Osnovi Računarske Inteligencije

Neuronske Mreže i Algoritam Propagacije Unazad (*Backpropagation*)

Predavač: Aleksandar Kovačević

Slajdovi preuzeti sa CS 231n, Stanford

http://cs231n.stanford.edu/

Šta smo do sada naučili...

$$s = f(x; W) = Wx$$

skor funkcija

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

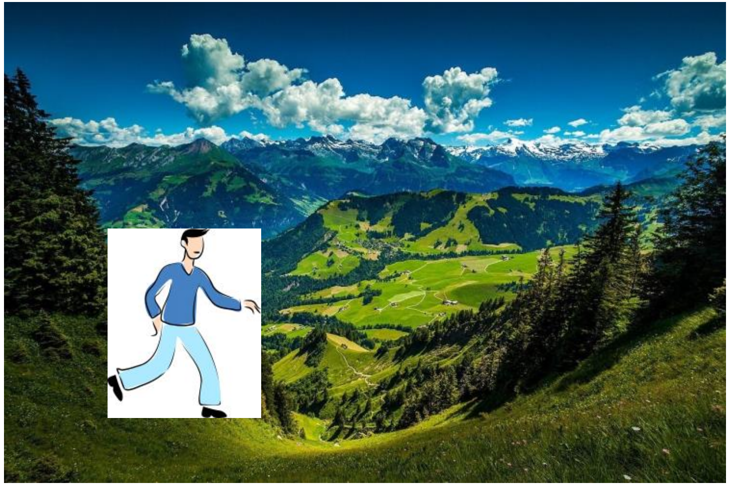
SVM funkcija greške

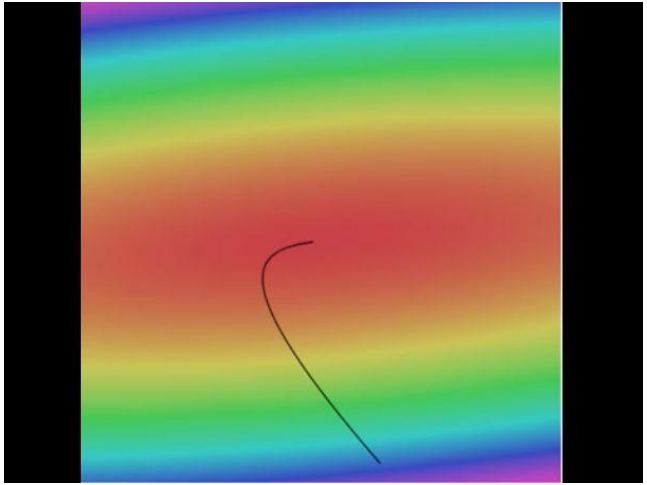
$$L = \frac{1}{N} \sum_{i=1}^{N} L_i + \sum_k W_k^2$$

greška + regularizacija

hoćemo $\boxed{\nabla_W L}$

Optimizacija





```
# Vanilla Gradient Descent

while True:
  weights_grad = evaluate_gradient(loss_fun, data, weights)
  weights += - step_size * weights_grad # perform parameter update
```

Gradijentni spust

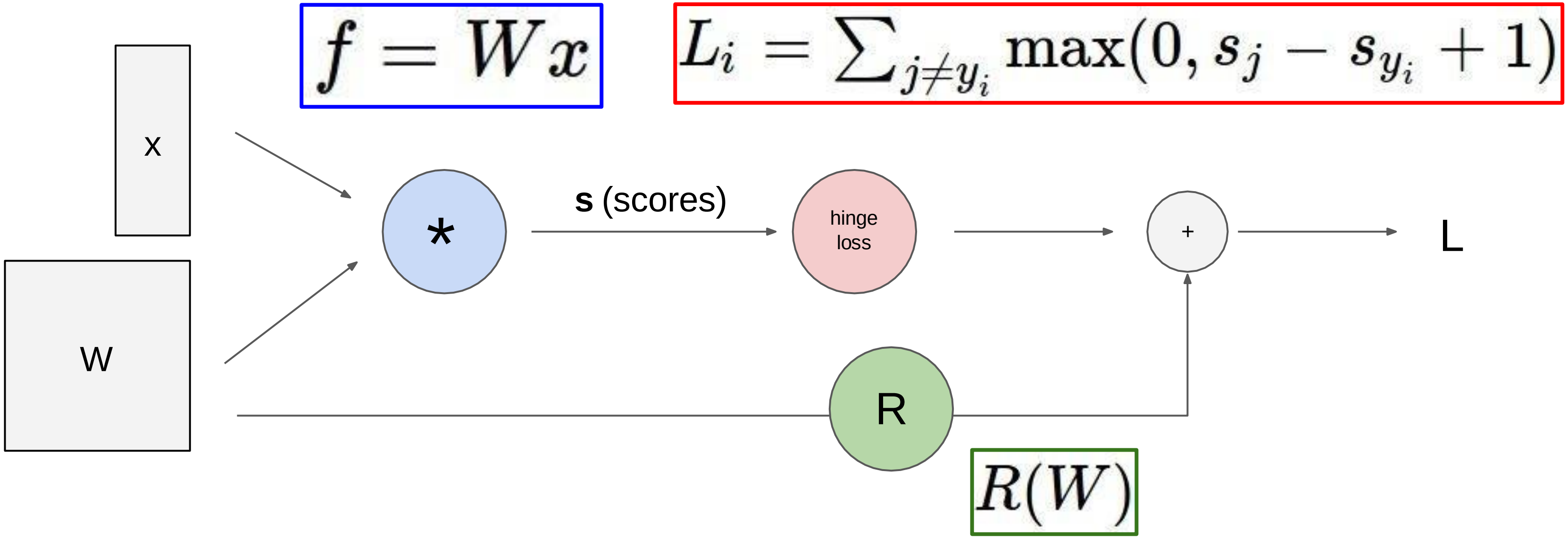
$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Numerički gradijent: spor :(, aproksimacija :(, lako se računa :)
Analitički gradijent: brz :), tačan :), teško se računa i moguće su greške :(

U praksi: Prvo simbolički izvedemo analitički gradijent, pa ga implemenitramo, a implementaciju proverimo pomoću numeričkog gradijenta

Grafovi Izračunavanja

Computational graphs

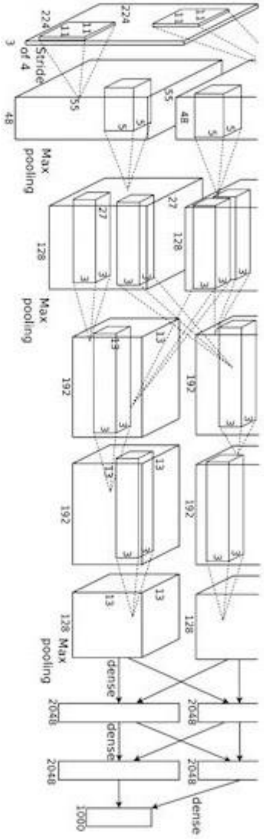


Konvolutivna Mreža (AlexNet)

ulazna slika

težine

greška



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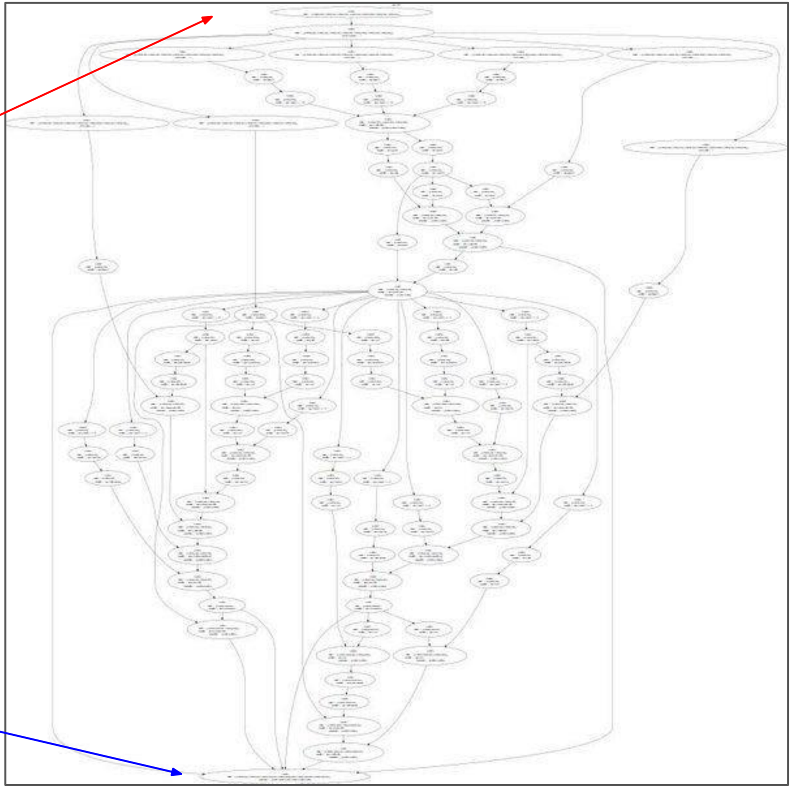
Neuronska Tjuringova Mašina

ulazna slika

greška



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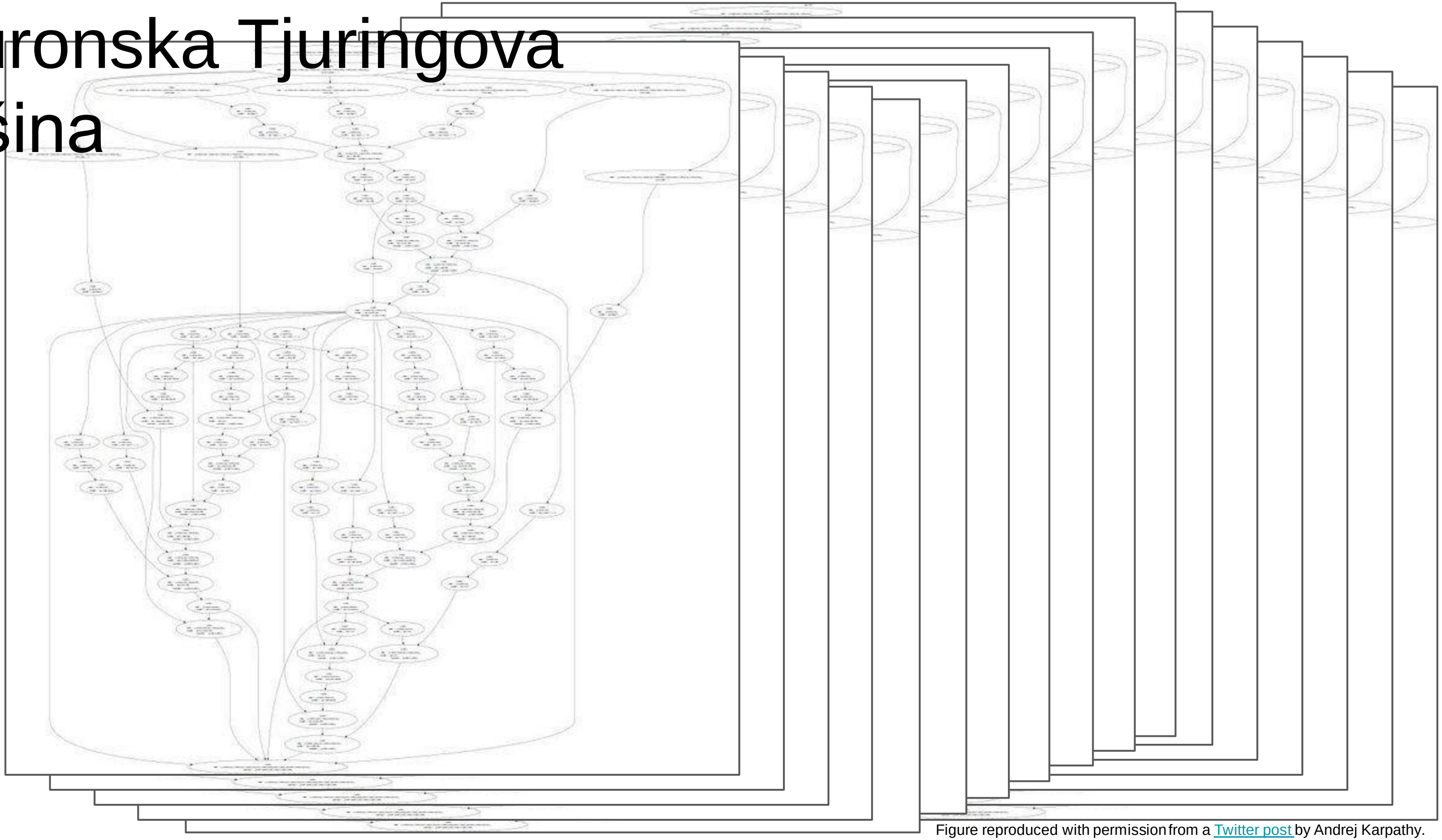
Neuronska Tjuringova Mašina



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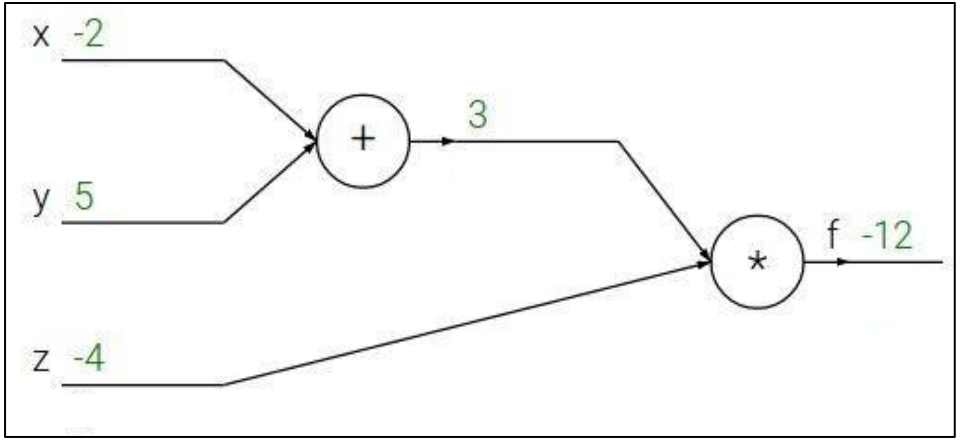
Backpropagation: jednostavan primer

$$f(x, y, z) = (x + y)z$$

npr. x = -2, y = 5, z = -4



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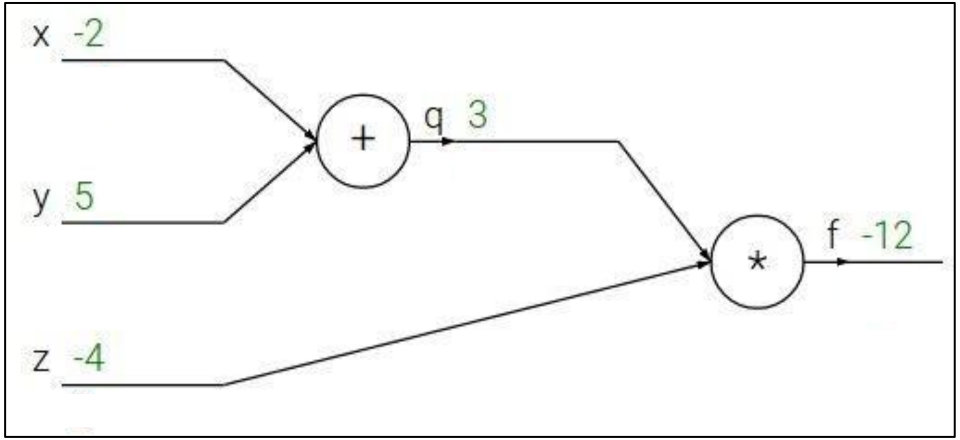

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Tražimo: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

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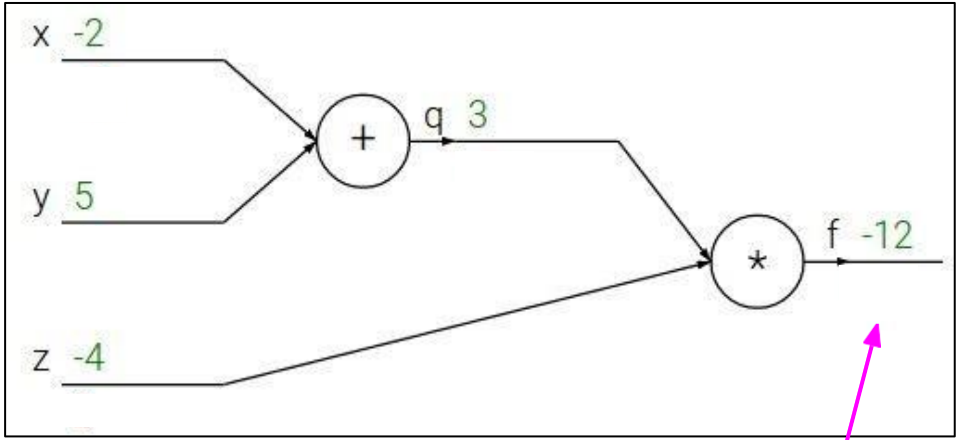

$$\frac{\partial f}{\partial f}$$

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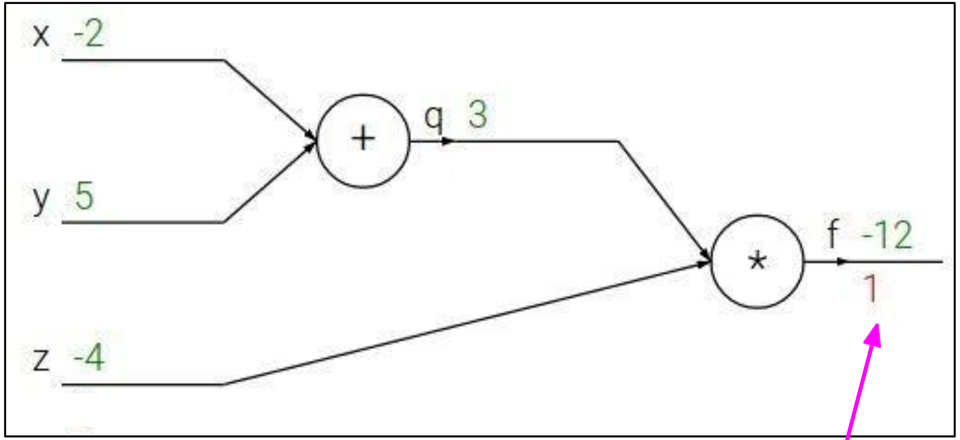

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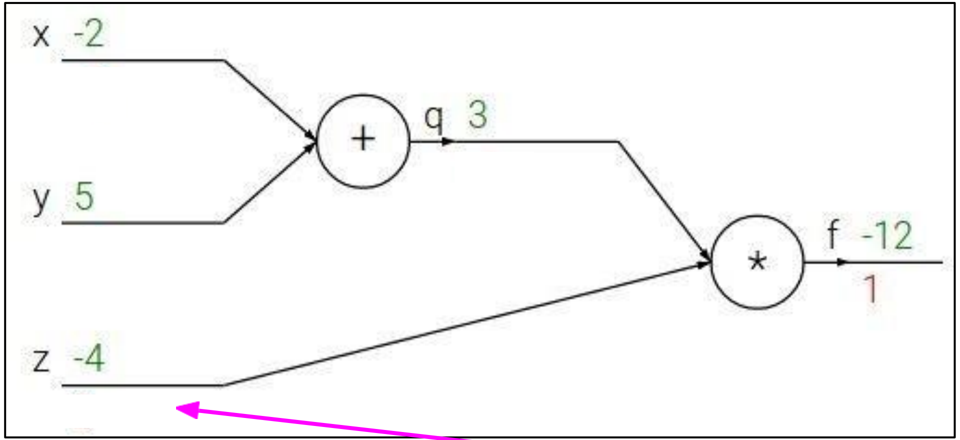

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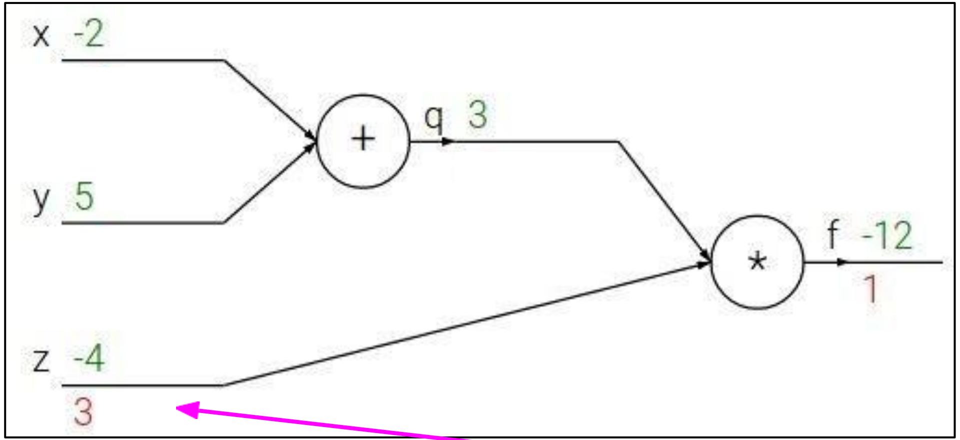

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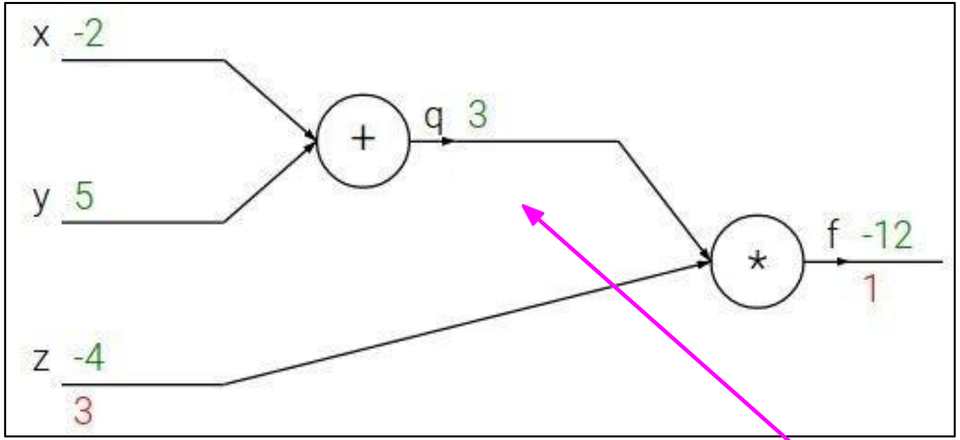

$$\frac{\partial f}{\partial q}$$

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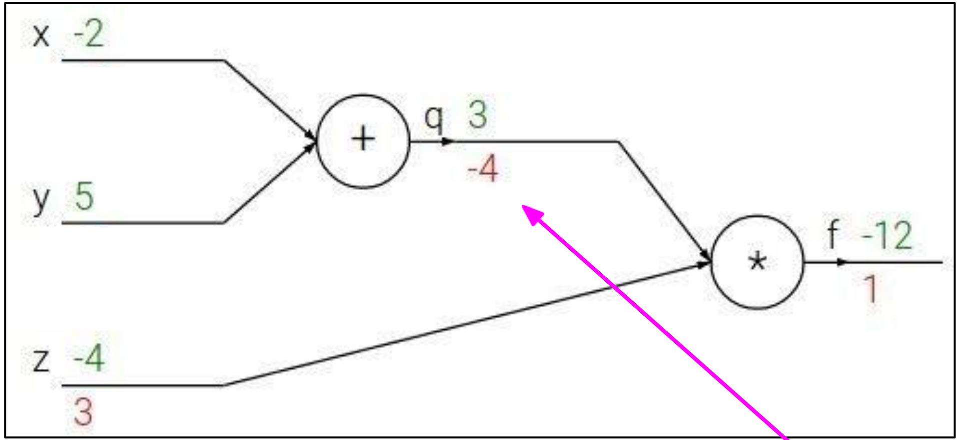

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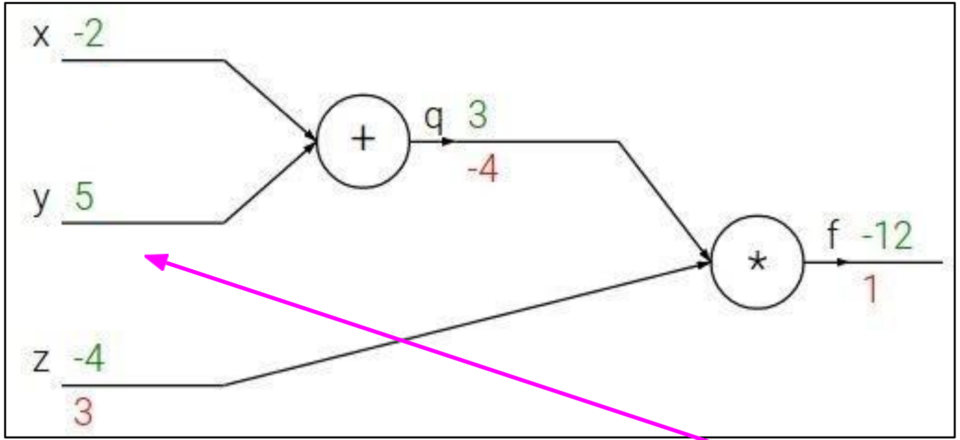

$$\frac{\partial f}{\partial y}$$

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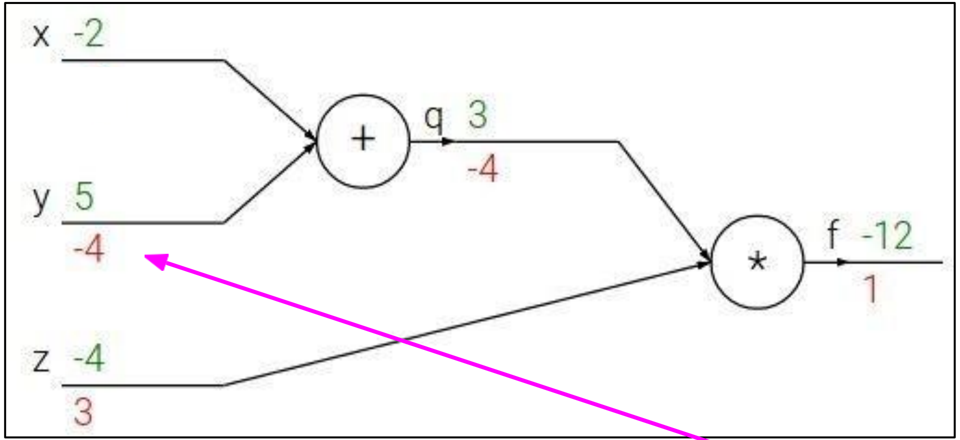

$$\frac{\partial f}{\partial y}$$

Ulačavanje izvoda:

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial y}$$

izvod složene funkcije (*Chain rule*):

Backpropagation: jednostavan primer

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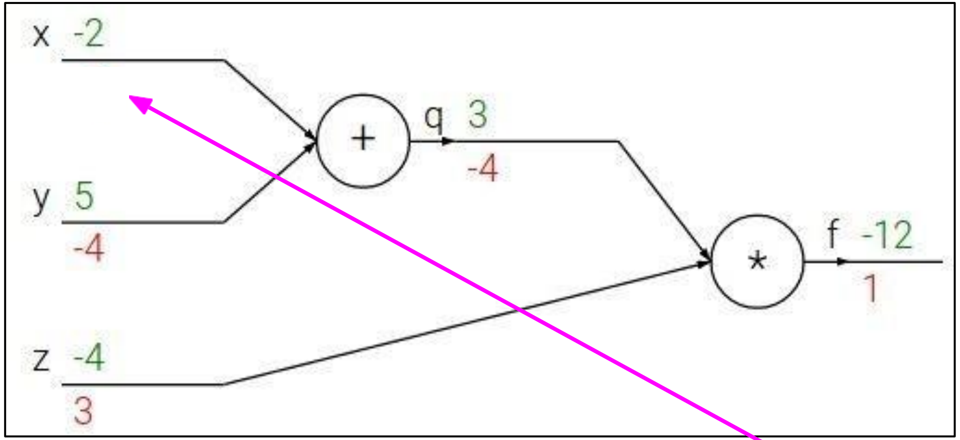

$$\frac{\partial f}{\partial x}$$

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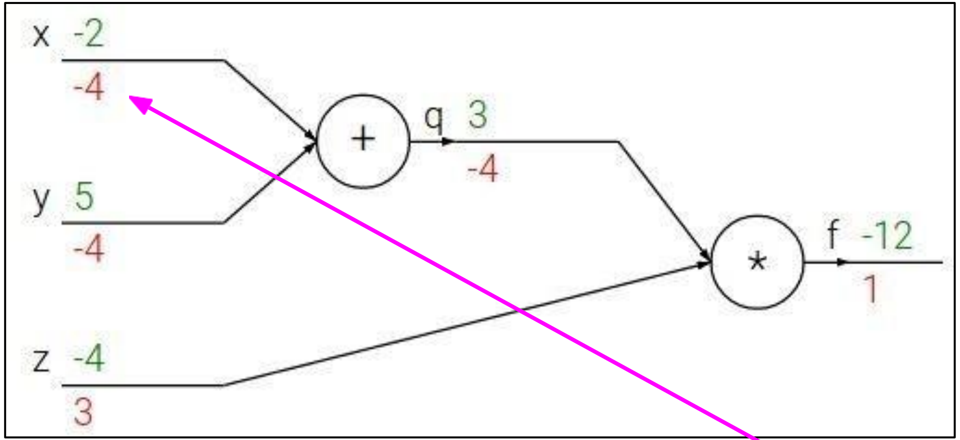
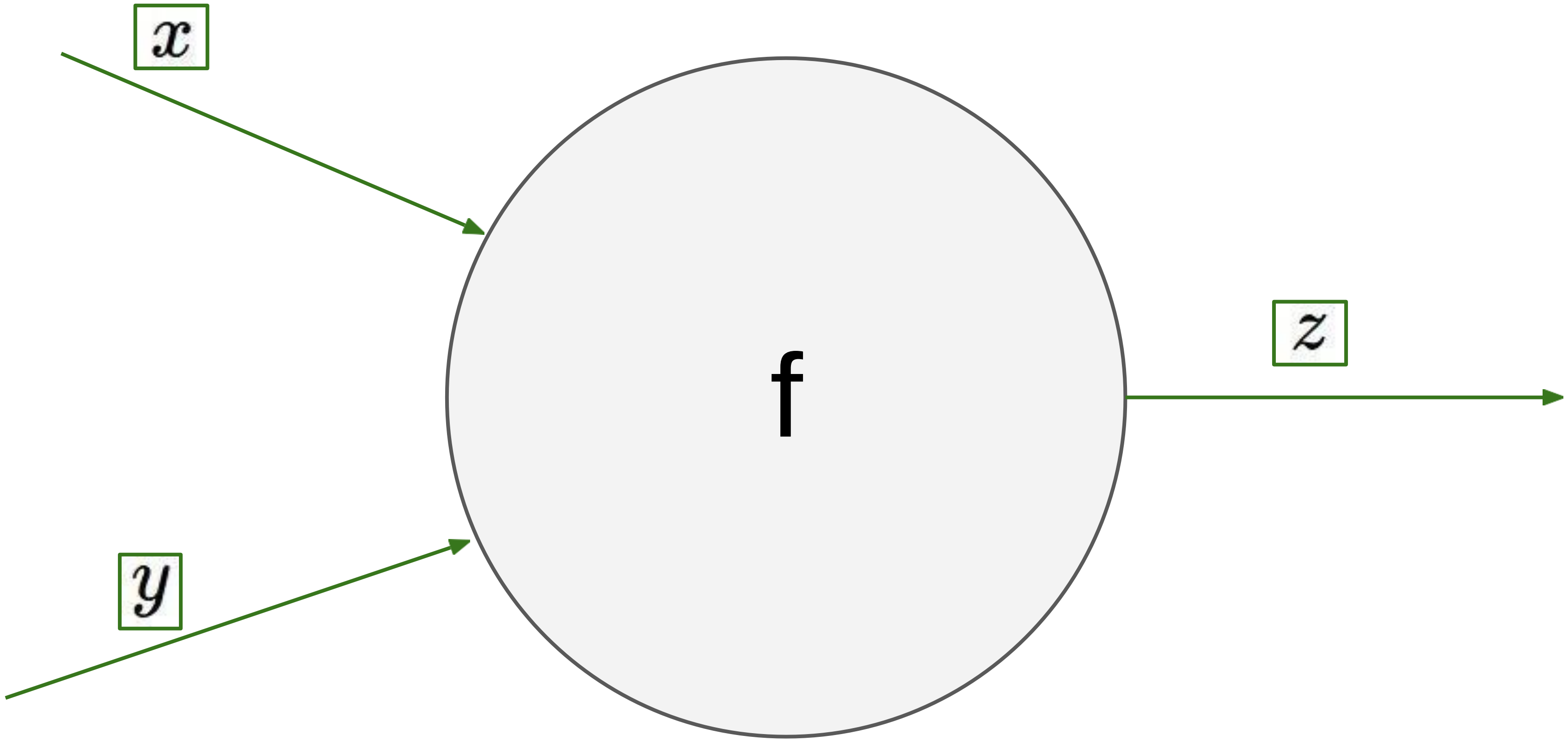
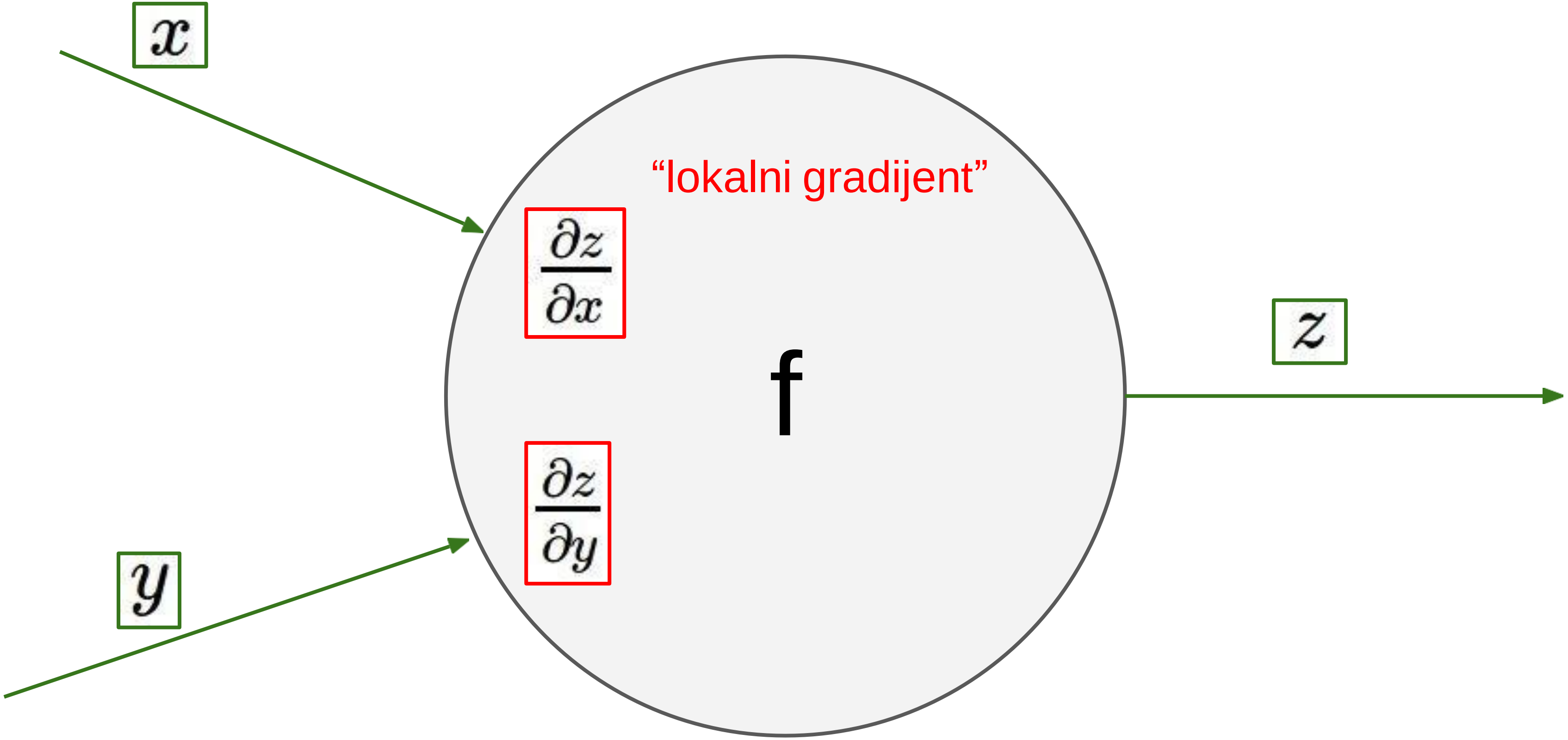


$$\frac{\partial f}{\partial x}$$

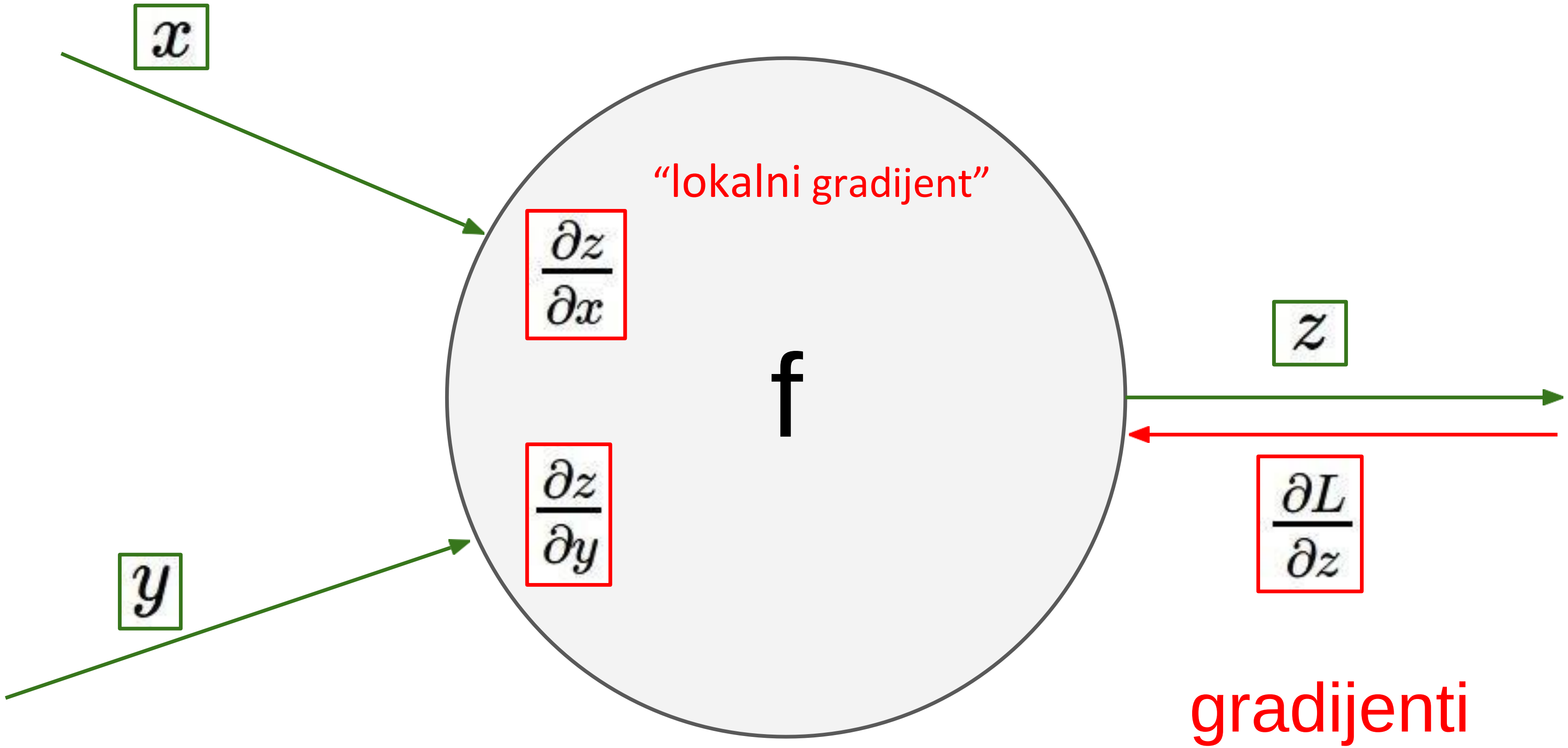
Ulačavanje izvoda:

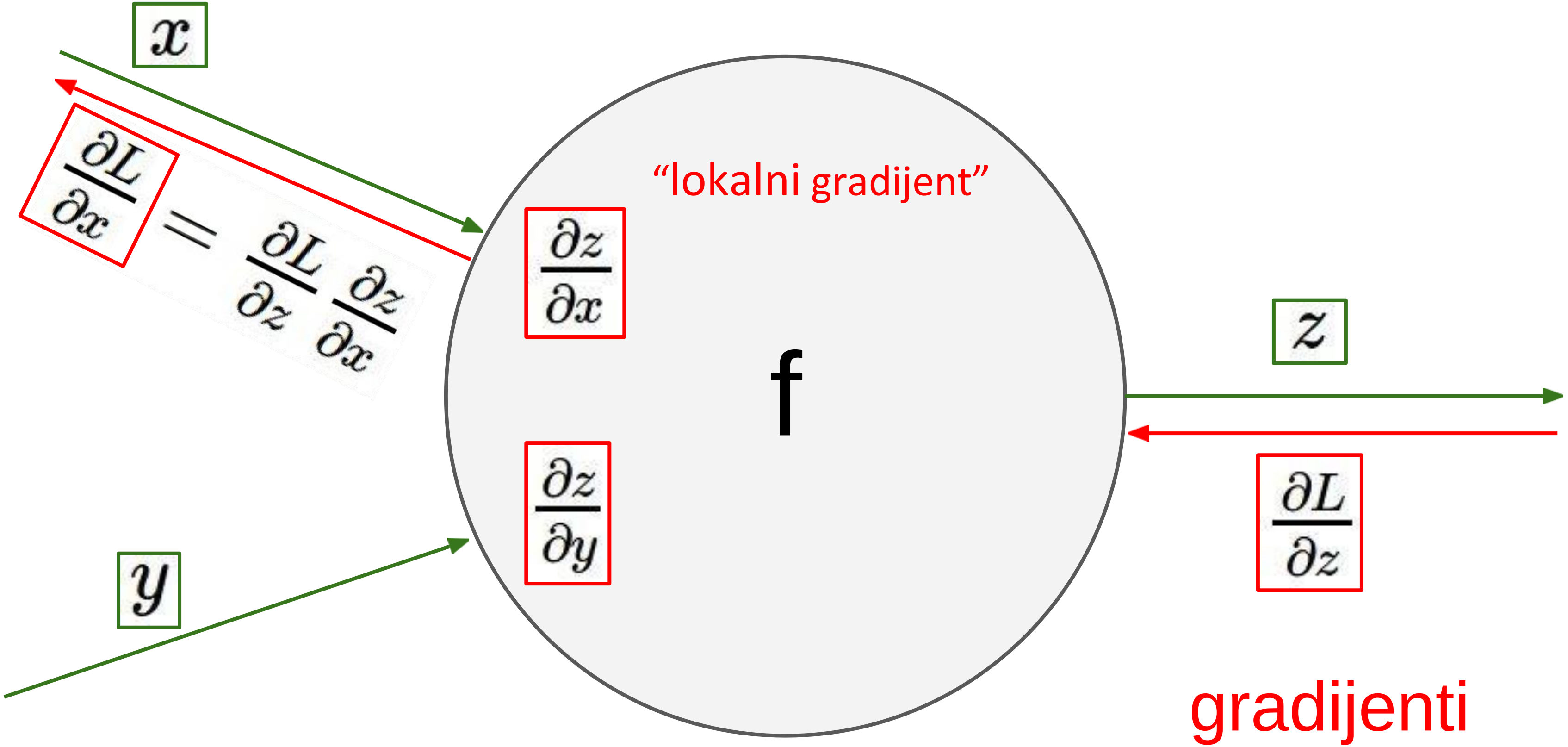
$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q}\frac{\partial q}{\partial x}$$

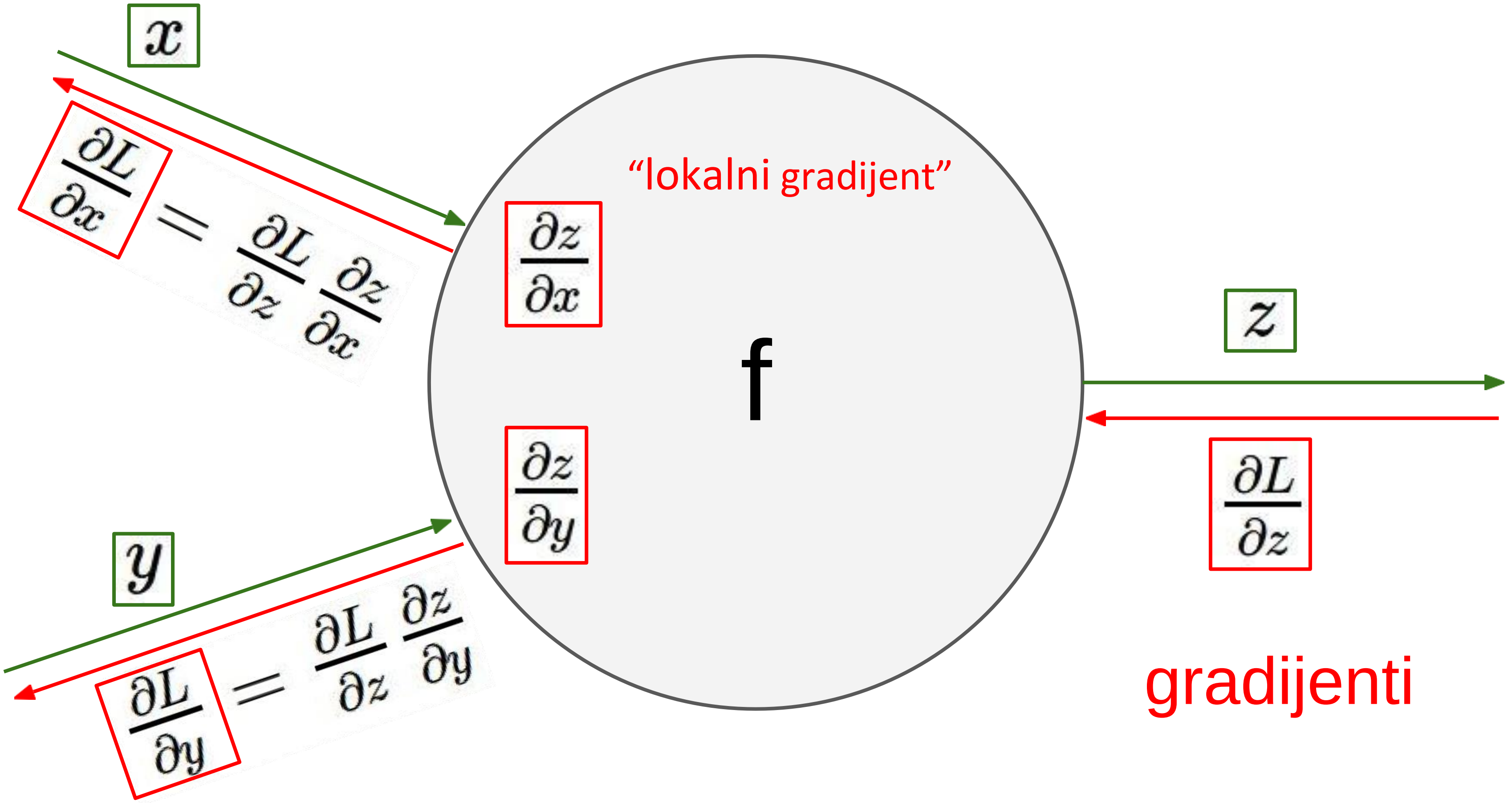

x
y
f
z



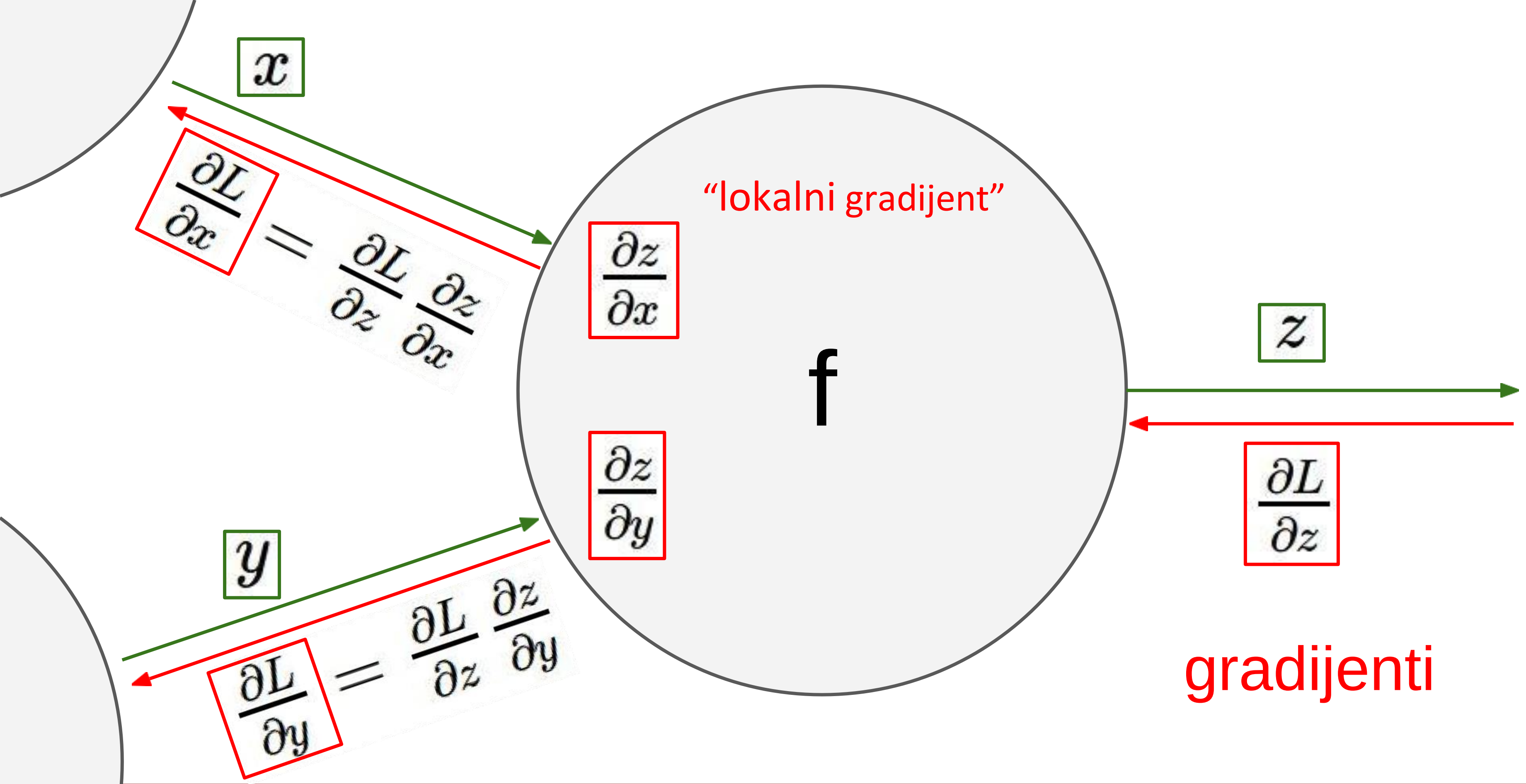
x
"lokalni gradijent"
$\frac{\partial z}{\partial x}$
f
z
$\frac{\partial z}{\partial y}$
y


x
“lokalni gradijent”
$\frac{\partial z}{\partial x}$
z
f
$\frac{\partial z}{\partial y}$
$\frac{\partial L}{\partial z}$
y
gradijenti


x
$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial x}$
"lokalni gradijent"
$\frac{\partial z}{\partial x}$
f
$\frac{\partial z}{\partial y}$
y
z
$\frac{\partial L}{\partial z}$
gradijenti



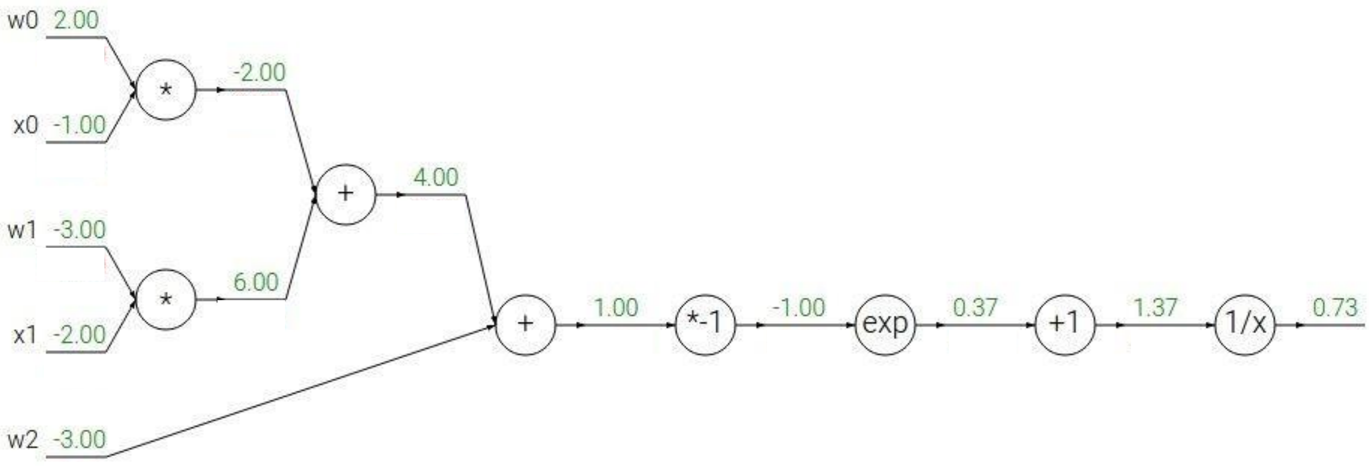
x
y
z
f
"lokalni gradijent"
$\frac{\partial z}{\partial x}$
$\frac{\partial z}{\partial y}$
$\frac{\partial L}{\partial z}$
$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial x}$
$\frac{\partial L}{\partial y} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial y}$
gradijenti



x
$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial x}$
“lokalni gradijent”
$\frac{\partial z}{\partial x}$
f
z
$\frac{\partial z}{\partial y}$
$\frac{\partial L}{\partial z}$
y
$\frac{\partial L}{\partial y} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial y}$
gradijenti

Još jedan primer:

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



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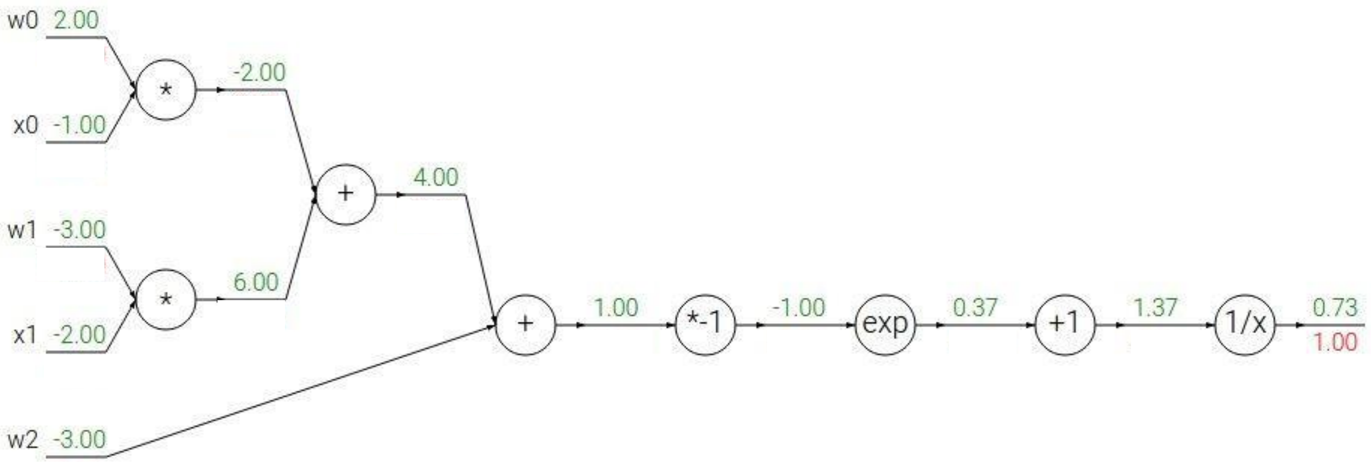

$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$$

$$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

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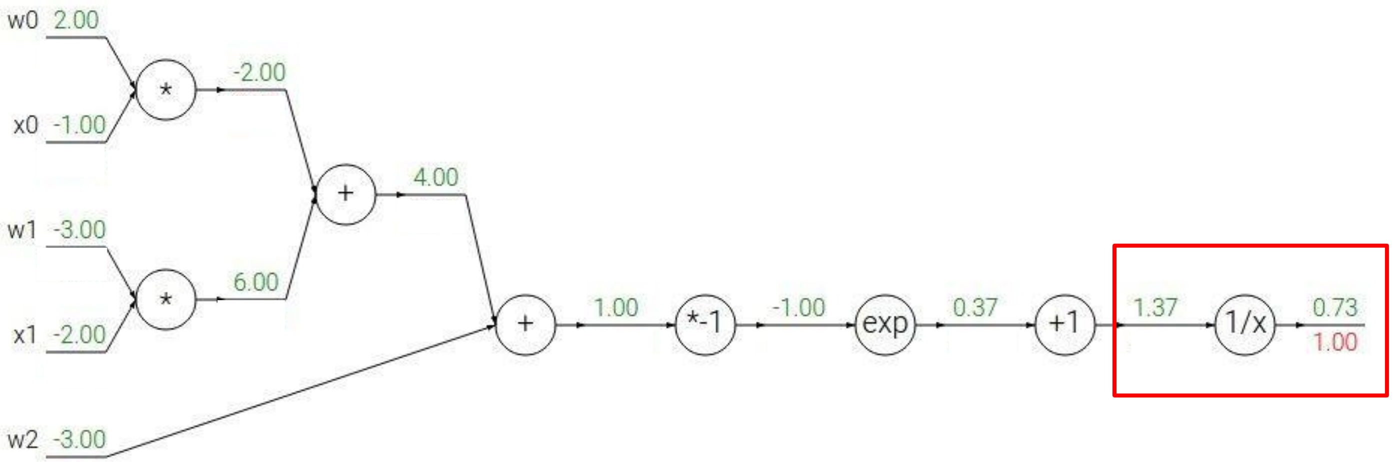

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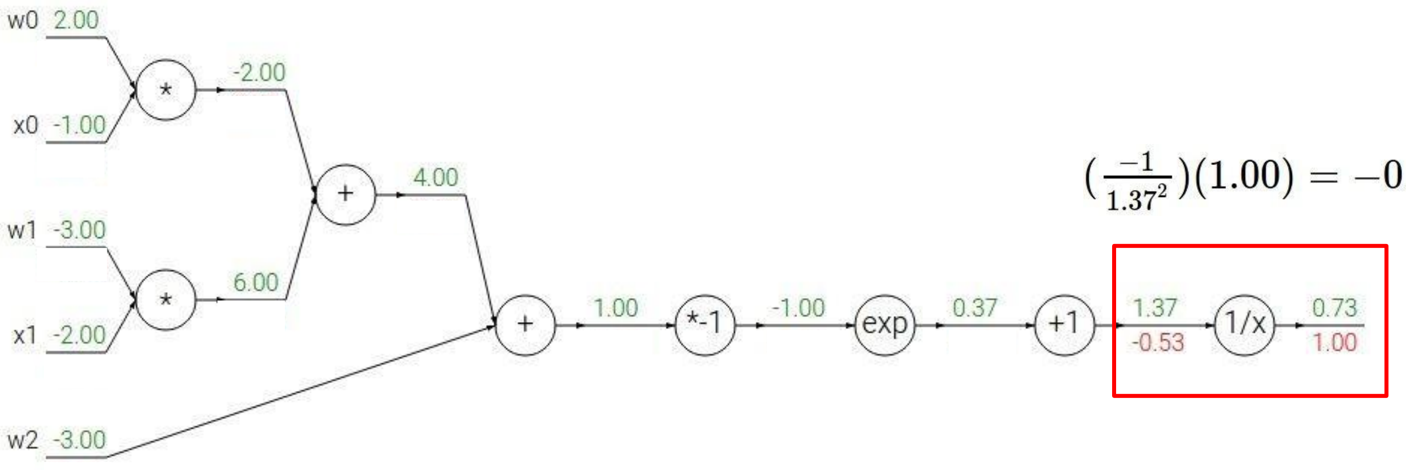

$$\left(\frac{-1}{1.37^2}\right)(1.00) = -0.53$$

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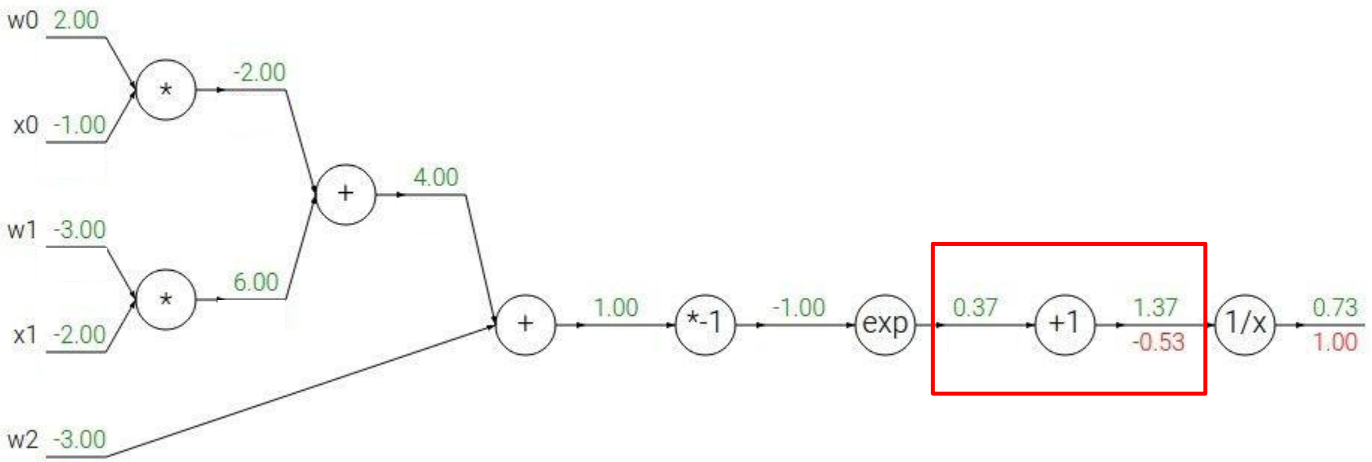

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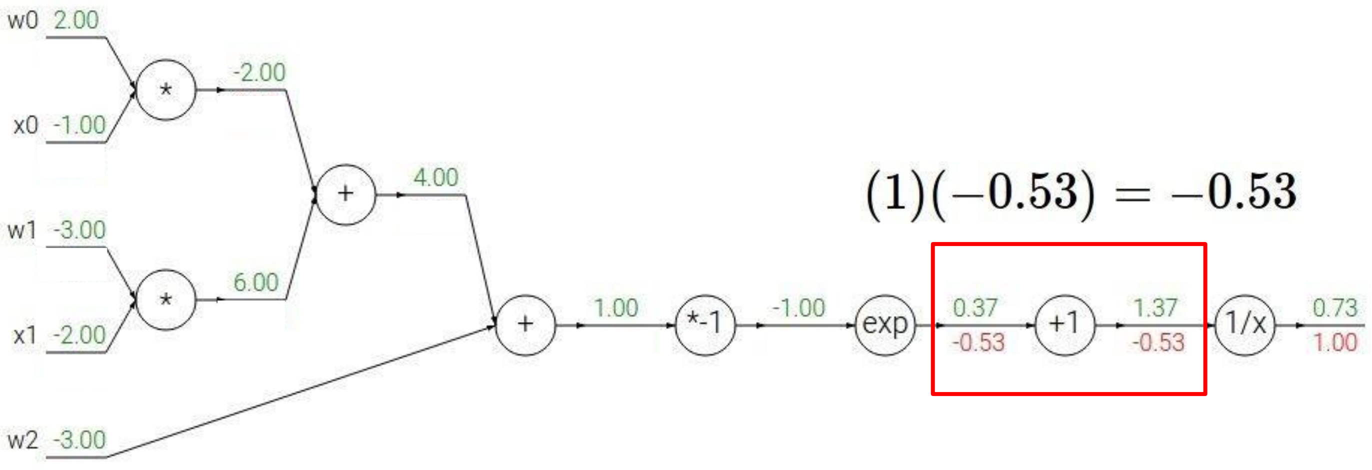

$$(1)(-0.53) = -0.53$$

$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$		$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$	
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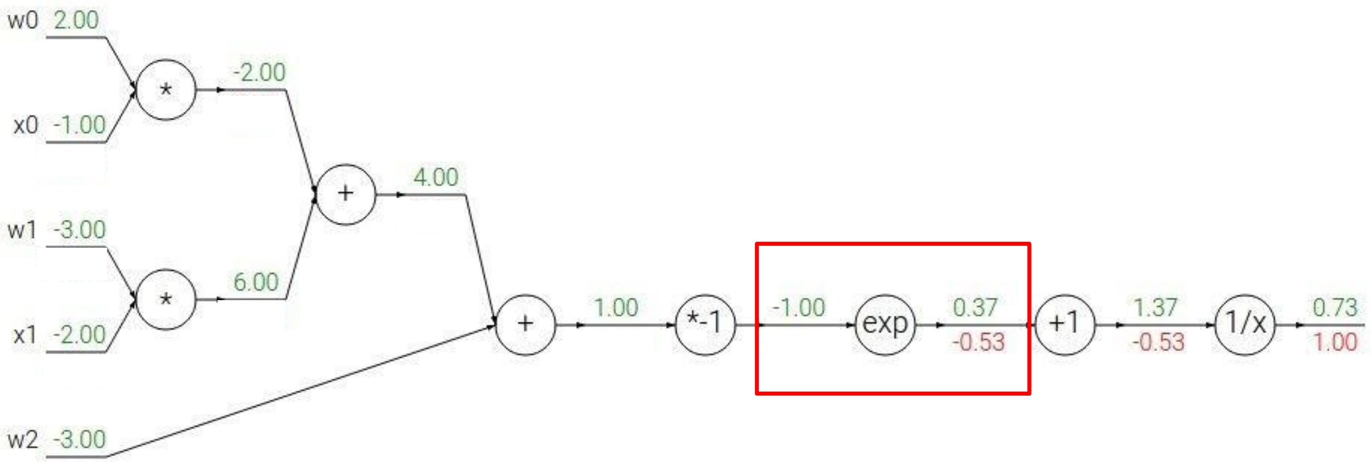

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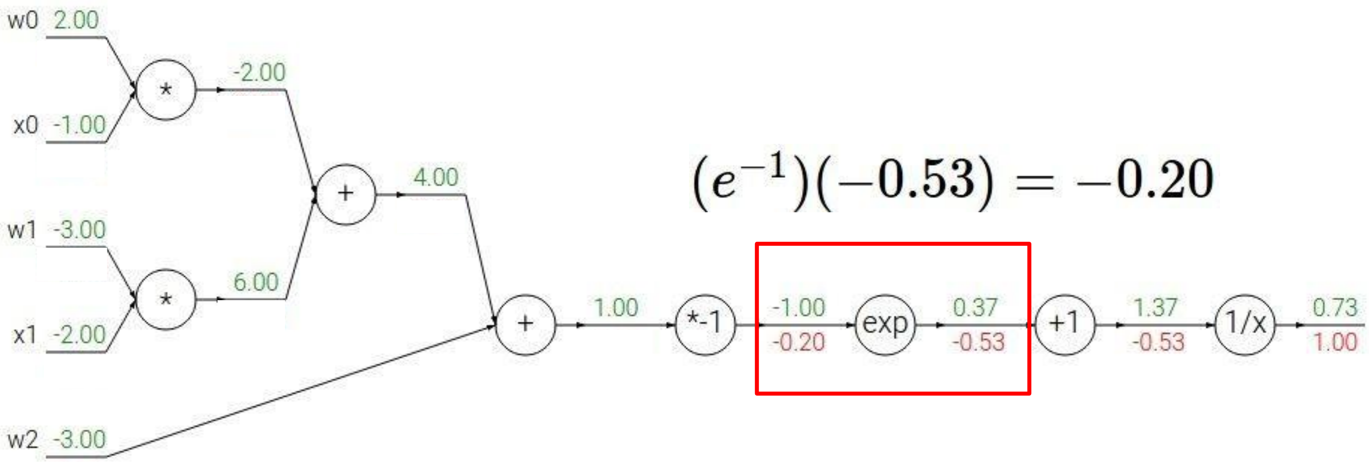

$$(e^{-1})(-0.53) = -0.20$$

$$\boxed{f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x}$$

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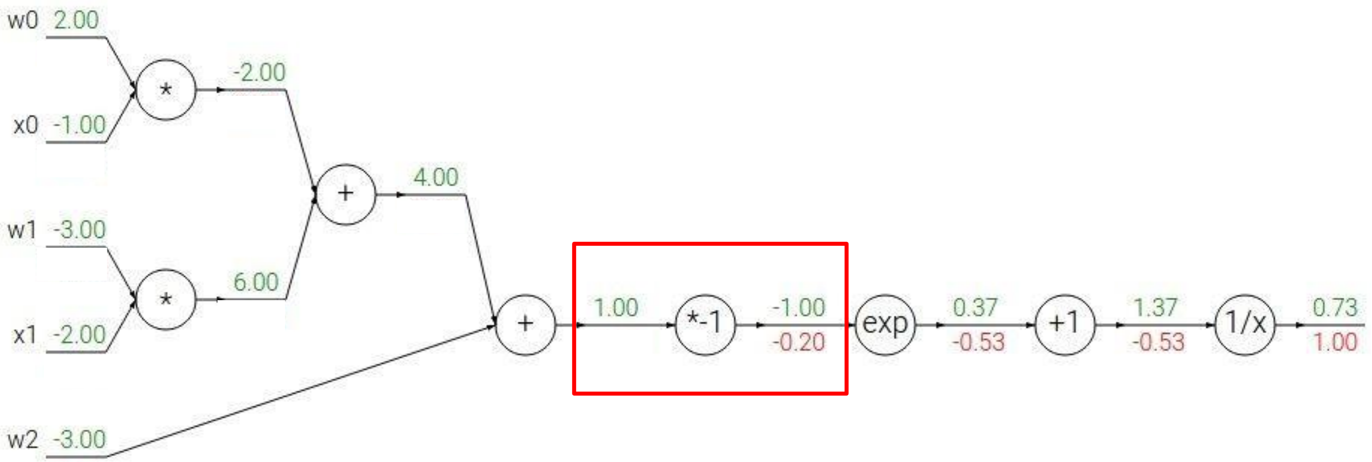

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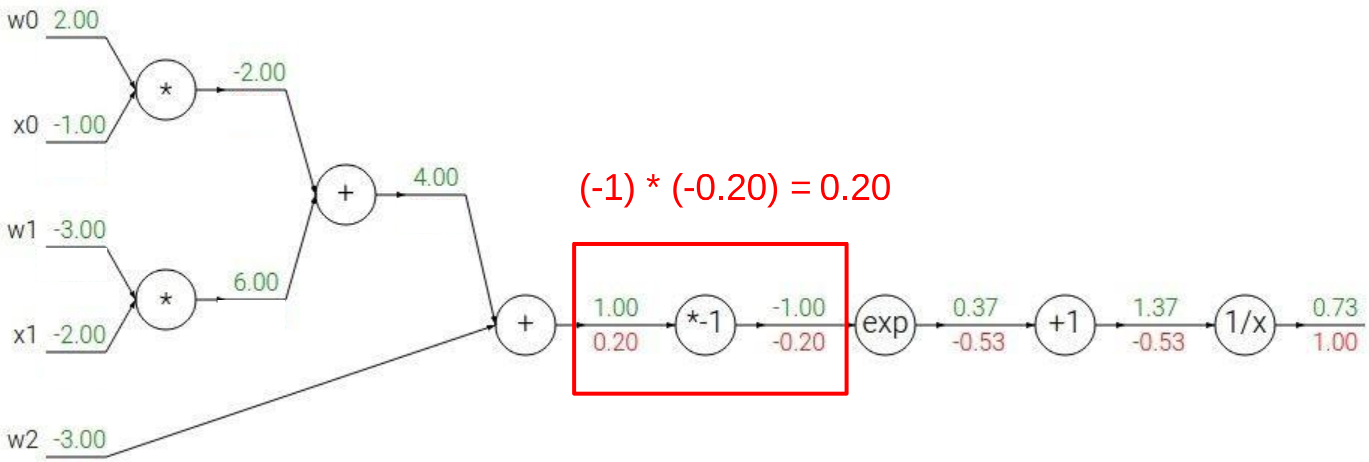

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Još jedan primer:

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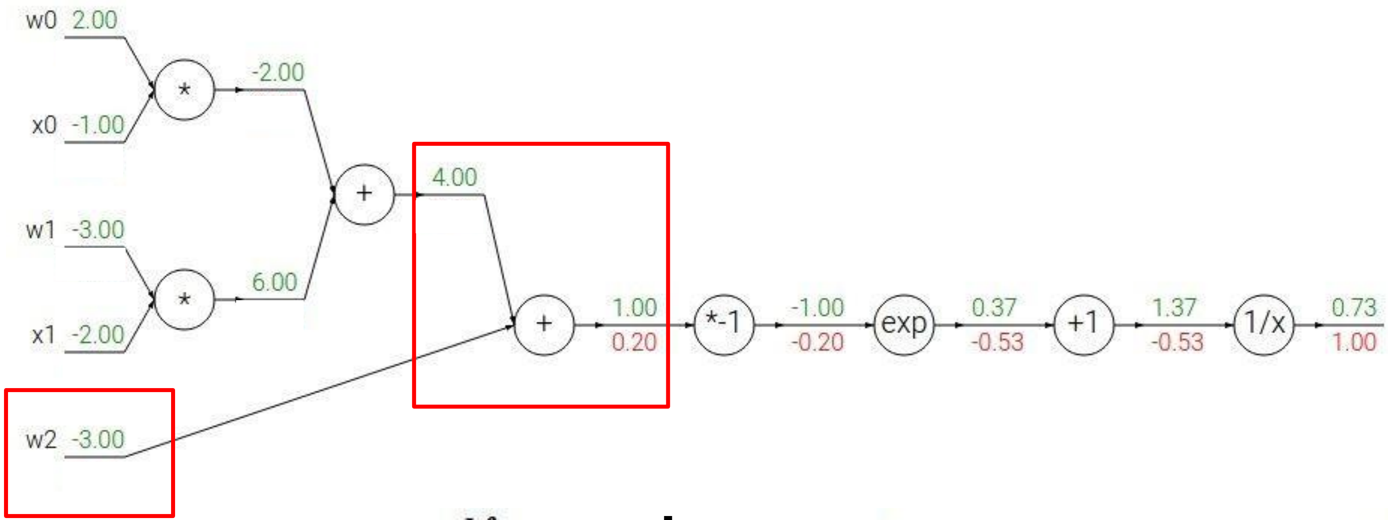


$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$$

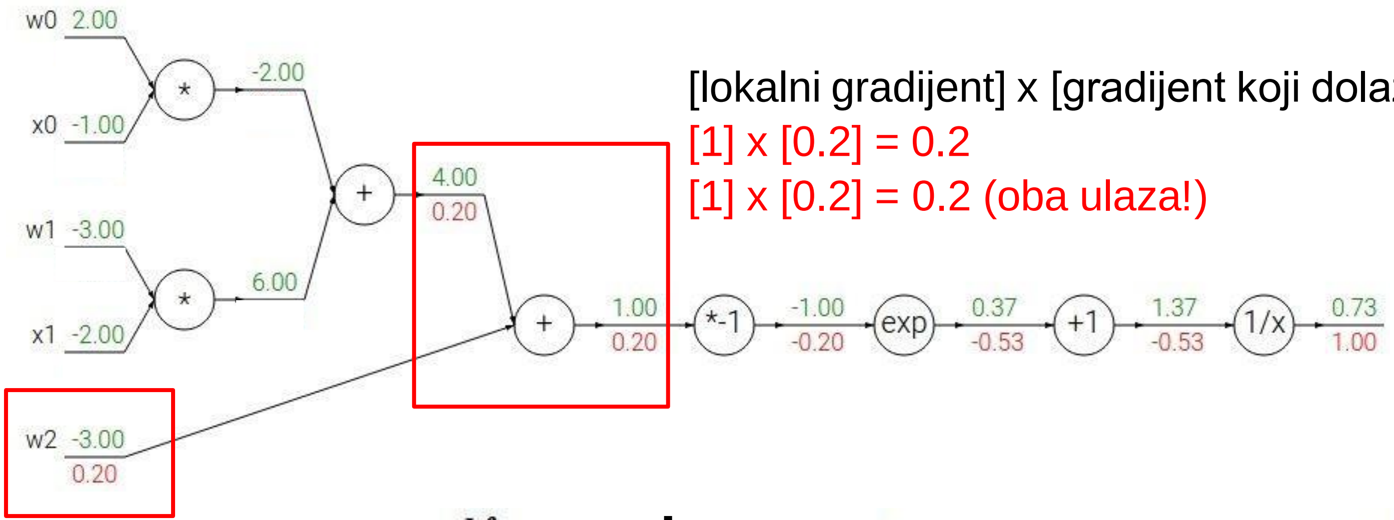
$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$$

$$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

$$f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$$

Još jedan primer:

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



[lokalni gradijent] x [gradijent koji dolazi od „gore“]

[1] x [0.2] = 0.2

[1] x [0.2] = 0.2 (oba ulaza!)

$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x \qquad\qquad f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a \qquad\qquad f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$$

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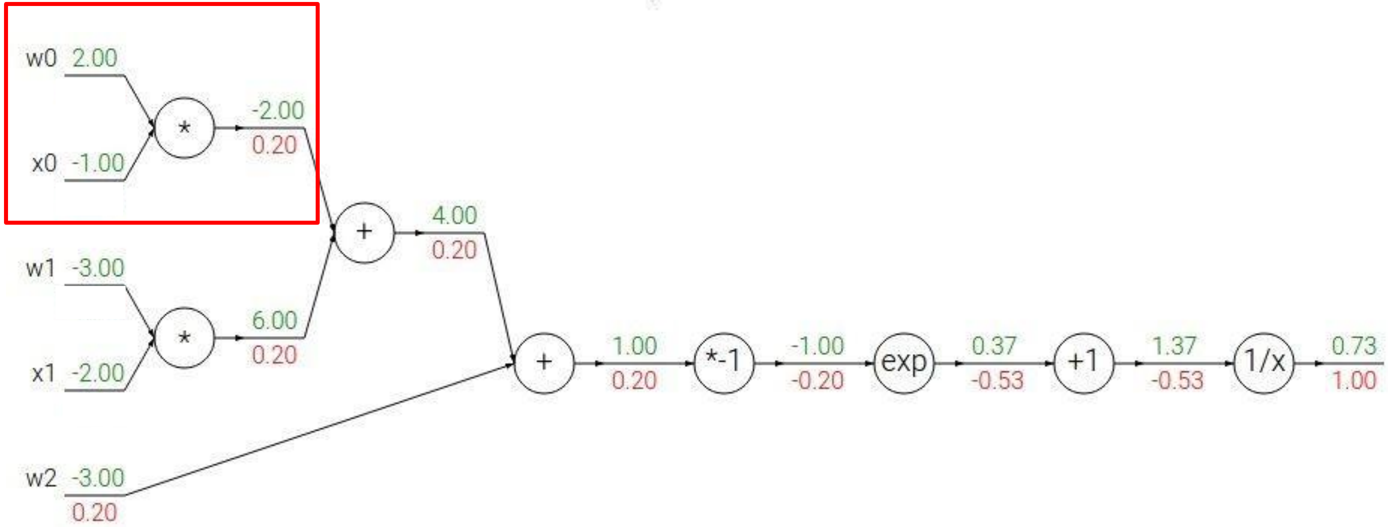

$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x \qquad\qquad f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

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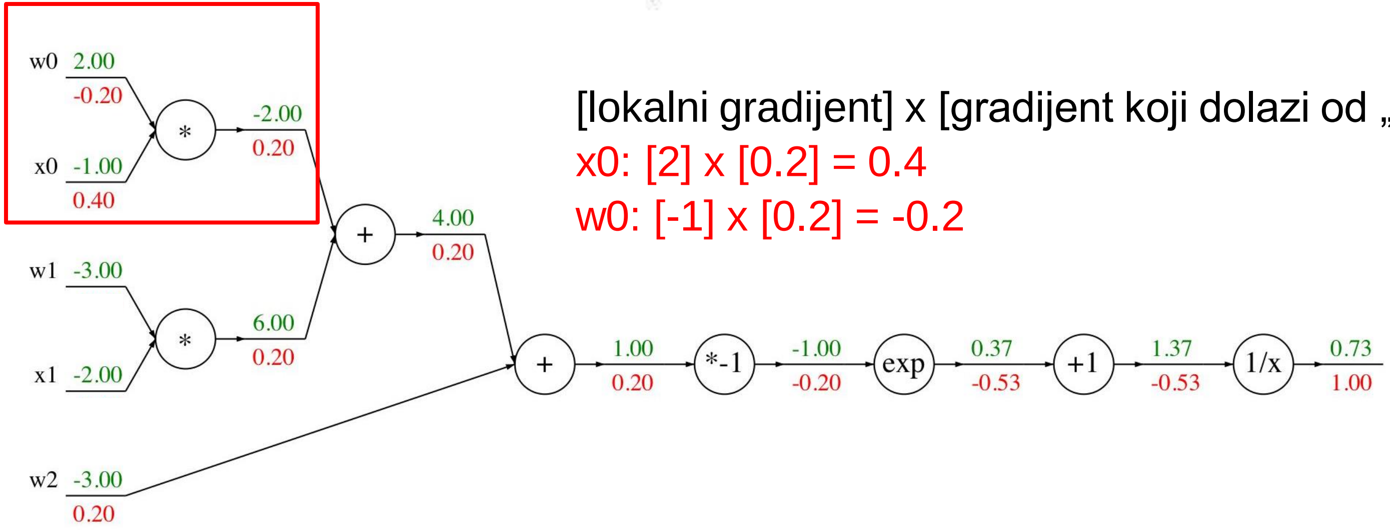
Još jedan primer:

$$f(w,x)=\frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



[lokalni gradijent] x [gradijent koji dolazi od „gore“]

x0: [2] x [0.2] = 0.4

w0: [-1] x [0.2] = -0.2

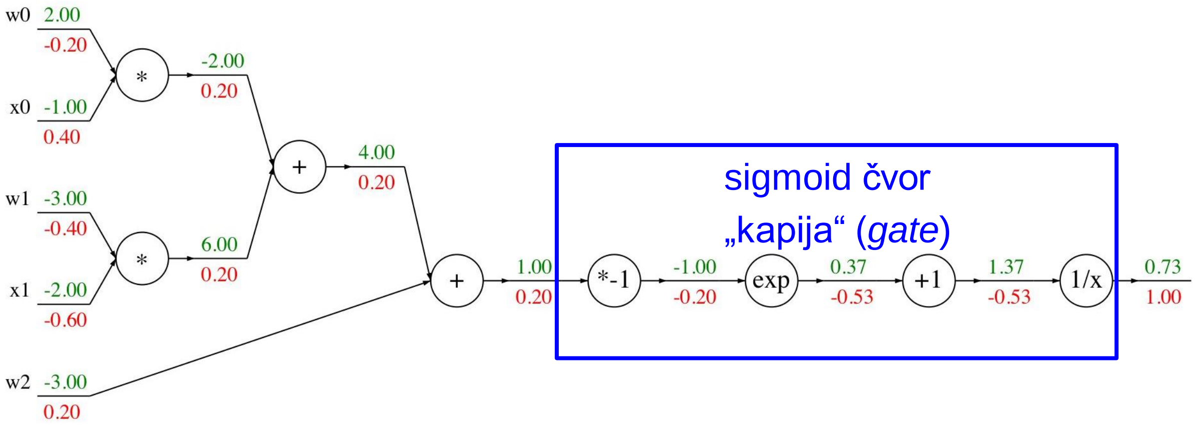
$$f(x)=e^x \quad \rightarrow \quad \frac{df}{dx}=e^x \qquad\qquad f(x)=\frac{1}{x} \quad \rightarrow \quad \frac{df}{dx}=-1/x^2$$

$$f_a(x)=ax \quad \rightarrow \quad \frac{df}{dx}=a \qquad\qquad f_c(x)=c+x \quad \rightarrow \quad \frac{df}{dx}=1$$

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

$$\boxed{\sigma(x) = \frac{1}{1+e^{-x}}}$$ sigmoid funkcija

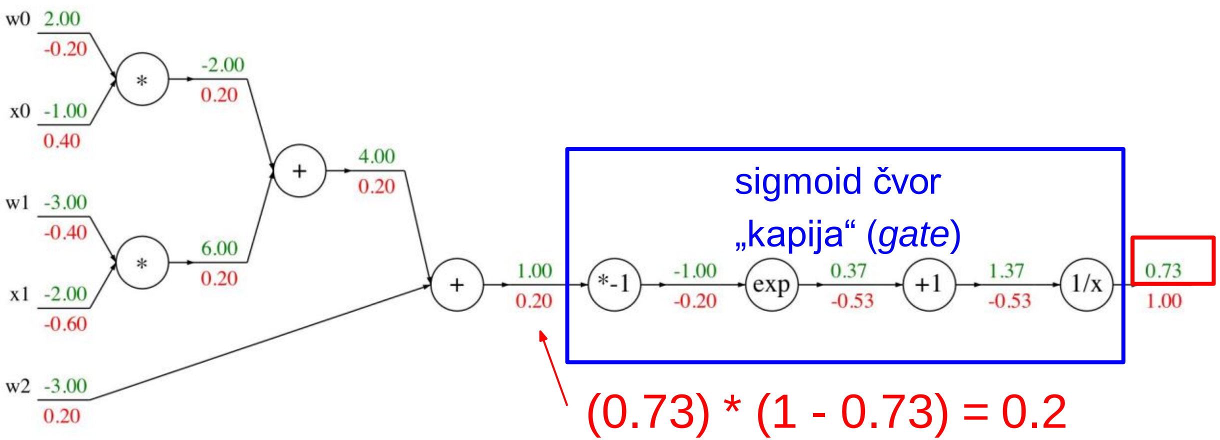
$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2} = \left(\frac{1+e^{-x}-1}{1+e^{-x}}\right)\left(\frac{1}{1+e^{-x}}\right) = (1-\sigma(x))\,\sigma(x)$$



$$f(w,x)=\frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

$$\boxed{\sigma(x)=\frac{1}{1+e^{-x}}}$$ sigmoid funkcija

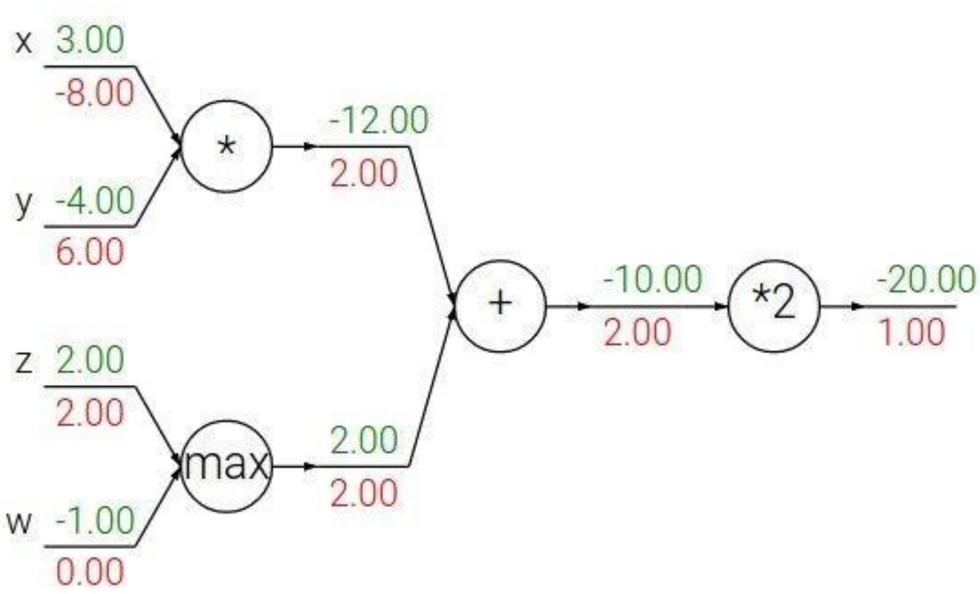
$$\frac{d\sigma(x)}{dx}=\frac{e^{-x}}{(1+e^{-x})^2}=\left(\frac{1+e^{-x}-1}{1+e^{-x}}\right)\left(\frac{1}{1+e^{-x}}\right)=(1-\sigma(x))\,\sigma(x)$$



(0.73) * (1 - 0.73) = 0.2

Ponašanja čvorova pri prolasku gradijenta unazad

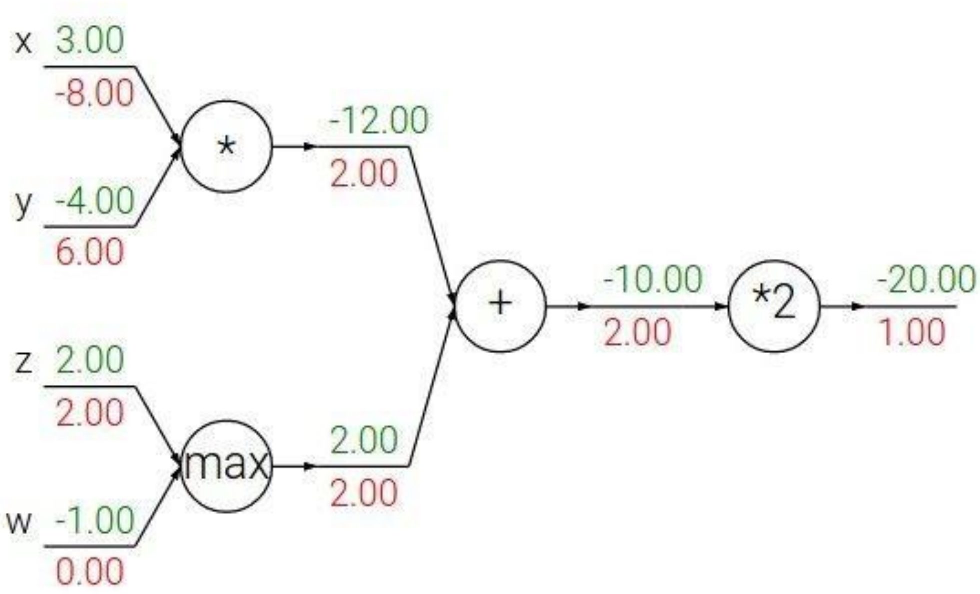
čvor **sabiranja**: distributor gradijenta



Ponašanja čvorova pri prolasku gradijenta unazad

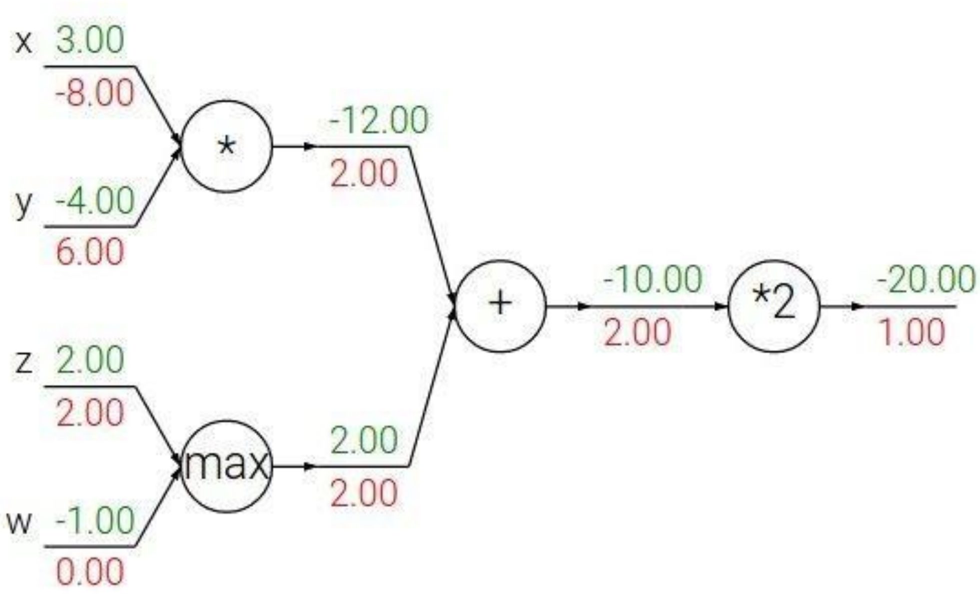
čvor **sabiranja**: distributor gradijenta

Šta radi **max** čvor?



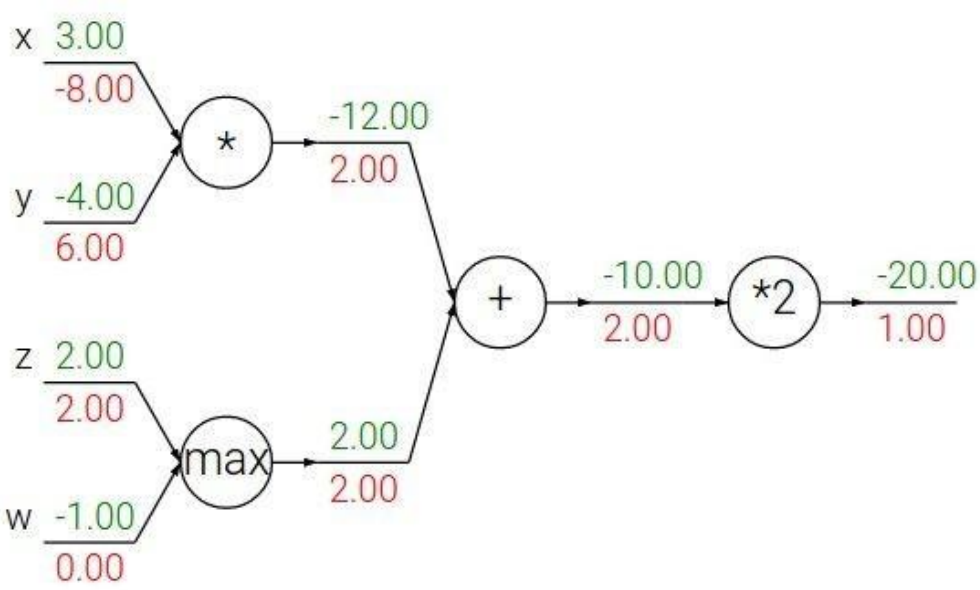
Ponašanja čvorova pri prolasku gradijenta unazad

čvor **sabiranja**: distributor gradijenta
max čvor: ruter gradijenta



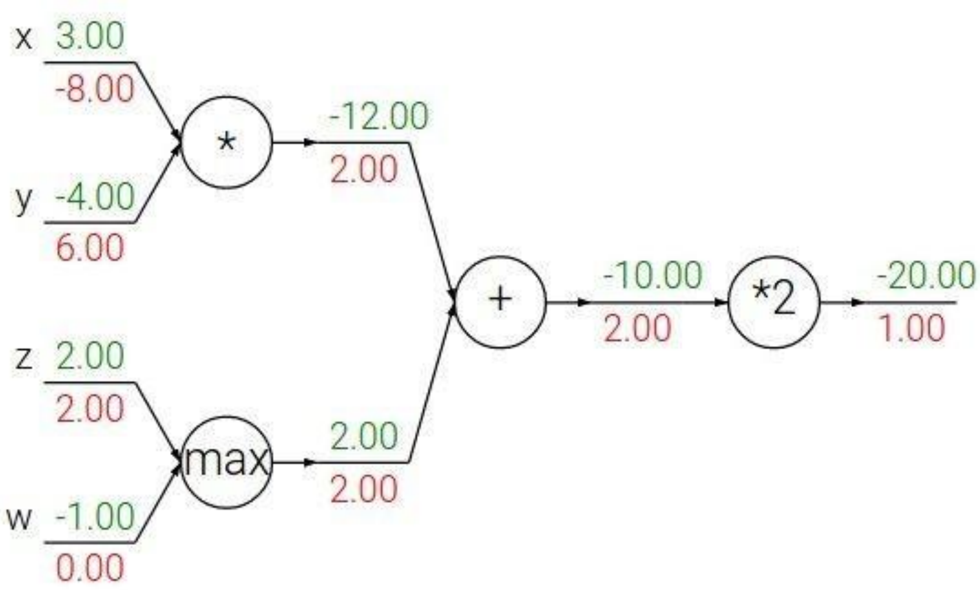
Ponašanja čvorova pri prolasku gradijenta unazad

čvor **sabiranja**: distributor gradijenta
max čvor: ruter gradijenta
Šta radi čvor **množenja**?

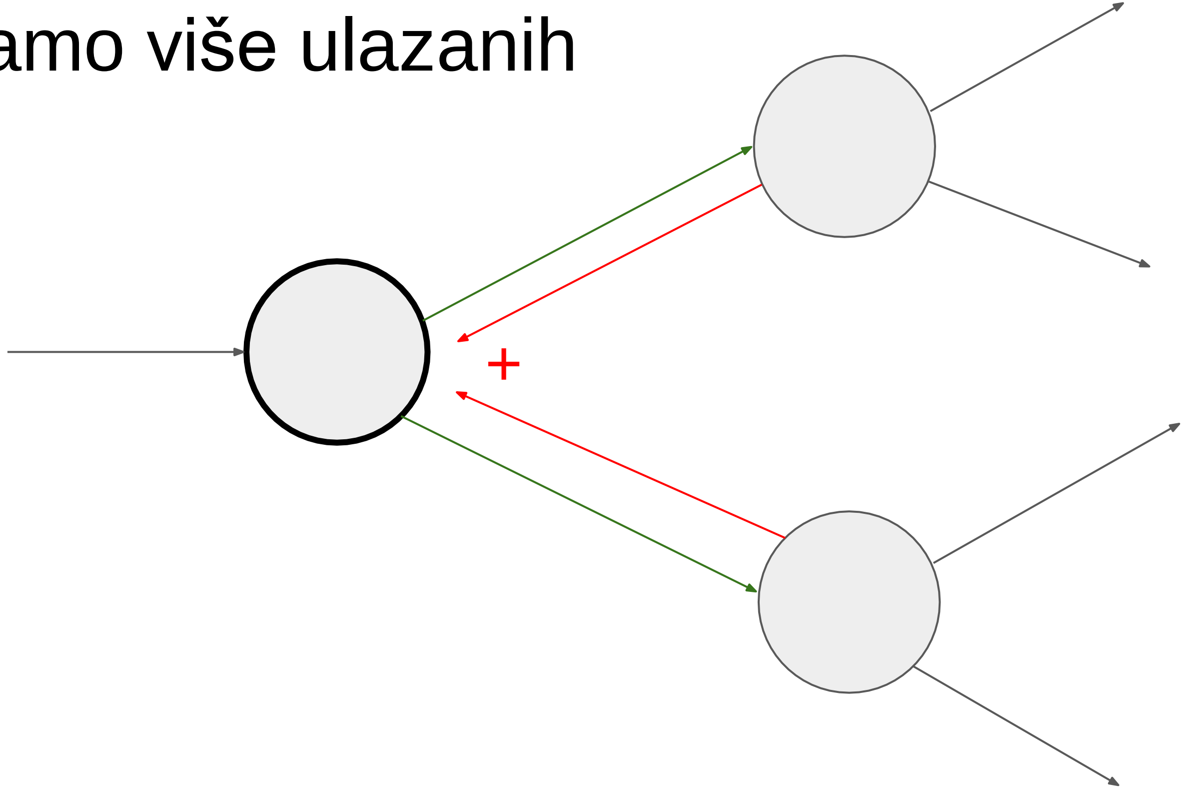


Ponašanja čvorova pri prolasku gradijenta unazad

čvor **sabiranja**: distributor gradijenta
max čvor: ruter gradijenta
čvor **množenja**: razmenjuje gradijente



Gradijenti se sabiraju kada imamo više ulazanih grana

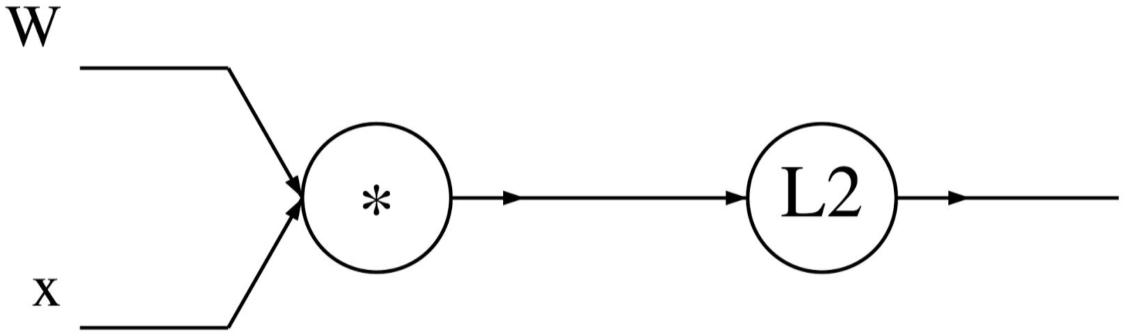


Primer sa vektorima $f(x, W) = ||W \cdot x||^2 = \sum_{i=1}^{n} (W \cdot x)_i^2$

Primer sa vektorima $f(x, W) = ||W \cdot x||^2 = \sum_{i=1}^{n} (W \cdot x)_i^2$

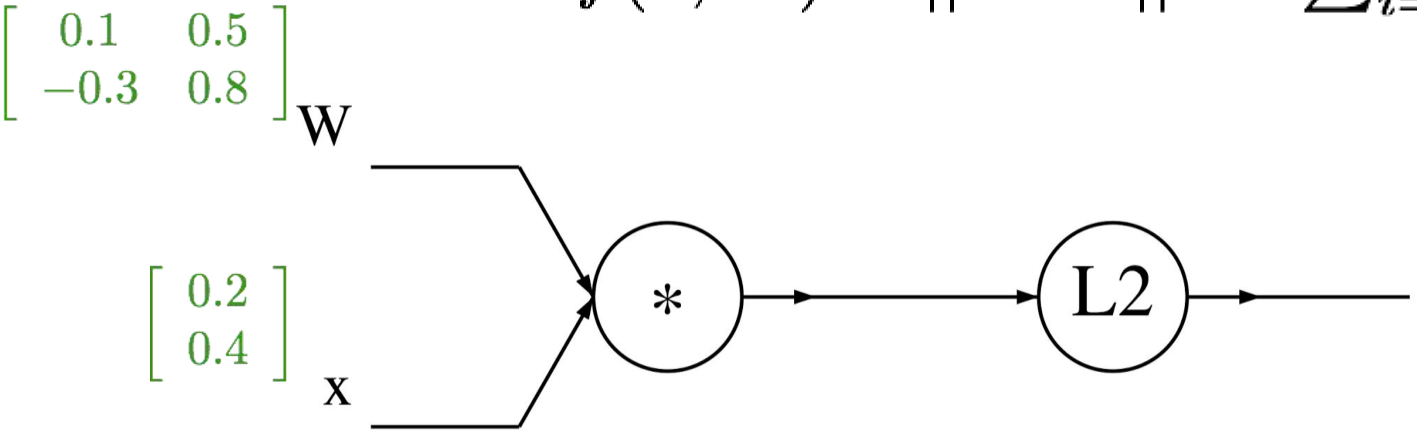
$\in \mathbb{R}^n$ $\in \mathbb{R}^{n \times n}$

Primer sa vektorima $f(x, W) = ||W \cdot x||^2 = \sum_{i=1}^{n} (W \cdot x)_i^2$



Primer sa vektorima

$$f(x, W) = ||W \cdot x||^2 = \sum_{i=1}^{n} (W \cdot x)_i^2$$



$$q = W \cdot x = \begin{pmatrix} W_{1,1}x_1 + \cdots + W_{1,n}x_n \\ \vdots \\ W_{n,1}x_1 + \cdots + W_{n,n}x_n \end{pmatrix}$$

$$f(q) = ||q||^2 = q_1^2 + \cdots + q_n^2$$

Primer sa vektorima

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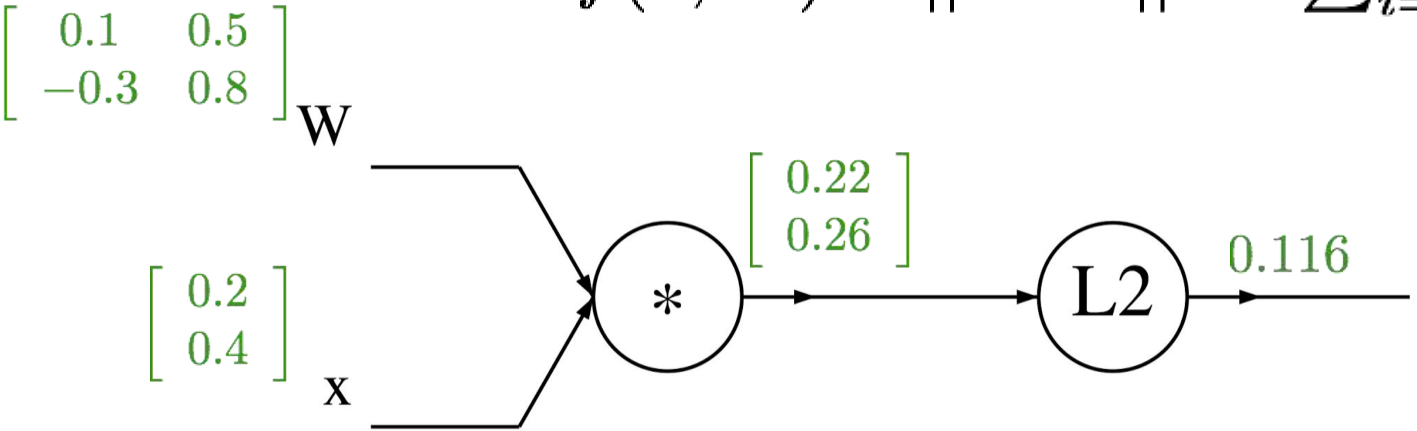

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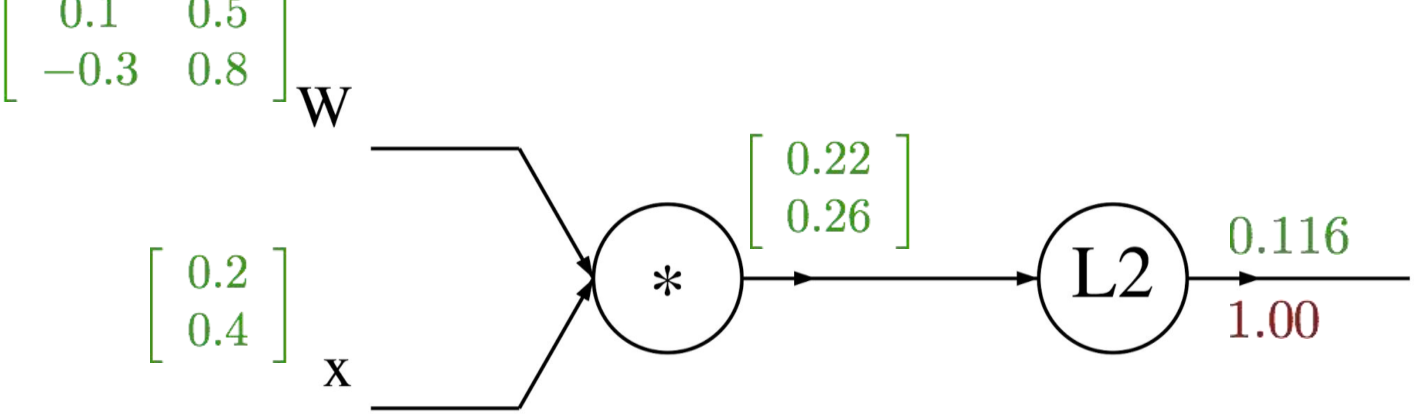

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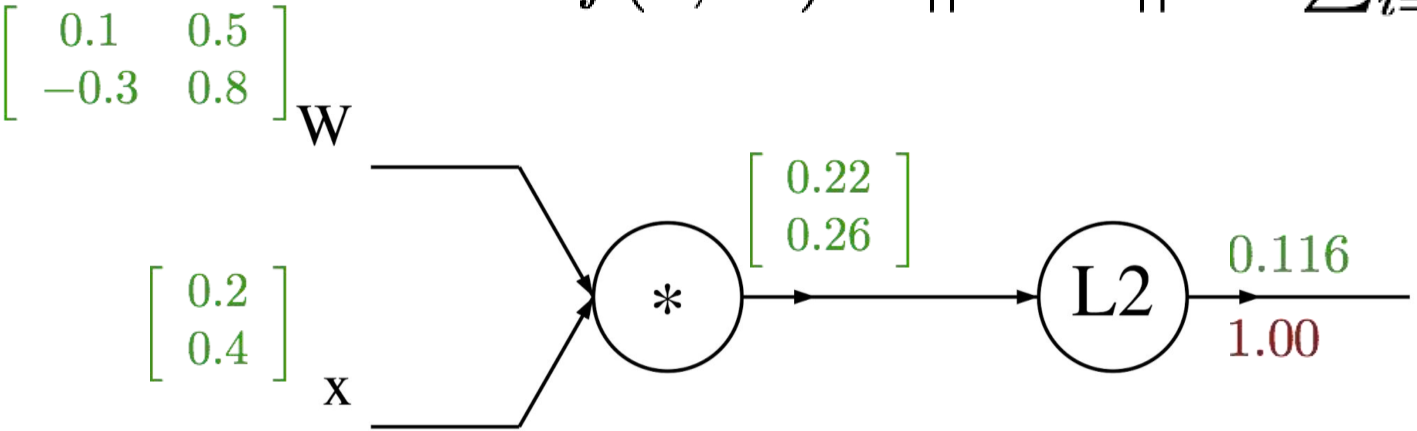

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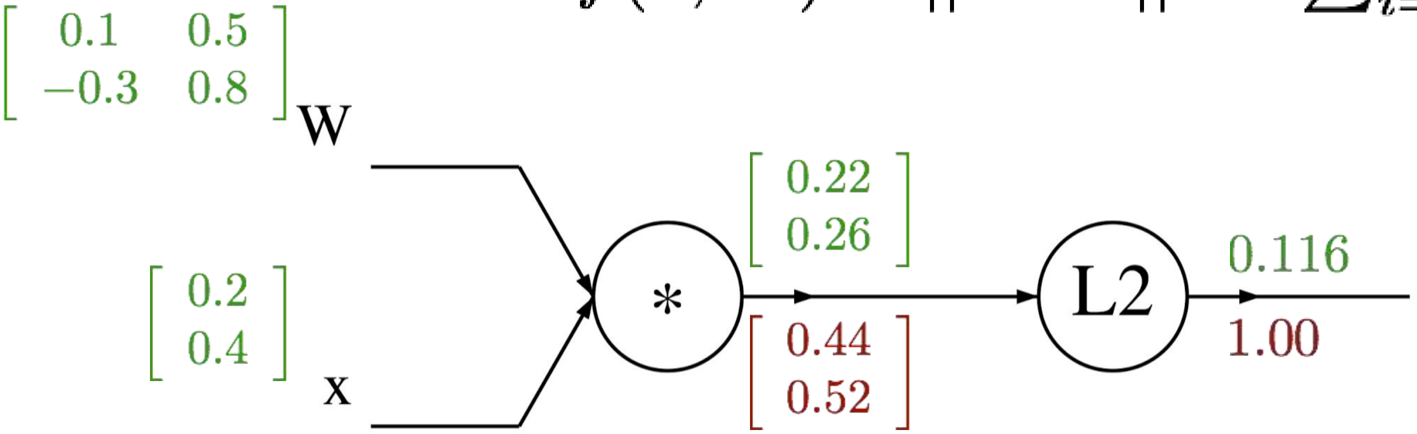
$$f(q) = ||q||^2 = q_1^2 + \cdots + q_n^2$$

$$\frac{\partial f}{\partial q_i} = 2q_i$$

$$\boxed{\nabla_q f = 2q}$$

Primer sa vektorima

$$f(x, W) = ||W \cdot x||^2 = \sum_{i=1}^{n} (W \cdot x)_i^2$$



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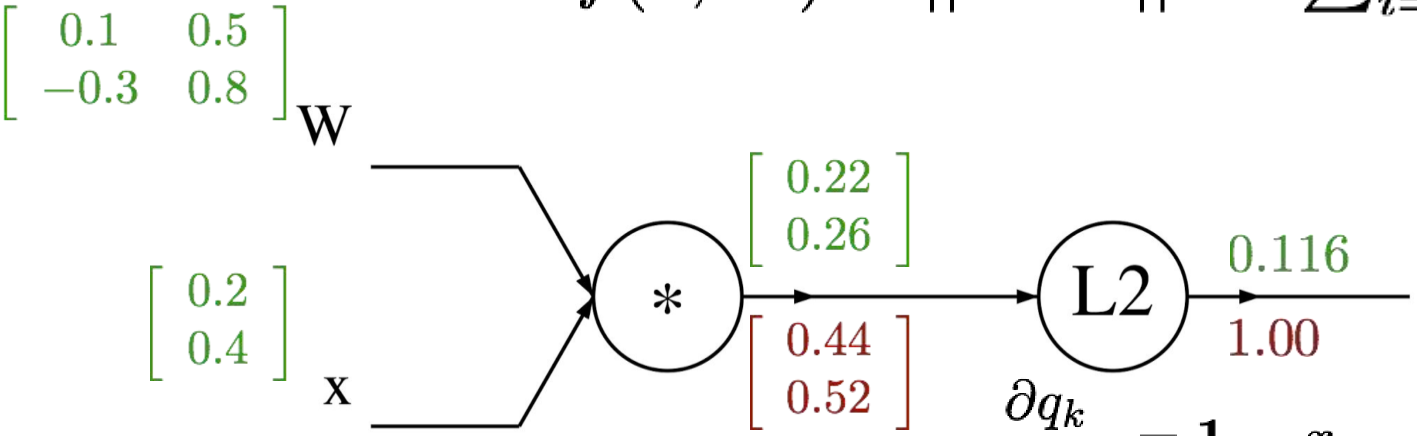

$$\frac{\partial q_k}{\partial W_{i,j}} = \mathbf{1}_{k=i} x_j$$

$$q = W \cdot x = \begin{pmatrix} W_{1,1}x_1 + \cdots + W_{1,n}x_n \\ \vdots \\ W_{n,1}x_1 + \cdots + W_{n,n}x_n \end{pmatrix}$$

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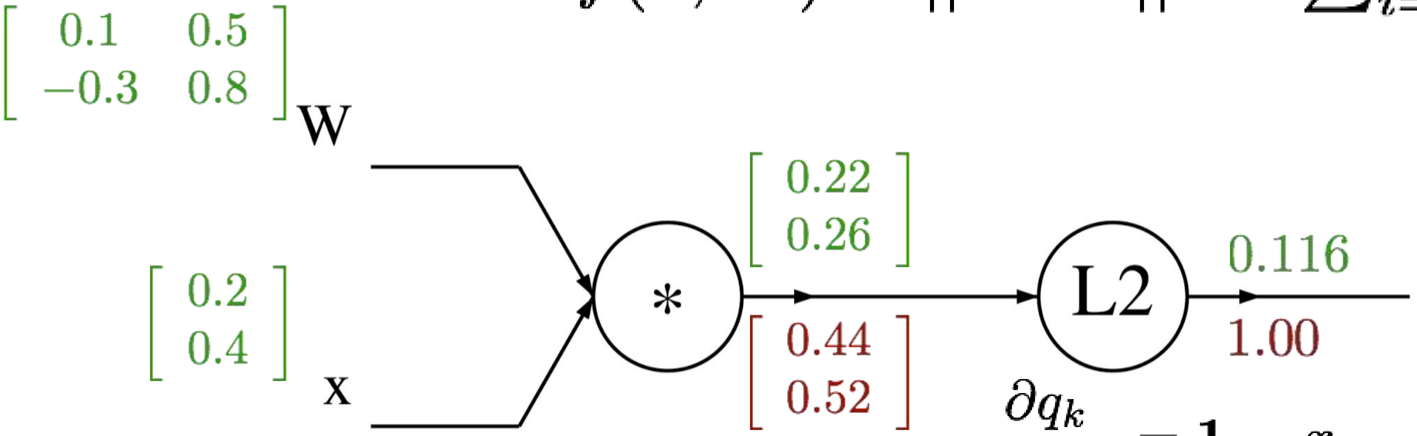


$$\frac{\partial q_k}{\partial W_{i,j}} = \mathbf{1}_{k=i} x_j$$

$$\frac{\partial f}{\partial W_{i,j}} = \sum_k \frac{\partial f}{\partial q_k} \frac{\partial q_k}{\partial W_{i,j}}$$

$$= \sum_k (2q_k)(\mathbf{1}_{k=i} x_j)$$

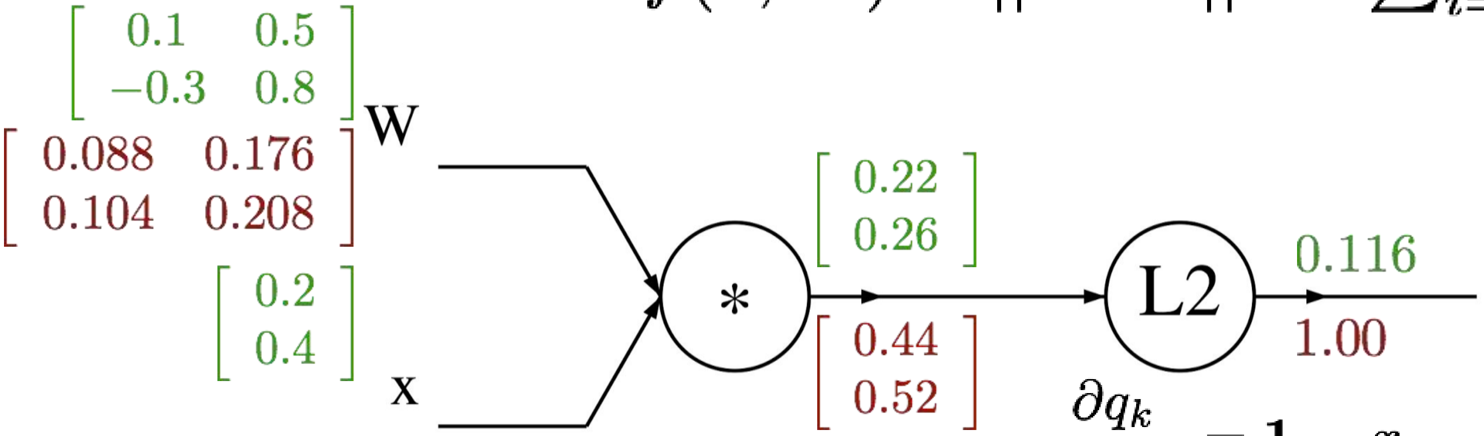
$$= 2q_i x_j$$

$$q = W \cdot x = \begin{pmatrix} W_{1,1}x_1 + \cdots + W_{1,n}x_n \\ \vdots \\ W_{n,1}x_1 + \cdots + W_{n,n}x_n \end{pmatrix}$$

$$f(q) = ||q||^2 = q_1^2 + \cdots + q_n^2$$

Primer sa vektorima

$$f(x, W) = ||W \cdot x||^2 = \sum_{i=1}^{n} (W \cdot x)_i^2$$



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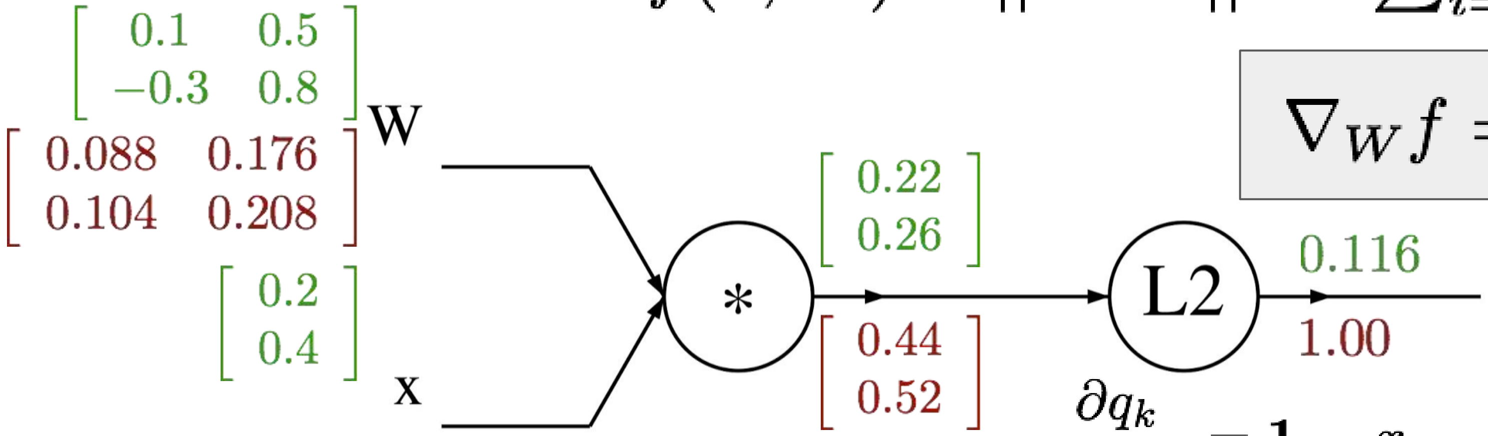
Primer sa vektorima

$$f(x, W) = ||W \cdot x||^2 = \sum_{i=1}^{n} (W \cdot x)_i^2$$

$$\boxed{\nabla_W f = 2q \cdot x^T}$$



$$\frac{\partial q_k}{\partial W_{i,j}} = \mathbf{1}_{k=i} x_j$$

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$$f(q) = ||q||^2 = q_1^2 + \cdots + q_n^2$$

Primer sa vektorima - dodatak

$$\nabla_W f = 2q \cdot x^T$$

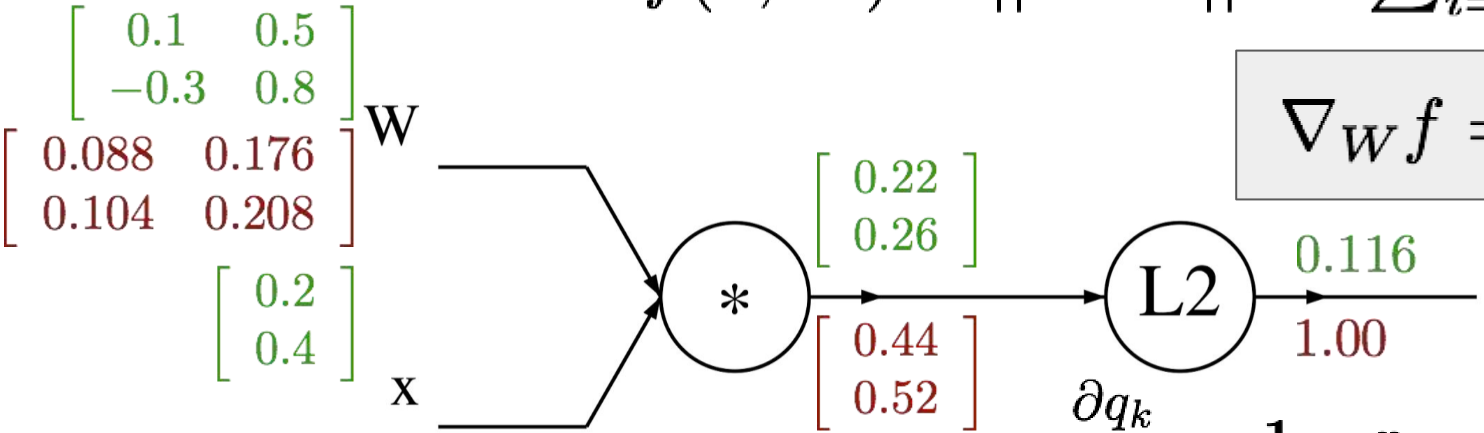
$$\begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} & a_{11}b_{12} \\ a_{21}b_{11} & a_{21}b_{12} \end{bmatrix}$$

$$\begin{bmatrix} 0.44 \\ 0.52 \end{bmatrix} \begin{bmatrix} 0.2 & 0.4 \end{bmatrix} = \begin{bmatrix} 0.44 * 0.2 & 0.44 * 0.4 \\ 0.52 * 0.2 & 0.52 * 0.4 \end{bmatrix}$$

$$\begin{bmatrix} 0.44 \\ 0.52 \end{bmatrix} \begin{bmatrix} 0.2 & 0.4 \end{bmatrix} = \begin{bmatrix} 0.088 & 0.176 \\ 0.104 & 0.208 \end{bmatrix}$$

Primer sa vektorima

$$f(x, W) = ||W \cdot x||^2 = \sum_{i=1}^{n} (W \cdot x)_i^2$$



$$\boxed{\nabla_W f = 2q \cdot x^T}$$

Uvek proverite: Gradijent po nekoj promenljivoj mora da ima isti oblik (dimenzije) kao ta promenljiva

$$\frac{\partial q_k}{\partial W_{i,j}} = \mathbf{1}_{k=i} x_j$$

$$\frac{\partial f}{\partial W_{i,j}} = \sum_k \frac{\partial f}{\partial q_k} \frac{\partial q_k}{\partial W_{i,j}}$$

$$= \sum_k (2q_k)(\mathbf{1}_{k=i} x_j)$$

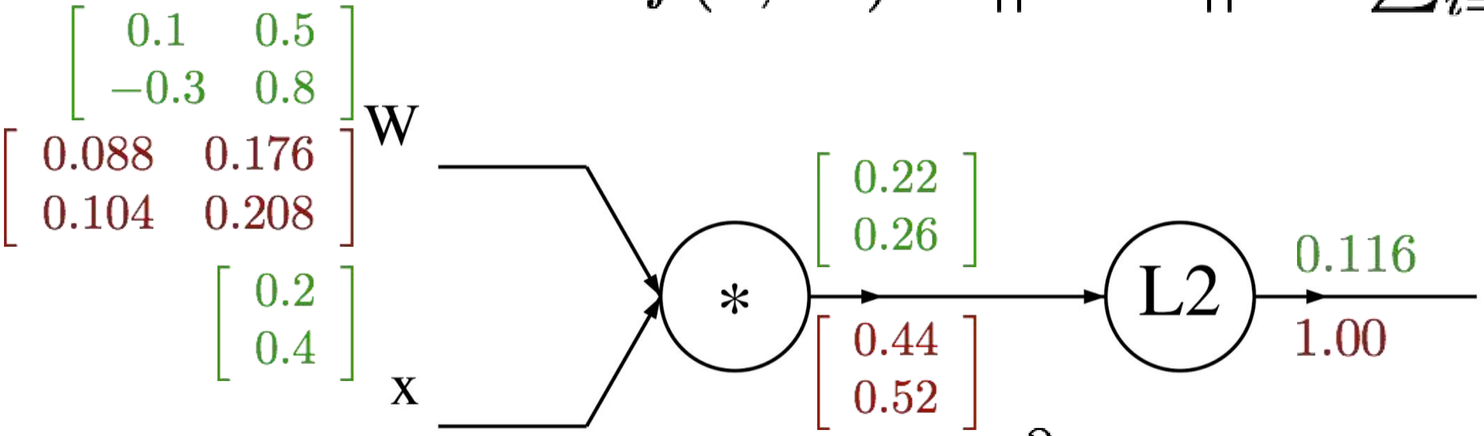
$$= 2q_i x_j$$

$$q = W \cdot x = \begin{pmatrix} W_{1,1}x_1 + \cdots + W_{1,n}x_n \\ \vdots \\ W_{n,1}x_1 + \cdots + W_{n,n}x_n \end{pmatrix}$$

$$f(q) = ||q||^2 = q_1^2 + \cdots + q_n^2$$

Primer sa vektorima

$$f(x, W) = ||W \cdot x||^2 = \sum_{i=1}^{n} (W \cdot x)_i^2$$



$$\frac{\partial q_k}{\partial x_i} = W_{k,i}$$

$$q = W \cdot x = \begin{pmatrix} W_{1,1}x_1 + \cdots + W_{1,n}x_n \\ \vdots \\ W_{n,1}x_1 + \cdots + W_{n,n}x_n \end{pmatrix}$$

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Primer sa vektorima

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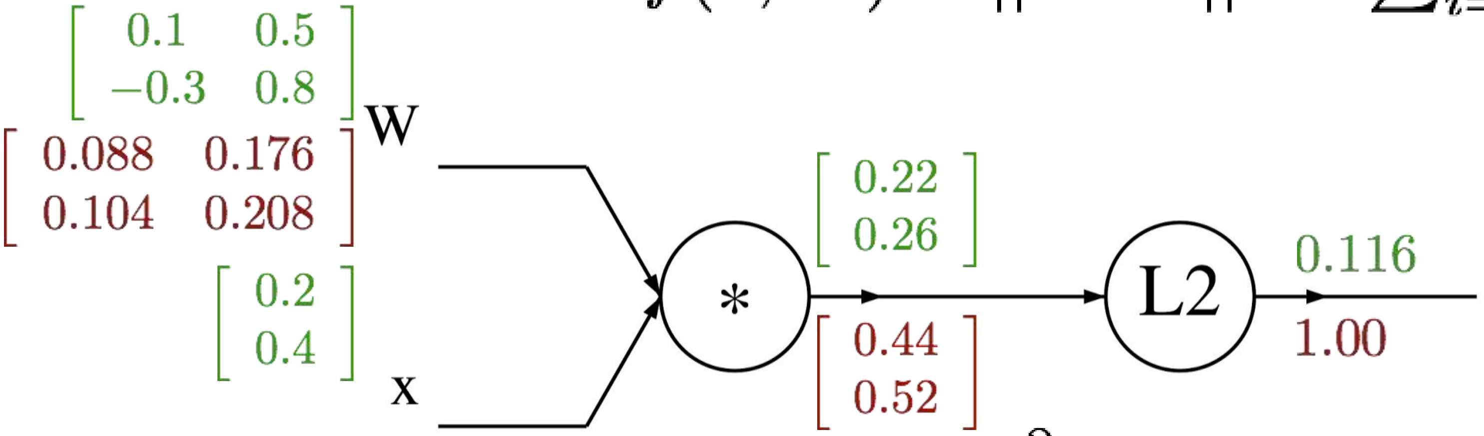

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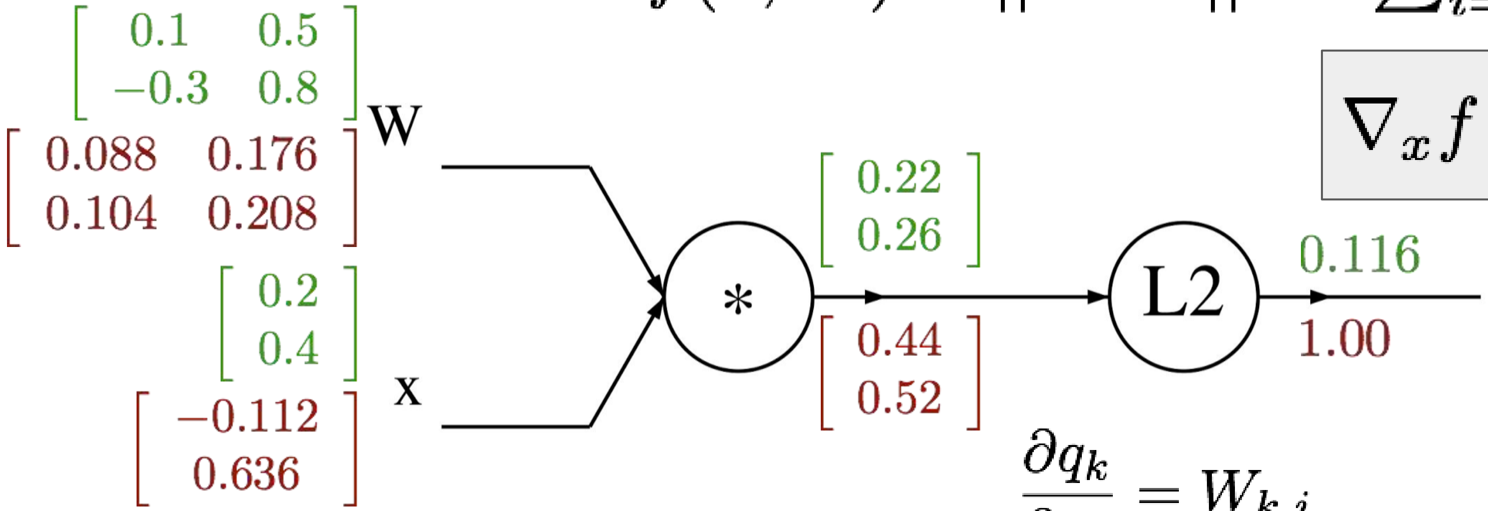
$$f(q) = ||q||^2 = q_1^2 + \cdots + q_n^2$$

$$\frac{\partial f}{\partial x_i} = \sum_k 2 q_k W_{k,i}$$

Primer sa vektorima

$$f(x, W) = ||W \cdot x||^2 = \sum_{i=1}^{n} (W \cdot x)_i^2$$

$$\boxed{\nabla_x f = 2W^T \cdot q}$$



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$$\frac{\partial f}{\partial x_i} = \sum_k 2 q_k W_{k,i}$$

Primer sa vektorima - dodatak

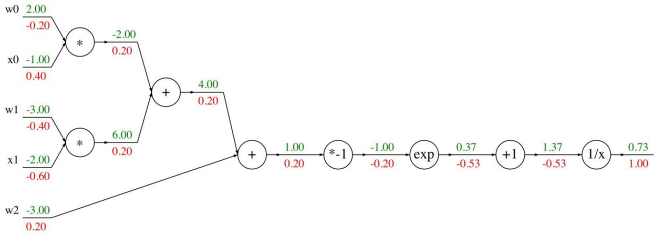
$$\nabla_x f = 2W^T \cdot q$$

$$2\begin{bmatrix} w_{11} & w_{21} \\ w_{12} & w_{22} \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} w_{11}q_1 + w_{21}q_2 \\ w_{12}q_1 + w_{22}q_2 \end{bmatrix}$$

$$2\begin{bmatrix} 0.1 & -0.3 \\ 0.5 & 0.8 \end{bmatrix}\begin{bmatrix} 0.44 \\ 0.52 \end{bmatrix} = \begin{bmatrix} 0.1 * 0.44 + (-0.3 * 0.52) \\ 0.5 * 0.44 + (0.8 * 0.52) \end{bmatrix}$$

$$\begin{bmatrix} -0.112 \\ 0.636 \end{bmatrix}$$

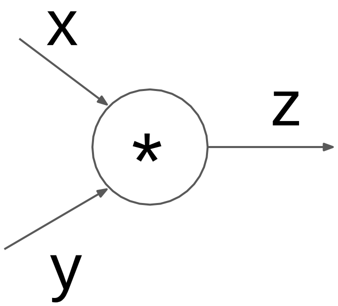
Modularizovana implementacija: *forward / backward API*



Graf (ili Mreža) objekat *(grubi pseudo-kod)*

```
class ComputationalGraph(object):
    #...
    def forward(inputs):
        # 1. [pass inputs to input gates...]
        # 2. forward the computational graph:
        for gate in self.graph.nodes_topologically_sorted():
            gate.forward()
        return loss # the final gate in the graph outputs the loss
    def backward():
        for gate in reversed(self.graph.nodes_topologically_sorted()):
            gate.backward() # little piece of backprop (chain rule applied)
        return inputs_gradients
```

Modularizovana implementacija: *forward / backward API*



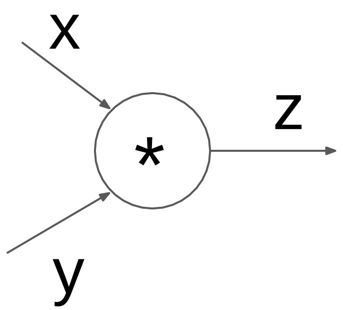
(x,y,z su skalari)

```
class MultiplyGate(object):
    def forward(x,y):
        z = x*y
        return z
    def backward(dz):
        # dx = ... #todo
        # dy = ... #todo
        return [dx, dy]
```

$$\boxed{\frac{\partial L}{\partial z}}$$

$$\boxed{\frac{\partial L}{\partial x}}$$

Modularizovana implementacija: *forward / backward API*



(x,y,z su skalari)

```python
class MultiplyGate(object):
    def forward(x,y):
        z = x*y
        self.x = x # must keep these around!
        self.y = y
        return z
    def backward(dz):
        dx = self.y * dz # [dz/dx * dL/dz]
        dy = self.x * dz # [dz/dy * dL/dz]
        return [dx, dy]
```

Primer: Slojevi u *Caffe* okruženju za *Deep Learning*

Branch: master ▾ caffe / src / caffe / layers / Create new file Upload files Find file History

shelhamer committed on GitHub Merge pull request #4630 from BlGene/load_hdf5_fix ... Latest commit e687a71 21 days ago

..

absval_layer.cpp	dismantle layer headers	a year ago
absval_layer.cu	dismantle layer headers	a year ago
accuracy_layer.cpp	dismantle layer headers	a year ago
argmax_layer.cpp	dismantle layer headers	a year ago
base_conv_layer.cpp	enable dilated deconvolution	a year ago
base_data_layer.cpp	Using default from proto for prefetch	3 months ago
base_data_layer.cu	Switched multi-GPU to NCCL	3 months ago
batch_norm_layer.cpp	Add missing spaces besides equal signs in batch_norm_layer.cpp	4 months ago
batch_norm_layer.cu	dismantle layer headers	a year ago
batch_reindex_layer.cpp	dismantle layer headers	a year ago
batch_reindex_layer.cu	dismantle layer headers	a year ago
bias_layer.cpp	Remove incorrect cast of gemm int arg to Dtype in BiasLayer	a year ago
bias_layer.cu	Separation and generalization of ChannelwiseAffineLayer into BiasLayer	a year ago
bnll_layer.cpp	dismantle layer headers	a year ago
bnll_layer.cu	dismantle layer headers	a year ago
concat_layer.cpp	dismantle layer headers	a year ago
concat_layer.cu	dismantle layer headers	a year ago
contrastive_loss_layer.cpp	dismantle layer headers	a year ago
contrastive_loss_layer.cu	dismantle layer headers	a year ago
conv_layer.cpp	add support for 2D dilated convolution	a year ago
conv_layer.cu	dismantle layer headers	a year ago
crop_layer.cpp	remove redundant operations in Crop layer (#5138)	2 months ago
crop_layer.cu	remove redundant operations in Crop layer (#5138)	2 months ago
cudnn_conv_layer.cpp	dismantle layer headers	a year ago
cudnn_conv_layer.cu	Add cuDNN v5 support, drop cuDNN v3 support	11 months ago

cudnn_lcn_layer.cpp	dismantle layer headers	a year ago
cudnn_lcn_layer.cu	dismantle layer headers	a year ago
cudnn_lrn_layer.cpp	dismantle layer headers	a year ago
cudnn_lrn_layer.cu	dismantle layer headers	a year ago
cudnn_pooling_layer.cpp	dismantle layer headers	a year ago
cudnn_pooling_layer.cu	dismantle layer headers	a year ago
cudnn_relu_layer.cpp	Add cuDNN v5 support, drop cuDNN v3 support	11 months ago
cudnn_relu_layer.cu	Add cuDNN v5 support, drop cuDNN v3 support	11 months ago
cudnn_sigmoid_layer.cpp	Add cuDNN v5 support, drop cuDNN v3 support	11 months ago
cudnn_sigmoid_layer.cu	Add cuDNN v5 support, drop cuDNN v3 support	11 months ago
cudnn_softmax_layer.cpp	dismantle layer headers	a year ago
cudnn_softmax_layer.cu	dismantle layer headers	a year ago
cudnn_tanh_layer.cpp	Add cuDNN v5 support, drop cuDNN v3 support	11 months ago
cudnn_tanh_layer.cu	Add cuDNN v5 support, drop cuDNN v3 support	11 months ago
data_layer.cpp	Switched multi-GPU to NCCL	3 months ago
deconv_layer.cpp	enable dilated deconvolution	a year ago
deconv_layer.cu	dismantle layer headers	a year ago
dropout_layer.cpp	supporting N-D Blobs in Dropout layer Reshape	a year ago
dropout_layer.cu	dismantle layer headers	a year ago
dummy_data_layer.cpp	dismantle layer headers	a year ago
eltwise_layer.cpp	dismantle layer headers	a year ago
eltwise_layer.cu	dismantle layer headers	a year ago
elu_layer.cpp	ELU layer with basic tests	a year ago
elu_layer.cu	ELU layer with basic tests	a year ago
embed_layer.cpp	dismantle layer headers	a year ago
embed_layer.cu	dismantle layer headers	a year ago
euclidean_loss_layer.cpp	dismantle layer headers	a year ago
euclidean_loss_layer.cu	dismantle layer headers	a year ago
exp_layer.cpp	Solving issue with exp layer with base e	a year ago
exp_layer.cu	dismantle layer headers	a year ago

Caffe Sigmoid Sloj

```cpp
#include <cmath>
#include <vector>

#include "caffe/layers/sigmoid_layer.hpp"

namespace caffe {

template <typename Dtype>
inline Dtype sigmoid(Dtype x) {
  return 1. / (1. + exp(-x));
}

template <typename Dtype>
void SigmoidLayer<Dtype>::Forward_cpu(const vector<Blob<Dtype>*>& bottom,
    const vector<Blob<Dtype>*>& top) {
  const Dtype* bottom_data = bottom[0]->cpu_data();
  Dtype* top_data = top[0]->mutable_cpu_data();
  const int count = bottom[0]->count();
  for (int i = 0; i < count; ++i) {
    top_data[i] = sigmoid(bottom_data[i]);
  }
}

template <typename Dtype>
void SigmoidLayer<Dtype>::Backward_cpu(const vector<Blob<Dtype>*>& top,
    const vector<bool>& propagate_down,
    const vector<Blob<Dtype>*>& bottom) {
  if (propagate_down[0]) {
    const Dtype* top_data = top[0]->cpu_data();
    const Dtype* top_diff = top[0]->cpu_diff();
    Dtype* bottom_diff = bottom[0]->mutable_cpu_diff();
    const int count = bottom[0]->count();
    for (int i = 0; i < count; ++i) {
      const Dtype sigmoid_x = top_data[i];
      bottom_diff[i] = top_diff[i] * sigmoid_x * (1. - sigmoid_x);
    }
  }
}

#ifdef CPU_ONLY
STUB_GPU(SigmoidLayer);
#endif

INSTANTIATE_CLASS(SigmoidLayer);


}  // namespace caffe
```

$$\boxed{\sigma(x) = \frac{1}{1+e^{-x}}}$$

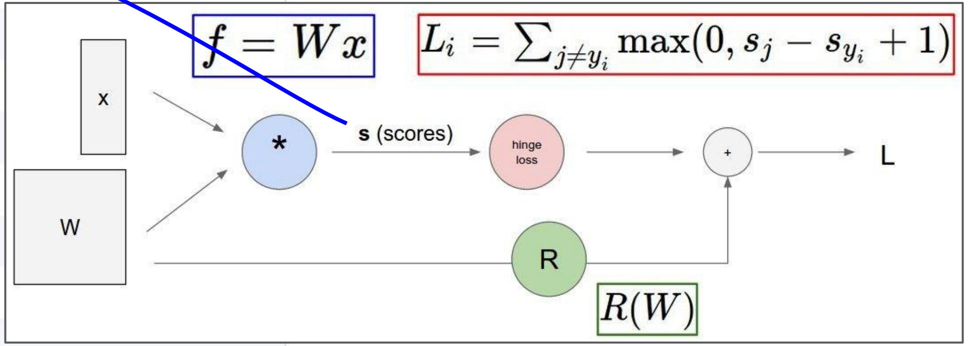
$$\boxed{(1-\sigma(x))\,\sigma(x)}$$ * top_diff (pravilo ulančavanja izvoda)

Kako bi izgledao SVM / Softmax klasifikator kao Graf Izračunavanja? Da li možemo obučavanje da organizujemo na forward/backward način?

Npr. SVM:

margine

```
# receive W (weights), X (data)
# forward pass (we have 8 lines)
scores = #...
margins = #...
data_loss = #...
reg_loss = #...
loss = data_loss + reg_loss
# backward pass (we have 5 lines)
dmargins = # ... (optionally, we go direct to dscores)
dscores = #...
dW = #...
```



Rezime...

- neuronske mreže su u praksi veoma velike: nije praktično da se ručno određuju gradijenti za svaki parametar (gradijente određujemo onkako kako smo to radili danas)
- **backpropagation** = rekurzivna primena ulančavanja izvoda kroz graf izračunavanja da bi izračunali sve potrebne gradijente
- implementacije neuronskih mreža koriste strukturu grafa gde svaki čvor implementira **forward**() / **backward**() API
- **forward**: izračunavamo odgovaraću operaciju i čuvamo rezultate u memoriji da bi mogli da ih iskoristimo za računanje gradijenata kasnije u **backpropagation**
- **backward**: primenjujemo pravilo ulančavanja da bi izračunali gradijente za sve parametre u odnosu na funkciju greške

Neuronske Mreže
Neural Networks

Neuronske Mreže: prvo bez analogije sa ljudskim mozgom

(**Ranije**) Linearna skor funkcija: $f = Wx$

Neuronske Mreže: prvo bez analogije sa ljudskim mozgom

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Neuronske Mreže: prvo bez analogije sa ljudskim mozgom

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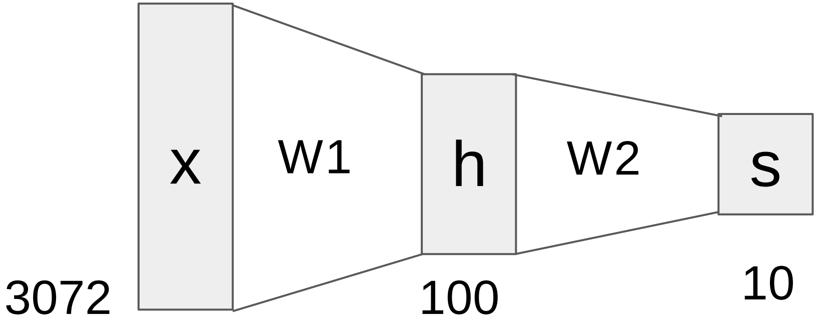
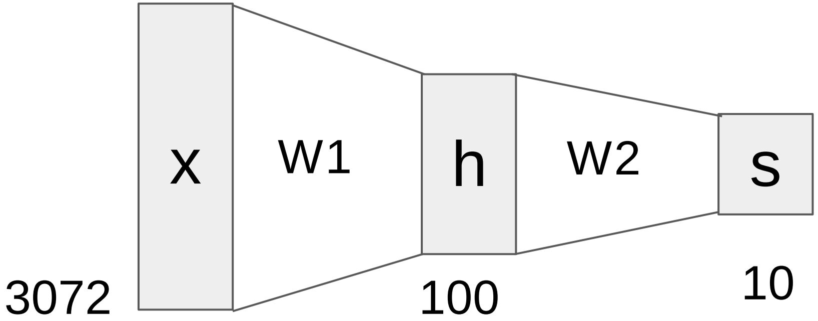


Neuronske Mreže: prvo bez analogije sa ljudskim mozgom

(**Ranije**) Linearna skor funkcija: $f = Wx$

(**Sada**) Neuroska mreža sa dva sloja: $f = W_2 \max(0, W_1 x)$



Neuronske Mreže: prvo bez analogije sa ljudskim mozgom

(**Ranije**) Linearna skor funkcija: $f = Wx$

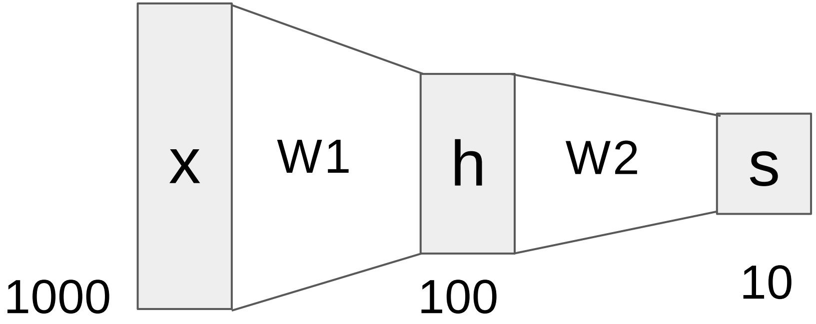
(**Sada**) Neuroska mreža sa dva sloja: $f = W_2 \max(0, W_1 x)$

Neuroska mreža sa tri sloja:

$$f = W_3 \max(0, W_2 \max(0, W_1 x))$$

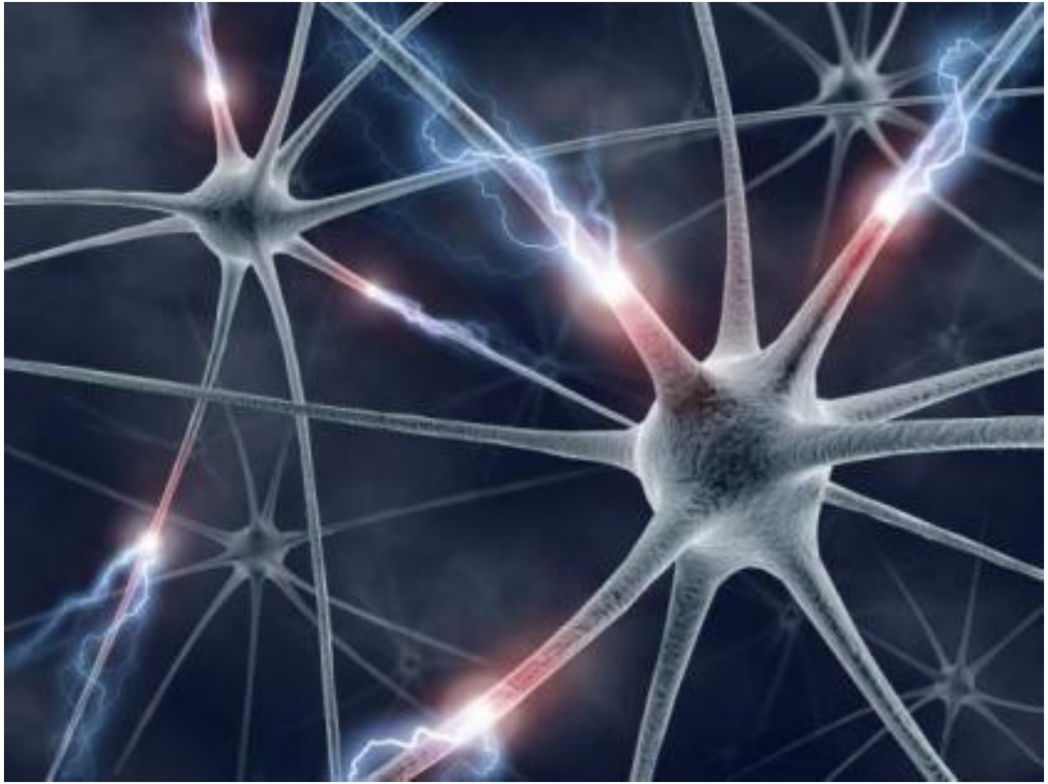
Kompletna implementacija dvo-slojne Neuronske Mreže ima ~20 linija koda:

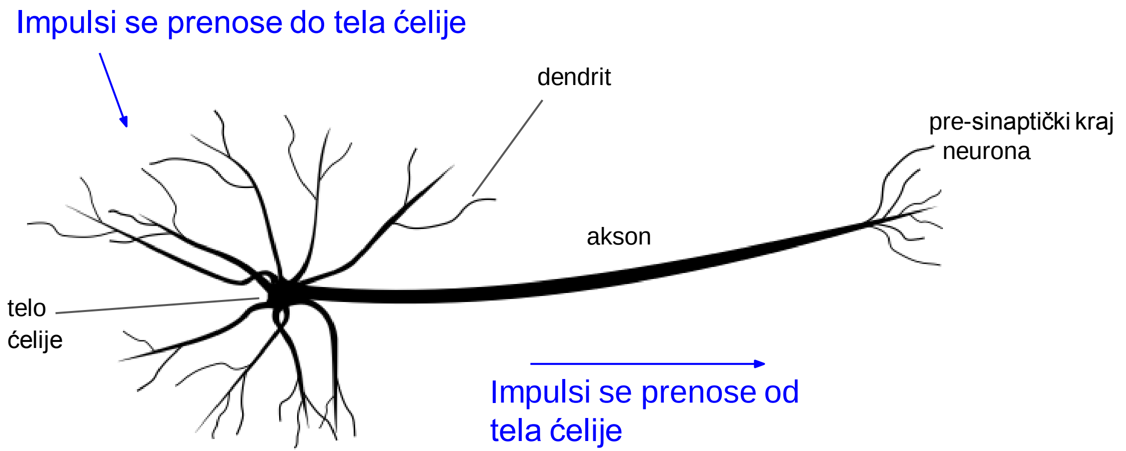
```python
import numpy as np
from numpy.random import randn

N, D_in, H, D_out = 64, 1000, 100, 10
x, y = randn(N, D_in), randn(N, D_out)
w1, w2 = randn(D_in, H), randn(H, D_out)

for t in range(2000):
  h = 1 / (1 + np.exp(-x.dot(w1)))
  y_pred = h.dot(w2)
  loss = np.square(y_pred - y).sum()
  print(t, loss)

  grad_y_pred = 2.0 * (y_pred - y)
  grad_w2 = h.T.dot(grad_y_pred)
  grad_h = grad_y_pred.dot(w2.T)
  grad_w1 = x.T.dot(grad_h * h * (1 - h))

  w1 -= 1e-4 * grad_w1
  w2 -= 1e-4 * grad_w2
```

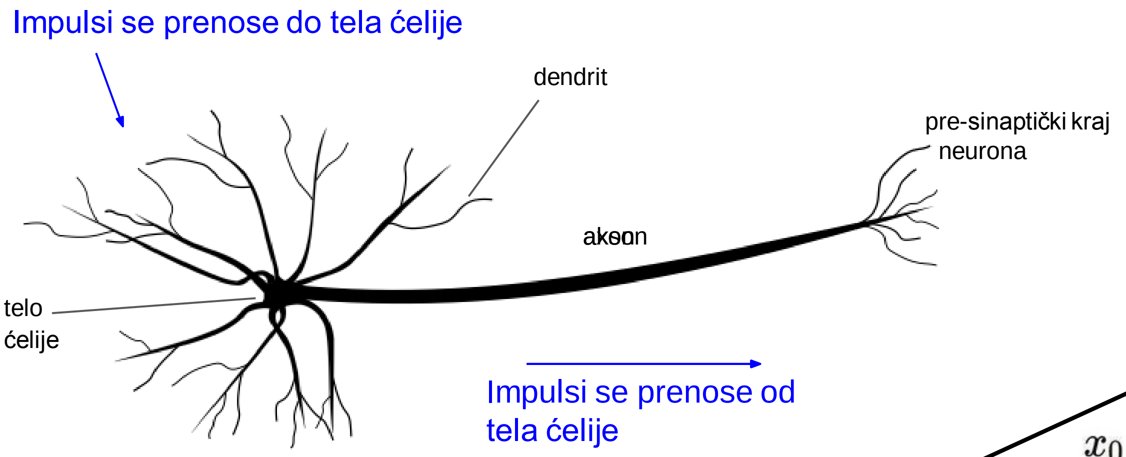


Kako bi ste to sami uradili?

```
# receive W1,W2,b1,b2 (weights/biases), X (data)
# forward pass:
h1 = #... function of X,W1,b1
scores = #... function of h1,W2,b2
loss = #... (several lines of code to evaluate Softmax loss)
# backward pass:
dscores = #...
dh1,dW2,db2 = #...
dW1,db1 = #...
```




Impulsi se prenose do tela ćelije
dendrit
pre-sinaptički kraj
neurona
akson
telo
ćelije
Impulsi se prenose od
tela ćelije



Impulsi se prenose do tela ćelije
dendrit
pre-sinaptički kraj
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akson
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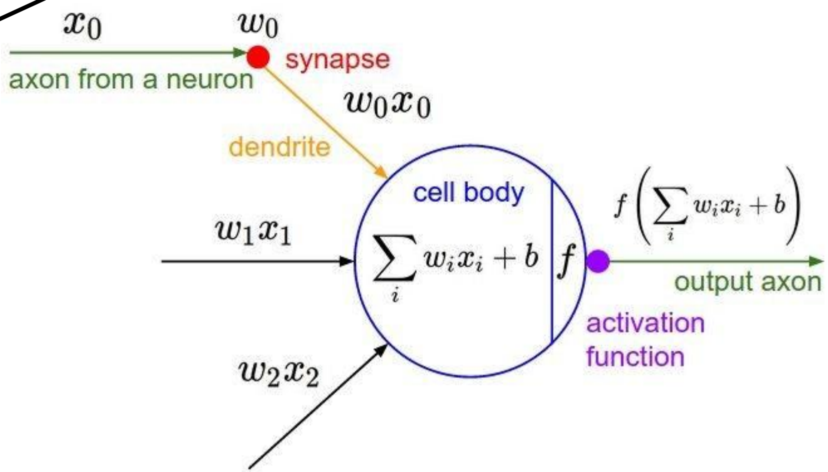
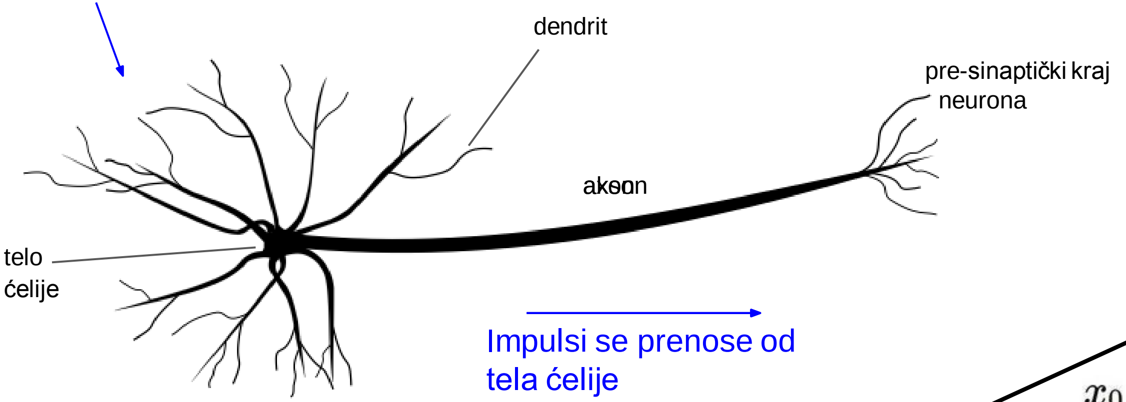
ćelije
Impulsi se prenose od
tela ćelije

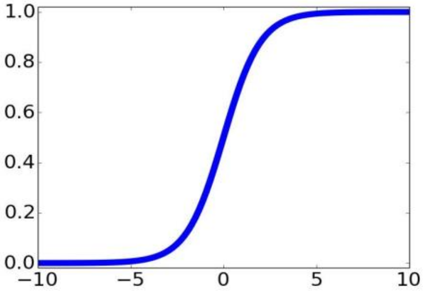


x_0
w_0
synapse
axon from a neuron
$w_0 x_0$
dendrite
cell body
$f\left(\sum_i w_i x_i + b\right)$
$w_1 x_1$
$\sum_i w_i x_i + b$
f
output axon
activation
function
$w_2 x_2$

Impulsi se prenose do tela ćelije

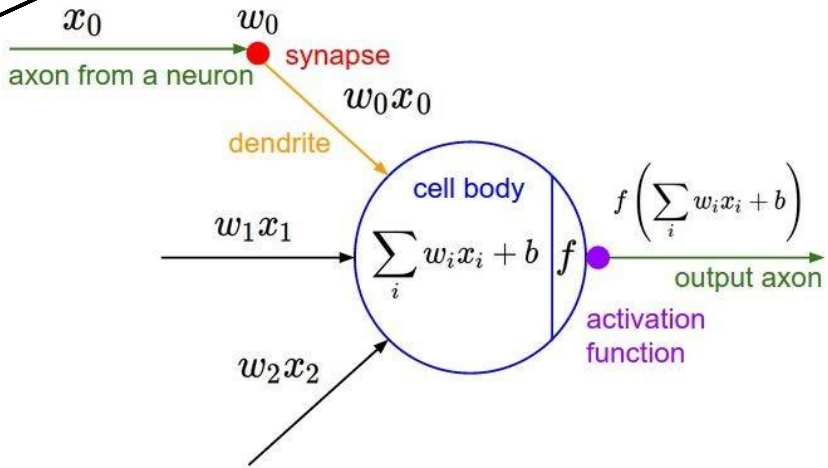


Impulsi se prenose od tela ćelije

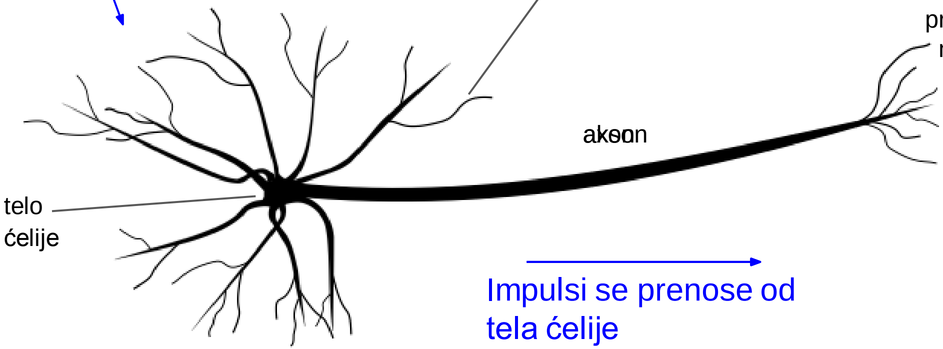


sigmoid aktivacija

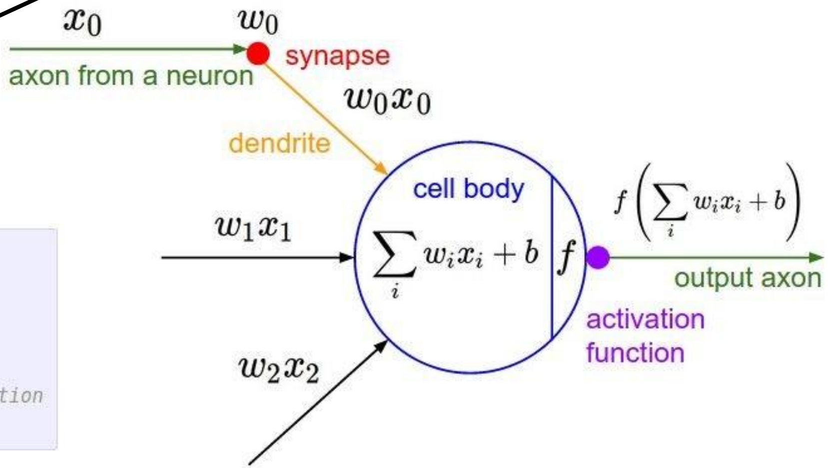
$$\frac{1}{1+e^{-x}}$$



Impulsi se prenose do tela ćelije



Impulsi se prenose od tela ćelije




```
class Neuron:
  # ...
  def neuron_tick(inputs):
    """ assume inputs and weights are 1-D numpy arrays and bias is a number """
    cell_body_sum = np.sum(inputs * self.weights) + self.bias
    firing_rate = 1.0 / (1.0 + math.exp(-cell_body_sum)) # sigmoid activation function
    return firing_rate
```

Treba biti pažljiv u pravljenju analogija sa mozgom!

Biološki neuroni:

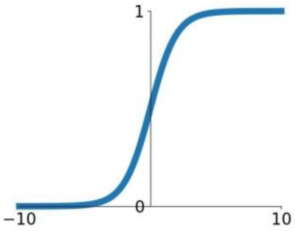
- Mnogo različitih tipova
- Dendriti vrše kompleksne ne-linearne operacije
- Sinapse nisu samo jedna težina već kompleksan ne-linearan dinamički sistem
- ....

[Dendritic Computation. London and Hausser]

Aktivacione funkcije

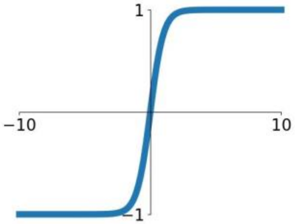
Sigmoid

$\sigma(x) = \frac{1}{1+e^{-x}}$



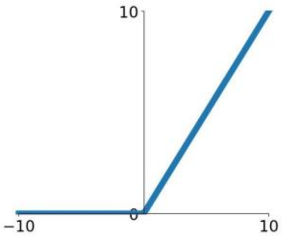
tanh

$\tanh(x)$



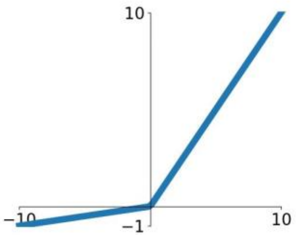
ReLU

$\max(0, x)$



Leaky ReLU

$\max(0.1x, x)$

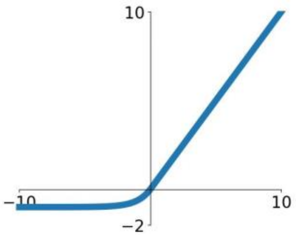


Maxout

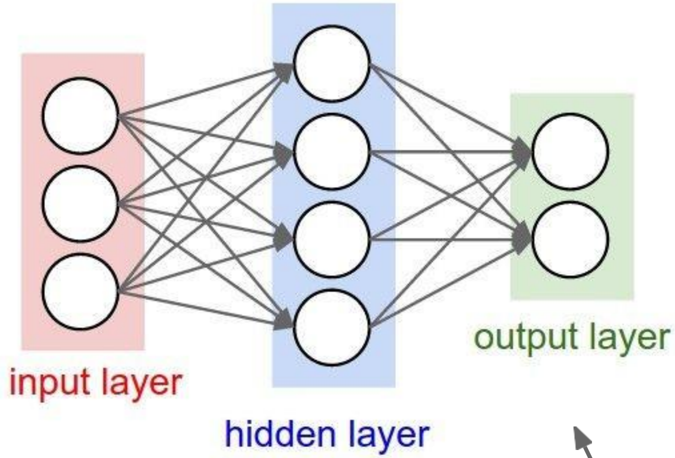
$\max(w_1^T x + b_1, w_2^T x + b_2)$

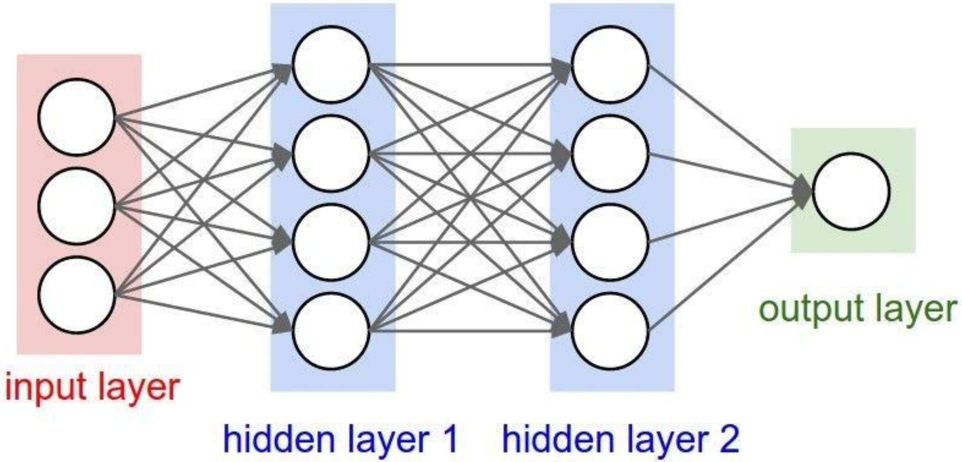
ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



Neuronske Mreže: Arhitekture





“2-slojna NN”, ili
“Neuronska mreža koja ima jedan skriveni sloj”

“Potpuno-povezani” slojevi

“3-slojna NN”, ili
“Neuronska mreža koja ima dva skrivena sloja”

Primer izračunavanja prenosa unapred (*feed-forward*) u Neuronskoj Mreži

```
class Neuron:
  # ...
  def neuron_tick(inputs):
    """ assume inputs and weights are 1-D numpy arrays and bias is a number """
    cell_body_sum = np.sum(inputs * self.weights) + self.bias
    firing_rate = 1.0 / (1.0 + math.exp(-cell_body_sum)) # sigmoid activation function
    return firing_rate
```

Možemo, vrlo efikasno, vektorskim operacijama da izračunamo aktivacije za jedan ceo sloj u mreži.

Rezime

- Uređujemo neurone u potputno-povezane slojeve
- Apstrakcija jednog sloja nam dozvoljava da koristimo vektorske operacije zarad efikasnosti (npr. množenje matrica)
- Neuronske mreže nisu baš stvarno *neuronske* u biloškom smislu