

1.) Algoritam za izracunavanje zatvaraca

$F = \{AB \rightarrow AC, CD \rightarrow E, A \rightarrow B, AE \rightarrow F\}$

$AB \rightarrow A \quad AB \rightarrow C \quad (AB \rightarrow AC)$

ZATVARAC

$(AD)^+ = AD$

$(AD)^+ = ADB \quad (A \rightarrow B)$

$(AD)^+ = ADBC \quad (AB \rightarrow C)$

$(AD)^+ = ADBCE \quad (CD \rightarrow E)$

$(AD)^+ = ADBCEF \quad (AE \rightarrow F)$

2.)

$F = \{AB \rightarrow C, C \rightarrow A, BC \rightarrow D, ACD \rightarrow B, D \rightarrow EG, BE \rightarrow C, CG \rightarrow BD, CE \rightarrow AG\}$

$D \rightarrow E \quad D \rightarrow G \quad (D \rightarrow EG)$

$CG \rightarrow B \quad CG \rightarrow D \quad (CG \rightarrow BD)$

$CE \rightarrow A \quad CE \rightarrow G \quad (CE \rightarrow AG)$

$(BD)^+ = BDEGCA$

3.)

$F = \{A \rightarrow B, A \rightarrow C, A \rightarrow E, D \rightarrow C, E \rightarrow I, BI \rightarrow J\}$

$(AI)^+ = AIBCEJ$

$(DJ)^+ = DJC$

$(BE)^+ = BEIJ$

4.)

algoritam za trazenje kljuca

$R = \{A, B, C, D, E\}$  skup obelezja

$F = \{AB \rightarrow CDE, E \rightarrow A, CD \rightarrow B\}$

$AB \rightarrow C, AB \rightarrow D, AB \rightarrow E \quad (AB \rightarrow CDE)$

$(ABCDE)^+ = ABCDE = R / E$

$(ABCD)^+ = ABCDE = R / D$

$(ABC)^+ = ABCDE = R / C$

$(AB)^+ = ABCDE = R / B$

$(A)^+ = A$

$(B)^+ = B$

$K = \{AB, EB, ACD, ECD\}$

5.)

$$U = \{A, B, C, D, E, F\}$$

$$F = \{AB \rightarrow C, C \rightarrow A, C \rightarrow D, AB \rightarrow E, AB \rightarrow F, E \rightarrow F\}$$

$$(ABCDEF)^+ = ABCDEF = U / F$$

$$(ABCDE)^+ = ABCDEF = U / E$$

$$(ABCD)^+ = ABCDEF = U / D$$

$$(ABC)^+ = ABCDEF = U / C$$

$$(AB)^+ = ABCDEF = U / B$$

$$(A)^+ = A$$

$$(B)^+ = B$$

$$K = \{AB, CB\}$$

6.)

$$F = \{AB \rightarrow CE, C \rightarrow B, ED \rightarrow F, F \rightarrow G\}$$

$$R = \{A, B, C, D, E, F, G, H\}$$

H i D svakako mora biti u ključu jer se ne javljaju na desnim stranama FZ

$$(ABCDEFGH)^+ = ABCDEFGH / G$$

$$(ABCDEFH)^+ = ABCDEFGH / F$$

$$(ABCDEH)^+ = ABCDEFGH / E$$

$$(ABCDH)^+ = ABCDEFGH / C$$

$$(ABDH)^+ = ABCDEFGH / B$$

$$(ADH)^+ = ADH$$

$$(BDH)^+ = BDH$$

$$K = \{ABDH, ACDH\}$$

7.) Dokazivanje da je FZ logicka posledica skupa FZ prema Armstrongovim pravilima

$$F = \{A \rightarrow B, B \rightarrow C, AC \rightarrow D, BD \rightarrow E, C \rightarrow E\}$$

$$U = \{A, B, C, D, E\}$$

**F | = A → D? Treba dokazati.**

Prvo se traži zatvarač nad levom stranom (A)

$$(A)^+ = ABCDE$$

D se nalazi u zatvaraču od A tako da važi FZ A → D.

$$\begin{array}{l} A \rightarrow B \\ B \rightarrow C \end{array} \} \Rightarrow_{A3} \quad \begin{array}{l} A \rightarrow C \\ A \in U \end{array} \} \Rightarrow_{A2} \quad \begin{array}{l} A \rightarrow AC \\ AC \rightarrow D \end{array} \} \Rightarrow_{A3} \quad A \rightarrow D$$

**F | = AD → E? Treba dokazati.**

$$(AD)^+ = ABCDE$$

$$\begin{array}{l} A \rightarrow B \\ D \in U \end{array} \} \Rightarrow_{A2} \quad \begin{array}{l} AD \rightarrow BD \\ BD \rightarrow E \end{array} \} \Rightarrow_{A3} \quad AD \rightarrow E$$

8.)

$F = \{ A \rightarrow F, AB \rightarrow CE, AC \rightarrow D, EB \rightarrow D, D \rightarrow A, F \rightarrow AE \}$

$AB \rightarrow C, AB \rightarrow E \text{ (} AB \rightarrow C \text{)}$

$F \rightarrow A, F \rightarrow E \text{ (} F \rightarrow AE \text{)}$

$U = \{A, B, C, D, E, F\}$

**$F \models AB \rightarrow D$ ? Treba dokazati.**

$(AB)^+ = ABFCED$

$AB \rightarrow E$	$\} \Rightarrow_{A3}$	$EB \rightarrow D$	$\} \Rightarrow_{A3}$	$AB \rightarrow D$
$EB \rightarrow D$				
$AB \rightarrow E$	$\} \Rightarrow_{A2}$	$AB \rightarrow EB$	$\} \Rightarrow_{A3}$	$AB \rightarrow D$
$B \in U$		$EB \rightarrow D$		

9.)

$F = \{ AB \rightarrow C, C \rightarrow A, BC \rightarrow D, ACD \rightarrow B, D \rightarrow EG, BE \rightarrow C \}$

$D \rightarrow E, D \rightarrow G \text{ (} D \rightarrow EG \text{)}$

$CG \rightarrow B, CG \rightarrow D \text{ (} CG \rightarrow BD \text{)}$

$CE \rightarrow A, CE \rightarrow G \text{ (} CE \rightarrow AG \text{)}$

$U = \{A, B, C, D, E, F, G\}$

**$F \models CE \rightarrow B$ ? Treba dokazati.**

$(CE)^+ = ABCDEFG$

$CE \rightarrow G$	$\} \Rightarrow_{A2}$	$CE \rightarrow CG$	$\} \Rightarrow_{A3}$	$CE \rightarrow B$
$C \in U$		$CG \rightarrow B$		

10.)

**$F \models BD \rightarrow C$ ? Treba dokazati.**

$(BD)^+ = ABCDEFG$

$D \rightarrow E$	$\} \Rightarrow_{A2}$	$BD \rightarrow BE$	$\} \Rightarrow_{A3}$	$BD \rightarrow C$
$B \in U$		$BE \rightarrow C$		

11.)

**$F \models CE \rightarrow D$ ? Treba dokazati.**

$(CE)^+ = ABCDEFG$

$CE \rightarrow G$	$\} \Rightarrow_{A2}$	$CE \rightarrow CG$	$\} \Rightarrow_{A3}$	$CE \rightarrow D$
$C \in U$		$CG \rightarrow D$		

12.)

**$F \models ABG \rightarrow E$ ? Treba dokazati.**

$(ABG)^+ = ABCDEFG$

$AB \rightarrow C$	$\} \Rightarrow_{A2}$	$ABG \rightarrow CG$	$\} \Rightarrow_{A3}$	$ABG \rightarrow D$	$\} \Rightarrow_{A3}$	$ABG \rightarrow E$
$G \in U$		$CG \rightarrow D$		$D \rightarrow E$		

13.)

**F** |= **CD**->**B**? Treba dokazati.

(CD)<sup>+</sup> = ABCDEFG

$$\begin{array}{l} D \rightarrow G \\ C \in U \end{array} \left. \vphantom{\begin{array}{l} D \rightarrow G \\ C \in U \end{array}} \right\} \Rightarrow_{A_2} \quad \begin{array}{l} CD \rightarrow CG \\ CG \rightarrow B \end{array} \left. \vphantom{\begin{array}{l} CD \rightarrow CG \\ CG \rightarrow B \end{array}} \right\} \Rightarrow_{A_3} \quad CD \rightarrow B$$

14.)

F={AB->AC, CD->E, A->B, AE->F}

U={A, B, C, D, E, F}

**F** |= **AD**->**F**? Treba dokazati.

(AD)<sup>+</sup> = ABCDEF

A->B A ∈ U	$\left. \vphantom{\begin{array}{l} A \rightarrow B \\ A \in U \end{array}} \right\} \Rightarrow_{A_2}$	A->AB AB->C	$\left. \vphantom{\begin{array}{l} A \rightarrow AB \\ AB \rightarrow C \end{array}} \right\} \Rightarrow_{A_3}$	A->C D ∈ U	$\left. \vphantom{\begin{array}{l} A \rightarrow C \\ D \in U \end{array}} \right\} \Rightarrow_{A_2}$	AD->CD CD->E	$\left. \vphantom{\begin{array}{l} AD \rightarrow CD \\ CD \rightarrow E \end{array}} \right\} \Rightarrow_{A_3}$	AD->E A ∈ U
$\left. \vphantom{\begin{array}{l} A \rightarrow B \\ A \in U \end{array}} \right\} \Rightarrow_{A_2}$	AD->AE AE->F	$\left. \vphantom{\begin{array}{l} AD \rightarrow AE \\ AE \rightarrow F \end{array}} \right\} \Rightarrow_{A_3}$	AD->F					