. H. Ferziger and M. Peric. *Computational Methods for Fluid Dynamics*. 3rd. Springer, 2002.

[8] J. F. Helgaker. "Modeling Transient Flow in Long Distance Offshore Natural Gas Pipelines". PhD thesis. 2013.

[9] J. F. Helgaker, B. Müller, and T. Ytrehus. "Transient Flow in Natural Gas Pipelines Using Implicit Finite Difference Schemes". In: *Journal of Offshore Mechanics and Arctic Engineering* 136.3 (2014).

[10] J. F. Helgaker et al. "Validation of 1D flow model for high pressure offshore natural gas pipelines". In: *Journal of Natural Gas Science and Engineering* 16 (2014).

[11] L. I. Langelandsvik. “Modeling of natural gas transport and friction factor for large-scale pipelines: Laboratory experiments and analysis of operational data”. PhD thesis. Norwegian University of Science and Technology, 2008.

[12] A. Oosterkamp, J. F. Helgaker, and T. Ytrehus. “Modelling of natural gas pipe flow with rapid transients - case study of effect of ambient model”. In: *Energy Procedia* 64.C (2015).

[13] A. Oosterkamp. “Heat transfer modelling of natural gas pipe flow - effect of yearly ambient temperature cycles”. In: *The Twenty-sixth (2016) International Ocean and Polar Engineering Conference*. 2016.

[14] K. E. Starling. *Fluid Thermodynamic Properties for Light Petroleum Systems*. Gulf Publishing Company, 1973.

[15] F. Sund, A. Oosterkamp, and S. M. Hope. “Pipeline Modeling - Impact of Ambient Temperature and Heat Transfer Modeling”. In: *Proceedings of the Twenty-fifth (2015) International Ocean and Polar Engineering Conference*. 2015.

[16] F. M. White. *Viscous Fluid Flow*. Third Edit. McGraw-Hill Education, 2006.

[17] R. Winterton. “Where did the Dittus and Boelter equation come from?” In: *International Journal of Heat and Mass Transfer* 41.4-5 (1998).