| $r$ | A | B | C |
|---|---|---|---|
| | $a_1$ | $b_1$ | $c_1$ |
| | $a_1$ | $b_2$ | $c_2$ |

| $\pi_{AB}(r)$ | A | B |
|---|---|---|
| | $a_1$ | $b_1$ |
| | $a_1$ | $b_2$ |

| $\pi_{Ac}(r)$ | A | C |
|---|---|---|
| | $a_1$ | $c_1$ |
| | $a_1$ | $c_2$ |

| $\pi_{AB}(r) \bowtie \pi_{Ac}(r)$ | A | B | C |
|---|---|---|---|
| | $a_1$ | $b_1$ | $c_1$ |
| | $a_1$ | $b_1$ | $c_2$ |
| | $a_1$ | $b_2$ | $c_1$ |
| | $a_1$ | $b_2$ | $c_2$ |

$$\pi_{AB}(r) \bowtie \pi_{Ac}(r) \neq r$$

$$r(\mathcal{U}), \quad R_1, R_2 \subseteq \mathcal{U}, \quad R_1 R_2 = \mathcal{U}$$

$$\pi_{R_1}(r) \bowtie \pi_{R_2}(r) \neq r$$

$$\pi_{R_1}(r) \bowtie \pi_{R_2}(r) \not\supseteq r$$

$$r \subseteq \pi_{R_1}(r) \bowtie \pi_{R_2}(r) \; ?$$

T:

$$r(\mathcal{U}) \subseteq \pi_{R_1}(r) \bowtie \pi_{R_2}(r), \quad R_1 R_2 = \mathcal{U}$$

$$t \in r \Rightarrow$$

$$t[R_1] \in \pi_{R_1}(r) \land t[R_2] \in \pi_{R_2}(r) \Rightarrow$$

$$\underbrace{t[R_1] \, t[R_2]}_{t} \in \pi_{R_1}(r) \bowtie \pi_{R_2}(r) \Rightarrow$$

$$t \in \pi_{R_1}(r) \bowtie \pi_{R_2}(r) \quad \square$$

$$\pi_{R_1}(r) \bowtie \pi_{R_2}(r) \subseteq r \; ?$$

| r | A | B | C |
|---|---|---|---|
| | $a_1$ | $b_1$ | $c_1$ |
| | $a_1$ | $b_1$ | $c_2$ |

| $\pi_{AB}(r)$ | A | B |
|---|---|---|
| | $a_1$ | $b_1$ |

| $\pi_{AC}(r)$ | A | C |
|---|---|---|
| | $a_1$ | $c_1$ |
| | $a_1$ | $c_2$ |

| $\pi_{AB}(r) \bowtie \pi_{AC}(r)$ | A | B | C |
|---|---|---|---|
| | $a_1$ | $b_1$ | $c_1$ |
| | $a_1$ | $b_1$ | $c_2$ |

$$\pi_{AB}(r) \bowtie \pi_{AC}(r) = r$$

$$r(\mathcal{U}) \not\models A \rightarrow B \; !$$

$$r \vDash X \to Y \implies \pi_{XY}(r) \bowtie \pi_{X(u \setminus Y)}(r) = r$$

T: gok: $\pi_{XY}(r) \bowtie \pi_{X(u \setminus Y)}(r) \subseteq r$

$$(\forall t)(t \in \pi_{XY}(r) \bowtie \pi_{X(u \setminus Y)}(r) \implies t \in r)$$

$$t \in \pi_{XY}(r) \bowtie \pi_{X(u \setminus Y)}(r) \implies$$

$$t[XY] \in \pi_{XY}(r) \land t[X(u \setminus Y)] \in \pi_{X(u \setminus Y)}(r) \implies$$

$$(\exists t_1 \in r)(t_1[XY] = t[XY]) \land$$

$$(\exists t_2 \in r)(t_2[X(u \setminus Y)] = t[X(u \setminus Y)]) \implies$$

$$t_1 \in r \land t_1[XY] = t[XY] \land$$

$$t_2 \in r \land t_2[X(u \setminus Y)] = t[X(u \setminus Y)] \implies$$

$$\left. \begin{array}{l} t_1[X] = t[X] \land \\ t_2[X] = t[X] \end{array} \right\} \implies t_1[X] = t_2[X] = t[X]$$

$$\left. \begin{array}{l} r \vDash X \to Y \\ t_1[X] = t_2[X] = t[X] \end{array} \right\} \implies t_1[Y] = t_2[Y] = t[Y]$$

$$\left. \begin{array}{l} t_2[Y] = t[Y] \\ t_2[X(u \setminus Y)] = t[X(u \setminus Y)] \end{array} \right\} \implies \left. \begin{array}{l} t_2 = t \\ t_2 \in r \end{array} \right\} \implies \underline{t \in r}$$

У општем случају:

$$\pi_{XY}(r) \bowtie \pi_{X(u \setminus Y)}(r) = r \nRightarrow r \vDash X \to Y$$

Багатозначна залежності

$$X \twoheadrightarrow Y, \quad X, Y \subseteq U$$

$$r \models X \twoheadrightarrow Y$$

$$(\forall u, v \in r)( u[X] = v[X] \Rightarrow$$

$$(\exists t \in r)( u[XY] = t[XY] \land$$

$$v[X(U \setminus Y)] = t[X(U \setminus Y)] )$$

$$)$$

| $r_1$ | A | B | C |
|-------|-----|-----|-----|
| | $a_1$ | $b_1$ | $c_1$ |
| | $a_1$ | $b_2$ | $c_2$ |

$$r_1 \not\models A \twoheadrightarrow B$$

$$r_1 \not\models A \twoheadrightarrow C$$

| $r_2$ | A | B | C |
|-------|-----|-----|-----|
| | $a_1$ | $b_1$ | $c_1$ |
| | $a_1$ | $b_2$ | $c_2$ |
| | $a_1$ | $b_1$ | $c_2$ |
| | $a_1$ | $b_2$ | $c_1$ |

$$r_2 \models A \twoheadrightarrow B$$

$$r_2 \models A \twoheadrightarrow C$$

T: $\quad r \models X \twoheadrightarrow Y \iff \pi_{XY}(r) \bowtie \pi_{X(U \setminus Y)}(r) = r$

dem:

($\Rightarrow$)

$r \vDash X \twoheadrightarrow Y \Rightarrow \pi_{XY}(r) \bowtie \pi_{X(u \setminus y)}(r) = r$

$\pi_{XY}(r) \bowtie \pi_{X(u \setminus y)}(r) \subseteq r$

$t \in \pi_{XY}(r) \bowtie \pi_{X(u \setminus y)}(r) \Rightarrow$

$t[XY] \in \pi_{XY}(r) \wedge t[X(u \setminus y)] \in \pi_{X(u \setminus y)}(r) \Rightarrow$

$(\exists t_1 \in r)(t_1[XY] = t[XY]) \wedge$

$(\exists t_2 \in r)(t_2[X(u \setminus y)] = t[X(u \setminus y)]) \Rightarrow$

$t_1 \in r \wedge t_1[XY] = t[XY] \wedge$

$t_2 \in r \wedge t_2[X(u \setminus y)] = t[X(u \setminus y)] \Rightarrow$

$\left.\begin{array}{l} t_1[X] = t_2[X] = t[X] \\ r \vDash X \twoheadrightarrow Y \end{array}\right\} \Rightarrow$

$(\exists u \in r)( u[XY] = t_1[XY] \wedge$
$\qquad\qquad u[X(u \setminus y)] = t_2[X(u \setminus y)])$

$u \in r \wedge u[XY] = t_1[XY] \wedge$
$\qquad\qquad u[X(u \setminus y)] = t_2[X(u \setminus y)]$

$\left.\begin{array}{l} u[XY] = t_1[XY] \\ t[XY] = t_1[XY] \end{array}\right\} = u[XY] = t[XY]$

$\left.\begin{array}{l} u[X(u \setminus y)] = t_2[X(u \setminus y)] \\ t[X(u \setminus y)] = t_2[X(u \setminus y)] \end{array}\right\} \Rightarrow u[X(u \setminus y)] = t[X(u \setminus y)]$

$$u[xy] = t[xy]$$

$$u[x(u\setminus y)] = t[x(u\setminus y)] \Bigg\} \Rightarrow \begin{array}{c} u = t \\ u \in r \end{array} \Bigg\} \Rightarrow \underline{\underline{t \in r}}$$

$$(\Leftarrow)\ \pi_{xy}(r) \bowtie \pi_{x(u\setminus y)}(r) = r \Rightarrow r \models x \twoheadrightarrow y$$

$$(\forall u, v \in r)(\ u[x] = v[x] \Rightarrow$$

$$(\exists t \in r)(\ u[xy] = t[xy] \wedge$$

$$v[x(u\setminus y)] = t[x(u\setminus y)]))$$

$$u, v \in r, \quad u[x] = v[x]$$

$$u \in r \Rightarrow \quad u[xy] \in \pi_{xy}(r)$$

$$v \in r \Rightarrow \quad v[x(u\setminus y)] \in \pi_{x(u\setminus y)}(r) \Bigg\} \Rightarrow$$

$$u[x] = v[x]$$

$$u[xy]\, v[x(u\setminus y)] \in \pi_{xy}(r) \bowtie \pi_{x(u\setminus y)}(r) \Big\}$$

$$\pi_{xy}(r) \bowtie \pi_{x(u\setminus y)}(r) = r \Bigg\} \Rightarrow$$

$$\underbrace{u[xy]\, v[x(u\setminus y)] \in r}_{t}, \qquad t \in r$$

$$t[xy] = u[xy]$$

$$t[x(u\setminus y)] = v[x(u\setminus y)] \Bigg\} \Rightarrow \text{доказано}$$
$$\text{лаврјеие}$$
$$t \in r \qquad\qquad (\exists t \in r)(\dots)$$

$$T: \quad x \twoheadrightarrow y \underset{\not\Leftarrow}{\overset{\not\Rightarrow}{\Rightarrow}} x \twoheadrightarrow y\ ?$$

# Armstrongova pravila izvođenja za fd i mvd:

1. $Y \subseteq X \vdash X \to Y$

2. $X \to Y, \ V \subseteq W \vdash XW \to YV$

3. $X \to Y, \ Y \to Z \vdash X \to Z$

4. $X \twoheadrightarrow Y \vdash X \twoheadrightarrow U \setminus XY$

5. $X \twoheadrightarrow Y, \ V \subseteq W \vdash XW \twoheadrightarrow YV$

6. $X \twoheadrightarrow Y, \ Y \twoheadrightarrow Z \vdash X \twoheadrightarrow Z \setminus Y$

7. $X \to Y \vdash X \twoheadrightarrow Y$

8. $X \twoheadrightarrow Y, \ W \to Z, \ Z \subseteq Y, \ W \cap Y = \emptyset \vdash X \to Z$

Последично:

$$X \twoheadrightarrow Y, \ X \twoheadrightarrow Z \vdash X \twoheadrightarrow YZ, \ X \twoheadrightarrow Y \setminus Z,$$
$$X \twoheadrightarrow Z \cup Y, \ X \twoheadrightarrow Z \setminus Y$$

Тривијалне mvd: akko $Y \subseteq X \ \lor \ XY = U$

Зависност споја:

$$\bowtie (X_1, \ldots, X_k)$$

$$r \models \bowtie (X_1, \ldots, X_k)$$

$$(\forall t_1, \ldots, t_k \in r)(\forall i, j \in \{1, \ldots, k\})$$

$$(t_i[X_i \cap X_j] = t_j[X_i \cap X_j] \Rightarrow (\exists t \in r)($$
$$(\forall i \in \{1, \ldots, k\})(t[X_i] = t_i[X_i]))$$

| r | A | B | C | D |
|---|---|---|---|---|
| | $a_1$ | $b_1$ | $c_1$ | $d_1$ |
| | $a_1$ | $b_1$ | $c_2$ | $d_2$ |
| | $a_2$ | $b_2$ | $c_2$ | $d_2$ |

$\bowtie(AB, BC, CD)$

| $\pi_{AB}(r)$ | A | B |
|---|---|---|
| | $a_1$ | $b_1$ |
| | $a_2$ | $b_2$ |

| $\pi_{BC}(r)$ | B | C |
|---|---|---|
| | $b_1$ | $c_1$ |
| | $b_1$ | $c_2$ |
| | $b_2$ | $c_2$ |

| $\pi_{CD}(r)$ | C | D |
|---|---|---|
| | $c_1$ | $d_1$ |
| | $c_2$ | $d_2$ |

| $\pi_{AB}(r) \bowtie \pi_{BC}(r)$ | A | B | C |
|---|---|---|---|
| | $a_1$ | $b_1$ | $c_1$ |
| | $a_1$ | $b_1$ | $c_2$ |
| | $a_2$ | $b_2$ | $c_2$ |

| $\pi_{AB}(r) \bowtie \pi_{BC}(r) \bowtie \pi_{CD}(r)$ | A | B | C | D |
|---|---|---|---|---|
| | $a_1$ | $b_1$ | $c_1$ | $d_1$ |
| | $a_1$ | $b_1$ | $c_2$ | $d_2$ |
| | $a_2$ | $b_2$ | $c_2$ | $d_2$ |

$$X_1 \cup .. \cup X_k = \mathcal{U}$$
$$r \models \bowtie (X_1, ..., X_k) \text{ akko } \pi_{X_1}(r) . \bowtie .. \bowtie \pi_{X_k}(r) = r$$
$$X \twoheadrightarrow Y \Leftrightarrow \bowtie(XY, X(\mathcal{U} \setminus Y))$$