r	A	B	C
	a_1	b_1	c_1
	a_1	b_2	c_2

$\pi_{AB}(r)$	A	B
	a_1	b_1
	a_1	b_2

$\pi_{AC}(r)$	A	C
	a_1	c_1
	a_1	c_2

$\pi_{AB}(r) \bowtie \pi_{AC}(r)$	A	B	C
	a_1	b_1	c_1
	a_1	b_1	c_2
	a_1	b_2	c_1
	a_1	b_2	c_2

$$\pi_{AB}(r) \bowtie \pi_{AC}(r) \neq r$$

$r(U)$, $\quad R_1, R_2 \subseteq U$, $\quad R_1 R_2 = U$

$$\pi_{R_1}(r) \bowtie \pi_{R_2}(r) \neq r$$

$$\pi_{R_1}(r) \bowtie \pi_{R_2}(r) \not\subseteq r$$

$$r \subseteq \pi_{R_1}(r) \bowtie \pi_{R_2}(r) \text{ ?}$$

T:

$r(\mathcal{U}) \subseteq \pi_{R_1}(r) \bowtie \pi_{R_2}(r), \quad R_1 R_2 = \mathcal{U}$

$t \in r \Rightarrow$

$t[R_1] \in \pi_{R_1}(r) \;\wedge\; t[R_2] \in \pi_{R_2}(r) \Rightarrow$

$\underbrace{t[R_1]\, t[R_2]}_{t} \in \pi_{R_1}(r) \bowtie \pi_{R_2}(r) \Rightarrow$

$t \in \pi_{R_1}(r) \bowtie \pi_{R_2}(r) \quad \square$

$\pi_{R_1}(r) \bowtie \pi_{R_2}(r) \subseteq r$?

r	A	B	C
	a_1	b_1	c_1
	a_1	b_1	c_2

$\pi_{AB}(r)$	A	B
	a_1	b_1

$\pi_{AC}(r)$	A	C
	a_1	c_1
	a_1	c_2

$\pi_{AB}(r) \bowtie \pi_{AC}(r)$	A	B	C
	a_1	b_1	c_1
	a_1	b_1	c_2

$\pi_{AB}(r) \bowtie \pi_{AC}(r) = r$

$r(\mathcal{U}) \models A \rightarrow B$!

$$r \models X \to Y \Rightarrow \pi_{XY}(r) \bowtie \pi_{X(U\setminus Y)}(r) = r$$

Т: док: $\pi_{XY}(r) \bowtie \pi_{X(U\setminus Y)}(r) \subseteq r$

$$(\forall t)(t \in \pi_{XY}(r) \bowtie \pi_{X(U\setminus Y)}(r) \Rightarrow t \in r)$$

$$t \in \pi_{XY}(r) \bowtie \pi_{X(U\setminus Y)}(r) \Rightarrow$$

$$t[XY] \in \pi_{XY}(r) \wedge t[X(U\setminus Y)] \in \pi_{X(U\setminus Y)}(r) \Rightarrow$$

$$(\exists t_1 \in r)(t_1[XY] = t[XY]) \wedge$$
$$(\exists t_2 \in r)(t_2[X(U\setminus Y)] = t[X(U\setminus Y)] \Rightarrow$$

$$t_1 \in r \wedge t_1[XY] = t[XY] \wedge$$
$$t_2 \in r \wedge t_2[X(U\setminus Y)] = t[X(U\setminus Y)] \Rightarrow$$

$$\left.\begin{array}{l} t_1[X] = t[X] \wedge \\ t_2[X] = t[X] \end{array}\right\} \Rightarrow t_1[X] = t_2[X] = t[X]$$

$$\left.\begin{array}{l} r \models X \to Y \\ t_1[X] = t_2[X] = t[X] \end{array}\right\} \Rightarrow t_1[Y] = t_2[Y] = t[Y]$$

$$\left.\begin{array}{l} t_2[Y] = t[Y] \\ t_2[X(U\setminus Y)] = t[X(U\setminus Y)] \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_2 = t \\ t_2 \in r \end{array}\right\} \Rightarrow \underline{\underline{t \in r}}$$

У општем случају:

$$\pi_{XY}(r) \bowtie \pi_{X(U\setminus Y)}(r) = r \not\Rightarrow r \models X \to Y$$

Вишезначна зависност

$X \twoheadrightarrow Y, \quad X, Y \subseteq U$

$r \models X \twoheadrightarrow Y$

$(\forall u, v \in r)(\, u[X] = v[X] \Rightarrow$

$\quad (\exists t \in r)(\, u[XY] = t[XY] \wedge$

$\qquad v[X(U \setminus Y)] = t[X(U \setminus Y)]\,)$

$\quad)$

r_1	A	B	C
	a_1	b_1	c_1
	a_1	b_2	c_2

$r_1 \not\models A \twoheadrightarrow B$

$r_1 \not\models A \twoheadrightarrow C$

r_2	A	B	C
	a_1	b_1	c_1
	a_1	b_2	c_2
	a_1	b_1	c_2
	a_1	b_2	c_1

$r_2 \models A \twoheadrightarrow B$

$r_2 \models A \twoheadrightarrow C$

T: $\; r \models X \twoheadrightarrow Y \iff \pi_{XY}(r) \bowtie \pi_{X(U \setminus Y)}(r) = r$

Dem:

$(\Rightarrow)$

$$r \models X \twoheadrightarrow Y \Rightarrow \pi_{XY}(r) \bowtie \pi_{X(U\setminus Y)}(r) = r$$

$$\pi_{XY}(r) \bowtie \pi_{X(U\setminus Y)}(r) \subseteq r$$

$$t \in \pi_{XY}(r) \bowtie \pi_{X(U\setminus Y)}(r) \Rightarrow$$

$$t[XY] \in \pi_{XY}(r) \wedge t[X(U\setminus Y)] \in \pi_{X(U\setminus Y)}(r) \Rightarrow$$

$$(\exists t_1 \in r)(t_1[XY] = t[XY]) \wedge$$
$$(\exists t_2 \in r)(t_2[X(U\setminus Y)] = t[X(U\setminus Y)]) \Rightarrow$$

$$t_1 \in r \wedge t_1[XY] = t[XY] \wedge$$
$$t_2 \in r \wedge t_2[X(U\setminus Y)] = t[X(U\setminus Y)] \Rightarrow$$

$$\left.\begin{array}{l} t_1[X] = t_2[X] = t[X] \\ r \models X \twoheadrightarrow Y \end{array}\right\} \Rightarrow$$

$$(\exists u \in r)(u[XY] = t_1[XY] \wedge u[X(U\setminus Y)] = t_2[X(U\setminus Y)])$$

$$u \in r \wedge u[XY] = t_1[XY] \wedge u[X(U\setminus Y)] = t_2[X(U\setminus Y)]$$

$$\left.\begin{array}{l} u[XY] = t_1[XY] \\ t[XY] = t_1[XY] \end{array}\right\} = u[XY] = t[XY]$$

$$\left.\begin{array}{l} u[X(U\setminus Y)] = t_2[X(U\setminus Y)] \\ t[X(U\setminus Y)] = t_2[X(U\setminus Y)] \end{array}\right\} \Rightarrow u[X(U\setminus Y)] = t[X(U\setminus Y)]$$

$$\left.\begin{array}{l} u[XY] = t[XY] \\ u[X(U\setminus Y)] = t[X(U\setminus Y)] \end{array}\right\} \Rightarrow \left.\begin{array}{l} u = t \\ u \in r \end{array}\right\} \Rightarrow \underline{\underline{t \in r}}$$

$(\Leftarrow)$ $\pi_{XY}(r) \bowtie \pi_{X(U\setminus Y)}(r) = r \Rightarrow r \models X \twoheadrightarrow Y$

$$(\forall u, v \in r)\big(u[X] = v[X] \Rightarrow (\exists t \in r)(u[XY] = t[XY] \wedge v[X(U\setminus Y)] = t[X(U\setminus Y)])\big)$$

$u, v \in r, \quad u[X] = v[X]$

$$\left.\begin{array}{l} u \in r \Rightarrow u[XY] \in \pi_{XY}(r) \\ v \in r \Rightarrow v[X(U\setminus Y)] \in \pi_{X(U\setminus Y)}(r) \\ \qquad\qquad u[X] = v[X] \end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{l} u[XY]\, v[X(U\setminus Y)] \in \pi_{XY}(r) \bowtie \pi_{X(U\setminus Y)}(r) \\ \pi_{XY}(r) \bowtie \pi_{X(U\setminus Y)}(r) = r \end{array}\right\} \Rightarrow$$

$$\underbrace{u[XY]\, v[X(U\setminus Y)]}_{t} \in r, \qquad t \in r$$

$$\left.\begin{array}{l} t[XY] = u[XY] \\ t[X(U\setminus Y)] = v[X(U\setminus Y)] \\ t \in r \end{array}\right\} \Rightarrow \text{доказано тврђење } (\exists t \in r)(\ldots)$$

T: $X \rightarrow Y \underset{\not\Leftarrow}{\Rightarrow} X \twoheadrightarrow Y$?

Armstrongova pravila izvodenja
Za fd i mvd:

1. $Y \subseteq X \vdash X \to Y$
2. $X \to Y,\ V \subseteq W \vdash XW \to YV$
3. $X \to Y,\ Y \to Z \vdash X \to Z$
4. $X \twoheadrightarrow Y \vdash X \twoheadrightarrow U \setminus XY$
5. $X \twoheadrightarrow Y,\ V \subseteq W \vdash XW \twoheadrightarrow YV$
6. $X \twoheadrightarrow Y,\ Y \twoheadrightarrow Z \vdash X \twoheadrightarrow Z \setminus Y$
7. $X \to Y \vdash X \twoheadrightarrow Y$
8. $X \twoheadrightarrow Y,\ W \to Z,\ Z \subseteq Y,\ W \cap Y = \emptyset \vdash X \to Z$

Последично:

$$X \twoheadrightarrow Y,\ X \twoheadrightarrow Z \vdash X \twoheadrightarrow YZ,\ X \twoheadrightarrow Y \setminus Z,$$
$$X \twoheadrightarrow Z \cap Y,\ X \twoheadrightarrow Z \setminus Y$$

Зависности споја:

$$\bowtie(X_1, \dots, X_k)$$

$$r \models \bowtie(X_1, \dots, X_k)$$

$$(\forall t_1, \dots, t_k \in r)(\forall i, j \in \{1, \dots, k\})$$
$$(t_i[X_i \cap X_j] = t_j[X_i \cap X_j] \Rightarrow (\exists t \in r)($$
$$(\forall i \in \{1, \dots, k\})(t[X_i] = t_i[X_i]))$$

$\bowtie(AB, BC, CD)$

r	A	B	C	D
	a_1	b_1	c_1	d_1
	a_1	b_1	c_2	d_2
	a_2	b_2	c_2	d_2

$\pi_{AB}(r)$	A	B
	a_1	b_1
	a_2	b_2

$\pi_{BC}(r)$	B	C
	b_1	c_1
	b_1	c_2
	b_2	c_2

$\pi_{CD}(r)$	C	D
	c_1	d_1
	c_2	d_2

$\pi_{AB}(r) \bowtie \pi_{BC}(r)$	A	B	C
	a_1	b_1	c_1
	a_1	b_1	c_2
	a_2	b_2	c_2

$\pi_{AB}(r) \bowtie \pi_{BC}(r) \bowtie \pi_{CD}(r)$	A	B	C	D
	a_1	b_1	c_1	d_1
	a_1	b_1	c_2	d_2
	a_2	b_2	c_2	d_2

$X_1 \cup .. \cup X_k = \mathcal{U}$

$r \models \bowtie(X_1, \ldots, X_k)$ akko $\pi_{X_1}(r) \bowtie \ldots \bowtie \pi_{X_k}(r) =$