Remember to record your attendance

1. Download the University of Bristol app

2. Sign in with your university user-name and password

3. Select 'Record attendance'
(or tap on the notification)

4. If prompted, enable location services

5. Tap 'Check in' for this class, then 'Finish'

6. Your attendance confirmation will be displayed

Or go to check-in.bristol.ac.uk for web check-in

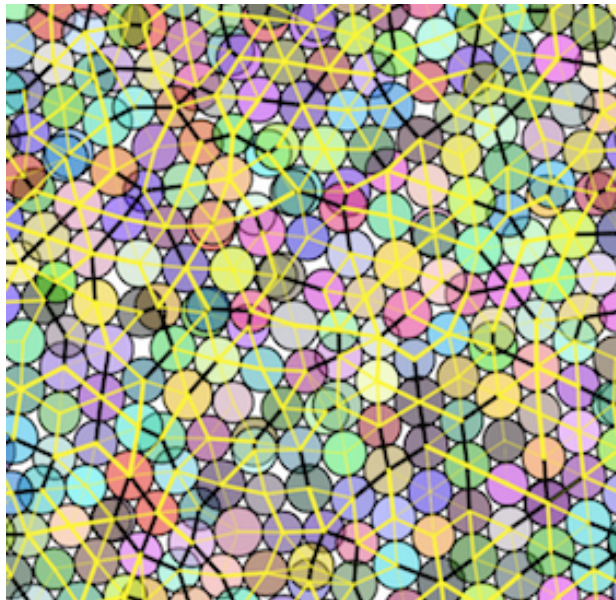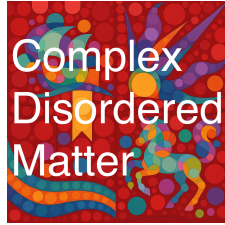For more information or help, search for 'record attendance' on www.bristol.ac.uk

Apple App Store

Google Play Store

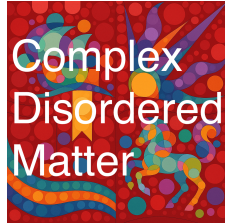# PHYS40071: Complex Disordered Matter

Nigel Wilding, Francesco Turci and Adrian Barnes

Welcome!

Course structure

TB 1 | 20 Credits | 3 Lectures per week + Problem Class

📌 **Unifying Concepts** (Weeks 1-5: Nigel Wilding)
- Statistical Mechanics: the energy-entropy competition
- Phase Transitions and Critical Phenomena
- Metastability & Dynamics
- Stochastic processes and fluctuations

🧩 **Complex Disordered Systems** (Weeks 7-11: Francesco Turci)
- Colloids and interactions
- Polymers & Surfactants
- Liquid Crystals, Gels, Glasses
- Active Matter

🔬 **Experimental Systems & Techniques** (Weeks 5 and 11: Adrian Barnes)
- Calorimetry
- Microscopy
- Diffraction

Teaching Block 1

Weeks	Commencing	Section	Lecturer	Assessment
1 - 4	22nd Sep	Unifying Concepts	Prof. Nigel Wilding	Coursework 1: Released Monday 13th October (Week 4) 12:30, due Monday 27th October, 09:30 (Week 6). Marks and feedback returned by Fri 14th November.
5	20th Oct	Unifying Concept and Experimental Techniques	Prof. Nigel Wilding and Dr. Adrian Barnes	
6	27st Oct	Consolidation Week	-	
7 - 10	3rd Nov	Complex Disordered Systems	Dr. Francesco Turci	Coursework 2: Released Thursday 13th Nov 12:30 (Week 8), due Thursday 27th November, 09:30. Marks and feedback returned Friday 12th December.
11	1st Dec	Complex Disordered Systems and Experimental Techniques	Dr. Francesco Turci and Dr. Adrian Barnes	
12	8th Dec	Revision Week	-	
	15th Dec	Assessment Period (1 week)	-	1.5 hour examination

Each teaching week, we shall meet as follows:

Tuesdays 10:00, Room: 3.34 (Physics)	Lecture
Wednesday 11:00, Room: 3.21 (Berry theatre)	Lecture
Thursdays 12:00, Room: Fry Building G.13	Problems class
Fridays 14:00, Room: 3.21 (Berry theatre)	Lecture

Assessment Breakdown

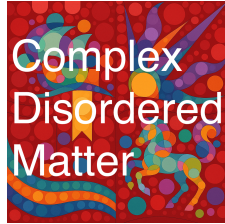🖥️ **Computational investigations** - 1 Assignment (30%)
🤔 **Problem set** (20%)
📖 **Final Exam** – Covering all lecture material (50%)

📌 **Total: 100% Assessment** – Equally balanced between coursework and final examination.

As this is a new course, you will be provided with a mock examination paper

Delivery and format (Unifying Concepts)

- Detailed e-notes (see Blackboard) can be viewed on a variety of devices.

- 'Traditional' lectures (Tues, Wed, Fri) in which I use slides to summarise and explain the lecture content. Questions welcome (within reason…)

- Try to read ahead in the notes, then come to lectures, listen to my explanations and then reread the notes.

- Rewriting the notes or slides to express your own understanding, or annotating a pdf copy can help wire the material into your own way of thinking.

- Problem class (Thurs) where you can try problem sheets and seek help. I will go over some problems with the class. **No classes week 6**.

1. Introduction to phase behaviour

- A wide variety of physical systems undergo rearrangements of their internal constituents in response to changes in thermodynamic conditions.

- Two classic examples of systems displaying such phase transitions are the ferromagnet and fluid systems.
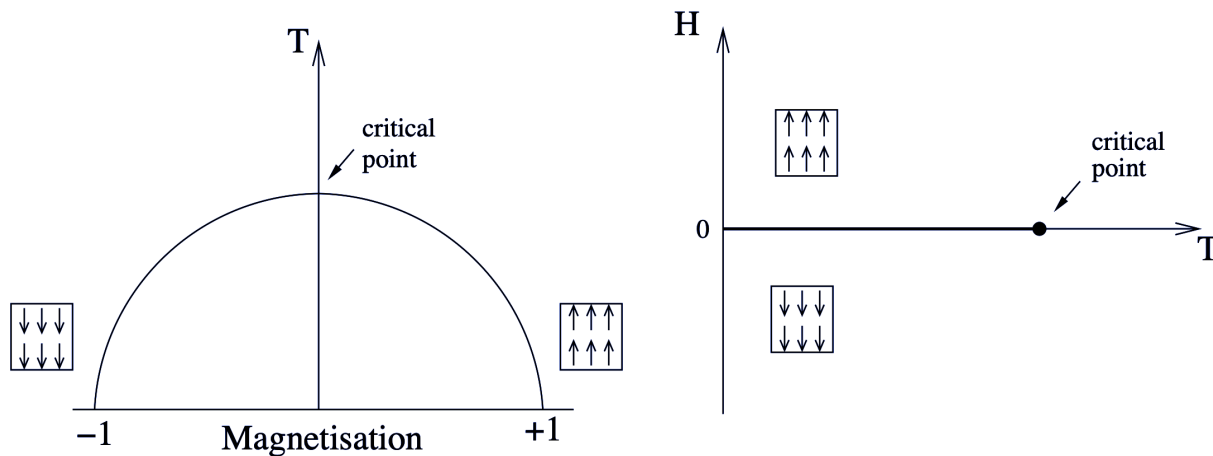
- As the temperature T of a ferromagnet is increased, its magnetic moment is observed to decrease smoothly, until at a certain **critical temperature**, it vanishes altogether

We define the magnetisation to be the **order parameter** of this phase transition.

- Similarly, we can induce a change of state from liquid to gas in a fluid simply by raising the temperature.
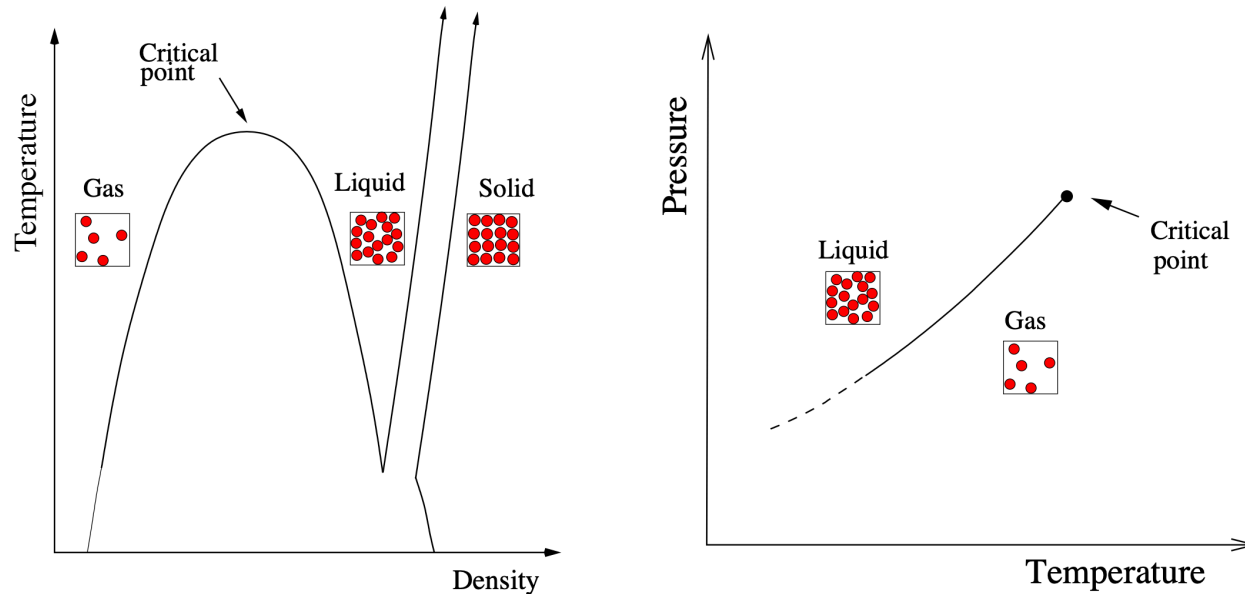
- Typically the liquid-gas transition is abrupt. However, abruptness can be reduced by applying pressure.

- At one particular p, T the jump in density at the transition vanishes. These conditions correspond to the critical point of the fluid.

- For H_2O: $T_c = 374$ K, $P_c = 218$ atm

- For CO_2: $T_c = 304$ K, $P_c = 73$ atm

- Density difference $\rho_{liquid} - \rho_{gas}$ is the order parameter for the liquid-gas phase transition.

- Close to its critical point, a system exhibits a variety of remarkable effects known as **critical phenomena**

- Examples are the divergence (infinite values) of thermal response functions such as the specific heat and the fluid compressibility or magnetic susceptibility

- Origin of the singularities in these quantities tracable to large-length-scale co-operative effects between the microscopic constituents of the system.

- Illustration: CO_2 in a pressurised container at P_c.

- As approach $T_c = 31°C$ from above, critical opalescence occurs.

- For $T < T_c$ a liquid-gas meniscus forms
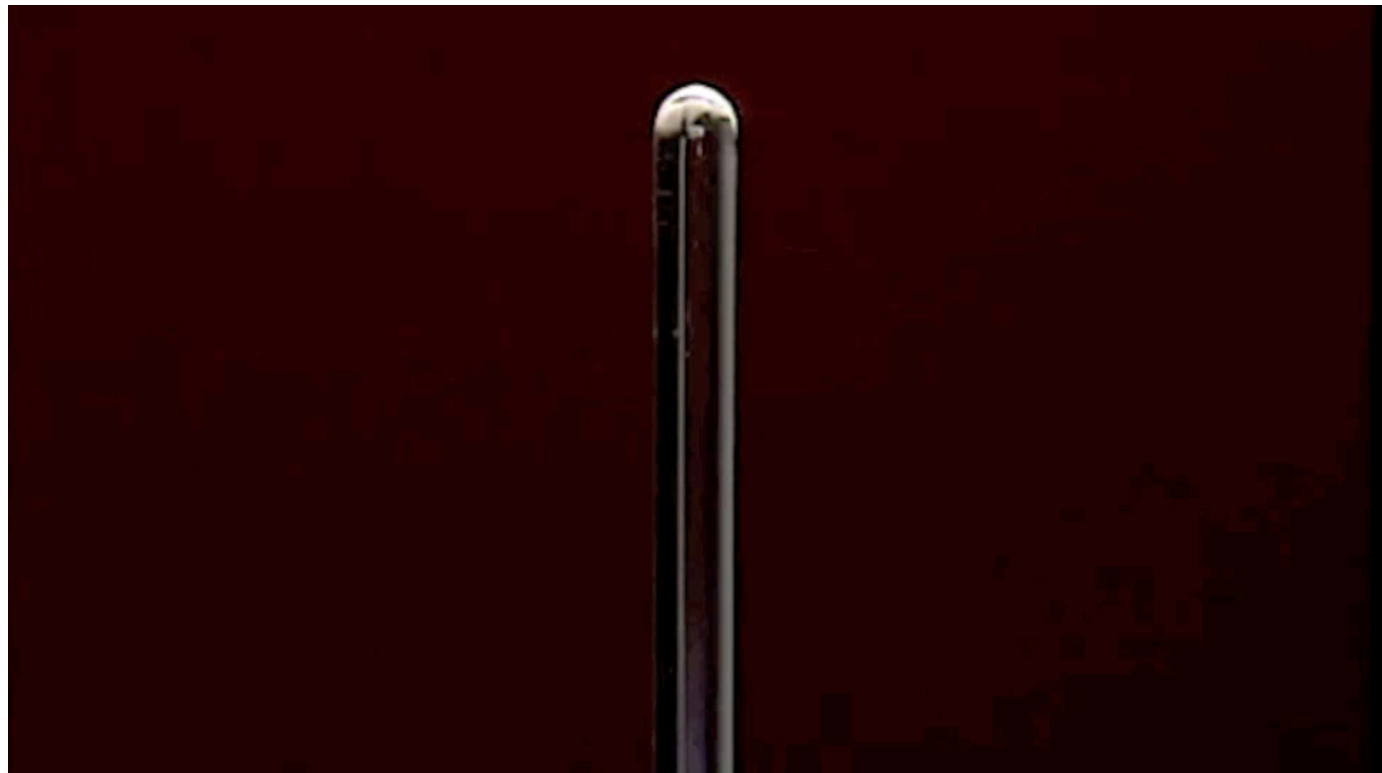
- Understanding the physics of critical points is important because often one observes **quantitatively identical** critical phenomena in a range of apparently quite disparate physical systems.

- This implies a profound underlying similarity among physical systems at criticality, regardless of many aspects of their distinctive microscopic nature.

- These ideas have found formal expression in the celebrated **universality hypothesis** (see later)

2. Background concepts

- Let us denote the order parameter as Q.

- Q provides a quantitative measure of the difference between the phases coalescing at the critical point:

- Reminder: For the fluid, $Q = \rho_{liq} - \rho_{gas}$. For the ferromagnet $Q = m$, the magnetisation.

- But why should a system exhibit a phase transition at all?
- Statistical Mechanics provides the answer!

- Probability p_a that a physical system at temperature T will have a particular microscopic arrangement ('configuration' or 'state'), labelled a, of energy E_a is

$$p_a = \frac{1}{Z}e^{-E_a/k_B T}$$

- Prefactor Z^{-1} is a T-dependent constant - the partition function (recall year 2 thermal physics). Since the system must always have some specific arrangement, the sum of the probabilities p_a must be unity, implying that

$$Z = \sum_a e^{-E_a/k_B T}$$

where the sum extends over all possible microscopic arrangements.

- Expectation (ie. average) value of an observable O is given by averaging O over all the arrangements a, weighting each contribution by p_a

$$\overline{O} \equiv \langle O \rangle = \frac{1}{Z}\sum_a O_a e^{-E_a/k_B T}$$

- The order parameter is a thermal average:

$$Q = \frac{1}{Z} \sum_a Q_a e^{-E_a/k_B T} \qquad \dagger$$

- Consider the ferromagnetic case $Q = m$. For T very small, system will be overwhelmingly likely to be in its minimum energy arrangements (ground states) having magnetisation +1, or -1.

- For T large, enhanced prob. of ground state arrangements is insufficient to offset the fact that the sum in eq. $\dagger$ contains a vastly greater number of arrangements in which Q_a has some intermediate value.

- In fact arrangements which have essentially zero magnetisation (equal populations of up and down spins) are by far the most numerous

- At high temperature, these disordered arrangements dominate the sum in eq. $\dagger$ and $Q \approx 0$.

- Thus T dependence of Q results from a competition between energy-of-arrangements weighting (or simply `energy') and the 'number of arrangements' weighting (or `entropy').

- The critical point is that T at which the system is forced to choose amongst a number of macroscopically different sets of microscopic arrangements i.e. the finite Q arrangements and the zero Q arrangements.

- Partition function provides bridge between stat. mech and thermodynamics via

$$F = - k_B T \ln Z$$

where F is the Helmholtz free energy (see year 2, and preparatory reading).

- All thermodynamic observables, eg. Order parameter Q, specific heat C_H, susceptibility χ and compressibility κ, are obtainable as appropriate derivatives of the free energy. Eg:

$$Z = \sum_a e^{-E_a/k_B T}$$

$$F = -k_B T \ln Z$$

$$\bar{E} = -\left(\frac{\partial(\beta F)}{\partial \beta}\right)_H$$

$$\bar{M} = -\left(\frac{\partial F}{\partial H}\right)_T \qquad (M = mN)$$

$$C_H = \left(\frac{\partial \bar{E}}{\partial T}\right)_H$$

$$\chi_T = \left(\frac{\partial m}{\partial H}\right)_T$$

Correlations

- Consider first **spatial** correlations.

- Two-point correlation function measures statistical relation between fluctuations at two spatial points

- For scalar field $\phi(\vec{R})$ eg. Local density or local magnetisation:

$$C(r) = \langle \phi(\vec{R})\phi(\vec{R}+\vec{r})\rangle - \langle \phi(\vec{R})\rangle^2 \qquad r = |\vec{r}| \text{ is spatial separation}$$

and the average is over configurations

- In homogeneous/isotropic systems, $C(r)$ depends only on r.

Behaviour of spatial correlations

- Correlations can be short ranged or long ranged:

  - Short-range: $C(r) \sim e^{-r/\xi}$, fast decay

  - Long-range (criticality): power-law decay
    $C(r) \sim r^{-x}$

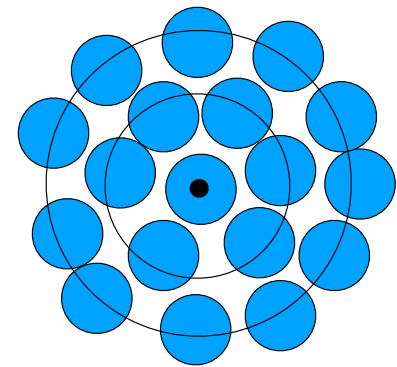- **Correlation length** ξ sets spatial scale.
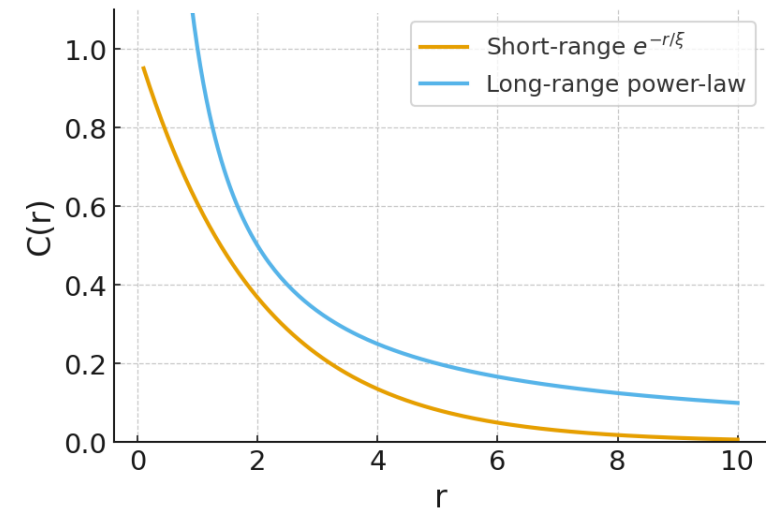
Pair Correlation Function
(a.k.a radial distribution function)

- $g(r)$ = probability of finding particle at distance r relative to ideal gas.

- It is related to density correlations via: $g(r) = \dfrac{1}{\rho^2}\langle \rho(0)\rho(r) \rangle$

$g(r)$

Radial distribution function for an atomic liquid

1.0

0

r
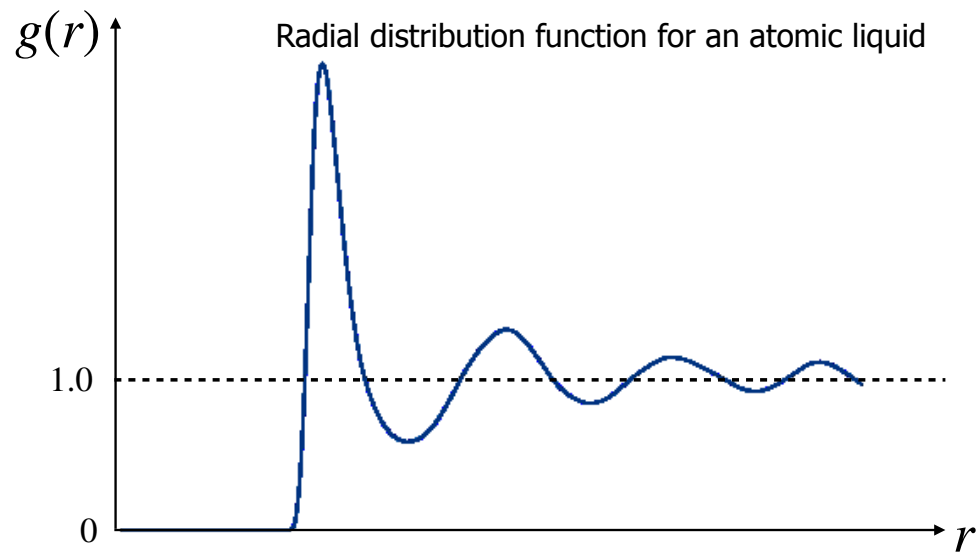
- At large r correlation die out, ie.

$$g(r) \to 1 \quad \text{as } r \to \infty$$

- Peaks in $g(r)$ indicate ordering (e.g. shells in liquids).

Structure Factor $S(k)$

- Fourier transform of correlations: $S(k) = \int d^3r \, e^{i\vec{k}\cdot\vec{r}} C(r)$

  k is the scattering wave vector (often also written as q)

- Measured in scattering experiments (see lecture by A. Barnes). Connects real-space correlations with momentum-space

- Peaks in $S(k)$ = characteristic length scales.

- Near criticality: divergence at small k (long-range fluctuations)

- For short ranged correlations, decay of $S(k)$ has a Lorentzian form.

- For long ranged correlations, decay has a power law form (see notes)

Structure factor for an atomic liquid

	ϕ
——	0.05
——	0.1
——	0.2
- - -	0.3
——	0.4
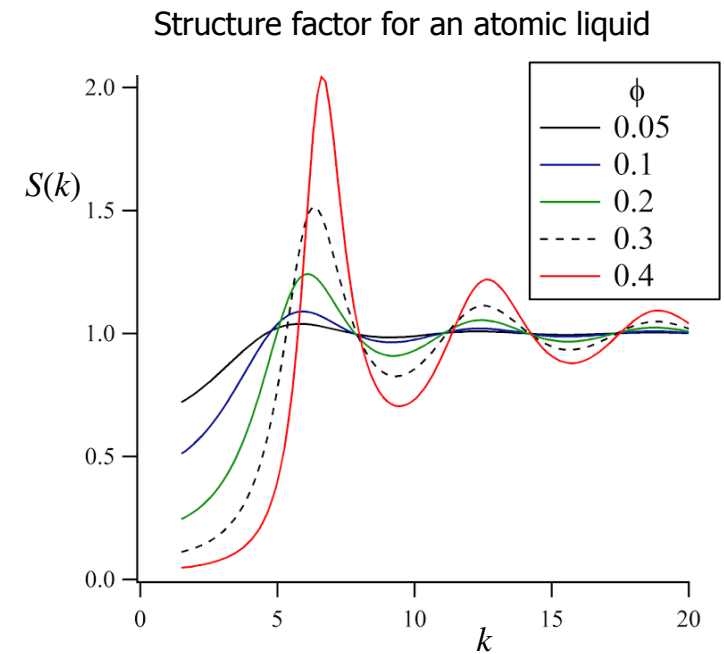
Temporal correlations

- Consider a thermodynamic variable x with zero mean that fluctuates over time.

- Temporal correlations are characterized by the two-time correlation function:

$$\langle x(\tau)x(\tau + t)\rangle$$

- In equilibrium: $\langle x(\tau)x(\tau + t)\rangle = M_{xx}(t)$

Typically (away from criticality):

$$M_{xx}(t) \sim \exp(-t/t_c)$$

ie. memory of fluctuations fades
exponentially with correlation time t_c